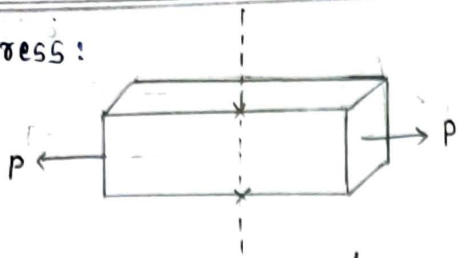
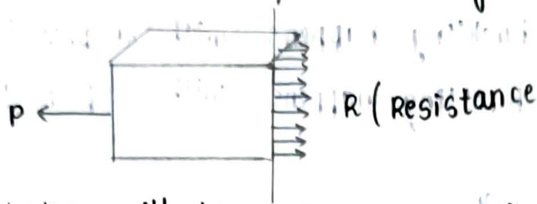
31/10/2019

Simple stress and strains***

* **Stress :**



Stress : The resistance / force acting per unit area



All bodies will have some amount of deformation.

★ **Definition :** — Independent

The force of resistance offered by a body against deformation is called stress.

$$\text{Stress } (\sigma) = \frac{R}{A} = \frac{P}{A}$$

A = cross - sectional area ; P = force or load

⇒ External force is called load.

* **Strain :** The ratio of change in length to the original length is called strain.

$$\text{Strain } (\varepsilon) = \frac{\text{Change in length}}{\text{Original Length}}$$

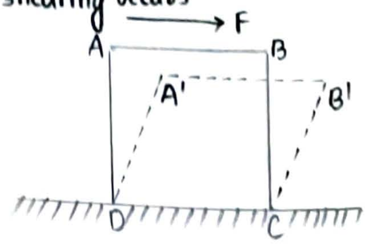
$$\varepsilon = \frac{\Delta l}{l} \text{ mm/mm}$$

⇒ No unit for strain.

⇒ There is no stress without strain. Stress exists from strain which can be measured.

* **Types of stress :**

1) Tensile stress
2) compressive stress
3) Shear stress or tangential stress — Because of twisting shearing occurs



Bending stress : Bending causes tensile and compression stress

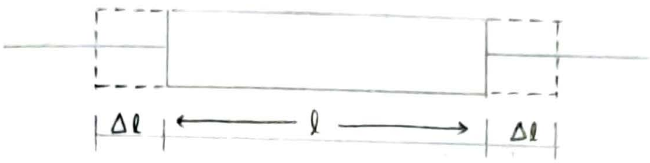
Torsion stress : It causes by shearing stress

Tensile, compression and shearing are the primary stress.

1) Tensile stress : Elongated body gets deformed.

$$\sigma = \frac{R}{A} = \frac{P}{A}$$

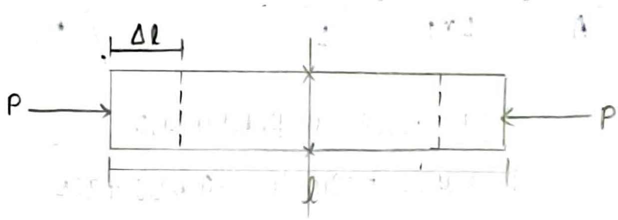
* Tensile strain :



$$\text{Tensile strain} = \frac{\text{Increased length}}{\text{Original length}}$$

$$\varepsilon = \frac{\Delta l}{l}$$

2) Compressive stress :



$R > P$

$$\text{compressive stress } (\sigma) = \frac{R}{A} = \frac{P}{A}$$

$$\text{Compressive strain} = \frac{\text{decreased length}}{\text{original length}}$$

$$\varepsilon = \frac{-\delta l}{l}$$

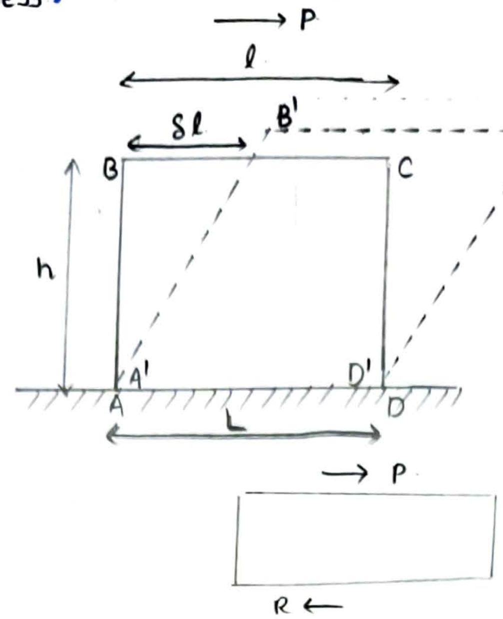
Units :
Stress → N/m^2 or pascal
Strain → No units or mm/mm or mm/m or m/m ⇒ $\frac{\text{Length}}{\text{Length}}$

8/11/2025

Stress - dependent quantity
Strain - independent quantity

$$\text{Stress} \propto \frac{1}{\text{cross-section \& area}}$$

3) Shear stress :



Shear stress (τ) $= \dfrac{\text{Shear resistance}}{\text{Shear area}}$

Thou.

Shear area = L × 1 – unit width inside the surface

Shear stress $= \dfrac{R}{A} = \dfrac{P}{L \times 1} = \dfrac{P}{L} =$ N/mm² / m

L → in the direction of shear force.

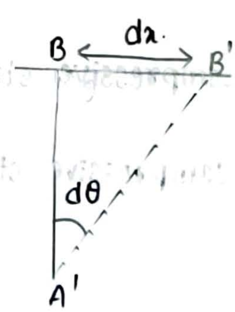
Shear strain (γ) $= \dfrac{\text{Transverse displacement}}{\text{Distance from the lower face}}$

$= \dfrac{\delta l}{h}$

$\gamma = \dfrac{dx}{x}$

$\tan\phi$ / $\tan\theta = \dfrac{dx}{x}$

$\theta = \dfrac{dx}{x} = \gamma$



* Hooke's law:

P - point of proportionality

e - point of yielding

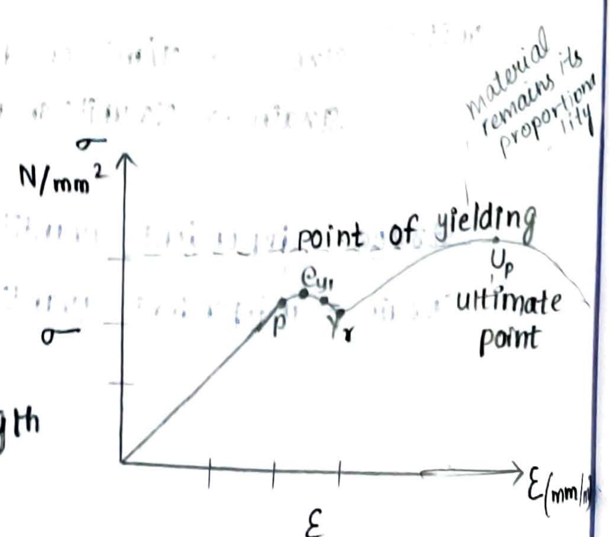
Hooke's law is not obeyed

Y_1 - upper yielding point

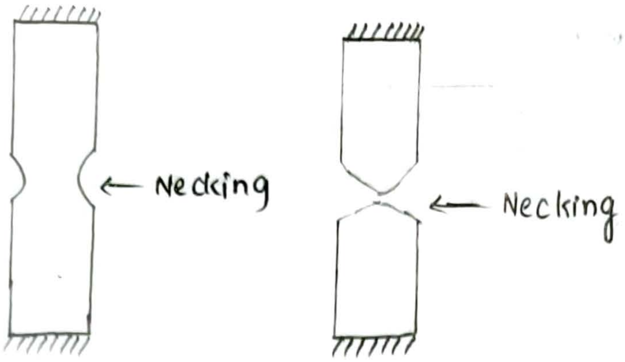
we can't get back original length

Y_2 - power yielding point

material can't take any load



Beyond up continue the elongation the material would break

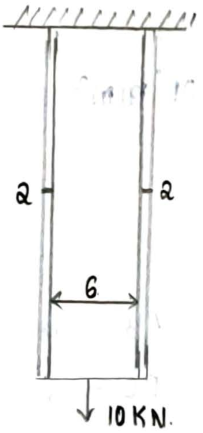


Break point (where upper part and lower part are separated)

04/11/2019

1) A pipe of 10 mm outer diameter and 2 mm the subjected to axial - tensive of 10 KN. Find the stress induced in the pipe.

a)



$A = \frac{\pi D^4}{4}$ (If inside is full)

$$A = \frac{\pi}{4} D_o^2 - \frac{\pi}{4} D_i^2$$

$$= \frac{\pi}{4} (D_o^2 - D_i^2)$$

$$\text{Stress} = \frac{\text{Load}}{\text{Area}}$$

$$= \frac{10 \times 10^3 \text{ N}}{\frac{\pi}{4} (10^2 - 6^2)} = \frac{10^4}{\frac{\pi}{4} (100 - 36)}$$

$$= \frac{10^4}{\frac{\pi}{4} \times 64}$$

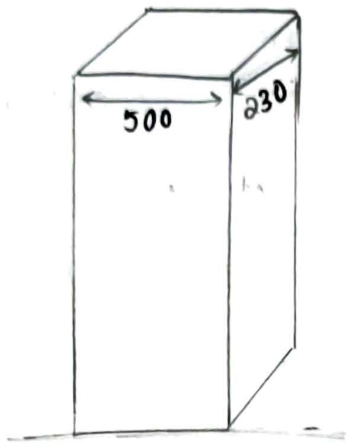
$$= \frac{10^4}{16 \pi}$$

$$= \frac{10000}{16 \times 3.14}$$

$$= \frac{10000}{50.24}$$

$$= 199.04 \text{ N/mm}^2$$

2) A column of 230 x 500 mm, cross-section is subjected to a compression force of 100 kN. Find the compressive stress induced in the column.

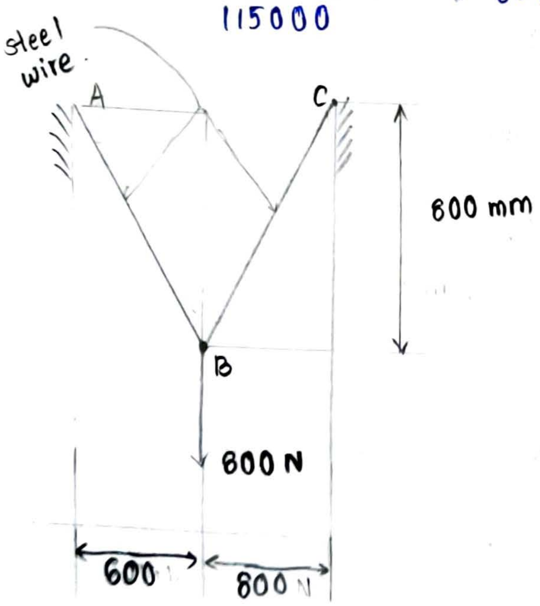


$$\text{Stress} = \frac{\text{load}}{\text{Area}}$$

$$= \frac{100\ \text{kN}}{500 \times 230}$$

$$= \frac{100 \times 10^3}{115000} = 0.8695\ \text{N/mm}^2$$

3)



AB = 2.68 mm ϕ

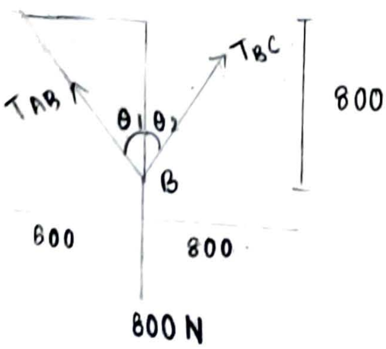
BC = 2.52 mm ϕ

Find force and stress.

$\tan\theta_2 = \frac{800}{800}$

$\tan\theta_2 = 1$

$\theta_2 = 45°$



$\tan\theta_1 = \frac{600}{800}$

$\tan\theta_1 = \frac{3}{4}$

$\theta_1 = \tan^{-1}\left(\frac{3}{4}\right)$

$\theta_1 = 36.86$

$\Sigma F_x = 0 \Rightarrow -T_A \sin 36.86° + T_C \sin 45° = 0$

$-T_A(0.5998) + T_C(0.7071) = 0$

$$T_A (0.5998) = T_C (0.7071)$$

$$T_A = \frac{T_C (0.7071)}{(0.5998)}$$

$$T_A = 1.1788\, T_C$$

$$\Sigma F_y = 0 \Rightarrow T_A \cos 36.86 - 800 + T_C \cos 45^\circ = 0$$

$$(0.8001) T_C \times 0.8001 - 800 + T_C (0.7071) = 0$$

$$(1.1788) T_C \times 0.8001 + T_C (0.7071) = 800$$

$$T_C (0.9431 + 0.7071) = 800$$

$$T_C = \frac{800}{1.6502}$$

$$T_C = 484.789 \text{ N}$$

$$\therefore T_A = 1.1788 \times 484.789$$

$$= 571.469 \text{ N}$$

$$\text{Stress at A} = \frac{\text{Load at A}}{\text{Area}} \qquad \text{Area} = \frac{\pi}{4}(D)^2$$

