	Utech
Name :	
Roll No.:	A Agency Of Exercising 2nd Explored
Invigilator's Signature :	

## **MATHEMATICS**

Time Allotted: 3 Hours Full Marks: 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

# GROUP – A ( Multiple Choice Type Questions )

 $1. \quad \hbox{Choose the correct alternatives for any $\it ten$ of the following:}$ 

 $10 \propto 1 = 10$ 

i) The value of 
$$\lim_{n \neq \infty} \left[ \frac{\sqrt{1} + \sqrt{2} + \sqrt{3} + \dots + \sqrt{n}}{n \sqrt{n}} \right]$$
 is

a) 0

b) 1

c)  $\frac{2}{3}$ 

d)  $\frac{3}{2}$ .

ii) Which of the following functions obeys Rolle's theorem in [ 0,  $\pi$  ] ?

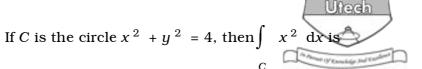
a) *x* 

b)  $\sin x$ 

c)  $\cos x$ 

d)  $\tan x$ .

11701 [ Turn over



a) 0 b)  $\frac{1}{3}$ 

c)

- The equation  $x^2 + y^2 = a^2$ , z = 0 represents
  - a) circle

b) cylinder

sphere c)

- d) right circular cylinder.
- If  $\theta$  be an angle between the vectors

$$\overset{\varnothing}{a} = 6\hat{i} + 2\hat{j} + 3\hat{k}$$
 and  $\overset{\varnothing}{b} = 2\hat{i} - 9\hat{j} + 6\hat{k}$ , then

- a)  $\theta = \cos^{-1}\left(\frac{12}{77}\right)$  b)  $\theta = \sin^{-1}\left(\frac{12}{77}\right)$
- c)  $\theta = \tan^{-1}\left(\frac{12}{77}\right)$
- d) none of these.
- If Cauchy's mean value theorem is applicable to the function f(x)x and  $g(x) = x^2$ , then the value of C is
  - 3/2 a)

b) 0

1/2 c)

- d) -3/2.
- vii) If  $y^2 = 4ax$  ( a is a real constant ), then  $\frac{d^2y}{dx^2} \cdot \frac{d^2x}{du^2}$  is

b)  $\frac{2a}{y^3}$ 

c)  $-\frac{2a}{u^3}$ 

d)  $-\frac{2a}{y}$ .



# viii) The law of mean is given by

a) 
$$\frac{f(b) + f(a)}{b - a} = f'(c)$$

b) 
$$\frac{f(b) + f(a)}{b + a} = f'(c)$$

c) 
$$\frac{f(b)-f(a)}{b-a} = f'(c)$$

d) 
$$\frac{f(b)-f(a)}{b-a} = f(c).$$

ix) If  $x = r \cos \theta$  and  $y = r \sin \theta$ , then the value of  $\frac{\partial (r, \theta)}{\partial (x, y)}$ 

is

a) (

b)

c)  $\frac{1}{r}$ 

- d) -r.
- x) The series  $\frac{1}{n} = \frac{2}{r} = \frac{2}{n}$  is
  - a) convergent
- b) divergent
- c) oscillatory
- d) none of these.

xi) The function 
$$f(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$
 is

- a) continuous and differentiable at x = 0
- b) continuous but not differentiable at x = 0
- c) neither continuous nor differentiable at x = 0
- d) none of these.



- xii) If  $f(x, y) = \tan(x / y)$ , then  $x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y}$ 
  - tan(x/y)a)
- b)  $\cot(x/y)$

c) 0

- d) none of these.
- xiii) The moment of inertia of a thin uniform rod of mass Mand length 2a about an axis perpendcular to the rod at its centre is

b)  $\frac{Ma^2}{2}$ 

c)  $Ma^2$ 

- d)  $\frac{Ma^2}{4}$ .
- xiv) The point of intersection of the line  $\frac{x-1}{2} = \frac{y}{3} = \frac{z+1}{-1}$ with the plane x + 2y - z = 5 is
  - a) (1, 1, 1)
- b) (0, 1, 3)
- c)  $\left(\frac{5}{3}, 1, \frac{-4}{3}\right)$
- d) none of these.
- xv) The reduction formula of  $I_n = \int \cos^n x \, dx$  is

  - a)  $I_n = \frac{n-1}{n} I_{n-1}$  b)  $I_n = \frac{n}{n-1} I_{n-1}$
  - c)  $I_n = \frac{n-1}{n} I_{n-2}$  d) none of these.



