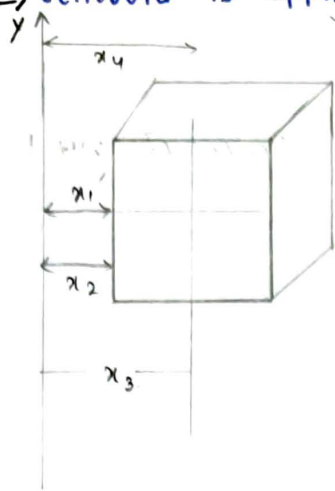


## \* Centroid :

centroid of an area lies on the axis of symmetry if it exists.

⇒ Centroid is applied to plane areas or lines (geometry).



CG

$$V\bar{x} = V_1x_1 + V_2x_2 + V_3x_3 + V_4x_4$$

$$V\bar{x} = V_1x_1 + V_2x_2 + V_3x_3 + V_4x_4$$

$$\sum V_i\bar{x} = \sum V_ix_i$$

$$V = A +$$

$$\therefore \sum A_i\bar{x} = \sum A_ix_i$$

$$\bar{x} = \frac{\sum V_ix_i}{\sum V_i} = \frac{\sum A_ix_i}{\sum A_i} = \frac{\sum L_ix_i}{\sum L_i}$$

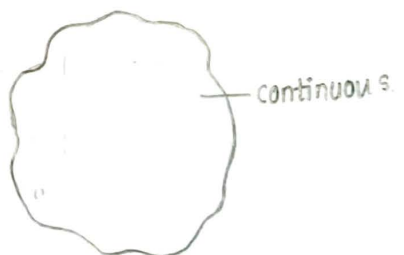
$$C_G = \frac{\text{Summation of all individual forces}}{\text{Summation of volumes/areas/lengths}} \times \text{distance}$$

## \* Areas :

For continuous body,

$$\bar{x}_c = \frac{\int_A x dA}{\int_A dA} = \int_0^1 \int_0^1 x dx dy$$

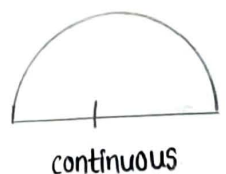
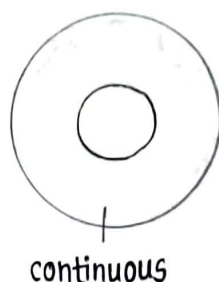
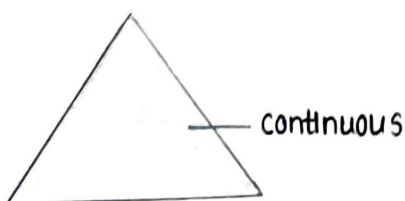
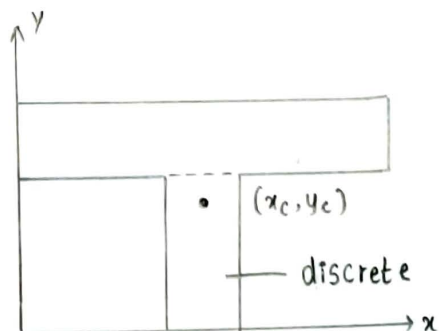
$$\bar{y}_c = \frac{\int_A y dA}{\int_A dA}$$



For discrete body,

$$\bar{x}_c = \frac{\sum A_ix_i}{\sum A_i}$$

$$\bar{y}_c = \frac{\sum A_iy_i}{\sum A_i}$$



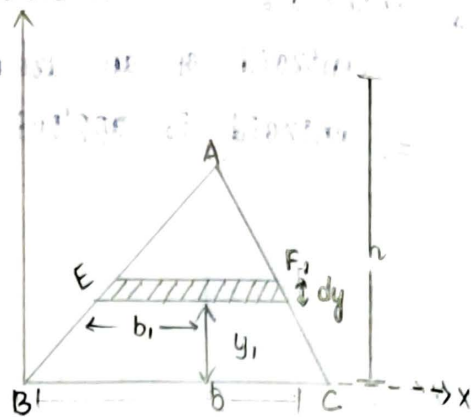
centroid of  $\Delta^{ic}$ .

$$y_c = \frac{\int_A y dA}{\int_A dA}$$

$$\Delta ABC \sim \Delta ABF$$

$$\frac{b_1}{b} = \frac{h-y}{h}$$

$$b_1 = b \left(1 - \frac{y}{h}\right)$$



$$\text{Area of element } EF = b_1 \times dy = dA$$

$$dA = b \left(1 - \frac{y}{h}\right) dy$$

$$N = \int y b \left(1 - \frac{y}{h}\right) dy = b \int_0^h \left(y - \frac{y^2}{h}\right) dy$$

$$= b \left[ \frac{y^2}{2} - \frac{y^3}{3h} \right]_0^h$$

$$= b \left[ \frac{h^2}{2} - \frac{h^3}{3h} \right] - b(0)$$

$$= b \left[ \frac{h^2}{2} - \frac{h^2}{3} \right]$$

$$= \frac{bh^2}{6}$$

$$D = \int_0^h dA$$

$$= b \int_0^h \left(1 - \frac{y}{h}\right) dy = b \left[ y - \frac{y^2}{2h} \right]_0^h$$

