

The Life and Death of Social Networks: A network formation model for opinion dynamics

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One of the most robust effects governing social life is homophily and triadic closure. We like people similar to ourselves, and we find new connections through our existing connections. These two effects can combine to self-reinforcing processes of shaping both social networks of social agents as well as their opinions. Here I investigate a stylized agent-based model of opinion dynamics. I find that the model can generate realistic social networks while simultaneously producing realistic opinion distributions.

opinion dynamics | computational social science | agent-based modeling

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Sections: Theory, Methods, Results, Discussion...

1. Theory

One of the most robust mechanisms governing social networks is the tendency of similarity to breed connection. This effect is often characterized as homophily. The effect of homophily results in neighbourhoods within social networks, which are homogeneous with regard to sociodemographic, behavioral and political characteristics. This has dramatic implications as one's social world defines what pieces of information, opinions and interactions that an individual have. The most clear example is echo chambers in social media networks. In echo chambers, homophily can lead to a distortion of what kind of information is presented to each individual.

It is examples like those from echo chambers that clearly illustrates that the principles of homophily not only influences who we make connections with, but also how we think. Several studies have pointed to the fact that one's peer group is an important influence on one's behavior. For instance, shared political beliefs predict more behavioral involvement with one another.

Homophily and its effects become especially potent as they can create self-reinforcing mechanisms when combined with tie-formation principles such as triadic closure. Triadic closure refers to tie formation between "friends of friends" in a network. For instance, if $A \leftrightarrow B$ and $B \leftrightarrow C$, then this small system would achieve triadic closure by forming the edge $C \leftrightarrow A$. When new ties are found via triadic closure,

To see why homophily and triadic closure can create self-

reinforcing homophily effects, notice the following. Due to homophily, the probability of A being similar to B will be higher than when they find new connections via triadic closure i.e. "friends of friends". In such cases, any social agent will have a propensity to like agents which are more similar to themselves. Moreover, they find new connections primarily by finding them via "friends of their friends".

Homophily and its effects become especially potent when social agents can be influenced by their peers.

A. Network Formation.

A.1. Social Networks. Social networks differ in key ways to more idealized networks. Here I outline the most important characteristics, which have been found across many different domains and networks.

The first of which is that the average path length in social networks tends to be small. This is best exemplified by the idea of "six degrees of separation", which refers to the notion that you are never more than five intermediaries away from any other person on the planet. Average path length is as the average of all shortest paths between nodes. CITE (Meeting strangers and friends) Recently, this notion has been studied empirically on social media networks, including 721 million people. They found that the average shortest path length was 4.74, corresponding to closer to four degrees of separation CITE (four degrees of separation). This feature is in part also the claim to fame of small-world networks. Simple lattice networks can be modified with random rewiring of connections, making some connections act as shortcuts across the network. CITE (small world)

The second characteristic is the fact that social networks tend exhibit clustering behavior. Small hubs of the network are well-connected locally, while often having limited connections to other hubs of the network. The level of clustering is often quantified with a clustering coefficient. Here, different

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coefficients can be used, but arguably the most common is the average clustering coefficient proposed by Watts and Strogatz in the seminal paper on small-world networks CITE(small world). The basic intuition is that with higher clustering coefficients, an agent's connections tend to be connected to each other. This is the reason why the literature often includes triangles and closed triangles in their terminology. A triangle is a set of three nodes, where either two of them are connected or three of them are connected. When all three are connected, the resulting set is a closed triangle. Clustering can also be thought of as the ratio between all closed triangles of the graph and all the triangles of the graph.

The third characteristic is that the degree distributions of networks tend to have "fat tails". In this case, the degree distribution refers to the probability distribution of degrees in a network. What is meant when degree distributions are described as having fat tails, is essentially two attributes. The distributions tend to have relatively few nodes with the average degree of the distribution, but instead have many low and high degree nodes. These degree distributions are also referred to as being "scale-free" or following a power-law distribution. In other words, most nodes have relatively few connections, but a few of them have disproportionally many connections compared to all the other nodes.

The fourth characteristic is that high degree nodes tend to connect with other high degree nodes. This is also referred to as nodes having positive assortativity.

Idealised networks such as the small world network and the scale-free network can generate a subset of these characteristics. However, none of these models can generate networks which exhibit all features simultaneously. When evaluating good candidate models for how social networks are generated, our candidate models should be able to generate networks which exhibit all characteristics at the same time using one underlying algorithm.