$$= \frac{571.469}{\frac{\pi}{4}(2.68)^2}$$

$$= \frac{571.469}{5.638} = 101.36 \text{ N/mm}^2$$

$$\text{Stress at B} = \frac{\text{Load at B}}{\text{Area}}$$

$$= \frac{484.789}{\frac{\pi}{4}(2.52)^2}$$

$$= \frac{484.789}{4.985} = 97.249 \text{ N/mm}^2$$

5/11/2019

$\frac{\text{Stress}}{\text{Strain}}$ = Young's Modulus

$\boxed{\frac{\sigma}{\varepsilon} = E}$

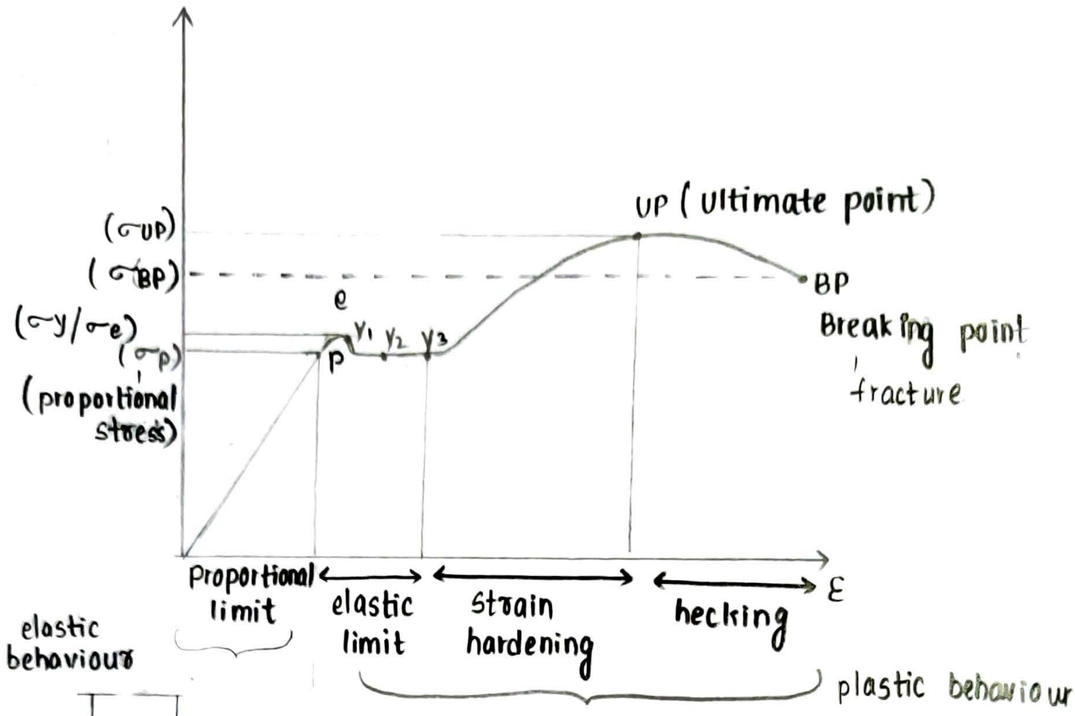
1 D → Rubber band increase in length only

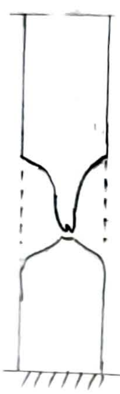
2 D → Thick Rubber band increase in length and decrease in thickness

3 D → Sponge → increase in x direction
decrease in y, z direction

$\boxed{\frac{\sigma}{\varepsilon} = E\gamma}$ poison's ratio

1 direction increase, other 2 decreases or Vice versa.



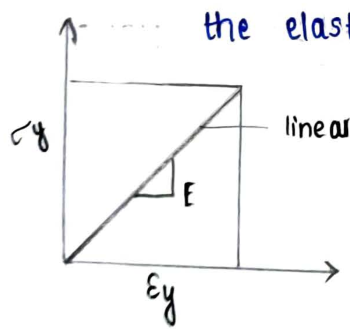


★ Materials:

1) Ductile material :
will have reducing cross-section

2) Frugile material :
will not have reducing cross-section.
Prefectly horizontal.

* Hooke's law : Strain is directly proportional to the stress up to the elastic limit.



$\sigma \propto \varepsilon$

$\sigma = E\varepsilon$

$E = \dfrac{\sigma}{\varepsilon}$

⇒ E - young's modulus / modulus of elasticity

⇒ It is the material property

⇒ Units : N/mm^2

E = 210 Gpa for steel

E = 70 Gpa for aluminium

E = 0.1 Gpa for wood

E = 20-40 Gpa for concrete

Gpa = Giga pascal
= $10^9\ N/m^2$

Mpa = Mega pascal
= $10^6\ N/m^2$

1 Mpa = 1 N/mm^2

$\frac{\sigma}{E} = E$

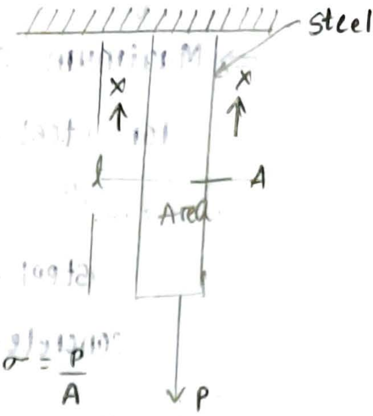
$\varepsilon = \frac{\sigma}{E}$

$\frac{\delta l}{l} = \frac{P}{AE}$

$\delta l = \frac{Pl}{AE}$

E = Young's modulus.

δl = Elongation



$\sigma = \frac{P}{A}$

$\varepsilon = \frac{\delta l}{l}$

1) A rod of diameter 5 mm is subjected to a tensile force of 200 kN. Assuming E (Young's modulus) = 200 Gpa. Find the stress in the material, elongation, strain. length of rod is 1m

$d = 5mm$; $F = 200kN$; $E = 200\ Gpa$; $l = 1m$

$= 10^3 mm$

$= 200 \times 10^3\ Mpa$

(or)

$= 200 \times 10^3\ N/mm^2$

$A = \frac{\pi}{4}(D)^2$

Stress $(\sigma) = \frac{P}{A}$

$= \frac{200 \times 10^3}{\frac{\pi}{4} \times (5)^2} = \frac{200000}{\frac{\pi}{4} \times 25}$

$= \frac{200000}{19.625} = 10191.08\ N/mm^2$

$\cong 10191\ N/mm^2$

Elongation $(\delta l) = \frac{Pl}{AE}$

$= \frac{10191 \times 1000}{200 \times 10^3}$

$= 50.955\ mm$

Strain $(\varepsilon) = \frac{\delta l}{l}$

$= \frac{50.955\ mm}{1m}$

$= \frac{50.955\ mm}{1000\ mm}$

$= 0.050955\ mm/mm$

$\cong 0.05\ mm/mm$

* Working stress

⇒ Maximum stress is yield stress (σ_y) only.

