CS/B.Tech/Even/Sem-2nd/M-201/2015

WEST BENGAL UNIVERSITY OF TECHNOLOGY

M-201

MATHEMATICS - II

Time Allotted: 3 Hours   Full Marks: 70

The questions are of equal value.
The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

GROUP A
(Multiple Choice Type Questions)

1.   Answer any *ten* questions. $10 \times 1 = 10$

(i) $L\{e^{-2t}\cos t\} =$

   (A) $\dfrac{s+2}{s^2+4s+5}$ (B) $\dfrac{s}{s^2+4s+5}$

   (C) $\dfrac{s+1}{s^2+4s+1}$ (D) none of theses

(ii) If $L\{f(t)\} = \tan^{-1}\left(\dfrac{1}{s}\right)$, then $L\{tf(t)\}$ is

   (A) $\tan^{-1}\left(\dfrac{1}{s^2}\right)$ (B) $\dfrac{1}{s^2+1}$

   (C) $\dfrac{1}{s+1}$ (D) $\tan^{-1}\left(\dfrac{2}{\pi s}\right)$

(iii) The value of $\beta\left(\dfrac{1}{2}, \dfrac{1}{2}\right)$ is

   (A) π (B) $\sqrt{\pi}$

   (C) $\dfrac{\sqrt{\pi}}{2}$ (D) $\dfrac{\pi}{2}$

(iv) The value of $\Gamma\left(\dfrac{1}{3}\right)\Gamma\left(\dfrac{2}{3}\right)$ is

   (A) $\dfrac{2\pi}{\sqrt{3}}$ (B) $\dfrac{3\pi}{\sqrt{2}}$

   (C) $\dfrac{\pi}{\sqrt{3}}$ (D) $\dfrac{\pi}{\sqrt{2}}$

(v) If a graph has 5 vertices and 7 edges, then the size of the adjacency matrix is

   (A) 5×5 (B) 5×7

   (C) 7×5 (D) 7×7

(vi) If n is a positive integer then $\Gamma(n+1) =$

   (A) $(n+1)!$ (B) $(n-1)!$

   (C) $n!$ (D) none of these

(vii) The integral $\int_a^\infty \dfrac{dx}{x^\mu}\,(a > 0)$ exists for

   (A) $\mu > 1$ (B) $\mu < 1$

   (C) $\mu = 1$ (D) for all μ

(viii) The number of pendent vertices in a binary tree with 9 vertices is

   (A) 2   (B) 3

   (C) 4   (D) 5

(ix) What is the order and degree of the differential equation $\frac{dy}{dx} + x = e^{\frac{d^2y}{dx^2}}$?

(A) 2, 1   (B) 2, 2

(C) 2, undefined   (D) undefined, 2

(x) The integrating factor of the differential equation $-\frac{dy}{dx} + Py = -Q$ is

(A) $e^{\int P dx}$ (B) $e^{\int Q dx}$

(C) $e^{\int -P dx}$ (D) $e^{\int -Q dx}$

(xi) The general solution of $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 0$ is

(A) $y = Ae^{-x} + Be^{-2x}$ (B) $y = e^{-x} + e^{-2x}$

(C) $y = e^{-x}(A + Bx)$ (D) none of these
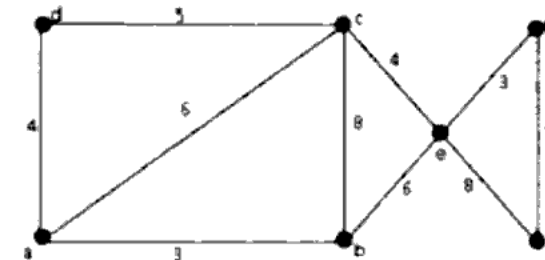
(xii) $\int_0^\infty \frac{\sin t}{t} dt =$

(A) $\frac{\pi}{4}$ (B) $\frac{2\pi}{3}$

(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

GROUP B
(Short Answer Type Questions)

