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Invigilator's Signature :	

# CS/B.Tech(N)/SEM-1/M-101/2012-13 2012

## **MATHEMATICS-I**

Time Allotted: 3 Hours Full Marks: 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

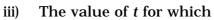
### **GROUP - A**

### (Multiple Choice Type Questions)

- 1. Choose the correct alternatives for any ten of the following :  $10 \times 1 = 10$ 
  - i) The sequence  $\left\{ (-1)^n \frac{1}{n} \right\}$  is
    - a) Convergent
- b) Oscillatory
- c) Divergent
- d) none of these.
- ii) The matrix  $\left[ \begin{array}{ccc} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{array} \right] is$ 
  - a) Symmetric
- b) Skew-symmetric
- c) Singular
- d) Orthogonal.

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The value of 
$$t$$
 for which  $\overrightarrow{f} = (x + 3y) \hat{i} + (y - 2z) \hat{j} + (x + tz) \hat{k}$  i

solenoidal is

a) 2

b) - 2

 $\mathbf{c}$ ) (

d) 1.

iv) The series  $\sum \frac{1}{n^p}$  is convergent if

a)  $p \ge 1$ 

b)  $p \le 1$ 

c) p > 1

d) p < 1.

v) The two eigenvalues of the matrix

$$A = \begin{bmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix} \text{ are 2 and - 2. The third}$$

eigenvalue is

a) 1

b) 0

c) 3

d) 2.

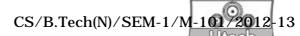
vi) If Rolles theorem is applicable to  $f(x) = x(x^2 - 1)$  in [ 0, 1 ], then c =

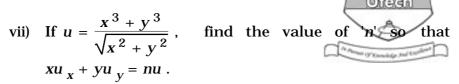
a) 1

b) (

c)  $-\frac{1}{\sqrt{3}}$ 

d)  $\frac{1}{\sqrt{3}}$ .





a) 0

b) 2

c)  $\frac{1}{2}$ 

d) none of these.

viii) n-th derivative of  $\sin(5x + 3)$  is

- a)  $5^n \cos(5x + 3)$
- b)  $5^n \sin\left(\frac{n\pi}{2} + 5x + 3\right)$
- c)  $5^n \cos\left(\frac{n\pi}{2} + 5x + 3\right)$
- d) none of these.
- ix) The value of  $\int\limits_C$  (  $x\mathrm{d}x$   $\mathrm{d}y$  ) where C is a line joining

(0,1) to (1,0) is

a) 0

b)  $\frac{3}{2}$ 

c)  $\frac{1}{2}$ 

d)  $\frac{2}{3}$ .

x) The value of 
$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 \theta \, d\theta \text{ is }$$

a) 0

b)  $\frac{6.4.2}{7.5.3.1}$ 

c)  $\frac{6!}{7!}$ 

d) none of these.



- xi) The characteristic equation of a matrix A is  $X^3 + 3X^2 + 5X + 9 = 0$ , then determinant of the matrix is
  - a) 7

b) 5

c) 6

- d) 9.
- xii) Let A and B be two square matrices and  $A^{-1}$ ,  $B^{-1}$ , exists. Then  $(AB)^{-1}$  is
  - a)  $A^{-1}B^{-1}$
- b)  $B^{-1}A^{-1}$

c) AB

d) none of these.

### **GROUP - B**

# (Short Answer Type Questions)

Answer any *three* of the following.

 $3 \times 5 = 15$ 

2. Verify Rolles theorem for the function

$$f(x) = |x|, -1 \le x \le 1.$$

- 3. A and B are orthogonal matrix and |A| + |B| = 0. Prove that A + B is singular.
- 4. Find the  $n^{\text{th}}$  derivative of  $\frac{x^2+1}{(x-1)(x-2)(x-3)}$ .
- 5. Let

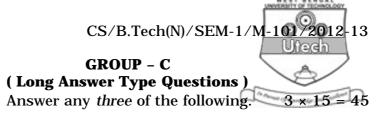
$$f(x, y) = \frac{xy}{x + y^2}, (x, y) \neq (0, 0)$$

$$= 0, (x, y) = (0, 0)$$

Evaluate  $f_{xy}$  ( 0, 0 ) and  $f_{yx}$  ( 0, 0 ).