A.2. Candidate Models. One model which generates all the characteristics using a simple network generation model is the model proposed by CITE(Jackson and Rogers). The model is a simple network formation model, which creates a network by constantly adding new nodes. The primary mechanism of interest for this paper is the mechanism used for generating new connections. In their model, links are primarily formed via "triadic closure" and randomness. Triadic closure refers to the idea of closing triangles. In other words, new connections are formed primarily by searching through "friends of friends". When they are not, they are added randomly between agents. Let us first consider why this mechanism of tie formation will likely generate the typical characteristics of social networks.

In terms of the average path length, the random component of tie formation decreases the average path length considerably. This will create the aforementioned shortcuts in the network as seen in small world networks CITE(small world). Clustering is almost guaranteed in this model, as the defining feature of the model is closing triangles. The degree distribution will also be quite similar to social networks and exhibit scale-free tendencies. To see this, notice that larger degrees will have a higher probability of gaining a new agent via triadic closure. The growth of an agent's degree is proportional to their existing degree. This relation will lead to the large disparities in degree observed in scale free degree distributions. Finally, positive assortativity is likely as older nodes in the network will tend

to have larger degrees as well. Older nodes will therefore also have more frequent opportunities to connect to each other, which will lead to positive assortativity.

This model was expanded upon by Ilany and Acay (CITE) which contributed in two notable ways. Their model was a model of social inheritance of different social systems in animals. They showed that their simple model could accurately capture the complex social structures of many different types of animals. This adds credence to the idea that triadic closure might be a common tie formation strategy throughout biology. The model also made an important alteration to the model by Jackson and Rogers (CITE) by making the network a finite size. The original model by Jackson and Rogers, the network grows indefinitely. Ilany and Acay modified the network to function more closely to biological networks. This is done by having a probability for agents to disappear from the network, which stabilizes the size.

Beyond being able to generate the attributes of social networks, the model proposed by Jackson and Rogers (CITE) is in accordance with existing theory. Triadic closure is the most common structural constraint of real life networks. In minimal dynamic social networks, triadic closure should be considered the main mechanism for tie formation. In more general terms, it has been found that the likelihood of forming a new tie is a monotonically decreasing function of distance. For instance, forming new links to people further than 4 degrees of separation from you is 2.500 times less likely than forming new links via triadic closure CITE(Origins of Homophily). Moreover, triadic closure represents a simple and natural mechanism - it lines up with common sense intuitions of how social networks develop.

A.3. The problems with current models. As mentioned previously, many of the idealized models for generating social networks can only partly generate the characteristics of social networks. Arguably, this is a symptom of a deeper problem in the field which is a loose connection between models and empirical data. An example of this is the fact that even though we refer to the distributions of social networks as "scale free", scale free networks don't match observed social networks. Few models even include data, or calculates how well their model approximates the system they are modeling. Often such measures of fit are eye-balled and not rigorously defined.

We therefore need better integration between models and data. This is true both for model creation but also true for model evaluation. As is always the case, a model has to simplify the world and make crude assumptions. A good model is a useful model with useful assumptions. The results from a model critically hinges upon these assumptions. Evaluating the assumptions of a model is therefore a crucial step to take before jumping to conclusion. This is especially noteworthy as very few of the current models of social influence are dynamic networks. This assumes that no new relations are created, and that no relations are deleted. This might not seem consequential, but it is. The model will assume that relations are kept between people even when they vermently disagree about everything. This fact in itself can account for much of the signal reported in classical models in the literature.

B. Social Influence. Including some of the basic literature (Axelrod)

B.1. Homophily. One of the most persistent facts regarding social networks is that similarity breeds connection. This tendency is known as homophily where individuals seem to be connected to individuals like themselves. This is not only true for humans, but is a pervasive fact for tie formation in numerous biological systems. From zebras to dolphins, homophily predicts which ties will companionships will form. In primates, similarity even predicts the quality of the tie you have with other people. The more you have in common, the more likely that person is a close friend. This is also true for humans. We exhibit high levels of homophily in tie formation. Personal networks in humans are homogeneous across sociodemographic, behavioral and intrapersonal characteristics. Characteristics like age, sex, race, education level, intelligence, attitude and aspirations have all been shown to exhibit high degrees of homophily. This also exemplifies what is meant when it is said that homophily is an extremely robust finding. It permeates almost every part of social life. Studies regarding the homophily of race in schools found that in middle school, only 10% of the expected cross race friendships were observed. A similar example comes from religion. A study concerning the social networks of Jews found that 80% of their friendships were with other Jews. In addition, 80% of marriages were with other Jews. At the time, Jews consisted of only 2% of the population.

