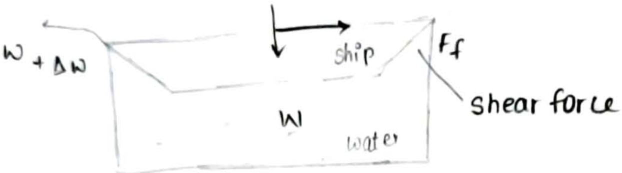
01/10/2019

Friction...

Friction

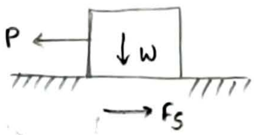
Fluid friction — bearings. grease. ship in water

Dry friction — friction between two solid bodies



$P > W$,

$P = F_S$ for equilibrium



$F_S \propto$ Normal

$F_S = \mu N$ — coefficient of friction

Dry friction

Static friction — friction b/w 2 bodies are in rest

Kinetic friction — friction b/w 2 bodies are not moving but in

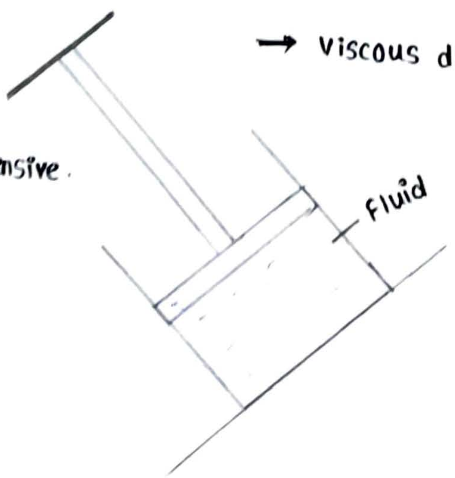
Taipe - 101 dampers is useful for buildings. This helps during times of earth quakes.

* **Friction Damper**
  movement of surface

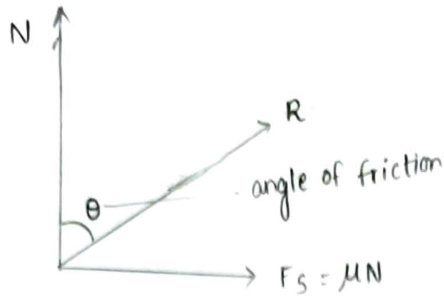
* **Material Damper**
  movement of particles

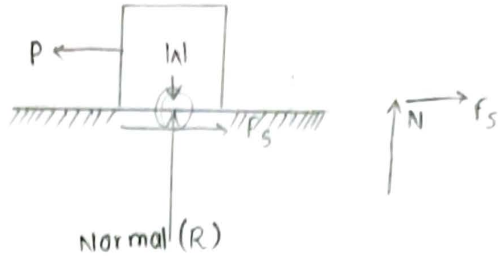
→ Viscous damper

It is very expensive.



* Angle of friction:





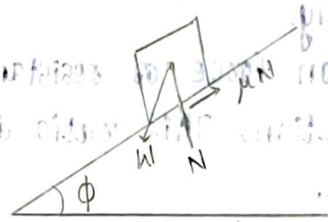
It is the angle between the normal force and the resultant of normal and frictional force.

$$\tan\theta = \frac{F_s}{N} = \frac{\mu N}{N} = \mu$$

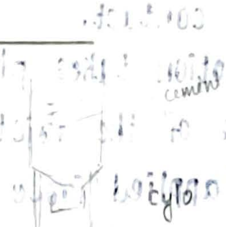
$$\mu = \tan\theta$$

* Angle of repose:

If a body kept in an inclined plane angle of repose is minimum inclination at which the body start to slip.

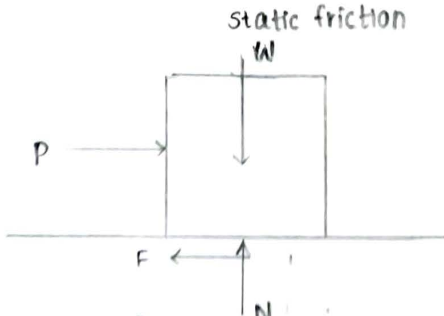


Application : forming of concrete.



