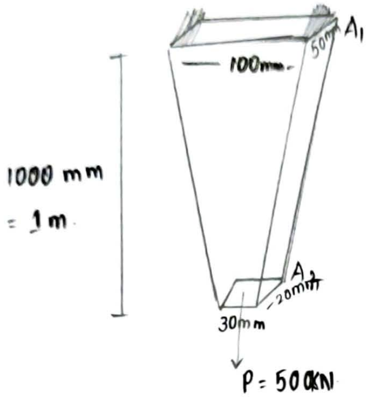
19/11/2019

1) Varying cross-section

2) Composite section.

21/11/2019
1)



On a varying cross-section of a bar point average stress in the bar,

Find the,

(i) average stress in the bar

(ii) elongation.

Assume, steel bar,

So. E = 200 Gpa

$= 200 \times 10^3$ Mpa

$= 200 \times 10^3$ N/mm²

(i) Average stress $= \dfrac{P}{A}$

$$= \frac{500 \times 10^3 \text{ N}}{\frac{(A_1 + A_2)}{2}}$$

$$= \frac{500 \times 10^3}{\frac{(100 \times 50 + 30 \times 20)}{2}}$$

$$= \frac{500 \times 10^3}{2800}$$

$= 178.5$ N/mm²

$= 178.5$ MPa

(ii) Total elongation, $\delta l = \dfrac{Pl}{AE} = \dfrac{\sigma l}{E}$

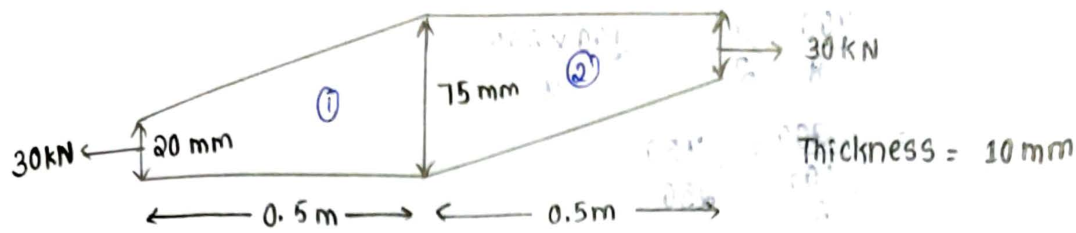
$$= \frac{178.5 \times 1000}{200 \times 10^3}$$

$$= \frac{178.5}{200}$$

$= 0.8925$

$\simeq 0.89$ mm

2)



Determine the elongation of a steel member when it is subjected to an axial force of 30 kN. The member is 10 mm thick. E = 200 Gpa

Elongation, $\delta l_1 = \frac{P l_1}{A_1 E} + \frac{P l}{A_2 E}$

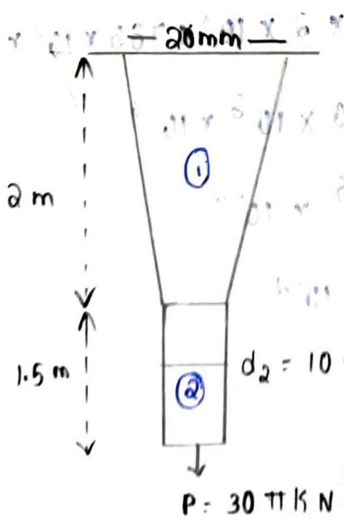
$$= \left[\frac{30 \times 10^3 \times 500}{\frac{20 \times 10 + 75 \times 10}{2}} + \frac{30 \times 10^3 \times 500}{\frac{20 \times 10 + 75 \times 10}{2}} \right] \frac{1}{200 \times 10^3}$$

$$= \left[\frac{15 \times 10^6}{\frac{200 + 750}{2}} \times 2 \right] \times \frac{1}{200 \times 10^3}$$

$$= \frac{15 \times 10^6}{475} \times \frac{1}{100 \times 10^3}$$

$$= \frac{15}{475} \times 10 = \frac{150}{475} = 0.31 \text{ mm}$$

3)



E = 200 Gpa

Elongation, $\delta l = \frac{P l_1}{A_1 E} + \frac{P l_2}{A_2 E}$

$$= \frac{30 \cancel{\pi} \times 2000 \times 10^3}{\frac{\cancel{\pi}}{4} (20)^2 \times \cancel{200} \times 10^3} + \frac{30 \cancel{\pi} \times 1500 \times 10^3}{\frac{\cancel{\pi}}{4} \times (10)^2 \times \cancel{200} \times 10^3}$$

$$\delta l = \frac{300}{400} + \frac{30 \times 15}{\frac{900}{4}}$$

13.8 mm

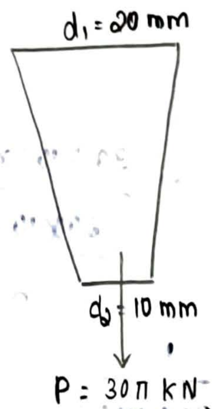
12 mm

$$= \frac{300}{100} + \frac{450}{50}$$

$$= 3 + 9$$

$$= 12 \text{ mm}$$

In the above question, if the self weight of rod is included what will be the total elongation



