

# Unfriending facilitates cooperation: Co-evolution of opinion and network dynamics

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## Abstract

Countries around the world are experiencing polarization of opinions and echo chambers of information (Del Vicario et al., 2016; Tsai, Tao, Chuan, & Hong, 2020; Sasahara et al., 2021). Discovering what conditions can prevent political polarization is a vital and pressing issue, as political polarization can critically harm democratic institutions (Boxell, Gentzkow, & Shapiro, 2020). Previous models studying opinion dynamics have not studied the co-evolution of network and opinion dynamics, despite the fact that co-evolution is known to have a large effect on how opinions are shared and formed (Flache et al., 2017; Galesic, Olsson, Dalege, van der Does, & Stein, 2021; McPherson, Smith-Lovin, & Cook, 2001). Moreover, the field of opinion dynamics suffers from not being able to incorporate empirical data in their theoretical models, which severely limits the fields ability to make reliable explanations and predictions (Flache et al., 2017; Galesic et al., 2021; Mäs, 2019). This paper tries to fill both of these gaps in the literature by developing and investigating a co-evolutionary agent-based model of opinion and networks dynamics, drawing on insights from social psychology, computational biology, data science and network science. The main interest of this paper is how co-evolution can prevent polarization of opinions. To incorporate empirical data, cutting edge techniques of hyperparameter optimization is used to investigate how well the generated networks of the model match up with 7 empirical social networks (Krivorotko, Sosnovskaia, Vashchenko, Kerr, & Lesnic, 2022; Akiba, Sano, Yanase, Ohta, & Koyama, 2019). The results of the hyperparameter optimization shows that the co-evolutionary model can explain the patterns of empirical networks well using very simple generating principles. The co-evolutionary model is found to be a much better model than models without opinion dynamics when the empirical social networks are large and highly opinionated. Contrary to recent findings (Sasahara et al., 2021), the main result of this paper is that avoiding polarization can be facilitated by the deletion of ties between dissimilar agents. This echoes the general principle from computational biology of positive assortments ability to facilitate cooperation (Pepper & Smuts, 2002; Dakin & Ryder, 2018; Santos, Pacheco, & Lenaerts, 2006).

**Keywords:** *agent-based modelling, opinion dynamics, social influence, co-evolution, social networks*

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# 1 Introduction

## 1.1 Polarization, echo chambers and the threat to democracy

Democracies around the world are experiencing increased amounts of polarization (Boxell et al., 2020; McCoy, Rahaman, & Somer, 2018; Somer & McCoy, 2018). As a result, making decisions by cooperating with other people becomes harder which can severely damage the ability of democratic systems to solve problems (Andris et al., 2015; Levin, Milner, & Perrings, 2021; McCoy et al., 2018). A striking recent example is the uniquely severe rise of polarization of the political system in the United States (Dimock & Wike, 2020). During the last two decades, the amount of overlap of opinions between the two political parties have decreased substantially, which has led to gridlock, government shutdowns and failure to enact new legislation (Andris et al., 2015; Center, 2014b). A similar increase can be observed in terms of affective polarization. Affective polarization refers to how much citizens carry more negative feelings towards other political parties than their own (Boxell et al., 2020; Iyengar, Lelkes, Levendusky, Malhotra, & Westwood, 2019). Once again, the United States is reported to have uniquely high amounts of affective polarization (Boxell et al., 2020). Although the United States is unique in the extent of affective polarization, they are not the only country experiencing an increase in affective polarization. Countries such as France, Switzerland and Denmark has seen increased levels of affective polarization in the last two decades (Boxell et al., 2020). Polarization is in other words a general trend, affecting countries around the world (McCoy et al., 2018; Somer & McCoy, 2018; Wilson, Parker, & Feinberg, 2020). With that said, polarization of opinions does not seem to be an inevitable state for democracies. During the last two decades, a number of European democracies have experienced higher degrees of consensus and less affective polarization, which includes Sweden, Norway and Germany (Boxell et al., 2020).

Although some countries are experiencing a decrease in polarization, there is reason to believe that the general problem of polarization might be getting worse due to technological advances. Specifically, polarization might be increasing due to the rise of the internet and social media. A growing literature has reported that information on the internet and on social media is not uniform across all its users (Taylor, Mantzaris, & Garibay, 2018; Sasahara et al., 2021; F. Baumann, Lorenz-Spreen, Sokolov, & Starnini, 2020; Tsai et al., 2020). Instead, information is catered to the individual based on their previous search history and behavior. Very different people will be exposed to very different types of information, as users are only exposed to small subset of the available information. It is this filtering of the information that is referred to as informational filters in the literature (Geschke, Lorenz, & Holtz, 2019). Informational filters can create situations where almost all opinions one is exposed to are congruent with one's belief due to confirmation bias, popularized under the term echo chambers on social media (F. Baumann et al., 2020; Sasahara et al., 2021; Tsai et al., 2020; Geschke et al., 2019). With regards to polarization, the internet can be seen as a "mixed blessing". The internet can cause opinions to be pulled closer together by diversifying information, but it also enables encounters of opposing opinions by chance, which can increase polarization (Lev-On & Manin, 2009). This points to a certain tension between how much exposure to different opinions affects polarization. When presented with diversifying views, people generally decrease negative affects to other political parties (Levy, 2021), but exposure to opposing views can cause opinions to become more polarized (Bail et al., 2018). Relatedly, higher involvement in one's echo

chamber correlates with more negative emotions in terms of valence, which suggests that echo chambers are driven largely by outrage (Del Vicario et al., 2016).

Although polarization is usually referred to as a political phenomenon, polarization can critically impair any social system's ability to cooperate successfully. It is therefore vital that we identify and understand what mechanisms can lead a system to polarization, to consensus or anywhere in between the two extremes. This is no easy task, as opinion formation is a highly complex phenomena (F. T. W. Baumann, 2021). Despite the complexity of the problem, an entire field has been dedicated to study how opinions change. The study of the dynamics shaping opinions is aptly named opinion dynamics. The models from opinion dynamics are often analogous models, drawing inspiration from statistical physics in how they build their models (Galesic et al., 2021). Opinion dynamical models include models of discrete and continuous opinions. The field has a rich history of research on how opinions change using experimental data to inform the assumptions of formal models (F. T. W. Baumann, 2021; Chacoma & Zanette, 2015; Flache et al., 2017; Friedkin & Johnsen, 1990; Noorazar, Vixie, Talebanpour, & Hu, 2020; Spears, 2021; Turner & Smaldino, 2018). Typically, these models take social influence as their central mechanism, where interactions between agents shape the agents' opinions over time (Flache et al., 2017). This reflects the idea that our opinions are shaped by the opinions of our peers (McPherson et al., 2001). What these models often neglect is the fact that agents can not only change their opinions but also change their peers. The models of opinion dynamics typically assume a static network structure in which agents are situated (Galesic et al., 2021). By doing so, these models fail to take into account that networks are dynamic and very likely co-evolves with opinions (Ferraz de Arruda et al., 2022; Galesic et al., 2021). The interdependence between the dynamics of social networks and opinion dynamics is largely understudied, despite the fact that the two processes are highly interacting (Asikainen, Iñiguez, Ureña-Carrión, Kaski, & Kivelä, 2020; Bruch & Atwell, 2015; Galesic et al., 2021; Kossinets & Watts, 2009; Noorazar et al., 2020). It is therefore not surprising that the dynamic relation between the network structure and social influence has often been reported as an important avenue for future research (Flache et al., 2017; Galesic et al., 2021).

Another important avenue to gain insights into is how to incorporate empirical data in the evaluation of theoretical models (Mäs, 2019). This is a problem for many theoretical models, but the issue seems especially pertinent in the field of opinion dynamics, which often fails to account for how the results of their models match empirical data (Galesic et al., 2021; Flache et al., 2017; Mäs, 2019). The lack of empirical validation is understandable, as opinions are notoriously hard to measure (Mäs, 2019). Moreover, most of the models of opinion dynamics are not meant to accurately predict how opinions evolve over time, but rather point out interesting interactions between variables of interest in formal models (Mäs, 2019). These models are extremely useful for theory building, but they often run the risk of being too detached from the real world to be useful (P. E. Smaldino, 2020; Mäs, 2019). If we want to get closer to the answers of how opinions are formed in social networks, we need to make better use of the empirical data available as well as evaluate how well our models can explain the observed patterns of opinionated social networks.

This paper seeks to fill these two critical gaps of data integration and co-evolution in the current literature on opinion dynamics. This paper starts by identifying and introducing the critical underlying mechanisms of opinion and network dynamics. These mechanisms are then included and manipulated in a co-evolutionary agent-based model of opinion

and network dynamics, drawing on insights from data science, social psychology, computational biology and network science (Asikainen et al., 2020; Flache et al., 2017; Ilany & Akçay, 2016; Jackson & Rogers, 2007; Santos et al., 2006). The model is specifically designed to allow for the considering of the impact that opinions have on social networks and vice versa. After this introduction and model specification, the paper falls into two parts. The first uses cutting-edge techniques from data science in to integrate empirical data in the assesment of the models. The analysis focuses on how the inclusion of opinion dynamics can improve models of network dynamics. It does so by using hyperparameter optimization to investigate whether the model can generate the patterns found in real empirical social networks. This paper expands upon previous use of this cutting-edge technique in agent-based modelling, by also implementing a functional ANOVA (fANOVA) of the hyperparameter importance. The second part investigates how co-evolution influences the results from the agent-based model. The analysis focuses on how network dynamics can alter opinion dynamics. This leads to the central question that this paper will try to provide answers to. Namely, this paper's central interest is to investigate what the effect of tie-deletion to dissimilar others has on facilitating cooperation. We begin by discussing why agent-based models are particularly well-suited to model opinion dynamics.

## 2 Agent-based modelling of complex phenomena

In order to get sufficiently good answers to our questions, we need to build sufficiently good models. When it comes to models of opinion dynamics, this is no easy task. The social systems underlying opinion dynamics are complex and complicated (Mäs, 2019). The change of opinions is both a cognitive matter of updating one's belief, but also a social matter of being exposed to different perspectives (Flache et al., 2017; Friedkin & Johnsen, 1990; Spears, 2021). Models that do incorporate realistic social and cognitive mechanisms are typically verbal models. Although these verbal models often provide rich and detailed accounts with realistic mechanisms, they lack the precision of formal models (Fogarty, Ammar, Holding, Powell, & Kandler, 2022; Galesic et al., 2021; P. E. Smaldino, 2020). Agent-based models are well-equipped to strike a balance between the two types of models (Flache, 2018; Galesic et al., 2021; Epstein, 1999; Mäs, Flache, & Kitts, 2014). This is done by investigating the macro-behaviors of a system, where the micro-behaviors are specified (Bruch & Atwell, 2015; Epstein, 1999; Flache, 2018). In particular, agent-based models are well suited for analyzing interactions between individuals instead of interactions between variables (Geschke et al., 2019). As is the case with any model, the results critically hinge on the assumptions of the model. It is therefore the role of the modeler to provide as empirically plausible mechanisms as possible for the system of study (Crooks & Heppenstall, 2012; Epstein, 1999; Page, 2010). At the same time, the value of a model is in its simplification of reality. The model should be as simple as it can be and as complicated as it needs to be in order to answer the questions of interest (P. E. Smaldino, 2020). The hope is that by simplifying the system to a sufficient extent, we can observe and understand some important features of even extremely complex systems (Fogarty et al., 2022; P. E. Smaldino, 2020; P. Smaldino, 2016).

The rest of the introduction will present previous work in related fields in order to identify the central mechanisms that shape how networks and opinions change over time. This is a vital step in model building, as these central

mechanisms will serve as the core of the agent-based model of this paper. We start by considering key features of social interactions. Here, we will focus on homophily as well as positive and negative social influence. Afterwards, we turn our attention to the central role of triadic closure in how networks are shaped over time.

## 3 Central Mechanisms

### 3.1 Homophily

One of the most robust findings of the social world is summed up in the now famous phrase "birds of a feather flock together" (McPherson et al., 2001). This phrase refers to the fact that assortment in humans is non-random, and is often based on similarity of traits between individuals (Asikainen et al., 2020; Crandall, Cosley, Huttenlocher, Kleinberg, & Suri, 2008; McPherson et al., 2001). This principle of assortment based on similarity is what is referred to as homophily (McPherson et al., 2001; Kossinets & Watts, 2009). The principle of homophily seems to permeate most social context (Taylor et al., 2018). Homophily is a central mechanism for almost all demographic variables which have been investigated, which includes gender, race, religion, and socioeconomic class (Asikainen et al., 2020; McPherson et al., 2001). The observation of assortment based on similarity is referred to as observed homophily (Peixoto, 2022). Observed homophily has mainly been explained by a psychological and a structural explanation. The psychological explanation refers to the finding that people seem to exhibit a psychological preference to be with people like themselves (Asikainen et al., 2020; McPherson et al., 2001; Winter & Kataria, 2020). Interacting primarily with similar people ensures easier communication and enhances the individual's ability to predict the other person's behavior (Kossinets & Watts, 2009; Winter & Kataria, 2020). Assorting based on similarity might facilitate cooperation, because coordination between similar individuals is simpler and less costly for the individual (Winter & Kataria, 2020; Carter, Lee, & Marshall, 2015). Although this psychological explanation is an important part of the puzzle of explaining homophily, part of the explanation seems also to be a structural explanation. Structural constraints could limit how dissimilar choices of interactions are available to a person (Peixoto, 2022). As an example, if you work as a female nurse chances are that most of your interactions at work will be with other females. In this case, even if you do not have a strong psychological preference for interacting with people like yourself, the social interactions available to you might primarily be people like yourself. In this scenario, there will be high levels of observed homophily by sheer chance. The existing assortment in the network makes the probability of interacting with similar individuals much higher than with non-similar individuals (Peixoto, 2022). These structural constraints are of course not exclusive to gender or workplaces. Other examples include socio-economic status and the neighborhood you live in and preference in music and what concerts you attend (McPherson et al., 2001). In other words, the psychological and the structural factor contributing to observed homophilic interactions are not mutually exclusive but might instead facilitate each other (Asikainen et al., 2020). Even a little psychological homophilic preference could turn into structural factors over the course of time (Asikainen et al., 2020; Kossinets & Watts, 2009; Taylor et al., 2018). Empirical work points to both psychological and structural homophily being important mechanisms, but suggests that structural homophily might be

a more powerful mechanism for finding new connections in a social network (Bener, Çağlayan, Henry, & Prałat, 2016; Kossinets & Watts, 2009). As mentioned, the observed general finding of assortment in networks are not only based on selecting friends based on similarity. It is also due to the fact that humans interact with their social ties, which typically further increases similarity (Friedkin & Johnsen, 1990; Spears, 2021). The combined mechanism of social interactions in shaping opinions is called social influence and is what we turn our attention to next.

Homophily is such a prevalent mechanism that it might seem very intuitive. It seems reasonable to assume that you have more in common with your friends than you have with strangers. Despite its prevalence, few studies have been made to gauge the importance of the effect of homophily on social interactions. With that said, large-scale studies on the effect of homophily has been made. One such study was made by Kossinets and Watts (2009) which estimated how observed homophily manifested in real dynamical social networks. They recorded social interactions on a large university over the course of a year. These interactions allowed the researchers to gauge not only how social ties changed over time, but also how similar individuals of the social networks were. Their findings not only suggests that the intuitive notion of homophily is true, but that it is a special case of a more general phenomenon. The findings suggest that distance in similarity is a function of distance in the social network (see Fig 1). The further you are removed from someone in the social network, the less you will have in common (Kossinets & Watts, 2009). The connection between distance in similarity and distance in the social network seems to be a robust finding of social networks (Bener et al., 2016; Crandall et al., 2008). Choice and strurucal homophily both contribute to the link between distance in similarity and distance in social networks. Moreover, the effect becomes more extreme as interactions between peers tend to make like-minded individuals even more like-minded (Friedkin & Johnsen, 1990; Spears, 2021). This influence that can be exerted through social interactions is called social influence, and it is on this topic that we now turn our attention.

