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Paper Code : BSM102 Mathematics –IB

UPID : 001035

Time Allotted : 3 Hours *Full Marks :70*

The Figures in the margin indicate full marks.
Candidate are required to give their answers in their own words as far as practicable

Group-A (Very Short Answer Type Question)

1. Answer any ten of the following : [1 x 10 = 10]

(I) Elementary operations on a matrix do not alter its rank.
(a) true (b) false

(II) The value of $\Gamma\left(\frac{5}{2}\right)$ is _____.

(III) Whether Rolle's theorem is applicable in [-1, 1] to the function f(x) = x^2 +1?

(IV) Find the period of the function f(x) = Sinx.

(V) I f(x, y) is a homogeneous function of degree 5 then $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} =$ _____.

(VI) Find the condition that the square matrix A will be idempotent.

(VII) The series $\sum \frac{1}{n^p}$ is convergent if p is _____ .

(VIII) Find the directional derivative of f(x, y, z) in the direction of positive x-axis.

(IX) If A is a symmetric as well as a skew symmetric matrix, then A must be a _____ matrix.

(X) The improper integral $\int_1^\infty \frac{x\,dx}{(1+x)^3}$ converges.

Whether the statement is TRUE or FALSE?

(XI) The function f(x) = Sinx obeys Rolle's theorem in [0, π]
(a) true (b) false

(XII) Find the period of the function f(x) = 2|Cos2x|.

Group-B (Short Answer Type Question)

Answer any three of the following [5 x 3 = 15]

2. Evaluate $\lim_{x\to 0}\left(\frac{1}{x^2} - \frac{1}{\sin^2 x}\right)$. [5]

3. Find $div\,\vec{F}$ and $curl\,\vec{F}$ where $\vec{F} = grad(x^3 + y^3 + z^3 - 3xyz)$. [5]

4. Use Mean Value Theorem to prove that $\frac{x}{1+x} < \log(1+x) < x$ if x>0. [5]

5. Show that the following series is absolutely convergent: $\sum_{n=1}^{\infty} \frac{\cos nx}{n^2}$. [5]

6. If $O(v^2 - x^2, v^2 - y^2, v^2 - z^2) = 0$, where v is a function of x,y,z, then show that [5]
$\frac{1}{x}\frac{\partial v}{\partial x} + \frac{1}{y}\frac{\partial v}{\partial y} + \frac{1}{z}\frac{\partial v}{\partial z} = \frac{1}{v}$.

Group-C (Long Answer Type Question)

Answer any three of the following [15 x 3 = 45]

7. (a) Show that the following integral is convergent. Hence find its value. [5]

$$\int_0^\infty \frac{dx}{(1+x)\sqrt{x}}$$

(b) Show that $\int_0^{\frac{\pi}{2}} \sin^4 x \cos^5 x dx = \frac{8}{315}$. [5]

(c) The arc of the parabola $y = x^2$ from $(1,1)$ to $(2,4)$ is rotated about the $y-$axis. Find the area of the resulting surface. [5]

8. (a) State Rolle's Theorem and show that it is not applicable to f(x) = tanx in [0,π], although f(0) = f(π). [5]

(b) Verify Lagrange's mean Value Theorem for f(x) = x(x-1)(x-2) in [0,1/2]. [5]

(c) Verify Cauchy's mean Value Theorem for f(x) = sinx and g(x) = cosx in [-π/2,0]. [5]

9. (a) Find the value of x for which the function $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 5$ has maximum and minimum. Find the maximum and minimum values. [7]

(b) Show that $\cos x \sin^3 x$ is maximum at $x = \frac{\pi}{3}$. [8]

10. (a) Show that the following series is conditionally convergent: $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots$. [7]

(b) Expand sinx in a power series in x. [8]

11. (a) Show that the largest rectangle inscribed in a circle is a square. [8]

(b) For a given volume of a right circular cone, show that the curved surface is minimum when the semi-vertical angle is $\sin^{-1}(1/\sqrt{3})$. [7]