Answer any *three* questions. $3 \times 5 = 1$

2. Using the method of variation of parameters solve. $\frac{d^2y}{dx^2} + y = \frac{1}{1+\sin x}$

3. Evaluate $L\{(\sin^3 t - \cos^3 t)\}$

4. By Kruskal's algorithm find a minimum spanning tree in the following graph:

5. Assuming the convergence of the integrals prove that

$$\int_0^\infty e^{-x^4} dx \times \int_0^\infty x^2 e^{-x^4} dx = \frac{\pi}{8\sqrt{2}}$$

6. Apply convolution theorem to prove that

$$\int_0^t \sin u \cos(t - u) du = \frac{t}{2} \sin t$$

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GROUP C
(Long Answer Type Questions)

Answer any *three* questions. $3 \times 15 = 45$

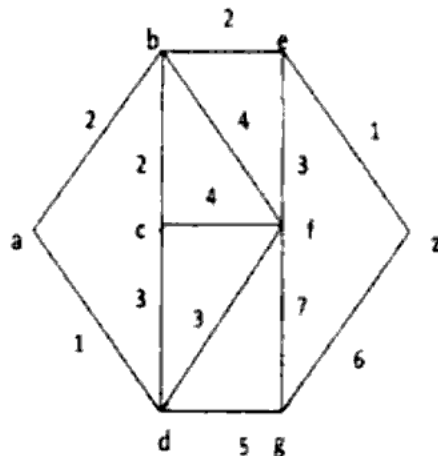7. (a) Solve $(x^2 D^2 - xD - 3)y = x^2 \log x$ 5+5+5

(b) Show that number of pendant vertices in a binary tree is $\frac{n+1}{2}$, where n is the number of vertices in the tree.

(c) If $L\{f(t)\} = F(p)$ then prove that $L\{t^n f(t)\} = (-1)^n \frac{d^n}{dp^n}\{F(p)\}$ where n is a positive integer.

8. (a) Applying Dijkstra's method find the shortest path and distance between the   7+5+3
two vertices a and z in the given graph

(b) Obtain the general solution and singular solution of the equation
$\sin(xp) \cos y = \cos(xp) \sin y + p$, p have its usual meaning.
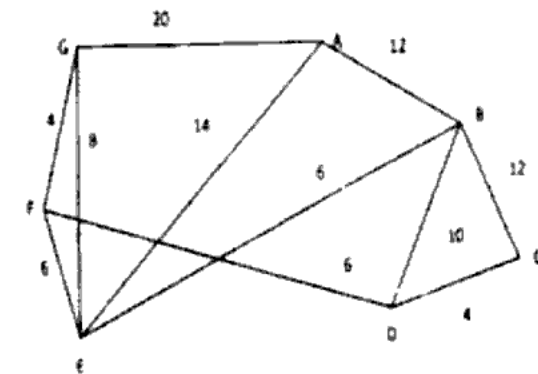
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(c) Find the Laplace transform of the function $f(t)$, where
$$f(t) = \begin{cases} t, & 0 < t < 1, \\ 0, & 1 < t < 2 \end{cases}$$
and $f(t+2) = f(t)$.

9. (a) Find the minimal spanning tree from the following graph using Prim's   5+5+5
algorithm.

(b) Examine the convergence of $\int_1^2 \frac{x\,dx}{\sqrt{2-x}}$

(c) State convolution theorem. Using convolution theorem prove that
$$L^{-1}\left(\frac{s}{(s^2 + a^2)^2}\right) = \frac{t \sin at}{2a}.$$

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10.(a) Find the incidence matrices of the following graphs, hence show that the two graphs are isomorphic   5+6+4

(b) Solve by Laplace transform the equation $\dfrac{d^2y}{dt^2} + y = 8\cos t$ where $y(0) = 1,\ y'(0) = -1$.

(c) If a simple regular graph has n vertices and 24 edges, find all possible values of n.

11.(a) Solve: $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} = e^x \sin x$.   5+5+5

(b) Prove that $B(m, n) = \displaystyle\int_0^\infty \dfrac{x^{m-1}}{(1+x)^{m+n}}\, dx$.

(c) Solve: $\dfrac{dx}{dt} + y = e^t,\ \dfrac{dy}{dt} - x = e^{-t}$.