$$= b \left[ h - \frac{h^2}{2h} \right]$$

$$= \frac{bh}{2}$$

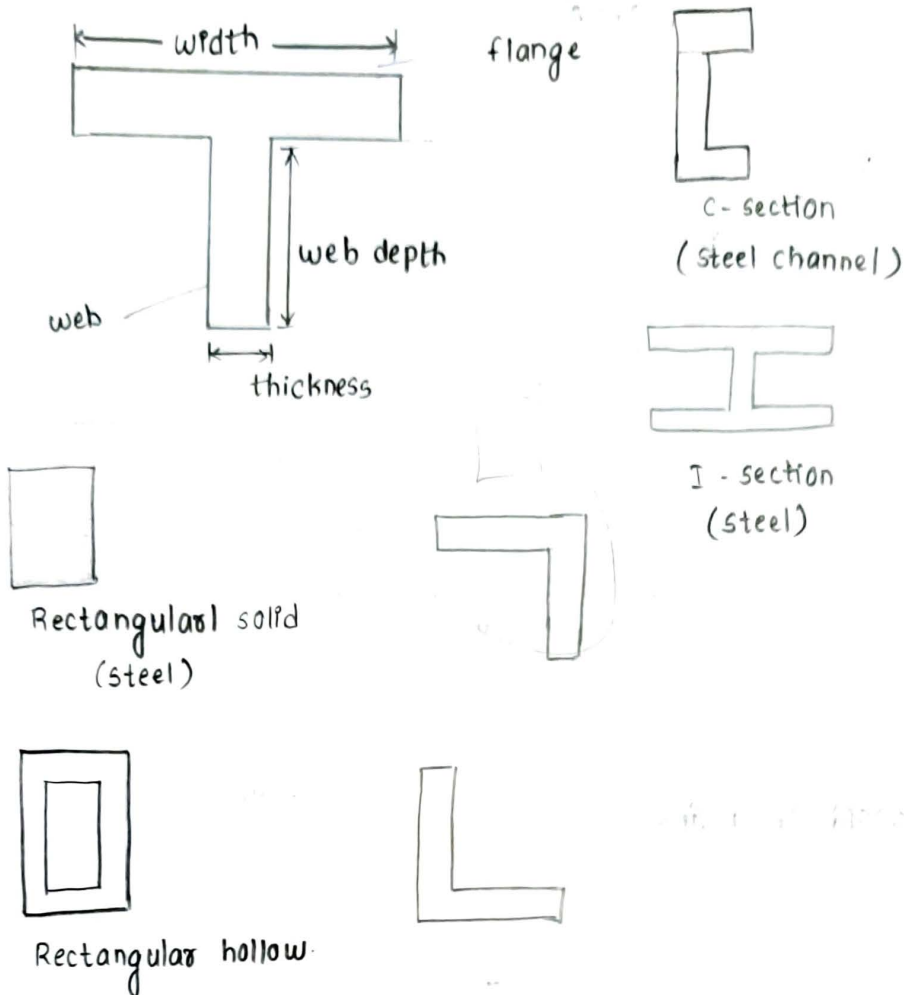
$$y_c = \frac{bh^2/6}{bh/2}$$

$$y_c = h/3$$

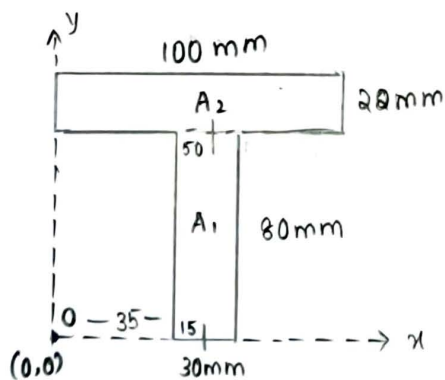
$$\therefore \text{Centroid} = (x_c, y_c)$$

$$= (b/2, h/3)$$

21/10/2014



\* Symmetry : For centroid  $\Rightarrow$  area is in same ratio.



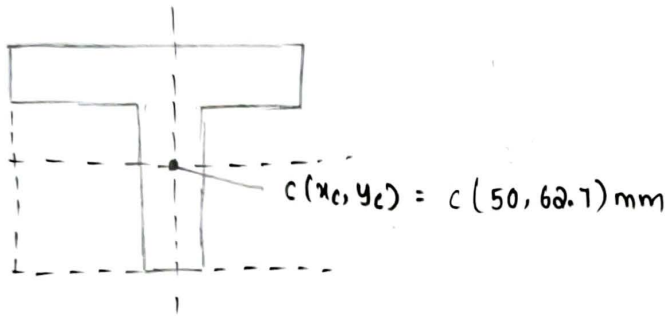
$$\begin{aligned}
 x_c &= \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2} \\
 &= \frac{30 \times 80 \times 15 + 100 \times 20 \times 50}{30 \times 80 + 100 \times 20} \\
 &= \frac{120000 + 100000}{2400 + 2000} \\
 &= \frac{220000}{4400} = 50 \text{ mm.}
 \end{aligned}$$

$$y_c = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

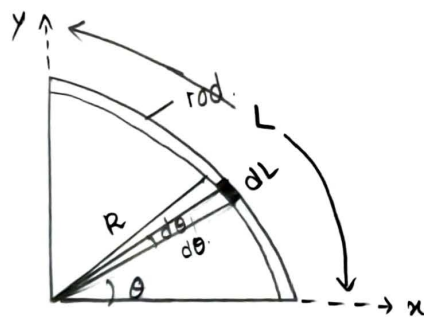
$$= \frac{30 \times 80 \times 40 + 100 \times 20 \times (80+10)}{30 \times 80 + 100 \times 20}$$

$$= \frac{96000 + 180000}{2400 + 2000}$$

$$= 62.7 \text{ mm}$$



\* Centroid of a line (rod)



