Answers of Sheet 1 on Chapter 26

1.

An air-filled spherical capacitor is constructed with inner- and outer-shell radii of 7.00 cm and 14.0 cm, respectively. (a) Calculate the capacitance of the device. (b) What potential difference between the spheres results in a 4.00-μC charge on the capacitor?

(a) For a spherical capacitor with inner radius *a* and outer radius *b*,

$$C = \frac{ab}{k_e(b-a)} = \frac{(0.0700\ \text{m})(0.140\ \text{m})}{(8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(0.140\ \text{m}-0.0700\ \text{m})}$$

$$= \boxed{15.6\ \text{pF}}$$

(b) $$\Delta V = \frac{Q}{C} = \frac{4.00\times 10^{-6}\ \text{C}}{1.56\times 10^{-11}\ \text{F}} = 2.57\times 10^5\ \text{V} = \boxed{257\ \text{kV}}$$

2.

A 50.0-m length of coaxial cable has an inner conductor that has a diameter of 2.58 mm and carries a charge of 8.10 μC. The surrounding conductor has an inner diameter of 7.27 mm and a charge of $-8.10\ \mu$C. Assume the region between the conductors is air. (a) What is the capacitance of this cable? (b) What is the potential difference between the two conductors?

(a) The capacitance of a cylindrical capacitor is

$$C = \frac{\ell}{2k_e \ln(b/a)}$$

$$= \frac{50.0\ \text{m}}{2(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\ln(7.27\ \text{mm}/2.58\ \text{mm})}$$

$$= \boxed{2.68\ \text{nF}}$$

(b) Method 1: $\Delta V = 2k_e\lambda \ln\left(\frac{b}{a}\right)$

$$\lambda = \frac{Q}{\ell} = \frac{8.10\times10^{-6}\ \text{C}}{50.0\ \text{m}} = 1.62\times10^{-7}\ \text{C/m}$$

$$\Delta V = 2(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.62\times10^{-7}\ \text{C/m})\ln\left(\frac{7.27\ \text{mm}}{2.58\ \text{mm}}\right)$$

$$= \boxed{3.02\ \text{kV}}$$

Method 2: $\Delta V = \frac{Q}{C} = \frac{8.10\times10^{-6}\ \text{C}}{2.68\times10^{-9}\ \text{F}} = \boxed{3.02\ \text{kV}}$

3.

(a) Regarding the Earth and a cloud layer 800 m above the Earth as the "plates" of a capacitor, calculate the capacitance of the Earth–cloud layer system. Assume the cloud layer has an area of 1.00 km^2 and the air between the cloud and the ground is pure and dry. Assume charge builds up on the cloud and on the ground until a uniform electric field of 3.00×10^6 N/C throughout the space between them makes the air break down and conduct electricity as a lightning bolt. (b) What is the maximum charge the cloud can hold?

(a) $$C = \frac{\kappa \epsilon_0 A}{d} = \frac{(1.00)(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(1.00\times10^3\ \text{m})^2}{800\ \text{m}}$$

$$= \boxed{11.1\ \text{nF}}$$

(b) The potential between ground and cloud is

$$\Delta V = Ed = (3.00\times10^6\ \text{N/C})(800\ \text{m}) = 2.40\times10^9\ \text{V}$$

$$Q = C(\Delta V) = (11.1\times10^{-9}\ \text{C/V})(2.40\times10^9\ \text{V}) = \boxed{26.6\ \text{C}}$$

4.

When a potential difference of 150 V is applied to the plates of a parallel-plate capacitor, the plates carry a surface charge density of 30.0 nC/cm^2. What is the spacing between the plates?

