Logical Design

CS 221

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1. Number Systems

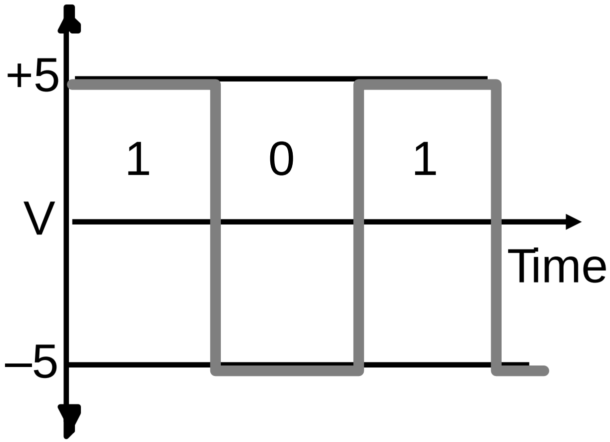
Location in course textbook

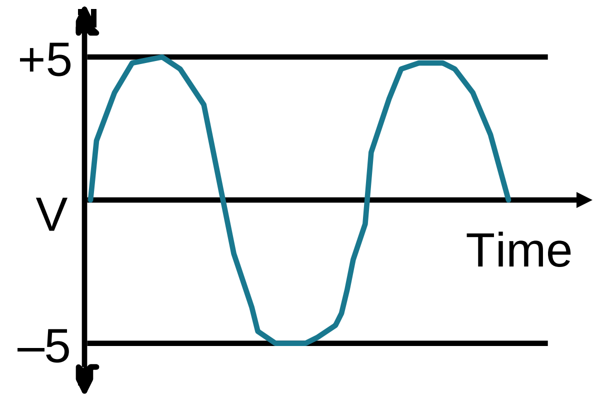
Chapt. 1

Digital Hardware Systems

Digital Systems

Digital vs. Analog Waveforms





Digital:
only assumes discrete values

Analog:
values vary over a broad range continuously

Digital Hardware Systems

- Digital Binary System
  - Two discrete values:
    - yes, on, 5 volts, current flowing, "1"
    - no, off, 0 volts, no current flowing, "0”
  - Advantage of binary systems:
    - Rigorous (exact) mathematical foundation based on logic
    - it’s easy to implement

IF the garage door is open
AND the car is running
THEN the car can be backed out of the garage

both the door must be open and the car running before I can back out

the preconditions must be true to imply the conclusion

Binary Bit and Group Definitions

- Bit - a single binary digit
- Nibble - a group of four bits
- Byte - a group of eight bits
- Word - depends on processor; 8, 16, 32, or 64 bits
- LSB - Least Significant Bit (on the right)
- MSB - Most Significant Bit (on the left)

Binary Representation of Information

- Information divided into groups of symbols
  - 26 English letters
  - 10 decimal digits
  - 50 states in USA
- Digital systems manipulate information as 1's & 0's
- The mapping of symbols to binary value is known as a "code"
- The mapping must be unique

Common Number Systems

System	Base	Symbols	Used by humans?	Used in computers?
Decimal	10	0, 1, … 9	Yes	No
Binary	2	0, 1	No	Yes
Octal	8	0, 1, … 7	No	No
Hexa-decimal	16	0, 1, … 9, A, B, … F	No	No

Quantities/Counting (1 of 3)

Decimal	Binary	Octal	Hexa-decimal
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7

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Quantities/Counting (2 of 3)

Decimal	Binary	Octal	Hexa-decimal
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