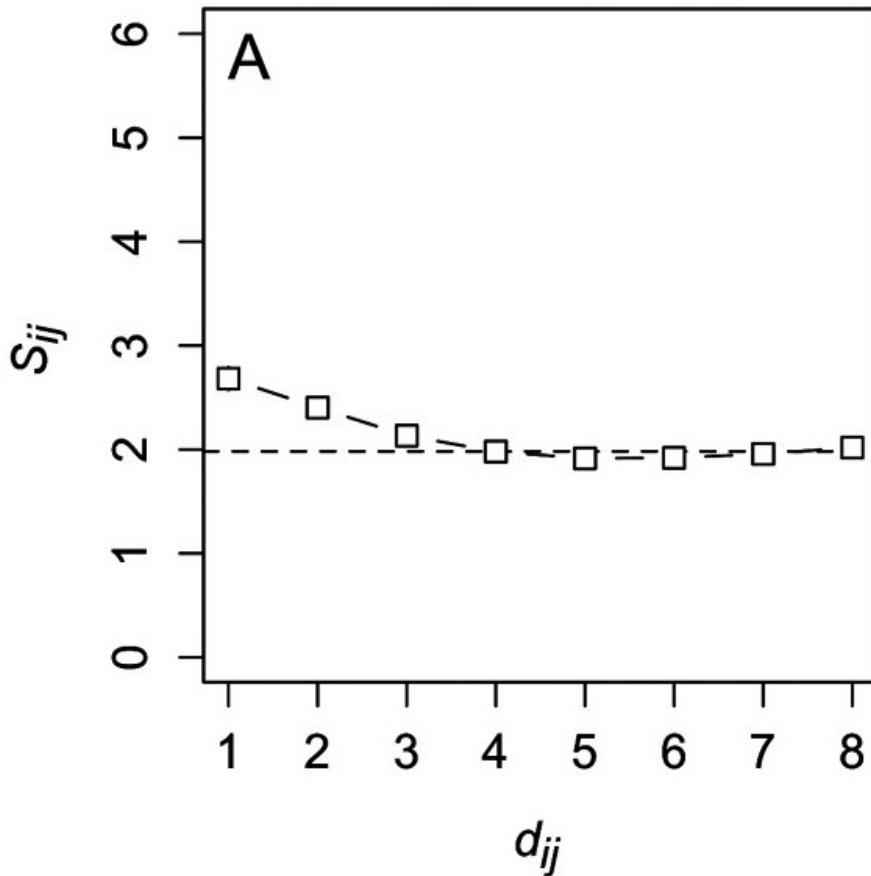


Figure 1: The link between distance and similarity. Adapted from Kossinets and Watts (2009). X-axis shows the distance in the network to other agents. Y-axis shows the average similarity between agents.

### 3.2 Social Influence

Humans are inherently a social species. We think and form our opinions by discussing them with other people. Either via social media, debates or discussions with friends, opinions are constantly being shared between individuals. As a result of these social interactions, the opinions of people involved in the interaction might change. The effect that social encounters and social relations has on the opinions of the involved agents is referred to as social influence. Social influence has been proposed as the most important effect in shaping an individual's opinions (Chacoma & Zanette, 2015). In this paper, we will distinguish between positive and negative social influence. Positive social influence refers to interactions that result in more similarity between agents after interacting (Flache et al., 2017; Levin et al., 2021). This can be thought of as reaching a compromise, finding common ground or seeing the other person's perspective on a topic. Negative social influence refers to interactions that result in less similarity between agents after interacting (Flache et al., 2017). Examples include moral outrage, outgroup aversion or similar distancing effects. It is important to note that although positive influence seems to be a robust empirical finding, negative influence is more elusive (Flache et al., 2017; Takács, Flache, & Maes, 2014; Turner & Smaldino, 2018). With that said, there is evidence that suggests that exposure to opposing views can lead to increased polarization (Bail et al., 2018).

## 4 Opinion Dynamics

The effects of homophily and social influence has been studied extensively with different types of formal models, primarily agent-based models (Flache et al., 2017; Flache, 2018; Noorazar et al., 2020). These simplified systems are normally studied to identify which conditions can give rise to polarization or consensus in terms of the opinions of simulated agents (Flache et al., 2017). Previous models have largely only focused on how social influence affects a system over time, without considering how the social interactions change as a result of changes in opinions (Galesic et al., 2021; Holme & Newman, 2006; Jalili, 2015). Central to most classical models of opinion dynamics is that agents are situated in static, theoretical networks (Flache et al., 2017). The assumption of static networks is not inconsequential. Social networks are inherently dynamic, with new ties being formed and deleted over time. But as has already been established, empirical work suggests that these two processes are largely interdependent (Bener et al., 2016; Kossinets & Watts, 2009). New social interactions are more likely between similar agents, and tie deletion is also more likely between dissimilar agents (Kossinets & Watts, 2009). Although all models do some violence on the world by making assumptions, models should try to include the vital mechanisms of the system they are representing, while simplifying the system enough to understand it (Epstein, 1999; P. Smaldino, 2016). Considering the empirical evidence (Bener et al., 2016; Crandall et al., 2008; Kossinets & Watts, 2009), it is one of this paper's central claims that coevolution should be considered a vital mechanism. In order to support this claim, we need to expand our understanding of the basic underlying mechanisms of social influence to also incorporate the underlying mechanisms of network formation. We therefore turn our attention to identifying empirically plausible micro-behaviors of network formation.

## 5 Network Formation

In order to identify realistic mechanisms of network behavior, it is fruitful to first survey previously considered networks in opinion dynamics. Classic examples of the static, theoretical networks used in most models include ring lattices, small-world networks and scale-free networks (Barabási & Bonabeau, 2003; Watts & Strogatz, 1998). Both small-world and scale-free networks are famous for being able to capture essential features of real social networks. Namely, small-world networks are able to capture the general tendency of large average clustering coefficient and small average path length (Watts & Strogatz, 1998). Famously, this is achieved by starting with a ring lattice and rewiring a small percentage of the ties randomly, which creates highways of information that decreases the average path length considerably (Watts & Strogatz, 1998). On the other hand, scale-free networks can capture the general tendency of long tails in degree distributions of social networks. That is, the degree distribution of social networks often follows a power law, where a few nodes have an extreme amount of edges, while most nodes only have a few (Barabási & Bonabeau, 2003). Recent work seems to call into question how universal power laws are in empirical social networks, and finds instead that most networks follow a lognormal distribution, and not a power law (Broido & Clauset, 2019). It is noteworthy that neither of the theoretical networks can capture the essential features of social

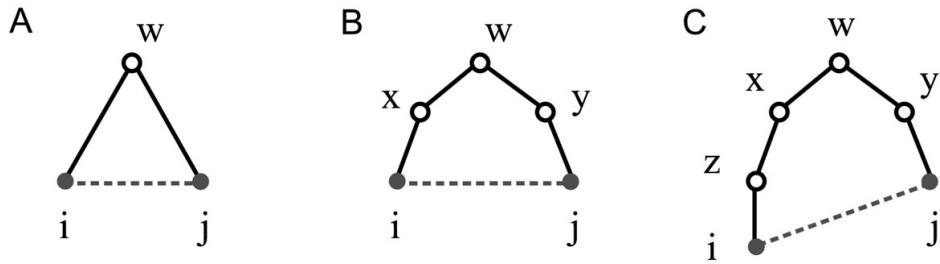
networks simultaneously (Jackson & Rogers, 2004). As these features are likely to be critical to the behavior of how the agents interact in such a network, we need more empirically plausible models of social network formation.

## 5.1 Triadic Closure

An alternative to the preferential attachment of scale-free networks and the rewiring of small-world networks is to consider what mechanisms govern how real social networks change over time. Both scale-free and small world networks attempt to answer the question of how networks characteristics could be generated, not why social networks have the characteristics that they do (Jackson & Rogers, 2004). An attempt at answering the why-question is the work by Jackson & Rogers (2004), which emphasizes the role of triadic closure in network generation. Reliably, triadic closure is found to be the most important and robust mechanism for how new connections are made in social networks (Asikainen et al., 2020; Bianconi, Darst, Iacovacci, & Fortunato, 2014; Kossinets & Watts, 2009; Peixoto, 2022). Triadic closure refers to generating new connections by selecting from “friends of friends”. Empirical studies on dynamical networks find that the probability of creating a new tie is a decreasing function of the distance in the network (Bener et al., 2016; Kossinets & Watts, 2009). The less separated you are from someone, the more likely you are to form a social tie to this person. Specifically, the empirical study suggests that when you are removed one rather than two degrees of separation from someone, you are 30 times more likely to form a connection. This increase in likelihood only increases with distance. When you are 5 degrees of separation away, you are 2.500 times less like to form a connection than you would have been were you only removed by one degree of separation (Kossinets & Watts, 2009).

Using triadic closure as the generating principle for network formation has shown great promise in explaining some of the key characteristics of social networks. In formal models, this is implemented by making new connections predominantly by triadic closure while letting a small percent of new ties be formed at random (Ilany & Akçay, 2016; Jackson & Rogers, 2004, 2007). These models can generate the important characteristic findings of high average clustering coefficient, low average path length and lognormal degree distributions (Jackson & Rogers, 2004, 2007). It is worthwhile to take time to understand why these measures are important and why these models generate the patterns that they do. Average clustering coefficient is a measure of connected local communities are in the network (Watts & Strogatz, 1998). Models of triadic closure are likely to exhibit high average clustering coefficients as triadic closure will increase the number of local links. This is closely linked to the fact that social networks are often made of tight-knitted, well-connected local communities (Peixoto, 2022). Average path length refers to average length of the shortest path between any node in the network. Here, the implemented formal models of triadic closure benefit from similar mechanisms of randomness as small-world networks to achieve low average path lengths (Jackson & Rogers, 2007; Watts & Strogatz, 1998). Having even a small amount of randomness can create long-range connections, that acts like shortcuts between nodes, decreasing the average path length substantially (Watts, 1999). Path length is important for how fast information or contagious diseases can spread between individuals in a network. Finally, triadic closure will lead to lognormal or scale free degree distributions, as high degree nodes have more possibilities for being selected for tie generation simply because they have larger degrees (Jackson & Rogers, 2007). The principle of triadic closure seems to not only characterize human social networks, but could be a robust organizing principle for many different

social species (Ilany & Akçay, 2016).



*Figure 2: Tie closure.* Taken from Kossinets and Watts (2009). Panels show different possible tie-closures. The first panel (A) depicts closing ties with a friend of a friend. This type of tie-closure is called triadic closure. Panel B and C shows longer cycles of closure.

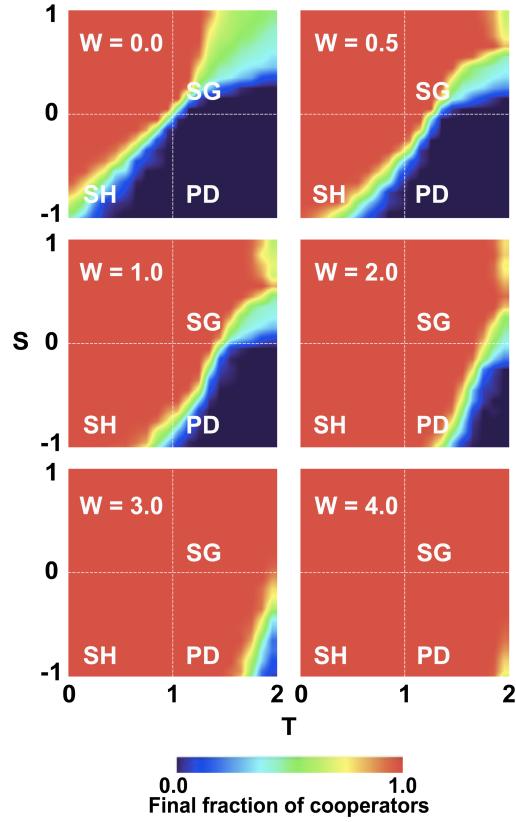
## 6 Co-evolution via tie-deletion

Now that we have established the fundamental principles of opinion and network dynamics, we now turn to the mechanisms that could enable them to co-evolve. It is easy to see how opinions could be influenced by network dynamics. As the social network changes, the available interactions for each agent will change. As one of the primary forces of opinion dynamics is social influence, changes in the social network will have large effects on the opinions of the social agents of the network. This is one side of the coin. As the social network already influences opinion dynamics, we need to identify mechanisms that can change the ties of the social network based on the opinions of its agents. Any mechanism that changes the ties in the network based on the opinions of the agents will cause opinion and network dynamics to co-evolve. There are good reasons to believe that there are many effects that live up to this criteria, as we have seen with the effect of observed homophily. Similarity is a strong predictor of which pairs form new ties (Bener et al., 2016; Kossinets & Watts, 2009). In the same vein, dissimilarity also predicts the breaking of social ties (Bener et al., 2016; Kossinets & Watts, 2009; Levin et al., 2021). Although both mechanisms are at play in social networks, we will focus our efforts on tie-deletion of dissimilar ties. The reason for this choice is that there are already existing work focusing on the effects of tie-deletion in similar models (Santos et al., 2006; Sasahara et al., 2021). This allows us to examine the robustness of the effect of tie-deletion on opinion dynamics. Understanding the previous findings concerning co-evolution and the effect of tie-deletion is critical for understanding this paper. Because of this, we turn to presenting the main findings from the previous literature.

### 6.1 Tie-deletions effect on cooperation

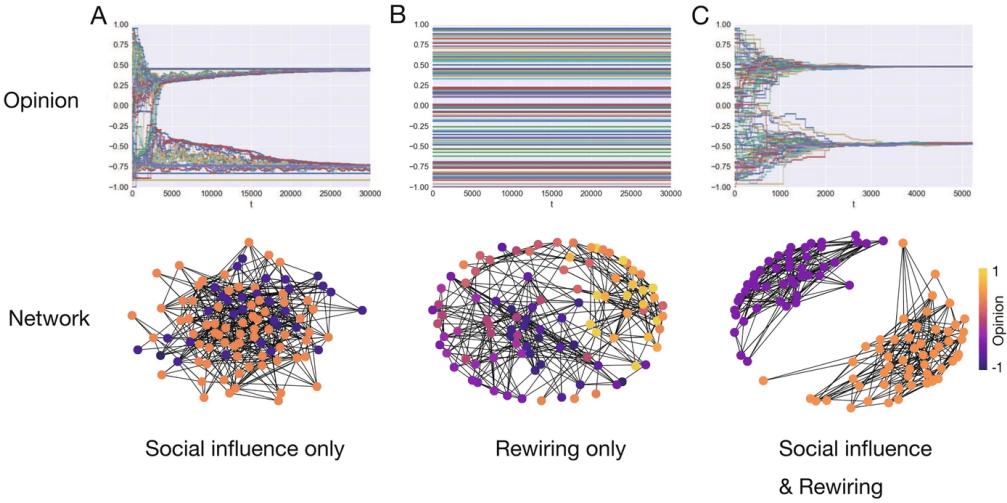
In addition to the empirical work investigating the interplay between social networks and the opinions of their constituents, theoretical work also points to the primacy of the co-evolution of networks (Holme & Newman, 2006). These

studies come specifically from another field than opinion dynamics interest in identifying the mechanisms that facilitate cooperation. The field I am referring to is the field of computational biology. In many ways, this field should be considered an adjacent field to opinion dynamics, as they share much in their interests and in their methods. The literature from computational biology suggests that co-evolution can play a vital role in facilitating cooperation (Dakin & Ryder, 2018; Melamed & Simpson, 2016; Pepper & Smuts, 2002; Santos et al., 2006). Intuitively, one might expect that when connectivity increases in such a network, cooperation increases as well. However, the opposite effect is observed. In their paper, Santos et al. (2006) considers agents playing game-theoretic games, where agents either cooperate or defect. In relation to the mechanism of tie-deletion, Santos et al. (2006) proposes a way for agents to adjust their ties based on interactions made with other agents. When cooperators interact with defectors, the social tie between cooperators and defectors has a chance of being rewired to a different agent. This keeps the number of edges constant, but decreases connectivity (Santos et al., 2006). Their results indicate that as the propensity for deleting ties between cooperators and defectors increases, cooperation flourishes. The more likely tie-deletion is, the more cooperation evolves in the system. The reason given for this result is that removing ties between cooperators and defectors increases positive assortment between cooperators and decreases positive assortment between defectors and cooperators. One of the striking attributes of this finding is that without any propensity for tie-removal, cooperation does not evolve for conditions equivalent to the Prisoner's Dilemma (Santos et al., 2006). More generally, positive assortment has been found to be a robust facilitator of cooperation in computational biology (Boyd, Gintis, & Bowles, 2010; Dakin & Ryder, 2018; Melamed & Simpson, 2016; Pepper & Smuts, 2002). The results also indicate that even relatively simple topological network dynamics that reflect the individual agent's reaction to their social interactions can produce realistic networks (Santos et al., 2006).