Density of steel,

$$\rho_{steel} = 7850 \text{ kg/m}^3$$

$$= 7850 \times 10 \text{ N/m}^3$$

$$= \frac{78500}{10^9} = 78500 \times 10^{-9} \text{ N/mm}^3$$

Self weight = volume × density (ρ)

$$= \text{Average area} \times l \times \rho$$

$$= \frac{\pi}{4}\left(\frac{20^2 + 10^2}{2}\right) \times 2000 \times 78500 \times 10^{-9}$$

$$= \frac{\pi}{4}\left[250\right] \times 2 \times 10^3 \times 785 \times 10^2 \times 10^{-9}$$

$$= \frac{\pi}{4} \times 39250 \times 10^6 \times 10^{-9}$$

$$= \frac{3.14 \times 3925}{4} \times 10^{-2}$$

$$= 3081.125 \times 10^{-2}$$

$$= 30.81 \text{ N}$$

$\therefore P_1 = P + \text{Self weight}$

$$= 30\pi \text{ kN} + 30.81 \text{ N}$$

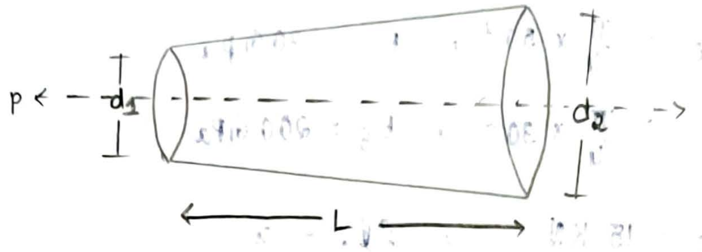
$$= 30 \times 3.14 \times 10^3 + 30.81 \text{ N}$$

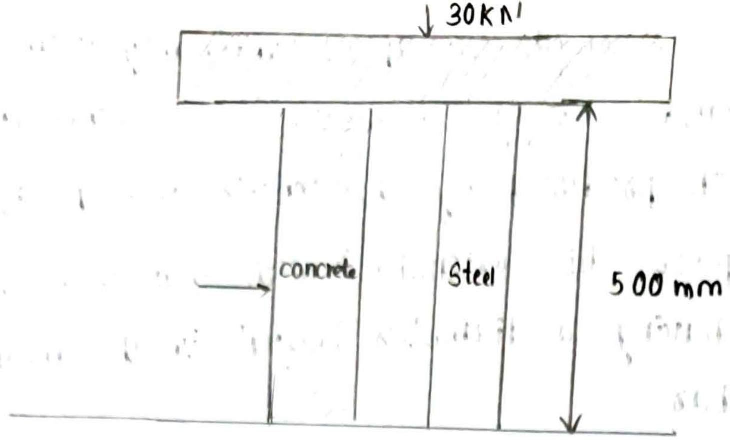
$$= 94230.8 \text{ N}$$

$$= 94.2308 \text{ kN}$$

$$\simeq 94.2 \text{ kN}$$

A round bar of length L tapers uniformly from small diameter d_1 at one end to bigger diameter d_2 at other end. Show that the extension produced by the tensile force 'p' is $\delta = \frac{4PL}{\pi d_1 d_2}$ if $d_2 = 2d_1$, compare this extension with that of a uniform cylindrical bar having a diameter equal to the mean diameter of the taper bar.





$A_{concrete} = \frac{\pi}{4} \times 50^2$, $E_c = 30\,GPa$

$A_{steel} = \frac{\pi}{4} \times 30^2$, $E_S = 200\,GPa$

$P_{concrete} = 15\ KN$, $\delta l_c = x$

$P_{steel} = 15\ KN$, $\delta l_s = y$

$x = y$
compalibility condition

$\delta l_c = \frac{P_c l_c}{A_c E_c}$ $\qquad$ $\delta l_s = \frac{P_s l_s}{A_s E_s}$

$\frac{\cancel{P_c}\cancel{l_c}}{A_c E_c} = \frac{\cancel{P_s}\cancel{l_s}}{A_s E_s}$ $\qquad$ $P_c = P_s$, $l_c = l_s$

$\sigma_{concrete} = \frac{P_1}{A_1}$

$= \frac{15 \times 10^3}{\frac{\pi}{4} \times 50^2} = \underline{15000}$

25/11/2019

* Method and Analyse a composite bar

1) Displacement

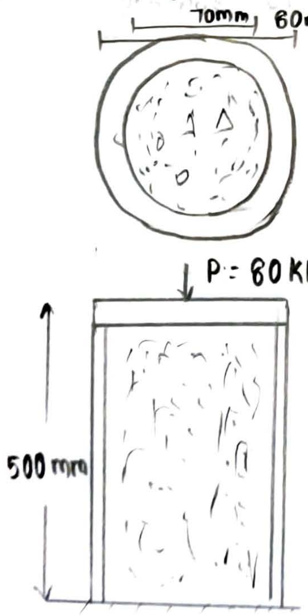
2) Force method

1) A steel pipe filled with a concrete having external diameter has 80 mm and internal diameter has 70 mm. It is subjected to a axial compressive force of 80 KN.

