Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/B.Tech (CSE/IT-OLD)/SEM-4/M-401/2013

2013

MATHEMATICS

Time Allotted : 3 Hours *Full Marks :* 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

   i) Product of two permutations is commutative

      a) True

      b) False.

   ii) A group contains 12 elements. Then the possible number of elements in a subgroup is

      a) 3 b) 5

      c) 7 d) 11.

   iii) A ring with zero divisors is called an integral domain

      a) True

      b) False.

   iv) The generators of the cyclic group $\{ 1, -1, i, -i \}$ with respect to usual multiplication is

      a) $\{ 1, -1 \}$ b) $\{ i, 1 \}$

      c) $\{ -1, -i \}$ d) $\{ i, -i \}$.

v) A vertex having no incident edge is called an isolated vertex

a) True

b) False

vi) If a is a generator of cyclic group then a^{-1} is also a generator of the group

a) True

b) False.

vii) A minimally connected graph is a

a) Binary tree b) Hamiltonian graph

c) Tree d) Regular graph.

viii) Tree contains at least

a) one vertex b) two vertex

c) three vertex d) four vertex.

ix) In the POset ($\left(Z^+, /\right)$, Z^+ represents set of all positive integers and / represents 'divides', which of the following pairs are not comparable ?

a) (4, 6) b) (5, 5)

c) (2, 4) d) (3, 15).

x) In a Boolean Algebra (B, + , . , ' , 0, 1), $a + 1 = 1$

a) True

b) False.

xi) Number of operations required in a Boolean Algebra is

a) 1 b) 2

c) 3 d) 4.

xii) The generating function of the following numeric function $\langle 1, 1, 1, \ldots\ldots \rangle$ is

a) $(1 + x)^{-1}$ b) $(1 - x)^{-1}$

c) $(1 - x)^2$ d) $(1 + x)^2$.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Show that the centre of a group G, given by $Z\,(G) = \{a \in G : ag = ga \;\forall\; g \in G\}$ is a normal subgroup of G.

3. Show that the ring of matrices of the form $\begin{bmatrix} 2\alpha & 0 \\ 0 & 2\beta \end{bmatrix}$, $\alpha, \beta \in Z$ contains divisors of zero. (Z = set of all integers and the operations are matrix addition and multiplication).

4. In a lattice (L, $\wedge$, $\vee$) prove that $a \wedge b = a$ if any only if $a \vee b = b$, $a, b \in L$.

5. Express $E = y' + z\left(x' + y\right)$ as a full disjunctive normal form.

6. Show that the maximum number of edges in a simple graph with n vertices is $\dfrac{n\,(n-1)}{2}$.

7. Prove that the maximum degree of any vertex in a simple graph with n vertices is ($n - 1$).

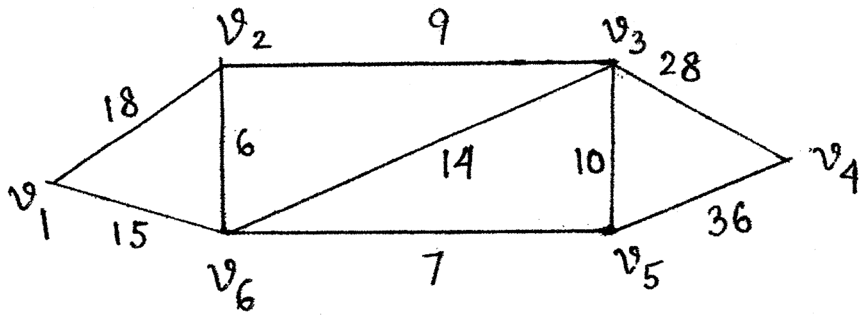
GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

8. a) Prove that in a simple graph with n (≥ 2) vertices must have at least one pair of vertices whose degrees are equal.

b) Applying Dijkstra's algorithm find the shortest path from the vertex v_1 to v_4 in the following simple graph :



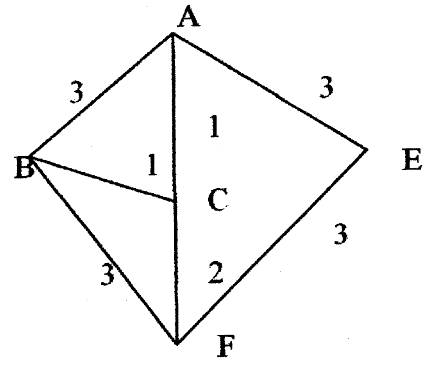
c) Draw the graph whose incidence matrix is :

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array}\begin{array}{c} \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

4 + 6 + 5

9. a) Prove that there exists no graph with four edges having vertices of degree 4, 3, 2, 1.

b) Find by Kruskal's algorithm a minimal spanning tree for the following graph :



c) If a simple regular graph has n vertices and 24 edges, find all possible values of n. 5 + 5 + 5

10. a) Prove that a ring R is commutative if and only if

$$(a+b)^2 = a^2 + 2ab + b^2 \quad \forall\, a, b \in R$$

b) If two operations $*$ and $\circ$ on the set Z of integers are defined as follows :

$$a * b = a + b - 1,\ a \circ b = a + b - ab$$

Prove that (Z, $*$, $\circ$)is a commutative ring with unit element.

c) Prove that (Z, +, •) is not an ideal of (Q, +, •) where + and • are usual addition and multiplication respectively.

[Q = set of all rational numbers]. 5 + 7 + 3

11. a) Show that "is congruent to" on the set of all triangles in a plane is an equivalence relation.

b) Let S be the set of all real $n \times n$ non-singular matrices A, with det $A = 1$ and G be the group of all $n \times n$ real non-singular matrices. Prove that $(S, \bullet)$ is a normal subgroup of $(G, \bullet)$ where $\bullet$ denotes matrix multiplication.

c) Let f be a homomorphism from a group G to $G^{\prime}$. Let $f (G)$ be the set of homomorphic images of G in $G^{\prime}$. Prove that $f (G)$ is a subgroup of $G^{\prime}$. 5 + 5 + 5

12. a) A light in a room is to be controlled by 3 switches located at three entrances. Design a simple series-parallel switching circuit, such that flicking any one of the switches will change the state of the light.

b) Construct the Boolean function and simplify it given the following table :

x	y	z	$f (x, y, z)$
1	1	1	0
1	1	0	1
1	0	1	1
1	0	0	0
0	1	1	0
0	1	0	1
0	0	1	1
0	0	0	0

c) Let $S = \{ 1, 2, 3, 4, 6, 8, 9, 12, 18, 24 \}$ be a set and '/' be a relation defined in S such that a/b mean b is divisible by a. Draw the Hasse diagram. 5 + 5 + 5

13. a) For any Boolean algebra B, prove that

$$(a + b)(b + c)(c + a) = ab + bc + ca \quad \forall \; a, b, c \in B.$$

b) Consider the lattice $L = \{ 1, 2, 3, 4, 6, 12 \}$ ordered by divisibility (/). Find the lower and upper bound of L. Is L a complemented lattice ?

c) Express the Boolean expression $z\,(x' \, y)'$ in a complete sum of product form. 5 + 5 + 5