*Figure 3:* Tie-deletions facilitates cooperation. Taken from Santos et al. (2006). Panels show simulations of different values of  $W$ . In this model,  $W$  controls the propensity to delete ties between defectors and cooperators. Y-axis shows the disadvantage of being cheated, x-axis shows the pay-off of cheating. Letters on the panel represent well-known game theoretic dilemmas (Prisoners Dilemma, Stag Hunt and Snow Drift Game). Colors show the final fraction of cooperators after simulations are completed. When  $W = 4.0$ , cooperation effectively wipes out defectors completely.

Although positive assortments effect on cooperation seems general in the computational biology literature, the opinion dynamics literature points to an opposite result. When combined with social influence, they report that tie deletion can lead to echo chambers, that stifle cooperation (Sasahara et al., 2021). This is more in line with a more intuitive explanation, where a decrease in the communication between agents leads to a decrease in cooperation. Obviously, computational biology and opinion dynamics are not same field, but it is still noticeable that the same underlying mechanism gives rise to effects of opposite directions on a networks ability to cooperate. Both studies are interested in how the dynamics of the network influences how well agents cooperate in the network, and they both find that tie-deletion is critical to developing cooperation (Santos et al., 2006; Sasahara et al., 2021). It is precisely because of this dispute that this paper is especially interested in how the deletion of negative ties can impact cooperation. Like the work of Santos et al. (2006), the probability of deleting negative ties will therefore be a key aspect of our model.



*Figure 4:* Tie-deletion accelerates echo chambers. Taken from Sasahara et al. (2021). First row shows the opinions for the simulation, second row shows the network after simulation. Columns show simulations of only including social influence, only including rewiring based on opinions or both respectively. Of special interest is the last panel, which shows that rewiring can accelerate echo chamber formation, as the opinions stabilize faster than when only social influence is included.

## 7 Model Description

Next, I present the description for the agent-based model used to investigate the co-evolution of opinion and network dynamics. The description first introduces the key assumptions of the model. Afterwards, the specifics of the underlying behavior of the agents at every stage of a time-step is explained. The section concludes by presenting the outcome metrics recorded from the model.

### 7.1 Conceptual overview

This agent-based model consists of agents situated in an undirected network. Nodes of the network represent social agents. Edges represent social ties to other agents. As the network is undirected, all relations between agents are assumed to be reciprocal. The model does not have agents with tasks to perform or strategies to follow. The system the model is trying to represent is a system where social agents interact with their friends and discuss their opinions with each other. Such a system is constantly changing, as social agents create ties with new friends, sever ties to old ones and change their opinions.

### 7.2 Assumptions

To investigate such a complex system as outlined above, several assumptions must be made in the agent-based model. It is assumed that the opinion of an agent is shaped only by her initial opinion and the influence of her peers. Agents

are not trying to find any "true opinion" or act in a way to hide their own opinion from their peers. Instead, agents are assumed to share their opinions truthfully to other agents, and to be able to perceive the exact opinion of their peers. In line with previous models of opinion dynamics, the model assumes positive social influence between agents of similar opinions. In other words, agents of similar opinions will reach a compromise, drawing their opinions closer together. Similarly, the model assumes negative or no social influence between dissimilar agents. Agents will either not be influenced at all, or will further distance themselves from agents of dissimilar opinions from their own. Regarding the social network of the model, it is assumed that connections between agents are not static but dynamic in nature. The model assumes that agents will find new connections primarily through their already existing connections via the mechanism of triadic closure. Finally, it is assumed that agents will tend to delete ties to dissimilar agents. All these assumptions can vary in the strength of the proposed effect. For instance, similar agents could reach a perfect compromise or only convince each other slightly. As the results of the model critically hinges on the strength of these assumptions, the model parameters which control the strength of these effects are varied to allow the investigation of their influence on each other.

### 7.3 The stages of the model

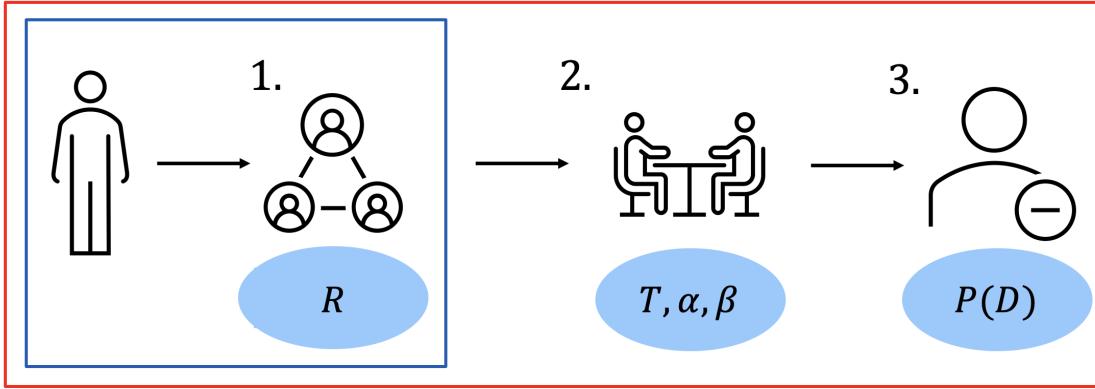
The agent-based model is divided into three distinct and sequential stages that are executed at every time-step. These stages is referred to as the network dynamic stage, the opinion dynamic stage and the co-evolutionary stage respectively. The network dynamic stage specifies how new edges are created in the social network of the model. The behavior of this stage of the model is controlled by one parameter,  $R$ .  $R$  specifies the probability of generating random ties rather than ties via triadic closure. The opinion dynamic stage describes how interactions between agents change the opinions. The opinion dynamics are controlled by three parameters:  $T$  specifies the threshold for what constitutes a similar agent,  $\alpha$  specifies the power of positive social influence and  $\beta$  specifies the power of negative social influence. Finally, the co-evolutionary stage specifies the tendency for agents to delete connections that are dissimilar from themselves. This is controlled by the parameter  $P(D)$ , which describes the probability of deleting ties to dissimilar agents. The specifics of each stage of the process is introduced in detail in the section called "Dynamics".

Before specifying the details of these stages, it is important to mention that while this paper mainly focuses on one agent-based models, it develops and compares two models. This is done to be able to investigate how the co-evolution of opinion and network dynamics differs from only considering network dynamics. One the models considered is therefore an agent-based model which only contains the network dynamic stage as described above. This model therefore doesn't include opinion dynamics and is best considered an algorithm of network formation. The aim of this model is to serve as a baseline for the full model, which includes all three stages described above. The comparison between these two models will become important in evaluating the agent-based model with empirical data. I will refer to the model that only includes the network dynamic stage as the Network Formation Model and the model with all three stages as the Co-evolutionary Model. In order for the Co-evolutionary model to be computationally feasible, only a subset of the possible values of each of the five parameter of the model is considered. All different possible combinations of each parameters are simulated, resulting in 3780 different parameter combinations. The considered

parameter values are:

$$\begin{aligned} R &\in \{0.1, 0.3, 0.5\} \\ \alpha &\in \{0.05, 0.10, 0.15, 0.20, 0.25\} \\ \beta &\in \{0.00, 0.05, 0.10, 0.15, 0.20, 0.25\} \\ T &\in \{0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2\} \\ P(D) &\in \{0.0, 0.2, 0.4, 0.6, 0.8, 1.0\} \end{aligned}$$

Some justification of the different parameters are in order. First, it is fair to assume that  $\alpha$  and  $\beta$  are powerful because social interactions are so frequent, but not because of the strength of social interactions themselves. One rarely reaches a perfect compromise when debating other people, which is what  $\alpha = 1$  would be. It therefore makes sense to assume that realistic values  $\alpha$  will be fairly low. The same argument applies to  $\beta$ . Negative social influence is as mentioned less empirically validated than positive social influence (Takács et al., 2014). I therefore also include simulations with  $\beta = 0$ .  $T$  was chosen as these parameters gave the most interesting results in preliminary analyses. For  $T > 1.2$ , the model more or less always converges to consensus no matter what the values of all other parameters are. In the same vein, only very few simulations avoid polarization for  $T < 0.6$ . The parameters of interest for this paper are therefore in between these two values, where  $0.6 \leq T \leq 1.2$ . For values of  $R$ , we have good reason to believe based on previous empirical work that most interactions are made via triadic closure and are not random (Kossinets & Watts, 2009). It is therefore fair to assume that new connections are primarily generated via triadic closure and are atmost randomly generated 50% of the time ( $R = 0.5$ ). Finally, as co-evolution is one of this papers primary interests, I study the whole range of the co-evolutionary force of  $P(D)$ , the probability of tie-deletion. I therefore consider a range of values resulting in different levels of co-evolution in the system. Values of interest range from no co-evolution to perfect co-evolution i.e.  $0 \leq P(D) \leq 1$ .



*Figure 5:* Flowchart of a single timestep. Numbers indicate different parts of the process of a single time-step. Squares of different colors indicate time-steps of different models. The blue square depicts a single time-step for the Network Formation Model, which does not include any opinion dynamics. The red square indicates the Co-evolutionary Model, which includes both network and opinion dynamics. Blue bubbles show what model parameters are relevant to the particular step in the process. A random agent,  $A_t$  is sampled from the network. Next, the network dynamic stage begins (1). With a probability of  $R$ , a new connection is made to another randomly sampled agent. With a probability of  $1 - R$ , a new connection is made via triadic closure. This is the only stage included in the Network Formation Model. After the network dynamics stage, the Co-evolutionary Model starts the opinion dynamic stage (2). The sampled agent,  $A_t$ , pushes and pulls iteratively on all the opinions of her neighboring agents. Whether opinions are pushed or pulled is determined by whether the distance between them is further than  $T$  apart. How much they are pulled or pushed is determined by  $\alpha$  and  $\beta$  respectively. After this process, the co-evolutionary stage begins (3). With a probability of  $P(D)$ , ties to agents with opinions further than  $T$  apart from  $A_t$ 's opinion are deleted. This concludes one time-step of the Co-evolutionary Model.

## 7.4 Initialization

The agent-based model initializes with first making a random Watts-Strogatz small-world graph with  $N = 500$ ,  $k = 7$ , and  $p = 0.5$  (Watts & Strogatz, 1998). This acts as the initial social network of the agents. For each node in the network, an agent is initialized with a value ( $O_I$ ) which is taken to represent the opinion of the agent. The opinion is modelled as a continuous number between -1 and 1. This is done to reflect that opinions are rarely binary, but are instead better represented as a spectrum between opinions that are extremely pro some notion and opinions that are extremely against some notion. In line with previous research the initial opinions of agents are assumed to be diverse. To implement diversity of initial opinions, all values of  $O_i$  are initialized by drawing from a uniform distribution between -1 and 1:

$$O_I \sim U(-1, 1)$$

As a consequence, no agent will be initialized with an opinion lower than -1 or higher than 1. Additionally, the expected mean of the distribution of initial opinions is 0.

## 7.5 Dynamics

### 7.5.1 Network dynamic stage

The model is an attempt at representing social encounters and their change over time. To represent this idea, agents interact with each other over a certain amount of time-steps. During such a time-step, an agent will always create a new connection. At every timestep ( $t$ ) a random agent ( $A_t$ ) is sampled from the network ( $N_t$ ). This agent creates a new edge to another agent in the network. As the dynamics also allows for tie-deletion, I control for the number of edges in the network. To keep the number of edges in the network approximately constant, edges are either created or rewired to account for deleted edges. To do this, let  $E_1$  be the number of edges of  $N_1$  and let  $E_t$  be the number of edges of  $N_t$ . If  $E_t < E_1$ ,  $A_t$  will not rewire one of its existing edges, but instead create a new edge. If  $E_t \geq E_1$ ,  $A_t$  will rewire one of its existing connections to the new agents. With a probability of  $R$ ,  $A_t$  connects to a random agent, that  $A_t$  is not currently connected to. With a probability of  $1 - R$ ,  $A_t$  connects to one of its edge's edges. Regardless of whether the connection was sampled randomly or through an edge's edges, let  $C_t$  define the newly connected agent. In order to calculate the average path length of the network, the model is restricted to being only one component. Simply put, having only one component guarantees that there exists a path between any two nodes of the network. To ensure this property of the network, we check the degree of  $A_t$  and  $C_t$ . If the degree of either  $A_t$  or  $C_t$  is 0, a new edge is created randomly to a new node from  $N_t$ . If both  $A_t$  and  $C_t$  have degrees larger than 0, and the network has more than one component, a new edge is created to restore the network. Specifically, we restore the edge that  $A_t$  rewired to  $C_t$ , while keeping the edge from  $A_t$  to  $C_t$ . I will refer to the process of ensuring only one connected component simply as component insurance.

### 7.5.2 Opinion dynamic stage

After finding a new connection, the agents interact with each other. This is meant to represent discussion or other social interactions between agents. Social interactions between similar individuals will lead to compromise between their initial opinion and their opinion after the encounter. As a consequence, the more you disagree with a person, the more malleable your opinion is. If you reach a compromise with a person with a very different opinion than yours, you will change your opinion more dramatically than if you reached a compromise with someone that you almost agree with. To model this kind of behavior, the randomly sampled agent,  $A_t$ , interacts with all its connected agents and changes its opinions according to the opinion of the connected agents. This is done by iteratively interacting with every edge of  $A_t$ . Let  $B$  denote an edge of  $A_t$ . Let  $O(\cdot)$  define a function with agents as inputs and their opinions as outputs. The interaction between two different agents is determined by a threshold value,  $T$ .

When  $T \geq |O(A_t) - O(B)|$ , the interaction will pull the opinions of the two agents closer to each other. The force with which they are pulled is defined as a fraction of their distance from each other:

$$V_p = (|O(A_t) - O(B)|) \cdot \frac{\alpha}{2}$$

Let  $O_{max} = \max(O(A_t), O(B))$  and  $O_{min} = \min(O(A_t), O(B))$  and update the values of opinions by:

$$O'_{max} = O_{max} - V_p$$

$$O'_{min} = O_{min} + V_p$$

where  $O'_{max}$  and  $O'_{min}$  are the respective updated values of  $O_{max}$  and  $O_{min}$  after interacting with each other. When  $T < |O(A_t) - O(B)|$ , the interaction will push the opinions of the two agents further apart by a similar principle as illustrated above:

$$V_n = (|O(A_t) - O(B)|) \cdot \frac{\beta}{2}$$

Using the same definitions as above, we update the opinions of each agent:

$$O'_{max} = O_{max} + V_n$$

$$O'_{min} = O_{min} - V_n$$

This process concludes when  $A_t$  has updated her values for all her edges.

