



Name : .....

Roll No. : .....

Invigilator's Signature : .....

**CS/B.TECH(OLD)/SEM-2/M-201/2011**

**2011**

**MATHEMATICS**

Time Allotted : 3 Hours

Full Marks : 70

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

**GROUP – A**

**( Multiple Choice Type Questions )**

1. Choose the correct alternatives for any *ten* of the following :

$$10 \times 1 = 10$$

i) The rank of the matrix  $\begin{bmatrix} 2 & 3 & 5 \\ 1 & -1 & 2 \\ 1 & 4 & 3 \end{bmatrix}$  is

- a) 3                                      b)  $5/2$   
c) 2                                      d) 1.

ii) In the Newton's forward interpolation formula the value of  $u = (x - x_0) / h$  lies between

- a) 1 and 2                              b) -1 and 1  
c) 0 and  $\infty$                           d) 0 and 1.



iii) The eigenvalues of a matrix  $A$  are 2 and 4. Then the eigenvalues of  $A^{-1}$  are

- a) 2, 4                                      b) 4, 4  
c) 0.25, 0.5                                d) 0.5, 0.25.

iv) The value of  $a$  for which the vectors  $(1, 2, 1)$ ,  $(a, 1, 1)$  and  $(1, 1, 2)$  are linearly dependent is

- a) 2    b)  $\frac{2}{3}$   
c) -1    d)  $\frac{3}{2}$ .

v) The value of the determinant  $\begin{vmatrix} 0 & -a & -b \\ a & 0 & c \\ b & -c & 0 \end{vmatrix}$  is

- a) 0    b)  $abc$   
c)  $-abc$                                         d)  $2abc$ .

vi) If the system of equations  $4x + 2y - 5z = 0$ ,  $x + \lambda y + 2z = 0$ ,  $2x + y - z = 0$  has a non-zero solution, then  $\lambda$  is

- a)  $\frac{1}{3}$     b)  $\frac{1}{2}$   
c) 1    d) none of these.

vii) The integrating factor of  $\frac{dy}{dx} + y = 1$  is

- a)  $e^x$     b)  $x$   
c)  $e^2$     d) 2.



dimension of

- of linear equations

## linear equation

- $$[m, r] - r.$$



5. Solve :

$$\begin{cases} \frac{dx}{dt} + y = e^t \\ \frac{dy}{dt} - x = e^{-t} \end{cases}$$

6. Evaluate :  $\int_0^2 (x^3 + 1) dx$  by Simpson's one-third rule

taking 4 intervals.

### GROUP – C

#### ( Long Answer Type Questions )

Answer any *three* of the following.  $3 \times 15 = 45$

7. a) Investigate for what values of  $\lambda$  and  $\mu$  the following equations

$$x + y + z = 6$$

$$x + 2y + 3z = 10$$

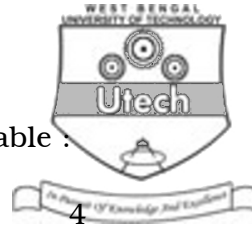
$$x + 2y + \lambda z = \mu$$

have (i) no solution (ii) a unique solution and (iii) an infinite number of solutions.

b) Prove that

$$\begin{vmatrix} bc - a^2 & ca - b^2 & ab - c^2 \\ ca - b^2 & ab - c^2 & bc - a^2 \\ ab - c^2 & bc - a^2 & ca - b^2 \end{vmatrix} = (a^3 + b^3 + c^3 - 3abc)^2$$

c) Show that every square matrix can be expressed as a sum of a symmetric and skew-symmetric matrix.



8. a) Compute  $f(1 \cdot 3)$  from the following table :

$x :$	0	1	2	3	4
$f(x) :$	1	1.5	2.2	3.1	4.3.

- b) Solve by the method of variation of parameters

$$\frac{d^2 y}{dx^2} + 3 \frac{dy}{dx} + 2y = 3e^{-x} + x$$

- c) If  $\alpha$ ,  $\beta$  and  $\gamma$  form a basis of a vector space  $V$ , then prove that  $\alpha + \beta + \gamma$ ,  $\beta + \gamma$  and  $\gamma$  also form a basis of  $V$ .

9. a) Show that the matrix

$$A = \frac{1}{3} \begin{pmatrix} -1 & 2 & -2 \\ -2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix} \text{ is orthogonal and hence obtain } A^{-1}.$$

- b) Using the method of separation of symbols, prove that

$$u_0 + u_1 + u_2 + \dots + u_n = {}^{n+1}C_1 u_0 + {}^{n+1}C_2 \Delta u_0 + {}^{n+1}C_3 \Delta^2 u_0 + \dots + {}^{n+1}C_{n+1} \Delta^n u_0.$$

- c) Show that

$$M = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} / \begin{matrix} a, b, c, d \in R \\ a = d \end{matrix} \right\} \text{ is a subspace of the vector space of } 2 \times 2 \text{ real matrices. Obtain a basis and dimension of } M.$$



10. a) If possible diagonalise the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 2 & 1 & 2 \end{pmatrix},$$

Specify the matrix which diagonalises  $A$  and the diagonal matrix to which  $A$  is changed after being diagonalised.

- b) Find  $L \left( \frac{1 - e^t}{t} \right)$ .
- c) Assuming orthogonal property of Legendre function, prove that

$$\int_{-1}^1 [P_n(x)]^2 dx = \frac{2}{2n+1}.$$

=====