#### **GROUP - B**

## (Short Answer Type Questions)

Answer any three of the following.



2. If  $y = (x^2 - 1)^n$ , then show that

$$(x^2-1)y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0.$$

3. If  $\overset{\varnothing}{a}$ ,  $\overset{\varnothing}{b}$ ,  $\overset{\varnothing}{c}$  are three vectors, then show the

 $\left[ \stackrel{\bigcirc}{a} \propto \stackrel{\bigcirc}{b}, \stackrel{\bigcirc}{b} \propto \stackrel{\bigcirc}{c}, \stackrel{\bigcirc}{c} \propto \stackrel{\bigcirc}{a} \right] = \left[ \stackrel{\bigcirc}{a}, \stackrel{\bigcirc}{b}, \stackrel{\bigcirc}{c} \right]^2$ , where symbols have their usual meanings.

- 4. Test the convergence of the series
- 5. A. B, C and D are points ( $\alpha$ , 3, -1), (3, 5, -3), (1, 2, 3,) and (3, 5, 7) respectively. If AB is perpendicular to CD, then find the value of  $\alpha$ .
- 6. If , then prove that
- 7. Verify Rolle's theorem for the function

$$f(x) = |x|, -1 \le x \le 1$$
.

#### **GROUP - C**



# (Long Answer Type Questions)

Answer any three of the following.



- 8. a) Examine continuity and differentiability of  $f(x) \text{ at } x = 0, \text{ when } f(x) = x \sin\left(\frac{1}{x}\right); (x \neq 0) \text{ and } f(0) = 0.$ 
  - b) Show that

is not continuous at (0,0)

c) Find the extrema of the function

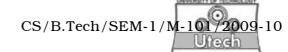
 $f(x, y) = x^3 + 3xy^2 - 3y^2 - 3x^2 + 4.$  5 + 5 + 5

9. a) Obtain a reduction formula for  $\int_{0}^{\pi/2} \sin^{n} x \, dx$  and

evaluate  $\int_{0}^{\pi/2} \sin^5 x \, dx.$ 

- b) If z = f(x, y) where  $x = e^u \cos v$ ,  $y = e^u \sin v$  then show that
- c) Prove that the function f(x) = |x-1|, 0 < x < 2, is continuous at x = 1, but not differentiable there. Is it continuous and derivable at x = 0? 5 + 5 + 5

11701



- 10. a) State Leibnitz's theorem for Alternating Series and test convergence of the series  $1 \frac{1}{2^2} + \frac{1}{3^2} \frac{1}{4^2} + \dots$ 
  - b) Define absolute and conditional convergence of Series. Also show that the series  $\sum_{n=1}^{\infty} \frac{\cos nx}{n^2}$  is absolutely convergent.
- 11. a) A particle moves on the curve  $x = 2t^2$ ,  $y = t^2 4t$ , z = 3t 5, where t is the time. Find the components of velocity and acceleration at time t = 1 in the direction  $\hat{i} 3\hat{j} + 2\hat{k}$ .
  - b) Find the angles between the lines whose direction cosines are given by the equations l+m+n=0 and  $l^2+m^2-n^2=0.$
  - c) Find the shortest distance between the lines

$$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$$
 and  $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ .

5 + 5 + 5



- Find the *n*-th derivative of  $y = (ax + b)^m$ 12. a) number.
  - Test the convergence of the series b)

$$1 + \frac{x}{2} + \frac{x^2}{5} + \frac{x^3}{10} + \dots$$

Find: c)

$$div\stackrel{\triangle}{F}$$
 and  $curl\stackrel{\triangle}{F}$  , where  $\stackrel{\triangle}{F}$  = grad (  $x^3 + y^3 + z^3 - 3xyz$  ) .

$$5 + 5 + 5$$

13. a) Find the whole length of the loop of the curve

$$9y^2 = (x-2)(x-5)^2$$
.

Evaluate b)

$$6 + 6 + 3$$