### 7.5.3 Co-evolutionary stage

After interacting with her connected agents,  $A_t$  might delete the connection to some of these agents based on their similarity. In this model, social interactions can result in opinions becoming more distant than they were initially. This can be because what the agents are discussing is divisive or controversial. When this happens, connections can be severed. I call this process tie-deletion. If  $T < |O(A_t) - O(B)|$ , there is a chance that their tie is deleted after interacting. This probability of tie deletion between dissimilar agents in the model is described by the parameter  $P(D)$ . When  $P(D) = 1$ , all dissimilar ties are severed. When  $P(D) = 0.5$ , there is 50% chance that dissimilar ties will be deleted. After ties are deleted, the process of component insurance is performed where  $C_t$  is replaced with  $B$ . When all edges of  $A_t$  are evaluated, the time-step concludes. This process repeats for 10.000 time-steps.

## 7.6 Outcome metrics

### 7.6.1 Time-dependent Metrics

Every 20th time-step, the current state of the network is recorded. To track the polarization of opinions over time, the mean and standard deviation of the absolute value of opinions are recorded. To evaluate the effect of tie-deletion, the cumulative frequency of deleted ties are recorded. For characterizing the network dynamics over time, I record the

average clustering coefficient and the average path length of the network. At every 500th time-step, the opinions of every agent is recorded. This is used to track how opinions change over time.

### 7.6.2 Final State Metrics

After 10.000 time-steps, the network reaches its final state. At its final state, I record the initial opinion of every agent ( $O_I$ ) and the final opinion of every agent ( $O_F$ ). I then calculate the difference between the absolute value of the final and the initial opinion of every agent ( $|O_F| - |O_I|$ ). This gives a measure of whether the final opinion of agents are more extreme than their initial values were.

### 7.6.3 Correlations

To measure how much initial opinions dictate the final opinion of agents, I calculate the Pearson Correlation Coefficient between the final and the initial opinions of the agents ( $\rho_{O_I, O_F}$ ). Similarly, I calculate the Pearson Correlation Coefficient of the estimated average path length and mean of the absolute value of opinions of all agents over time ( $\rho_{|O|, APL_*}$ ). This gives a measure for how the polarization of the agents influences the distance in the network.

## 8 Model Evaluation

### 8.1 Fitting the model

As mentioned, previous models have often not integrated empirical data of social networks sufficiently to test the validity of their models (Flache et al., 2017). What is meant by this critique? Arguably, the most pressing issue is that models rarely consider how the results of their computational models compare with the real world. This is largely a product of the types of agent-based models typical in opinion dynamics as well as the phenomena of opinion dynamics themselves. Most models of opinion dynamics are low-dimensional (Bener et al., 2016), in that they don't seek to explain exhaustively how opinions are formed, but rather to point out interesting interactions between key variables of the process. This makes empirical validation especially tricky for these models. Compounding to this problem is that the time-scales considered by these models are massive, and that kind of data is very hard to come by (Mäs, 2019). And even if they were available to us, opinions are very hard to measure. Any self-reported measure is going to be subject to the usual problems of the unreliability of self-reports. Using political voting patterns might be a usable proxy for polarization, but these data are very hard to come by. This does not however mean that incorporation of empirical data is impossible. Because the model considered in this paper is coevolutionary, it allows us to investigate how the generated networks compare to real empirical social networks. By borrowing methodology from computer science, we will treat this as a machine learning problem, and investigating the goodness of fit using hyperparameter optimization (Akiba et al., 2019; Bergstra et al., 2011). What we really want to know is to what degree the model can explain the

patterns observed in real world social networks. To do this, we want to estimate the model parameters by generating networks from the model, and then see how much these generated networks resemble real networks. I will refer to the process of finding these parameter values as agent-based model fitting, as it is analogous to estimation of parameter values in statistics. Beyond knowing whether the model can generate the observed structure of real-world networks, this paper is specifically interested in how critical co-evolution is to the explanatory power of these models. Ideally, we want to compare models which include co-evolution to models without co-evolution and investigate whether co-evolutionary models are better explanations than their network formation counterparts. I will refer to this process as agent-based model comparison, which is again reasoned by its analogy to model comparison in statistics. Although these steps are analogous to model fitting and model comparison, there are important differences worth mentioning. Especially important is the fact that model comparison normally penalizes the inclusion of additional parameters in the model (Vrieze, 2012). In other words, simple explanations are preferred when the performance is comparable (Emiliano et al., 2014; Vrieze, 2012). The primary reason for this is that including additional parameters allows the model to overfit to the data. In classical regression models, models with additional parameters will always give a better fit, as the scalar for the new parameter can simply be set very close to zero. In this case, the model becomes mathematically identical to the simpler explanation. Although we don't have a formal way of calculating agent-based equivalents to Akaike Information Criteria or Bayesian Information Criteria to determine which model is the better explanation (Vrieze, 2012), we can try to avoid situations of mathematical identity and overfitting. This is done by framing the problem of agent-based model fitting as a hyper parameter optimization problem.

## 8.2 Data

To fit the model to data, we consider 7 empirical social networks; a network of representing social networks of Dolphins (Lusseau, 2003), the Karate Club Network (Zachary, 1977), a citation network (Newman, 2006), co-purchase network of political books on Amazon (Shi et al., 2017), political blogs (Adamic & Glance, 2005) and network of politicians based on shared likes on Facebook (Rozemberczki et al., 2018). For all networks, only the main component of the network was considered. This was done as average path length is not well-defined for unconnected graphs. As the model assumes undirected networks, the network of political blogs was transformed from a directed to an undirected network. This was done by making any directed link between two agents be undirected.

## 8.3 Hyperparameter Optimization

To evaluate how closely the generated networks resemble the real networks, we must first define a distance metric between networks. Although many features are important for characterizing a network, we will follow previous suggestions that identify the average path length, the average clustering coefficient and the degree distribution as the most defining aspects of a network (Jackson & Rogers, 2004, 2007). We therefore seek to minimize the mean difference on these parameters between the generated and the real networks. Before we can simply calculate the mean of difference, there are some specifics that needs to be clarified. These primarily concern the average path length and how to

operationalize the difference in degree distributions. To identify the distance of degree distributions of networks, we calculate the Jensen Shannon Divergence between the two distributions. The Jensen Shannon Divergence is a distance metric between probability distributions (Fuglede & Topsoe, 2004). In terms of the average path length, there is a problem of computational costs. Calculating the average path length can be very expensive computationally, especially for large networks (Matsumura et al., 2018). Instead of calculating the actual average path length, we approximate it by taking 1000 samples of random pairs of nodes and calculating the average path length of those 1000 nodes. Although this is still only an estimation of the actual average path length, this method generally approximates the average path length fairly well (Matsumura et al., 2018). Both the correlation coefficient and the Jensen Shannon Divergence are values between 0 and 1. Average path length on the other hand is not restricted in the same way. If it is not normalized, it will therefore be weighted much higher than the correlation coefficient and the Jensen Shannon Divergence in the distance metric. We therefore normalize the average path length by

$$APL* = \frac{|APL(G) - APL(A)|}{APL(A) + 2}$$

where  $APL*$  is the normalized average path length,  $APL(G)$  is the approximated average path length of the generated network and  $APL(A)$  is the apprioximated average path length for the actual network. Adding two to the denominator is somewhat arbitrary. The choice was made to be more certain that  $APL*$  would still be below 1. With this in place, the final distance metric considered between the actual network,  $A$  and the generated network  $G$  is

$$O(A, G) = \frac{1}{3} \cdot (JSD(D(A), D(G)) + |C(A) - C(G)| + APL*)$$

where  $O(A, G)$  is the distance between the two networks,  $JSD$  is the Jensen Shannon Divergence,  $D(A)$  is the degree distribution of  $A$ ,  $D(G)$  is the degree distibution of  $G$ ,  $C(A)$  is the average clustering coefficient of  $A$  and  $C(G)$  is the average clustering coefficient of  $G$ . This distance metric is just the average difference on the considered network characteristics between  $A$  and  $G$ . This is also why the sum of the difference is multiplied by  $\frac{1}{3}$ . This distance metric between networks,  $O(A, G)$  will serve as the objective function that we want to minimize. The hyperparameter optimization algorithm search the parameter space for the combination of parameters that results in the lowest values of  $O(A, G)$  (Akiba et al., 2019; Wu et al., 2019).

All models were fit by first calculating the ratio between edges,  $E$ , and vertices,  $V$ , and calculating  $K = \lfloor (2 \cdot \frac{E}{V}) \rfloor$ . The value  $K$  is then used as the  $K$ -parameter in the initially generated small world network to ensure that the generated network has approximately the same number of edges per vertex as the target network. For both models, we let randomness be a free parameter between 0 and 1. For the co-evolutionary model, we restrict the threshold to be between 0.1 and 1.3. The lower limit is chosen as values of threshold close to 0 results in agents that don't agree with anyone else but themselves. The upper limit was chosen to ensure that the behavior of the model differs significantly from the model without opinion dynamics. The values of positive learning rate are restricted to be between 0.05 and 0.5. This is justified by values close to 0 being akin to no positive social influence and values above 0.5 assumes social

influence to be an unrealistically powerful force. Similarly, we let the negative learning rate be restricted to be between 0 and 0.5. Here, we allow values close to 0 to reflect that negative social influence is still a less established fact. Finally, we force the model to exhibit co-evolution by restricting the probability of tie-deletion to be between 0.1 and 1.

Fitting the model to the different networks estimates the best combination of parameters for each given network. These parameters are then used to simulate 10 different networks using different random seeds but the same parameter values. This is done as the model is largely stochastic. Having 10 simulations of the same parameters gives a more robust estimate of how closely the model can recreate the patterns from the empirical networks. After finding the best parameter values, the importance of each parameter value and each network is calculated using a fANOVA (Hutter, Hoos, & Leyton-Brown, 2014).

## 8.4 Results

The results indicate that the network formation algorithm of triadic closure can explain many of the considered characteristics of social networks. This is argued by the fact that the distance between simulated and empirical networks are below 0.25 for six out of seven considered empirical networks (see Fig 9). The Network Formation Model is on par with the Co-evolutionary Model for only the two smallest networks considered, the Dolphins network and the Karate Club Network (see Fig 9 & Table 2). The Co-evolutionary Model drastically outperforms the Network Formation model on larger networks, especially when large networks are expected to be highly opinionated. This is the case for the two networks Politicians and Political Blogs (see Fig 9 & Table 2). These two networks outperform the Network Formation Model primarily due to lower Jensen-Shannon Divergence as well as lower difference in average clustering coefficient between simulated and empirical networks (see Fig 6, 8). A fANOVA was performed for all considered networks of the Co-evolutionary Model. The fANOVA indicates that the  $R$ -parameter was the most important parameter for all networks except for the Political Blogs network. The Political Blogs network has the  $T$ -parameter as the most important instead. The parameter importance of  $T$  are highest for the three political networks (Political Books, Political Blogs, Politicians) (see Fig 10). The best parameters of the models show that all networks except the Karate Club network are characterized by low values of  $R$ . Similarly, we see that most networks show low values of  $\beta$  and relatively low values of  $T$ . This is especially the case for the Politicians and Political Blogs networks, who have  $T \approx 0.1$  and  $\beta \approx 0.01$  (see Table 1).

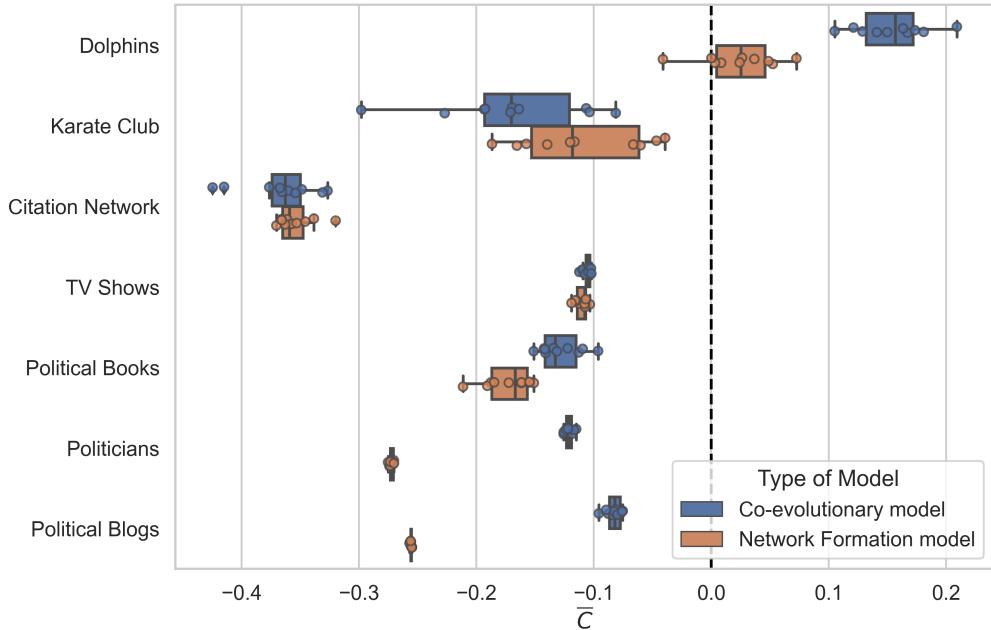


Figure 6: Difference between models and target regarding average clustering coefficient. X-axis shows the average correlation coefficient ( $\bar{C}$ ) of the different networks. Y-axis shows the different empirical networks considered. Colors shows the different models as well as the target value. Black lines indicate 95% confidence intervals.

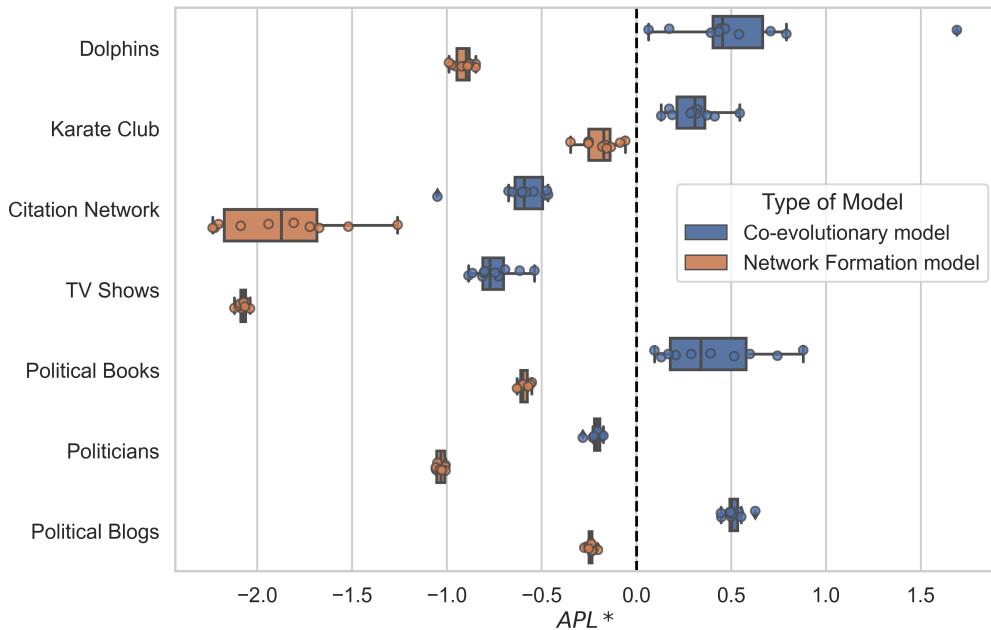


Figure 7: Difference between models and target regarding average path length. X-axis shows the estimated average path length ( $APL^*$ ) of the different networks. Y-axis shows the different empirical networks considered. Colors shows the different models as well as the target value. Black lines indicate 95% confidence intervals.

