

Engineer Mechanic

29/8/19

* Structural Engineering

It deals with construction, designing, surveying

It covers 60% of weightage in the field of civil engineering

* Geotechnical E

It deals with ground, soil strength

* Transportation

It deals with traffic, roads, runways

* Environmental

It deals with water quality — Impurities removal

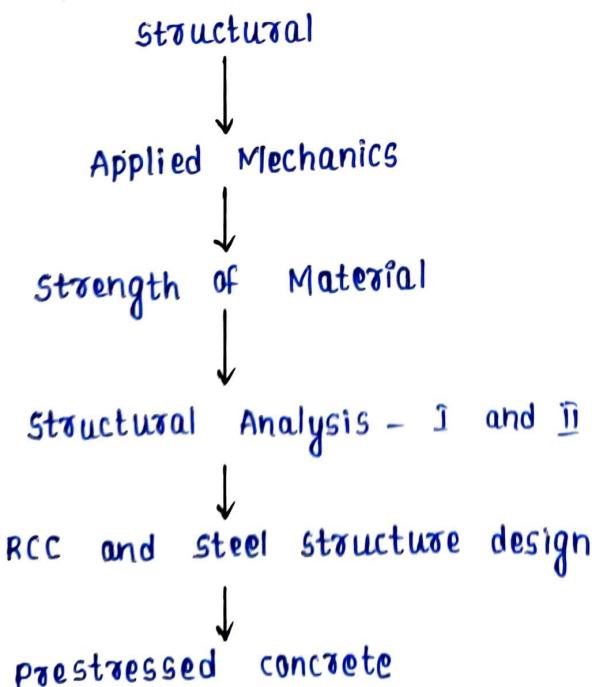
air quality

sewage system

* Water Resources

It deals with utilize and supply of water

05/9/19



Textbook : Applied Mechanics — R.C. Hibbler

- 1) Equilibrium of forces (force system)
- 2) Centroid and moment of inertia^{isogeny}
- 3) Friction
- 4) Simple stresses and strains - I
- 5) Simple stresses and strains - II

Applied Mechanics

* Static

- It is understanding of the force induced by the external body.
- The study is about finding forces or reactions of a resting body.
- Example:
 - ★ A book placed on a table
 - ★ Lift moving up and down

* Kinematics

- It is the study of the motion of body or the object.
- Body is in motion.
- Example:
 - ★ Throwing bomb from the aeroplane.

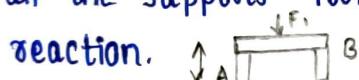
* Dynamics

- It is the study about both forces and its motion.
- Body is in motion.
- Example:
 - ★ Earthquakes

Forces acting on a body

→ Coplanar forces, Parallel forces
→ Collinear forces, concurrent forces

Free body diagram of a body is a diagram where we remove all the supports from the body and replace them with their reaction.



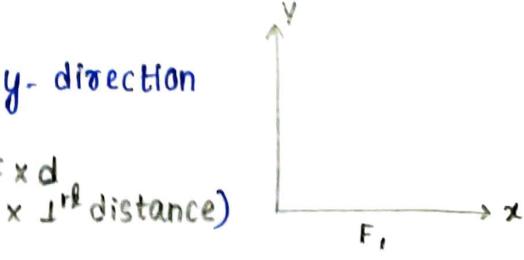
The two approaches available to solve force systems on a rigid body are : (i) Equilibrium of forces
(ii) Lami's theorem

(i) Equilibrium of forces :

The summation of all forces in x-direction must be equal to zero i.e., $\sum F_x = 0$

The summation of all forces in y-direction must be equal to zero i.e., $\sum F_y = 0$

The summation of moment (force \times distance) at a given point must be equal to zero i.e., $\sum \text{Moment} = 0$.



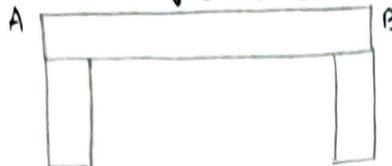
$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$F_x \cdot d \sin \theta - F_y \cdot d \cos \theta = 0$$

$$F_x \cdot d \sin \theta = F_y \cdot d \cos \theta$$

13/09/2019
Engineering Mechanics



y-axis

W (Newton)

known quantity

V_A and V_B are unknowns

V_A

V_B

Equilibrium equation.

$$\sum F_x = 0 \Rightarrow 0$$

$$\sum F_y = 0 \Rightarrow V_A + V_B - W = 0 \quad \text{--- (1)}$$

$$V_A + V_B = W$$

\Rightarrow Any force or displacement indicated in the direction of axes are considered as positive and opposite direction is negative.

$$\sum M_A = 0 \Rightarrow -V_B \times l + W \times \frac{l}{2} + V_A \times 0 = 0 \quad \text{--- (2)}$$

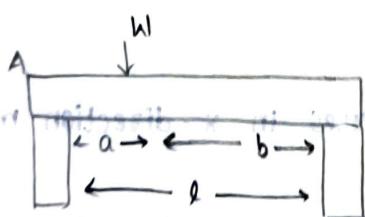
$$V_B \times l' = W \times l' \quad \text{Anticlockwise moment} \rightarrow -ve$$

$$V_B = \frac{W}{2}$$

Putting equation (1),

$$V_A + \frac{W}{2} = W$$

$$V_A = W - \frac{W}{2} = \frac{W}{2}$$



y-axis

W

V_A

V_B

$$a + b = l$$

$$\sum F_x = 0 \Rightarrow 0$$

$$\sum F_y = 0 \Rightarrow V_A + V_B - W = 0$$

$$V_A + V_B = W \quad \text{--- (1)}$$

$$\sum M_A = -V_B \times l + W \times a = 0$$

$$V_B \times l = Wa$$

$$V_B(a+b) = Wa$$

$$V_B = \frac{Wa}{l}$$

By equation ①,

$$V_A + \frac{Wa}{l} = W$$

$$V_{AB} = W \left(\frac{l-a}{l} \right)$$

$$V_A = W \left(\frac{l-a}{l} \right)$$

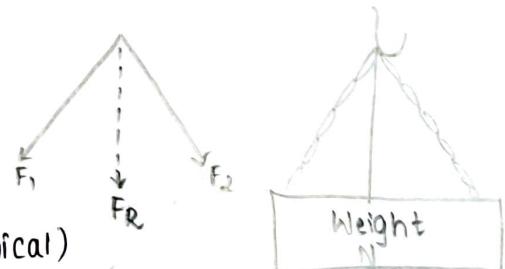
$$V_A = \frac{Wb}{l}$$

Quantity	Dimensional symbol	Unit	Symbol
Mass	M	kilogram	kg
Length	L	meter	m
Time	T	second	s
Force	F	Newton	N

* Resultant force

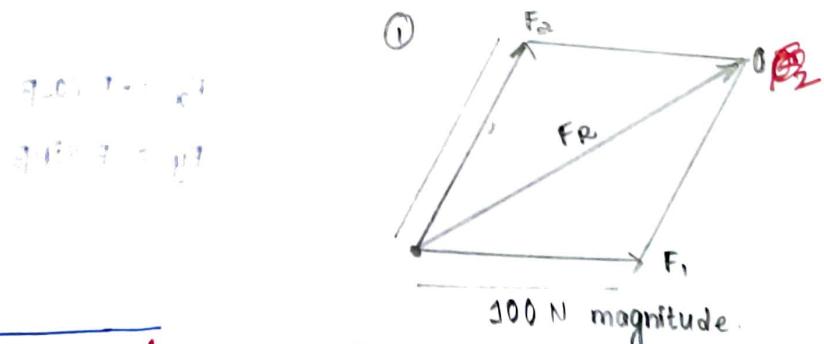
$F_1, F_2 \rightarrow$ component forces

$F_R \rightarrow$ resultant force



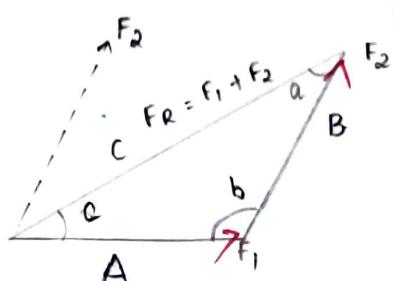
Method - 1 : Parallelogram law (Graphical)

Method - 2 : Trigonometry

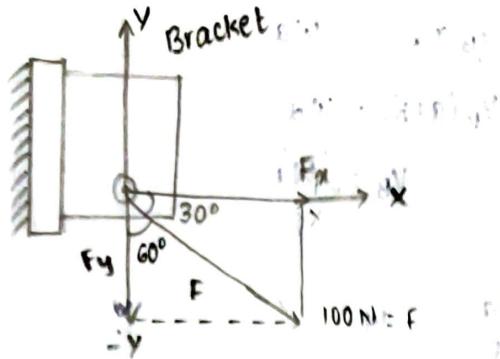


$$② C = \sqrt{A^2 + B^2 - 2AB \cos \theta}$$

$$\frac{A}{\sin a} = \frac{B}{\sin B} = \frac{C}{\sin c}$$



* Component of forces from resultant



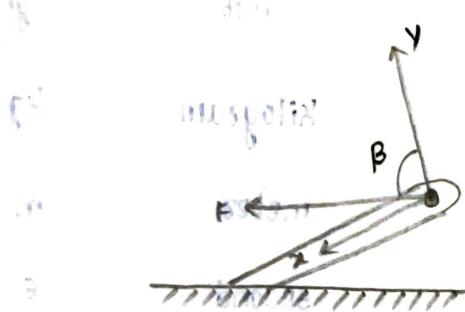
$$\cos \theta = \frac{\text{Base}}{\text{Hypo}} \quad \cos 60^\circ = \sin 30^\circ = \frac{F_x}{F}$$

$$\cos 30^\circ = \frac{F_x}{F} \quad F_x = F \cos 30^\circ$$

$$F_x = F \cos 30^\circ$$

* Component force

①

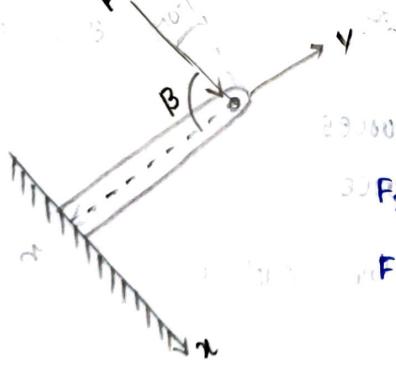


component along $\text{along } y$ directions = ?

