*Name* : .....

*Roll No.* : .....

*Invigilator's Signature* : .....



**2010**

# DIGITAL SIGNAL PROCESSING

*Time Allotted* : 3 Hours *Full Marks* : 70

*The figures in the margin indicate full marks.*
*Candidates are required to give their answers in their own words as far as practicable.*

## GROUP – A
## ( Multiple Choice Type Questions )

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) If $x_1(n)$ and $x_2(n)$ are finite length sequences of sizes $L$ and $M$ respectively, their linear convolution has the length

a) $L + M - 2$ b) $L + M - 1$

c) $L + M$ d) none of these.

ii) The digital systems in $y(n) = x\left(n^2\right)$ is

a) linear and causal

b) non-linear and causal

c) linear and non-causal

d) non-linear and non-causal.

iii) Zero padding of a signal

a) reduces aliasing

b) increases frequency

c) increases time resolution

d) has no effect.

iv) In a system $y(-n) = x(n-1)$

a) the system is causal for all

b) the system is linear and causal for all

c) the system is stable, linear and causal for all

d) none of these.

v) If $x[n] = \{1, 0, 0, 1\}$, the DFT value $X(0)$ is

a) 2 b) $1 + j$

c) 0 d) $1 - j$.

vi) The Fourier transform of an aperiodic discrete time sequence is

a) discrete and periodic function of frequency

b) discrete and aperiodic function of frequency

c) continuous and periodic function of frequency

d) continuous and aperiodic function of frequency.

vii) The convolution of $u[n]$ with $u[n-4]$ at $n = 5$ is

a) 5 b) 2

c) 1 d) 0.

viii) A digital filter is said to be IIR

a) if present output depends on previous output only

b) if system function $H(z)$ has one or more non-zero denominator co-efficients

c) if all the poles lie outside the unit circle

d) if system function has only zeros.

ix) FIR filter is

a) recursive and linear

b) non-recursive and linear

c) recursive and non-linear

d) none of these.

x) System function of digital filter expressed as $H(z) = b_k z^{-k}$ represents

a) IIR filter b) FIR filter

c) Butterworth filter d) Chebyshev filter.

xi) Stability criteria for a discrete time LTI system is

a) $h(n) > 1$ b) $h(n) < 1$

c) $h(n) = 1$ d) $h(n) = 0$.

xii) A digital filter has $h(n) = \{-3, -2, 0, 2, 3\}$ then it has

a) no linear phase

b) symmetric linear phase

c) anti-symmetric linear phase

d) none of these.

xiii) $\left(\frac{1}{2}\right)^n u(n)$ is

a) energy signal b) power signal

c) both (a) and (b) d) none of these.

xiv) The system $y(n) = x(n) + nx(n+1)$ is

a) linear time invariant

b) non-linear time invariant

c) linear time variant

d) none of these.

xv) The mapping from analog to digital domain in impulse invariant method is

a) one to many b) many to one

c) one to one d) none of these.

## GROUP – B

### ( Short Answer Type Questions )

Answer any *three* of the following. $3 \times 5 = 15$

2. Given the following transfer function $T(z)$ of a DSP system, write the difference equation :

$$T(z) = \frac{3 + 2z^{-1} + z^{-2}}{1 - 4z^{-1} + 5z^{-2}}.$$

3. If Fourier transform of $x(n)$ is $X(j\omega)$, then prove that

a) $x(-t) \leftrightarrow X(-j\omega)$.

b) $x(at) \leftrightarrow \frac{1}{|a|} X\left(\frac{j\omega}{a}\right)$.

4. State the properties of convergence for the $Z$-transform.

5. Apply bilinear transformation to $H(s) = \frac{2}{(s+1)(s+3)}$ with $T = 0{\cdot}1$ s.

6. Define phase delay and group delay.

The length of an FIR filter is 13. If the filter has a linear phase, show that

$$\sum_{n=0}^{\frac{M-1}{2}} h(n) \sin \omega(\tau - n) = 0.$$

## GROUP - C
## ( Long Answer Type Questions )

Answer any *three* of the following. $3 \times 15 = 45$

7. a) What is ROC ? State its properties.

   b) Find the system function and impulse response of the system described by

   $y(n) = x(n) + 2x(n-1) - 4x(n-2) + x(n-3)$

   c) Find the inverse $Z$-transform of

$X(z) = z\left(z^2 - 4z + 5\right) / (z-3)(z-2)(z-1) ; 2 < z < 3$

   d) Prove that an LTI system is BIBO stable if the ROC of system function includes the unit circle. $2 + 5 + 5 + 3$

8. a) Sketch the magnitude response of Butterworth LPF filter and derive an expression for order of such a filter.

   b) Design a digital Butterworth filter using the following specifications using Impulse Invariant method

   $0{\cdot}9 < H(j\omega) < 1$ for $0 < \omega < 0{\cdot}2$ pi

   $H(j\omega) < 0{\cdot}2$ for $0{\cdot}4$ pi $< \omega <$ pi

   c) What are the advantages and disadvantages of bilinear transformation ? $5 + 5 + 5$

9. a) Compute 8-point DFT of the sequence

   $x(n) = \{0{\cdot}5,\ 0{\cdot}5,\ 0{\cdot}5,\ 0{\cdot}5,\ 0,\ 0,\ 0,\ 0\}$ using any FFT algorithm.

b) Find the linear convolution using circular convolution for the two sequences

$x(n) = \{1, 2, -1, 2, 3, -2, -3, -1, 1, 1, 2, -1\}$

$h(n) = \{1, 2\}$

c) Compute the circular convolution of two sequences

$x(n) = \{1, 2, 0, 1\}$

$x(n) = \{2, 2, 1, 1\}$ 6 + 5 + 4

10. a) Obtain the mapping formula for the approximation of derivatives method using backward difference :

b) Determine $H(z)$ for a Butterworth filter satisfying the following constraints :

$$\sqrt{0.5} \leq \left| H\left(e^{j\omega}\right) \right| \leq 1, \; 0 \leq \omega \leq \pi/2$$

$$\left| H\left(e^{j\omega}\right) \right| \leq 0.2, \; 3\pi/4 \leq \omega \leq \pi$$

with $T = 1s$. Apply impulse invariant transformation.

6 + 9

11. Write short notes on any *three* of the following : 3 × 5

a) Gibbs phenomenon

b) CCS6713 architecture

c) IIR and FIR filters

d) Periodic and aperiodic signals.