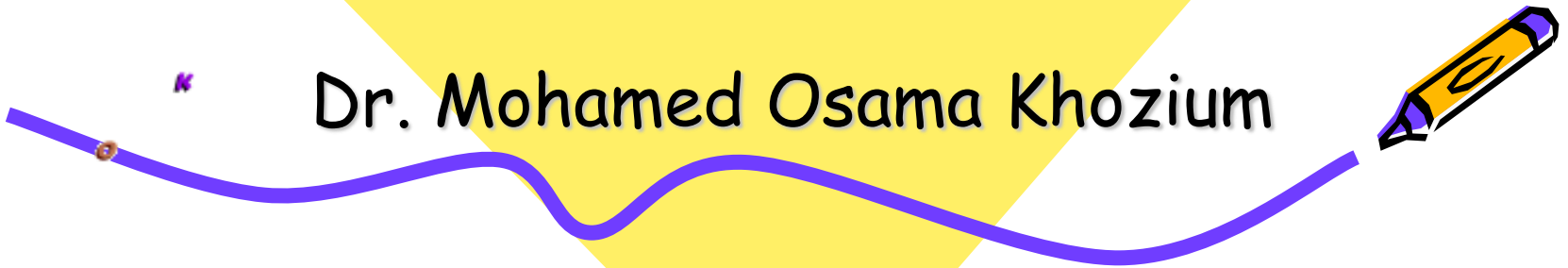




# Logic Design

CSE 221

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| Decimal | Binary | Octal | Hexadecimal |
|---------|--------|-------|-------------|
| 31      |        |       |             |
|         | 11101  |       |             |
|         |        | 23    |             |
|         |        |       | 1A          |





| Decimal | Binary | Octal | Hexadecimal |
|---------|--------|-------|-------------|
| 31      | 11111  | 37    | 1F          |
| 29      | 11101  | 35    | 1D          |
| 19      | 10011  | 23    | 13          |
| 26      | 11010  | 32    | 1A          |









# Exercise - 2's C conversions



- What is -20 expressed as an 8-bit binary number in 2's complement notation?  
- Answer: \_\_\_\_\_
- 1100011 is a 7-bit binary number in 2's complement notation. What is the decimal value?  
- Answer: \_\_\_\_\_





# Exercise - 2's C conversions

Answer

- What is -20 expressed as an 8-bit binary number in 2's complement notation?  
- Answer: 11101100
- 1100011 is a 7-bit binary number in 2's complement notation. What is the decimal value?  
- Answer: -29

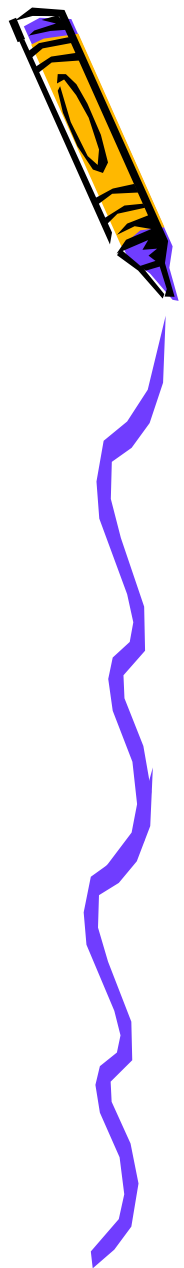




One's Complement subtraction

01010 (10)  
-  
00111 (7)  
One's complement for subtrahend  
then add  
01010  
+  
11000  
-----  
100010  
Neglect carry and add 1  
00011 (+3)

Two's Complement subtraction  
01010 (10)  
-  
00111 (7)  
Two's complement for subtrahend  
then add  
01010  
+  
11001  
-----  
100011  
Neglect carry  
00011 (+3)