$$F_x = F \sin \beta$$

$$F_y = F \cos \beta$$

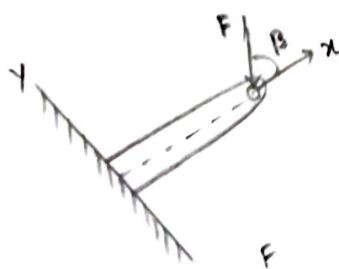
②



$F_x = F \sin \beta$

$$F_y = F \cos \beta$$

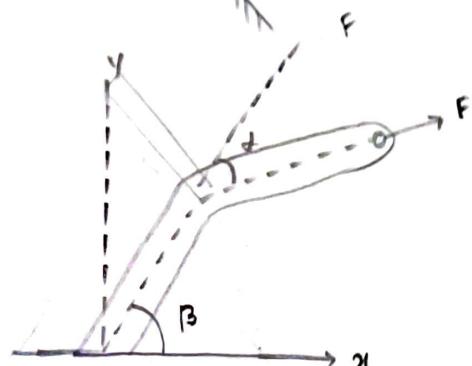
③



$$F_x = -F \cos \beta$$

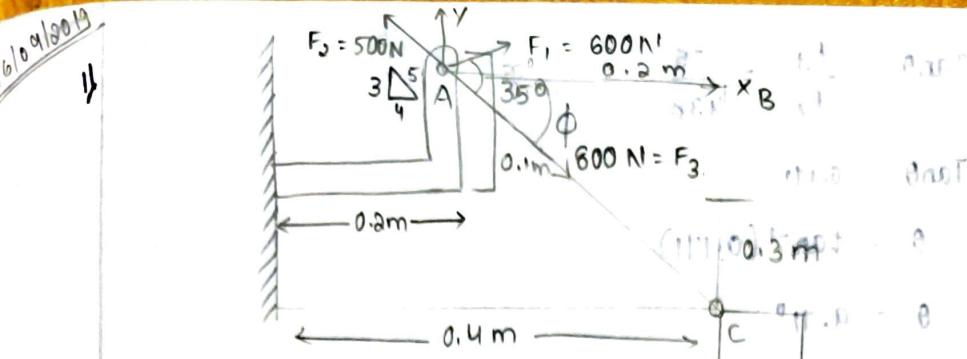
$$F_y = -F \sin \beta$$

④

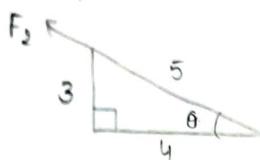


$$F_x = F \cos (\beta - \alpha)$$

$$F_y = F \sin (\beta - \alpha)$$



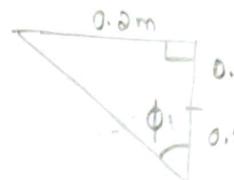
Find component of F_1 , F_2 , F_3



$$\tan \theta = \frac{3}{4}$$

$$\theta = \tan^{-1}(\frac{3}{4})$$

$$\theta = 36.85^\circ$$



$$\tan \phi = \frac{0.1 + 0.3}{0.4 - 0.2}$$

$$\tan \phi = \frac{0.4}{0.2} = 2$$

$$\phi = \tan^{-1}(2)$$

$$\phi = 63.4^\circ$$

$$\sin 35^\circ = 0.57$$

$$\cos 35^\circ = 0.81$$

$$\sin 36.85^\circ = 0.59$$

$$\cos 36.85^\circ = 0.80$$

$$\sin 63.4^\circ = 0.89$$

$$\cos 63.4^\circ = 0.44$$

$$F_{1x} = F_1 \cos 35^\circ = 486 \text{ N}$$

$$\frac{0.57 \times 600}{342}$$

$$F_{1y} = F_1 \sin 35^\circ = +342 \text{ N}$$

$$\frac{0.81 \times 600}{486}$$

$$F_{2x} = -F_2 \cos 36.85^\circ = -400 \text{ N}$$

$$\frac{-0.59 \times 600}{400}$$

$$F_{3x} = F_3 \cos 63.4^\circ = 352 \text{ N}$$

$$\frac{0.89 \times 600}{352}$$

$$F_{3y} = -F_3 \sin 63.4^\circ = -712 \text{ N}$$

$$\frac{-0.44 \times 600}{-712}$$

$$F_x = F_{1x} + F_{2x} + F_{3x} = (486 - 400 + 352) \text{ N} = 438 \text{ N} \rightarrow$$

$$F_y = F_{1y} + F_{2y} + F_{3y} = (342 + 0 - 712) \text{ N} = -75 \text{ N} \downarrow$$

$$0.08 \times 438 \text{ N} = 35 \text{ N}$$

$$F_x = 438 \text{ N}$$

$$0.08 \times 6.62 \text{ N} = 0.53 \text{ N}$$

$$F_R = \sqrt{F_x^2 + F_y^2}$$

$$= \sqrt{(438)^2 + (-75)^2}$$

$$1.68 = \sqrt{191844 + 5625}$$

$$= \sqrt{197469} = 444.34 \text{ N}$$

$$F_R = \sqrt{F_x^2 + F_y^2}$$

$$0.08 \times 6.62 \text{ N} = 0.53 \text{ N}$$

$$\tan \theta = \frac{F_y}{F_x} = \frac{75}{438}$$

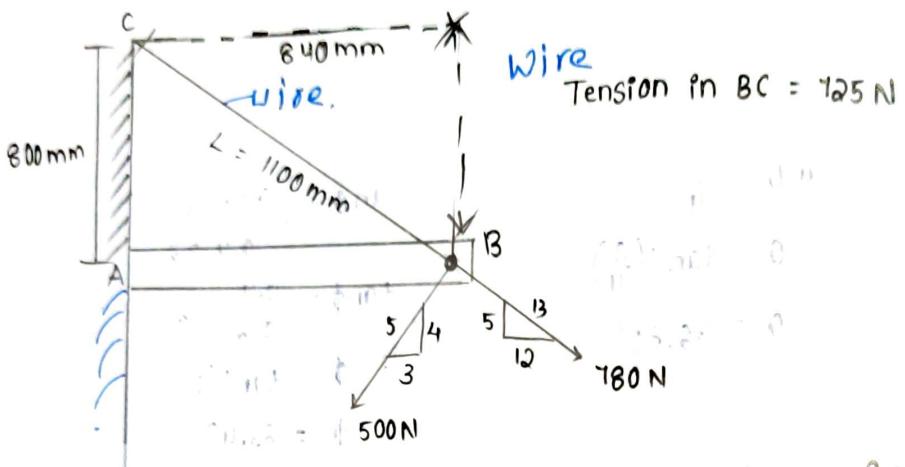
$$\tan \theta = 0.171$$

$$\theta = \tan^{-1}(0.171)$$

$$\theta = 9.71^\circ$$

Actual angle of resultant force $F_R = 360^\circ - 9.71^\circ$
 $= 350.29^\circ$

2)



Find the resultant force F_R .

$$\cos 53.2 = 0.599$$

$$\cos 43.6 = 0.724$$

$$\sin 53.2 = 0.800$$

$$\sin 43.6 = 0.689$$

$$\cos 46.4 = 0.689$$

$$\cos 22.7 = 0.920$$

$$\sin 46.4 = 0.724$$

$$\sin 22.7 = 0.385$$

$$\cos 36.8 = 0.800$$

$$\cos 67.3 = 0.385$$

$$\sin 36.8 = 0.599$$

$$\sin 67.3 = 0.920$$

$$F_x = \frac{F_{1x}}{1160} = \cos 43.6$$

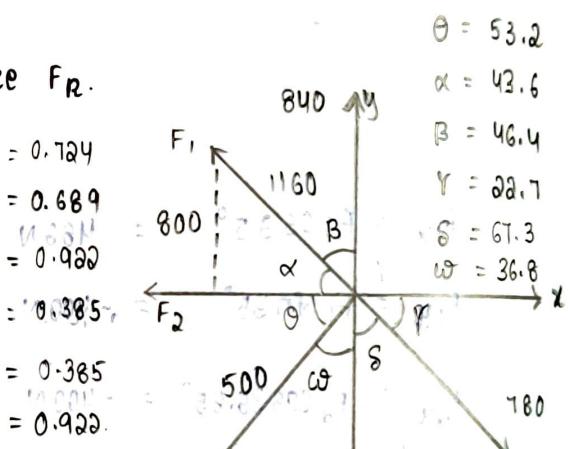
$$F_{1x} = \cos 43.6 \times 1160 \\ = 0.724 \times 1160 \\ = 839.84 \text{ N}$$

$$F_{2x} = \frac{F_{2x}}{500} = \cos 53.2$$

$$F_{2x} = \cos 53.2 \times 500 \\ = 0.599 \times 500 \\ = 299.5 \text{ N}$$

$$F_{3x} = \frac{F_{3x}}{780} = \cos 22.7$$

$$F_{3x} = \cos 22.7 \times 780 \\ = 0.920 \times 780 \\ = 719.16 \text{ N}$$



$$F_y = \frac{F_{1y}}{1160}$$

$$= \cos 46.4 \times 1160$$

$$= 0.689 \times 1160$$

$$= 799.24 \text{ N}$$

$$F_y = \frac{F_{2y}}{500} = \cos 36.8$$

$$= 0.800 \times 500$$

$$= 400 \text{ N}$$

$$F_y = \frac{F_{3y}}{780} = \cos$$

$$F_{3y} = \cos 67.3 \times 780$$

$$= 296.4 \text{ N}$$

$$\begin{aligned}
 F_{Rx} &= -F_1x - F_2x + F_3x \\
 &= -839.84 - 299.5 + 719.16 \\
 &= -420.18 \text{ N}
 \end{aligned}$$