$$x_c = \frac{\int x dL}{\int dL}$$

$$L = R\theta$$

$$dL = R d\theta$$

$$x = R \cos \theta$$

$$\int x dL = \int_0^{\pi/2} R \cos \theta \cdot R d\theta$$

$$= R^2 \int_0^{\pi/2} \cos \theta d\theta$$

$$= R^2 [\sin \theta]_0^{\pi/2} = R^2 (1) = R^2$$

$$\int dL = \int_0^{\pi/2} R d\theta$$

$$= R \int_0^{\pi/2} d\theta$$

$$= R [\theta]_0^{\pi/2}$$

$$= R \frac{\pi}{2}$$

$$x_c = \frac{R^2}{R \pi/2}$$

$$= \frac{2R}{\pi}$$

$$y_c = \frac{\int y dL}{\int dL}$$

$$y = R \sin \theta$$

$$\int y dL = \int_0^{\pi/2} R \sin \theta \cdot R d\theta$$

$$= R^2 \int_0^{\pi/2} \sin \theta d\theta$$

$$= -R^2 [\cos \theta]_0^{\pi/2}$$

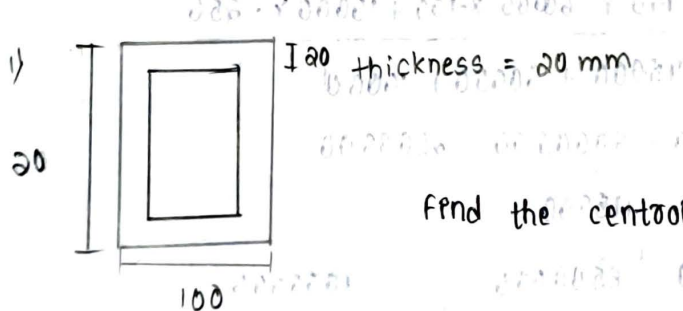
$$= -R^2 [0 - 1] = R^2$$

$$\int dL = \int_0^{\pi/2} R d\theta$$

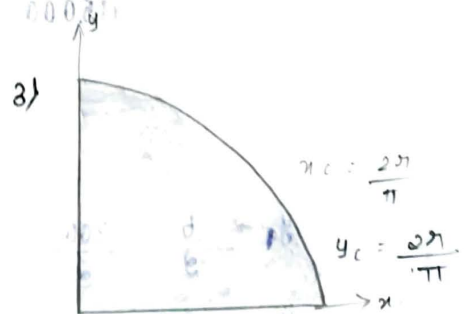
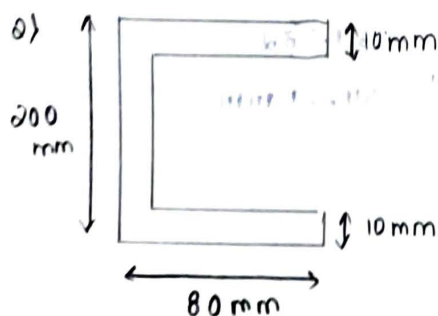
$$= \frac{R\pi}{2}$$

$$y_c = \frac{R^2}{R\pi/2} = \frac{2R}{\pi}$$

$$\therefore (x_c, y_c) = \left( \frac{2R}{\pi}, \frac{2R}{\pi} \right)$$



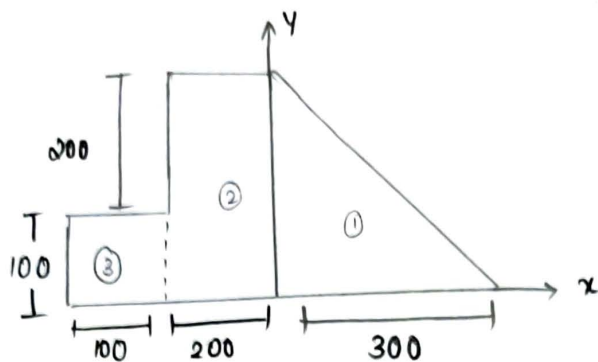
Find the centroid.  $(x_c, y_c)$



$$x_c = 0 \quad y_c = \frac{2R}{\pi}$$

\* Composite area or hybrid area centroid.

1)

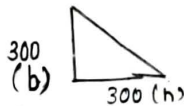


$$A_1 = \frac{1}{2} \times 300 \times 300 = 45000 \text{ mm}^2$$

$$A_2 = 200 \times 300 = 60000 \text{ mm}^2$$

$$A_3 = 100 \times 100 = 10000 \text{ mm}^2$$

$$x_1 = \frac{h}{3} = \frac{300}{3} = 100 \text{ mm}$$



$$x_2 = \frac{b}{2} = \frac{200}{2} = 100 \text{ mm}$$

$$x_3 = 200 + \frac{100}{2} = 250 \text{ mm}$$

$$x_c = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3}{A_1 + A_2 + A_3}$$

$$= \frac{45000 \times 100 + 60000 \times 100 + 10000 \times 250}{45000 + 60000 + 10000}$$

$$= \frac{4500000 + 6000000 + 2500000}{115000}$$

$$= \frac{13000000}{115000}$$

$$= 11304.35 \text{ mm}$$

$$= \frac{4500000 + 6000000 + 2500000}{115000} = \frac{13000000}{115000}$$

$$= 11304.35 \text{ mm}$$

$$= 11304.35 \text{ mm}$$

$$y_1 = \frac{b}{3} = \frac{300}{3} = 100 \text{ mm}$$

$$y_2 = \frac{300}{2} = 150 \text{ mm}$$

$$y_3 = \frac{100}{2} = 50 \text{ mm}$$

$$y_c = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$$= \frac{45000 \times 100 + 60000 \times 150 + 10000 \times 50}{45000 + 60000 + 10000}$$