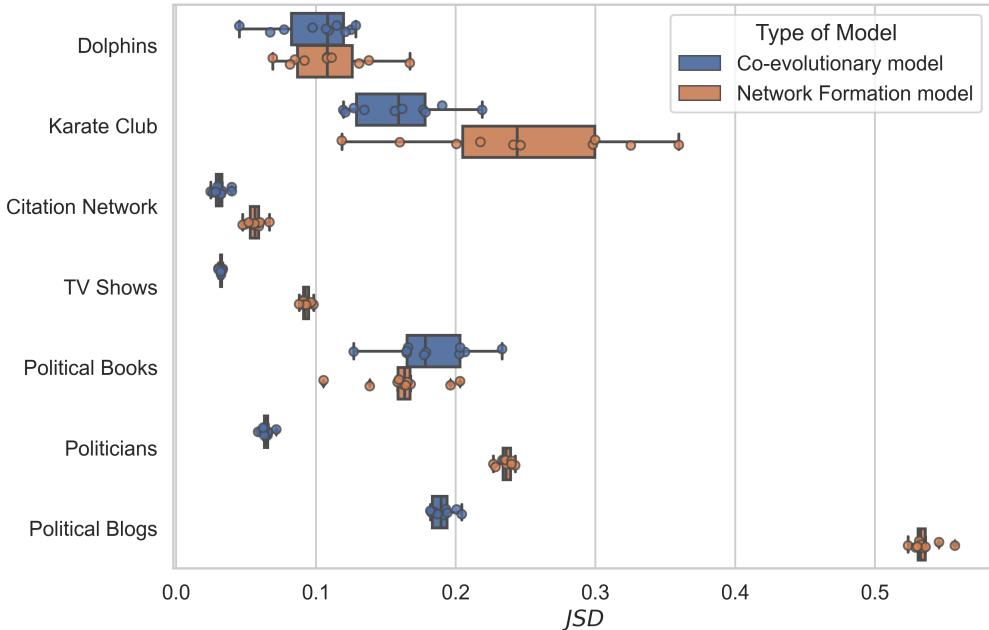


Figure 8: Difference between models and target regarding Jensen Shannon Divergence. X-axis shows the Jensen Shannon Divergence of the degree distribution between the generated and the target network. Notice that as Jensen Shannon Divergence is a distance metric, lower values indicate less difference in degree distributions. Y-axis shows the different empirical networks considered. Colors shows the different models. Black lines indicate 95% confidence intervals.

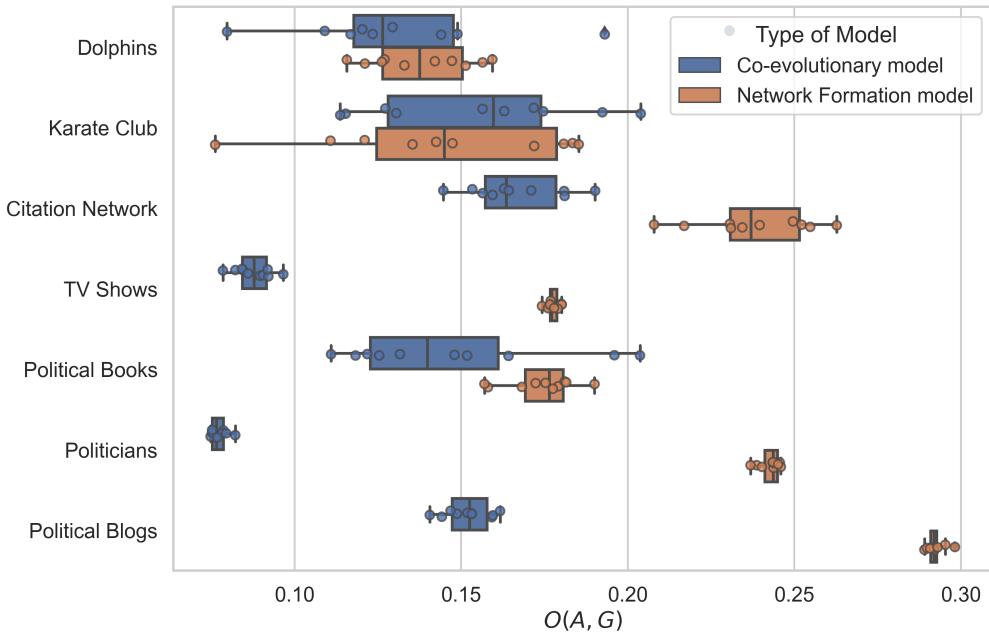


Figure 9: Difference in model performance. X-axis shows the mean difference between the generated network and actual networks, ( $O(A, G)$ ). Y-axis shows the different empirical networks considered. Colors indicate whether the model includes or excludes opinion dynamics. Black lines indicate 95% confidence intervals.

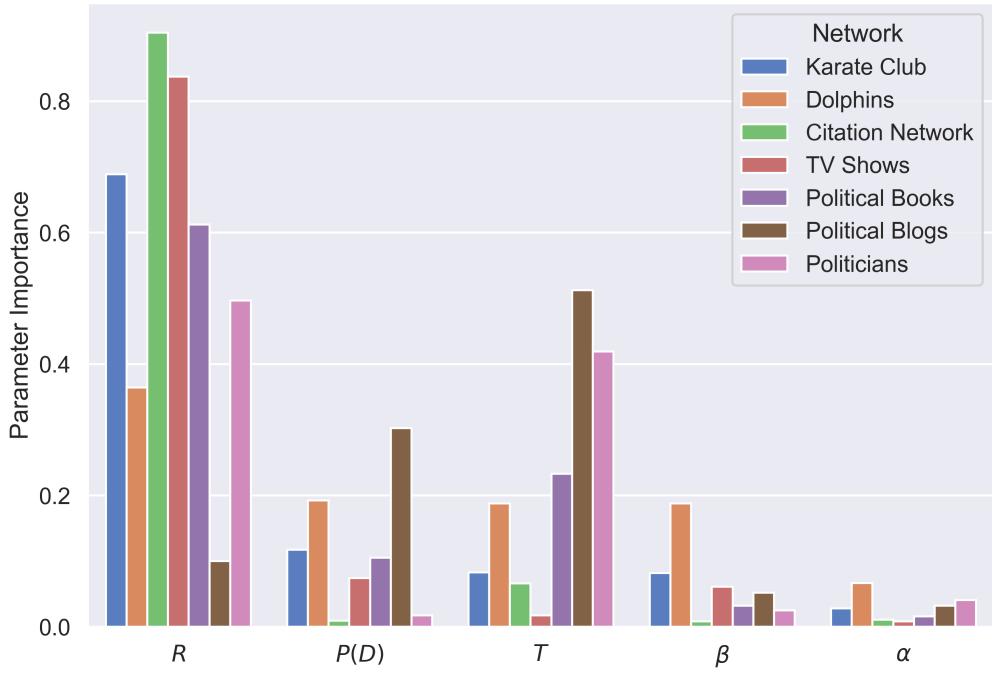


Figure 10: Hyper Parameter Importance. X-axis shows different hyperparameters. Y-axis shows parameter importance. Colors indicate different networks. Parameter importance is calculated using an fANOVA (Hutter et al., 2014).

Table of best parameter values					
Network	R	T	$\alpha$	$\beta$	P(D)
Dolphins	0.003	0.531	0.202	0.095	0.111
Karate Club	0.870	0.137	0.282	0.090	0.911
Citation Network	0.060	0.655	0.389	0.493	0.150
TV Show	0.038	0.157	0.349	0.093	0.751
Political Books	0.004	0.222	0.075	0.144	0.508
Politicians	0.014	0.101	0.050	0.014	0.229
Political Blogs	0.109	0.100	0.246	0.007	0.249

Table 1: Table of best parameter values for the Co-evolutionary model.

Table of network characteristics						
Network	$N$	$E$	$D_\mu$	$D_\sigma$	$\bar{C}$	$APL$
Dolphins	62	159	5.129	2.932	0.259	3.257
Karate Club	34	78	4.588	3.820	0.571	2.326
Citation Network	379	914	4.823	3.927	0.741	6.052
TV Shows	3892	17262	8.871	12.557	0.374	6.324
Political Books	105	441	8.400	5.449	0.488	3.015
Politicians	5908	41729	14.126	20.096	0.385	4.628
Political Blogs	1222	16717	27.360	38.402	0.320	2.737

Table 2: Table of network characteristics.  $N$  is the number of nodes,  $E$  is the number of edges,  $D_\mu$  is the mean degree,  $D_\sigma$  is the standard deviation of the degree distribution,  $\bar{C}$  is the average clustering coefficient, and  $APL$  is the average path length.

## 8.5 Discussion of hyperparameter optimization results

The results from the hyperparameter optimization reiterates the point that triadic closure works well as a basic generating principle of social networks (Jackson & Rogers, 2007; Kossinets & Watts, 2009; Bianconi et al., 2014). Considering that the Network Formation Model only has one parameter,  $R$ , it is able to explain the patterns of social networks fairly well. It seems that this is true especially when the size of the network is small as it is in the Dolphins network and the Karate Club network. It is interesting in its own right that the Network Formation model is on par with the Co-evolutionary model on the Karate Club network, as this network is notoriously a polarized network (Zachary, 1977). The fact that the two models show similar performance on this network seems to suggest that the Karate Club network is not large enough for the two models to perform significantly different. This is argued by the fact that the size of the network seems to be an important attribute difference between the networks where the models achieve similar performance and where the Co-evolutionary model performs better than the Network Formation model.

### 8.5.1 Why the Co-evolutionary model is a better explanation

As the Co-evolutionary model is a more complicated model, it is to be expected that it can fit more complicated patterns. The important question to answer is what the added capabilities of the more complicated model is. In other words, what has the model gained at the cost of higher complexity? The most plausible answer seems to be that it enables the Co-evolutionary model to have multiple neighborhoods within the network. By neighborhoods, I mean a set of nodes that are well-connected locally but not globally. The Co-evolutionary model can achieve this because of the co-evolution between opinion and network dynamics. When there is a propensity to delete ties to dissimilar neighbors ( $P(D) \gg 0$ ), neighborhoods of like-minded individuals will be created. This is especially the case when new ties are primarily generated via triadic closure ( $R \ll 0.5$ ), as this will make new connections primarily between agents of the

same neighborhood. If these two conditions are met, the Co-evolutionary model can in effect manipulate how many neighborhoods the network contains by adjust the parameter  $T$ . The lower  $T$  is, the less open-minded agents are as they will have stricter criteria for who to maintain ties with. With lower values of  $T$ , the resulting network will therefore contain more neighborhoods, where each neighborhoods contains fewer agents. Notice that for all large networks ( $N > 1000$ ), the best parameters of the Co-evolutionary model have  $P(D) \gg 0$ ,  $(R \ll 0.5)$  and  $(T < 0.2)$ .

### 8.5.2 The impact of neighborhoods

The ability to systematically create neighborhoods is important because it is highly unlikely that larger networks only contain one neighborhoods. It is far more likely that it contains several neighborhoods or subgroups, where social agents mostly interact with a small subset of all the agents of the network. This might explain the large difference between the Co-evolutionary model and the Network Formation model, as this ability to generate neighborhoods is something that the Network Formation model doesn't have. This will obviously have large effect on the performance of the model, as having more multiple neighborhoods in the network will have a strong influence on all the considered network metrics. Let's start by considering the impact of neighborhoods on the average path length. Having more neighborhoods will increase the average path length of the network. Neighborhoods are very well-connected locally, but not very well-connected globally. Reaching distant neighborhoods will therefore require more jumps than if there had been no neighborhoods at all. This is also evident from the results. On all networks considered, the average path length of the Co-evolutionary model is consistently higher than the average path length of the Network Formation model (see Fig 7). Next, let us turn our attention to the degree distributions of the networks. Having multiple neighborhoods within the network will most likely have a large effect on the degree distribution of a network. Having multiple neighborhoods in a network will tend to cause fewer very low and very high degree nodes. In other words, the network will have less extreme disparity in degrees of its nodes. Extremely high degree nodes become less plausible simply due to the fact, that an agent is very unlikely to be part of many neighborhoods at the same time. It will therefore sever connections to more agents than had there not been any neighborhoods. Having multiple neighborhoods will tend to reduce the number of extremely low degree nodes, because these neighborhoods are locally very well-connected. Low degree nodes will have more connections within these neighborhoods than they would have if there had not been any neighborhoods at all. These two effects combined will tend to make the degree distribution of the network less scale-free and more log-normal, which is more in-line with the degree distribution of most social networks (Broido & Clauset, 2019). It should be expected that the separation between groups becomes more stark in highly opinionated networks. This is also congruent with the much lower Jensen Shannon Divergence of the Co-evolutionary model, where the large political networks (Political Blogs and Politicians) show large disparities between the Network Formation model and the Co-evolutionary model in Jensen Shannon Divergence of the degree distributions (see Fig 8). Finally, the clustering coefficient is also affected by the development of neighborhoods. For large networks, one should expect the average clustering coefficient to increase for networks with neighborhoods. This is because the neighborhoods are well-connected locally, and exhibit very similar features to that of caveman-networks (Watts, 1999). Again, the difference between the Network Formation model and the Co-evolutionary model is most vivid for the large, opinionated

networks of Political Blogs and Politicians. Surprisingly, the average clustering coefficient for the TV Shows network is almost identical (see Fig 6).

This neighborhood explanation for the difference in performance between the two models is also supported by the fANOVA analysis, which specifies the importance of each model parameter (Hutter et al., 2014). Here we see that the political networks have  $T$  as a more important parameter than all other networks. This suggests that clearly delineated neighborhoods are a more important part of the explanation for political networks than they are for the other considered networks.

### 8.5.3 Including empirical data in agent-based models

Although this explanation should be considered highly plausible, the results of this analysis should not be considered completely exhaustive of the possible explanations for what the underlying difference between the models are. Mostly, this is due to the fact that although an attempt to integrate more empirical data is made, we can always do better. For instance, we should improve in regards to the quantity of data. Only 7 empirical networks are considered. Moreover, controlling for size was done by including only one network, namely the TV Show network. This was done as it was created in the same way that the Politicians network was made, and was of a similar size. This doesn't however control the variable of size explicitly. This could potentially be done by varying the size of a network by for instance sampling a subset of the nodes of the network. However, it is not clear how such sampling should be done while maintaining the patterns of the original social network. The analysis would also be exhaustive by improving the quality of the data. In this analysis, we are comparing a static network to the final result of a generative model. In essence, we are only really interested in the match between the end result of the generative process and the static empirical social network, because it lends credence to the idea that these empirical networks could be generated by the principles of the generative model. If we had dynamic data over time, where the network changes and warps, we could get closer to how well the proposed mechanisms describe the dynamics of real world networks. In the case of the model proposed in this paper, a dynamical dataset containing the network evolution over time as well as the change of opinions of each agent would allow for a much better integration of model and empirical data. However, such data is of course extremely hard to come by. Although most researchers lack such high resolution in their data, there is still ample possibility to include empirical data in theoretical models. The analysis of this paper shows that we can use recent insights from the field of data science to not only answer how well our models match up to reality but also why they perform well or not. This is possible specifically through the use of frontier methods such as the hyperparameter optimization used here. To the authors knowledge, the use of a fANOVA to investigate the importance of each parameter has not previously been done with agent-based models. It is the hope of this paper that future papers will make use of this method, as it dramatically increases the interpretability of the results of hyperparameter estimation.

In conclusion, the agent-based model fitting and model comparison suggests two important notions. The first notion is that triadic closure alone seems to be able to generate many of the characteristic patterns of social network. This echoes previous findings concluding the same (Ilany & Akçay, 2016; Jackson & Rogers, 2007, 2004). The sec-

ond notion is that including co-evolutionary mechanisms makes for a much better model of social networks, especially when these networks are large and opinionated. This second notion points to the fact that there is a clear interdependence between the effects of network and opinion dynamics. We have seen that opinion dynamics can drastically alter the dynamics of networks. We now turn to the other side of this interplay by considering how network dynamics can change opinions.