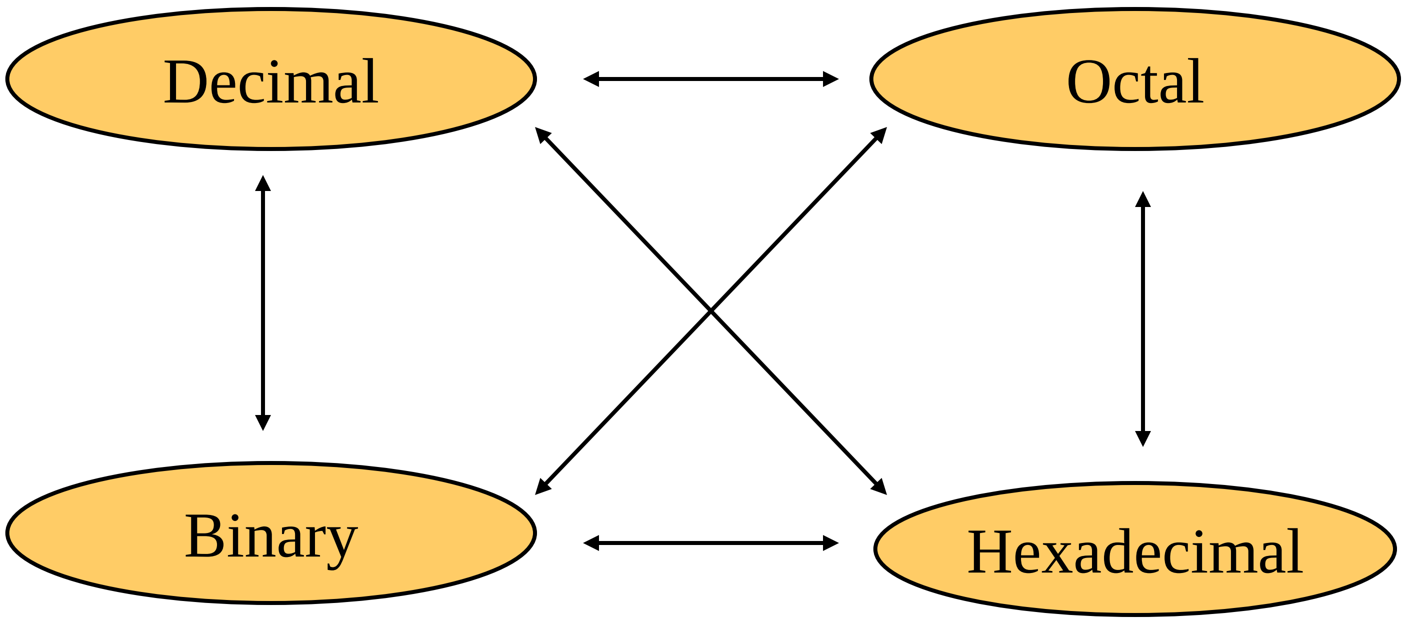
Quantities/Counting (3 of 3)

Decimal	Binary	Octal	Hexa-decimal
16	10000	20	10
17	10001	21	11
18	10010	22	12
19	10011	23	13
20	10100	24	14
21	10101	25	15
22	10110	26	16
23	10111	27	17

Etc.

Conversion Among Bases

- The possibilities:

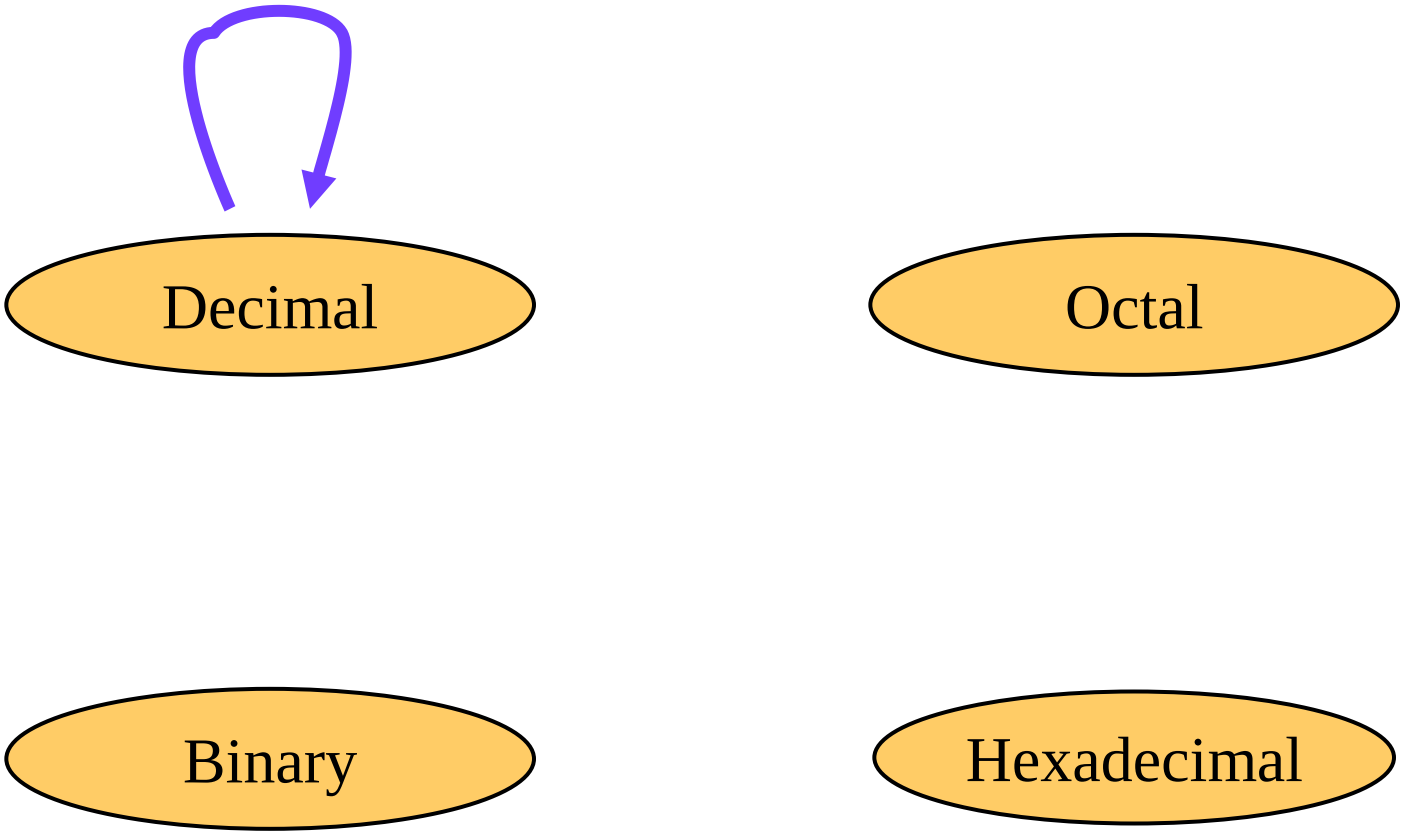


Quick Example

$$25_{10} = 11001_2 = 31_8 = 19_{16}$$

Base

Decimal to Decimal (just for fun)



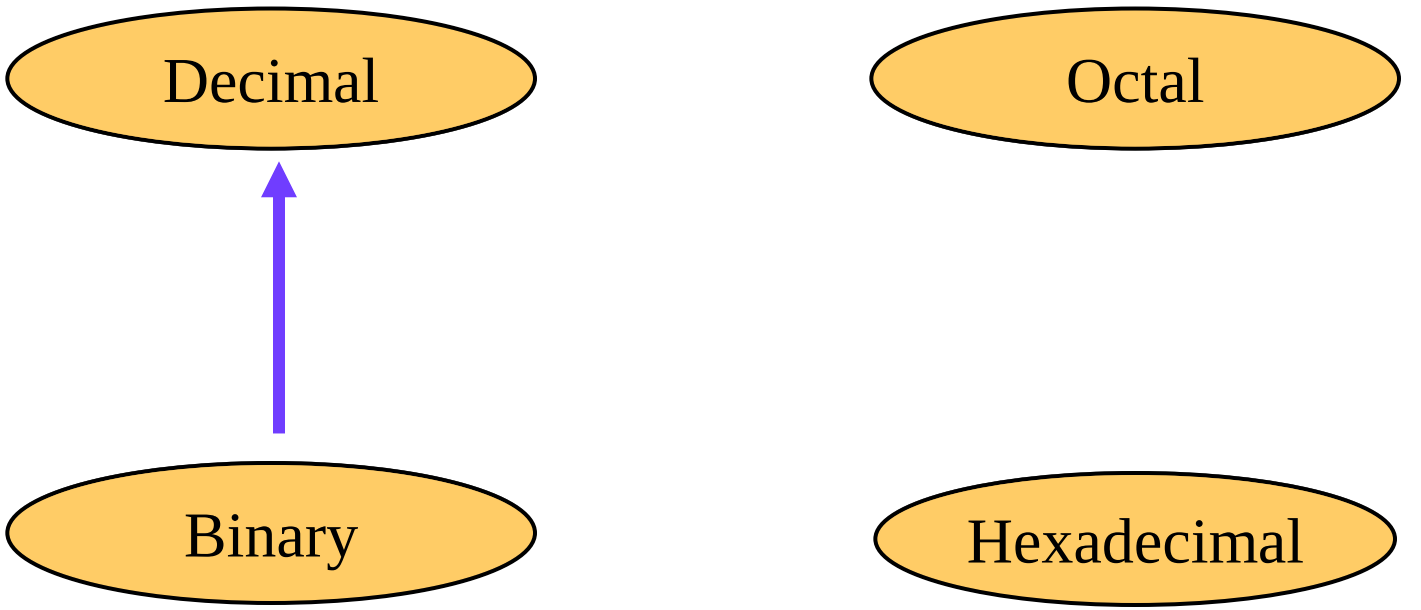
Next slide…

$$125_{10} \Rightarrow \begin{array}{rcr} 5 \times 10^0 & = & 5 \\ 2 \times 10^1 & = & 20 \\ 1 \times 10^2 & = & 100 \\ \hline & & 125 \end{array}$$