Assume, $E_{steel} = 200$ GPa

$E_{conc.} = 24$ GPa

Find the stresses in each of the material.



$$\delta_c = \delta_s$$

$$\frac{P_c l_c}{A_c E_c} = \frac{P_s l_s}{A_s E_s}$$

$$\frac{P_c \times \cancel{500}}{\cancel{\frac{\pi}{4}} \times (70)^2 \times 24} = \frac{P_s \times \cancel{500}}{\cancel{\frac{\pi}{4}} \times (80^2 - 70^2) \times 200}$$

$$\frac{P_s}{P_c} = \frac{(80^2 - 70^2) \times 200}{70^2 \times 24}$$

$$\frac{P_s}{P_c} = 2.55 \text{ --- (1)}$$

$$P_s + P_c = 80 \text{ --- (2)}$$

$$2.55 P_c + P_c = 80$$

$$P_c = \frac{80}{2.55}$$

$$P_c = 22.53 \text{ KN}$$

$P_s = 60 - P_c$

$= 60 - 22.53$

$= 57.47$ KN

$$\sigma_c = \frac{P_c}{A_c} = \frac{22.53 \times 10^3}{\frac{\pi}{4} \times 70^2}$$

$$= \frac{22530}{3846.5} = 5.85 \text{ N/mm}^2 \text{ (MPa)}$$

$$\sigma_s = \frac{P_s}{A_s} = \frac{57.47 \times 10^3}{\frac{\pi}{4} \times (80^2 - 70^2)}$$

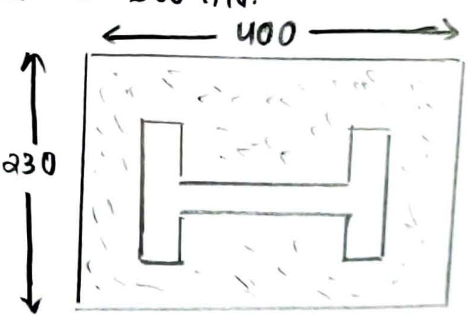
$$= \frac{57470}{1500} = 46.8 \text{ N/mm}^2 \text{ (MPa)}$$

2) Determine the required area of steel so that the force is shared equally between the steel and concrete.

Assume, $E_{conc.} = 24$ GPa

$E_{steel} = 200$ GPa

The load applied on the section is 300 KN.



$P = 300$ KN

$A_s = ?$

$A_c = 400 \times 230 - A_s$

$\frac{P_s}{P_c} = 1$

$P_s + P_c = 300$ KN

$$\frac{P_c L_c}{A_c E_c} = \frac{P_s L_s}{A_s E_s}$$

$$A_c E_c = A_s E_s$$

$$(400 \times 230 - A_s) \times 24 = A_s \times 200$$

$$400 \times 230 \times 24 - 24 A_s = 200 A_s$$

$$224 A_s = 400 \times 230 \times 24$$

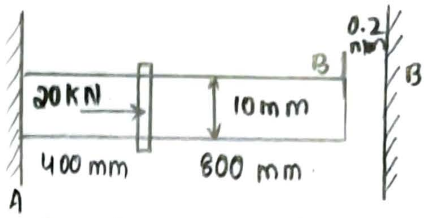
$$224 A_s = 2208000$$

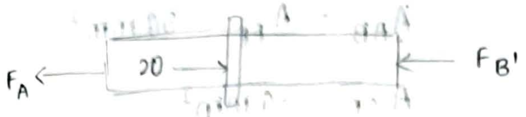
$$A_s = 9857 \text{ mm}^2$$

3) A steel rod shown in figure has a diameter of 10 mm. It is fixed to wall at A and before it is loaded there is a gap of 0.2 mm between the wall B' and the rod.

Determine the reactions at A and B' if the rod is subjected to an axial force of P = 20 kN. Neglect the size of collon.

Assume, $E_{steel} = 200$ GPa





$$-F_A - F_B' + 20 = 0$$

$$F_A + F_B' = 20 \text{ kN} \quad \text{——— (i)}$$

$$\delta_{AB} = \frac{PL_1}{AE} - \frac{F_{B'}L_2}{AE} = 0.2 \text{ mm}$$

$$0.2 = \frac{20 \times 10^3 \times 400}{\frac{\pi}{4} \times (10)^2 \times 200 \times 10^3} - \frac{F_B' \times 400 + 800 \times 10^3}{\frac{\pi}{4} \times (10)^2 \times 200 \times 10^3}$$

$$0.2 = \frac{8000}{0.785 \times 2 \times 10^4} - \frac{F_B' \times 1200}{0.785 \times 2 \times 10^4}$$

$$\frac{F_B' \times 1200}{0.785 \times 2 \times 10^4} = \frac{8000}{0.785 \times 2 \times 10^4} - 0.2$$

$$\frac{F_B' \times 1200}{15700} = \frac{8000}{15700} - 0.2$$

$$\frac{F_B' \times 1200}{15700} = \frac{8000 - 3140}{15700}$$

$$F_B' = \frac{4860}{1200}$$

$$F_B' = 4.05 \text{ kN}$$

Then,

$$F_A + F_B' = 20 \text{ kN}$$

$$F_A = 20 \text{ kN} - F_B'$$

$$F_A = 20 \text{ kN} - 4.05 \text{ kN}$$

$$F_A = 15.95 \text{ kN}$$

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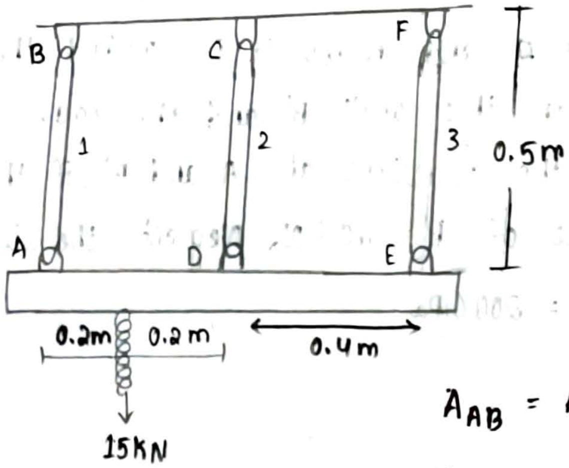
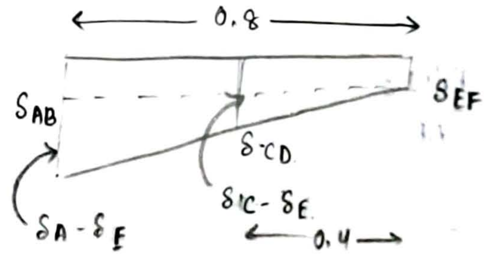
4)