23/10/2019

Static friction

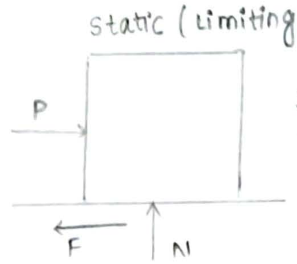


$F_{max} = \mu N$

$P < F_{max}$

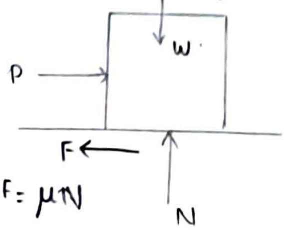
Static (Limiting friction)

Just about to move



$P = F_{max} = \mu_s N$

(moving) Kinetic friction



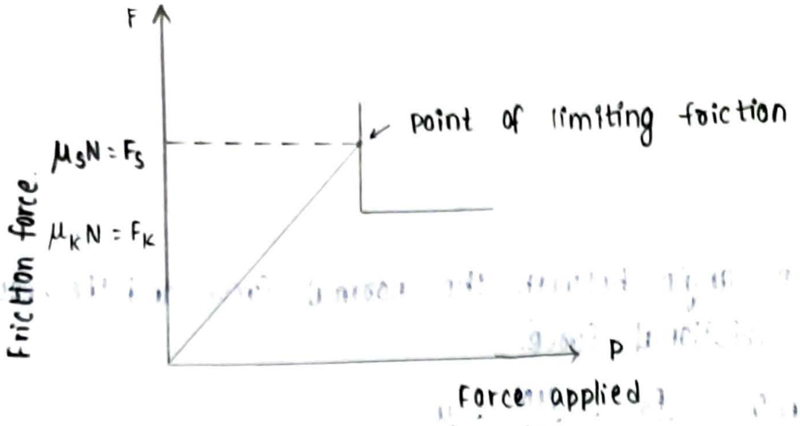
$F = \mu N$

$P > F_{max}$

μ_s → Static coefficient of friction

μ_k → Kinetic coefficient of friction

$$\mu_s > \mu_k$$



* **Laws of friction :**

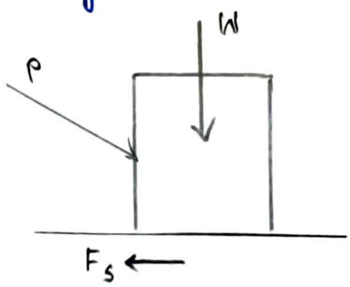
(I) Friction always opposes the motion and comes into existence/play when the body is urged/impend to move.

(II) The magnitude of the friction force is just sufficient to prevent the body from moving.

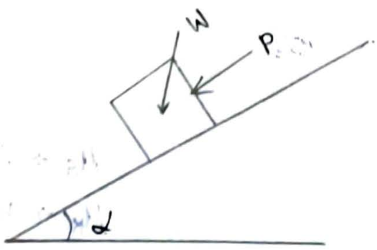
(III) The limiting friction force or resistance bears a constant ratio with the normal reaction. This ratio depends upon the nature of surface in contact.

(IV) When a motion takes place as the one body slides over other, the magnitude of the friction force will be slightly less than the offered force/applied force at that condition of limiting equilibrium

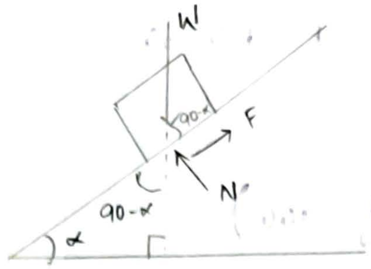
Case-1: Body on horizontal ground and force required to push or pull on plane ground.