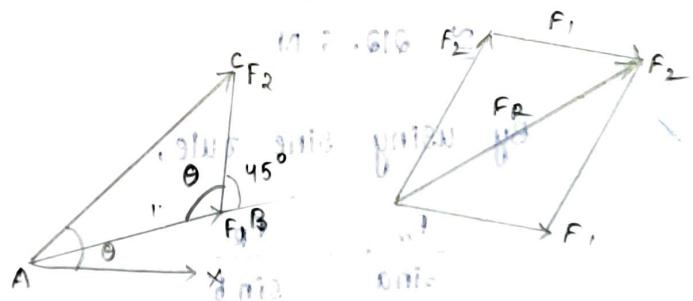
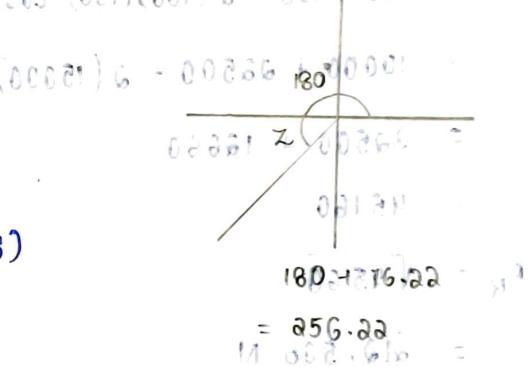
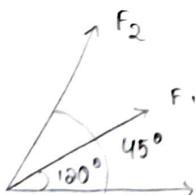
$$\begin{aligned}
 F_{Ry} &= F_1y - F_2y - F_3y \\
 &= 799.24 - 400 - 296.4 \\
 &= 102.84 \text{ N} \\
 F_R &= \sqrt{F_{Rx}^2 + F_{Ry}^2} \\
 &= \sqrt{(-420.18)^2 + (102.84)^2} \\
 &= \sqrt{176551.23 + 10516.06} \\
 &= \sqrt{187187.29} \\
 &= 432.58 \text{ N.}
 \end{aligned}$$

$$\tan(z) = \frac{F_{Ry}}{F_{Rx}}$$

$$\tan(z) = \frac{420.18}{102.84}$$

$$\tan(z) = 4.08$$

$$\begin{aligned}
 z &= \tan^{-1}(4.08) \\
 &= 76.22^\circ
 \end{aligned}$$



Cosine rule : $AC^2 = F_R^2 = F_1^2 + F_2^2 - 2F_1F_2 \cos B$

Sine rule : $\frac{F_1}{\sin C} = \frac{F_2}{\sin A} = \frac{F_R}{\sin B}$

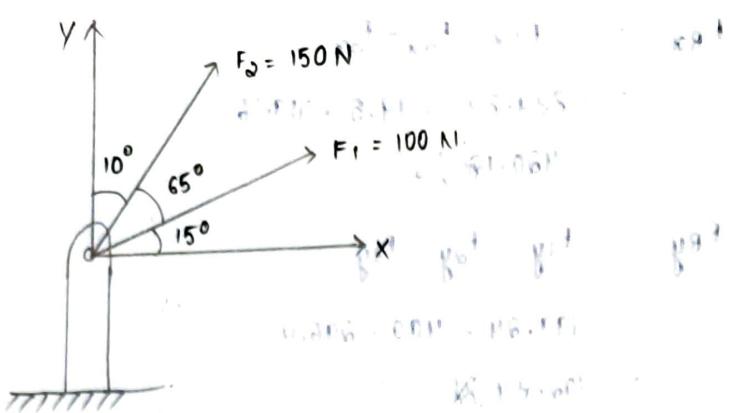
$$\begin{aligned}
 \frac{F_1}{\sin C} &= \frac{F_2}{\sin A} = \frac{F_R}{\sin B} \\
 \frac{420.18}{\sin 13.78^\circ} &= \frac{299.5}{\sin 103.78^\circ} = \frac{432.58}{\sin 45^\circ}
 \end{aligned}$$

$$\sin 13.78^\circ = 0.231$$

$$\sin 103.78^\circ = 0.961$$

$$420.18 = 299.5$$

$$8.48 = 8.48$$



Find the magnitude and directions of resultant force by using the cosine and sine rule

$$F_1 = 100 \text{ N} \quad F_2 = 150 \text{ N}$$

$$\angle b = 115^\circ$$

By using cosine rule,

$$F_R^2 = F_1^2 + F_2^2 - 2F_1F_2 \cos \theta$$

$$= 100^2 + 150^2 - 2(100)(150) \cos 115^\circ$$

$$= 10000 + 22500 - 2(15000)(-0.422)$$

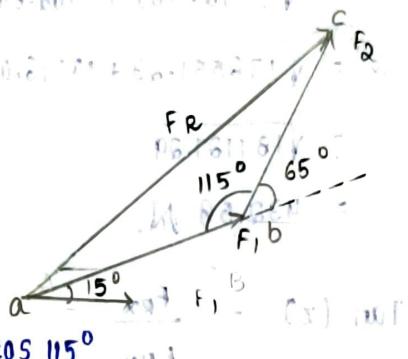
$$= 32500 + 12660$$

$$= 45160$$

$$F_R = \sqrt{45160}$$

$$= 212.508 \text{ N}$$

$$\approx 212.6 \text{ N}$$



By using Sine rule,

$$\frac{F_2}{\sin a} = \frac{F_R}{\sin b}$$

$$\frac{150}{\sin a} = \frac{212.6}{\sin 115^\circ}$$

$$\sin a = \frac{150}{212.6} \times \sin 115^\circ$$

$$= \frac{150}{212.6} \times 0.906 = \frac{135.9}{212.6} = 0.639$$

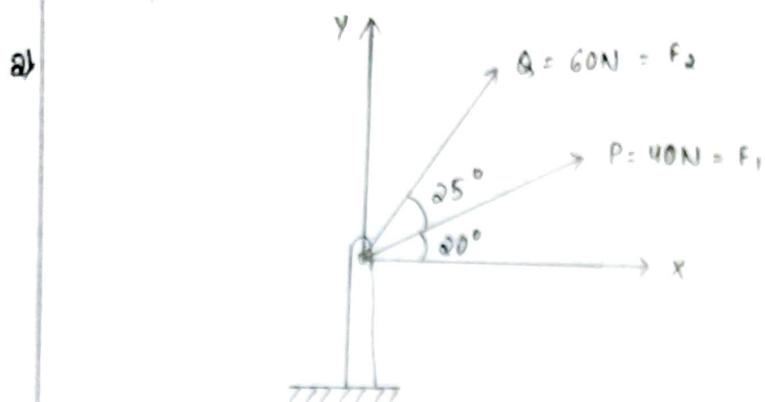
$$\sin a = 0.639$$

$$a = \sin^{-1}(0.639)$$

$$a = 39.111$$

$$a \approx 39.0$$

Resultant of F_R has direction of, $39.8^\circ + 15^\circ = 54.8^\circ$



Find the magnitude and directions of resultant force.

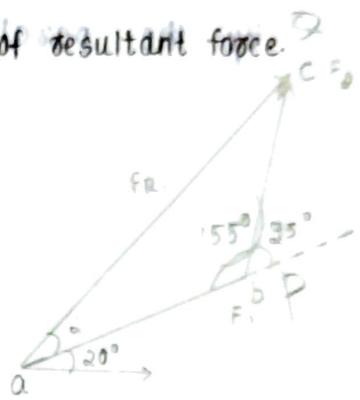
$$F_1 = 40 \text{ N} \quad F_2 = 60 \text{ N} \quad \angle b = 105^\circ$$

By using cosine rule,

$$\begin{aligned} F_R^2 &= 40^2 + 60^2 - 2(40)(60) \cos 105^\circ \\ &= 1600 + 3600 - 2(2400)(-0.90) \\ &= 5200 + 4320 \\ &= 9520 \end{aligned}$$

$$F_R = \sqrt{9520}$$

$$= 97.57 \text{ N} \quad 97.73 \text{ N}$$



By using sine rule,

$$\frac{F_2}{\sin a} = \frac{97.57}{\sin 155^\circ}$$

$$\sin a = \frac{F_2 \times \sin 155^\circ}{97.57}$$

$$\sin a = \frac{60 \times 0.422}{97.57} = 0.259$$

$$a = \sin^{-1}(0.259)$$

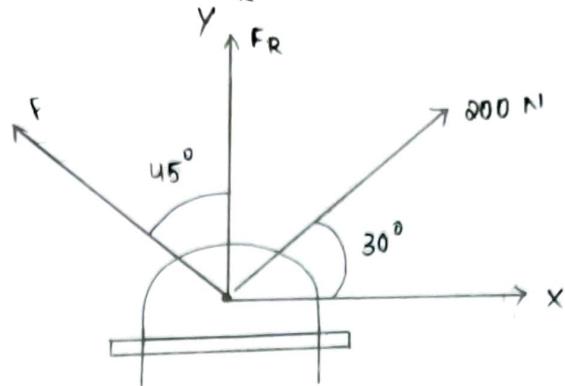
$$a = 15.0107^\circ$$

Resultant of F_R has direction of,

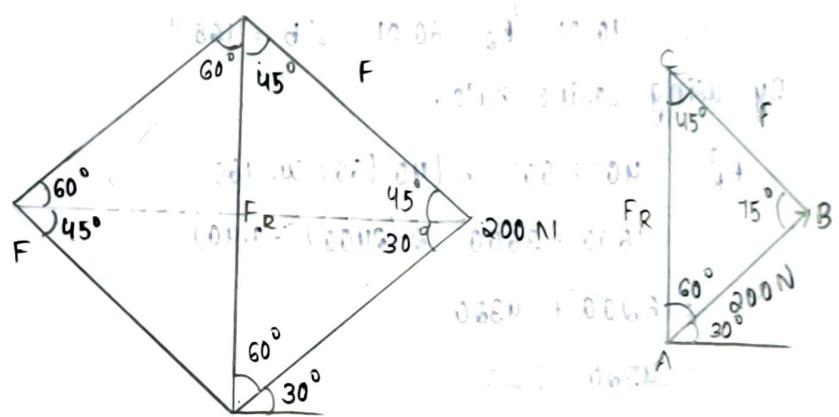
$$= 15.01^\circ + 20^\circ$$

$$= 35.01^\circ$$

- 3) Determine the magnitude of the component force F_x and magnitude of the resultant force F_R . If F_R is directed along the positive y-axis.