For steel, $\sigma_y = 250\ N/mm^2$
structural steel (Max. stress)

Torque steel (Max. stress)
steel, $\sigma_y = 415\ N/mm^2$

concrete, $\sigma_y = 20\ N/mm^2$

* Factor of safety:

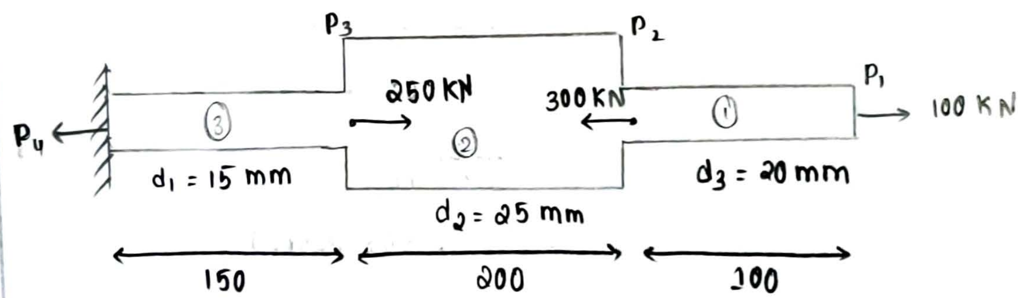
1) Incidental load
2) Material quality

$$\text{Working stress} = \frac{\sigma_y}{FOS}\ N/mm^2$$ FOS - Factor of safety

FOS → 1.8 to 2.0

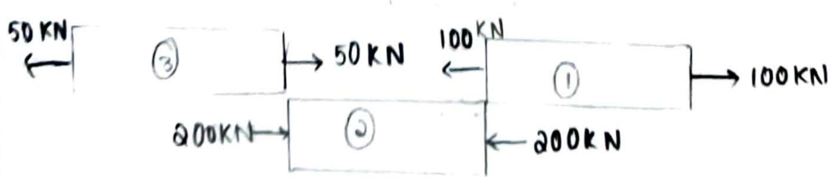
$$W.S = \frac{250}{2} = 125\ N/mm^2$$

1)



Determine the stresses in varies segments of circular part. Compute total elongation assuming young's modulus, E = 195 Gpa

$$\Sigma F_x = 0 \Rightarrow -P_4 + 250 - 300 + 100 = 0$$
$$P_4 = 50\ KN$$



$$\text{Stress at } ① = \frac{\text{Load } (P_1)}{\text{Area } (A_1)}$$

$$= \frac{100\ KN}{\frac{\pi}{4} \times (20)^2} = \frac{100 \times 10^3}{314}$$

$$= 318.47\ N/mm^2$$

Stress at ② $= \dfrac{P_2}{A_2}$

$= \dfrac{200\ KN}{\frac{\pi}{4} \times (25)^2}$

$= \dfrac{200 \times 10^3}{490.625}$

$= 407.643\ N/mm^2$

Stress at ③ $= \dfrac{P_3}{A_3}$

$= \dfrac{50\ KN}{\frac{\pi}{4} \times (15)^2}$

$= \dfrac{50 \times 10^3}{176.625}$

$= 283.085\ N/mm^2$

at ① $\Rightarrow$ $\delta l_1 = \dfrac{P_1 l_1}{A_1 E}$ $[l_1 = 100\ mm]$

$= 318.47 \times \dfrac{100}{195 \times 10^3}$

$= \dfrac{31847}{195000}$

$= 0.1633\ mm$ (elongation)

at ② $\Rightarrow$ $\delta l_2 = \dfrac{P_2 l_2}{A_2 E}$

$= 407.643 \times \dfrac{200}{195 \times 10^3}$

$= \dfrac{81528.6}{195000}$

$= -0.4180\ mm$ (compression)

at ③ $\Rightarrow$ $\delta l_3 = \dfrac{P_3 l_3}{A_3 E}$

$= 283.085 \times \dfrac{150}{195 \times 10^3}$

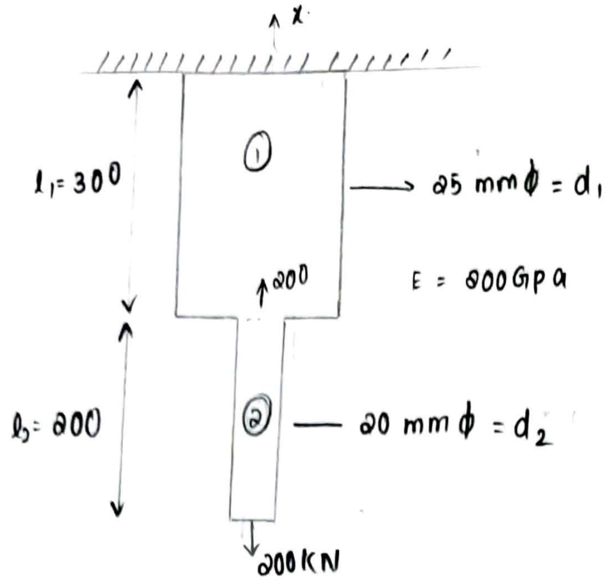
$= \dfrac{42462.75}{195000} = 0.2177\ mm$ (elongation)

Total elongation $= \delta_1 + \delta_2 + \delta_3$

$= 0.1633 - 0.4180 + 0.2177$

$= -0.037\ mm$

a)



Stress at ① $= \dfrac{\text{Load } (P_1)}{\text{Area } (A_1)}$

$= \dfrac{200 \text{ KN}}{\frac{\pi}{4} \times (25)^2}$

$= \dfrac{200 \times 10^3}{490.625}$

$= 407.643 \text{ N/mm}^2$

Stress at ② $= \dfrac{\text{Load } (P_2)}{\text{Area } (A_2)}$

$= \dfrac{200 \text{ KN}}{\frac{\pi}{4} \times (20)^2}$

$= \dfrac{200 \times 10^3}{314} = 636.942 \text{ N/mm}^2$

At ① $\Rightarrow$ $\delta l_1 = \dfrac{P_1 l_1}{A_1 E}$

$= 407.643 \times \dfrac{300}{200 \times 10^3}$

$= \dfrac{122292.9}{200000} = 0.6114645$

$\cong -0.611$ mm (compression)

At ② $\Rightarrow$ $\delta l_2 = \dfrac{P_2 l_2}{A_2 E}$

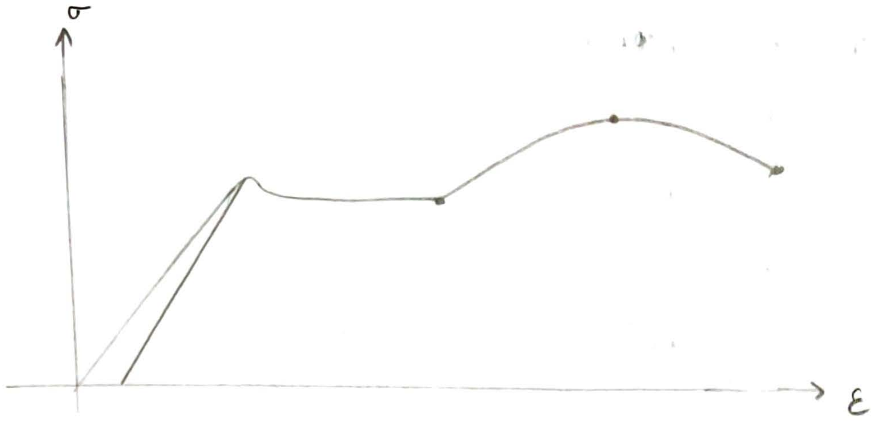
$= 636.942 \times \dfrac{200}{200 \times 10^3} = 0.636942$