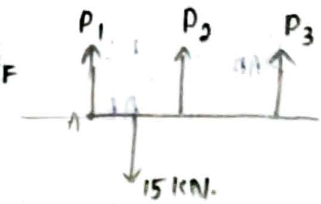


$A_{AB} = A_{EF} = 50\ mm^2$

$A_{CD} = 30\ mm^2$

Determine the force developed in each bar





$$\delta_{AB} = \frac{P_1 L}{A_1 E}$$

$$\delta_{CD} = \frac{P_2 L}{A_2 E}$$

$$\delta_{EF} = \frac{P_3 L}{A_3 E}$$

By similar triangles,

$$\frac{\delta_C - \delta_E}{0.4} = \frac{\delta_A - \delta_E}{0.8}$$

$$2\delta_C - 2\delta_E = \delta_A - \delta_E$$

$$2\delta_C - \delta_E = \delta_A$$

$$\frac{P_2 L}{A_2 E} - \frac{P_3 L}{A_3 E} = \frac{P_1 L}{A_1 E}$$

$$\frac{2P_2}{A_2} - \frac{P_3}{A_3} = \frac{P_1}{A_1}$$

$A_1 = A_3 = 50\ mm^2$

$$\frac{2P_2}{30} - \frac{P_3}{50} = \frac{P_1}{50}$$

$\Sigma F_y = 0 \Rightarrow \quad P_1 + P_2 + P_3 = 15 \quad \text{——} \quad (A)$

$\Sigma M_A = 0 \Rightarrow \quad P_2 \times 0.4 + P_3 \times 0.8 - 15 \times 0.2 = 0$

$P_2 + 2P_3 = 7.5 \quad \text{——} \quad ①$

$$\frac{2P_2}{30} - \frac{P_3}{50} = \frac{P_1}{50}$$

$$\frac{2P_2}{3} - \frac{P_3}{5} = \frac{P_1}{5}$$

$$\frac{2P_2}{3} = \frac{P_1 + P_3}{5}$$

$$10P_2 = 3P_1 + 3P_3$$

$3P_1 - 10P_2 + 3P_3 = 0 \quad \text{——} \quad ②$

$3(A) \Rightarrow \quad 3P_1 + 3P_2 + 3P_3 = 45 \quad \text{——} \quad ③$

$② - ③ \Rightarrow$

$$3P_1 - 10P_2 + 3P_3 = 0$$
$$3P_1 + 3P_2 + 3P_3 = 45$$
$$- \quad - \quad - \quad -$$
$$-13P_2 = -45$$
$$P_2 = 3.46 \text{ KN}$$

$① \Rightarrow \quad 3.46 + 2P_3 = 7.5$

$$2P_3 = 4.04$$
$$P_3 = 2.04 \text{ KN}$$

$Ⓐ \Rightarrow \quad P_1 + 3.46 + 2.04 = 15$

$$P_1 + 5.5 = 15$$
$$P_1 = 15 - 5.5$$
$$P_1 = 9.5 \text{ KN}$$

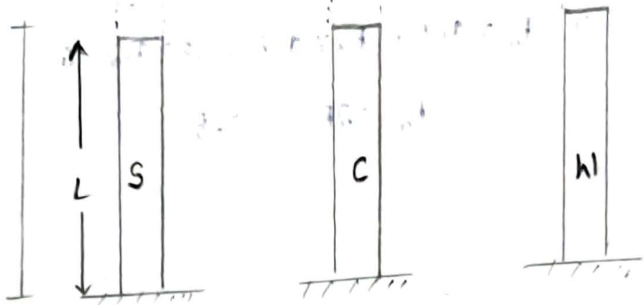
$\therefore \quad P_1 = 9.5 \text{ KN}$

$P_2 = 3.46 \text{ KN}$

$P_3 = 2.04 \text{ KN}.$

28/11/19

* Thermal stresses - occurs only when restrain the member



$$\Delta L = \alpha \Delta T L$$

α — Linear coefficient of thermal expansion

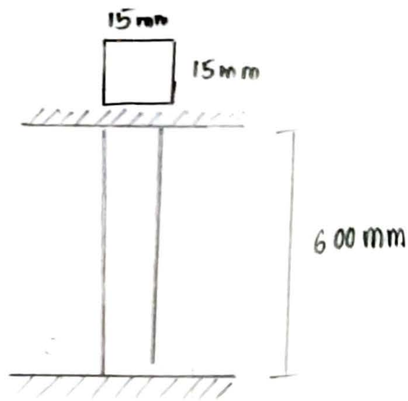
$$\alpha_{steel} = 6.6 \times 10^{-6} /°C$$

$$\text{Strain} = \frac{\Delta L}{L} = \varepsilon = \alpha \Delta T$$

1) A steel bar shown in figure is constrained (restrict) to just fix between two supports.