Weight

Base

Binary to Decimal



Binary to Decimal

- Technique
  - Multiply each bit by 2^n, where n is the "weight" of the bit
  - The weight is the position of the bit, starting from 0 on the right
  - Add the results

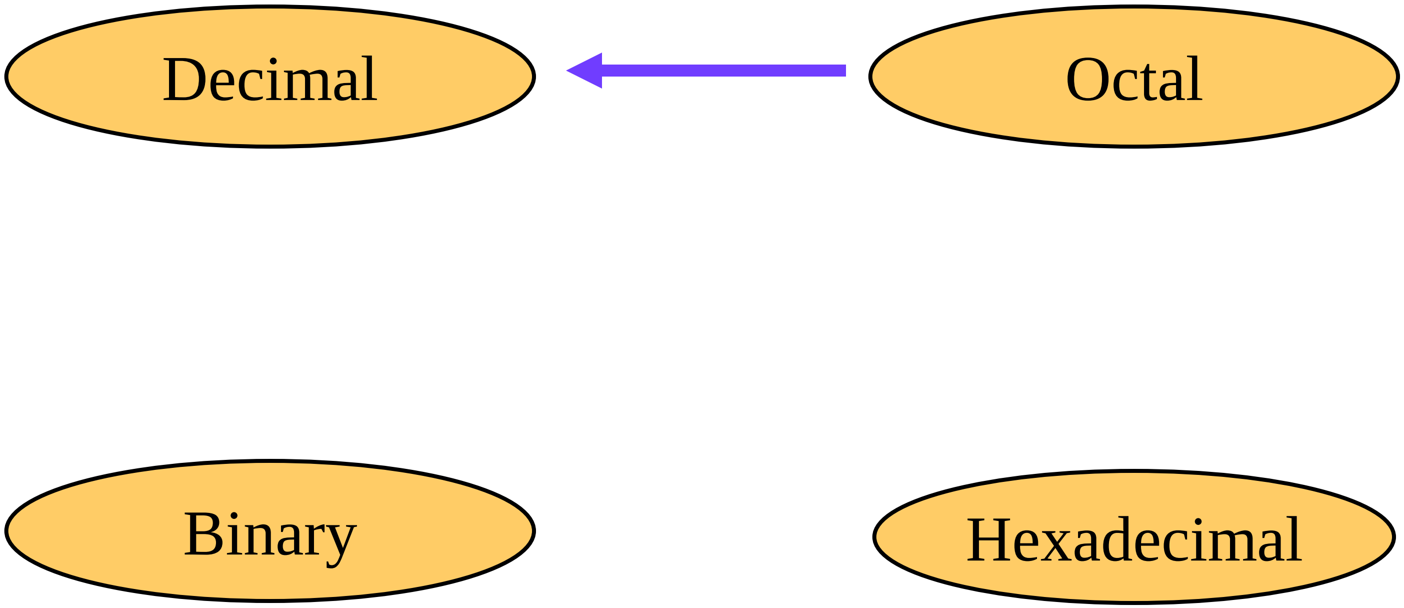
Example

Bit “0”

101011_2 =>

1	x	2^0	=	1
1	x	2^1	=	2
0	x	2^2	=	0
1	x	2^3	=	8
0	x	2^4	=	0
1	x	2^5	=	32
				43_{10}

