Assignment 3

G.Soujanya

Download all python codes from

https://github.com/G.Soujanya/Assignment3/tree/main/CODES

and latex-tikz codes from

https://github.com/G.Soujanya/Assignment3/tree/main

1 QUESTION No-2.45 (Linear forms)

show that the line joining the origin to the point $\begin{pmatrix}2\\1\\1\end{pmatrix}$ is perpendicular to the line determined by the points $\begin{pmatrix}3\\5\\-1\end{pmatrix}$, $\begin{pmatrix}4\\3\\-1\end{pmatrix}$

2 Solution

Let's take the given ponits P=$\begin{pmatrix}2\\1\\1\end{pmatrix}$, Q=$\begin{pmatrix}3\\5\\-1\end{pmatrix}$ and R=$\begin{pmatrix}4\\3\\-1\end{pmatrix}$

case(1):Now, we find the line equation passing through the origin to point $\begin{pmatrix}2\\1\\1\end{pmatrix}$

Let origin point be O=$\begin{pmatrix}0\\0\\0\end{pmatrix}$ and other pont be P=$\begin{pmatrix}2\\1\\1\end{pmatrix}$

The vector form of the line passing through O and P, which is the line passing through the point O and along direction vector A is given by

$$\mathbf{r} = \mathbf{O} + k\mathbf{A} \tag{2.0.1}$$

$$\mathbf{r} = \begin{pmatrix}0\\0\\0\end{pmatrix} + k\begin{pmatrix}2\\1\\1\end{pmatrix} \tag{2.0.2}$$

$$\mathbf{r} = r\begin{pmatrix}2\\1\\1\end{pmatrix} \tag{2.0.3}$$

Here the direction vector is

$$\mathbf{d_1} = \begin{pmatrix}2\\1\\1\end{pmatrix} \tag{2.0.4}$$

case(2) :Now let we find the line equation passing through the points

$$\mathbf{Q} = \begin{pmatrix}3\\5\\-1\end{pmatrix} and \mathbf{R} = \begin{pmatrix}4\\3\\-1\end{pmatrix} \tag{2.0.5}$$

Direction vector A of the points Q and R is given by

$$\mathbf{A} = \mathbf{R} - \mathbf{Q} = \begin{pmatrix}4\\3\\-1\end{pmatrix} - \begin{pmatrix}3\\5\\-1\end{pmatrix} \tag{2.0.6}$$

Therefore A=$\begin{pmatrix}1\\-2\\0\end{pmatrix}$

the equation of the line is

$$\mathbf{x} = \mathbf{Q} + \lambda\mathbf{A} \tag{2.0.7}$$

$$\mathbf{x} = \begin{pmatrix}3\\5\\-1\end{pmatrix} + \lambda\begin{pmatrix}1\\-2\\0\end{pmatrix} \tag{2.0.8}$$

Here, the direction vector is

$$\mathbf{d_2} = \begin{pmatrix}1\\-2\\0\end{pmatrix} \tag{2.0.9}$$

Two lines are perpendicular each other when the dot product of their direction vectors is 0

Dot product ofdirection vectors $\mathbf{d_1}$ and $\mathbf{d_2}$

$$\mathbf{d_1}^T\mathbf{d_2} = (2 \times 1) + (1 \times (-2)) + (1 \times 0) \tag{2.0.10}$$
$$= 2 + (-2) + 0 \tag{2.0.11}$$
$$= 2 - 2 \tag{2.0.12}$$
$$= 0 \tag{2.0.13}$$
$$\boxed{\mathbf{d_1}^T\mathbf{d_2} = 0} \tag{2.0.14}$$

as the dot product of direction vector of the lines is $0(\mathbf{d_1}^T\mathbf{d_2})$, we can say that the lines are perpendicular to each other

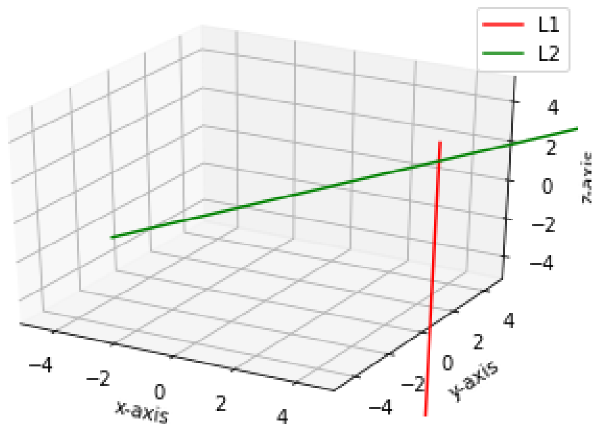

Fig. 2.1: lines perpendicular to each other