



Name :

Roll No. :

Invigilator's Signature :

CS/B.TECH (OLD)/SEM-2/M-201/2012

2012
MATHEMATICS

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for the following :

$$10 \times 1 = 10$$

i) Rank of the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 3 & 1 & 1 \end{pmatrix}$ is

- a) 4 b) 3
c) 2 d) 1

ii) The value of t for which the matrix $A = \begin{pmatrix} 2 & 0 & 1 \\ 5 & t & 3 \\ 0 & 3 & 1 \end{pmatrix}$ is

singular is

- a) $\frac{3}{2}$ b) 2
c) 1 d) $\frac{1}{3}$.



- iii) The equation $x + y + z = 0$ has
- infinite number of solutions
 - no solution
 - unique solution
 - two solutions.
- iv) The value of k for which the vectors $(1, 2, 1)$, $(k, 1, 1)$ and $(1, 12)$ are linearly dependent is
- 2
 - $\frac{2}{3}$
 - 1
 - $\frac{3}{2}$.
- v) The eigenvalues of the matrix A and B are a and b . then The eigenvalues of A^2 are
- ab, b^2
 - a^2, b
 - a^2, b^2
 - a, b .
- vi) If a linear transformation $T: R^2 \rightarrow R^2$ be defined by $T(x_1, x_2) = (x_1, x_2)$, then $\text{Ker}(T)$ is
- $\{(-1, -1), (1, 1)\}$
 - $\{(1, 2), (1, \frac{1}{2})\}$
 - $\{(1, 0), (0, 1)\}$
 - $\{(0, 0)\}$.
- vii) The interpolation formula which can be used to find a polynomial from the following data
- $X: 0 \ 1 \ 2 \ 4$
 $Y: 3 \ 9 \ 17 \ 22$
- is
- Newton's forward interpolation formula
 - Gaussian interpolation formula
 - Lagrange's interpolation formula
 - Newton's backward interpolation formula.



viii) Which of the following is not true ?

- a) $\Delta = E - 1$ b) $\Delta \cdot \nabla = \Delta - \nabla$
 c) $\frac{\Delta}{\nabla} = \Delta + \nabla$ d) $\nabla = 1 - E^{-1}$.

ix) The degree and order of the differential equation

$$\left(\frac{d^2 y}{dx_2} \right)^{\frac{3}{2}} = x \frac{dy}{dx} \text{ are}$$

- a) $\left(\frac{3}{2}, 2 \right)$ b) $(2, 3)$
 c) $(3, 2)$ d) $(2, 1)$.
 x) $L \left\{ \frac{\sin t}{t} \right\} = \tan^{-1} \left(\frac{1}{s} \right)$. Then $L \left\{ \frac{\sin at}{t} \right\}$ is
 a) $\tan^{-1} \left(\frac{1}{s^2} \right)$ b) $\tan^{-1} \left(\frac{a}{s} \right)$
 c) $\tan^{-1} \left(\frac{1}{as} \right)$ d) $\tan^{-1} \left(\frac{1}{a^2 + s^2} \right)$.

GROUP - B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Solve the differential equation by Laplace Transformation :

$$\frac{d^2 y}{dt^2} + 9y = 1, y(0) = 1, y\left(\frac{\pi}{2}\right) = -1$$



3. Solve by the method of variation of parameter :

$$\frac{d^2 y}{dx^2} + 9y = \sec 3x$$

4. Solve the system of equations if possible :

$$x + y + z = 1$$

$$2x + y + 2z = 2$$

$$3x + 2y + 3z = 5$$

5. If $W = \{ (x, y, z) \in R^3 : x + y + z = 0 \}$, show that W is a subspace of R^3 , and find a basis of W .

6. Examine whether the mapping $T : R^2 \rightarrow R^2$ defined by $T(x, y) = (2x - y, x)$ is linear.

7. Evaluate $\int_0^1 \frac{dx}{1+x}$ using Trapezoidal rule, taking four equal subintervals.

GROUP - C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

8. a) Evaluate $\left(\frac{\Delta^2}{E} \right) x^3$.

- b) Find the missing data in the following table :

x	-2	-1	0	1	2
$f(x)$	6	0	?	0	6



- c) Show that $\int_0^{\infty} \frac{\sin t}{t} dt = \frac{\pi}{2}$, by Laplace transform.
9. a) Show that $(3, 1, -2)$, $(2, 1, 4)$ and $(1, -1, 2)$ form a basis of R^3 .
- b) Apply convolution theorem to prove that $\int_0^t \sin u \cos (t - u) dt = \frac{t}{2} \sin t$.
- c) Solve $\frac{dy}{dx} - \frac{\tan y}{(1+x)} = (1+x) e^x \sec y$.
10. a) Solve by Cramer's rule :
- $$x + y + z = 7$$
- $$x + 2y + 3z = 15$$
- $$x - y + z = 3$$
- b) Find general solution of $p = \cos (y - px)$, where $p = \frac{dy}{dx}$.
- c) Solve $\frac{dy}{dx} + \frac{y \log y}{x} = \frac{y (\log y)^2}{x^2}$
11. a) Solve $(D^2 - 2D) y = e^x \sin x$, where $D \equiv \frac{d}{dx}$.
- b) Solve $\frac{dx}{dt} - 7x + y = 0$ and $\frac{dy}{dt} - 2x - 5y = 0$
- c) Solve by Gauss-elimination method :
- $$2x + 2y + z = 12$$
- $$3x + 2y + 2z = 8$$
- $$5x + 10y - 8z = 10$$



12. a) Use Lagrange's interpolation formula to find $f(x)$, where $f(0) = -18$, $f(3) = 0$, $f(5) = -248$, $f(6) = 0$ and $f(9) = 13104$.

- b) Apply appropriate interpolation formula to calculate $f(2.1)$, correct up to two significant figures from the following data :

x	0	2	4	6	8	10
$f(x)$	1	5	17	37	45	51

- c) Apply method of variation of parameter to solve the equation

$$\frac{d^2 y}{dx^2} + y = \sec^3 x \cdot \tan x$$

13. a) Apply Simpson's $\frac{1}{3}$ rd rule to evaluate $\int_0^6 \frac{dx}{(1+x)^2}$ taking six equal intervals from (0, 6) and correct up to three decimal places. 5 + 10

- b) Expand by Laplace method

$$\begin{vmatrix} 0 & a & b & c \\ -a & 0 & d & e \\ -b & -d & 0 & f \\ -c & -e & -f & 0 \end{vmatrix} = (af - be + cd)^2$$

- c) Given that

$$L\left\{\frac{\sin t}{t}\right\} = \tan^{-1}\left(\frac{1}{s}\right), \text{ Find } L\left\{\frac{\sin at}{t}\right\}.$$



14. a) Assuming the orthogonal properties of Legendre function, prove that

$$\int_{-1}^{+1} [P_n(x)]^2 dx = \frac{2}{2n+1}$$

- b) Show that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$.

- e) State Cayley-Hamilton theorem and show that the matrix

$$A = \begin{pmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{pmatrix} \text{ satisfies the above theorem.}$$

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