When temperature $T_1 = 20°C$ if the temperature is raised to $60°C$, determine the average normal thermal stress developed in the bar.

Assume, $\alpha_{steel} = 6.6 \times 10^{-6} /°C$.



$T_1 = 20°C$ $T_2 = 60°C$

$\Delta T = 40°C$

If not constrained, $\Delta L = \alpha \Delta T \times L$

$= 6.6 \times 10^{-6} \times 40 \times 600$

$= 0.15$ mm

The bar is constrained.

$$\delta_{A/B} = 0$$

$$\alpha \Delta T L - \frac{F_A L}{AE} = 0$$

$$\frac{F_A L}{AE} = \alpha \Delta T L$$

$$F_A = \frac{0.15 \times AE}{L}$$

$$F_A = \frac{0.15 \times 200 \times 10^3 \times 15 \times 15}{600}$$

$$= \frac{6750000}{600}$$

$$= 11250$$

$$\Rightarrow 11.25 \text{ KN}$$

$$\boxed{F = \alpha \Delta T E A}$$

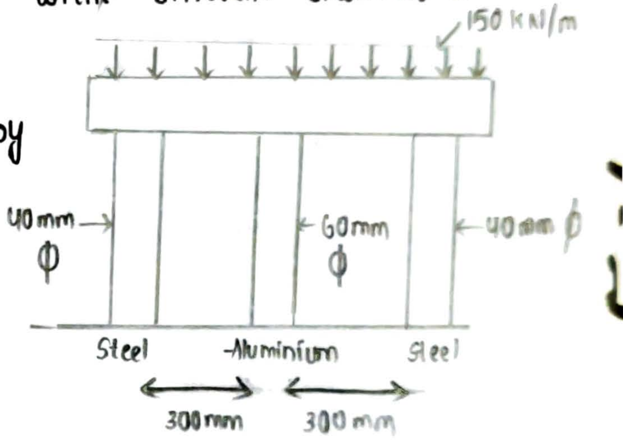
$$\frac{F}{A} = \sigma = \alpha \Delta T E$$

$$\sigma = 6.6 \times 10^{-6} \times 40 \times 200 \times 10^3$$

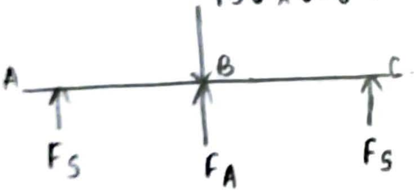
$$= 52.8 \text{ N/mm}^2$$

2) Three pillars on Bridge stand with different diameter of cross section.

Determine the force supported by each pillar and the initial temperature $T_1 = 20°c$, final temperature $T_2 = 80°c$.



$150 \times 0.6 = 90$ KN



$\Sigma F_y = 0 \Rightarrow F_S + F_A + F_S = 90$ KN

$F_A + 2F_S = 90$ KN ——— (1)

~~$\Sigma M_A = 0 \Rightarrow + F_A \times 300 + F_S \times 600 = 90$ KN~~

~~$F_A \times 10 + F_S \times 20 = 3$ KN~~

$\alpha_{steel} = 12 \times 10^{-6} / °c$

$\alpha_{Aluminium} = 23 \times 10^{-6} / °c$

$\delta_{Al} = \delta_{St} \Rightarrow [\delta_{al\,load} - \delta_{al\,temp.}] = [\delta_{st\,load} - \delta_{st\,temp.}]$

$$\frac{F_A L}{A_A E_A} = \frac{F_S L}{A_S E_S}$$

$$\frac{F_A}{\frac{\pi}{4}(60)^2 \times 70 \times 10^3} = \frac{F_S}{\frac{\pi}{4}(40)^2 \times 200 \times 10^3}$$

