

Name : .....

Roll No. : .....

Invigilator's Signature : .....

**CS/B.TECH(BME(N)/ECE(N)/EE(N)/EEE(N)/EIE(N)/  
ICE(N)/PWE(N) )/SEM-3/M(CS)-301/2011-12  
2011**

**NUMERICAL METHODS**

Time Allotted : 3 Hours

Full Marks : 70

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words  
as far as practicable.*

**GROUP – A**

**( Multiple Choice Type Questions )**

1. Choose the correct alternatives for any *ten* of the following :

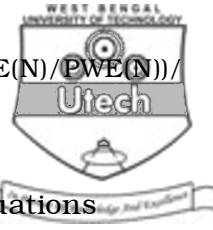
$$10 \times 1 = 10$$

i) Which of the following is not a computational error ?

- |                     |                    |
|---------------------|--------------------|
| a) Truncation error | b) Round-off error |
| c) Inherent error   | d) None of these.  |

ii) Newton-Raphson method fails when

- |                 |                   |
|-----------------|-------------------|
| a) $f'(x) = 1$  | b) $f'(x) = 0$    |
| c) $f'(x) = -1$ | d) $f''(x) = 0$ . |



iii) Finite difference method is used to solve

- a) a system of linear simultaneous equations
- b) a system of non-linear simultaneous equations
- c) partial differential equations
- d) non-linear equations.

iv) Regula-falsi method has a convergence rate of the order of

- a) 2
- b) 1.62
- c) 1
- d) none of these.

v) Gauss-Seidel method for solution of a system of linear simultaneous equations converges if

a)  $|a_{ii}| \geq \sum_{\substack{j=1 \\ j \neq i}}^n |a_{ij}|$

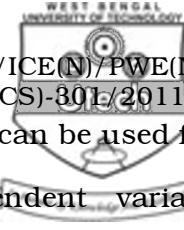
b)  $|a_{ii}| > \sum_{\substack{j=1 \\ j \neq i}}^n |a_{ij}|$

c)  $|a_{ii}| / |a_{nn}| = 1$

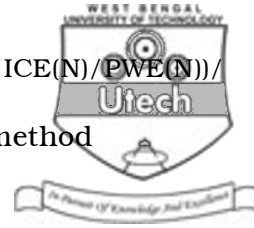
d) none of these.

vi) Modified Euler's method has a truncation error of the order of

- a)  $h$
- b)  $h^2$
- c)  $h^4$
- d)  $h^3$ .



- vii) Divided difference interpolation formula can be used for
- the tabular values with independent variable unequally spaced
  - inverse interpolation
  - both (a) and (b)
  - none of these.
- viii) Truncation error in Simpson's  $\frac{1}{3}$  rd rule is given by
- $\frac{b-a}{180} h^4 f^{iv}(\xi)$ ,  $a \leq \xi \leq b$
  - $\frac{b-a}{90} h^5 f^{iv}(\xi)$ ,  $a \leq \xi \leq b$
  - $\frac{b-a}{6} h^4 f'''(\xi)$ ,  $a \leq \xi \leq b$
  - $\frac{b-a}{90} h^4 f^{iv}(\xi)$ ,  $a \leq \xi \leq b$ .
- ix) Which of the following relations is true ?
- $E = 1 - \Delta$ ,  $\Delta - \square = \Delta \square$
  - $E = 1 - \Delta$ ,  $\Delta + \square = \Delta \square$
  - $E = 1 + \Delta$ ,  $\Delta + \square = \Delta \square$
  - $E = 1 + \Delta$ ,  $\Delta - \square = \Delta \square$ .
- x) Trapezoidal method can be used to integrate numerically a function represented in tabular form
- with odd number of intervals only
  - with even number of intervals only
  - both (a) and (b)
  - none of these.



xi) Condition of convergence for Euler's method

a)  $|1 + hf'(x_i, y_i)| < 1$

b)  $|1 + hf'(x_i, y_i)| \leq 1$

c)  $|1 + hf'(x_i, y_i)| > 1$

d)  $|1 + hf'(x_i, y_i)| \geq 1.$

xii) Milne's corrector formula is

a)  $y_{n+1} = y_n + \frac{h}{3} (y'_{n-1} + 4y'_n + 4y'_{n+1})$

b)  $y_{n+1} = y_{n-1} + \frac{h}{3} (y'_{n-1} + 4y'_n + 4y'_{n+1})$

c)  $y_{n+1} = y_n + \frac{4h}{3} (y'_{n-1} + 4y'_n + 4y'_{n+1})$

d) none of these.