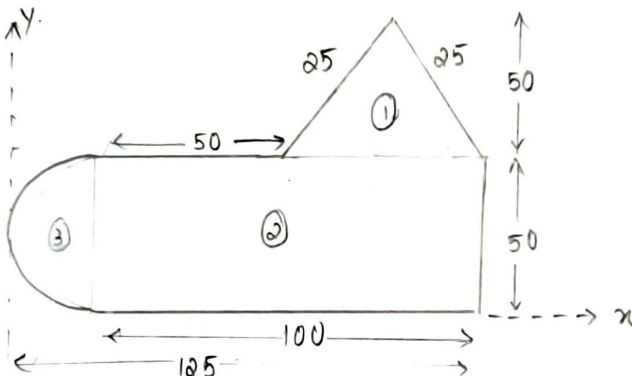
$$= \frac{4500000 + 9000000 + 500000}{115000}$$

$$= \frac{14000000}{115000}$$

$$= 121.739 \text{ mm}$$

$$\approx 121.7 \text{ mm}$$

$$\therefore (x_c, y_c) = (-34.79, 121.7) \text{ mm}$$



$$A_1 = \frac{1}{2} \times 50 \times 50 = 1250 \text{ mm}^2$$

$$A_2 = 100 \times 50 = 5000 \text{ mm}^2$$

$$A_3 = \frac{1}{2} \pi r^2$$

$$= \frac{1}{2} \times 3.14 \times 25^2 = 981.25 \text{ mm}^2$$

$$x_1 = 25 + 50 + 25 = 100 \text{ mm}$$

$$x_2 = \frac{100}{2} + 25 = 75 \text{ mm}$$

$$x_3 = R - \frac{4R}{3\pi} = R \left(1 - \frac{4}{3\pi}\right)$$

$$= 25 \left(1 - \frac{4}{3\pi}\right) = 14.39 \text{ mm}$$

$$x_c = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3}{A_1 + A_2 + A_3} = 14.4 \text{ mm}$$



$$x_c = R - \frac{4R}{3\pi}$$

$$= \frac{1250 \times 100 + 5000 \times 75 + 981.25 \times 14.4}{1250 + 5000 + 981.25}$$

$$= 14.4 \text{ mm}$$



$$= \frac{125000 + 375000 + 14130}{7231.25}$$

$$= \frac{514130}{7231.25}$$

$$= 71.09 \text{ mm}$$

$$y_1 = \frac{50}{3} + 50 = 66.66 \text{ mm}$$

$$y_2 = \frac{50}{2} = 25 \text{ mm}$$

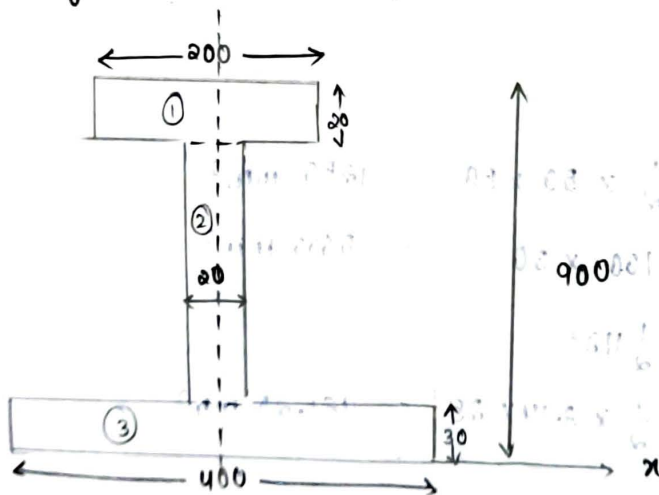
$$y_3 = R = 25 \text{ mm}$$

$$y_c = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$$= \frac{1250 \times 66.66 + 5000 \times 25 + 981.25 \times 25}{1250 + 5000 + 981.25}$$

$$= \frac{83325 + 125000 + 24531.25}{7231.25} = \frac{232856.25}{7231.25} = 32.201 \text{ mm}$$

$$\therefore (x_c, y_c) = (71.09, 32.201) \text{ mm}$$



$$A_1 = 20 \times 200 = 4000 \text{ m}^2$$

$$A_2 = 20 \times 850 = 17000 \text{ m}^2$$

$$A_3 = 30 \times 400 = 12000 \text{ m}^2$$

$$x_1 = 0$$

$$x_2 = 0$$

$$x_3 = 0$$



$$x_c = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3}{A_1 + A_2 + A_3}$$

$$= 0$$

$$y_1 = \frac{90}{2} + 850 + 30 = 10 + 880 = 890 \text{ mm}$$

$$y_2 = \frac{850}{2} + 30 = 425 + 30 = 455 \text{ mm}$$

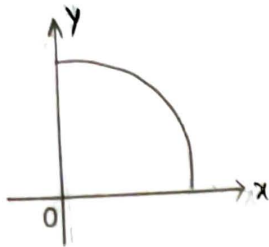
$$y_3 = \frac{30}{2} = 15 \text{ mm}$$

$$y_c = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

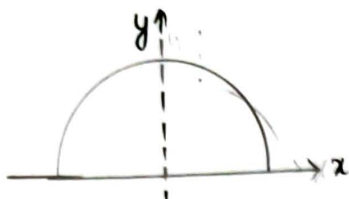
$$= \frac{4000 \times 890 + 17000 \times 455 + 12000 \times 15}{4000 + 17000 + 12000}$$

$$= \frac{3560000 + 7735000 + 180000}{33000} = \frac{11475000}{33000} = 347.72 \text{ mm}$$