$$\frac{F_A}{252000} = \frac{F_S}{320000}$$

$$80F_A - 63F_S = 0 \quad \text{—— (2)}$$

(1) × 80 ⇒ $80(2F_S + F_A) = 90 \times 80$

$$160F_S + 80F_A = 7200 \quad \text{—— (3)}$$

(2) − (3) ⇒

$$80F_A - 63F_S = 0$$

$$80F_A + 160F_S = 7200$$

$$- \quad -$$

$$223F_S = 7200$$

$$F_S = 32.28 \text{ KN}$$

(1) ⇒ $F_A + 2F_S = 90 \text{ KN}$

$$F_A + 64.56 = 90$$

$$F_A = 25.44 \text{ KN}$$

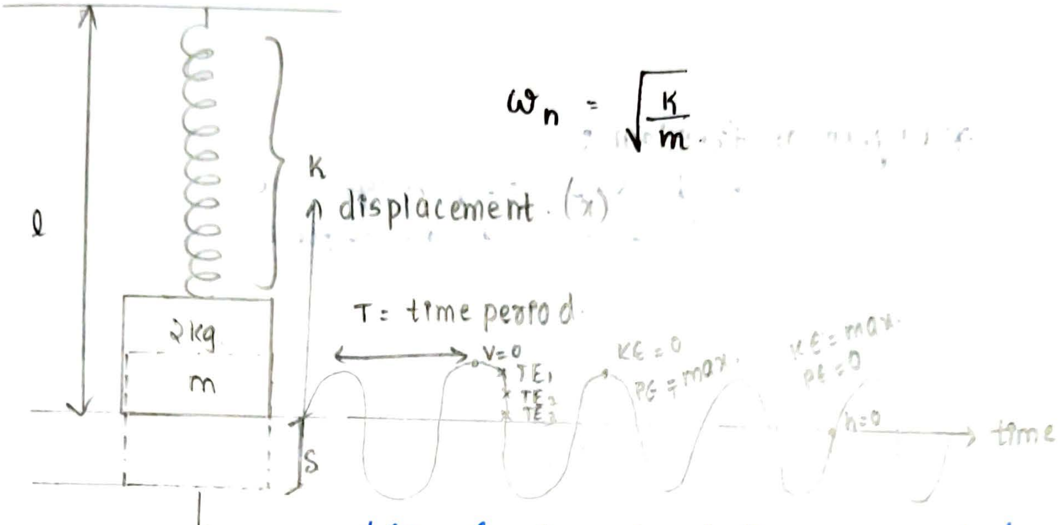
$$S_A - \alpha_A \Delta T l = S_{St} - \alpha_{St} \Delta T l$$

Natural frequency of spring mass system

$$\text{Stiffness} = \frac{\text{Load}}{\text{Static displacement}}$$

$$K = \frac{W \text{ (Newton)}}{\delta \text{ (meter)}}$$

$$\omega_n = \sqrt{\frac{K}{m}}$$



after leaving the body, motion starts.

Time period : The distance between any two crusts or troughs.

$$T = \frac{2\pi}{\omega_n}$$ subscription

ω_n is measured in radians/sec or cycles/sec = Hertz (Hz)

$KE = \frac{1}{2}mv^2$ $\quad PE = mgh$

Total energy = KE + PE

= constant (for perfectly elastic spring)

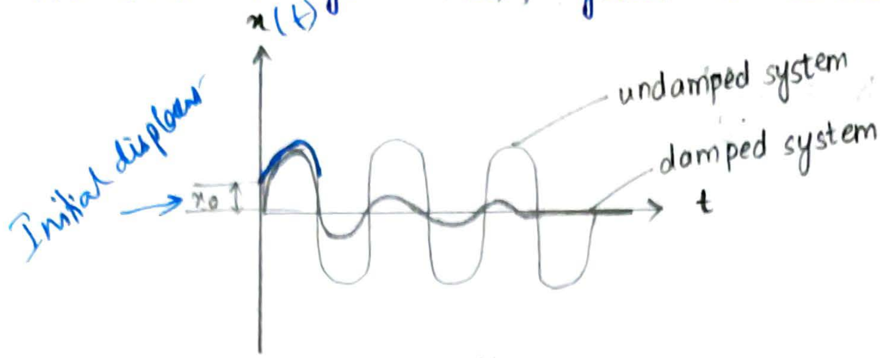
This is only possible for perfectly elastic spring only then spring mass system is continuous oscillating.

$$\frac{1}{2}mv^2 + mgh = \text{Constant}$$

Free vibration : A spring mass system given an initial displacement (stretched) and left free the mass to oscillate. This is called free vibration.

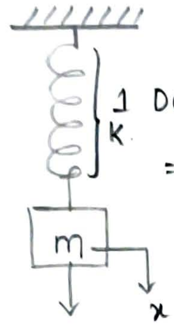
* Undamped system :

When oscillation amplitude remains constant forever in the life time of system that system is called undamped system.



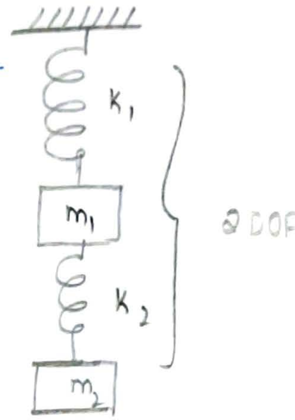
* Degree of freedom :

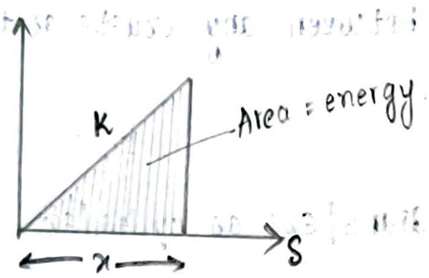
Defined as no. of independent ways a mass can displace.



1 Degree of freedom (DOF)

= Single degree of freedom (SDOF)





$P \propto S$

$P = KS$

$\frac{P}{S} = K$ - spring constant

$\sigma \propto \epsilon$

$\sigma = \epsilon E$

$$TE = KE + PE$$

$$= \frac{1}{2}mv^2 + mgh$$

$$PE = \frac{1}{2}kx^2$$

$$KE = \frac{1}{2} \times mv^2$$

$$= \frac{1}{2} \times \frac{W}{g}\left(\frac{dx}{dt}\right)^2$$

$$TE = \frac{1}{2}\frac{W}{g}\left(\frac{dx}{dt}\right)^2 + \frac{1}{2}kx^2$$

Differentiating w.r.t dt.

$$\frac{1}{2}\frac{W}{g} \cdot 2\frac{dx}{dt}\frac{d^2x}{dt^2} + \frac{1}{2}k.2x.\frac{dx}{dt} = 0$$

$$\frac{w}{g}\frac{d^2x}{dt^2} + kx = 0$$

$$m\frac{d^2x}{dt^2} + kx = 0.$$

$m\frac{d^2x}{dt^2} + kx = 0$ is equation of motion for single degree freedom system i.e, undamped system