Case-2: Body on inclined ~~plan~~ ground and force required to motion of the body.



stable
motion upward
motion backward



$\Sigma F_x = 0 \Rightarrow \quad F_S - W\sin\alpha = 0$

$F_S = W\sin\alpha = \mu_S N$

$\Sigma F_y = 0 \Rightarrow \quad -W\cos\alpha + N = 0$

$N = W\cos\alpha$

$\mu_S N = W\sin\alpha$

$\mu_S = \dfrac{W\sin\alpha}{W\cos\alpha} = \tan\alpha$

$\tan\alpha = \mu_S$

ϕ = Angle of friction

$\tan\phi = \dfrac{F}{N} = \dfrac{\mu N}{N} = \mu_S$

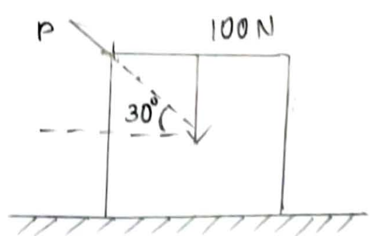
A ~~bod~~ block of weight 100 N is kept on a horizontal surface a force P is applied to the block in order to move the block. Find the minimum force required P to move the block either,

(i) Pushing the block

(ii) Pulling the block

Consider static coefficient of friction $\mu_S = 0.3$

(i) Pushing the block



$F_{max} = \mu_S N$

$= 0.3 \times N = 0.3N$

$\Sigma F_y = 0 \Rightarrow -P\sin 30° - 100 + N = 0$

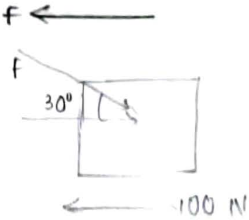
$-P\cos 30° - F_{max} = 0$

$P\cos 30 = F_{max}$

$N = P\sin 30 + 100$

$P = \dfrac{30}{\cos 30°}$

$N = \dfrac{P}{2} + 100$



$= 34.64\ N$

$\Sigma F_x = 0 \Rightarrow P\cos 30^\circ - F = 0$

$F = P\cos 30^\circ$

$F_{max} = \mu_s N$

$F = 0.3 \times \left(\frac{P}{2} + 100\right)$

$P\cos 30^\circ = 0.3 \times \left(\frac{P}{2} + 100\right)$

$0.86P = 0.15P + 30$

$0.71P = 30$

$P = \frac{30}{0.71}$

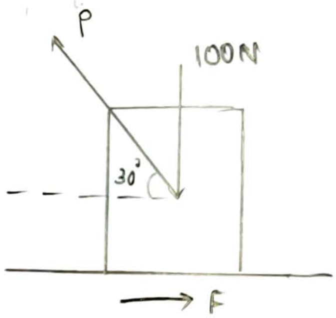
$P = 41.89\ N$

(II) Pulling the block

$F = 0.3 N$

$\Sigma F_x = 0 \Rightarrow -P\cos 30 + F = 0$ —— ①

$F = P\cos 30^\circ$



$\Sigma F_y = 0 \Rightarrow P\sin 30^\circ - 100 + N = 0$ —— ②

from ① $\Rightarrow 0.86P = 0.3N$

From ② $\Rightarrow \frac{P}{2} + N = 100$

$\frac{P}{2} + \frac{0.86P}{0.3} = 100$

$P\left(\frac{1}{2} + \frac{0.86}{0.3}\right) = 100$

$P(0.5 + 2.87) = 100$

$P = \frac{100}{3.37}$

$P = 29.52\ N$

15/10/2019

wedge problems

wedge friction

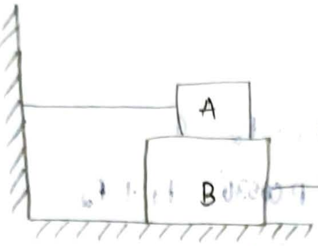
Sliding / lipping

1) A block A weigh 1000 N and block B weight 2000 N. When block A is tied with rope to a wall horizontally, If the coefficient of friction between blocks is $\mu_s = 2.5$. and coefficient of friction between block B and ground is $\mu_s = \frac{1}{3}$.