$\cong 0.636$ mm

Total elongation $= \delta l_1 + \delta l_2$

$= -0.611 + 0.636 = 0.025$ mm

1/11/2019

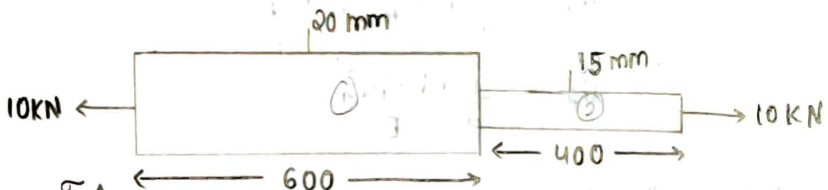


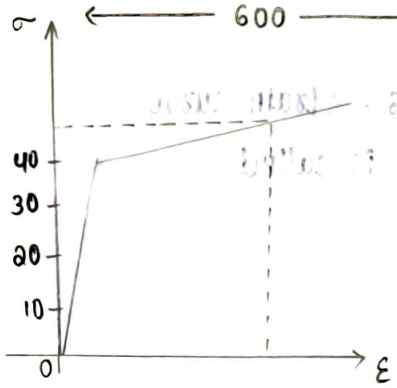
$0.2\% = \varepsilon =$ proof strain

$0.002 = 0.2\% = \varepsilon$ strain — proof stress

⇒ A strain equal to 0.2% is called proof strain at which material gets yielding. Then draw a parallel line to the proportional limit, which cuts " because of elongation

⇒ At a point it cuts on stress and strain curve, the corresponding reading on y-axis is called yield stress.





Determine the approximate elongation of the rod when load is applied

$E = 70\ \text{Gpa}$

$$\delta l_1 + \delta l_2 = \frac{P_1 l_1}{A_1 E_1} + \frac{P_2 l_2}{A_2 E_2}$$

$$= \frac{P}{E}\left[\frac{l_1}{A_1} + \frac{l_2}{A_2}\right]$$

$$\varepsilon = \frac{\delta l}{l} \Rightarrow \delta l_1 = l\varepsilon$$

$$= 400 \times 0.0450$$

$$= 18\ \text{mm}$$

$$\delta l_2 = \frac{P_2 l_2}{A_2 E} = \frac{31.8 \times 600}{70 \times 10^3}$$

$$= 0.272\ \text{mm}$$

$$\text{Stress } \sigma_1 = \frac{\text{Load } P_1}{\text{Area } A_1}$$

$$= \frac{10 \times 10^3}{\frac{\pi}{4} \times (20)^2}$$

$$= \frac{10000}{314}$$

$$= 31.8\ \text{N/mm}^2$$

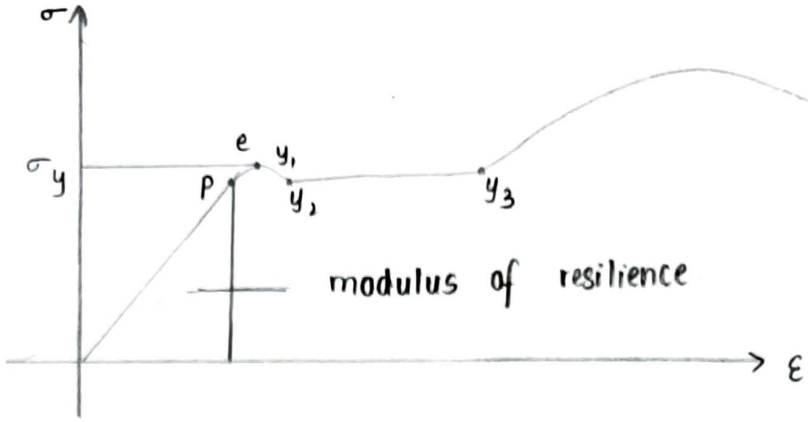
$$\sigma_2 = \frac{P_2}{A_2}$$

$$= \frac{10 \times 10^3}{\frac{\pi}{4} \times (15)^2}$$

$$= \frac{10^4}{176}$$

$$= 56\ \text{N/mm}^2$$

* Modulus of resilience:



⇒ The area under stress-strain diagram upto proportional limit is called modulus of resilience.

$$U_r = \frac{1}{2}\sigma_{PL} \times \varepsilon_{PL}.$$

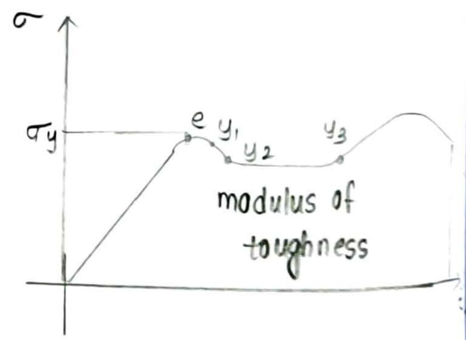
⇒ This property represents the ability of the material to absorb energy without any permanent damage to the material.

$$U_r = \frac{1}{2} \times \sigma_{PL} \times \frac{\sigma_{PL}}{E}$$

$$U_r = \frac{1}{2}\frac{(\sigma_{PL})^2}{E}$$

* Modulus of Toughness

⇒ The area under stress-strain curve upto fracture (break point) is called modulus of toughness.



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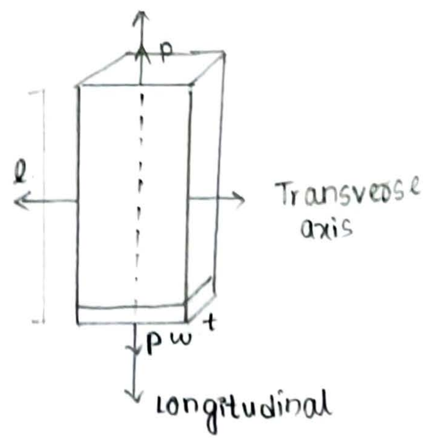
* Poison's ratio:

$$\underset{(\text{new})}{\nu} = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

$$= \frac{\delta w/w}{\delta l/l}$$

$$= \frac{+\varepsilon_t}{\varepsilon_l} \quad \text{(compressive force)}$$



'-' indicates the dimension is getting reduced

$$= \frac{-\varepsilon_t}{\varepsilon_l} \quad \text{(tensile force)}$$

It is a symmetric property.