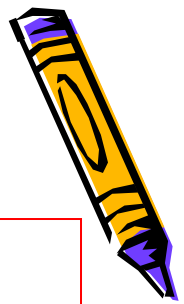


One's Complement subtraction

00111 (7)  
-  
01010 (10)  
One's complement for subtrahend  
then add  
00111  
+  
10101  
-----  
11100  
NO carry and get one's  
complement again...put -  
-00011 (-3)

Two's Complement subtraction

00111 (7)  
-  
01010 (10)  
Two's complement for subtrahend then  
add  
00111  
+  
10110  
-----  
11101  
No carry Two's complement again  
- 00011 (-3)





# BCD conversion

Ex 1 : convert each of the following decimal to BCD code :

a) 35

b) 98

c) 170

d) 2469





# BCD conversion



Solution

|       |      |      |
|-------|------|------|
| a) 35 | 3    | 5    |
|       | 0011 | 0101 |

Then  $35 \rightarrow 00110101$

|       |      |      |
|-------|------|------|
| b) 98 | 9    | 8    |
|       | 1001 | 1000 |

Then  $98 \rightarrow 10011000$





# BCD conversion



|        |      |      |      |
|--------|------|------|------|
| c) 170 | 1    | 7    | 0    |
|        | 0001 | 0111 | 0000 |

Then 170  $\rightarrow$  000101110000

|         |      |      |      |      |
|---------|------|------|------|------|
| d) 2469 | 2    | 4    | 6    | 9    |
|         | 0010 | 0100 | 0110 | 1001 |

Then 2469  $\rightarrow$  0010010001101001





# BCD conversion



Ex2 Convert each of the following BCD code to decimal :

a) 10000110    b) 001101010001

c) 1001010001110000

Solution

a) Start from right and group each four digits

|      |      |
|------|------|
| 1000 | 0110 |
|------|------|

8

6

Then 10000110  $\rightarrow$  86





# BCD conversion



b) 001101010001

0011

0101

0001

3

5

1

→ 351

c) 1001

0100

0111

0000

9

4

7

0

→ 9470





# BCD addition



1 - use binary addition rules

2 - if the 4-bit sum is greater than 9 then it is not a BCD valid number ....add 6(0110) to the 4-bit sum.





# BCD addition



Add the following BCD numbers

a)  $0011 + 0100$

b)  $001000111 + 00010101$

c)  $1001 + 0100$

d)  $00010110 + 00010101$

e)  $01100111 + 01010011$







# BCD addition



## Solutions

$$\begin{array}{rcl} \text{a)} & 0011 & \rightarrow 3 \\ & + 0100 & \rightarrow 4 \end{array}$$

---

$$0111 \rightarrow 7$$

$$\begin{array}{rcl} \text{b)} & 00100011 & \rightarrow 23 \\ & + 00010101 & \rightarrow 15 \end{array}$$

---

$$00111000 \rightarrow 38 \quad (\text{each number} < 9)$$





# BCD addition



## Solutions

$$\begin{array}{rcl} \text{c)} & 1001 & \rightarrow 9 \\ & + 0100 & \rightarrow 4 \\ \hline \end{array}$$

$$\begin{array}{rcl} & 1101 & \rightarrow 13 \rightarrow \text{invalid BCD number} > 9 \\ & + 0110 & \rightarrow \text{Add 6 (0110)} \\ \hline \end{array}$$

$$\begin{array}{rcl} & 10011 & \rightarrow 0001\ 10011 \rightarrow 13 \text{ in BCD} \end{array}$$





# BCD addition



## Solutions

$$\text{d) } 00010110 \rightarrow 16$$

$$+ 00010101 \rightarrow 15 \quad 6+5 = 11 > 9$$

---

$$0010\underline{1011} \rightarrow 1011 > 9 \text{ then add } 6(0110)$$

$$+ 0110 \rightarrow \text{Add } 6(0110)$$

---

$$00110001 \rightarrow 0011 \ 00001 \rightarrow 31 \text{ in BCD}$$





# BCD addition



## Solutions

$$\begin{array}{rcll} \text{e)} & 01100111 & \rightarrow & 6 \quad 7 \\ + & 01010011 & \rightarrow & 5 \quad 3 \quad 6+5 = 11 > 9 \text{ \& } 7+3 > 9 \end{array}$$