What should be the value of P to move the block B, if

(i) P is horizontal

(ii) P is inclined 30° upward to horizontal



(i) P is horizontal

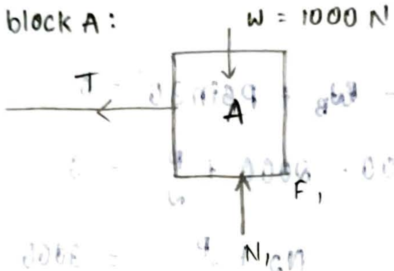
A = 1000 N

B = 2000 N

$\mu_{s_1} = 0.25$

$\mu_{s_2} = \frac{1}{3}$

FBD of block A:



$F_1 = \mu_s N$

$\Sigma F_x = 0 \Rightarrow F_1 - T = 0$

$F_1 = T$

$\Sigma F_y = 0 \Rightarrow N_1 - 1000 = 0$

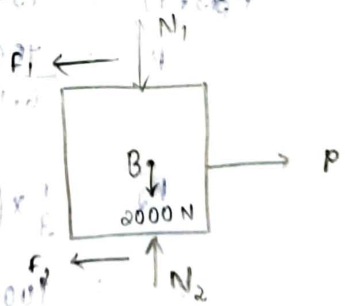
$N_1 = 1000$ N

$F_1 = 0.25 \times 1000$

$F_1 = 250$ N

$T_1 = 250$ N

FBD of block B:



$\Sigma F_x = 0$

$P - F_1 - F_2 = 0$

$P = F_1 + F_2$

$\Sigma F_y = 0$

$N_2 - N_1 - 2000 = 0$

$N_2 = N_1 + 2000$

$= 3000$ N

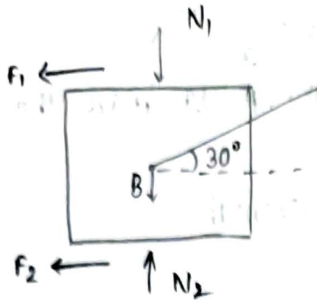
$F_2 = \mu_{s_2} N_2$

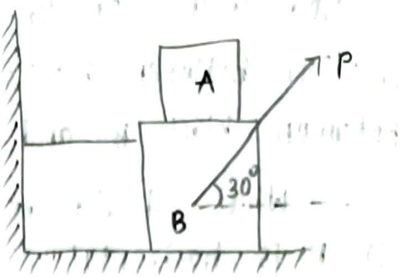
$= \frac{1}{3} \times 3000 = 1000$ N

$\therefore P = F_1 + F_2$

$= 250 + 1000 = 1250$ N

(ii) P is inclined 30° upward to horizontal





$\xrightarrow{+} \Sigma F_x = 0 \Rightarrow P\cos 30° - F_1 - F_2 = 0$

$$P\cos 30° = F_1 + F_2$$

$$\frac{\sqrt{3}P}{2} = 250 + F_2$$

$+\uparrow \Sigma F_y = 0 \Rightarrow \quad N_2 - N_1 - W_B + P\sin 30° = 0$

$$N_2 - 1000 - 2000 + \frac{P}{2} = 0$$

$$N_2 + \frac{P}{2} = 3000$$

$$F_2 = \mu_{s_2} N_2$$

$$= \frac{1}{3} \times \left(3000 - \frac{P}{2}\right)$$

$$\frac{\sqrt{3}P}{2} = 250 + 1000 - \frac{P}{6}$$

$$\frac{\sqrt{3}P}{2} + \frac{P}{6} = 1250$$

$$3\sqrt{3}P + P = 1250 \times 6$$

$$P(3\sqrt{3} + 1) = 7500$$

$$P = \frac{7500}{6.19} = 1211.6$$

$$F_2 = \frac{1}{3} \times \left(3000 - \frac{1211.6}{2}\right)$$

$$= 1000 - 201.93$$

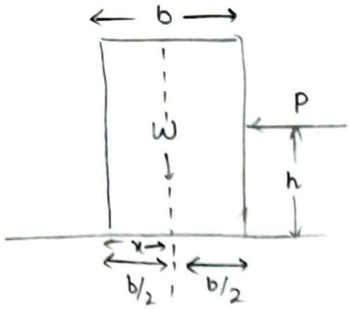
$$= 798.07 \text{ N}$$

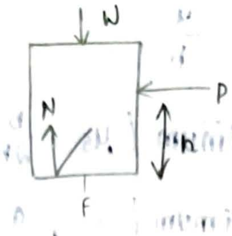
$$N_2 = 3000 - \frac{1211.6}{2}$$

$$= 3000 - 605.8$$

$$= 2394.2 \text{ N}$$

* Sliding or tipping





$$R \cos\phi = W$$
$$R \sin\phi = P$$
$$\tan\phi = \frac{P}{W} \le \mu_s$$ ← applied force & weight of body