By the parallelogram law of forces,



$$\frac{F}{\sin 60^\circ} = \frac{200}{\sin 45^\circ} = \frac{F_R}{\sin 75^\circ}$$

$$F = \frac{200 \times \sin 60^\circ}{\sin 45^\circ}$$

$$F = \frac{200 \times \sqrt{3}/2}{1/\sqrt{2}}$$

$$F = 244.94$$

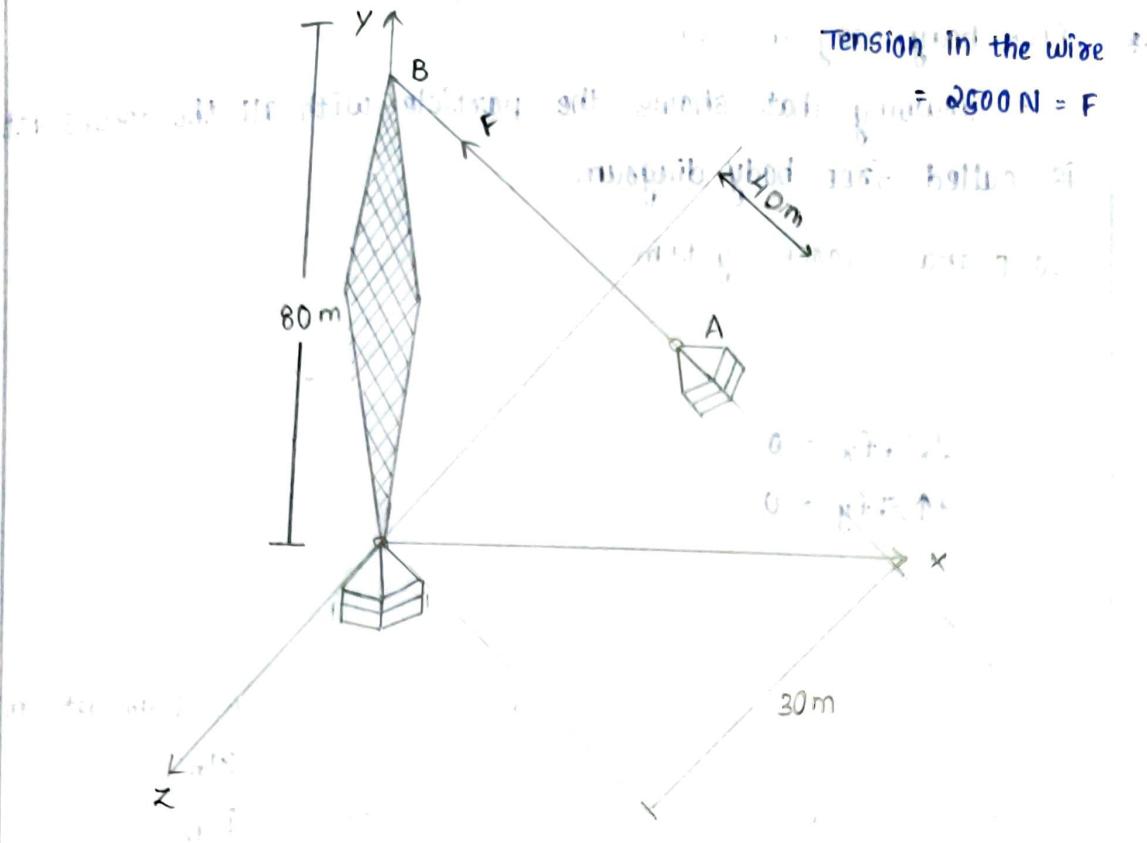
$$\frac{F_R}{\sin 45^\circ} = \frac{244.94}{\sin 60^\circ}$$

$$F_R = \frac{244.94 \times \sin 45^\circ}{\sin 60^\circ}$$

$$F_R = \frac{244.94 \times 1/\sqrt{2}}{\sqrt{3}/2}$$

$$F_R = 173.19$$

* Component of forces in space



Determine F_x, F_y, F_z at bolt A

$$\overrightarrow{AB} = -40\mathbf{i} + 80\mathbf{j} + 30\mathbf{k}$$

$$|AB| = \sqrt{40^2 + 80^2 + 30^2}$$

$$\begin{aligned} \text{Resultant magnitude} &= \sqrt{1600 + 6400 + 900} \\ &= \sqrt{8900} = 94.3 \end{aligned}$$

$$\lambda = \frac{\overrightarrow{AB}}{|AB|}$$

$$= \frac{-40\mathbf{i}}{94.3} + \frac{80\mathbf{j}}{94.3} + \frac{30\mathbf{k}}{94.3}$$

$$= -0.424\mathbf{i} + 0.848\mathbf{j} + 0.318\mathbf{k} \quad \text{— components of forces}$$

Actual force, $F = \lambda \text{ Tension in the wire}$

$$= 2500 (-0.424\mathbf{i} + 0.848\mathbf{j} + 0.318\mathbf{k})$$

$$= -1060\mathbf{i} + 2120\mathbf{j} + 795\mathbf{k}$$

$$F_x = -1060 \text{ N}$$

$$F_y = 2120 \text{ N}$$

$$F_z = 795 \text{ N}$$

Equilibrium of force system

free body diagram (FBD)

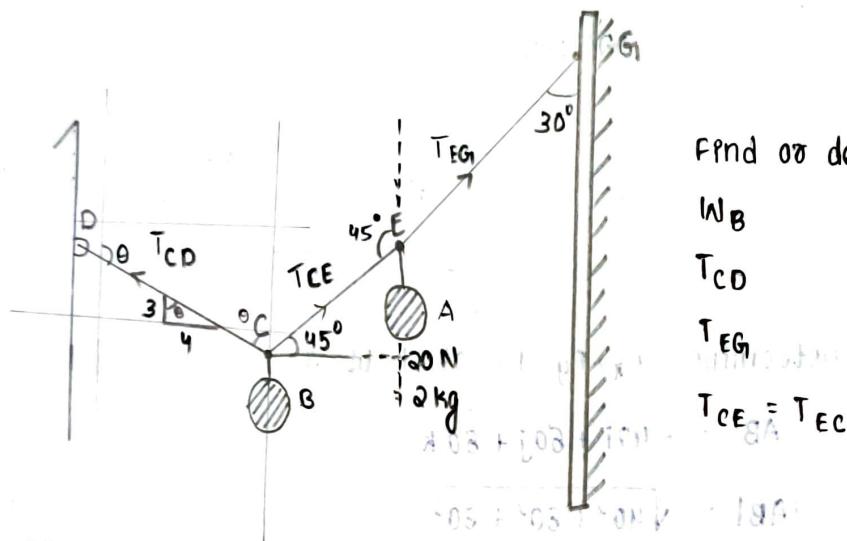
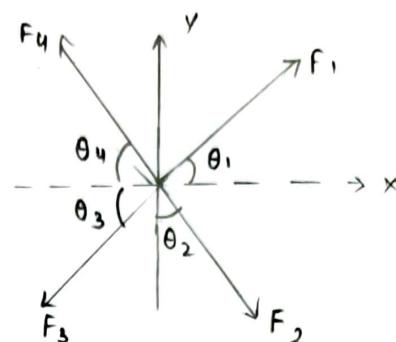
- A drawing that shows the particle with all the forces act on it is called free body diagram

Co-planar force system

All the forces of a body are in one plane.

$$\rightarrow \sum f_x = 0$$

$$\uparrow \sum f_y = 0$$



Find or determine,

W_B

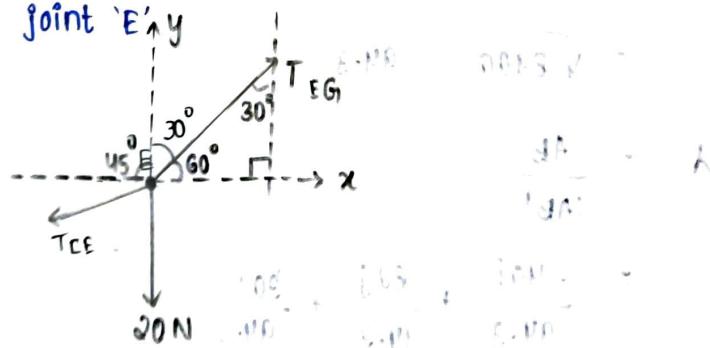
T_{CD}

T E G

$$T_{CE} = T_{EC}$$

Consider equilibrium of joint 'E'