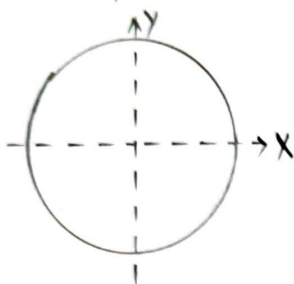
$$\therefore (x_c, y_c) = (0, 347.72) \text{ mm}$$



$$x_c = \frac{2r}{\pi}, \quad y_c = \frac{2r}{\pi}$$



$$x_c = 0, \quad y_c = \frac{2r}{\pi}$$



$$x_c = 0, \quad y_c = 0$$

\*

# Centroids of common shapes of areas

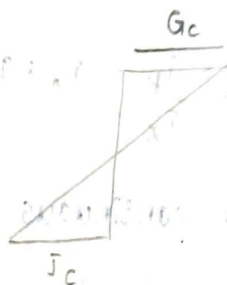
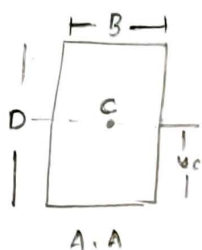
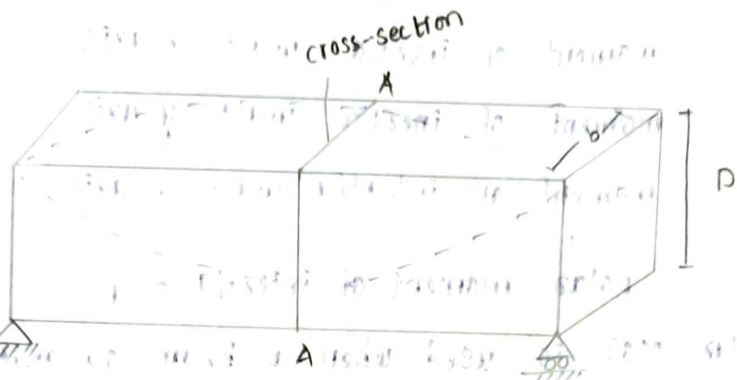
Shape	Diagram	$\bar{x}$	$\bar{y}$	Area
Triangular area			$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter-elliptical area		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Semi elliptical area		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
Quarter-circular arc		$\frac{2r}{\pi}$	$\frac{2r}{\pi}$	$\frac{\pi r}{2}$
Semicircular arc		0	$\frac{2r}{\pi}$	$\frac{\pi r}{2}$
Arc of circle		$\frac{8r \sin \frac{\alpha}{2}}{3\alpha}$	0	$2\alpha r$

# Moment of Inertia

It is a geometrical property.

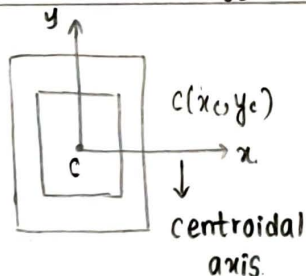
It is used for finding the bending capacity of a structure.

It is also used for finding the deflection of a structure.



$$\frac{\text{Moment (M)}}{\frac{\text{Moment of Inertia (I)}}{\text{Centroid of } y_c}} = \text{stress}$$

depth is more  
= moment of inertia is more



The moment of inertia can be calculated by both base axis and centroidal axis as a reference.

Moment of inertia is also known as second moment of Area

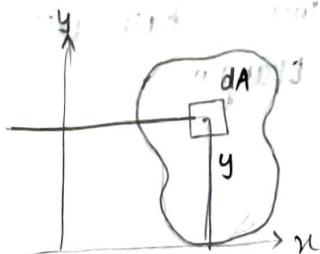
$$\text{2nd moment of area} = A \times y \times y$$

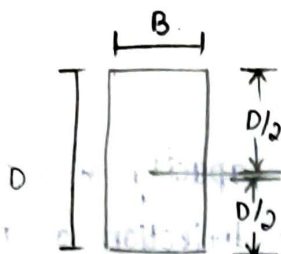
$$I_x = Ay^2$$

where,  $I_x$  = moment of inertia

$A$  = moment of area

$$I_x = \int_A y^2 dA$$





$I_x$  - moment of inertia w.r.t x-axis

$I_y$  - moment of inertia w.r.t y-axis

$I_x$  - moment of inertia w.r.t x-axis

↓  
polar moment of inertia =  $I_p$

polar MOI is used when a beam or member is twisting

$$I_x = I_p = I_x + I_y$$

for a continuous body,

$$I_x = \int y^2 dA$$

$$I_y = \int x^2 dA$$



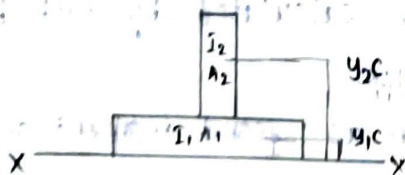
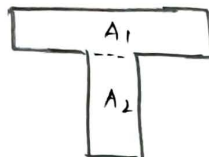
for a discontinuous body,

Individual MOI is used by adding area

$$I_{yx} = I_1 + I_2$$

$$= \frac{b_1 D_1^3}{12} + \frac{b_2 D_2^3}{12} + \dots$$

parallel axis theorem



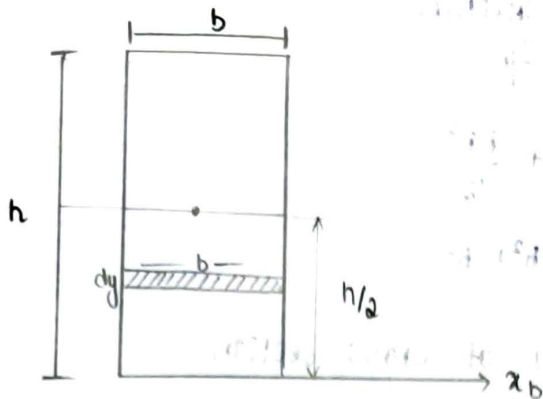
$$MOI = I_1 + I_2 + A_1 x_{1c}^2 + A_2 x_{2c}^2$$

1) Determine the Moment of inertia for the rectangular section shown in figure

(a) w.r.t centroidal axis - x

(b) w.r.t base axis passing through base of the rectangle.

