Assignment-2

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1 QUESTION NO-2.14 (linear forms)

Find the equation of the line satisfying the following conditions.

1) passing through the point $\begin{pmatrix}-1\\1\end{pmatrix}$ and $\begin{pmatrix}2\\-4\end{pmatrix}$
2) perpendicular distance from the origin is 5 and the angle made by the perpendicular with the positive x-axis is 30°.

2 Solution

1) Let $\mathbf{A} = \begin{pmatrix}-1\\1\end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix}2\\-4\end{pmatrix}$ direction vector $\mathbf{m}$ of the points $\mathbf{A}$ and $\mathbf{B}$ is give by,

$$\mathbf{m} = \mathbf{B} - \mathbf{A} = \begin{pmatrix}3\\-5\end{pmatrix} \tag{2.0.1}$$

Hence the normal vector

$$\mathbf{n} = \begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix} m \tag{2.0.2}$$

$$= \begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix}\begin{pmatrix}3\\-5\end{pmatrix} \tag{2.0.3}$$

$$\mathbf{n} = \begin{pmatrix}5\\3\end{pmatrix} \tag{2.0.4}$$

The equation of the line in terms of the normal vector is then obtained as

$$\mathbf{n}^T(\mathbf{x} - \mathbf{A}) = 0 \tag{2.0.5}$$

$$\implies \begin{pmatrix}5 & 3\end{pmatrix}\mathbf{x} = -2 \tag{2.0.6}$$

$$\mathbf{X} = \begin{pmatrix}-1\\1\end{pmatrix} + \lambda\begin{pmatrix}3\\-5\end{pmatrix} \tag{2.0.7}$$

2) the foot of the perpendicular P is the intersection of the lines L and M. Thus,

$$\mathbf{n}^T\mathbf{P} = c \tag{2.0.8}$$

$$\mathbf{P} = \mathbf{A} + \lambda\mathbf{n} \tag{2.0.9}$$

$$\text{or, } \mathbf{n}^T\mathbf{P} = \mathbf{n}^T\mathbf{A} + \lambda\,\|\mathbf{n}\|^2 = c \tag{2.0.10}$$

$$\implies -\lambda = \frac{\mathbf{n}^T\mathbf{A} - c}{\|\mathbf{n}\|^2} \tag{2.0.11}$$

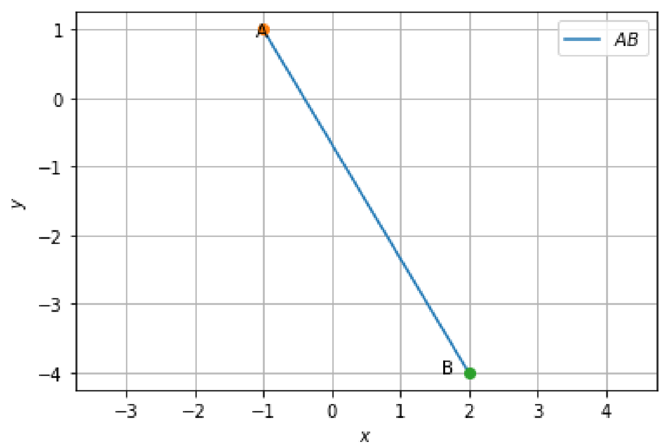

Fig. 2.1: Plot of Line AB (Part-1)

From (2.0.11)

$$\|\mathbf{P} - \mathbf{A}\| = \frac{\left|\mathbf{n}^T\mathbf{A} - c\right|}{\|\mathbf{n}\|} \tag{2.0.12}$$

$$\mathbf{n} = \begin{pmatrix}1\\\tan 30^\circ\end{pmatrix} \tag{2.0.13}$$

$\because \mathbf{A} = \mathbf{0}$,

$$5 = \frac{|c|}{\|\mathbf{n}\|} \implies c = \pm 5\sqrt{1 + \tan^2 30^\circ} \tag{2.0.14}$$

$$= \pm 5 \sec 30^\circ \tag{2.0.15}$$

where

$$\sec\theta = \frac{1}{\cos\theta} \tag{2.0.16}$$

This follows from the fact that

$$\cos^2\theta + \sin^2\theta = 1 \tag{2.0.17}$$

$$\implies 1 + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta} \tag{2.0.18}$$

It is easy to verify that

$$\frac{\sin\theta}{\cos\theta} = \tan\theta \tag{2.0.19}$$

$$\implies 1 + \tan^2\theta = \sec^2\theta \tag{2.0.20}$$

$Thus$, the equation of the line is

$$\begin{pmatrix} 1 & \tan 30^\circ \end{pmatrix} \mathbf{c} = \pm 5 \sec 30^\circ \qquad (2.0.21)$$

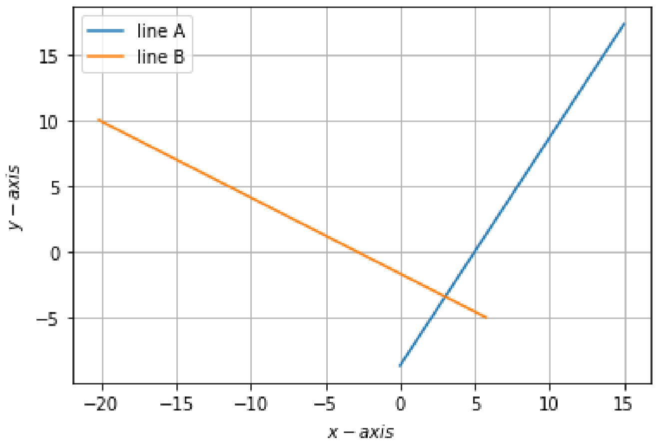

Fig. 2.2: Plot of Line AB(Part-2)