The underlying mechanism of what causes homophily is not completely clear. The most plausible explanations are individualistic and structuralistic explanations. The individualistic explanation to this effect is the psychological claim that people will tend to prefer people who are similar to themselves. It is in other words the psychological preferences of the individual. The structuralistic explanation to this effect is that structure of the environment will make certain choices more or less available to the individual. If most of the people in your neighborhood are similar to you, chances are that you will form ties to people similar to you, regardless of your psychological preference. These two different causes of homophily are important to distinguish, and I will follow the definitions proposed by Kossinets and Watts. I will define the effect of the individual's psychological preference as choice homophily and the structuralistic effect as induced homophily. The literature suggests that neither induced nor choice homophily is enough in themselves to explain the patterns of similarity in social communities. In the study by Kossinets and Watts, highly similar pairs were 50 times more likely to form a tie than dissimilar pairs, and 13 times more likely than average similarity pairs. However, when controlling for structural constraints (i.e. induced homophily), both numbers drop significantly (4 times as likely than dissimilar pairs, 2.5 times more likely than average similarity pairs). In other words, induced homophily is responsible for much of the observed homophily, but there is also a strong effect of choice homophily regardless.

Many of the personal characteristics that show the strongest homophilic effect (i.e. race, background, religion) is always or often inherited by parents of the individual. This fact also suggests that even small amounts initial choice homophily can lead to extreme levels of induced homophily by amplifying the effect over the course of generations. More similar individuals are more likely to interact and interaction between them is likely to make them even more similar through social influence. This is also in line with findings suggesting that peer groups are an important source of influence for the behavior of people.

This points to the possibility that powerful feedback loops between homophily and social influence are likely to be an underlying mechanism shaping our opinions and our social networks.

B.2. Xenophobia.

B.3. Shaping opinions. Introduce the evidence from psychology and computational literature to show why the assumptions in the model make sense

B.4. Models of Social Influence. Report the evolution of models and where to place this model in all of the literature

C. A network formation model for social influence. Explain the importance of making both a network formation and opinion dynamics model in one go

2. Methods

A. Model specification. The model is a network formation algorithm, relying on the opinion of the agents to determine how tie formation and tie dissolution happens. These opinions in turn rely on the opinion of other agents via social influence. The network is therefore truly dynamic over time in regards to both its connections and the opinions of the agents.

For clarity, we will here give a more rigorous mathematical formulation of the model. Let G be the graph of the network and t be the timestep of the model. We denote G_t to refer to the state of graph, G , at timestep, t . Let N be current number of nodes in G and N' be the target number of nodes. By target number of nodes, I mean the number of nodes we want the final network to contain. As the model is a network formation model, the number of nodes N for G_0 will always be 0. We then model the probability of agent genesis, $P(G)$ and agent death $P(D)$ simply as:

$$P(D) = \frac{N}{2N'}$$

$$P(G) = 1 - P(D)$$

At each t , a node is either added or deleted from G according to the probabilities specified by $P(G)$ and $P(D)$. By modeling node genesis and death this way, we ensure that for a large enough t , $N \approx N'$. When an agent is deleted from G , it is done so by sampling a node from G randomly. All nodes are initialized with an "opinion"-value, O . This value is drawn from uniform distribution, with a lower limit of -1 and upper limit of 1 :

$$O \sim \mathcal{U}(-1, 1)$$

This ensures that agents are initialized without any bias in opinions. Moreover, this conceptualization of opinions offers logical interpretations, as 0 will be the neutral middle of the opinion space.

In the special case of $N = 0$, a new node is added without adding extra connections. For $N \leq 1$, a new node is added and connected randomly to another node in G . With probability

$P(E)$, the new node will add another connection to a randomly sampled neighbor's neighbor. With probability $1 - P(E)$, the new node will add another connection randomly.

After nodes have been added or deleted, an agent is sampled to be on turn. We denote this agent by A_t . A_t will add connections exactly as if it was a new agent. After establishing new connections, A_t now updates her opinion based on the opinions of her neighbors.