We have $Q = C\Delta V$ and $C = \epsilon_0 A/d$. Thus, $Q = \epsilon_0 A\Delta V/d$

The surface charge density on each plate has the same magnitude, given by

$$\sigma = \frac{Q}{A} = \frac{\epsilon_0 \Delta V}{d}$$

Thus,

$$d = \frac{\epsilon_0 \Delta V}{Q/A} = \frac{(8.85\times10^{-12}\ C^2/N\cdot m^2)(150\ V)}{(30.0\times10^{-9}\ C/cm^2)}$$

$$d = \left(4.43\times10^{-2}\frac{V\cdot C\cdot cm^2}{N\cdot m^2}\right)\frac{(1\ m^2)}{(10^4\ cm^2)}\frac{J}{V\cdot C}\frac{N\cdot m}{J} = \boxed{4.43\ \mu m}$$

5.

For the system of four capacitors shown in Figure P26.19, find (a) the equivalent capacitance of the system, (b) the charge on each capacitor, and (c) the potential difference across each capacitor.

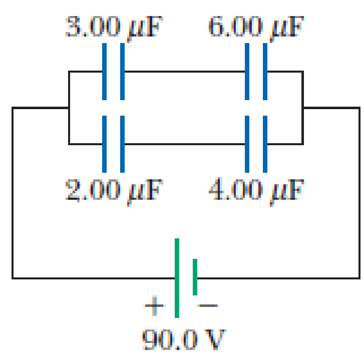


Figure P26.19

(a) The equivalent capacitance of the series combination in the upper branch is

$$\frac{1}{C_{upper}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{3.00\ \mu F} + \frac{1}{6.00\ \mu F} \rightarrow C_{upper} = 2.00\ \mu F$$

Likewise, the equivalent capacitance of the series combination in the lower branch is

$$\frac{1}{C_{lower}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{2.00\ \mu F} + \frac{1}{4.00\ \mu F} \rightarrow C_{lower} = 1.33\ \mu F$$

These two equivalent capacitances are connected in parallel with each other, so the equivalent capacitance for the entire circuit is

$$C_{eq} = C_{upper} + C_{lower} = 2.00\ \mu F + 1.33\ \mu F = \boxed{3.33\ \mu F}$$

(b) Note that the same potential difference, equal to the potential difference of the battery, exists across both the upper and lower branches. Each of the capacitors in series combination holds the same charge as that on the equivalent capacitor. For the upper branch:

$$Q_3 = Q_6 = Q_{upper} = C_{upper}(\Delta V) = (2.00\ \mu F)(90.0\ V) = 180\ \mu C\ \text{s}$$

so, 180 □C on the 3.00-□F and the 6.00-□F capacitors

For the lower branch:

$$Q_2 = Q_4 = Q_{lower} = C_{lower}(\Delta V) = (1.33\ \mu F)(90.0\ V) = 120\ \mu C$$

so, $\boxed{120\ \mu C \text{ on the } 2.00\text{-}\mu F \text{ and } 4.00\text{-}\mu F \text{ capacitors}}$

(c) The potential difference across each of the capacitors in the circuit is:

$$\Delta V_2 = \frac{Q_2}{C_2} = \frac{120\ \mu\text{C}}{2.00\ \mu\text{F}} = \boxed{60.0\ \text{V}}$$

$$\Delta V_3 = \frac{Q_3}{C_3} = \frac{180\ \mu\text{C}}{3.00\ \mu\text{F}} = \boxed{60.0\ \text{V}}$$

$\boxed{\text{60.0 V across the 3.00-}\mu\text{F and the 2.00-}\mu\text{F capacitors}}$

$$\Delta V_4 = \frac{Q_4}{C_4} = \frac{120\ \mu\text{C}}{4.00\ \mu\text{F}} = \boxed{30.0\ \text{V}}$$

$$\Delta V_6 = \frac{Q_6}{C_6} = \frac{180\ \mu\text{C}}{6.00\ \mu\text{F}} = \boxed{30.0\ \text{V}}$$

$\boxed{\text{30.0 V across the 6.00-}\mu\text{F and the 4.00-}\mu\text{F capacitors}}$

6.

Find the equivalent capacitance between points a and b in the combination of capacitors shown in Figure P26.25.