Geometry. $\tan\phi = \frac{x}{h} \le \frac{b}{2}/h.$

$\tan\phi = \text{minimum of } (\mu_s, \frac{b}{2h})$

*** If sliding to happen the coefficient of friction is,

$$\mu_s \le \frac{b}{2h}.$$

$$\boxed{h_{max} \le \frac{b}{2\mu_s}}$$

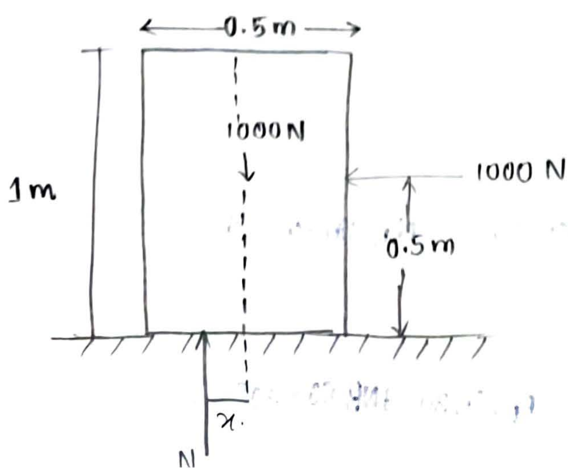
⇒ If tipping to happen the coefficient of friction is,

$$\mu_s > \frac{b}{2h}$$

$$\boxed{h > \frac{b}{2\mu_s}}$$

1) In the figure block of weight 1000 N having width of 0.5 m and height of 1m of which force 1000 N is applied. and above is 0.5 m.

Assuming that μ_s is not known and the block is just start to slipping / slide. Find out the location of vertical reaction on the block.



μ_s - not known

block just start to slipping

$\tan\phi = \frac{P}{W} = 1$

$\tan\phi = \frac{x}{h}$

$\tan\phi = \text{Minimum}\left(\mu_s , \frac{b}{2h}\right)$

$= \text{Minimum}\left(1 , \frac{0.5}{2.5}\right)$

$= \text{Minimum}\,(1, 0.50)$

$= 0.50$

$x = 0.5 \times 0.5$

$= 0.25$

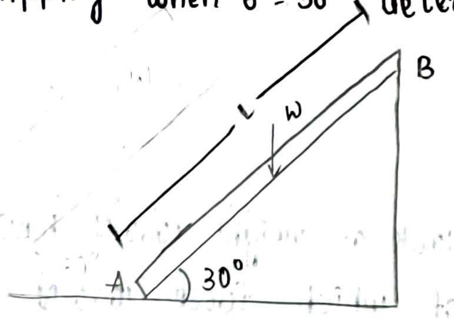
$\frac{x}{h} \leq \frac{b/2}{h}$

$x \leq b/2$

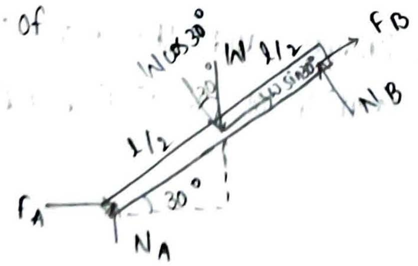
$x \leq \frac{0.5}{2} \leq 0.25$

2) Ladder problems.

A uniform rod having a weight W and length L is supported at its end taken as surface A and B. If the rod is on the verge of slipping when $\theta = 30°$, determine μ_s at A and B.