MOI is applied to area or cross section only but not applicable for line or surface.



$$(a) \quad I_x = \int_A y^2 dA$$

$$= \int_{-h/2}^{h/2} y^2 \cdot b \cdot dy$$

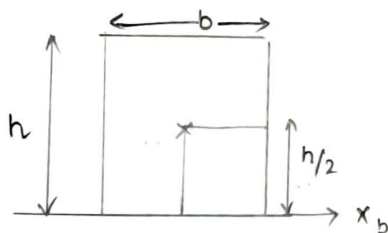
$$= b \left[ \frac{y^3}{3} \right]_{-h/2}^{h/2} = \frac{b}{3} \left[ \frac{h^3}{8} + \frac{h^3}{8} \right]$$

$$= \frac{2bh^3}{3 \cdot 8} = \frac{bh^3}{12}$$

for scale =  $\frac{2 \times 30^3}{12} = 4500$

$$\left[ \epsilon_0 + \frac{\epsilon_0}{\epsilon} \right] \frac{x_0}{\epsilon} \Rightarrow \frac{30 \times 20^3}{12} = 20$$

(b)



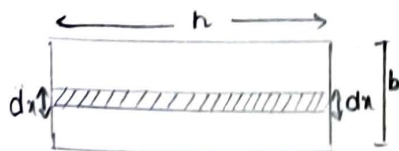
Parallel axis theorem,  $I_{x_b} = I_x + A \times \left( \frac{h}{2} \right)^2$

centroid

$$= \frac{bh^3}{12} + b \times h \times \frac{h^2}{4}$$

$$= \frac{bh^3}{12} + \frac{bh^3}{4} = \frac{bh^3}{3}$$

MOI w.r.t y-axis :



$$I_y = \int_A x^2 dA = \int_{-b/2}^{b/2} x^2 \times dx \times h = h \left[ \frac{x^3}{3} \right]_{-b/2}^{b/2}$$

$$= \frac{h}{3} \left[ \frac{b^3}{8} + \frac{b^3}{8} \right]$$

$$= \frac{hb^3}{12}$$



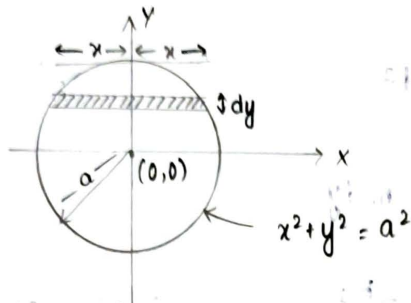
Polar moment of inertia,

$$I_p = I_x + I_y$$

$$= \frac{bh^3}{12} + \frac{hb^3}{12}$$

$$= \frac{bh}{12} (h^2 + b^2)$$

Solid circular beam of cross section



$$\begin{aligned} I_x &= \int_A y^2 dA = \int_A y^2 \cdot 2x dy \\ &= \int_{-a}^a 2xy^2 dy \\ &= 2 \int_{-a}^a \sqrt{a^2 - y^2} \cdot y^2 dy \\ &= \frac{a^4}{8} \int_{-\pi/2}^{\pi/2} (\cos 4\theta - 1) d\theta \\ &= \frac{a^4}{8} \left[ \frac{\sin 4\theta}{4} - \theta \right]_{-\pi/2}^{\pi/2} \end{aligned}$$

$$\text{Let } y = a \sin \theta$$

$$\frac{dy}{d\theta} = a \cos \theta$$

$$dy = a \cos \theta d\theta$$

$$\begin{aligned} &\int \sqrt{a^2 - a^2 \sin^2 \theta} \cdot a^2 \sin^2 \theta \cdot a \cos \theta d\theta \\ &= \int a^2 \sin^2 \theta \cdot a^2 \cos^2 \theta d\theta \\ &= a^4 \int \sin^2 \theta \cos^2 \theta d\theta \\ &= a^4 \int (\sin \theta \cos \theta)^2 d\theta \\ &= a^4 \int \left( \frac{\sin 2\theta}{2} \right)^2 d\theta \\ &= \frac{a^4}{4} \int \sin^2 2\theta d\theta \\ &= \frac{a^4}{4} \int \frac{\cos 4\theta - 1}{2} d\theta \\ &= \frac{a^4}{8} \left[ \frac{\sin 4\theta}{4} - \theta \right] \end{aligned}$$

$$\text{U.L} \Rightarrow y = a \Rightarrow a = a \sin \theta \Rightarrow \sin \theta = 1$$

$$\theta = \pi/2$$

$$\text{L.L} \Rightarrow y = -a \Rightarrow -a = a \sin \theta \Rightarrow \sin \theta = -1$$

$$\theta = -\pi/2$$

$$\begin{aligned} \Rightarrow \frac{a^4}{8} \left[ \sin 4\theta - \theta \right]_{-\pi/2}^{\pi/2} &= \frac{a^4}{32} \left[ \sin^2 \left( \frac{\pi}{2} \right) - \frac{\pi}{2} - \left( \sin^2 \left( -\frac{\pi}{2} \right) + \frac{\pi}{2} \right) \right] \\ &= \frac{a^4}{32} \left[ 1 - \frac{\pi}{2} + 1 - \frac{\pi}{2} \right] \end{aligned}$$



