Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/B.Tech(N)/SEM-1/M-101/2012-13

2012

MATHEMATICS-I

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : 10 × 1 = 10

i) The sequence $\left\{ (-1)^n \frac{1}{n} \right\}$ is

a) Convergent b) Oscillatory

c) Divergent d) none of these.

ii) The matrix $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ is

a) Symmetric b) Skew-symmetric

c) Singular d) Orthogonal.

iii) The value of t for which

$\vec{f} = (x + 3y)\hat{i} + (y - 2z)\hat{j} + (x + tz)\hat{k}$ is solenoidal is

a) 2 b) – 2

c) 0 d) 1.

iv) The series $\Sigma \frac{1}{n^p}$ is convergent if

a) $p \geq 1$ b) $p \leq 1$

c) $p > 1$ d) $p < 1$.

v) The two eigenvalues of the matrix

$$A = \begin{bmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$$ are 2 and – 2. The third eigenvalue is

a) 1 b) 0

c) 3 d) 2.

vi) If Rolles theorem is applicable to $f(x) = x(x^2 - 1)$ in $[0, 1]$, then c =

a) 1 b) 0

c) $-\frac{1}{\sqrt{3}}$ d) $\frac{1}{\sqrt{3}}$.

vii) If $u = \dfrac{x^3 + y^3}{\sqrt{x^2 + y^2}}$, find the value of '$n$' so that $xu_x + yu_y = nu$.

a) 0   b) 2

c) $\frac{1}{2}$ d) none of these.

viii) n-th derivative of $\sin(5x + 3)$ is

a) $5^n \cos(5x + 3)$

b) $5^n \sin\left(\dfrac{n\pi}{2} + 5x + 3\right)$

c) $5^n \cos\left(\dfrac{n\pi}{2} + 5x + 3\right)$

d) none of these.

ix) The value of $\int_C (x\,dx - dy)$ where C is a line joining $(0,1)$ to $(1, 0)$ is

a) 0   b) $\frac{3}{2}$

c) $\frac{1}{2}$ d) $\frac{2}{3}$.

x) The value of $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7\theta \, d\theta$ is

a) 0   b) $\dfrac{6.4.2}{7.5.3.1}$

c) $\dfrac{6!}{7!}$ d) none of these.

xi) The characteristic equation of a matrix A is $X^3 + 3X^2 + 5X + 9 = 0$, then determinant of the matrix is

a) 7 b) 5

c) 6 d) 9.

xii) Let A and B be two square matrices and A^{-1}, B^{-1}, exists. Then $(AB)^{-1}$ is

a) $A^{-1}B^{-1}$ b) $B^{-1}A^{-1}$

c) AB d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Verify Rolles theorem for the function

$$f(x) = |x|, -1 \le x \le 1.$$

3. A and B are orthogonal matrix and $|A| + |B| = 0$. Prove that $A + B$ is singular.

4. Find the n^{th} derivative of $\dfrac{x^2 + 1}{(x-1)(x-2)(x-3)}$.

5. Let

$$f(x, y) = \frac{xy}{x + y^2}, \ (x, y) \ne (0, 0)$$

$$= 0, \ (x, y) = (0, 0)$$

Evaluate $f_{xy}(0, 0)$ and $f_{yx}(0, 0)$.

6. Find $div\ \vec{F}$ and $curl\ \vec{F}$ where

$$\vec{F} = grad\left(x^3 + y^3 + z^3 - 3xyz\right).$$

GROUP – C
(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) If $u = x^2 - 2y$, $v = x + y + z$, $w = x - 2y + 3z$, find $\dfrac{\partial(u, v, w)}{\partial(x, y, z)}$.

b) Prove that $\begin{vmatrix} 1 & \alpha & \alpha^2 - \beta\gamma \\ 1 & \beta & \beta^2 - \gamma\alpha \\ 1 & \gamma & \gamma^2 - \alpha\beta \end{vmatrix} = 0$.

c) If $v = f\left(x^2 + 2yz, y^2 + 2zx\right)$, prove that

$$\left(y^2 - zx\right)\frac{\partial v}{\partial x} + \left(x^2 - yz\right)\frac{\partial v}{\partial y} + \left(z^2 - xy\right)\frac{\partial v}{\partial z} = 0.$$

5 + 5 + 5

8. a) If $\theta = t^n e^{\frac{-r^2}{4t}}$, find what value of n will make $\dfrac{1}{r^2}\dfrac{\partial}{\partial r}\left(r^2 \dfrac{\partial \theta}{\partial r}\right) = \dfrac{\partial \theta}{\partial t}$.

b) Using mean value theorem prove that

$$0 < \frac{1}{x}\log\left(\frac{e^x - 1}{x}\right) < x.$$

c) If $I_n = \displaystyle\int_0^{\frac{\pi}{2}} x^n \sin x \, dx \; (n > 1)$, then show that

$I_n + n(n-1) I_{n-2} = n\left(\dfrac{\pi}{2}\right)^{n-1}$. 5 + 5 + 5

9. a) State D'Alembert's ratio test for convergence of an infinite series. Examine the convergence or divergence of the series $\left(\frac{1}{3}\right)^2 + \left(\frac{1.2}{3.5}\right)^2 + \left(\frac{1.2.3}{3.5.7}\right)^2 + \ldots$

b) If $y = e^{\tan^{-1}x}$, then show that $\left(1 + x^2\right) y_{n+2} + (2nx + 2x - 1)\, y_{n+1} + n(n+1)\, y_n = 0$.

c) Find the extreme value of the function

$$f(x, y) = x^3 + y^3 - 3x - 12y + 20. \qquad 5 + 5 + 5$$

10. a) If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, then verify that A satisfies its own characteristic equation. Hence find A^{-1} and A^9.

b) If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$, then show that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = (1 - 4\sin^2 u)\sin 2u.$$

c) Given the system of equation :

$x_1 + 4x_2 + 2x_3 = 1$, $2x_1 + 7x_2 + 5x_3 = k$, $4x_1 + mx_2 + 10x_3 = 2k + 1$. Find for what values of k and m, the system has (i) an unique solution, (ii) no solution (iii) many solution.

11. a) Show that $\vec{\nabla} r^n = nr^{n-2}\vec{r}$,

where $\vec{r} = \vec{i}x + \vec{j}y + \vec{k}z$.

b) Evaluate $\int\int\sqrt{4x^2 - y^2}\, dxdy$ over the triangle formed by the straight lines $y = 0$, $x = 1$ and $y = x$.

c) Verify Stokes theorem for

$\vec{F} = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. 5 + 5 + 5