Let O_1 be the opinion of A_t and O_2 be the opinion of one of A_t 's neighbors. Let B be the boundary threshold. The boundary threshold defines when two opinions either pull each other closer together or push each other further apart. For cases where $B \geq |O_1 - O_2|$, we have a case of positive learning where agents pull each other closer. We define a positive learning rate, $PLR \in [0, 1]$. Let V be a fraction of the distance between O_1 and O_2 so that

$$V = (|O_1 - O_2|) \cdot PLR$$

The opinions of the agents are then updated using V . Let O_{max} be the maximum of the two opinions and O_{min} be the minimum of the two opinions:

$$\begin{aligned} O_{max} &= \max(O_1, O_2) \\ O_{min} &= \min(O_1, O_2) \end{aligned}$$

We then pull the two values closer together by V :

$$\begin{aligned} O'_{max} &= O_{max} - V \\ O'_{min} &= O_{min} + V \end{aligned}$$

For cases where $B \leq |O_1 - O_2|$, we have a case of negative learning where agents push each other further away. Similar to the positive learning, we define a negative learning rate $NLR \in [0, 1]$ and let V be given by

We then push the values further apart by V :

Notice that the only difference between positive and negative learning is the learning rate and whether V pushes or pulls the opinions apart.

The process of updating values is done iteratively for each neighbor of A_t . Notice that updating is symmetric in the sense that both A_t and the neighbor of A_t will be pushed or pulled by V .

After the process of updating is finished for A_t , all ties to agents which are outside of the boundary threshold ($B \geq |O_1 - O_2|$), are disconnected. This concludes one timestep, t , in the model.

B. Model fitting. explain how the model was calibrated (Bayesian Hyperparameter Optimization)

3. Model investigation

Get familiar with the different parameters and their interpretations

$$V = (|O_1 - O_2|) \cdot NLR$$

$$\begin{aligned} O'_{max} &= O_{max} + V \\ O'_{min} &= O_{min} - V \end{aligned}$$

A. The effect of randomness. How randomness affects the distribution of opinions, as well as the network

B. The effect of the boundary threshold. How the boundary threshold affects the distribution of opinions, as well as the network

C. The effect of homophily. How homophily affects the distribution of opinions, as well as the network

D. Important interactions. Point to some of the important interactions (possible Golden zones)

4. Results

A. Network generation.

B. Opinion generation.

5. Discussion

6. Conclusion

7. Presentation

A. Intro - Why should people care? Much of our time spent in life is spent sharing and receiving ideas. Many of these ideas are best described not as objective answers but rather as opinions. If you have spent any amount of time on Twitter, you will know that people have opinions on more or less anything. Whether it is whether cats or dogs are the best, to which person should govern country, opinions are everywhere. Discovering the mechanisms of how opinions are shaped, distributed and consumed is therefore of interest to us as scientists but to society as a whole. The best example of the possible societal impact comes perhaps not surprisingly from politics. With a better understanding of the underlying mechanisms, we could better understand echo chambers, opinion diversity and opinion polarization. The dream is that from understanding the underlying phenomena, we might be able to intervene or guide communication to be better shaped to the kind of discourse we want, instead of just accepting the discourse we have.

And this dream is becoming more relevant by the day. During the last decade, political opinions are becoming more and more polarized around the world - but especially in the US. This polarization makes passing legislation harder, in fact any kind of cooperation is made harder by such a distance in opinion space. This polarization can be seen when looking at the increase in partisan voting patterns as well as the rise of overtly partisan news media. And there might be reason to assume that there could be even more polarization coming. One of the big factors to consider here is that technology can enable and accelerate polarization. This has been widely

discussed under the name of echo chambers on social media. In these echo chambers, information is heavily filtered as most of people inside the echo chamber shares the same opinion. And social media has made it easier to find people that share our opinion. The "Friend suggestion" tool on Facebook for instance is largely based on the principle of similarity. The more you have in common, the more likely you are to be friends. As technology becomes more powerful and information becomes more vast, these "Friend suggestion" algorithms might become even better at finding similarities.

With that said, we have not reached doomsday yet. Polarization does not seem to be our only fate. Although Europe is no stranger to polarization, some countries have much more agreement and diversity in political opinions. Of course, part of that explanation is to due with the 2-party system, which will not be the focus here. Beyond the historical evidence from Europe, psychological research on the effect of social influence seems to indicate that the effect is largely assimilative rather than distancing. And just as technology has the power to polarize, it also has the power to diversify. Technology has the potential to liberate you from geography, connecting with more diverse people than just the people from your town. You have the power to be a part of communities spanning the globe instead of spanning the town hall. Moreover, the amount of information available to you is also much more diverse and abundant thanks to technology.