Octal to Decimal



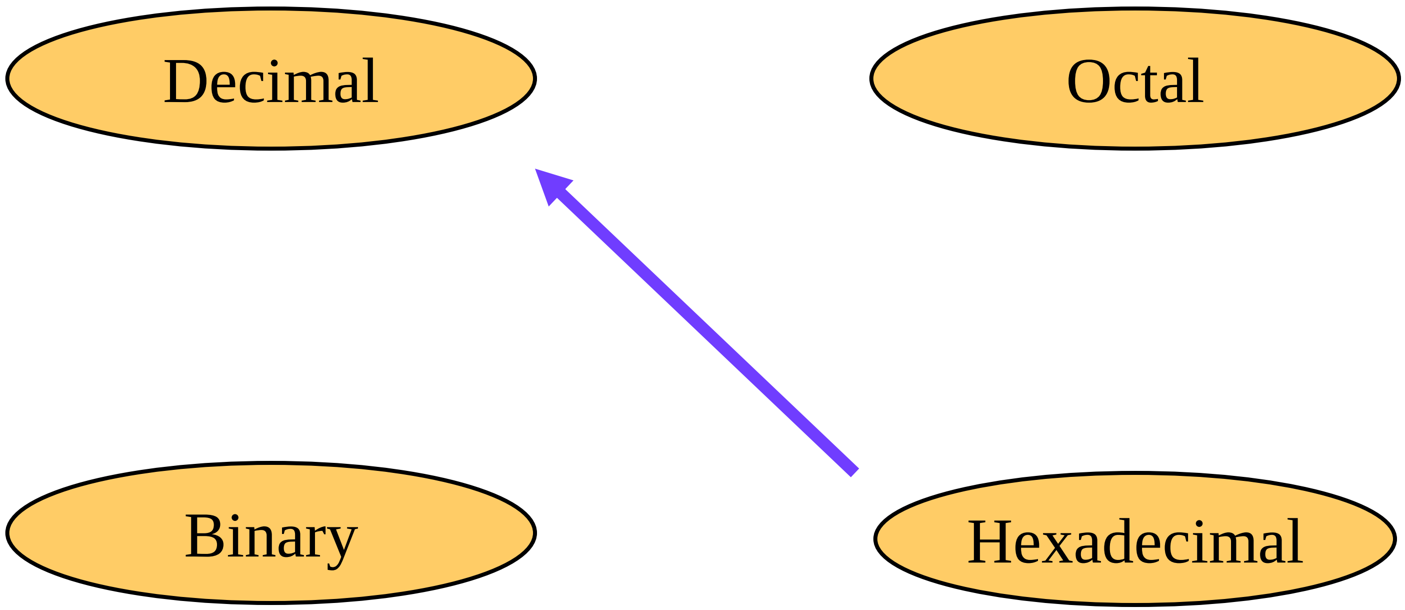
Octal to Decimal

- Technique
  - Multiply each bit by 8^n, where n is the "weight" of the bit
  - The weight is the position of the bit, starting from 0 on the right
  - Add the results

Example

$$
\begin{array}{lrcl}
724_8 \Rightarrow & 4 \times 8^0 = & 4 \\
& 2 \times 8^1 = & 16 \\
& 7 \times 8^2 = & \underline{448} \\
& & 468_{10}
\end{array}
$$

Hexadecimal to Decimal



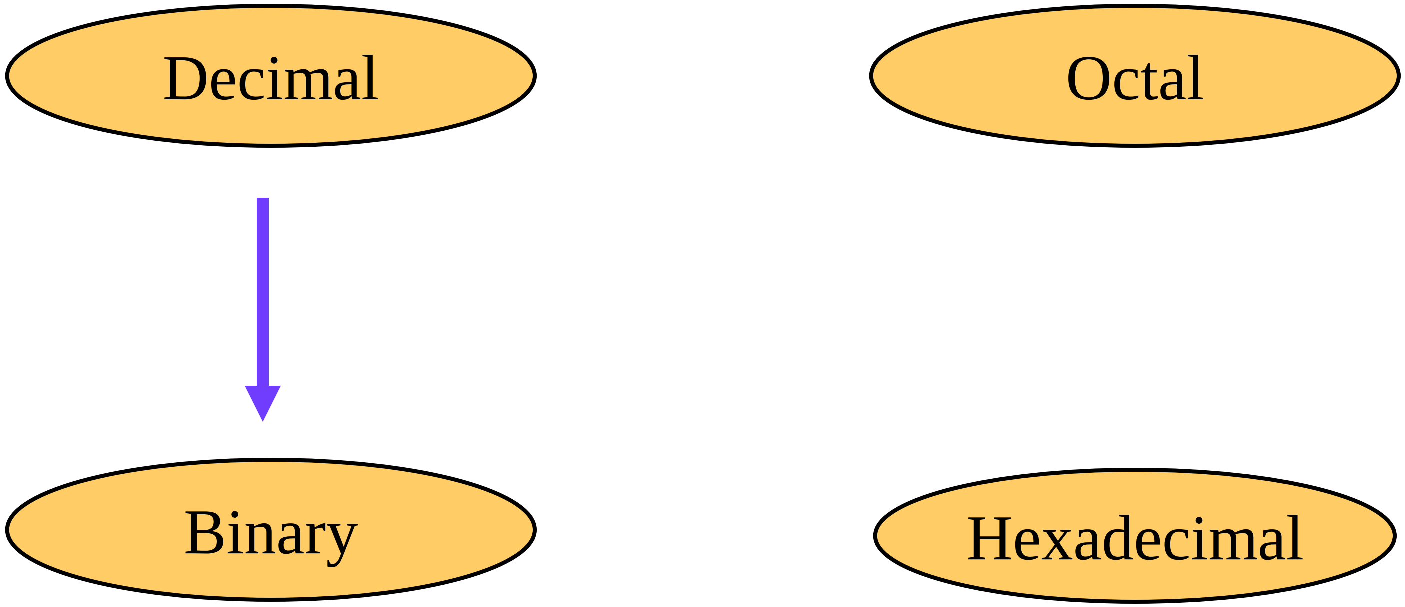
Hexadecimal to Decimal

- Technique
  - Multiply each bit by 16^n, where n is the "weight" of the bit
  - The weight is the position of the bit, starting from 0 on the right
  - Add the results