FBD of joint 'E' \hat{y}



$$\sum F_x = 0 \Rightarrow T_{EG} \sin 30^\circ - T_{EC} \cos 45^\circ = 0$$

$$+\uparrow \sum F_y = 0 \Rightarrow T_{EG} \cos 30^\circ - T_{EC} [\sin 45^\circ - 20] = 0$$

$$T_{EG} \sin 30^\circ = T_{EC} \cos 45^\circ$$

$$T_{EG} \cos 30^\circ - T_{EO_1} \sin 30^\circ = 20$$

$$T_{EG} = \frac{20}{\cos 30^\circ - \sin 30^\circ}$$

$$= \frac{20}{\frac{\sqrt{3}}{2} - \frac{1}{2}}$$

$$T_{EG} = \frac{20 \times 2}{0.732} = \frac{40}{0.732} \quad \sqrt{3} = 0.732.$$

$$= 54.64 \text{ N}$$

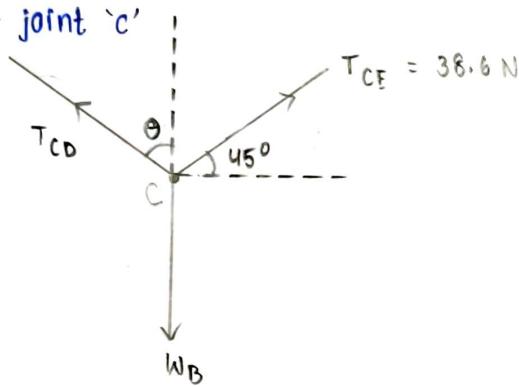
$$T_{EC} = \frac{T_{EG} \sin 30^\circ}{\cos 45^\circ}$$

$$= \frac{54.6 \times \frac{1}{2}}{\frac{1}{\sqrt{2}}}$$

$$= 38.6 \text{ N}$$

Consider equilibrium of joint 'c'

FBD of joint 'c'



$$\begin{aligned} \tan \theta &= 4/3 \\ \theta &= \tan^{-1}(4/3) \\ &= 53.1^\circ \end{aligned}$$

$$\therefore \sum F_x = 0 \Rightarrow T_{CE} \cos 45^\circ - T_{CD} \sin \theta = 0$$

$$T_{CE} \cos 45^\circ = T_{CD} \sin 53.1^\circ$$

$$T_{CD} = \frac{38.6 \times \frac{1}{\sqrt{2}}}{\sin 53.1^\circ}$$

$$T_{CD} = \frac{4.144 \times 0.8909}{0.799}$$

$$= 34.15 \approx 34.2 \text{ N}$$

$$+ \uparrow \sum F_y = 0 \Rightarrow T_{CD} \cos 53.1 + 38.6 \sin 45^\circ - W_B = 0$$

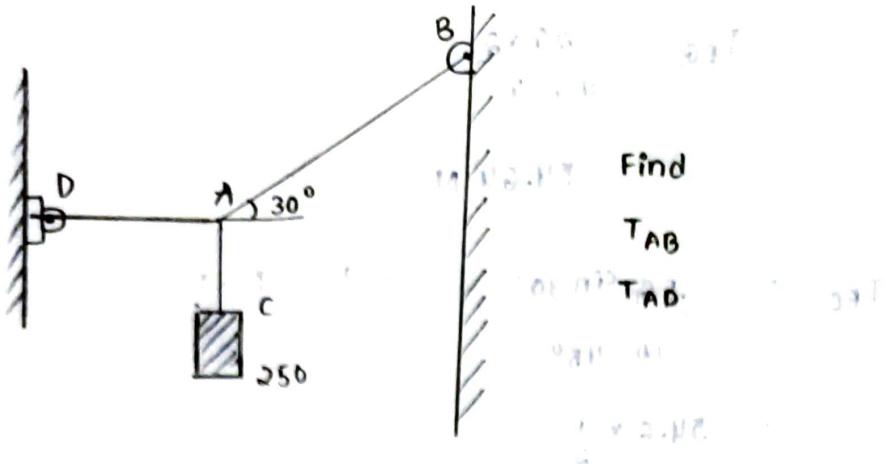
$$34.2 \cos 53.1 + 38.6 \sin 45^\circ = W_B$$

$$W_B = 34.2 \times 0.600 + 38.6 \times \frac{1}{\sqrt{2}}$$

$$= 20.52 + 27.29$$

$$= 47.81$$

14.54



$$\rightarrow F_x = 0$$

$$T_{AB} \cos 30^\circ - T_{AD} \sin 30^\circ = 0 \quad \text{Eq 1}$$

$\uparrow \rightarrow F_y = 0 \rightarrow$ Resultant force in vertical upwards direction

$$T_{AB} \sin 60^\circ - T_{AD} \cos 60^\circ - 250 = 0 \quad \text{Eq 2}$$

$$\textcircled{1} \Rightarrow T_{AB} \cos 30^\circ = T_{AD} \sin 30^\circ$$

$$\textcircled{2} \Rightarrow T_{AB} \sin 60^\circ = T_{AD} \cos 60^\circ + 250$$

$$\textcircled{1} \Rightarrow T_{AB} \left(\frac{\sqrt{3}}{2}\right) = T_{AD} \left(\frac{1}{2}\right)$$

$$\textcircled{2} \Rightarrow T_{AB} \left(\frac{\sqrt{3}}{2}\right) - T_{AD} \left(\frac{1}{2}\right) - 250 = 0$$

$$\textcircled{1} + \textcircled{2} \Rightarrow T_{AB} \left(\frac{\sqrt{3}}{2}\right) + T_{AD} \left(\frac{1}{2}\right) - 250 = 0$$

$$\begin{aligned} & \text{Let } T_{AB} = 500 \text{ N} \\ & \therefore T_{AD} = \frac{500 + 250}{2} \times \frac{\sqrt{3}}{2} \\ & \therefore T_{AD} = 250 \times \frac{\sqrt{3}}{2} \end{aligned}$$

$$\begin{aligned} & \text{Let } T_{AD} = 1500 \text{ N} \\ & \therefore T_{AB} = \frac{1500 + 250}{2} \times \frac{\sqrt{3}}{2} \\ & \therefore T_{AB} = \frac{1750}{2} \times \frac{\sqrt{3}}{2} \end{aligned}$$

$$\textcircled{1} \Rightarrow T_{AB} \left(\frac{\sqrt{3}}{2}\right) - T_{AD} \left(\frac{1}{2}\right) + 0 = 0 \quad \text{Eq 3}$$

$$\therefore \frac{1750\sqrt{3}}{4} + T_{AD} \left(\frac{1}{2}\right) - T_{AD} \left(\frac{1}{2}\right) = 0$$

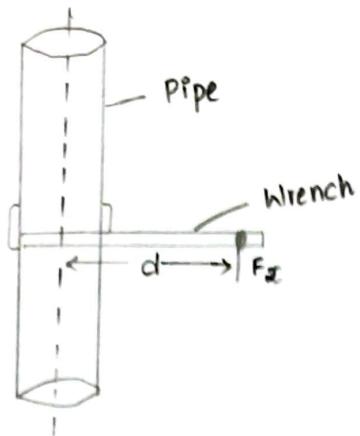
$$\therefore 437.5 + 750 = 1187.5 \text{ N}$$

$$\textcircled{1} \Rightarrow T_{AB} \left(\frac{\sqrt{3}}{2}\right) - \frac{1500}{2} = 0$$

$$T_{AB} = \frac{1500}{\sqrt{3}}$$

$$T_{AB} = 866.02$$

* Moment of forces



$$\text{Moment} = \text{Force} \times \text{distance} (\text{fr})$$

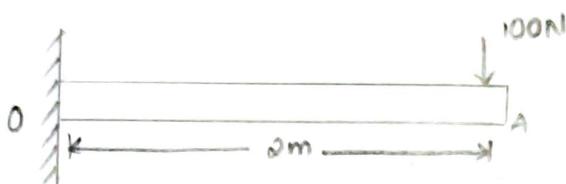
\downarrow
N - m

\downarrow
N

$$G + M \quad (-M)$$

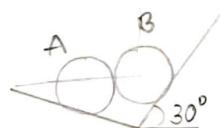
\downarrow
A.C.W

\downarrow
C.m.t



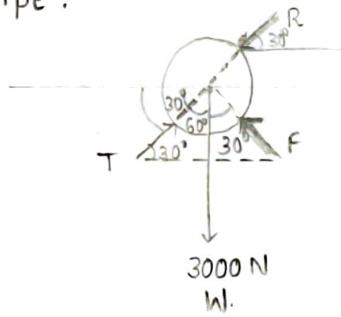
$$M_O = -100 \times 2 = -200 \text{ N} \cdot \text{m}$$

24/02/2018
Two smooth pipes each having a mass of 300kg are supported by forks of tractor. Draw free body diagram for each pipe and both pipe together.

