Quantum Computing - Final project

Solving satisfiability problems using Grover's Algorithm

Grover's algorithm is a quantum algorithm for unstructured search that finds with high probability the input to a black box that produces a particular output value. The canonical application of the algorithm is the problem of searching for an element in an unstructured array. For an array with N elements, Grover's algorithm takes $\mathcal{O}(\sqrt{N})$ steps, leading to a quadratic improvement compared with the classical counterpart.

The oracle in Grover's algorithm is a black box that can be considered a function. Grover's algorithm can find multiple inputs of a function that produce a particular output. If the black box has r inputs in a total N, that produce such output, Grover's algorithm finds a solution in $\mathcal{O}(\sqrt{N/r})$ steps.

A Boolean satisfiability problem determines if a Boolean formula is satisfied, i.e., evaluates TRUE given a specific assignment of its binary variables. If this is the case, the formula is called satisfiable. On the other hand, if no such assignment exists, the function expressed by the formula is FALSE for all possible variable assignments, and the formula is unsatisfiable.

$$f(v1,v2,v3) = (\neg v1 \vee \neg v2 \vee \neg v3) \wedge (v1 \vee \neg v2 \vee v3) \wedge (v1 \vee v2 \vee \neg v3) \wedge (v1 \vee \neg v2 \vee \neg v3) \wedge (\neg v1 \vee v2 \vee v3)$$

Equation 1: Example of boolean formula with five clauses with three literals each - 3-SAT

The Boolean formula in Equation 1 is an example of a 3-SAT problem, i.e., each clause has three literals. The goal is to find an assignment for each literal such that the function evaluates TRUE. Such a problem can be seen as a search problem, where the solution is the assignment where the Boolean formula is satisfied.

Remember that the algorithm starts with a uniform superposition of all possible assignments. The oracle will be composed of gates that evaluate each clause and invert the amplitude of assignments that validate it.

- *Tasks*:
  - Design a solvable 3-SAT boolean formula.
  - Implement Grover's algorithm for solving the satisfiability problem.
  - Assess the quality of the solution employed by the quantum algorithm.

- Study the complexity associated with the algorithm applied to your problem, I.e., the optimal number of Grover iterations needed to reach a solution.

***References*:**

- Satisfiability with Grover - https://qiskit.org/textbook/ch-applications/satisfiability-grover.html
- Grover's algorithm - Qiskit - https://qiskit.org/textbook/ch-algorithms/grover.html