## 9 Results

### 9.1 The main effects of the different model parameters

Higher values of the threshold decrease polarization (see Figure 11). Higher values of alpha decreases polarization (see Figure 12). Higher values of beta increases polarization(see Figure 13). Notably, when beta is zero, almost all simulations reach consensus, regardless of all other parameters (see Figure 17). Similarly, the is no effect of negative tie-deletion when  $\beta = 0$  (see Figure 16). The drivers of polarization are low values of  $T$  and high values of  $\beta$ . When  $T$  is sufficiently small and  $\beta$  is sufficiently large, all agents will have more extreme opinions at the end of the simulations than they had in the beginning. With intermediate values of  $\beta$  and  $T$ , whether the opinions of a simulation becomes polarized or not is largely dictated by  $P(D)$  (see Figure 17). The results indicate that higher values of  $P(D)$  results in lower values of  $|O|$  on average. The strength of the effect is modulated by how random new connections are made in the network. When randomness is higher, the effect of cooperation facilitation by high tie-deletion is diminished (see Figure 14). When  $P(D) = 0$  and  $\beta > 0$ , the network almost always polarizes (see Figure 17). The effect of how random new ties are, is also critical for how many negative ties are deleted. The more random new ties are, the more ties will be deleted on average. Of course, the probability of negative tie-deletion,  $P(D)$ , also has a large impact on the number of deleted ties. Interestingly, higher probabilities of tie-deletion almost always result in a lower frequency of deleted ties over time (see Figure 15).

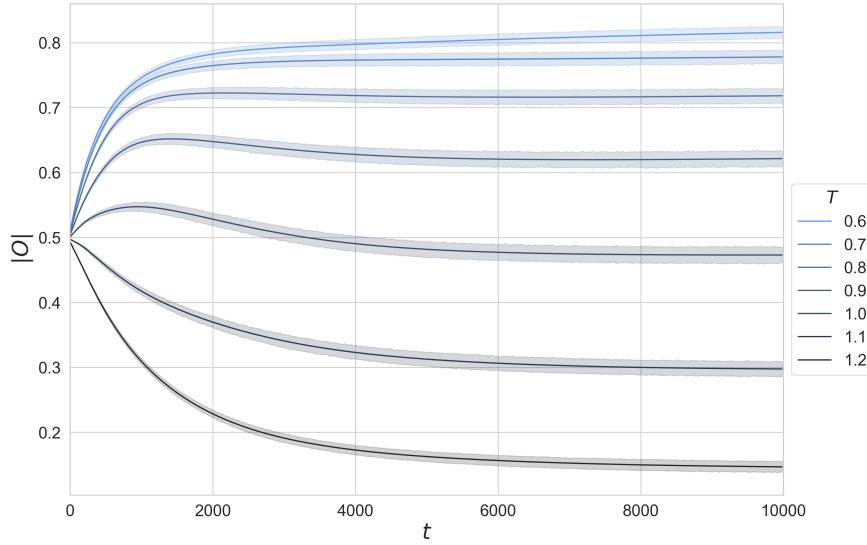


Figure 11:  $T$ 's effect on polarization over time. The x-axis shows the time-steps and the y-axis shows the absolute value of the opinions of agents. Colors indicate different values of  $T$ . Lines indicate the mean value of each time-step, aggregated over all parameters excluding  $T$ . Shaded regions are the 95% confidence intervals.

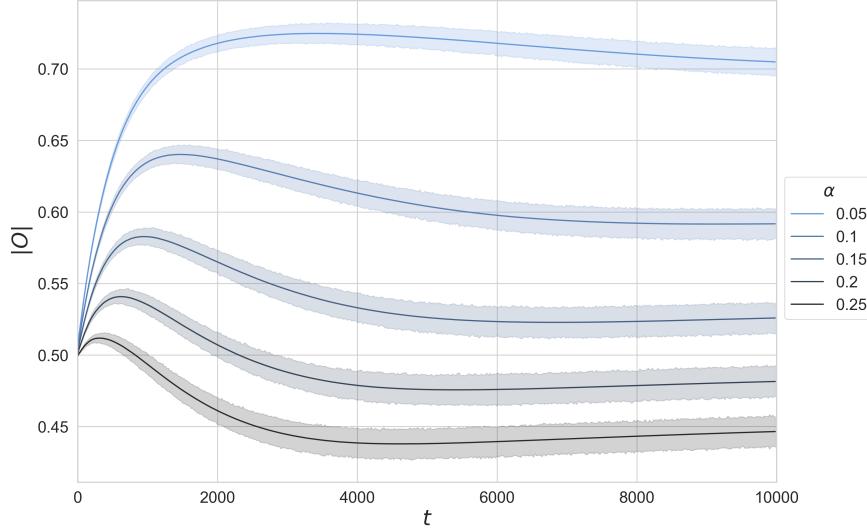


Figure 12:  $\alpha$ 's effect on polarization over time. The x-axis shows the time-steps and the y-axis shows the absolute value of the opinions of agents. Colors indicate different values of  $\alpha$ . Lines indicate the mean value of each time-step, aggregated over all parameters excluding  $\alpha$ . Shaded regions are the 95% confidence intervals.

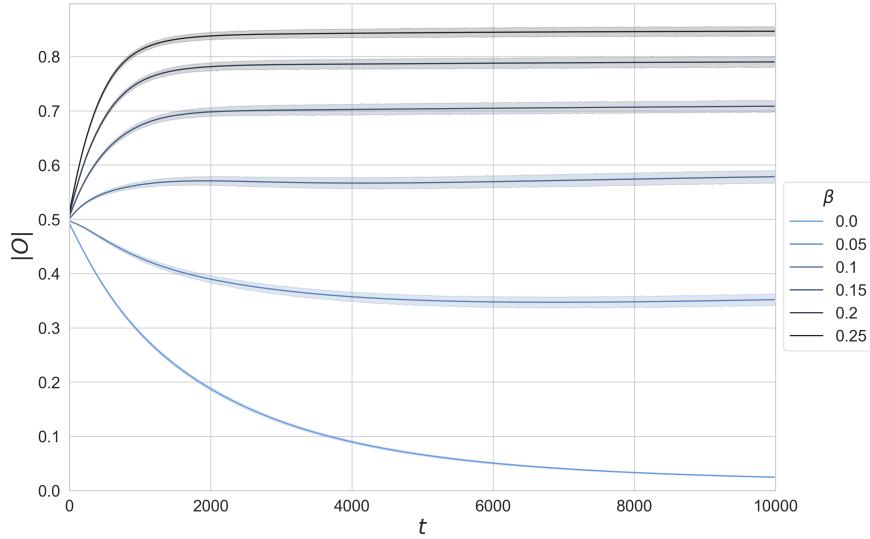


Figure 13:  $\beta$ 's effect on polarization over time. The x-axis shows the time-step and the y-axis shows the absolute value of the opinions of agents. Colors indicate different values of  $\beta$ . Lines indicate the mean value of each time-step, aggregated over all parameters excluding  $\beta$ . Shaded regions are the 95% confidence intervals.

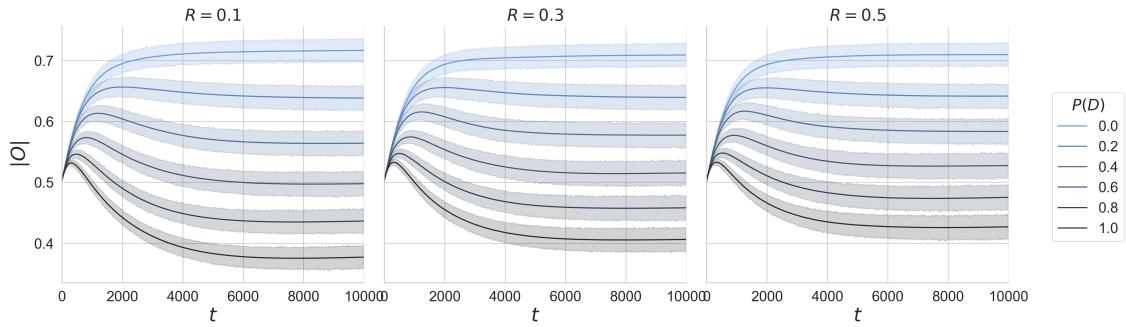
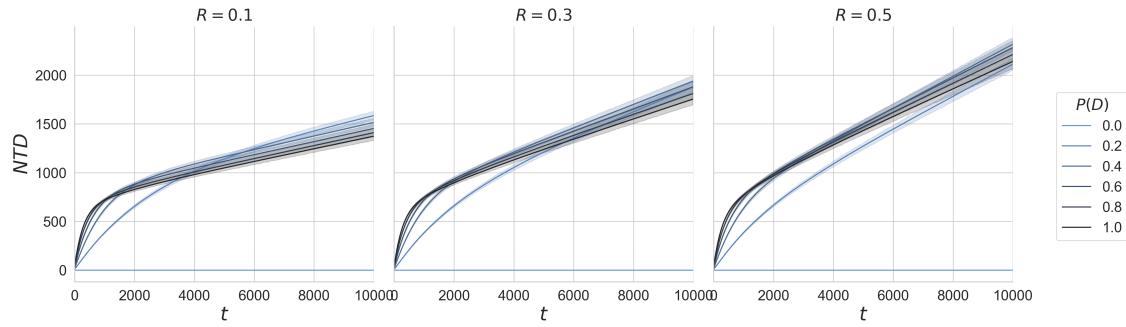
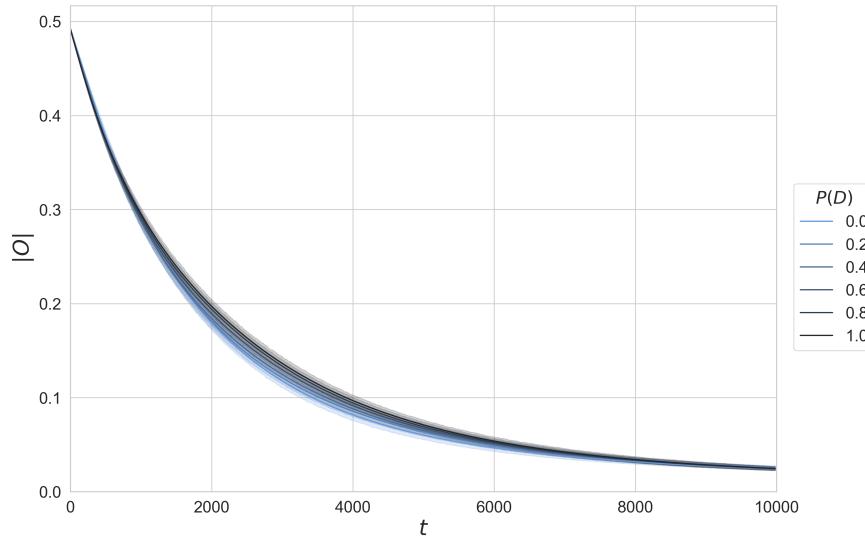


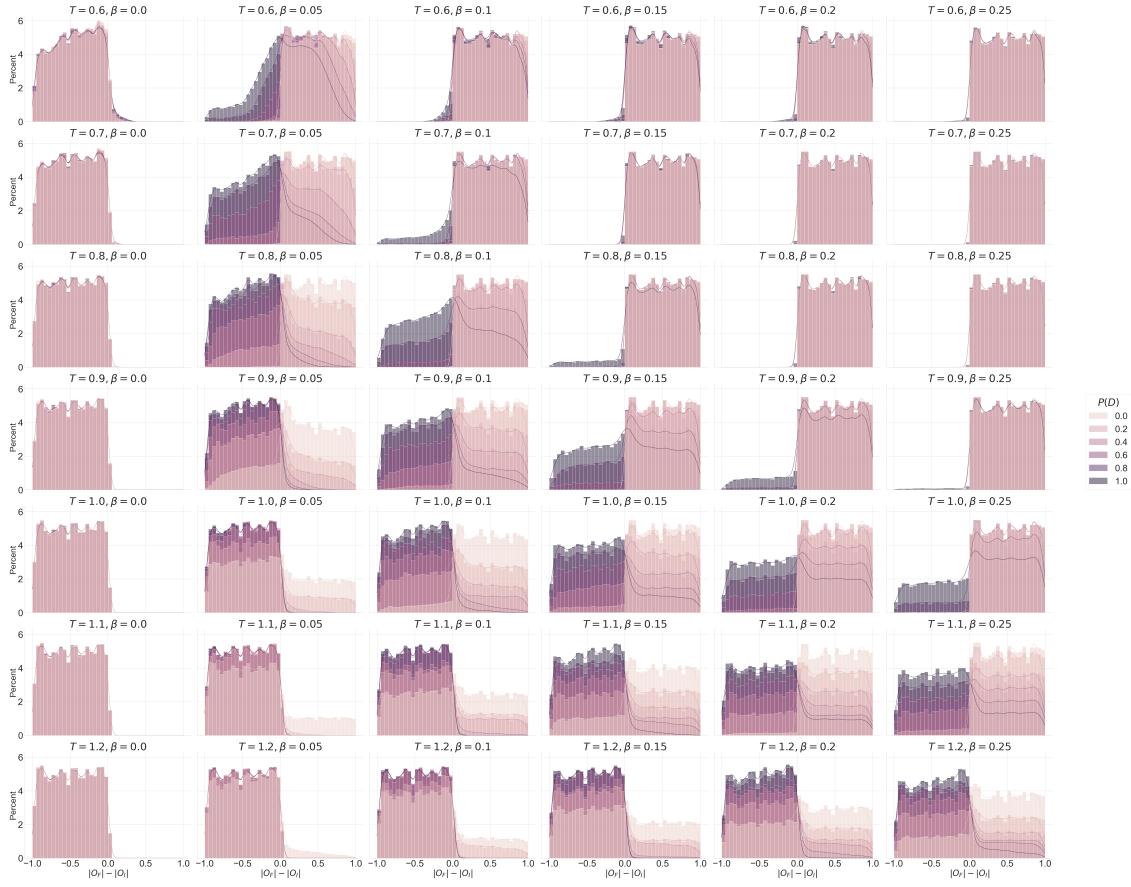
Figure 14:  $P(D)$ 's effect of polarization over time. The x-axis shows the time-steps and the y-axis shows the absolute value of the opinions of agents. Colors indicate different probabilities of tie-deletion of dissimilar agents. Lines indicate the mean value of each time-step, aggregated over all parameters excluding the probability of tie-deletion and randomness. Shaded regions are the 95% confidence intervals. The three different plots show the different values of probability of new connections to be generated randomly.



*Figure 15:* Negative ties deleted. The x-axis shows the time-steps and the y-axis shows the average cumulative frequency of deleted ties. Colors indicate different probabilities of tie-deletion of dissimilar agents. Lines indicate the mean value of each time-step, aggregated over all parameters excluding the probability of tie-deletion and randomness. Shaded regions are the 95% confidence intervals. The three different plots show the different values of probability of new connections being generated randomly. Notice that when  $P(D) = 0$ , no ties are deleted, which results in a bright blue line on the X-axis on all of the plots.