11/2019

* Equivalent stiffness

Springs in series,

$$\frac{1}{K_{eq}} = \frac{1}{K_1} + \frac{1}{K_2}$$

$$K_{eq} = \frac{K_1K_2}{K_1+K_2}$$

Springs in parallel,

$$K_{eq} = K_1 + K_2$$

Equation of motion

$$m\frac{d^2x}{dt^2} + Kx = 0$$

$$m\ddot{x} + Kx = 0$$

$\dot{x} = \frac{dx}{dt}$, $\ddot{x} = \frac{d^2x}{dt^2}$ Representation

x is a function in terms of time

$$m\ddot{x}(t) + kx(t) = 0$$

where, $x(t)$ = Response of system

↳ At what time function is displaced

2/2/19

Derive the ~~response~~ frequency for undamped single degree of freedom (SDOF) free vibration

Equation of motion

$$m\frac{d^2x}{dt^2} + Kx = 0$$

$$m\ddot{x} + kx = 0$$

Assume a solution,

$$x(t) = A\sin\omega_n t \quad \text{——} \quad (1)$$

where, A = Amplitude of sine

$$\dot{x}(t) = A\omega_n \cos\omega_n t$$

$$\ddot{x}(t) = -A\omega_n^2 \sin\omega_n t \quad \text{--- (2)}$$

Now,

$$m(-A\omega_n^2 \sin\omega_n t) + kA\sin\omega_n t = 0$$

$$A\sin\omega_n t\left[-m\omega_n^2 + k\right] = 0$$

$A\sin\omega_n t \neq 0$ since there is some amplitude for a vibrational body

$$-m\omega_n^2 + k = 0$$

$$m\omega_n^2 = k$$

$$\omega_n^2 = \frac{k}{m}$$

$$\boxed{\omega_n = \sqrt{\frac{k}{m}}}$$

where, ω_n = natural frequency

★ Units : radian/second | $k \to N/m$, $m \to kg$.

Derive the response for SDOF for undamped system

Assume a solution

$$x(t) = A\sin\omega_n t + B\cos\omega_n t$$

where, A, B are constants obtained from initial solutions

$x(t)$ at time $t=0$ is x_0

$\dot{x}(t)$ at time $t=0$ is $\underline{U_0}$ - initial velocity

$x(t)$ at $t=0$ is $x_0 = A\sin\omega_n \times 0 + B\cos\omega_n(0)$

$$x_0 = B \times 1 = B$$

$$\dot{x}(t) = A\omega_n \cos\omega_n t - B\omega_n \sin\omega_n t$$

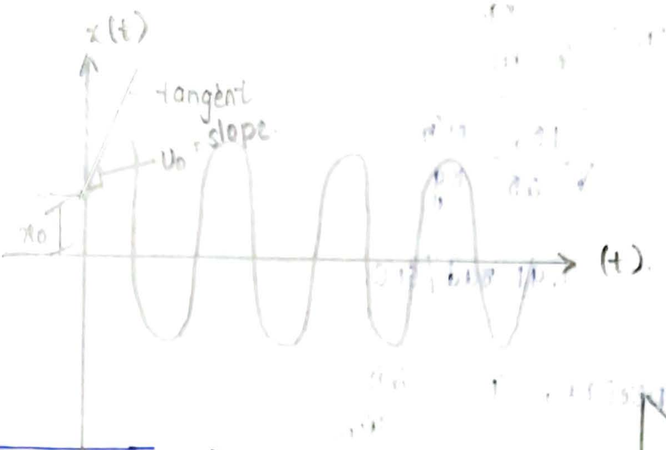
$$U_0 = A\omega_n \cos\omega_n(0) - B\omega_n \sin\omega_n(0)$$

$$U_0 = A\omega_n$$

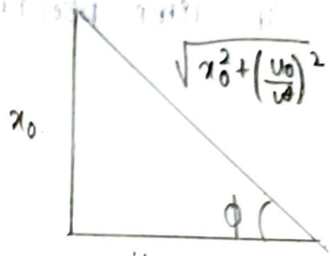
$$A = \frac{U_0}{\omega_n}$$

$\therefore \quad x(t) = A \sin \omega_n t + B \cos \omega_n t$

$$\boxed{x(t) = \frac{v_0}{\omega} \sin \omega_n t + x_0 \cos \omega_n t}$$



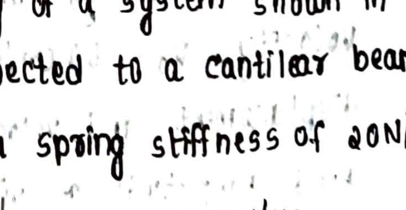
$$x(t) = \sqrt{x_0^2 + \left(\frac{v_0}{\omega_n}\right)^2} \quad \frac{v_0/\omega_n}{\sqrt{x_0^2 + \left(\frac{v_0}{\omega_n}\right)^2}} \sin \omega t + \frac{x_0}{\sqrt{x_0^2 + \left(\frac{v_0}{\omega_n}\right)^2}} \cos \omega t$$



$$x(t) = \sqrt{x_0^2 + \left(\frac{v_0}{\omega_n}\right)^2} \left[\cos\phi \sin \omega_n t + \sin\phi \cos \omega_n t\right]$$

$$x(t) = \sqrt{x_0^2 + \left(\frac{v_0}{\omega_n}\right)^2} \sin(\omega t_n + \phi)$$