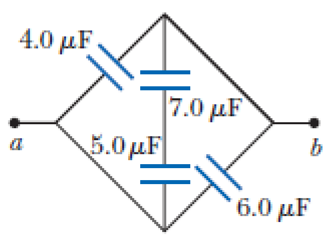


Figure P26.25

$$C_s = \left(\frac{1}{5.00} + \frac{1}{7.00}\right)^{-1} = 2.92\ \mu\text{F}$$

$$C_p = 2.92 + 4.00 + 6.00 = \boxed{12.9\ \mu\text{F}}$$

7.

Find (a) the equivalent capacitance of the capacitors in Figure P26.26, (b) the charge on each capacitor, and (c) the potential difference across each capacitor.

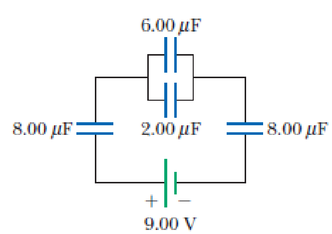
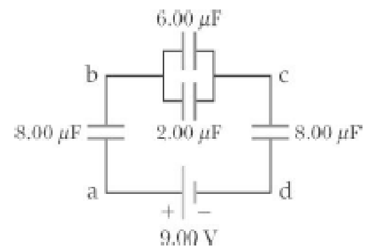


Figure P26.26

(a) First, we replace the parallel combination between points b and c by its equivalent capacitance,
$C_{bc} = 2.00\ \mu F + 6.00\ \mu F = 8.00\ \mu F$.
Then, we have three capacitors in series between points a and d. The equivalent capacitance for this circuit is therefore



$$\frac{1}{C_{eq}} = \frac{1}{C_{ab}} + \frac{1}{C_{bc}} + \frac{1}{C_{cd}} = \frac{3}{8.00\ \mu F}$$

giving

$$C_{eq} = \frac{8.00\ \mu F}{3} = \boxed{2.67\ \mu F}$$

(b) The charge on each capacitor in the series is the same as the charge on the equivalent capacitor:

$$Q_{ab} = Q_{bc} = Q_{cd} = C_{eq}\left(\Delta V_{ad}\right) = (2.67\ \mu F)(9.00\ V) = 24.0\ \mu C$$

Then, note that $\Delta V_{bc} = \frac{Q_{bc}}{C_{bc}} = \frac{24.0\ \mu C}{8.00\ \mu F} = 3.00\ V$. The charge on each capacitor in the original circuit is:

On the 8.00 μF between a and b:

$$Q_8 = Q_{ab} = \boxed{24.0\ \mu C}$$

On the 8.00 μF between c and d:

$$Q_8 = Q_{cd} = \boxed{24.0\ \mu C}$$

On the 2.00 μF between b and c:

$$Q_2 = C_2\left(\Delta V_{bc}\right) = (2.00\ \mu F)(3.00\ V) = \boxed{6.00\ \mu C}$$

On the 6.00 μF between b and c:

$$Q_6 = C_6\left(\Delta V_{bc}\right) = (6.00\ \mu F)(3.00\ V) = \boxed{18.0\ \mu C}$$

(c) We earlier found that $\Delta V_{bc} = 3.00\ V$. The two 8.00 μF capacitors have the same voltage: $\Delta V_8 = \Delta V_8 = \frac{Q}{C} = \frac{24.0\ \mu C}{8.00\ \mu F} = 3.00\ V$, so we conclude that the potential difference across each capacitor is the same: $\Delta V_8 = \Delta V_2 = \Delta V_6 = \Delta V_8 = \boxed{3.00\ V}$.

8.

A parallel-plate capacitor in air has a plate separation of 1.50 cm and a plate area of 25.0 cm^2. The plates are charged to a potential difference of 250 V and disconnected from the source. The capacitor is then immersed in distilled water. Assume the liquid is an insulator. Determine (a) the charge on the plates before and after immersion, (b) the capacitance and potential difference after immersion, and (c) the change in energy of the capacitor.