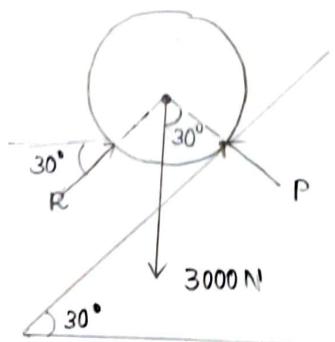


$$300 \text{ kg} = 300 \times 10 \text{ N} \\ = 3000 \text{ N}$$

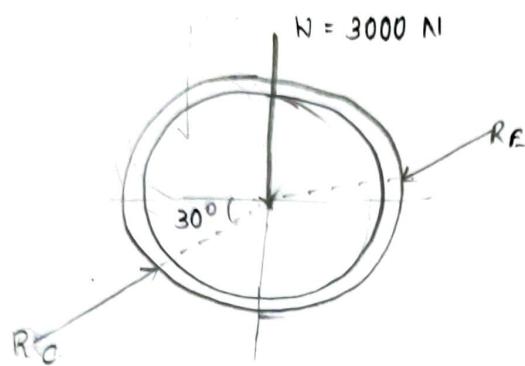
FBD of 'A' pipe:



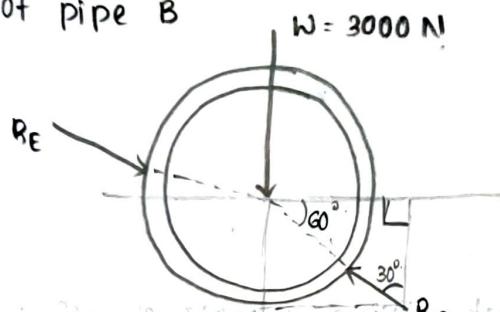
FBD of 'B' pipe:



FBD of pipe A

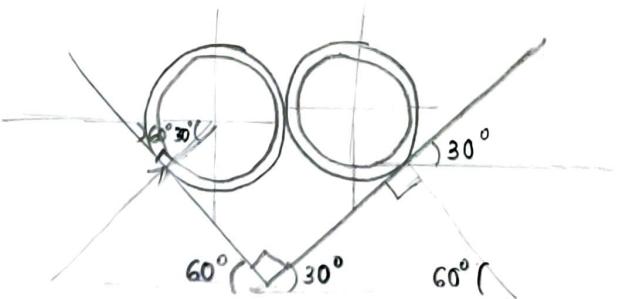


FBD of pipe B

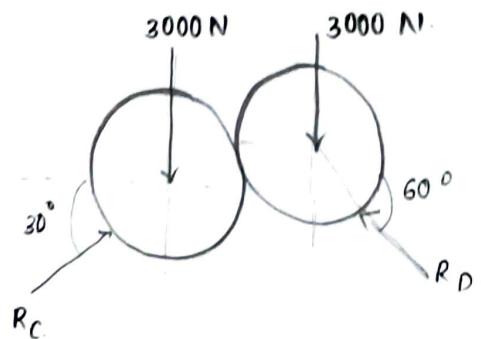


Whenever we need to draw the FBD of balls or pipes it is good to draw local x-axis & y-axis passing through the centre of circle.

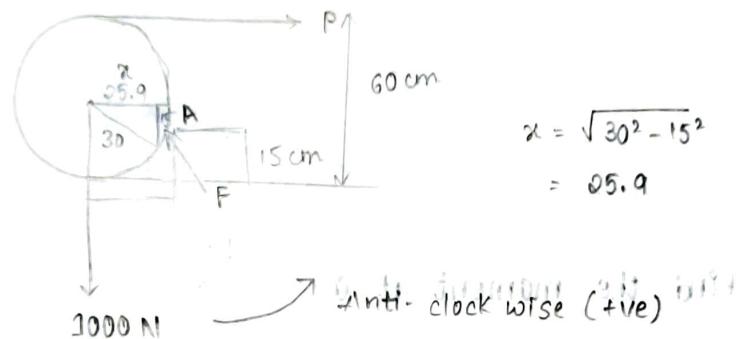
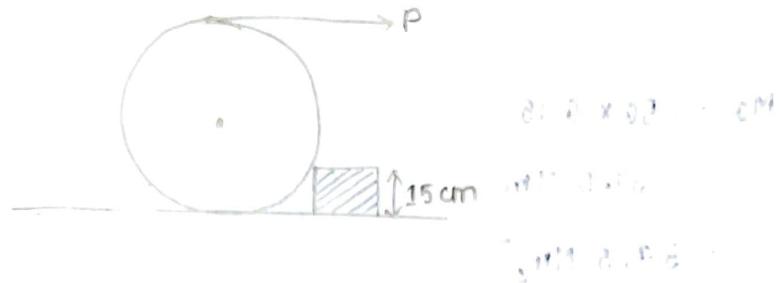
Angles :



Both pipes FBD



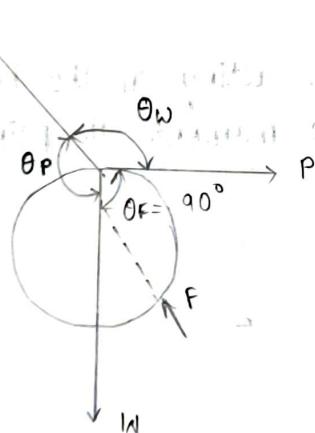
a) Find the minimum value of P which will just make wheel move. Wheel diameter is 60 cm. P is the force applied here. Weight of the wheel is 1000 N.



$$\text{Momentum at } A \Rightarrow P \times (60 - 15) + 1000 \times 25.9 = 0$$

$$P \times 45 = 1000 \times 25.9$$

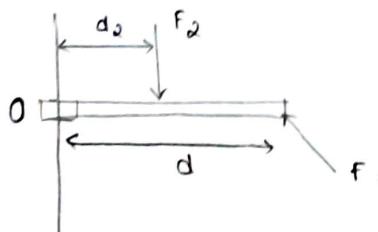
$$P = \frac{1000 \times 25.9}{45} = 575.6 \text{ N}$$



Lami's Theorem

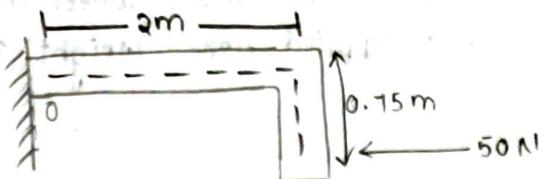
$$\frac{P}{\sin \theta_P} = \frac{F}{\sin \theta_F} = \frac{W}{\sin \theta_W}$$

* Moment of forces / couple

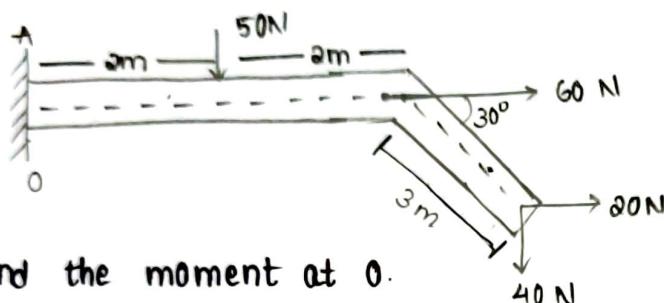


$$M_O = F_2 \times d_2 + F_1 \times d_1$$

$$= \sum F_i d_i$$

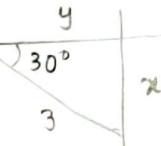


$$\begin{aligned}
 M_O &= -50 \times 0.75 \\
 &= -37.5 \text{ Nm} \\
 &= 37.5 \text{ Nm}
 \end{aligned}$$



* Find the moment at O.

$$\begin{aligned}
 M_O &= -50 \times 2 + 60 \times 0 + 20 \times 1.5 + 40 \times (4 \times 0.59) \text{ N.m} = 3 \cos 30 \\
 &= -100 + 0 + 30 - 40 \times (6.59) \\
 &= -100 + 30 - 263.6 \\
 &= 334 \text{ N.m}
 \end{aligned}$$



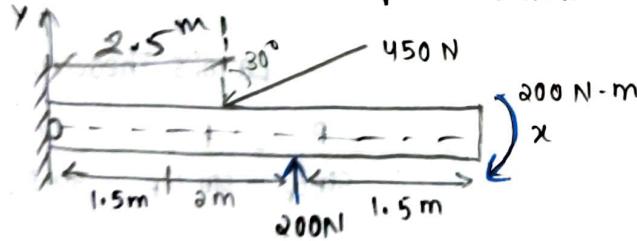
$$\cos 30 = \frac{y}{3}$$

$$\sin 30 = \frac{x}{3}$$

$$x = 3 \sin 30$$

$$= \frac{3}{2} = 1.5 \text{ m}$$

- * Replace the loading system acting on the beam by equivalent resultant force and couple moment at point 'O'.



$$\begin{aligned}
 M_O &= 450 \cos 30 \\
 &= 450 \times 0.866
 \end{aligned}$$

$$450 \times 0.866$$

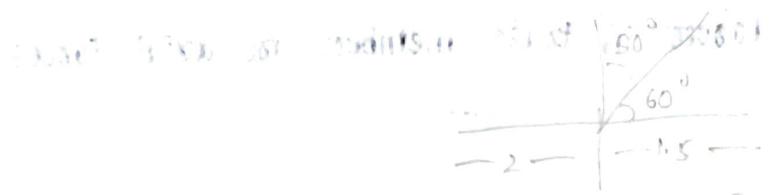
$$450 \cos 30$$

$$450 \times 0.866$$

$$F_x =$$

$$F_y =$$

<math display="block



$$y = 0.75$$

$$2.12^{\circ}$$

$$\sin 60^{\circ} = \frac{y}{1.5}$$

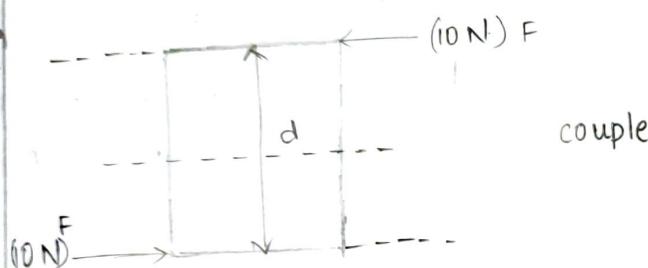
$$y = 1.29$$

$$\cos 60^{\circ} = \frac{x}{1.5}$$

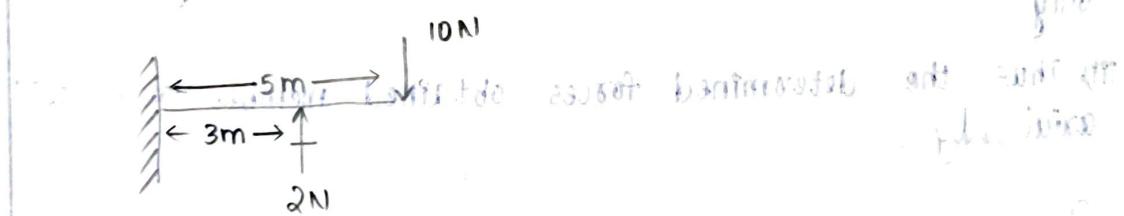
$$x = 0.75$$

angle in bottom

principle to bottom



Moment produced by this couple = $F \times d$
 $= 10 \times d \text{ KNm}$



$$\begin{aligned}\text{Moment} &= -10 \times 5 + 2 \times 3 \\ &= -50 + 6 \\ &= -44 \text{ N.m}\end{aligned}$$

Tusses are of two types :

Roof tusses - used buildings, industries.

used for - Bridge tusses

Deck type

Through type



Through type.

