# Assignment-2

G.Soujanya

## 1 Question No-2.14 (Linear forms)

Find the equation of the line satisfying the following conditions.

1) passing through the point $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$
2) perpendicular distance from the origin is 5 and the angle made by the perpendicular with the positive x-axis is 30°.

## 2 Solution

1) Let $\mathbf{A} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$
direction vector $\mathbf{m}$ of the points A and B is give by,

$$\mathbf{m} = \mathbf{B} - \mathbf{A} \qquad = \begin{pmatrix} 3 \\ -5 \end{pmatrix} \tag{2.0.1}$$

Parametric equation is given by,

$$\mathbf{P} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -5 \end{pmatrix} \tag{2.0.2}$$

Plot of the line $AB$

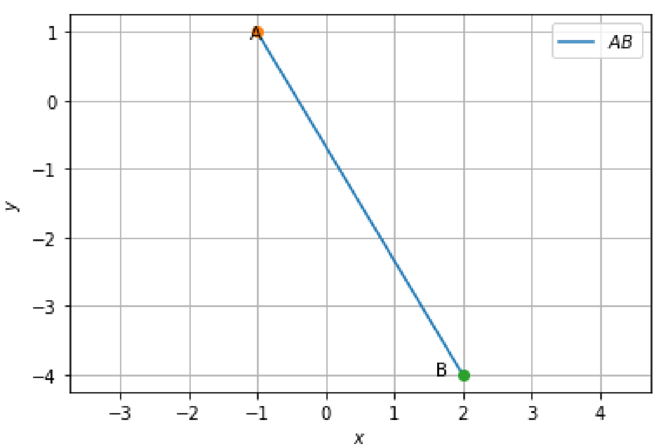


Fig. 2.1: Plot of Line $AB$ (Part-1)

**Solution:** In Fig. 2.2, the foot of the perpendicular $P$ is the intersection of the lines $L$ and $M$. Thus,

$$\mathbf{n}^T \mathbf{P} = c \tag{2.0.3}$$

$$\mathbf{P} = \mathbf{A} + \lambda \mathbf{n} \tag{2.0.4}$$

$$\text{or, } \mathbf{n}^T \mathbf{P} = \mathbf{n}^T \mathbf{A} + \lambda \left\|\mathbf{n}\right\|^2 = c \tag{2.0.5}$$

$$\implies -\lambda = \frac{\mathbf{n}^T \mathbf{A} - c}{\left\|\mathbf{n}\right\|^2} \tag{2.0.6}$$

Also, the distance between $\mathbf{A}$ and $L$ is obtained from

$$\mathbf{P} = \mathbf{A} + \lambda \mathbf{n} \tag{2.0.7}$$

$$\implies \left\|\mathbf{P} - \mathbf{A}\right\| = |\lambda| \left\|\mathbf{n}\right\| \tag{2.0.8}$$

From (2.0.6) and (2.0.8)

$$\left\|\mathbf{P} - \mathbf{A}\right\| = \frac{\left|\mathbf{n}^T \mathbf{A} - c\right|}{\left\|\mathbf{n}\right\|} \tag{2.0.9}$$

Fig. 2.2

$$\mathbf{n} = \begin{pmatrix} 1 \\ \tan 30° \end{pmatrix} \tag{2.0.10}$$

$\because \mathbf{A} = \mathbf{0}$,

$$5 = \frac{|c|}{\left\|\mathbf{n}\right\|} \implies c = \pm 5 \sqrt{1 + \tan^2 30°} \tag{2.0.11}$$

$$= \pm 5 \sec 30° \tag{2.0.12}$$

where

$$\sec \theta = \frac{1}{\cos \theta} \tag{2.0.13}$$

This follows from the fact that

$$\cos^2 \theta + \sin^2 \theta = 1 \tag{2.0.14}$$

$$\implies 1 + \frac{\sin^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta} \tag{2.0.15}$$

It is easy to verify that

$$\frac{\sin\theta}{\cos\theta} = \tan\theta \tag{2.0.16}$$

$$\implies 1 + \tan^2\theta = \sec^2\theta \tag{2.0.17}$$

*Thus*, the equation of the line is

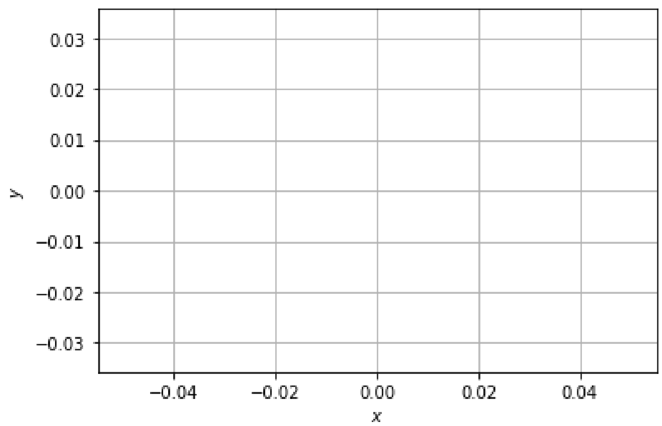


Fig. 2.3: Plot of Line $AB$ (Part-2)