FBD of



$\Sigma F_x = 0$
$\Sigma F_y = 0$ } Equilibrium eqn's
$\Sigma M_A = 0$

$F_A = \mu_{sA} N_A$
$F_B = \mu_{sB} N_B$ } Friction eqn's

$\xrightarrow{+} \Sigma F_x = 0$

$F_A + F_B \cos 30° - N_B \sin 30° = 0$ — (1)

$+\uparrow \Sigma F_y = 0$

$N_A - W + F_B \sin 30° + N_B \cos 30° = 0$ — (2)

$$F_A = \mu_{SA} \times N_A = \mu_S N_A$$
$$F_B = \mu_{SB} \times N_B = \mu_S N_B.$$

$$\Sigma M_A = 0$$
$$N_B \times l - W \cos 30^\circ \times \frac{l}{2} = 0$$
$$N_B = W \times \frac{\sqrt{3}}{4}$$
$$N_B = 0.433\, W$$

$$F_B = \mu_S \times 0.433\, W$$

(1) $\Rightarrow$ $$\mu_S N_A + \mu_S \times 0.433\, W \times \frac{\sqrt{3}}{2} - \frac{0.433}{2} W = 0. \quad \text{— (3)}$$

(2) $\Rightarrow$ $$N_A - W + 0.433\, W \mu_S \times \frac{1}{2} + 0.433\, W \frac{\sqrt{3}}{2} = 0. \quad \text{— (4)}$$

Solving, (3), (4)

$$2 \mu_S N_A + \mu_S\, 0.433\, W \times \sqrt{3} - 0.433\, W = 0$$

$$N_A = W - 0.433\, W \mu_S \cdot \frac{1}{2} - 0.433\, W \frac{\sqrt{3}}{2}.$$
$$N_A = W\left(1 - \frac{0.433}{2} \mu_S - 0.433 \frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow \mu_S \left[W\left(1 - \frac{0.433}{2}\mu_S - 0.433 \frac{\sqrt{3}}{2}\right)\right] + \mu_S\, 0.433 W \frac{\sqrt{3}}{2} - \frac{0.433}{2} W = 0.$$

$$\mu_S \left[\left(1 - \frac{0.433}{2}\mu_S - 0.433 \cdot \frac{\sqrt{3}}{2}\right)\right] + \mu_S\, 0.433 \frac{\sqrt{3}}{2} - \frac{0.433}{2} = 0.$$

$$\mu_S - \frac{0.433}{2}\mu_S^2 - \cancel{\frac{0.433\sqrt{3}}{2}\mu_S} + \cancel{\mu_S \frac{0.433\sqrt{3}}{2}} - \frac{0.433}{2} = 0$$

$$\mu_S - \frac{0.433}{2}\mu_S^2 - \frac{0.433}{2} = 0$$

$$\frac{0.433}{2}(\mu_S^2 + 1) = \mu_S.$$

$$\mu_S^2 + 1 = \mu_S\, 4.619$$

$$\mu_S^2 - \mu_S\, 4.619 + 1 = 0.$$

$$\mu_S = \frac{4.619 \pm \sqrt{(4.619)^2 - 4}}{2}.$$
$$= \frac{4.619 \pm \sqrt{21.335161 - 4}}{2}$$
$$= \frac{4.619 \pm \sqrt{17.335161}}{2.} = \frac{4.619 \pm 4.163}{2}$$

$$\mu_s = \frac{4.619 - 4.163}{2}$$

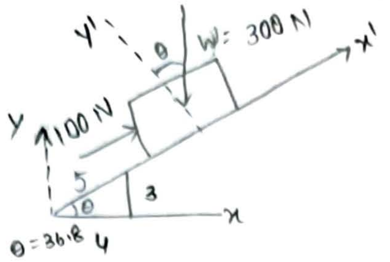
$$= 0.456$$

$$= 0.228$$

$$F_B = \mu_s \times N_B$$

$$= 0.228 \times 0.433$$

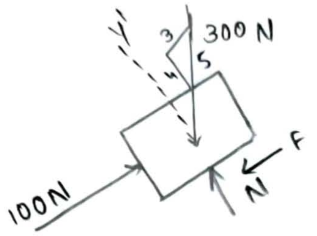
16/10/19 Q) A 100 N force acts as shown on a 300 N block placed on an inclined ~~problem~~ plane. The μ_s between $= 0.25$ $\mu_k = 0.2$ Determine whether the block is in equilibrium and find the value of frictional force.