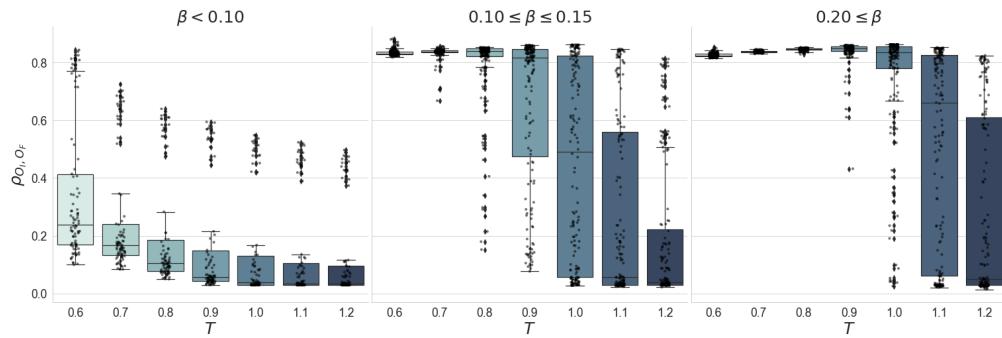
*Figure 16:* The effect of  $P(D)$  without negative learning. The x-axis shows the time-steps and the y-axis shows the absolute value of the opinions of agents. Colors indicate different probabilities of tie-deletion of dissimilar agents. Lines indicate the mean value of each time-step, aggregated over all parameters with  $\beta = 0$ , excluding the probability of tie-deletion. Shaded regions are the 95% confidence intervals.



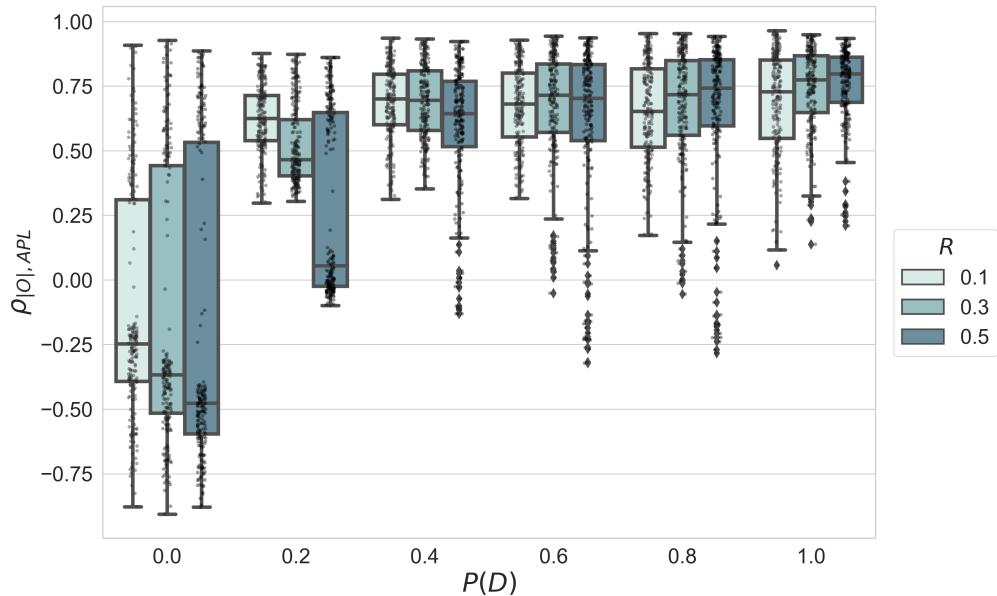
*Figure 17:* Distribution of opinion changes across different conditions. Rows display different values of thresholds and columns indicate different values of negative influence. Different colors show different values of the probability of tie-deletion. The x-axis shows the difference between the absolute value of the final and initial opinions of agents. Higher values indicate that the final value of agents is more extreme than the initial values, and vice versa. Y-axis exhibits the percent of values within each bin.

## 9.2 Correlations

I calculate two Pearson Correlation Coefficients of interest for all simulations. The first of which is the correlation coefficient between the initial opinions and the final opinions of agents,  $\rho_{O_I, O_F}$ . The second is the correlation between the absolute value of opinions and the average path length,  $\rho_{|O|, APL}$ . We start by reporting the findings for  $\rho_{O_I, O_F}$ . When  $\beta$  is low and  $T$  is high, there is little correlation between initial and final opinions. When the opposite is true, final and initial values are highly correlated (see Figure 18). Next, we turn to  $\rho_{|O|, APL}$ . As the probability of tie-deletion increases, so does the correlation between the average path length of the network and the absolute value of opinions. When the probability of tie-deletion is 0, there is no consistent correlation between the two variables (see Figure 19).



*Figure 18:* Correlations between initial opinions and final opinions. X-axis shows different values of threshold and the y-axis shows the Pearson Correlation Coefficient between the initial and final opinion of agents. Boxes indicate the quartiles and whiskers indicate the 1.5 of the inter-quartile range outside of the quartiles. Points outside this range is depicted as diamonds. Smaller dark dots show the correlation coefficient for individual simulations. Columns show low, medium and high values of  $\beta$ .



*Figure 19:* Correlations between absolute opinion and average path length. The Pearson Correlation Coefficient between the initial and final opinion of agents was calculated for all simulations. Boxes indicate the quartiles and whiskers indicate the 1.5 of the inter-quartile range outside of the quartiles. Points outside this range is depicted as diamonds. Smaller dark dots show the correlation coefficient for individual simulations. First plot has negative learning rate on the x-axis. Second plot shows the threshold on the x-axis.

## 10 Discussion

### 10.1 Reiterating previous literature

We first discuss the main effects of  $\alpha$ ,  $T$  and  $\beta$ . All of the effects of these parameters are intuitive and are in line with previous research on similar models (Flache et al., 2017). When  $\alpha$  is high, there is a stronger force pulling the opinion of agent's closer together. Higher values of  $\alpha$  therefore results in lower values of polarization (see Figure 12). But as can be seen from the scale of the y-axis, although this is large effect, it is small in comparison to the effect of  $\beta$  and  $T$ . Higher values of  $\beta$  have more drastic effects on the polarization of the population (see Figure 13). This is primarily caused by the considered values of  $T$  and its operationalization. When  $\beta = 0$ , all simulations reach a consensus (see Figure 17). If this simulation had considered lower values of  $T$ , this would most likely not be true. Lower values of  $T$  are known to be able to produce clusters of opinions (Flache et al., 2017; Sasahara et al., 2021). When  $\beta$  is large, almost all simulations polarize (see Figure 17). The reason for this is partly that in its operationalization, it is a stronger force than  $\alpha$ . The effect of pushing and pulling opinions is defined as being proportional to the distance between agents. By definition, agents that are pushed away from each other are further apart than agents that are pulling closer together. In this sense,  $\beta$  is a stronger force than  $\alpha$ . Only in aggregate can the effect of  $\alpha$  become stronger than the effect of  $\beta$ . This is only possible when the number of positive interactions is much higher than the number of negative interactions. The effect of  $T$  should therefore not be surprising. With larger values of  $T$ , agents are more "open-minded" - they can cooperate with more diverse agents, and will therefore have much more positive interactions than negative. To see this, notice that if an agent has an opinion of 0 and  $T \geq 1$ , this agent will not be able to have any negative interactions in this model. The opposite effect is witnessed when  $T$  is low. This limits the subset of agents the agent can cooperate with, which makes negative interactions much more plausible.

### 10.2 The importance of co-evolution

As mentioned in the introduction, previous models have focused almost exclusively on static networks. From the results of the model of this paper, we see that this is a very consequential assumption. This can be seen by the fact that when there is no co-evolution of the network and there are some negative social influence in the system ( $P(D) = 0$  and  $\beta > 0$ ), the network always polarizes. This is not the case when the network is dynamic and co-evolves, which can often prevent negative social influence from causing polarization in the network. This in and of itself proves that the co-evolution of network and opinion dynamics is a vital piece of the puzzle, and should be considered in the models of opinion dynamics.

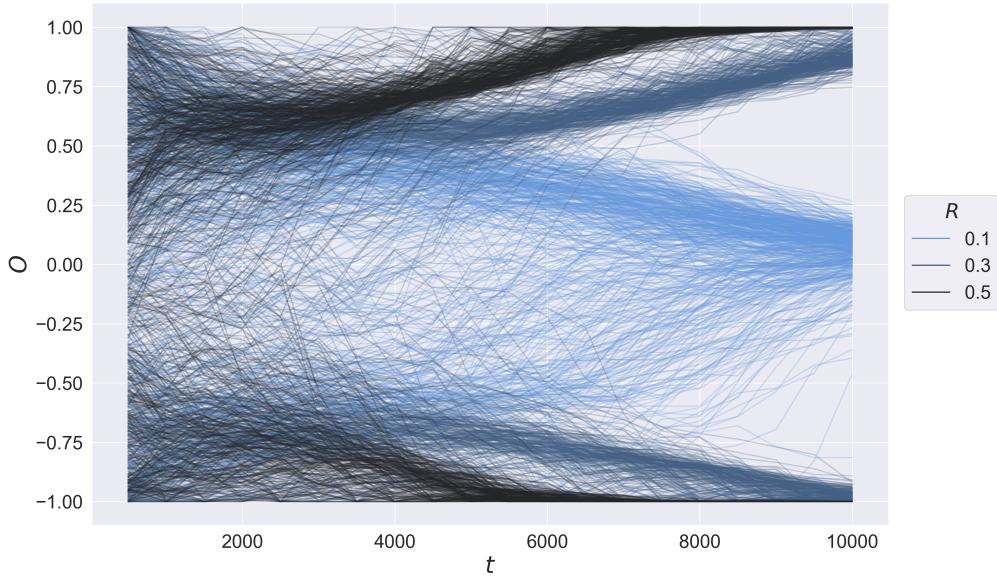
### 10.3 The effect of initial opinions

The results indicate that in some conditions, there is a strong positive correlation between starting opinion of an agent and the final opinion of the agent. These are conditions of high negative influence and of low thresholds (see Figure 18). These conditions are likely to lead to high degrees of polarization. The results indicate that in polarized conditions, agents will tend to move less than in conditions where they agents reach consensus. In conditions where agents do reach consensus, your starting position does not matter much. All agents will tend to be close to zero, no matter the starting condition. In polarizing conditions, agents will be very unlikely to move far from their starting position and will instead be pushed to the nearest extreme.

### 10.4 Tie-deletion facilitates cooperation

Finally, one of the main findings of this paper is that higher probabilities of tie-deletion can prevent polarization and facilitate cooperation. This is especially the case when new ties are created via triadic closure and not randomly (see Figure 14). This happens because of two effects in the system. The first is that deleting negative ties will cause fewer negative social interactions, which limits the effect of  $\beta$ . When negative ties are deleted, agents will assort based on the their opinions. Deleting negative ties is important in its own right because it deletes ties that would otherwise cause further polarization. But because of assortment, the effect of tie-deletion interacts with how random new ties are created. When assortment on opinions is high, the heuristic that "a friend of a friend is also friend" is very likely true. Triadic closure will result in even more positive interactions, because similar to what is found in empirical networks, distance in the social network is highly indicative of distance in similarity (Kossinets & Watts, 2009). When randomness is high, the probability of connecting to a very dissimilar agent is much higher than through triadic closure. In other words, increasing randomness decreases assortment, which leads to more negative encounters (see Figure 15). This also echoes the finding from the opinion dynamics literature stating that when new connections are made randomly, opinions will tend to polarize more quickly when they include negative influence (Flache & Macy, 2006, 2011; Turner & Smaldino, 2018). Here we show that reason for this effect is largely due to positive assortment of opinions. Positive assortment of opinions is also closely linked to the reason for the correlation between the absolute value of opinions and the average path length when tie-deletion is high. Because the network co-evolves with opinions when the average distance between opinions increase, the average distance in the network will increase as well. When opinions polarize, negative ties will tend to be deleted. Because of positive assortment, long-range ties will suffer especially when the network polarizes. This will make highways of information which connects distant parts of the network much less likely, which increases the average path length substantially. When the network depolarizes, the opposite happens. Some long range ties will be made by random, and because the network is depolarized, these will stay intact. This will make further long-range connections more probable, as the two clusters can connect via triadic closure.

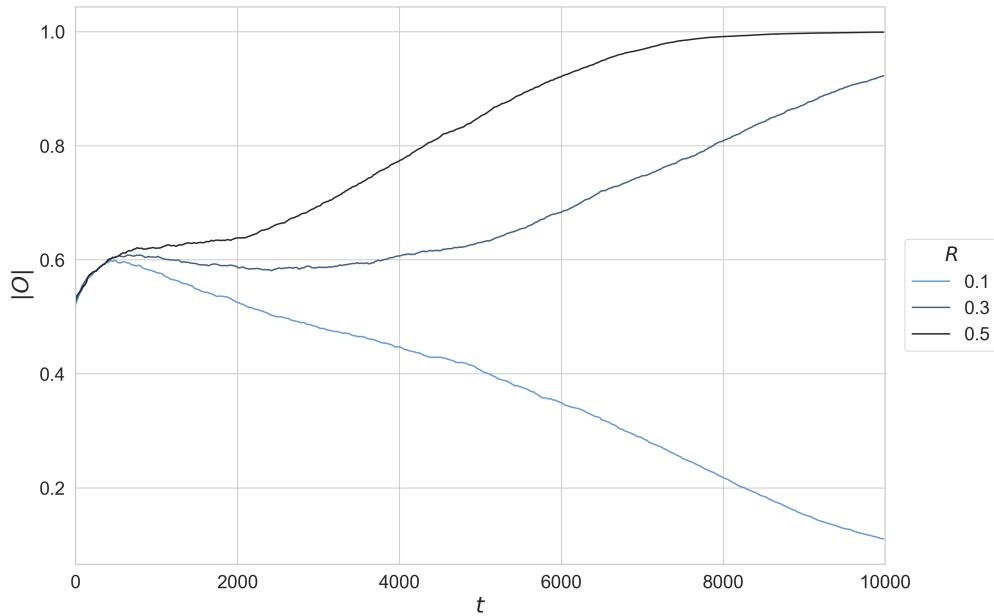
## 10.5 Distance in opinion and network space



*Figure 20:* Lineplot of opinions over time. X-axis shows time-steps of the simulation, y-axis shows the opinion of agents in the simulation. Colors show different values of  $R$ . All simulations were  $T = 0.8$ ,  $\alpha = 0.15$ ,  $\beta = 0.1$ ,  $P(D) = 1$  and with the same random seed.

To further appreciate why tie-deletion facilitates cooperation, it is beneficial to examine representative simulations in more detail. Here we investigate one of the conditions, where threshold is relatively low ( $T = 0.8$ ), there is some negative social influence ( $\beta = 0.1$ ) and positive social influence ( $\alpha = 0.15$ ). This combination of parameters only avoids polarization when tie-deletion is high for the reasons highlighted in the previous section. By focusing on these cases, we can inspect directly how the network evolves over time in relation to the opinions of its agents. The first thing to notice is the stark difference between the trajectory of opinions of different levels of randomness (see Figure 20). Only the simulation with  $R = 0.1$  avoids polarization. Additionally, when  $R = 0.5$ , the polarization is much faster than when  $R = 0.3$ . The second thing to notice is how closely the average path length follows the absolute value of opinions (see Figure 21, 22). As mentioned, the mechanism that links these two variables is positive assortment. This can clearly be seen by inspecting the opinions of the agents in the network directly (see Figure 24). After only 2000 time-steps, there is a clear gradient in the networks, which is the result of positive assortment. Positive assortment is achieved directly by negative tie-deletion in this model. Additionally, positive assortment is increased indirectly by triadic closure. This is the same mechanism referred to as structural homophily in the literature (Asikainen et al., 2020; Peixoto, 2022). In other words, tie-deletion is not necessarily an important mechanism in itself. Rather, it is important because it increases positive assortment. I presume that any mechanism that will increase positive assortment in these kinds of models will decrease polarization. This point is also strengthened by the difference between the simulations considered here. Increasing randomness causes the network to polarize, because it decreases positive assortment. Counterintuitively, tie deletion in combination with triadic closure facilitates cooperation because it isolates groups, not despite it. Here we can also clearly see that polarization in opinions causes the network to become less connected,

which increases the path length. To see why this happens, one can use a spring as an analogy for the network. As long-range ties are deleted, the tension in the spring rises, and the network starts to be better characterized as a network of two factions, connected mostly by a group of agents with values around 0. As distant parts of the network reconnect, the tension of the spring is lowered, and all the parts of the network become closely linked again. When polarization increases, there will be less of these long-range ties, but there might also be fewer middlemen to facilitate cooperation across the aisle. These middlemen are the nodes of cyan colors, that lie in the middle of the two major clusters of dark and light colors. These agents are critical for cooperation. Without them, the spring snaps. It does so because the different clusters cannot have positive interactions with each other as they are too far apart. By interacting only through these middlemen, there is a chance of bridging the gap. If these middlemen become part of either cluster before polarization has decreased sufficiently for the different groups to have positive interactions, the only possible outcome is polarization. Middlemen are therefore necessary conditions for cooperation in this model. It is also possible to give an alternative interpretation of what the probability of tie-deletion represents. The probability of tie-deletion can also be seen as how quickly the structure of the network is adjusted by opinions. In this interpretation, consensus can be achieved if the social network adjusts fast enough. Having faster adjustment of the structure of social interactions is critical, because the system is highly path dependent (Turner & Smaldino, 2018). When the network doesn't adjust fast enough, agents will already have had several negative interactions, which will have pushed them further towards an extreme value of opinions. This will make them more likely to polarize other agents in the future. This will also be the case for middlemen. Adjusting too slowly can therefore burn the only bridge that is available for facilitating cooperation.



