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Assignment 3

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Download all python codes from

https://github.com/G.Soujanya/Assignment3/tree/main/CODES

and latex-tikz codes from

https://github.com/G.Soujanya/Assignment3/tree/main

1 QUESTION No-2.45 (Linear forms)

show that the line joining the origin to the point $\begin{pmatrix} 2\\1\\1 \end{pmatrix}$ is perpendicular to the line determined by the

points
$$\begin{pmatrix} 3 \\ 5 \\ -1 \end{pmatrix}$$
, $\begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix}$

2 Solution

Let O be the origin and P be the given point.if A and B the other points

$$\mathbf{O} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \mathbf{P} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}, \mathbf{A} = \begin{pmatrix} 3 \\ 5 \\ -1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix} \quad (2.0.1)$$

case(1):Now, we find the line equation passing through the origin to point $\begin{pmatrix} 2\\1\\1 \end{pmatrix}$

Let origin point be $\mathbf{O} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ and other point be

$$\mathbf{P} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$

The vector form of the line passing through **O**and

Here the direction vector is

$$\mathbf{O} - \mathbf{P} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \tag{2.0.2}$$

case(2): Now let we find the line equation passing through the points

$$\mathbf{A} = \begin{pmatrix} 3 \\ 5 \\ -1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix} \tag{2.0.3}$$

Direction vector \mathbf{m} of the points \mathbf{A} and \mathbf{B} is given by

$$\mathbf{m} = \mathbf{A} - \mathbf{B} = \begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix} - \begin{pmatrix} 3 \\ 5 \\ -1 \end{pmatrix}$$
 (2.0.4)

Therefore $\mathbf{m} = \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}$

Here the direction vector is

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} \tag{2.0.5}$$

Two lines are perpendicular each other when the dot product of their direction vectors is 0

Dot product of direction vectors $\mathbf{d_1}$ and $\mathbf{d_2}$

$$(\mathbf{O} - \mathbf{P})^{T}(\mathbf{A} - \mathbf{B}) = (2 \times 1) + (1 \times (-2)) + (1 \times 0)$$

$$(2.0.6)$$

$$= 2 + (-2) + 0$$

$$(2.0.7)$$

$$= 2 - 2$$

$$(2.0.8)$$

$$= 0$$

$$(2.0.9)$$

$$\implies (\mathbf{O} - \mathbf{P})^{T}(\mathbf{A} - \mathbf{B}) = 0$$

$$(2.0.10)$$

as the dot product of direction vector of the lines is $0(\mathbf{d_1}^T\mathbf{d_2})$, we can say that the lines are perpendicular to each other

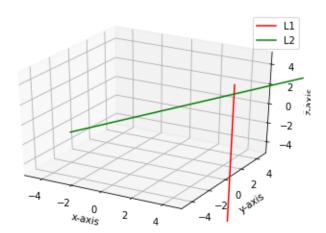


Fig. 2.1: lines perpendicular to each other