

ASSIGNMENT-5

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Download all python codes from

<https://github.com/behappy0604/Summer-Internship-IITH/tree/main/Assignment-5>

and latex-tikz codes from

<https://github.com/behappy0604/Summer-Internship-IITH/tree/main/Assignment-5>

1 QUESTION NO. 2.70 (B)

Find the equation of the Parabola that satisfy the following condition: Focus $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$, Directrix $(0 \ 1) = 3$

2 SOLUTION

Lemma 2.1. *Parabola is a conic whose eccentricity, $e=1$*

Lemma 2.2. *The distance of a point \mathbf{P} from a line $\mathbf{n}^T \mathbf{x} = c$ is given by:*

$$\frac{|c - \mathbf{P}^T \mathbf{n}|}{\|\mathbf{n}\|} \quad (2.0.1)$$

Lemma 2.3. *The equation for parabola assuming $\lambda = \|\mathbf{n}\|^2$ will be:*

$$\mathbf{x}^T (\lambda \mathbf{I} - \mathbf{n} \mathbf{n}^T) \mathbf{x} + 2(c \mathbf{n} - \lambda \mathbf{F})^T \mathbf{x} + \lambda \|\mathbf{F}\|^2 - c^2 = 0 \quad (2.0.2)$$

Given information:

$$\mathbf{F} = \begin{pmatrix} 0 \\ -3 \end{pmatrix}, \mathbf{n} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, c = 3, \lambda = 1 \quad (2.0.3)$$

Substituting values of $\mathbf{F}, \mathbf{n}, c, \lambda$ from (2.0.3):

$$\mathbf{x}^T \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} 6 & 0 \end{pmatrix} \mathbf{x} + 0 = 0 \quad (2.0.4)$$

Replacing \mathbf{x} by $\begin{pmatrix} x \\ y \end{pmatrix}$ in (2.0.4) gives:

$$y^2 = -12x \quad (2.0.5)$$

The general equation of parabola we got in (2.0.2) is of form:

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (2.0.6)$$

$$\mathbf{V} = \lambda \mathbf{I} - \mathbf{n} \mathbf{n}^T \quad (2.0.7)$$

$$\mathbf{u} = c \mathbf{n} - \lambda \mathbf{F} \quad (2.0.8)$$

$$f = \lambda \|\mathbf{F}\|^2 - c^2 \quad (2.0.9)$$

$$\mathbf{n} = \begin{pmatrix} x \\ y \end{pmatrix}, \lambda = \|\mathbf{n}\|^2 = x^2 + y^2; \mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \text{Now,}$$

$$|\mathbf{V}| = \begin{vmatrix} \lambda - x^2 & -xy \\ -xy & \lambda - y^2 \end{vmatrix} \quad (2.0.10)$$

$$= \begin{vmatrix} y^2 & -xy \\ -xy & x^2 \end{vmatrix} \quad (2.0.11)$$

$$= 0 \quad (2.0.12)$$

Also characteristic equation of \mathbf{V} is given by:

$$|\alpha \mathbf{I} - \mathbf{V}| = 0 \quad (2.0.13)$$

$$\begin{vmatrix} \alpha - x^2 & xy \\ xy & \alpha - y^2 \end{vmatrix} = 0 \quad (2.0.14)$$

$$\begin{vmatrix} \alpha - y^2 & xy \\ xy & \alpha - x^2 \end{vmatrix} = 0 \quad (2.0.15)$$

$$\alpha^2 - \alpha(x^2 + y^2) = 0 \quad (2.0.16)$$

$$\alpha(\alpha - \lambda) = 0 \quad (2.0.17)$$

$$\alpha_1 = 0 \quad (2.0.18)$$

$$\alpha_2 = \lambda = x^2 + y^2 = \|\mathbf{n}\|^2 \quad (2.0.19)$$

So, (2.0.12) and (2.0.18) shows that (2.0.2) is an equation of parabola.

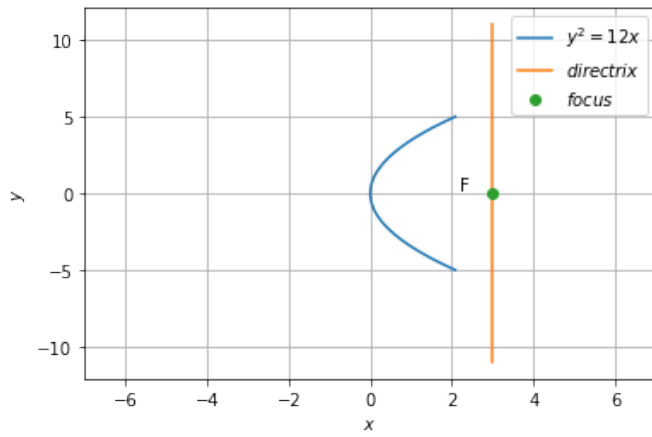


Fig. 2.1: Parabola