

# Assignment-2

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## 1 QUESTION NO-2.14 (LINEAR FORMS)

Find the equation of the line satisfying the following conditions.

- 1) passing through the point  $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$  and  $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$
- 2) perpendicular distance from the origin is 5 and the angle made by the perpendicular with the positive x-axis is  $30^\circ$ .

## 2 SOLUTION

- 1) Let  $\mathbf{A} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$  and  $\mathbf{B} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$   
direction vector  $\mathbf{m}$  of the points A and B is given by,

$$\mathbf{m} = \mathbf{B} - \mathbf{A} = \begin{pmatrix} 3 \\ -5 \end{pmatrix} \quad (2.0.1)$$

The line parallel to the X-axis has direction vector  $\mathbf{m} = \begin{pmatrix} 3 \\ -5 \end{pmatrix}$ . Hence its equation is obtained as

$$\mathbf{X} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -5 \end{pmatrix} \quad (2.0.2)$$

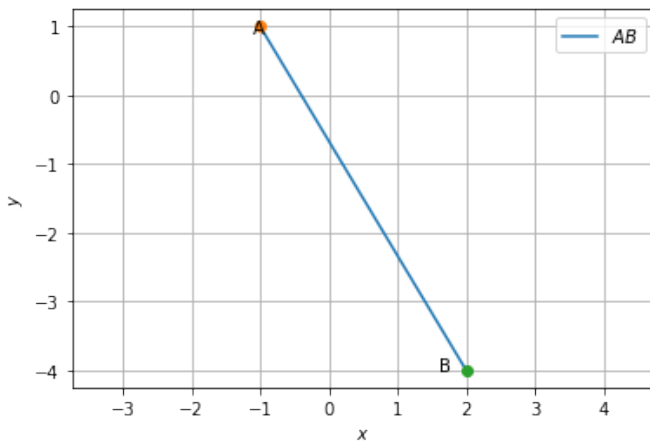


Fig. 2.1: Plot of Line AB (Part-1)

- 2) the foot of the perpendicular  $P$  is the intersection of the lines  $L$  and  $M$ . Thus,

$$\mathbf{n}^T \mathbf{P} = c \quad (2.0.3)$$

$$\mathbf{P} = \mathbf{A} + \lambda \mathbf{n} \quad (2.0.4)$$

$$\text{or, } \mathbf{n}^T \mathbf{P} = \mathbf{n}^T \mathbf{A} + \lambda \|\mathbf{n}\|^2 = c \quad (2.0.5)$$

$$\Rightarrow -\lambda = \frac{\mathbf{n}^T \mathbf{A} - c}{\|\mathbf{n}\|^2} \quad (2.0.6)$$

From (2.0.6)

$$\|\mathbf{P} - \mathbf{A}\| = \frac{|\mathbf{n}^T \mathbf{A} - c|}{\|\mathbf{n}\|} \quad (2.0.7)$$

$$\mathbf{n} = \begin{pmatrix} 1 \\ \tan 30^\circ \end{pmatrix} \quad (2.0.8)$$

$\therefore \mathbf{A} = \mathbf{0}$ ,

$$5 = \frac{|c|}{\|\mathbf{n}\|} \Rightarrow c = \pm 5 \sqrt{1 + \tan^2 30^\circ} \quad (2.0.9)$$

$$= \pm 5 \sec 30^\circ \quad (2.0.10)$$

where

$$\sec \theta = \frac{1}{\cos \theta} \quad (2.0.11)$$

This follows from the fact that

$$\cos^2 \theta + \sin^2 \theta = 1 \quad (2.0.12)$$

$$\Rightarrow 1 + \frac{\sin^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta} \quad (2.0.13)$$

It is easy to verify that

$$\frac{\sin \theta}{\cos \theta} = \tan \theta \quad (2.0.14)$$

$$\Rightarrow 1 + \tan^2 \theta = \sec^2 \theta \quad (2.0.15)$$

Thus, the equation of the line is

$$(1 \pm \tan 30^\circ) c = \pm 5 \sec 30^\circ$$

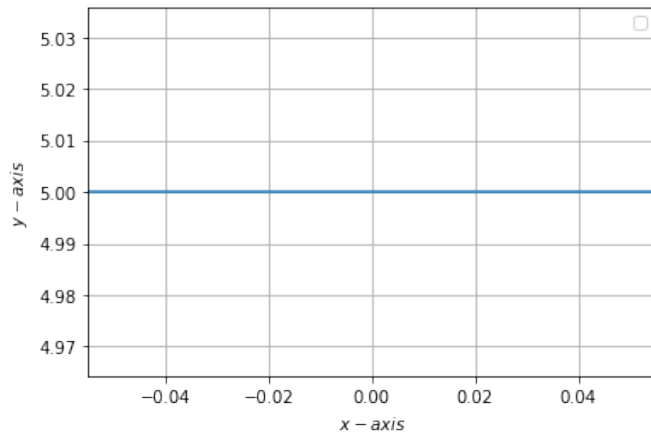


Fig. 2.2: Plot of Line  $AB$  (Part-2)