(a) The charge is the same before and after immersion, with value

$$Q = C_i(\Delta V)_i = \frac{\epsilon_0 A(\Delta V)_i}{d}$$

$$Q = \frac{(8.85\times10^{-12}\ C^2/N\cdot m^2)(25.0\times10^{-4}\ m^2)(250\ V)}{1.50\times10^{-2}\ m}$$

$$= \boxed{369\ pC}$$

(b) Finally,

$$C_f = \frac{\kappa\epsilon_0 A}{d} = \frac{Q}{(\Delta V)_f}:$$

$$C_f = \frac{(80)(8.85\times10^{-12}\ C^2/N\cdot m^2)(25.0\times10^{-4}\ m^2)}{1.50\times10^{-2}\ m}$$

$$= \boxed{1.20\times10^{-10}\ F}$$

and $$(\Delta V)_f = \frac{Q}{C_f} = \frac{C_i(\Delta V)_i}{C_f} = \frac{(\epsilon_0 A/d)}{(\kappa\epsilon_0 A/d)}(\Delta V)_i = \frac{(\Delta V)_i}{\kappa} = \frac{250\ V}{80}$$

$$= \boxed{3.10\ V}$$

(c) Originally, $U_i = \frac{1}{2}C_i(\Delta V)_i^2 = \frac{\epsilon_0 A(\Delta V)_i^2}{2d}$.

Finally, $U_f = \frac{1}{2}C_f(\Delta V)_f^2 = \frac{1}{2}\left(\frac{\kappa\epsilon_0 A}{d}\right)\left(\frac{(\Delta V)_i}{\kappa}\right)^2 = \frac{\epsilon_0 A(\Delta V)_i^2}{2d\kappa}$, where, from Table 26.1, $\kappa = 80$ for distilled water. So,

$$\Delta U = U_f - U_i$$

$$= \frac{\epsilon_0 A(\Delta V)_i^2}{2d\kappa} - \frac{\epsilon_0 A(\Delta V)_i^2}{2d}$$

$$= \frac{\epsilon_0 A(\Delta V)_i^2}{2d}\left(\frac{1}{\kappa} - 1\right) = \frac{\epsilon_0 A(\Delta V)_i^2(1-\kappa)}{2d\kappa}$$

$$\Delta U = \frac{\left(8.85\times10^{-12}\ C^2/N\cdot m^2\right)\left(25.0\times10^{-4}\ m^2\right)(250\ V)^2(1-80)}{2\left(1.50\times10^{-2}\ m\right)(80)}$$

$$= -4.55\times10^{-8}\ J = \boxed{-45.5\ nJ}$$

9.

The voltage across an air-filled parallel-plate capacitor is measured to be 85.0 V. When a dielectric is inserted and completely fills the space between the plates , the voltage drops to 25.0 V. (a) What is the dielectric constant of the inserted material?
(b) Can you identify the dielectric? If so, what is it?
(c) If the dielectric does not completely fill the space between the plates, what could you conclude about the voltage across the plates?

(a) Note that the charge on the plates remains constant at the original value, Q_0, as the dielectric is inserted. Thus, the change in the potential difference, $\Delta V = Q/C$, is due to a change in capacitance alone. The ratio of the final and initial capacitances is

$$\frac{C_f}{C_i} = \frac{\kappa \in_0 A/d}{\in_0 A/d} = \kappa$$

and $$\frac{C_f}{C_i} = \frac{Q_0/(\Delta V)_f}{Q_0/(\Delta V)_i} = \frac{(\Delta V)_i}{(\Delta V)_f} = \frac{85.0\text{ V}}{25.0\text{ V}} = 3.40$$

Thus, the dielectric constant of the inserted material is $\boxed{\kappa = 3.40}$.

(b) The material is probably $\boxed{\text{nylon}}$ (see Table 26.1).