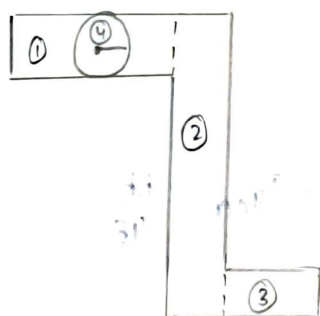
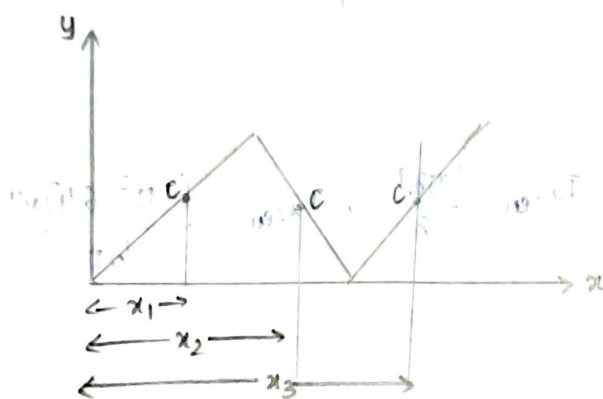
21/10/2019

Centroid :

Line  $\Rightarrow x_c = \frac{\sum x_i L_i}{\sum L_i}, y_c = \frac{\sum y_i L_i}{\sum L_i}$

Area  $\Rightarrow x_c = \frac{\sum x_i A_i}{\sum A_i}, y_c = \frac{\sum y_i A_i}{\sum A_i}$

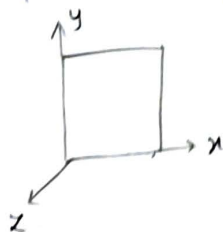
Volume  $\Rightarrow x_c = \frac{\sum x_i V_i}{\sum V_i}, y_c = \frac{\sum y_i V_i}{\sum V_i}$



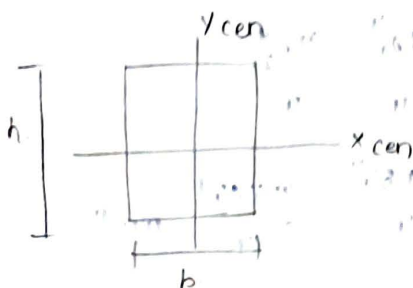
$$x_c = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3 - A_4 x_4}{A_1 + A_2 + A_3 + A_4}$$

Moment of inertia :

It is meant only for areas.



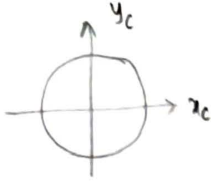
$I_x, I_y, I_z$



$$I_{x_{cen}} = \frac{bh^3}{12}$$

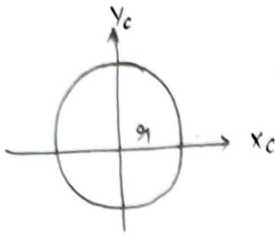
$$I_{y_{cen}} = \frac{hb^3}{12}$$

$$(x_c, y_c) = \left(\frac{b}{2}, \frac{h}{2}\right)$$

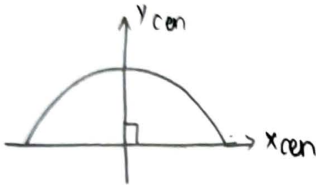


$$I_{x_{cen}} = \frac{\pi a^4}{4}$$

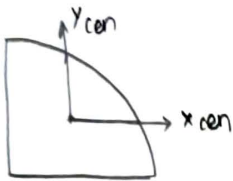
$$I_{y_{cen}} = \frac{\pi a^4}{4}$$



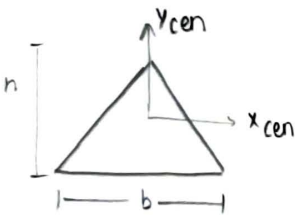
$$I_x = I_y = \frac{\pi a^4}{4}$$



$$I_{y_{cen}} = \frac{\pi a^4}{8}, \quad I_{x_{cen}} = \frac{(9\pi^2 - 64)a^4}{128\pi}$$

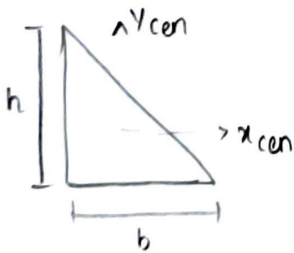


$$I_{x_{cen}} = \frac{\pi a^4}{16} = I_{y_{cen}}$$



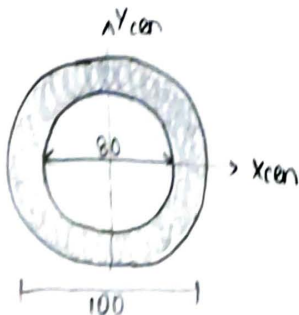
(Isosceles  $\Delta$ )

$$I_{x_{cen}} = \frac{bh^3}{36}, \quad I_{y_{cen}} = \frac{hb^3}{48}$$



$$I_{x_{cen}} = \frac{bh^3}{36}, \quad I_{y_{cen}} = \frac{hb^3}{36}$$

Composite area MOI

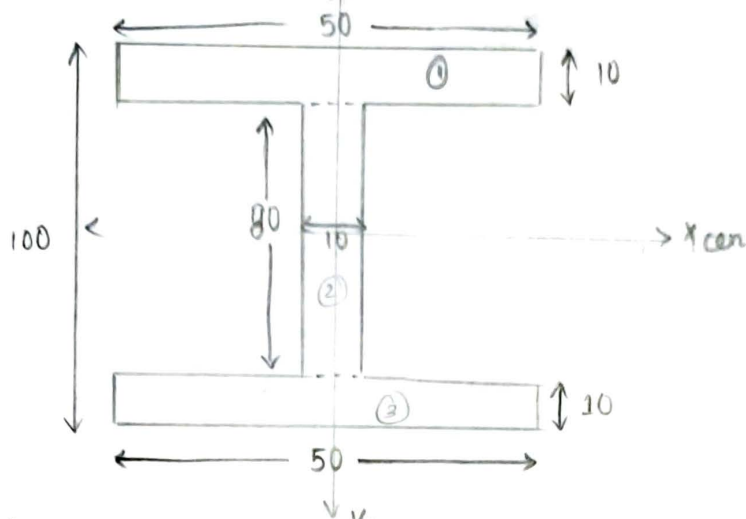


$$\begin{aligned} I_{x_{cen}} &= \frac{\pi}{4} (r_1^4 - r_2^4) \\ &= \frac{\pi}{4} \left( \frac{d_1^4}{16} - \frac{d_2^4}{16} \right) \\ &= \frac{\pi}{16} (d_1^4 - d_2^4) \end{aligned}$$