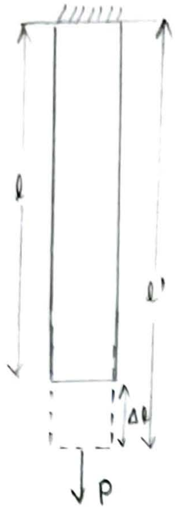
For Rubber, $\nu = 0.5$

Steel, $\nu = 0.3$

Al, $\nu = 0.22$

concrete, $\nu = 0.15$

*



Changed in length of the bar,

$$l' = l + \Delta l$$

$$= \frac{l}{l}\left[l + \Delta l\right]$$

$$= l\left[1 + \frac{\Delta l}{l}\right]$$

$$= l(1+\varepsilon)$$

$$l' = l(1+\varepsilon)$$

Changed length $= l(1+\varepsilon)$ $\qquad \varepsilon_t$ - trasverse strain

Change in diameter $\Delta d = \varepsilon_t d = \nu \varepsilon d$

Changed in diameter of the bar is, $\qquad \varepsilon_t = \frac{\Delta d}{d}$

$$d' = d - \Delta d$$

$$= d - \nu \varepsilon d$$

$$= d(1 - \nu\varepsilon)$$

The volume of bar after loading is;

Changed in volume, $V' = \frac{\pi}{4} d'^2 \times l'$

$$= \frac{\pi}{4}\left[d^2(1-\nu\varepsilon)^2\right] \times l\,(1+\varepsilon)$$

$$= \frac{\pi}{4} d^2 l\,(1 + \nu^2\varepsilon^2 - 2\nu\varepsilon)(1+\varepsilon)$$

negligible

$$= \frac{\pi}{4} d^2 l\,(1 + \nu^2\varepsilon^2 - 2\nu\varepsilon + \varepsilon + \nu^2\varepsilon^3 - 2\nu\varepsilon^2)$$

$$= \frac{\pi}{4} d^2 l\,(1 - 2\nu\varepsilon + \varepsilon)$$

Change in volume $\Delta V = V' - V$

$$= \frac{\pi}{4} d^2 l\,(1 - 2\nu\varepsilon + \varepsilon) - \frac{\pi}{4} d^2 l$$

$$= \frac{\pi}{4} d^2 l\left[1 - 2\nu\varepsilon + \varepsilon - 1\right]$$

$$= \underbrace{\frac{\pi}{4} d^2 l}_{V}\,\varepsilon\left[1 - 2\nu\right]$$

Volumetric strain, $\varepsilon_v = \frac{\Delta V}{V} = \varepsilon(1 - 2\nu)$

1) Determine the change in the volume of steel bar of 25mm diameter and 500 mm long when subjected to a tensile stress $\sigma_t = 200$ Mpa. The young's modulus of steel $E_s = 200$ Gpa. The poisson's ratio of steel $\nu = 0.3$

$d = 25$ mm $\quad \sigma_t = 200$ Mpa $\quad \nu = 0.3$

$l = 500$ mm $\quad E_s = 200$ Gpa

$= 200 \times 10^3$

Axial stress $= 200$ N/mm^2

$$\frac{\sigma_t}{\varepsilon} = E$$

$$\varepsilon = \frac{\sigma_t}{E} = \frac{200}{200 \times 10^3} = 10^{-3}$$

$$\Delta V = V \varepsilon (1 - 2\nu)$$

$$= \frac{\pi}{4} \times 25^2 \times 500 \times 10^{-3} (1 - 2 \times 0.3)$$

$$= \frac{3.14}{4} \times 625 \times 500 \times 10^{-3} (1 - 0.6)$$

$$= 0.785 \times 312.5 \times 0.4$$

$$= 98.125 \text{ mm}^3$$

8/11/2019

Volumetric strain, $= \frac{dv}{V} = \varepsilon_v$

$\varepsilon = \varepsilon_l$ = axial strain

ε_t = transverse strain

ε_v = volumetric strain

$$\frac{\varepsilon_t}{\varepsilon} = \nu$$

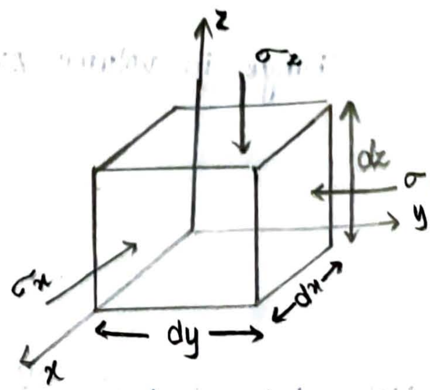
$$\varepsilon_v = \varepsilon(1 - 2\nu)$$

* Hydrostatic pressure

$\sigma_x = \sigma_y = \sigma_z = \sigma = \cancel{\phi}\ P$

After the pressure on the cube

Changed dimension $= (1 + \varepsilon_x)dx$, $(1 + \varepsilon_y)dy$, $(1 + \varepsilon_z)dz$



Change of volume $(V') = (1+\varepsilon_x)\,dx \cdot (1+\varepsilon_y)\,dy \cdot (1+\varepsilon_z)\,dz$

$$\Delta V = V' - V \simeq (\varepsilon_x + \varepsilon_y + \varepsilon_z)\,dx\,dy\,dz$$

$$\Delta V = (\varepsilon_x + \varepsilon_y + \varepsilon_z)\,V$$

$$\frac{\Delta V}{V} = \underbrace{\varepsilon_V = \varepsilon_x + \varepsilon_y + \varepsilon_z}_{\text{force/pressure in all directions}} \quad / \quad \underbrace{\frac{\Delta V}{V} = \varepsilon(1-2\nu)}_{\text{valid for when body is subjected in one direction}}$$

* **Bulk Modulus (K) :**

It is defined as the ratio between volumetric stress by volumetric strain upto proportional limit only.

$$K = \frac{\text{Volumetric stress}}{\text{Volumetric strain}}$$

When the body is subjected to uniform stress from all directions it exists upto proportional limit. then,

$$K = \frac{\sigma_x + \sigma_y + \sigma_z}{dV/V}$$

volumetric stress $= \sigma_x + \sigma_y + \sigma_z = 3\sigma$

volumetric strain $= \frac{dV}{V} = \varepsilon(1-2\nu)$

$$\frac{dV}{V} = \frac{\sigma_x + \sigma_y + \sigma_z}{E}(1-2\nu)$$

$$\varepsilon_V = \frac{3\sigma}{E}(1-2\nu)$$

$$\frac{E}{3(1-2\nu)} = \frac{\sigma}{\varepsilon_V}$$

$$K = \frac{E}{3(1-2\nu)} \qquad (K, E, \nu)$$

material property

⇒ The Bulk Modulus is caused only by normal strain not by shear strain.

⇒ It is a measure of stiffness of a volume of a material.