6. Find  $\overrightarrow{div} \overrightarrow{F}$  and  $\overrightarrow{curl} \overrightarrow{F}$  where

$$\vec{F} = grad(x^3 + y^3 + z^3 - 3xyz).$$



$$3 \times 15 = 43$$

a) If  $u = x^2 - 2y$ , v = x + y + z, w = x - 2y + 3z, find  $\frac{\partial (u, v, w)}{\partial (x, y, z)}$ . 7.

b) Prove that 
$$\begin{vmatrix} 1 & \alpha & \alpha^2 - \beta \gamma \\ 1 & \beta & \beta^2 - \gamma \alpha \\ 1 & \gamma & \gamma^2 - \alpha \beta \end{vmatrix} = 0.$$

c) If  $v = f(x^2 + 2yz, y^2 + 2zx)$ , prove that

$$\left(y^2-zx\right)\frac{\partial v}{\partial x}+\left(x^2-yz\right)\frac{\partial v}{\partial y}+\left(z^2-xy\right)\frac{\partial v}{\partial z}=0.$$

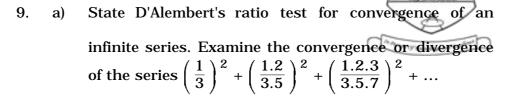
$$5 + 5 + 5$$

- 8. a) If  $\theta = t^n e^{\frac{-r^2}{4t}}$ , find what value of n will make  $\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial \theta}{\partial r} \right) = \frac{\partial \theta}{\partial t}$ .
  - b) Using mean value theorem prove that

$$0 < \frac{1}{x} \log \left( \frac{e^{x} - 1}{x} \right) < x.$$

c) If  $I_n = \int_0^{\frac{\pi}{2}} x^n \sin x \, dx$  ( n > 1 ), then show that  $I_n + n(n-1)I_{n-2} = n\left(\frac{\pi}{2}\right)^{n-1}$ . 5 + 5 + 5

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b) If 
$$y = e^{\tan^{-1}x}$$
, then show that  $(1 + x^2) y_{n+2} + (2nx + 2x - 1) y_{n+1} + n(n+1) y_n = 0$ .

c) Find the extreme value of the function

$$f(x, y) = x^3 + y^3 - 3x - 12y + 20.$$
 5 + 5 + 5

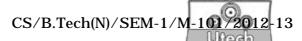
10. a) If  $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ , then verify that A satisfies its

own characteristic equation. Hence find  $A^{-1}$  and  $A^{9}$ .

- b) If  $u = \tan^{-1}\left(\frac{x^3 + y^3}{x y}\right)$ , then show that  $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = (1 4 \sin^2 u) \sin 2u.$
- c) Given the system of equation :

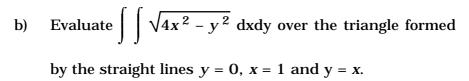
$$x_1 + 4x_2 + 2x_3 = 1$$
,  $2x_1 + 7x_2 + 5x_3 = k$ ,  $4x_1 + mx_2 + 10x_3 = 2k + 1$ . Find for what values of  $k$  and  $m$ , the system has (i) an unique solution, (ii) no solution (iii) many solution.

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Show that  $\overrightarrow{\nabla} r^n = nr^{n-2} \overrightarrow{r}$ ,

where 
$$\vec{r} = \vec{i}x + \vec{j}y + \vec{k}z$$
.



Verify Stokes theorem for c)

> $\vec{F} = (2x - y) \hat{i} - yz^2 \hat{j} - y^2 z \hat{k}$ , where S is the upper half surface of the sphere  $x^2 + y^2 + z^2 = 1$  and *C* is its boundary. 5 + 5 + 5