### GROUP – B

#### ( Short Answer Type Questions )

Answer any *three* of the following.

3 × 5 = 15

2. Given the function  $y = \frac{1}{x}$ , show that the divided difference of  $n^{\text{th}}$  order

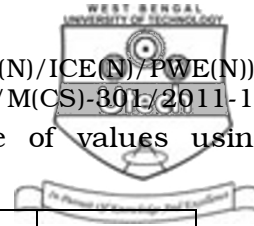
$$y[x_0, x_1, x_2, \dots, x_n] = (-1)^n / (x_0 x_1 x_2 \dots x_n)$$

3. Solve the following system of linear equations by Gauss-Seidel iterative method :

$$9x + 2y + 3z = -7$$

$$x - 6y + 2z = -2$$

$$x + y + 3z = 5.$$



4. Fit a polynomial to the following table of values using Lagrange interpolation formula :

|            |      |   |   |    |
|------------|------|---|---|----|
| <b>x :</b> | 0    | 1 | 3 | 4  |
| <b>y :</b> | - 12 | 0 | 6 | 12 |

Find the value of  $y$  when

- a)  $x = 2$   
b)  $x = 3.5$ .
5. Find the value of  $\frac{1}{23}$  using Newton-Raphson method. Result is required to be corrected up to 4 decimal places.
6. Solve the following equation using bisection method :

$$3x + \sin x - e^x = 0$$

Take  $x_0 = 1$  and  $x_1 = 0$ .

Result is required to be corrected up to 2 decimal places.

### GROUP – C

#### ( Long Answer Type Questions )

Answer any *three* of the following.  $3 \times 15 = 45$

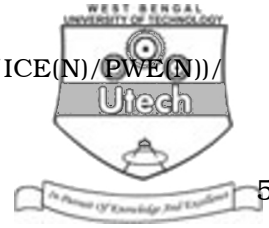
7. a) Derive the order of convergence for Newton-Raphson method. 5  
b) Solve the following initial value problem using Euler's method :

$$\frac{dy}{dx} = x^2 + y \text{ with } y(0) = 1.$$

Compute the first 5 steps of the solution with  $h = 0.1$ .

Compare the results ( % relative error ) with those obtained from the exact solution

$$y = 3e^x - x^2 - 2x - 2. \quad 10$$



8. a) Prove by the method of induction :

$$\Delta^m y_r = \square^m y_{r+m}.$$

5

- b) Use Newton's formula to find the area of a circle of diameter 98 cm.

5

|                             |      |      |      |      |      |
|-----------------------------|------|------|------|------|------|
| <b>D ( cm ) :</b>           | 80   | 85   | 90   | 95   | 100  |
| <b>A ( cm<sup>2</sup> )</b> | 5026 | 5674 | 6362 | 7088 | 7854 |

- c) Derive Lagrange interpolation formula. 5
9. a) Derive the expression for total truncation error associated with Simpson's  $\frac{1}{3}$  rd method. 8
- b) Evaluate the following integral using trapezoidal method :

$$I = \int_0^2 \left( 1 / (x^2 + 4) \right) dx.$$

Take  $h = 0.125$ . Hence obtain the value of  $\pi$ . 7

10. a) Solve the following system of equations using LU factorization method. 8

$$3x - y + 2z = 12$$

$$x + 2y + 3z = 11$$

$$2x - 2y - z = 2.$$

- b) Find the inverse of the following matrix : 7

$$A = \begin{bmatrix} 8 & -4 & 0 \\ -4 & 8 & -4 \\ 0 & -4 & 8 \end{bmatrix}.$$



11. a) Define  $\Delta$ ,  $\square$  and  $E$ . 5
- b) Derive Newton's Backward difference interpolation formula. 5
- c) Derive 4th order Runge-Kutta formula for solution of initial value problem of ordinary differential equation. 5
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