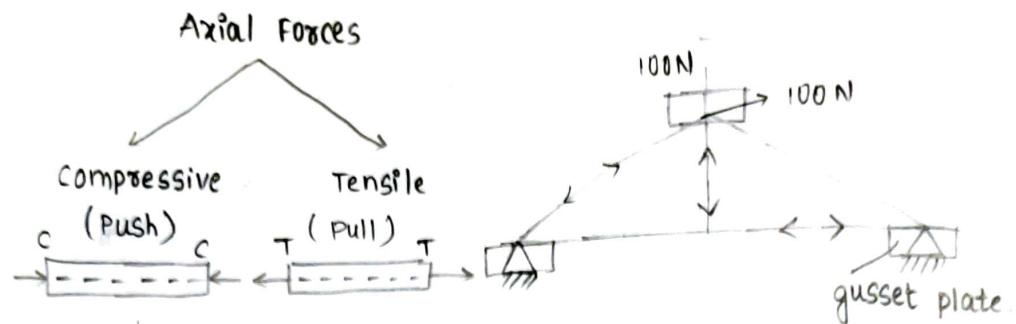
⇒ Provide shelter downward

Deck type.

⇒ not safer

* Roof Trusses : (Analysis) : finding member forces

Forces in truss members are axial forces. Centroid of the member → any force applied along the



Finding member forces

Method of joints

(all member forces)

It is the method to find out all member force.

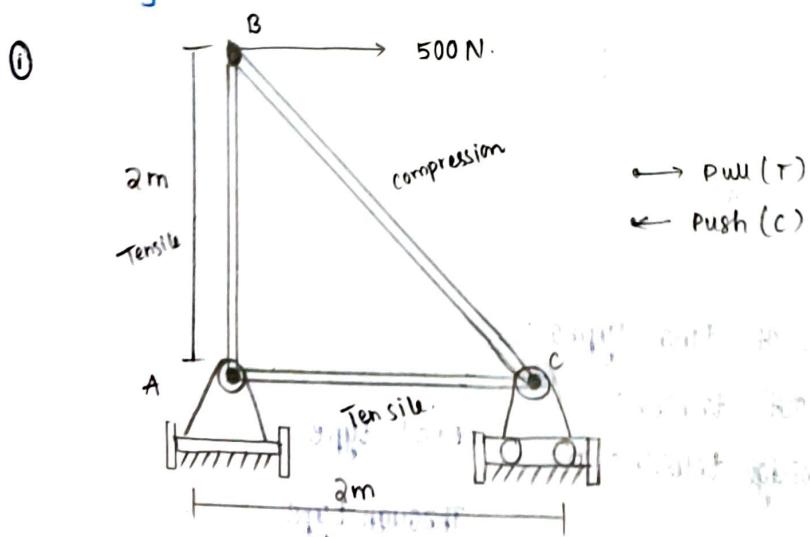
Method of sections

(particular member force)

It is the method to find out particular member force interested only.

* Assumptions to be made in the analysis of trusses:

- i) All the joints of truss are connected by pin
- ii) All the external loads must be applied through the joints only.
- iii) Thus the determined forces / obtained member forces will be axial only.



Find the member forces.

Unknowns :

Reactions from supporting Member forces.

$$F_{AB}, F_{BC}, F_{AC}$$

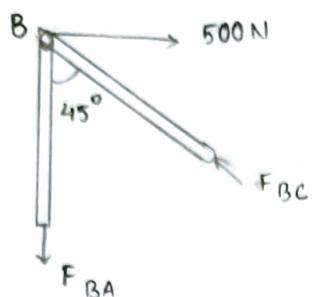
Restrained

$$\begin{cases} A_x \\ A_y \\ C_y \end{cases}$$

Using method of joints,

take joint 'B' equilibrium.

Draw FBD of joint B.



direction can be any side but
the answer is -ve, it should be
wrong assumption

$$\rightarrow \sum F_x = 0$$

$$-F_{BC} \sin 45^\circ + 500 = 0$$

$$F_{BC} = \frac{500}{\sin 45^\circ}$$

$$= 500 \times \sqrt{2}$$

$$= 707.110 \text{ N}$$

$$+\uparrow \sum F_y = 0$$

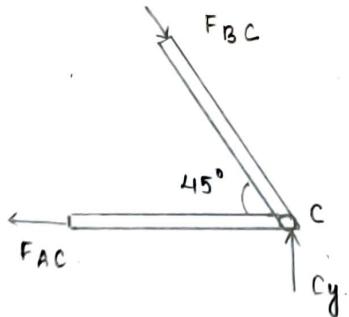
$$F_{BC} \cos 45^\circ - F_{BA} = 0$$

$$F_{BA} = 707 \times \cos 45^\circ$$

$$= 499.9 \approx 500 \text{ N}$$

taking joint 'C'

Draw FBD of joint C



$$\sum F_x = 0$$

$$-F_{AC} + 707 \cos 45^\circ = 0$$

$$F_{AC} = 707 \cos 45^\circ$$

$$= 500 \text{ N}$$

$$\sum F_y = 0$$

$$-F_{BC} \sin 45^\circ + C_y = 0$$

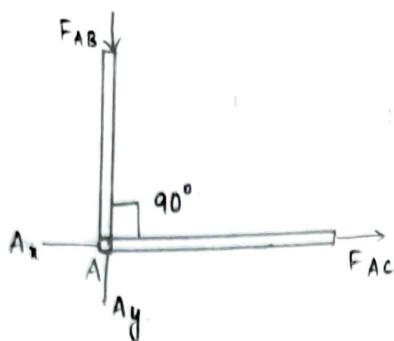
$$C_y = F_{BC} \sin 45^\circ$$

$$= 707 \cdot 1/\sqrt{2}$$

$$= 500 \text{ N}$$

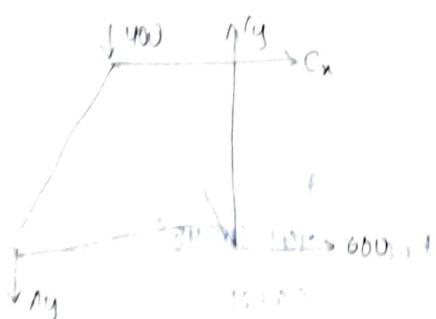
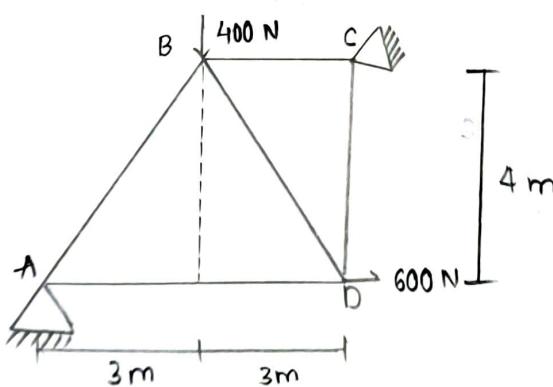
taking joint A,

draw FBD of joint A



Point	Force	Nature
AB	500 N	Tensile
BC	770 N	Compression
AC	500 N	Tensile

②



$$-A_x + C_x + 600 = 0$$

$$A_y + C_y - 400 = 0$$

$$\sum M_A = 0$$

$$-400 \times 3 + C_y \times 6 - C_x \times 4 = 0$$

$$-1200 + 6C_y - 4C_x = 0$$

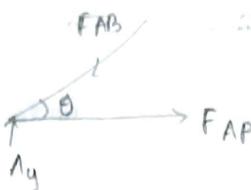
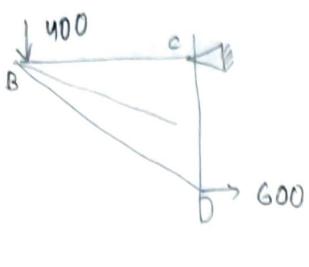
$$A_x = 0 \quad C_x = -600$$

$$1200 + 6C_y - 4(-600) = 0$$

$$1200 + 6C_y + 2400 = 0$$

$$6C_y = -3600 \text{ N}$$

$$C_y = -600 \text{ N}$$



$$\tan \theta = 4/3$$

$$\theta = 53.1^\circ$$

$$\sum F_x = 0$$

$$\sum F_y = 0$$

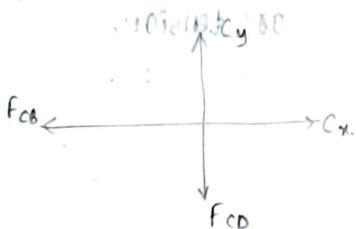
$$F_{AB} - F_{AB} \cos 53.1^\circ = 0$$

$$Ay - F_{AB} \sin 53.1^\circ = 0$$

$$F_{AB} = 150 \cdot 16 \text{ N}$$

$$F_{AB} = 250 \text{ N}$$

At point C, $\sum F_x = 0$



$$\sum F_x = 0$$

$$-Cx - F_{CB} = 0$$

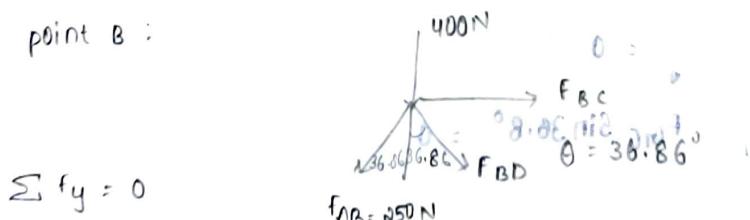
$$F_{CB} = -600 \text{ N}$$

$$\sum F_y = 0$$

$$Cy - F_{CD} = 0$$

$$F_{CD} = 600 \text{ N}$$

At point B:



