MAULANA ABUL KALAM AZAD UNIVERSITY OF TECHNOLOGY, WEST BENGAL

CS-503

DISCRETE MATHEMATICS

Time Allotted: 3 Hours Full Marks: 70

The questions are of equal value.
The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.

GROUP A
(Multiple Choice Type Questions)

1. Answer any *ten* questions. $10 \times 1 = 10$

(i) The truth value of the statement $x'^2 + 4 = 0$ hold for some real values of x' is

(A) true (B) false
(C) both (A) and (B) (D) none of these

(ii) For every integer x, gcd $(x, x+2) =$

(A) 0 (B) 2
(C) 1 (D) none of these

(iii) A poset S is a lattice if every pair of elements of it has

(A) greatest lower bound in S
(B) greatest lower bound and least upper bound in S
(C) greatest and least element in S
(D) maximal and minimal element in S

(iv) If p: 'Anil is rich' and q: 'Kanchan is poor' then the symbolic form of the statement 'Either Anil or Kanchan is rich' is

(A) $p \vee q$ (B) $p \vee \neg q$
(C) $\neg p \vee q$ (D) $\neg(p \wedge q)$

(v) The sequence represented by the function $\dfrac{1}{1-5x}$ is

(A) $\{3^n\}$ (B) $\{5^n\}$
(C) $\{5^n + 1\}$ (D) $\{4^n\}$

(vi) Solution of the recurrence relation $a_n = 2a_{n-1} + 1$ with $a_0 = 0$ is

(A) $1 - 2^n$ (B) $2^n - 2$
(C) $2^{n-1} - 1$ (D) $2^n - 1$

(vii) The number of ways a null graph having 4 vertices can be properly coloured with 5 colour is

(A) 256 (B) 1024
(C) 625 (D) 125

(viii) A two chromatic graph is

(A) a tree
(B) bi partite graph
(C) a cycle with odd number of vertices
(D) none of these

(ix) The number 9420544 is divisible by

(A) 36 (B) 28
(C) 24 (D) none of these

(x) If $7x \equiv 3 (mod\ 5)$, then x can take the value

(A) 17 (B) 19
(C) 21 (D) 22

(xi) If $ap = bq$ and a is prime to b then

(A) $a|p$ and $b|q$ (B) $a|b$ and $p|q$
(C) $a|q$ and $b|q$ (D) none of these

(xii) The number of positive divisors of 252 is

(A) 9 (B) 5 (C) 18 (D) 10

GROUP B
(Short Answer Type Questions)

Answer any *three* questions. $3\times5 = 15$

2. Show that the following pair of propositions are logically equivalent:
(i) $\sim((\sim p \wedge q) \vee (\sim p \wedge \sim q)) \vee (p \wedge q)$ and p.
(ii) $p \rightarrow (q \rightarrow r)$ and $(p \wedge q) \rightarrow r$.

3. Show that (D(24),|) represents a POset. Draw its corresponding Hasse Diagram.

4. Show that the inverse of an element n in $\mathbf{Z}_m$ will exist if and only if $\gcd(n, m) = 1$.

5. Use theory of congruence to prove that for $n \geq 1$, $17|(2^{3n+1} + 3.5^{2n+1})$.

6. Consider the family of finite sets $S = \{A_1, A_2, A_3, A_4\}$ where $A_1=\{a, b, d, e\}$; $A_2=\{b, c, d, e, f\}$; $A_3=\{c, f\}$; $A_4=\{b, c, f\}$. Show whether S satisfies the marriage condition. If yes, find two valid SDR of S.

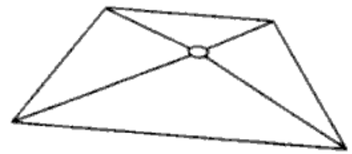
GROUP C
(Long Answer Type Questions)

Answer any *three* questions. $3\times15 = 45$

7. (a) Show that $((p \vee q) \wedge \sim(\sim p \wedge (\sim q \vee \sim r))) \vee (\sim p \wedge \sim q) \vee (\sim p \wedge \sim r)$ is a tautology. 5
(b) Let us consider the discrete mathematics class. If a student S_1 is late, then another student S_2 is late, and if both S_1 and S_2 are late, then the class becomes boring. Suppose that the class is not boring! What conclusion can be drawn about the student S_1 using truth table? 5
(c) Show that s is the valid conclusion from the premises $p \rightarrow \sim q$, $q \vee r$, $\sim s \rightarrow p$. 5

8. (a) State Pigeonhole Principal and solve : 5
A box contains 10 blue balls, 20 red balls, 8 green balls, 15 yellow balls and 25 white balls. How many balls must we choose to ensure that we have 12 balls of the same colour?
(b) Let P be a set of n points lying on the circumference of a circle such that if lines are drawn connecting every point to every other point, then no three of these lines intersect in a single point inside the circle. Let A be the set of all points of intersection of the lines in the interior of the circle. Note that, the points of P are not included in A. Find the cardinality of the set A. 10

9. Find the chromatic polynomial for the following graph : 5



(b) Let $G = (V, E)$ with $|V| = n$ be a connected graph. Let the maximum independent set of G be $\beta(G)$ and the chromatic number of G be $\chi(G)$. 5
Prove that $n \leq \beta(G)\chi(G)$. Use this result to show that $\beta(G) \geq \frac{n}{4}$ for a planar graph.
(c) Prove that the graph consisting of simply one circuit with $n \geq 3$ vertices is 2-Chromatic. 5

10. (a) Find the number of integers between 1 and 1000 both inclusive that are not divisible by any of the integers 2, 3 and 7. 6
(b) Find all possible value of x, for $345x \equiv 18 \pmod{912}$. 5
(c) Use the Mathematical Induction to prove that : 4
$A \cap (B_1 \cup B_2 \cup \ldots \cup B_n) = (A \cap B_1) \cup (A \cap B_2) \cup \ldots \cup (A \cap B_n)$.

11. (a) Obtain the conjunctive normal form of the following statement : 5
$$(p \rightarrow (q \wedge r)) \wedge (\sim p \rightarrow (\sim q \wedge \sim r))$$
(b) If $(L, \wedge, \vee)$ be a complemented distributed lattice, then prove that 5
$(a \vee b)' = a' \wedge b'$, $(a \wedge b)' = a' \vee b'$
(c) Find the number of integer's n, $1 \leq n \leq 1000$ that are not divisible by 5, 6 and 8. 5