

No. :

Student's Signature :

CS/B.TECH(N)/SEM-2/M-201/2013

2013

MATHEMATICS - II

• Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

Students are required to give their answers in their own words
as far as practicable.

GROUP - A

(Multiple Choice Type Questions)

Choose the correct alternatives for any ten of the following :

$$10 \times 1 = 10$$

The general solution of $y = px - \log p$ is

- a) $y = cx - \log c$ b) $y = 1 + \log x$
c) $y = 1 + \log x + c$ d) none of these.

ii) The particular integral of $\frac{d^2y}{dx^2} + y = \cos x$ is

- a) $\frac{1}{2} \sin x$ b) $\frac{1}{2} \cos x$
c) $\frac{1}{2} x \sin x$ d) $\frac{1}{2} x \cos x$.

iii) $\frac{1}{D-1} x^2$ is equal to

- a) $x^2 + 2x + 2$ b) $-(x^2 + 2x + 2)$
 c) $2x - x^2$ d) $-(2x - x^2)$

iv) The general solution of $\frac{d^2 y}{dx^2} + y = 0$ is

- a) $Ae^x + Be^{-x}$ b) $(A + Bx)e^x$
 c) $(A + Bx)\cos x$ d) $A\cos x + B\sin x$

v) A simple graph can have

- a) no pendant vertex b) no isolated vertex
 c) no circuit d) none of these

vi) A simple graph with 20 vertices and 5 components has at least

- a) 15 edges b) 10 edges
 c) 190 edges d) 120 edges

vii) Which of the following is incorrect about a tree T with n vertices?

- a) There exist multiple paths between every pair of vertices in T
 b) T is minimally connected
 c) T is connected and circuitless
 d) T has $(n - 1)$ edges

If the incidence matrix of a graph has five identical columns, the G has

- a) five loops b) five isolated vertices
 c) five parallel edges d) five edges in series

$$\mathcal{L}^{-1}\left(\frac{s}{s^2 - a^2}\right) =$$

- a) $\sin at$ b) $\sinh at$
 c) $\cos at$ d) $\cosh at$

$\mathcal{L}\{H(t - a)\}$, H being Heavyside unit step function, is

- a) e^{-as} b) se^{-as}
 c) $\frac{e^{-as}}{s}$ d) none of these

xii) Laplace transform of $\frac{\sin 2t}{t}$ is

- a) $\cot^{-1} \frac{s}{2}$ b) $\cot^{-1} \frac{2}{s}$
 c) $\frac{2}{s^2 + 4}$ d) $\frac{s}{s^2 - 4}$

xii) $\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{2}{3}\right)$ equals to

- a) $\frac{2\pi}{\sqrt{3}}$ b) $\frac{3\pi}{\sqrt{2}}$
c) $\frac{\pi}{\sqrt{3}}$ d) $\frac{\pi}{\sqrt{2}}$

xiii) $\int_{-\infty}^{\infty} x e^{-x^2} dx =$

- a) -1 b) 0
c) 1 d) none of these.

GROUP - B

(Short Answer Type Questions)

Answer any three of the following 3 × 5 =

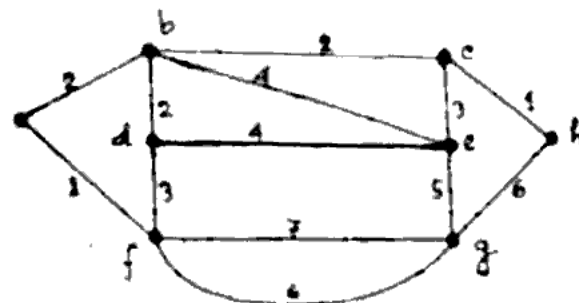
- Solve: $(x^2y - 2xy^2)dx + (3x^2y - x^3)dy = 0$.
- Solve the following simultaneous ODE:
 $\frac{dx}{dt} - 7x + y = 0, \frac{dy}{dt} - 2x + 5y = 0$
- Prove that the number of edges in a simple graph can exceed $\frac{n(n-1)}{2}$.
- Prove that a graph is a tree if and only if it is minimally connected.
- Define Gamma function. Show that $\Gamma(n+1) = n\Gamma(n)$. 2

GROUP - C

(Long Answer Type Questions)

Answer any three of the following. 3 × 15 = 45

1) Use Dijkstra's algorithm to find shortest path between vertices a and h in the following graph: 7



2) Solve: $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x$ 5

3) Construct a diagram from the following incidence matrix: 3

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & -1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \end{bmatrix}$$

4) Prove that a tree with n vertices has (n-1) edges. 6

5) Solve the following by the method of variation of parameters:

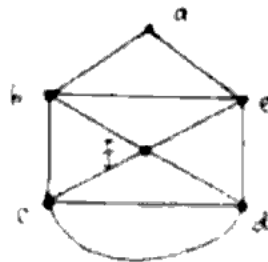
$$\frac{d^2y}{dx^2} + y = \tan x \quad 5$$

c) Solve the following differential equation by Laplace Transform:

$$(D^2 + 6D + 9)y = 0, y(0) = y'(0) = 1 \quad 4$$

a) i) Define Euler circuit. Write the necessary and sufficient condition for a graph to contain an Euler circuit.

- ii) Find, if possible, an Euler circuit in the following graph :



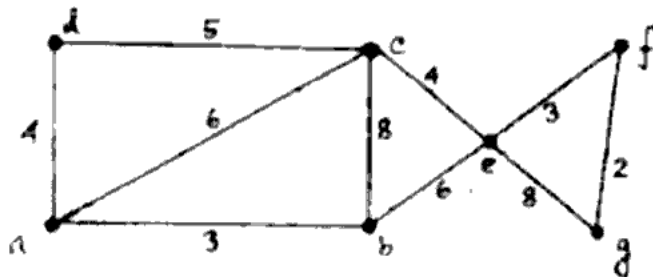
(2 + 1)

- b) Using convolution theorem prove that

$$L^{-1}\left\{\frac{s}{(s^2 + a^2)^2}\right\} = \frac{t \sin t}{2a}$$

- c) Prove that : $\int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$

10. a) By Kruskal's algorithm find a minimal spanning tree in the following graph :



- b) Find the Laplace Transform of $f(t)$ defined as :

$$f(t) = \begin{cases} \frac{t}{k}, & \text{when } 0 < t < k \\ 1, & \text{when } t > k \end{cases}$$

- c) Solve : $x^2 \frac{d^2 y}{dx^2} - 3x \frac{dy}{dx} + 4y = 2x^2$

Evaluate : $L^{-1}\left\{\frac{s^2}{(s^2 + a^2)(s^2 + b^2)}\right\}$

5

Examine whether the following graphs are isomorphic or not :

5



Solve : $y = px + \sqrt{a^2 p^2 + b^2}$

5

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