



Name : .....

Roll No. : .....

Invigilator's Signature : .....

**CS/B.TECH(OLD)/PWE/BME/ICE/EE/ECE/EIE/EEE**  
**/SEM-3/M-302/2011-12**

**2011**

**MATHEMATICS**

Time Allotted : 3 Hours

Full Marks : 70

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words  
as far as practicable.*

**GROUP – A**  
**( Multiple Choice Type Questions )**

1. Choose the correct alternatives for any *ten* of the following :

$$10 \times 1 = 10$$

i) Let  $C$  be a circle  $|z - i| = 2$ . Then  $\int_C \frac{dz}{(z^2 + 4)^2} =$

a)  $\frac{\pi}{16}$

b)  $\frac{\pi}{4}$

c)  $\frac{\pi}{8}$

d)  $\frac{\pi}{32}$ .

ii) The minimum number of edges in a simple graph with  $n$  vertices and  $k$  connected components is

a)  $n$

b)  $k$

c)  $n - k$

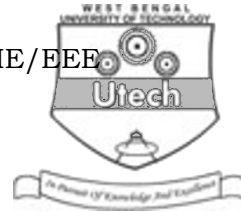
d)  $n + k$ .

3153 (O)

[ Turn over

- 2

- 3153 (O)



xii) The value of  $k$  for which

$$f(x) = kx(1-x), \quad 0 \leq x \leq 1$$

$$= 0, \quad \text{otherwise}$$

will be a p.d.f. of a random variable  $X$  is

a) 6

b) 2

c) 1

d) 3.

### GROUP - B

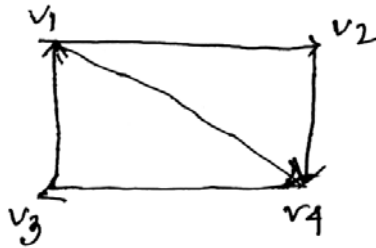
#### ( Short Answer Type Questions )

Answer any *three* of the following  $3 \times 5 = 15$

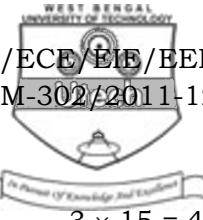
2. Find the Fourier series to represent  $x - x^2$  from  $x = -\pi$  to  $x = \pi$ . Hence find the value of the series

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$$

3. Define a binary tree. Prove that the number of vertices in a binary tree is always odd.
4. Define digraphs. Find the indegree and outdegree of the vertices of the following digraph. Write the adjacency matrix.



5. State and prove Baye's theorem.
6. Find the bilinear transformation which maps  $z = 1, i, -1$  respectively to  $w = i, 0, -1$ . Also find the image of  $|z| \leq 1$  under the transformation.



**GROUP – C**  
**( Long Answer Type Questions )**

Answer any *three* of the following.  $3 \times 15 = 45$

7. a) Examine the nature of the function

$$f(z) = \begin{cases} \frac{x^2 y^5 (x + iy)}{x^4 + y^{10}}, & z \neq 0 \\ 0, & z = 0 \end{cases}$$

in the region including the origin.

- b) Find the Taylor's series expansion of  
 $f(z) = \frac{(z+1)}{(z-3)(z-4)}$  about  $z = 2$ .

Find the region of convergence.

- c) Evaluate  $\int_C \frac{z^2 e^{zt}}{z^2 + 1} dz$ , where  $C$  is the circle  $|z|=2$  and  $t$

is a quantity independent of  $z$ .  $5 + 5 + 5$

8. a) Prove that mean and variance of Binomial distribution is  $np$  and  $npq$  respectively.  
b) If  $x = 4y + 5$  and  $y = kx + 4$  are two regression equations of  $x$  on  $y$  and  $y$  on  $x$  respectively, obtain the interval in which  $k$  lies.  
c) A batch of 1000 lamps is known to have 10% defectives. If 9 lamps are chosen at random and are tested, what is the probability that none of them will be defective? What is the probability that exactly 5 defectives will be found?  $5 + 5 + 5$

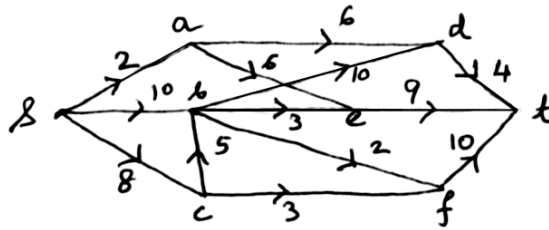
9. a) Find the Fourier transform of the function

$$f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$$

and hence evaluate  $\int_{-\infty}^{\infty} \frac{\sin \alpha a \cos \alpha x}{\alpha} d\alpha$ .

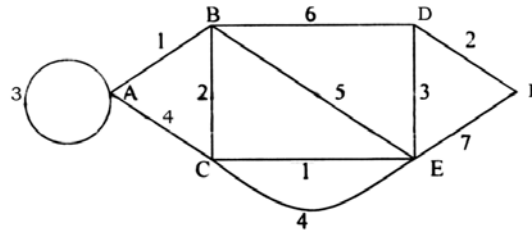


- b) Find a maximum flow in the directed network shown in the following figure and prove that it is maximum.

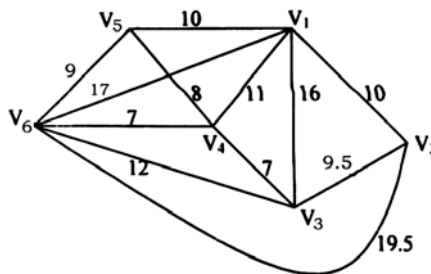


7 + 8

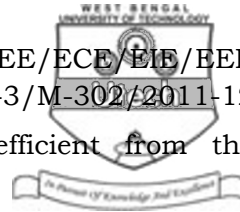
10. a) Prove that a tree with  $n$  vertices contains exactly  $(n - 1)$  edges.  
b) Find the shortest path from  $A$  to  $F$  applying Dijkstra's algorithm to the graph given below :



- c) Describe Kruskal's algorithm to find the shortest spanning tree in a graph  $G$ . Use this algorithm to find the shortest spanning tree for the following graph :



3 + 6 + 6



11. a) Calculate Pearson's correlation coefficient from the following data :

Height (cm) :	162	165	167	168	170	175
Weight (kg) :	58	60	65	67	72	75

- b) A random variable follows Normal distribution such that  $P(9.6 \leq X \leq 13.8) = 0.7008$ ,  $P(X \geq 9.6) = 0.8504$ , where

$$1/\sqrt{(2\pi)} \int_{-\alpha}^{0.9} e^{-t^2/2} dt = 0.8159, \quad 1/\sqrt{(2\pi)} \int_{-\alpha}^{1.2} e^{-t^2/2} dt = 0.8849.$$

Find the mean & variance of X.

6 + 9

=====