---

$$\begin{array}{rcll} & \underline{1011} \ \underline{1010} & \rightarrow & 1011 > 9 \text{ then add } 6(0110) \\ + & 0110 \ 0110 & \rightarrow & \text{Add } 6 \ (0110) \ \& \ 6(0110) \end{array}$$

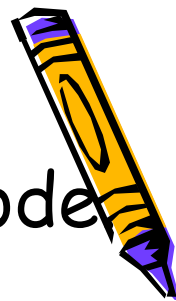
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$$1 \ 0010 \ 0000 \rightarrow 0001 \ 0010 \ 0000 \rightarrow 120$$





# Binary to Gray Code



Convert the binary number 11000110 to Gray code

Sol.

|        |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
|--------|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Binary | 1 | + | 1 | + | 0 | + | 0 | + | 0 | + | 1 | + | 1 | + | 0 |
|        | ↓ |   | ↓ |   | ↓ |   | ↓ |   | ↓ |   | ↓ |   | ↓ |   | ↓ |
| Gray   | 1 |   | 0 |   | 1 |   | 0 |   | 0 |   | 1 |   | 0 |   | 1 |

Shortcut

The first number will be the same...the second number in gray = first + second in binary..... the third in gray = second + third in binary and go on neglect carry.





# Gray Code to binary



Convert the gray code number 10100101 to binary.

Sol.

|        |   |       |       |       |   |   |   |   |
|--------|---|-------|-------|-------|---|---|---|---|
| Gray   | 1 | 0     | 1     | 0     | 0 | 1 | 0 | 1 |
|        | ↓ | ↓     | ↓     | ↓     | ↓ | ↓ | ↓ | ↓ |
| Binary | 1 | +0= 1 | +1= 0 | +0= 0 | 0 | 1 | 1 | 0 |

## Shortcut

The first number will be the same...the second number in binary = first (binary) + second (gray).. the third in binary = second (binary)+ third (gray) and go on ...neglect carry.







# Simplification review





| OR |                     |
|----|---------------------|
| 1  | $x+1 = 1$           |
| 2  | $x+x' = 1$          |
| 3  | $x+x = x$           |
| 4  | $x+0 = x$           |
| 5  | $(x')' = x$         |
| 6  | $x+y = y+x$         |
| 7  | $x+(y+z) = (x+y)+z$ |
| 8  | $x.(y+z) = x.y+x.z$ |
| 9  | $(x+y)' = x'.y'$    |
| 10 | $x+(x.y) = x$       |

| AND |                       |
|-----|-----------------------|
| 1   | $x.1 = x$             |
| 2   | $x.x' = 0$            |
| 3   | $x.x = x$             |
| 4   | $x.0 = 0$             |
| 5   | $(x')' = x$           |
| 6   | $x.y = y.x$           |
| 7   | $x.(y.z) = (x.y).z$   |
| 8   | $x+y.z = (x+y).(x+z)$ |
| 9   | $(x.y)' = x'+y'$      |
| 10  | $x.(x+y) = x$         |





