Logical Design

CS 221

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Complement of a Function

- Obtained by interchange of 1's to 0's and 0's to 1's for the values of F in the truth table
- Can be derived algebraically by applying DeMorgan's theorem
- Complement of an expression is obtained by interchanging AND and OR and complementing each variable and constant
- Example:

$$F = X'YZ' + X'Y'Z$$

$$F' = (X + Y' + Z)(X + Y + Z')$$

Standard Forms

- Variety of possible ways of writing a boolean function algebraically
- Hard to see unambiguously that it is the same function we are talking about
- Standard forms have been developed
- Standard forms also generate more desirable logic circuits
- Product terms and Sum terms
  - Product: $XY'Z$
  - Sum: $X + Y' + Z$

Canonical and standard forms

- Two Standard forms

1) Sum of product .. SOP …minterms .. Σ

2) Product of sum .. POS … maxterms.. Π

Minterms and maxterms of three binary variables

X	Y	Z	SOP Term	Designation	POS Term	Designation
0	0	0	X'Y'Z'	m0	X+Y+Z	M0
0	0	1	X'Y'Z	m1	X+Y+Z'	M1
0	1	0	X'YZ'	m2	X+Y'+Z	M2
0	1	1	X'YZ	m3	X+Y'+Z'	M3
1	0	0	XY'Z'	m4	X'+Y+Z	M4
1	0	1	XY'Z	m5	X'+Y+Z'	M5
1	1	0	XYZ'	m6	X'+Y'+Z	M6
1	1	1	XYZ	m7	X'+Y'+Z'	M7

Canonical standard SOP form : each minterm is having all the variable in normal or complimented form .... " Normally from the truth table "

$$\text{Ex } F = \overline{A}\,B + A\,B + \overline{A}\,\overline{B}$$

Minimal SOP form : Each minterm does not have all the variables in normal or complimented form ...

$$\text{Ex } G = A + \overline{B}\,C$$

Que. For the given truth table, minimize the SOP expression

B	A	Y
0	0	0
1	0	1
0	1	0
1	1	1

Sol.

Note in SOP 1 = A & 0 = $\overline{A}$

We see when Y = 1 and write the expression

Then $Y = \overline{A}\,B + A\,B$ This is the standard or canonical form

(each minterm is having all the variable in normal or complimented form)

We will convert the standard form to minimal form by simplification.

$$Y = \overline{A}\,B + A\,B = B\left(\overline{A} + A\right) = B * 1 = B$$

Then Y = B .... Minimal form

In the truth table Y = B

Que. Simplify the expression for

$$Y(A,B) = \sum m(0,2,3)$$

Sol. from the truth table

m0 means 0 0 ..... $\overline{A}\,\overline{B}$

m2 means 1 0 ..... $A\,\overline{B}$

m3 means 1 1 ..... $A\,B$

Then $Y = \overline{A}\,\overline{B} + A\,\overline{B} + A\,B$ *canonical / standard SOP Form*

$Y = \overline{B}\,(\overline{A} + A) + A\,B$

$= \overline{B} + A\,B = (\overline{B} + A)\quad(\overline{B} + B)$

$= A + \overline{B}$ *minimal SOP form*

A	B	Y from TT	$\overline{B}$	Y = A + $\overline{B}$
0	0	1	1	1
0	1	0	0	0
1	0	1	1	1
1	1	1	0	1

Express the Boolean function $f = A + B'C$ in sum of product

We have two methods ..algebraic & truth table

1) Algebraic method (two approaches)

a) Algebraic first approach

$F = A + B'C$

$A = A*1 = A(B+B') = AB + AB' = AB + AB' (C + C')$

$\rightarrow A = ABC + ABC' + AB'C + AB'C'$.

$B'C = B'C * 1 = B'C (A + A') = B'CA + B'CA' \rightarrow AB'C + A'B'C$

… $F = ABC + ABC' + \underline{AB'C} + AB'C' + \underline{AB'C} + A'B'C$

$F = ABC + ABC' + AB'C + AB'C' + A'B'C$

$F = m1 + m4 + m5 + m6 + m7$ → look the previous slide

… $F (A, B, C) = \sum (1, 4, 5, 6, 7)$

b) Algebraic second approach

$F = A + B'C$ (complete each part by all the rest available symbols)

$F = A (BC + B'C + BC' + B'C') + B'C (A + A')$

$\rightarrow F = ABC + \underline{AB'C} + ABC' + AB'C' + \underline{AB'C} + A'B'C.$

Erase the repeated symbols

… $F = ABC + ABC' + AB'C + AB'C' + A'B'C$

$F = m1 + m4 + m5 + m6 + m7$ → look the previous slide

… $F (A, B, C) = \sum (1, 4, 5, 6, 7)$

2) Truth table method

F = A + B'C

A	B	C	B'	B'C	A+B'C	Designation
0	0	0	1	0	0	m0
0	0	1	1	1	1	m1
0	1	0	0	0	0	m2
0	1	1	0	0	0	m3
1	0	0	1	0	1	m4
1	0	1	1	1	1	m5
1	1	0	0	0	1	m6
1	1	1	0	0	1	m7

Solution

F (A, B, C) = ∑ (1, 4, 5, 6, 7)

means at these points results = 1

Example : express the boolean function $F = XY + X'Z$ in product of sum

1 – Algebraic Method

$F = XY + X'Z = (XY + X')(XY + Z)$

$= (X' + X)(X' + Y)(Z + X)(Z + Y)$

$= (X' + Y)(Z + X)(Y + Z)$

MISSING Z Y X

$X' + Y = X'+Y+0 = X'+Y+ZZ' = (X'+Y+Z)(X'+Y+Z')$

$Z + X = X+Z+0 = X+Z+YY' = (X+Z+Y)(X+Z+Y')$

$Y + Z = Y+Z+0 = Y+Z+XX' = (X+Y+Z)(X'+Y+Z)$

$F = (X'+Y+Z)\ (X'+Y+Z')\ \underline{(X+Y+Z)}\ (X+Y'+Z)\ \underline{(X+Y+Z)}\ (X'+Y+Z)$

$F(x, y, z) = XY + X'Z = m0\ m2\ m4\ m5 = \Pi(0, 2, 4, 5)$

Truth table method
F = XY + X'Z

X	Y	Z	X'	XY	X'Z	XY+X'Z	Designation
0	0	0	1	0	0	0	m0
0	0	1	1	0	1	1	m1
0	1	0	1	0	0	0	m2
0	1	1	1	0	1	1	m3
1	0	0	0	0	0	0	m4
1	0	1	0	0	0	0	m5
1	1	0	0	1	0	1	m6
1	1	1	0	1	0	1	m7

F (x, y, z)= XY + X'Z = m0 m2 m4 m5 = Π (0, 2, 4, 5)

Means at these points results = zeros