(c) The presence of a dielectric weakens the field between plates, and the weaker field, for the same charge on the plates, results in a smaller potential difference. If the dielectric only partially filled the space between the plates, the field is weakened only within the dielectric and not in the remaining air-filled space, so the potential difference would not be as small. $\boxed{\text{The voltage would lie somewhere between 25.0 V and 85.0 V.}}$

10.

A small, rigid object carries positive and negative 3.50-nC charges. It is oriented so that the positive charge has coordinates (−1.20 mm, 1.10 mm) and the negative charge is at the point (1.40 mm, −1.30 mm). (a) Find the electric dipole moment of the object. The object is placed in an electric field $\vec{E} = (7.80 \times 10^3\,\hat{i} - 4.90 \times 10^3\,\hat{j})$ N/C. (b) Find the torque acting on the object. (c) Find the potential energy of the object–field system when the object is in this orientation. (d) Assuming the orientation of the object can change, find the difference between the maximum and minimum potential energies of the system.

(a) The displacement from negative to positive charge is

$$2\vec{a} = \left(-1.20\hat{i} + 1.10\hat{j}\right) \text{ mm} - \left(1.40\hat{i} - 1.30\hat{j}\right) \text{ mm}$$
$$= \left(-2.60\hat{i} + 2.40\hat{j}\right) \times 10^{-3} \text{ m}$$

The electric dipole moment is $\vec{p} = 2\vec{a}q$

$$\vec{p} = \left(3.50 \times 10^{-9} \text{ C}\right)\left(-2.60\hat{i} + 2.40\hat{j}\right) \times 10^{-3} \text{ m}$$
$$= \boxed{\left(-9.10\hat{i} + 8.40\hat{j}\right) \times 10^{-12} \text{ C} \cdot \text{m}}$$

(b) The torque exerted by the field on the dipole is

$$\vec{\tau} = \vec{p} \times \vec{E}$$
$$= \left[\left(-9.10\hat{i} + 8.40\hat{j}\right) \times 10^{-12} \text{ C} \cdot \text{m}\right] \times \left[\left(7.80\hat{i} - 4.90\hat{j}\right) \times 10^{3} \text{ N/C}\right]$$
$$= \left(+44.6\hat{k} - 65.5\hat{k}\right) \times 10^{-9} \text{ N} \cdot \text{m} = \boxed{-2.09 \times 10^{-8}\hat{k} \text{ N} \cdot \text{m}}$$

(c) Relative to zero energy when it is perpendicular to the field, the dipole has potential energy

$$U = -\vec{p} \cdot \vec{E}$$
$$= -\left[\left(-9.10\hat{i} + 8.40\hat{j}\right) \times 10^{-12} \text{ C} \cdot \text{m}\right] \cdot \left[\left(7.80\hat{i} - 4.90\hat{j}\right) \times 10^{3} \text{ N/C}\right]$$
$$= (71.0 + 41.2) \times 10^{-9} \text{ J} = \boxed{112 \text{ nJ}}$$

(d) For convenience we compute the magnitudes

$$|\vec{p}| = \sqrt{(9.10)^2 + (8.40)^2} \times 10^{-12}\ \mathrm{C \cdot m} = 12.4 \times 10^{-12}\ \mathrm{C \cdot m}$$

and $|\vec{E}| = \sqrt{(7.80)^2 + (4.90)^2} \times 10^3\ \mathrm{N/C} = 9.21 \times 10^3\ \mathrm{N/C}$

The maximum potential energy occurs when the dipole moment is opposite in direction to the field, and is

$$U_{max} = -\vec{p} \cdot \vec{E} = -|\vec{p}||\vec{E}|(-1) = |\vec{p}||\vec{E}| = 114\ \mathrm{nJ}$$

The minimum potential energy configuration is the stable equilibrium position with the dipole aligned with the field. The value is $U_{min} = -114\ \mathrm{nJ}$

Then the difference, representing the range of potential energies available to the dipole, is $U_{max} - U_{min} = \boxed{228\ \mathrm{nJ}}$.