Example

$$
\begin{array}{lllllll}
ABC_{16} \Rightarrow & C \times 16^0 & = & 12 \times 1 & = & 12 \\
 & B \times 16^1 & = & 11 \times 16 & = & 176 \\
 & A \times 16^2 & = & 10 \times 256 & = & 2560 \\
\hline
 & & & & & 2748_{10}
\end{array}
$$

Decimal to Binary



Decimal to Binary

- Technique
  - Divide by two, keep track of the remainder
  - First remainder is bit 0 (LSB, least-significant bit)
  - Second remainder is bit 1
  - Etc.

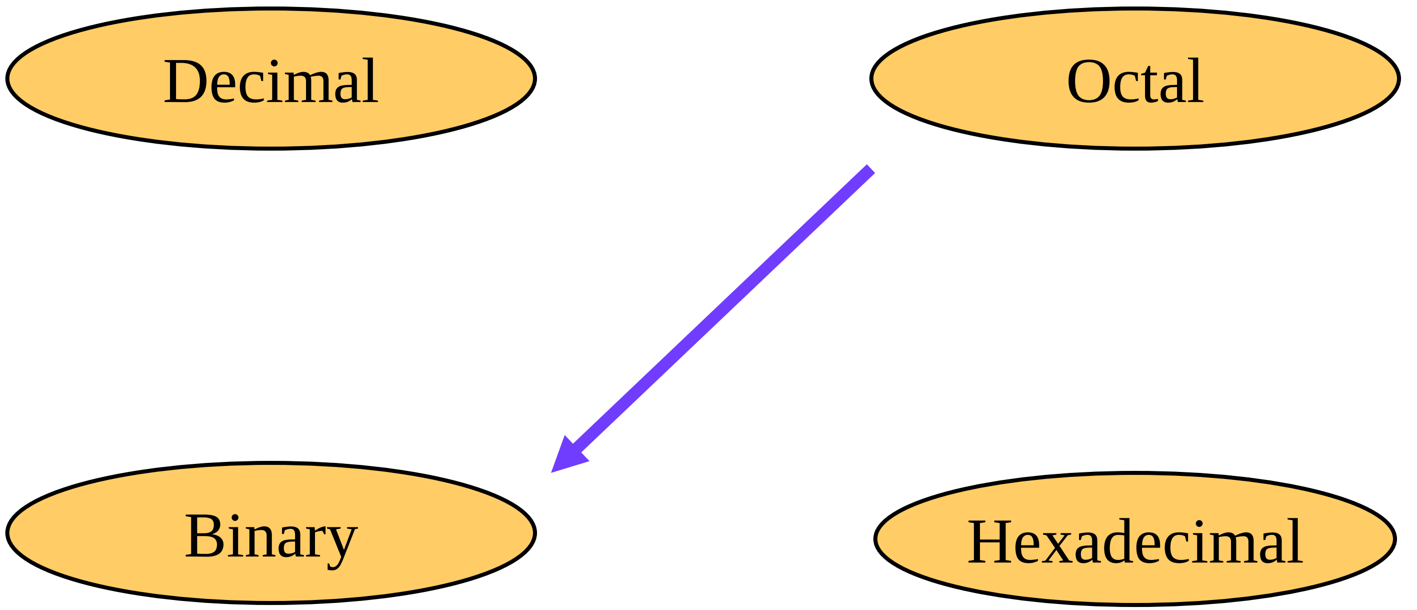
Example

$125_{10} = ?_2$

2	125	
2	62	1
2	31	0
2	15	1
2	7	1
2	3	1
2	1	1
	0	1

$125_{10} = 1111101_2$

Octal to Binary



Octal to Binary

- Technique
  - Convert each octal digit to a 3-bit equivalent binary representation