*Figure 21:* Absolute opinions of representative simulations over time. X-axis shows time-steps, and the y-axis shows the absolute opinion of agents. Colors indicate different levels of randomness. Lines show single simulations over time which were simulated using the same random seed. These simulations had parameter values of  $T = 0.8$ ,  $\alpha = 0.15$ ,  $\beta = 0.1$  and  $P(D) = 1$ .

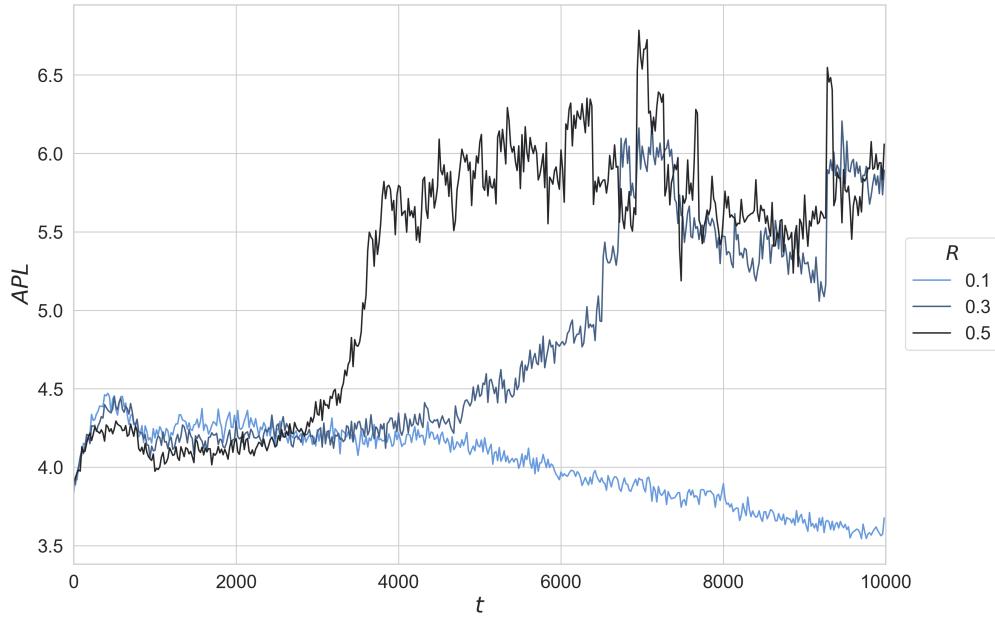


Figure 22: Average path length of representative simulations over time. X-axis shows time-steps, and the y-axis shows the average path length of the network. Colors indicate different levels of randomness. Lines show single simulations over time which were simulated using the same random seed. These simulations had parameter values of  $T = 0.8$ ,  $\alpha = 0.15$ ,  $\beta = 0.1$  and  $P(D) = 1$ .

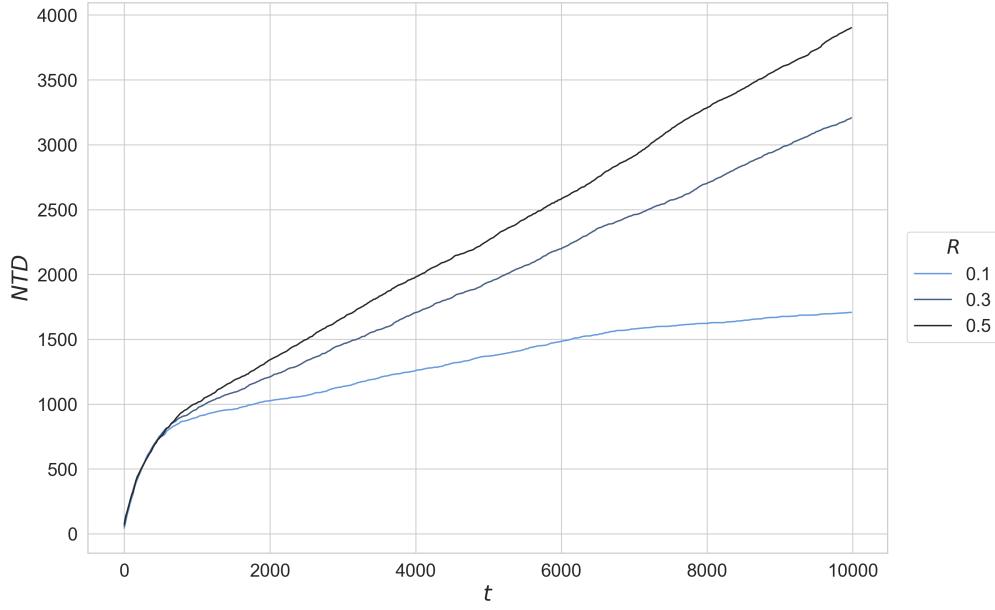


Figure 23: Negative ties deleted in representative simulations over time. X-axis shows time-steps, and the y-axis shows the cumulative frequency of negative ties deleted. Colors indicate different levels of randomness. Lines show single simulations over time which were simulated using the same random seed. These simulations had parameter values of  $T = 0.8$ ,  $\alpha = 0.15$ ,  $\beta = 0.1$  and  $P(D) = 1$ .



*Figure 24:* Network evolutions for different levels of randomness. Columns show different values of randomness,  $R$ . Rows are different time-steps of the simulations, increasing in increments of 2000. Nodes represent agents, lines represent edges between agents. Agents are colored by their opinion. Dark blue colors represent values close to -1, and lighter colors represent opinions close to 1. The size of the nodes of agents reflects their degree. The larger an agent's degree is, the larger their node is.

## 11 Contrast to previous work

It is important to understand why the results found here are in so stark contrast to the recent opinion dynamics models, which proposes that unfollowing dissimilar individuals accelerates polarization (Sasahara et al., 2021). First, the network they are considering is a directed network, meaning that relations need not be reciprocal. This could have important effects for the effects observed here, because it allows for social influence to be asymmetric. Secondly, the number of agents considered is only 100 instead of the 500 considered here. This could also be consequential. Tie-deletions can only facilitate cooperation if there are "middlemen", that can reunite the different clusters. Having enough diversity in opinions is critical for bridging the gap between distant opinions. Having more agents in the situations increases the number of expected agents with values close to 0, which can help in facilitating cooperation between agents of too different opinions. Finally, a critical difference between the model proposed here and the model that they considered is that this model includes negative social influence,  $\beta$ . Because of this, social relations that are outside of your boundary of confidence have no influence on you opinion. In other words, there are no "costs" associated with negative social interactions. For the parameter values considered in the model of this paper, tie-deletion is only effectively facilitating cooperation when there is negative social influence in the system. The reason for this is primarily that this paper does not consider very low values of the confidence boundary as it somewhat obscures the analysis, as the system's behavior changes qualitatively below these values. Although their model is also a variation of a classic confidence boundary model (Flache et al., 2017; Sîrbu, Loreto, Servedio, & Tria, 2017), like the one proposed in this paper, they analyze much lower levels of boundaries of confidence. This has benefits and drawbacks. First, this allows more than two clusters of opinions. Second, it allows for the clusters of opinion to be centered at other values than the extreme of the opinion spectrum (Sasahara et al., 2021). These are definite positives, and similar features can also be generated by the model of this paper. However, the behavior of the system changes drastically when values of very low levels of boundary thresholds are studied, which makes a comprehensive review of the system as a whole much more complex to study. This echoes previous findings from two different literatures. First, facilitating cooperation via tie deletion recapitulates findings from computational biology, which states that positive assortment leads to increased levels of cooperation (Boyd et al., 2010; Dakin & Ryder, 2018; Pepper & Smuts, 2002; Santos et al., 2006). Second, more random new connections leading to more polarization is a stable finding in opinion dynamics models which include negative influence (Flache & Macy, 2006, 2011; Turner & Smaldino, 2018). Here we show that these processes are interlinked.

## 12 Short comings and future work

### 12.1 On agent-based modelling

A typical retort for agent-based models in general is that the researcher can generate any type of outcome which is congruent with their theory (Geschke et al., 2019). I do not agree with this general conclusion. To allow for full transparency of the model and its conclusion, the entire code is available for anyone to reproduce. The code is fully documented and specifically designed to be easily readable. The hope is that other researchers will be able to build, expand, validate or refute the proposed framework.

### 12.2 Including Empirical Data

Previous review articles have pointed to the fact that the field needs to include empirical data more in their studies of these dynamical systems (Flache et al., 2017; Noorazar et al., 2020; Sîrbu et al., 2017). Although I make the attempt to integrate and test the model with real empirical networks in this paper, we still need more data. One of the main problems with integrating the data into the models seems to be that these models require very specific data to be properly evaluated. What we really need is dynamical networks of social networks of opinions over time. Previous results have focused on measuring dynamical networks (Kossinets & Watts, 2009), but including the opinions of the agents would be critical information to have for these types of models. If this data was available, it could be possible to identify how the patterns of the networks evolve over time. Although this paper shows that co-evolutionary models can better explain empirical opinionated networks, it does so by comparing the network to a snapshot of a real-world network. In other words, the fact that the co-evolutionary model fits better does not necessarily mean that it does so by implementing the real underlying mechanisms that gives rise to the patterns in the data. By having access to dynamical networks, it would be possible to identify the realism of the proposed mechanisms for both social influence and network formation. This data would also allow researchers to test more specific findings within this paper. Specifically, it would be possible to check whether increases in tie-deletion to dissimilar people decreases polarization.

### 12.3 Accurate perceptions of opinions

An important limitation to this study is that interactions are assumed to be symmetrical and accurate. By interactions being symmetrical, I mean that agents push and pull on each other's opinions by the same factor. This is most likely not an accurate reflection of reality. Individuals of an interacting dyad will probably move more or less than its counterpart. This has been modelled previously in related models to the one proposed here as either "stubbornness", or situations where more extreme agents will be less likely to move (Flache et al., 2017; Ghaderi & Srikant, 2014; Yildiz, Ozdaglar, Acemoglu, Saberi, & Scaglione, 2013). Although I believe the inclusion of this mechanism will increase the realism of this model, the mechanism of stubbornness is not a key variable of interest to this study. Stubbornness is nonetheless

worth mentioning because it could interact with some of the variables of interest to this paper. If extreme agents are more stubborn, the system could become even more path dependent than it already is (Turner & Smaldino, 2018). To see this, initial increases of polarization by chance encounters would lead to agents being more unlikely to change, which would make reaching consensus more implausible for the system. This could decrease the effect of tie-deletion on facilitating consensus, as agents would be more unlikely to overcome initial increases in polarization.

A perhaps more critical flaw in the assumptions of the underlying mechanisms of this model is the lack of noise in interactions. It is highly unlikely that humans perceive or share opinions accurately. Instead, it is more reasonable to assume that people will sometimes misunderstand or misrepresent opinions (Jussim & Osgood, 1989). Opinion formation is likely skewed by the same cognitive heuristics and biases that shape other parts of how we include information into our decisions (Arceneaux, 2012). This includes effects such as confirmation bias, which might play a vital role in how we update our opinions (Allahverdyan & Galstyan, 2014). Typically, models include noise in the interactions between agents to emulate imperfect communication (Sîrbu et al., 2017; Su, Chen, & Hong, 2017). Noisy interactions could potentially play a vital role in facilitating cooperation, as it can allow agents that would not normally interact positively to perceive each other as more similar by chance (Allahverdyan & Galstyan, 2014; Su et al., 2017). Including more realistic cognitive mechanisms might change the dynamics of the system in important ways which are not considered in this model. One way of implementing this would be to reimagine updating one's opinion as Bayesian updating considering other's opinions as new evidence (Allahverdyan & Galstyan, 2014). This can be done by letting the opinion of an agent be described by a distribution instead of a single number. The distribution could be a normal distribution, where the mean gives the general opinion of an agent, and the standard deviation describes the certainty of the agent's opinion.

## 12.4 Shrinking Variance

Because no new agents are added to the network and because interactions are non-noisy, the variance in opinions shrinks over the course of time. At first glance, this might not seem very problematic. People tend to conform to the majority of their social groups (McPherson et al., 2001), but we wouldn't expect near perfect consensus on most issues facing us. Introducing mechanisms that keep a sufficient amount of variance would be more realistic but might also decrease the path dependency of the system. SAY SOMETHING ABOUT MODELS HAVING AGENTS TRYING TO BE DIFFERENT

## 12.5 Social influence exclusively

The model investigated here assumes that one's opinion is only changed by local interactions. Although this is in line with most models of opinion dynamics, there are good reasons to believe that global trends and tendencies are powerful forces that shapes opinion (Bener et al., 2016). These global tendencies might shape our opinions more and more, as media consumption rises. Global perturbations to the system could be powerful forces that potentially could

have large impacts on whether the system can reach consensus. These forces would be especially counterproductive for reaching consensus, if they only influence certain parts of the system. The power of such effects is not hard to imagine – partisan news media is a vital piece of the puzzle of how opinions are formed (Pennycook & Rand, 2019). As media consumption rises and becomes a more and more important force for what information we receive, these force might already be a more important variable to consider than local interactions of agents (Bener et al., 2016; Strömbäck, Djerf-Pierre, & Shehata, 2013).

## 13 A broader perspective

The model investigated in this paper points to a possible mechanism for cooperation. As we have seen that mechanism is the coevolution of networks and opinions, which leads to distance in opinion space being closely linked to distance in network space. Whenever there is a possibility for negative interactions in the system, the speed of co-evolution becomes tantamount for whether the system reaches consensus or not. It is therefore interesting to see whether how connected we are and how much we disagree corresponds in empirical networks. As was pointed out in the introductory paragraph, polarization has been on the rise in many western democracies over the last two decades (Boxell et al., 2020; Center, 2014a). On the other hand, estimations of the average path length between users on Facebook has decreased from 2011 to 2016 (Bhagat, Burke, Diuk, Edunov, & Filiz, 2016). The coevolutionary model investigated in this paper suggests that the increase in connectivity and the simultaneous increase in polarization could be a bad omen. If anything like negative influence characterizes interactions on social media (i.e. outrage), the decrease in average path length will make negative encounters more likely, and most likely increase rather than decrease polarization. Although being more connected might sound good on paper, it might be detrimental to our ability to cooperate in the long run. This effect is congruent with previous literature, where the introduction of long-range ties sharply increases polarization of opinions in the population (Flache & Macy, 2011; Turner & Smaldino, 2018).

## 14 Conclusion

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