MAULANA ABUL KALAM AZAD UNIVERSITY OF TECHNOLOGY, WEST BENGAL

Paper Code : BSM-202(N)

MATHEMATICS – II

Time Allotted : 3 Hours | *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) $\iint f(x, y)dxdy$ represents

a) Surface area b) Volume

c) Work done d) Arc length.

ii) the value of $\int_{-1}^{+1} P_{17}(x)P_{19}(x)$ is

a) 17 b) 19

c) $\frac{17!}{19!}$ d) 0.

iii) The number of independent solutions in a second order differential equation are

a) infinitely many b) 2

c) 1 d) 0.

iv) For the 2nd order ODE $y''(x)+xy'(x)-y=0$, the point $x=0$ is

a) ordinary point
b) isolated point
c) point of discontinuity
d) regular singular point.

v) $\lim_{z\to 0}\frac{\bar{z}}{z}=$

a) ∞ b) $-\infty$
c) 0 d) does not exist.

vi) The transformation $w=z^2$ maps the line $x=a$ to a

a) elliptical path b) hyperbolic path
c) parabolic path d) circular path.

vii) The complete primitive of the equation $y=px+\sqrt{a^2p^2+b^2}$, $p\equiv\frac{dy}{dx}$ is

a) $y=cx$ b) $y=cx-\sqrt{a^2c^2+b^2}$
c) $y=cx+\sqrt{a^2c^2+b^2}$ d) $y=\frac{cx}{\sqrt{a^2p^2+b^2}}$.

viii) $\oint\frac{\cos 3z}{z-5}dz$, over C, where $C: |z| = 5$ is

a) 1. b) 5
c) π d) 0.

ix) $P_n(-1)=$

a) n b) $n-1$
c) $(-1)^n$ d) 0.

x) The residue of $\frac{z^3}{(z-1)^4(z-2)(z-3)}$ at $z=1$ is

a) $\frac{101}{16}$ b) $\frac{101}{15}$
c) $\frac{101}{19}$ d) $\frac{101}{17}$.

xi) $J_{-\frac{1}{2}}(x) =$

a) $\sqrt{\frac{2}{\pi x}}\sin x$ b) $\sqrt{\frac{2}{\pi x}}\cos x$

c) $\sqrt{\frac{2x}{\pi}}\cos x$ d) $\sqrt{\frac{2x}{\pi}}\sin x$.

xii) $\int_2^3\int_4^6 dxdy = ?$ where $R = \{2 \le x \le 3 \ \& \ 4 \le y \le 6\}$

a) 1 b) 12

c) 2 d) None of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. 3 × 5 = 15

2. Solve $\frac{d^2y}{dx^2} + a^2y = \sin ax$, using D operator method.

3. Evaluate $\iint (x^2 + y^2)\,dxdy$, over the region bounded by $xy = 1$, $y = 0$, $y = x$ and $x = 2$.

4. Find the pole and residue of $f(z) = \frac{z^2}{z^2 + 9}$

5. Determine the analytic function $f(z) = u + iv$, whose imaginary part is $v = x^4 - 6x^2y^2 + y^4$.

6. Prove that $\int_{-1}^{+1} P_m(x)P_n(x)dx = 0, m \ne n$.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. 3 × 15 = 45

7. a) Evaluate $\int_0^a\int_0^{\sqrt{a^2-y^2}} (x^2 + y^2)dy\,dx$ changing to polar coordinates. 5

b) Evaluate $\oint \bar{F}.d\bar{r}$ by Stoke's theorem, where $\bar{F} = y^2\hat{i} + x^2\hat{j} - (x+z)\hat{k}$ where C is the boundary of the triangle with vertices (0, 0, 0), (1, 0, 0) and (1, 1, 0). 5

c) Find the general solution of $\frac{d^2y}{dx^2} + a^2y = \sec ax$ using the method of variation of parameters. 5

8. a) Find the power series solution of the differential equation $(1-x^2)\frac{d^2y}{dx^2} + 2y = 0, y(0) = 4, y'(0) = 5.$ 7

b) Prove that $P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n.$ 8

9. a) Solve $y = px + ap(1-p), p \equiv \frac{dy}{dx}.$ 5

b) Solve $(x^2D^2 + 4xD + 2)y = e^x.$ 5

c) Solve $\frac{dy}{dx} - \frac{dx}{dy} = \frac{x}{y} - \frac{y}{x}.$ 5

10. a) Show that if $w = f(z)$ is an analytic function of z, then

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\log|f'(z)| = 0.$$ 5

b) Show that the circle $|w| = 1$ corresponds to the circle $x^2 + y^2 \pm 2y - 1 = 0.$ 5

c) Expand the function $f(z) = \frac{z^2-1}{(z+2)(z+3)}$ when (i) $|z| < 2$, (ii) $2 < |z| < 3$, (iii) $|z| > 3.$ 5

11. a) Using the contour integration method, prove that $\int_0^\infty \frac{\sin t}{t}dt = \frac{\pi}{2}$ 8

b) Evaluate $\iiint (x + y + z + 1)^{-3}\, dxdydz$ over the tetrahedron bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $x + y + z = 1.$ 7