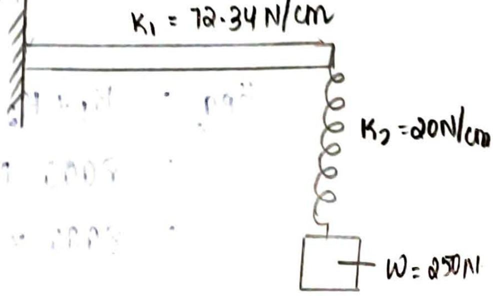
$$\boxed{x(t) = R \sin(\omega_n t + \phi)}$$



where, R = maximum amplitude

3/12/18

1) Determine the nature of frequency of a system shown in figure. A weight of 250 N is connected to a cantilear beam of stiffness k = 72.34 N/cm through a spring stiffness of 20N/cm



$$K_{eq} = \frac{K_1 K_2}{K_1 + K_2}$$

$$= \frac{72.34 \times 20}{72.34 + 20}$$

$$= \frac{1446.8}{92.34}$$

$$= 15.67 \text{ N/cm} = 15.67 \text{ N}/10^{-2}\text{m}$$

$$= 1567 \text{ N/m}$$

$$m = \frac{w}{g}$$

$$= \frac{250}{10} = 25 \text{ kg}$$

$$\omega_n = \sqrt{\frac{K_{eq}}{m}}$$

$$= \sqrt{\frac{1567 \text{ N/m}}{25 \text{ kg}}}$$

$$= 7.91 \text{ rad/sec.}$$

In Time period, $T = \frac{2\pi}{\omega_n}$

$$= \frac{2 \times 3.14}{7.91}$$

$$= 0.79 \text{ sec.}$$

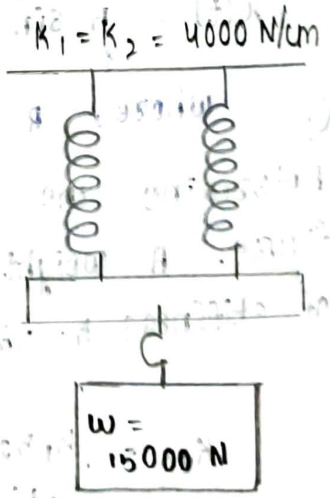
Cycles per second, $f = \frac{1}{T}$

$$= \frac{1}{0.79}$$

$$= 1.26 \text{ cycles/sec (Hz)}$$

2) Find the natural period and frequency of oscillations for the system shown in figure.

If the weight has initial displacement $y_0 = 2.5$ cm and an initial velocity $V_0 = 50$ cm/sec determine the displacement and velocity after 1 second. Find Amplitude of vibration.



$$K_{eq} = K_1 + K_2$$

$$= 8000 \text{ N/cm}$$

$$= 8000 \times 10^2 \text{ N/m}$$

$$\omega_n = \sqrt{\frac{K_{eq}}{m}}$$

$$m = \frac{w}{g}$$

$$= \frac{15000}{10} = 1500 \text{ kg}$$

$$\omega_n = \sqrt{\frac{8 \times 10^5}{1500}}$$

$$\omega_n = 23.09 \text{ rad/sec.}$$

Time period, $T = \dfrac{2\pi}{\omega_n}$

$= \dfrac{2 \times 3.14}{23.09}$

$= 0.27$ sec

Frequency, $f = \dfrac{1}{T}$

$= \dfrac{1}{0.27}$

$= 3.7$ cycles/sec.

$$x(t) = \frac{v_0}{\omega_n} \sin \omega_n t + x_0 \cos \omega_n t$$

$$x(t=1) = \frac{50 \times 10}{23.09} \times \sin(23.09)(1) + 2.5 \times 10 \cos(23.09)(1)$$

$= 21.65 \times \sin(23.09) + 25 \cos(23.09)$

$= 21.65 \times [-0.892] + 25 [-0.45]$

$= -19.3118 - 11.25$

$= -30.56$ mm

$$\dot{x}(t=1) = \frac{v_0}{\omega_n} \cos \omega_n t \times \omega_n - x_0 \sin \omega_n t \times \omega_n$$

$= 500 \cos(23.09 \times 1) - 25 \times 23.09 \times \sin(23.09 \times 1)$

$= 500 [-0.45] - 577.25 [-0.892]$

$= -225 + 514.907$

$= 286.75$ mm/sec

Maximum amplitude,

$$R = \sqrt{x_0^2 + \left(\frac{v_0}{\omega}\right)^2}$$

$$R = \sqrt{(25)^2 + \left(\frac{500}{23.09}\right)^2}$$

$$= \sqrt{625 + 468.8}$$

$$= \sqrt{1093.8}$$

$$= 33.09 \text{ mm}$$

12/12/2019

Impact loading

Weight $= W$

Height $= h$

Maximum deflection, $\Delta_{max} = \Delta_{st}\left[1 + \sqrt{1 + 2\left(\frac{h}{\Delta_{st}}\right)}\right]$

It is maximum displacement in a spring because of weight 'W' falling from height 'h'.

Work done = PE of spring

$$W(h + \Delta_{max}) = \frac{1}{2} K \Delta^2_{max}$$

Δ_{st} = static displacement $= \frac{W}{K}$

K = stiffness in N/m

W = Weight in N