Assignment 3

G.Soujanya

Download all python codes from

```
https://github.com/G.Soujanya/Assignment3/tree/main/CODES
```

and latex-tikz codes from

```
https://github.com/G.Soujanya/Assignment3/tree/main
```

1 QUESTION No-2.45 (Linear forms)

show that the line joining the origin to the point $\begin{pmatrix}2\\1\\1\end{pmatrix}$ is perpendicular to the line determined by the points $\begin{pmatrix}3\\5\\-1\end{pmatrix}, \begin{pmatrix}4\\3\\-1\end{pmatrix}$

2 Solution

Let O be the origin and P be the given point.if A and B the other points

$$\mathbf{O} = \begin{pmatrix}0\\0\\0\end{pmatrix}, \mathbf{P} = \begin{pmatrix}2\\1\\1\end{pmatrix}, \mathbf{A} = \begin{pmatrix}3\\5\\-1\end{pmatrix}, \mathbf{B} = \begin{pmatrix}4\\3\\-1\end{pmatrix} \tag{2.0.1}$$

case(1):Now, we find the line equation passing through the origin to point $\begin{pmatrix}2\\1\\1\end{pmatrix}$

Let origin point be $\mathbf{O} = \begin{pmatrix}0\\0\\0\end{pmatrix}$ and other point be $\mathbf{P} = \begin{pmatrix}2\\1\\1\end{pmatrix}$

The vector form of the line passing through **O**and **P**, which is the line passing through the point **O** and along direction vector **A** is given by

$$\mathbf{r} = \mathbf{O} + k\mathbf{A} \tag{2.0.2}$$

$$\implies \mathbf{r} = \begin{pmatrix}0\\0\\0\end{pmatrix} + k\begin{pmatrix}2\\1\\1\end{pmatrix} \tag{2.0.3}$$

$$\implies \mathbf{r} = r\begin{pmatrix}2\\1\\1\end{pmatrix} \tag{2.0.4}$$

Here the direction vector is

$$\mathbf{O} - \mathbf{P} = \begin{pmatrix}2\\1\\1\end{pmatrix} \tag{2.0.5}$$

case(2) :Now let we find the line equation passing through the points

$$\mathbf{A} = \begin{pmatrix}3\\5\\-1\end{pmatrix}, \mathbf{B} = \begin{pmatrix}4\\3\\-1\end{pmatrix} \tag{2.0.6}$$

Direction vector **m** of the points **A** and **B** is given by

$$\mathbf{m} = \mathbf{A} - \mathbf{B} = \begin{pmatrix}4\\3\\-1\end{pmatrix} - \begin{pmatrix}3\\5\\-1\end{pmatrix} \tag{2.0.7}$$

Therefore $\mathbf{m} = \begin{pmatrix}1\\-2\\0\end{pmatrix}$

the equation of the line is

$$\mathbf{x} = \mathbf{A} + \lambda\mathbf{m} \tag{2.0.8}$$

$$\mathbf{x} = \begin{pmatrix}3\\5\\-1\end{pmatrix} + \lambda\begin{pmatrix}1\\-2\\0\end{pmatrix} \tag{2.0.9}$$

Here, the direction vector is

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix}1\\-2\\0\end{pmatrix} \tag{2.0.10}$$

Two lines are perpendicular each other when the dot product of their direction vectors is 0

Dot product ofdirection vectors $\mathbf{d_1}$ and $\mathbf{d_2}$

$$(\mathbf{O}-\mathbf{P})^T(\mathbf{A}-\mathbf{B}) = (2\times 1)+(1\times(-2))+(1\times 0) \tag{2.0.11}$$
$$= 2+(-2)+0 \tag{2.0.12}$$
$$= 2-2 \tag{2.0.13}$$
$$= 0 \tag{2.0.14}$$
$$\implies \boxed{(\mathbf{O}-\mathbf{P})^T(\mathbf{A}-\mathbf{B}) = 0} \tag{2.0.15}$$

as the dot product of direction vector of the lines is $0(\mathbf{d_1}^T\mathbf{d_2})$, we can say that the lines are perpendicular to each other

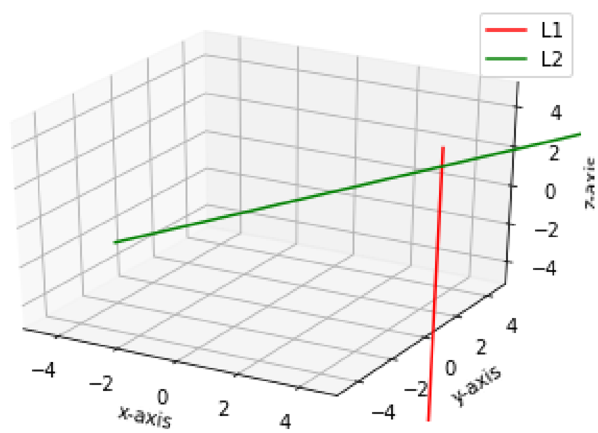

Fig. 2.1: lines perpendicular to each other