Note

$$XX' = 0$$

$$XX'Y = 0$$

$$X'XYZ'ABCD = 0$$

$$X + x' = 1$$

$$Xy + (xy)' = 1$$

$$Xyz + (xyz)' = 1$$





EX 1

$$F = AB + A(B + C) + B(B + C)$$





EX 1

$$F = AB + A(B + C) + B(B + C)$$

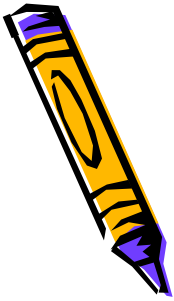
$$AB + AB + AC + BB + BC$$

$$AB + AC + B(1 + C)$$

$$B + AB + AC$$

$$B(1 + A) + AC$$

$$= B + AC$$





EX 2

$$F = [A\bar{B}(C + BD) + \bar{A}\bar{B}]C$$





EX 2

$$F = [A\bar{B}(C + BD) + \bar{A}\bar{B}]C$$

$$(A\bar{B}C + \textcolor{red}{A\bar{B}BD} + \bar{A}\bar{B})C$$

$$(A\bar{B}C + \bar{A}\bar{B})C$$

$$CA\bar{B}\textcolor{red}{C} + C\bar{A}\bar{B}$$

$$\bar{B}CA + \bar{B}C\bar{A}$$

$$\bar{B}C(\textcolor{red}{A + \bar{A}})$$

$$\bar{B}C$$





EX 3

$$F = \overline{AB} + \overline{AC} + \overline{A} \overline{B} \overline{C}$$





EX 3

$$F = \overline{AB} + \overline{AC} + \overline{A} \overline{B} \overline{C}$$

$$\overline{A} + \overline{B} + \overline{A} + \overline{C} + \overline{A} \overline{B} \overline{C}$$

$$\overline{A} + \overline{B} + \overline{C} + \overline{A} \overline{B} \overline{C}$$

$$\overline{A} + \overline{B} + \overline{C} (1 + \overline{A} \overline{B})$$

$$\overline{A} + \overline{B} + \overline{C}$$

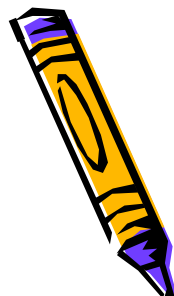






Ex 4

$$F = \overline{A}B + A\overline{C} + \overline{A}\overline{B}C$$





Ex 4

$$F = \overline{A B} + \overline{A C} + \overline{A} \overline{B} C$$

$$(\overline{A B})(\overline{A C}) + \overline{A} \overline{B} C$$

$$(\overline{A} + \overline{B})(\overline{A} + \overline{C}) + \overline{A} \overline{B} C$$

$$\overline{A} \overline{A} + \overline{A} \overline{C} + \overline{A} \overline{B} + \overline{B} \overline{C} + \overline{A} \overline{B} C$$

$$\overline{A} (1 + \overline{B} + \overline{C} + \overline{B} C) + \overline{B} \overline{C}$$

$$\overline{A} + \overline{B} \overline{C}$$





Ex 5

$$F = \left[ AB(C + \overline{BD}) + \overline{AB} \right] CD$$





$$F = \left[ AB(C + \overline{BD}) + \overline{AB} \right] CD$$

$$\left[ AB(C + \overline{B} + \overline{D}) + \overline{AB} \right] CD$$

$$\left[ ABC + \textcolor{red}{AB}\overline{B} + AB\overline{D} + \overline{AB} \right] CD$$

$$ABC\textcolor{red}{C}D + \textcolor{red}{ABC}\overline{D}D + \overline{AB}CD$$

$$ABCD + \overline{AB}CD$$

$$CD (AB + \overline{AB})$$

$$= CD$$





ANOTHER APPROACH



$$F = \left[ AB(C + \overline{BD}) + \overline{AB} \right] CD$$

$$[\overline{AB} + AB(C + \overline{B} + \overline{D})] CD$$

$$[(\overline{AB} + AB)(\overline{AB} + (C + \overline{B} + \overline{D}))] CD$$

$$[(\overline{AB} + (C + \overline{B} + \overline{D}))] CD$$

$$[\overline{AB} + C + \overline{B} + \overline{D}] CD$$

$$[\overline{A} + \overline{B} + C + \overline{B} + \overline{D}] CD$$

$$[\overline{A}CD + \overline{B}CD + C\overline{C}D + \overline{B}\overline{C}D + \overline{D}CD]$$

$$CD(1 + A + B)$$

$$CD$$





Thank you