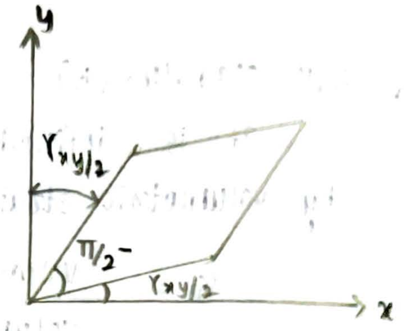
⇒ This material property provides an upper max. limit to Poisson's ratio of $\nu = 0.5$

* **Shear Modulus or Rigidity Modulus**

The constant of the proportionality of shear stress and shear strain is known as rigidity modulus of the material.

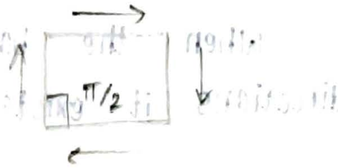
⇒ It is denoted by G.

$$G = \frac{\tau_{pc}}{\gamma_{pc}} = \frac{E}{2(1+\nu)}$$



⇒ This rigidity modulus changes its shape but not its volume

$$E = \frac{9GK}{G+3K}$$



$$\nu = \frac{E}{2G} - 1$$

$$\nu = \frac{1 - E/3K}{2}$$

1) An aluminium specimen has diameter of 25 mm and gauge length of 250 mm. If a force of 165 kN elongates the gauge length 1.20 mm, determine the modulus of elasticity (E) and also determine by how much the force causes the diameter of specimen to contract. Taking G = 26 Gpa (rigidity modulus) and yield stress σ_y = 440 Mpa.

$d = 25$ mm

$l_i = 250$ mm

$P = 165$ KN

$\Delta l = 1.20$ mm

$G = 26$ Gpa

$= 26 \times 10^3$ Mpa

$\sigma_y = 440$ Mpa

$E = ?$

$$\text{Stress} = \sigma = \frac{P}{A} \quad < \sigma_y \rightarrow \text{proportional limit}$$

$$= \frac{165 \times 10^3}{\frac{\pi}{4}(25)^2} =$$

$$= \frac{165000}{0.785 \times 625} = \frac{165000}{490.625}$$

$$= 336.1 \text{ N/mm}^2$$

$$\text{strain} = \varepsilon = \frac{\Delta l}{l}$$

$$= \frac{1.20}{250} = 4.8 \times 10^{-3} \text{ mm/mm}$$

$$= 0.0048 \text{ mm/mm}$$

$$E = \frac{\sigma}{\varepsilon} = \frac{336.1}{0.0048}$$

$$= 70020.8 \text{ N/mm}^2$$

$$= 70.0208 \text{ Gpa}$$

$$\cong 70.02 \text{ Gpa}$$

$$G = \frac{E}{2(1+\nu)}$$

$$\nu = \frac{E}{2G} - 1$$

$$= \frac{70}{2 \times 26} - 1$$

$$= \frac{70}{52} - 1 = 1.34615 - 1$$

$$= 0.34615$$

$$\cong 0.347$$

$$\nu = \frac{\varepsilon_t}{\varepsilon}$$

$$\varepsilon_t = \nu \varepsilon$$

$$= 0.347 \times 0.0048$$

$$= 1.6656 \times 10^{-3}$$

$$= 0.00166 \text{ mm/mm.}$$

$$\frac{\delta d}{d} = \varepsilon_t$$

$$\delta d = \varepsilon_t \times d$$

$$= 0.00166 \times 25$$

$$= 0.0415 \text{ mm}$$

14/11/2019

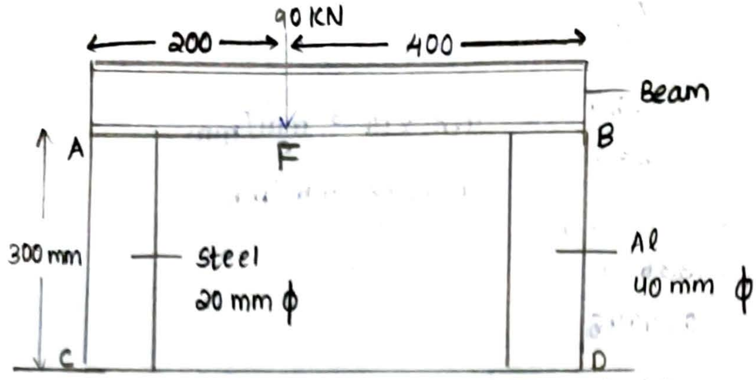
$\sigma - \varepsilon$ relation - different points are identified.

$\left.\begin{matrix}\sigma \\ \varepsilon\end{matrix}\right\} E$ $\quad P/A \Rightarrow \sigma$, $\quad \Delta l/l \Rightarrow \varepsilon$

Elongation

Poisson's ratio.

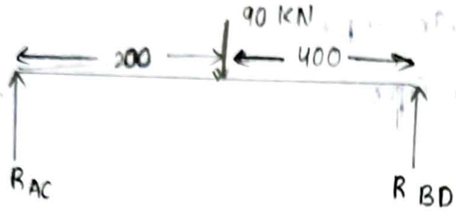
1)



$E_{st} = 200$ Gpa

$E_{Al} = 70$ Gpa

Determine the displacement of point F.

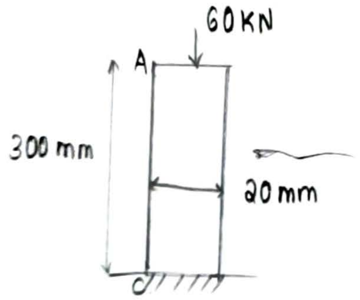


$\Sigma F_y = 0 \Rightarrow \quad R_{AC} + R_{BD} = 90$

$\Sigma M_A = 0 \Rightarrow \quad R_{BD} \times 600 - 90 \times 200 = 0$

$$R_{BD} = \frac{90 \times 200}{600} = 30 \text{ KN}$$

$$R_{AC} = 90 - 30 = 60 \text{ KN}$$



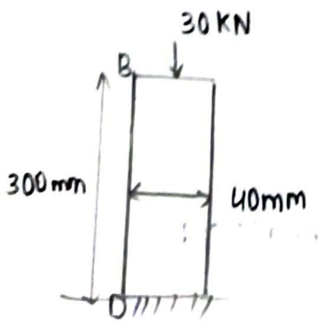
$$\delta l_{st} = \frac{Pl}{AE_{st}}$$

$$= \frac{60 \times 10^3 \times 300}{\frac{\pi}{4} \times (20)^2 \times 200 \times 10^3}$$

$$= \frac{18 \times 10^6}{\frac{\pi}{4} \times 80000000} = \frac{18 \times 10^6}{\pi \times 2 \times 10^7}$$