$$\begin{aligned} I_{x_{cen}} &= \frac{\pi r_1^4}{4} - \frac{\pi r_2^4}{4} \\ &= \frac{\pi \times 50^4}{4} - \frac{\pi \times 40^4}{4} \text{ mm}^4 \\ &= \frac{\pi}{4} [6250000 - 2560000] \\ &= \frac{\pi}{4} \times 3690000 \\ &= 2896650 \text{ mm}^4 \end{aligned}$$

## Parallel axis theorem

$$I_{C \text{ cen.}} = I_{x \text{ cen.} y \text{ cen.}} + A \times y_c^2$$



MOI with respect to centroid:

$$I_{x_{cen}} = \frac{50 \times 10^3}{12} + 50 \times 10 \times (50 - 5)^2 + 10 \times \frac{80^3}{12} + \frac{50 \times 10^3}{12} + (50 - 5)^2 \times 50 \times 10$$

$$= 4166.66 + 1012500 + 426666.66 + 4166.66 + 1012500$$

$$= 245999.98 \text{ mm}^4$$

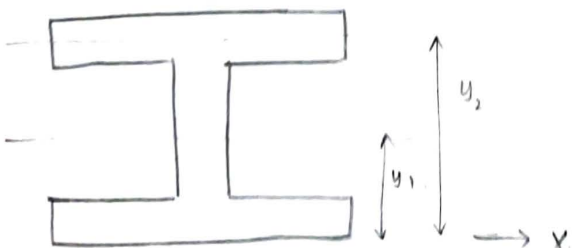
$$I_{y_{cen}} = \frac{10 \times 50^3}{12} + 50 \times 10 \times (0)^2 + \frac{80 \times 10^3}{12} + \frac{10 \times 50^3}{12} + 50 \times 10 \times 0$$

$$= \frac{10 \times 50^3}{12} + 0 + \frac{80 \times 10^3}{12} + \frac{10 \times 50^3}{12} + 0$$

$$= 1041666.66 + 6666.66 + 1041666.66$$

$$= 2089999.98 \text{ mm}^4$$

MOI with respect to base axis:



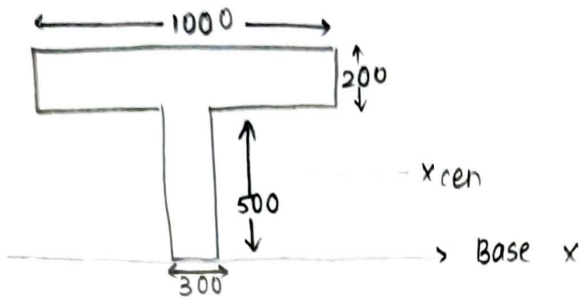
$$I_x = I_{x_1 \text{ cen.}} + A_1 y_c^2 + I_{x_2 \text{ cen.}} + A_2 y_{c_2}^2 + I_{x_3 \text{ cen.}} + A_3 y_{c_3}^2$$

$$= \frac{50 \times 10^3}{12} + 50 \times 10 \left( \frac{10}{2} + 90 \right)^2 + \frac{10 \times 80^3}{12} + 80 \times 10 \left( \frac{80}{2} + 10 \right)^2 + \frac{50 \times 10^3}{12} + 10 \times 50 \left( \frac{10}{2} \right)^2$$

$$= 4166.66 + 4512500 + 426666.66 + 2000000 + 4166.66 + 12500$$

$$= 6959999.98 \text{ mm}^4$$

2)



31/10/2019

# Radius of Gyration

$$r = \sqrt{\frac{I}{A}}$$

$r$  - determines in which the structure buckles

$$r_x = \sqrt{\frac{I_x}{A}}$$

$$r_y = \sqrt{\frac{I_y}{A}}$$

If  $r_y < r_x$  the buckling happens with respect to y-axis

$r_x < r_y$  the buckling happens with respect to x-axis

$$I_x = 3.8 \times 10^{-5} \text{ m}^4$$

$$I_y = 4.5 \times 10^{-6} \text{ m}^4$$

$$A = 1.5 \text{ m}^2$$

$$r_x = \sqrt{\frac{I_x}{A}}$$

$$r_y = \sqrt{\frac{I_y}{A}}$$

$$= \sqrt{\frac{3.8 \times 10^{-5} \text{ m}^4}{1.5 \text{ m}^2}}$$

$$= \sqrt{\frac{4.5 \times 10^{-6} \text{ m}^4}{1.5 \text{ m}^2}}$$

$$= \sqrt{2.533 \times 10^{-5} \text{ m}}$$

$$= 1.7320 \times 10^{-3} \text{ m}$$

$$= 5.0332 \times 10^{-3} \text{ m}$$

$$= 0.0017320 \text{ m}$$

$$= 0.0050332 \text{ m}$$

$$\therefore r_x = 0.0050332$$

$$r_y = 0.0017320$$

$\therefore r_y < r_x$  buckling happens with respect with y-axis