Unfriending facilitates cooperation:
Co-evolution of opinion and network dynamics

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1 Conceptual overview

The model consists of agents situated in an undirected network. Nodes on the network represent social agents. Edges represent social ties to other agents. The model simulates social interactions between the agents without agents having any strategy or task.

2 Components and their properties

The model is specified by five different parameters. The first three describe how interactions change opinions (α, β, T). The fourth specifies the probability of creating new edges randomly instead of through triadic closure (R). Finally, the fifth paramater specifies the probability of dissoluting negative ties (D). The specifics of how they influence the model is specified in ”Dynamics”.

To limit the combinations of different values, we limit the possible values of the different parameters with the following:

$$\alpha \in \{0.05, 0.10, 0.15, 0.20, 0.25\}$$
$$\beta \in \{0.00, 0.05, 0.10, 0.15, 0.20, 0.25\}$$
$$T \in \{0.8, 0.9, 1.0, 1.1, 1.2\}$$
$$R \in \{0.1, 0.3, 0.5\}$$
$$D \in \{0.0, 0.2, 0.4, 0.6, 0.8, 1.0\}$$

3 Initialization

A random Watts-Strogatz small-world graph is created with $N = 500$, $k = 7$, and $p = 0.1$. For each node in the network, an agent is initialized with a value (O_i) which is taken to represent the opinion of the agent. All values of O_i are initialized by drawing from a uniform distribution between -1 and 1:

$$O_i \sim U(-1, 1)$$

4 Dynamics

4.1 Sampling agents and interactions

At every timestep (t) a random agent (A_t) is sampled from the network (N_t). This agent creates a new edge to another agent in the network. The network occationally deletes edges. To keep the number of edges in the network approximately constant, edges are either created or rewired to account for deleted edges. To do this, let E_1 be the number of edges of N_1 and let E_t be the number of edges of N_t. If $E_t < E_1$, A_t will not rewire one of its existing edges, but instead create a new edge. If $E_t \geq E_1$, A_t will rewire one of its existing connections to the new agents. With a probability of R, A_t connects to a random agent, that A_t is not currently connected to. With a probability of $1-R$, A_t connects to one of its edge's edges. Regardless of whether the connection was sampled randomly or through an edge's edges, let C_t define the newly connected agent.

In other to calculate the average path length of the network, the model is restricted to being only one component. To ensure this property, we check the degree of A_t and C_t. If the degree of either A_t or C_t is 0, a new edge is created randomly to a new node from N_t. If both A_t and C_t have degrees larger than 0, and the network has more than one component, a new edge is created to restore the network. Specifically, we restore the edge that A_t rewired to C_t, while keeping the edge from A_t to C_t. I will refer to the process of ensuring only one connected component simply as component ensurance.

4.2 Interaction between neighbors

After a new connection has been formed to A_t, A_t interacts with all its edges. This is done by iteratively interacting with every edge of A_t. Let B denote an edge of A_t. Let $O(\cdot)$ define a function with agents as inputs and their opinions as outputs. The interaction between two different agents is determined by a threshold value, T.

When $T \geq |O(A_t) - O(B)|$, the interaction will pull the opinions of the two agents closer to each other. The force with which they are pulled is defined as a fraction of their distance from each other:

$$V_p = \left(|O(A_t) - O(B)|\right) \cdot \frac{\alpha}{2}$$

Let $O_{max} = \max(O(A_t), O(B))$ and $O_{min} = \min(O(A_t), O(B))$ and update the values of opinions by:

$$O_{max} = O_{max} - V_p$$

$$O_{min} = O_{min} + V_p$$

When $T < |O(A_t) - O(B)|$, the interaction will push the opinions of the two agents further apart by a similar principle as illustrated above:

$$V_n = \left(|O(A_t) - O(B)|\right) \cdot \frac{\beta}{2}$$

Using the same definition for O_{max} and O_{min}, we update the opinions of each agent:

$$O_{max} = O_{max} + V_n$$

$$O_{min} = O_{min} - V_n$$

This process concludes when A_t has updated her values for all her edges.

4.3 Tie-dissolution

When A_t has finished interacting with all her edges, the tie dissolution process begins. If $T < |O(A_t) - O(B)|$, then dissolute the edge between the two agents with a probability of D. After ties are dissoluted, the process of component ensurance is performed where C_t is replaced with B. When all edges of A_t have been evaluated, the time-step concludes. This process is repeated for 10.000 time-steps.

5 Outcome metrics

5.1 Time-dependent Metrics

Every 20th time-step, the current state of the network is recorded. To track the polarization of opinions over time, the mean and standard deviation of the absolute value of opinions are recorded. To track the similarity of an agent's opinion to the opinion of their neighors, the average distance to all edges are recorded. To evaluate the effect of tie-dissolution, the cumulative frequency of tie-dissolutions are recorded. For characterizing the network, I record the average clustering coefficient, average path length and degree assortativity coefficient.

5.2 Final State Metrics

After 10.000 time-steps, the network reaches its final state and its characteristics are recorded. For every agent in the network, I record:

- The initial opinion of the agent
- The opinion of the agent at the final state of the network
- The mean distance to all neighbors' opinions
- The degree of the agent
- The betweeness centrality of the agent
- The clustering coefficient of the agent

The measures related to the opinion of agents can then be correlated with clustering, centrality and degrees to see how changes in network topology relate to changes in opinions.