$$= \frac{18}{62.8}$$

$$= 0.286 \text{ mm}$$

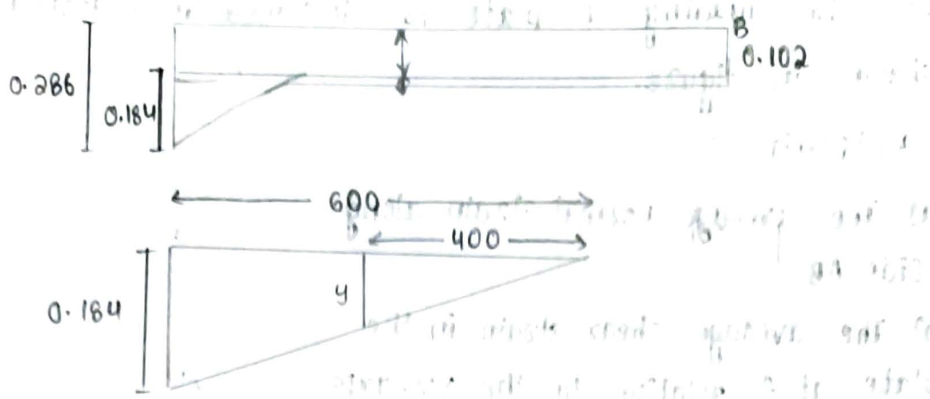


$$\delta l_{Al} = \frac{Pl}{AE_{Al}}$$

$$= \frac{30 \times 10^3 \times 300}{\frac{\pi}{4} \times (40)^2 \times 70 \times 10^3}$$

$$= \frac{9 \times 10^6}{\frac{\pi}{4} \times 112 \times 10^6} = \frac{9}{\pi \times 28} = \frac{9}{87.92}$$

$$= 0.1023 \text{ mm}$$



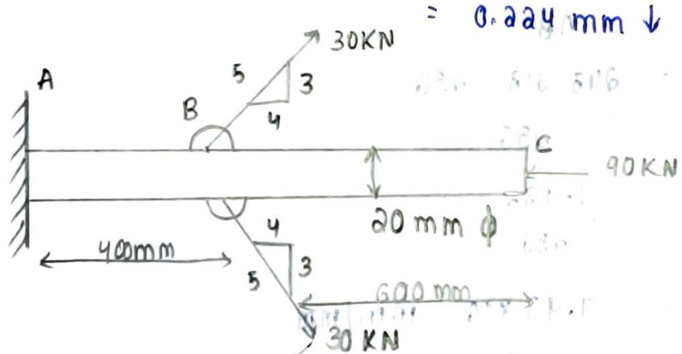
$$\frac{y}{0.184} = \frac{400}{600}$$

$$y = \frac{4}{6} \times 0.184$$

$$= 0.122 \text{ mm}$$

$\therefore$ Deflection of point F $= 0.102 + 0.122$

$= 0.224$ mm ↓

2) 

Determine the displacement of end 'C' w.r.t end 'A'.

$$\Sigma F_x = 30 \times \frac{4}{5} + 30 \times \frac{4}{5} - 90$$

$$= 24 + 24 - 90 = -42 \text{ KN} \leftarrow \text{(compression)}$$

$$\delta l = \frac{Pl}{AE}$$

$$= \frac{42 \times 10^3 \times 1000}{\frac{\pi}{4} \times (20)^2 \times 200 \times 10^3}$$

$$= \frac{42 \times 10^3}{0.785 \times 8 \times 10^4}$$

$$= \frac{42}{62.8}$$

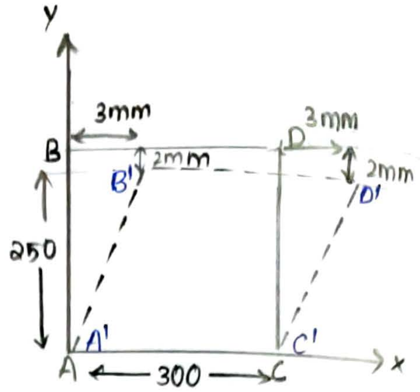
$$= 0.6687 \text{ mm}$$

3) Due to loading a plate is deformed into dotted shape as shown in figure.

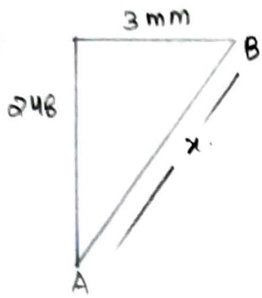
Determine :

(a) The average normal strain along side AB

(b) The average shear strain in the plate at A relative to the xy-axis.



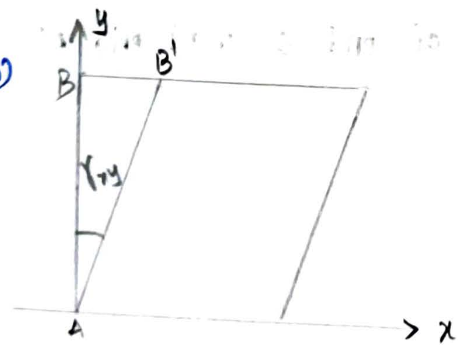
(a)

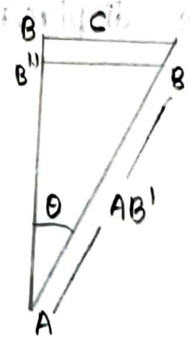


$$AB' = \sqrt{3^2 + 248^2}$$
$$= 248.018 \text{ mm}$$

$$\gamma = \frac{AB' - AB}{AB}$$
$$= \frac{248.018 - 250}{250}$$
$$= \frac{-1.982}{250}$$
$$= -7.93 \times 10^{-3} \text{ mm/mm}$$

(b)





$C = \gamma\theta$

$\theta = \frac{C}{\gamma}$

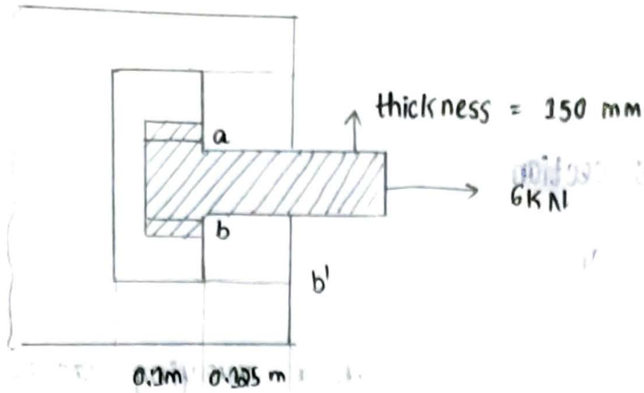
$$\tan\theta = \frac{3}{250-2}$$

$$\gamma_{xy} = \frac{3}{248} = 0.0121$$

$$\theta = \tan^{-1}\left(\frac{3}{248}\right)$$
$$= 0.6875$$

In radians,

$$\gamma_{xy}(\theta) = \frac{0.6875}{57.25}$$
$$= 0.0121 \text{ radians}$$



$$\sigma_{aa} = \frac{P}{A_1}$$

$$= \frac{3 \times 10^3}{100 \times 150}$$

$$= \frac{3}{15} = 0.2 \ N/mm^2$$

$$\sigma_{bb} = \frac{P}{A_2}$$

$$= \frac{3 \times 10^3}{125 \times 150}$$

$$= \frac{3 \times 10^2}{1875} = 0.16 \ N/mm^2$$