$$\sum F_y = 0$$

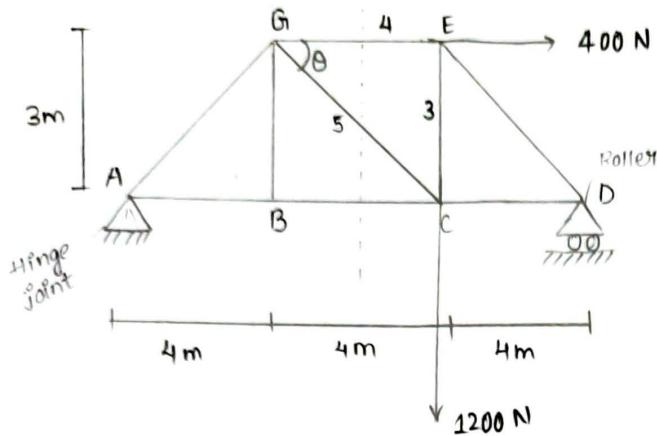
$$-400 - 150 \cos 53.1^\circ \cos 36.86^\circ + F_{BD} \cos 36.86^\circ = 0 \Rightarrow X$$

$$F_{BD} = 150 \text{ N}$$

numbers	force	Nature
AB	250 N	compression
BC	-600 N	tensile
CD	600 N	tensile
BD	150 N	compression

* Method of sections

- ⇒ This method is used to find member forces in few members.
- ⇒ This method is based on the principle that if a body is in equilibrium any point of the body is in equilibrium.

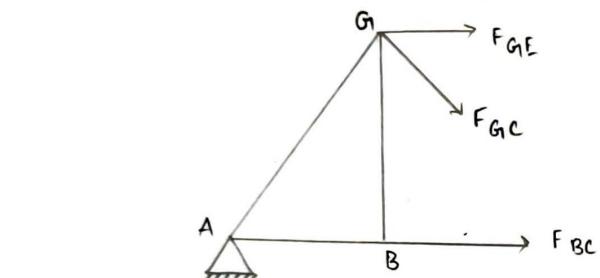


*** When section is cut it should not pass more than 3 members.

Unknowns:

$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M_A = 0 \end{cases} \begin{array}{l} A_x \\ A_y \\ D_y \end{array}$$

Determine the forces in members GE, GC, BC. Indicate whether the members are in compression or tension. Supports



Hinge joint - 2 Reaction - A_x, A_y
Roller - 1 Reaction - D_y
FP & Rd - 3 Reaction - E_x, E_y, M_B

part body in equilibrium
Equilibrium of joint B,

$$\sum F_x = 0$$

Draw FBD of joint B.

$$- A_x + F_{BC} + F_{GE} + F_{GC} \cos 36.8^\circ = 0 \quad \text{--- ①}$$

$$\sum F_y = 0$$

$$A_y - F_{GC} \sin 36.8^\circ = 0 \quad \text{--- ②}$$

$$\sum M_A = 0$$

$$- F_{GC} \times 3 - F_{GC} \cos 36.8^\circ \times 3 - F_{GC} \sin 36.8^\circ \times 4 = 0 \quad \text{--- ③}$$

For total diagram,

$$F_{BC} \times 3 - A_y \times 4 - A_x \times 3 = 0$$

$$* \sum F_x = 0$$

$$* \sum M_A = 0$$

$$- A_x + 400 = 0$$

$$- 400 \times 3 - 1200 \times 8 + D_y \times 12 = 0$$

$$A_x = 400 \text{ N}$$

$$- 1200 - 9600 + D_y \times 12 = 0$$

$$* \sum F_y = 0$$

$$A_y + D_y - 1200 = 0$$

$$A_y + 900 - 1200 = 0$$

$$A_y = 300 \text{ N}$$

$$\cos \theta = 4/5$$

$$\theta = \cos^{-1}(4/5)$$

$$= 36.8^\circ$$

$$F_{GC} \cos 36.8^\circ$$

$$F_{GC} \sin 36.8^\circ$$

$$12 D_y = 10800$$

$$D_y = 900 \text{ N}$$

From equation ②,

$$300 - F_{G,C} \sin 36^\circ 8' = 0$$

$$F_{GIC} \sin 36^\circ 8' = 300$$

$$F_{GIC} = \frac{300}{0.599}$$

= 500,8 N

from equation ③,

$$F_{BC} \times 3 - 300 \times 4 - 400 \times 3 = 0$$

$$3F_{BC} - 1200 - 1200 = 0$$

$$F_{BC} = 800$$

From equation ①,

$$-400 + F_{BC} + F_{GE} + F_{GC} \cos 36^\circ 8' = 0$$

$$-400 + 800 + F_{GE} + 500 \cdot 8 \times 0.807 = 0$$

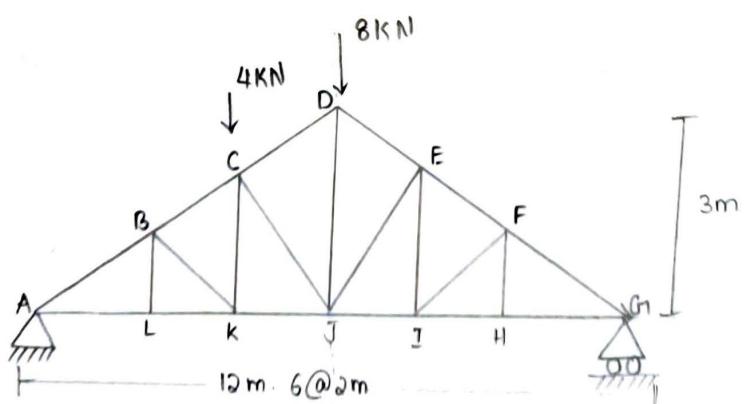
$$400 + F_{GE} + 404.1456 = 0$$

$$F_{GE} + 804.1456 = 0$$

$$F_{G,E} = -804.1456 \text{ N}$$

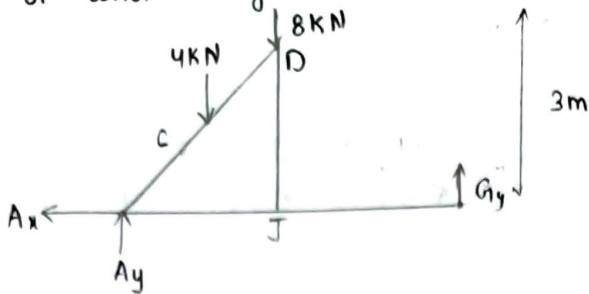
Member	Force	Nature
GC	500.8 N	Tension
GE	804.1 N	compression
BC	800 N	Tension

2) Find the member forces in a truss BC, CK and KJ



Indicate whether the members are in compression or tension.

FBD of whole body,



$$\sum F_x = 0 \Rightarrow -A_x = 0 \\ A_x = 0$$

$$\sum F_y = 0 \Rightarrow A_y + G_y - 4000 - 8000 = 0$$

$$A_y + G_y = 12000 - \textcircled{1} \quad \frac{y_1}{4} = \frac{3}{6} \Rightarrow y_1 = 2m$$

$$\sum M_A = 0$$

$$\Rightarrow G_y \times 12 - 8000 \times 6 - 4000 \times 4 = 0$$

$$G_y \times 12 - 48000 - 16000 = 0$$

$$12G_y - 64000 = 0$$

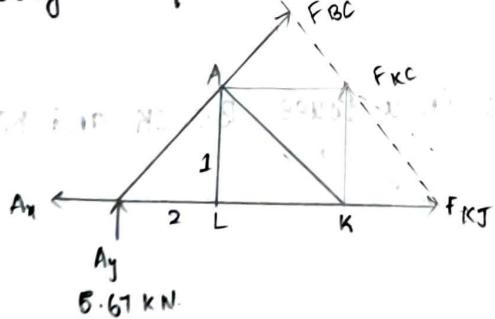
$$G_y = \frac{64000}{12}$$

$$= 5333.33 N$$

From $\textcircled{1}$, $A_y = 12000 - 5333.33$
 $= 6666.67 N$

$$A_y = 6666.67 \downarrow N \Rightarrow 6.67 k \downarrow N$$

Part body in equilibrium



$$\tan \theta = \frac{1}{2} \quad \theta = \tan^{-1}(\frac{1}{2}) \\ = 26.56^\circ$$

$$\sum F_x = 0 \Rightarrow F_{KJ} + F_{BC} \cos 26.56^\circ = 0 \quad \text{--- } \textcircled{1}$$

$$\sum F_y = 0 \Rightarrow F_{CK} + F_{BC} \sin 26.56^\circ + 6.67 = 0 \quad \text{--- } \textcircled{2}$$

$$\sum M_A = 0 \Rightarrow F_{KJ} \times 0 + F_{BC} \times 0 + F_{CK} \times 4 = 0 \quad \text{--- } \textcircled{3}$$

$$\text{from } \textcircled{3} \Rightarrow 0 + F_{BC} \times 0.452 + 6.67 = 0$$

$$F_{BC} = -6.67$$

$$F_{BC} = \frac{-6.67}{0.452}$$

$$= -14.917 KN$$

$$\text{From } ① \Rightarrow F_{KJ} + 16.46 = 0$$

$$F_{KJ} + F_{BC} \times 0.894 = 0$$

$$F_{KJ} = 14.917 \times 0.894$$

$$= 13.33 \text{ KN}$$

Member	Force	Nature
BC	-14.917 K	Compressive
CK	0	Tensile
KJ	13.33 K	Tensile