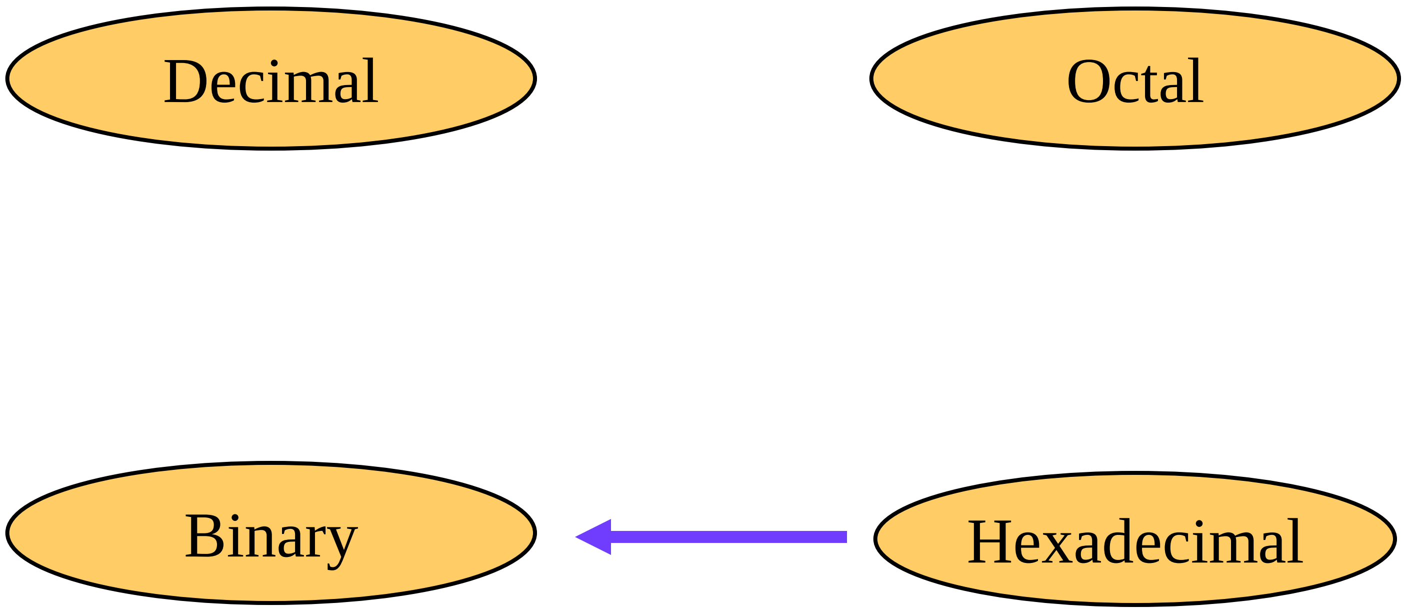
Example

$705_8 = ?_2$

$$\begin{array}{ccc} 7 & 0 & 5 \\ \downarrow & \downarrow & \downarrow \\ 111 & 000 & 101 \end{array}$$

$705_8 = 111000101_2$

Hexadecimal to Binary



Hexadecimal to Binary

- Technique
  - Convert each hexadecimal digit to a 4-bit equivalent binary representation