From this onset, we are ready to start asking and tackling the questions of opinion dynamics.

B. Models of Opinion Dynamics. There is a long tradition, going back all the way to the 1950s of building models of opinion dynamics. Many of the classic models are analytical models, but the field has largely transitioned to agent-based models. In these models, modellers have tried to incorporate some of the knowledge already discussed in the introduction. Specifically, these models are normally based on the idea of social influence and homophily. In other words, agents influence each other through interactions and they prefer to interact with agents that are similar to themselves. Beyond assimilative forces, modellers have also introduced "negative" social influence. Agents can sometimes cause their opinions to drift further apart. It's important to mention that negative influence is weakly supported by evidence, whereas positive influence is a robust finding. One of these models goes by the name of a "bounded confidence model". The work I am doing builds specifically on this type of model, so we will spend some time introducing it in some detail.

Thankfully, it is a very simple and intuitive agent-based model. We conceptualize opinions as a continuous value between -1 and 1. We have agents initialized with opinions drawn from a uniform distribution:

$$O \sim \mathcal{U}(-1, 1)$$

We situate the agents in a network, typically a fully connected network or a lattice structure. We then sample an agent, who updates their opinion based on the opinions of her peers. To model the homophilic nature of human interactions, we introduce a bound on the confidence on the agents, hence

the name "bounded confidence". We can think of this as a threshold for when agent's are similar enough to interact:

$$B \geq |O_1 - O_2|$$

where B is the threshold for confidence and O_1 and O_2 refers to opinions of two interacting agents. When agents are similar, they interact positively, meaning that they pull each other's opinions closer together. When they are too dissimilar, they push each other further away. This is formalized by letting the agents update their values based on the distance between them:

$$V_p = \frac{1}{2}(|O_1 - O_2|) \cdot \alpha$$

$$V_n = \frac{1}{2}(|O_1 - O_2|) \cdot \beta$$

where V_p is the positive influence value, V_n is the negative influence value, α is the positive learning rate and β is the negative learning rate. The fraction $\frac{1}{2}$ is only there to ease the interpretation of the learning rates. When $\alpha = 1$, agents will take the mean of O_1 and O_2 to be their opinion. They meet each other halfway. When $\alpha = \frac{1}{2}$, will both go half the distance from their onset to the mean of O_1 and O_2 . β is less intuitive, but it's exactly the same idea as α . The distance that opinions are pushed is proportional to the distance of opinions. β defines how much of that distance agents are pushed away from each other.

This does not outline the specific implementation of these models, but the principle is the exact same for these types of models. And these models have been well-studied and are a famous class of models within the opinion dynamics field. But is this a good model of opinion dynamics? To answer that, it is fruitful to ask what the model assumes. All models make assumptions, but some models abstract away crucial pieces of the puzzle. And in this case, a crucial point is that social networks determine opinions and opinions determine social networks. Connections to other people determines largely what kind of discussions we have, what opinions we are exposed to. But opinions and ideas also determine what people we are drawn to and who we keep as friends. Connections are not set in stone. Connections and opinions are in an interplay, where they co-evolve dynamically. In this case, bounded confidence models typically assume static networks. This assumes that opinions will not affect the underlying social structure. More specifically, the model assumes that although people can annoy you enough to change to the "opposite" opinions, you will still stay friends with them.

I will therefore argue that tie dissolution and tie creation should be an important feature of any opinion dynamic model. In related game-theoretic work on cooperation, tie dissolution has been shown to be an extremely important mechanism for generating cooperation. By deleting ties with non-cooperators, the defectors are isolated, and cooperation soars in populations. In other words, by defriending toxic people, you are helping making the world a more cooperative place. And something similar might be happening in opinion dynamics. By deleting ties with agents of negative influence, you might

avoid radicalization and polarization. We therefore need to develop a good model for tie generation and tie dissolution in a network.

C. Models of Network Formation. A consistent problem in the field of network formation is that idealized networks don't look like social networks. Typically, the networks can replicate one or two of the typical features of social networks, but not all of them at the same time. Ideally, we want a model that can generate the general pattern of real social networks, and not just one or two of their characteristics. Some of the most prominent social network characteristics are short average path length, high clustering coefficient, degree assortativity and scale-free degree distributions. Based on empirical networks studied over time, a good candidate model seems to be one based on the principle of triadic closure. Triadic closure is a fancy way of saying "finding new connections through friends of friends". The study found that connections based on triadic closure was 30 times more likely than finding through "friends of friends of friends", in other words one handshake more away from you. Models based on triadic closure has shown success in generating many of the classical patterns of social networks, as well as being able to describe social networks in animals. For this reason, I assume that most new connections are made through triadic closure and that the rest can be modelled as random connections in the network.