$\mu_s = 0.25$

$\mu_k = 0.2$

FBD :



$\overset{+}{\rightarrow} \Sigma F_{x'} = 0 \Rightarrow \quad 100 - F - \overset{W}{300}\ \cancel{\cos}\sin\theta = 0$

$100 - F - \overset{60}{\cancel{300}} \times \frac{3}{\cancel{5}} = 0$

$F = -80\ N$

$+\uparrow \Sigma F_{y'} = 0 \Rightarrow \quad N - \overset{W}{300}\ \overset{\cos\theta}{\frac{4}{5}} = 0$

$N - 240 = 0$

$N = 240\ N$

$F_{max} = \mu_s N$

$= 0.25 \times 240$

$= 60\ N$

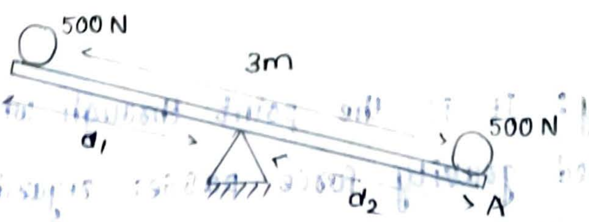
$F_{max} < F$ Block is at rest

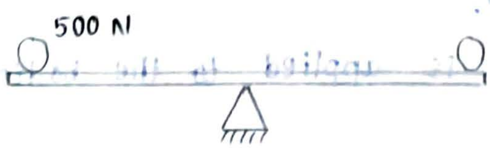
$F_{max} > F$ Block is moving / about to move.

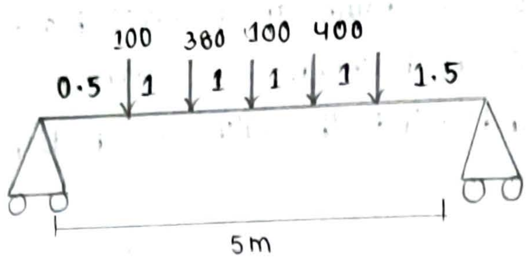
$F_k = \mu_k N$

$= 0.2 \times 240 = 48\ N$

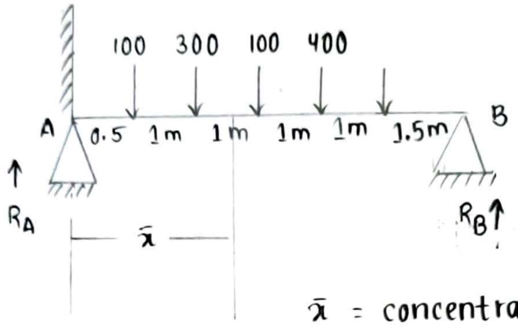
17/10/2019







→ Centre of gravity



$\bar{x}$ = concentration of distances.

Assume,

$$R_B \times 5 - 100 \times 0.5 - 300 \times 1.5 - 100 \times 2.5 - 400 \times 3.5 = 0$$

$$5R_B = 50 + 450 + 250 + 1400$$

$$5R_B = 2150$$

$$R_B = 430\,N$$

$$\Sigma F_y = (100 + 300 + 100 + 400) = 900\,N$$

$$\Sigma F_x = 0$$

$$-100 \times 0.5 - 300 \times 1.5 - 100 \times 2.5 - 400 \times 3.5 + 900 \times \bar{x} = 0$$

$$900\bar{x} = 2150$$

$$\bar{x} = 2.389$$

$$\Sigma F_i \bar{x} = F_1 x_1 + F_2 x_2 + F_3 x_3 + F_4 x_4$$

$$= \Sigma F_i x_i$$

$$\bar{x} = \frac{\Sigma F_i x_i}{\Sigma F_i}$$

It is a point where all the forces passes through this of irrespect geometry.

---

* **Centre of gravity:** It is the point through which the resultant of the distributed gravity force passes regardless of the orientation of the body in space.

⇒ Centre of gravity is applied to the bodies with mass and weight.