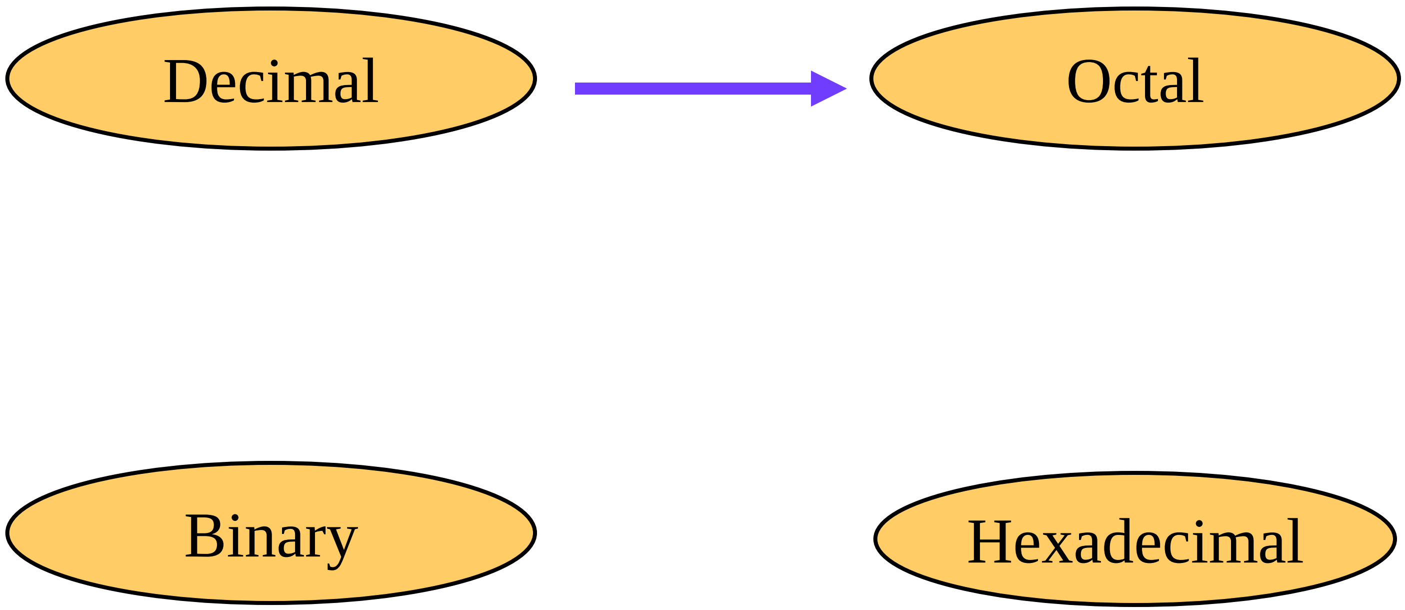
Example

$10AF_{16} = ?_2$

1	0	A	F
0001	0000	1010	1111

$10AF_{16} = 0001000010101111_2$

Decimal to Octal



Decimal to Octal

- Technique
  - Divide by 8
  - Keep track of the remainder

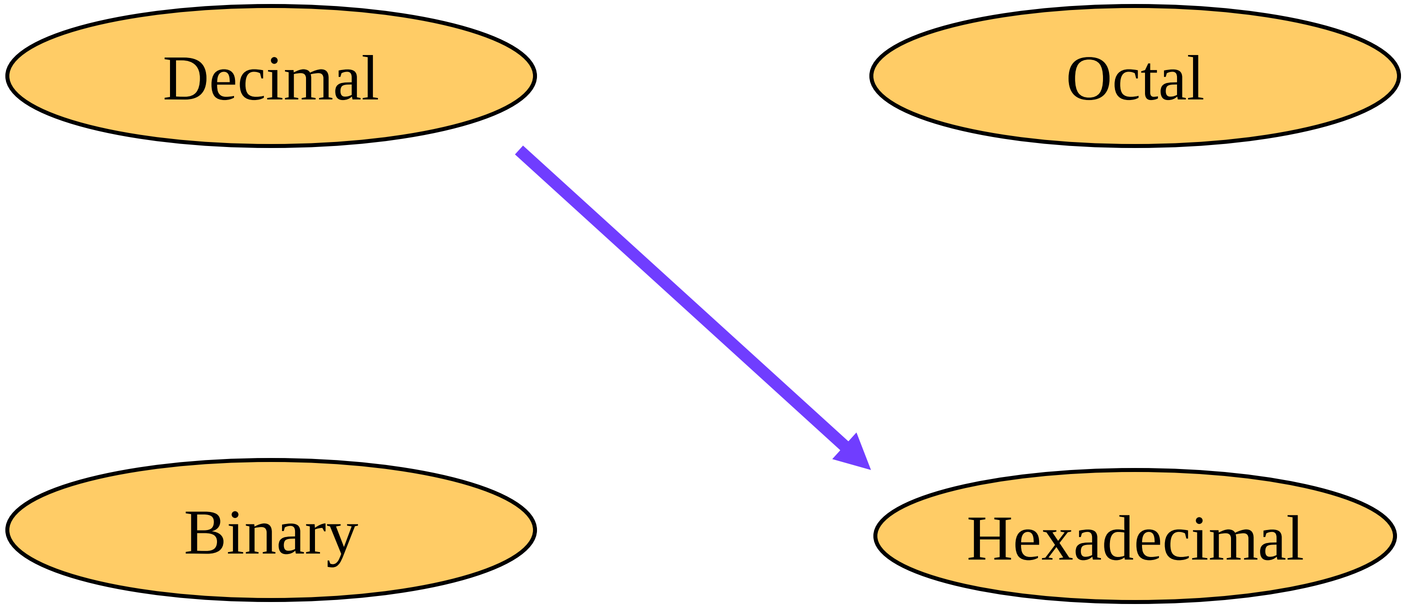
Example

$1234_{10} = ?_8$

8	1234	
8	154	2
8	19	2
8	2	3
	0	2

$1234_{10} = 2322_8$

Decimal to Hexadecimal



Decimal to Hexadecimal

- Technique
  - Divide by 16
  - Keep track of the remainder

Example

$1234_{10} = ?_{16}$

16	1234	
16	77	2
16	4	13 = D
	0	4

$1234_{10} = 4D2_{16}$

End of the first week

H A K

T N S

Thank you