D. Summary. We consider a model of opinion dynamics, where agents are initialized in a small-world network and interact with each other as described by the bounded confidence model. However, we let the connections of the network be changed over time based on the opinions of agents. At every time-step, an agent is sampled to be on turn. This agent makes new connections via triadic closure or via connection to a randomly sampled agent. After making this new connection, the sampled agent interacts with all their neighbors iteratively. When all neighbors have been interacted with, negative ties have some probability of being removed.

To keep the number of edges more or less constant, the model keeps track of how many edges have been disconnected. When there is no surplus of edges, agents create new connections by rewiring existing connections. When there is a surplus, these connections are generated without rewiring.

At some point, we have to stop running the simulation. We let the number of timesteps be 10 times the number of agents. On average, each agent will have 10 times where it is the agent on turn. This is where you can debate whether the model should be run for longer amounts of time, as many of the classic models are run more or less until all agents reach equilibrium values, where no opinion change is happening. We simplify and note that for most simulations, the overall direction of the simulation will be apparent after these runs.

E. Training the model. As mentioned, previous models have had problems with generating the combination of characteristic patterns of social networks. Moreover, many of them have problems of generalizability and for the most part only work in one context. To be part of the solution and not the problem, I fit the model to different real world social networks and evaluate how well the model can capture the observed network characteristics. Beyond evaluating the model, we can compare it to the network formation model which does not include

any opinion dynamics. This serves as a more abstract model comparison, to see how much better the model captures the network characteristics by including opinion dynamics.

To fit the models, we run through the parameters of the model (randomness, confidence boundary, learning rate, tie dissolution rate) and find the best combination using Bayesian Hyperparameter Optimization. Bayesian Hyperparameter Optimization is just an algorithm for efficiently searching the parameter space to find some minimum. What we want to minimize here is the difference between the generated network and the target network. Therefore, I minimize the Euclidean norm of the differences between the generated network and the target network:

$$\left[\begin{array}{c} \textit{CorrelationCoefficient} \\ \textit{DegreeAssortativity} \\ \textit{AveragePathLength} \\ \textit{Jensen - ShannonDivergence} \end{array} \right]$$

Although getting as close as possible numerically is what we want, we are more interested in whether the model is able to capture the patterns qualitatively. Of special interest to my purpose of getting more information regarding the co-evolution of networks and opinion dynamics, the relative difference between the model with and without opinion dynamics is interesting. If the co-evolution of networks does influence networks, we would expect the model with opinion dynamics to be better than the model without the opinion dynamics.

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Table 1. Character Level Combat Outcomes

	<i>Dependent variable:</i>		
	Combat Amount	Combat Variability	Combat Skill
	(1)	(2)	(3)
Man - Male	0.042*** (0.002)	5.659*** (0.056)	0.031*** (0.0004)
Woman - Female	-0.026*** (0.005)	1.529*** (0.143)	0.011*** (0.001)
Woman - Male	0.010 (0.009)	0.375 (0.272)	0.005* (0.002)
Player Age	-0.077*** (0.001)		-0.003*** (0.0002)
Mil. Label	0.135*** (0.002)		0.060*** (0.0004)
Constant		-97.425*** (0.046)	
Char. Order FEs	Y	N	Y
Create Date FEs	Y	N	Y
Observations	576,430	576,430	576,430
R ²	0.028	0.018	0.089

p<0.05, ** p<0.01, *** p<0.001

This table reports coefficients and standards errors from ordinary least squares regressions. In all models we can reject the null that *Woman - Female* and *Woman - Male* are equivalent with $p < .01$. In models 2 and 3 we can reject the null that the gender gaps within sex are equivalent ($(\text{Woman} - \text{Male}) - (\text{Woman} - \text{Female}) = \text{Man} - \text{Male}$) with $p < .001$.

$$\begin{aligned}
 (x + y)^3 &= (x + y)(x + y)^2 \\
 &= (x + y)(x^2 + 2xy + y^2) \\
 &= x^3 + 3x^2y + 3xy^2 + y^3.
 \end{aligned}
 \tag{1}$$

References

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