Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/B.Tech (CSE)/SEM-7/CS-704E/2010-11

2010-11
COMPUTATIONAL GEOMETRY

Time Allotted : 3 Hours *Full Marks :* 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for the following : $10 \times 1 = 10$

   i) A *conflict graph* is a

   a) line graph b) bipartite graph

   c) complete graph d) directed graph.

   ii) Let V, E and F are the number of vertices, edges and faces (including the infinite faces) respectively. Euler's formula says

   a) $V - E + F = 2$ b) $V + E = F$

   c) $V + E - F = 2$ d) $V - E - F = 2$.

iii) A simple polygon is a polygon

a) whose vertices are finite

b) with only one reflux angle

c) with only one convex angle

d) whose boundaries do not intersect.

iv) Triangulation means to

a) divide a region into triangles in respect to planner point location problem

b) prepare a triangular search window

c) partition a polygon into non-overlapping triangles using diagonals only

d) generate triangles in incremental hall algorithm.

v) Every triangulation of an n-gon has exactly ..... triangles.

a) $n - 2$ b) $n^2 - 2$

c) n d) n^2.

vi) In an art gallery with n vertices, necessary numbers of cameras are the

a) smallest integer not less than ($n/3$)

b) largest integer not greater than ($n/3$)

c) smallest integer not less than ($n/2$)

d) largest integer not greater than ($n/2$).

vii) *kd* tree is used in

a) line segment searching

b) point location searching

c) range searching

d) searching indexes.

viii) A Voronoi cell is the intersection of $n - 1$ ..... (n is the number of sites).

a) polylines b) contours

c) half-planes d) parabolas.

ix) In Voronoi diagram, equi-distant sites are

a) co-circular b) co-linear

c) co-triangular d) impossible to exist.

x) Chan's algorithm for computing a convex hull is

a) an output-sensitive algorithm

b) an input-sensitive algorithm

c) a neutral algorithm

d) a computation-sensitive algorithm.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. 3 × 5 = 15

2. Suppose we have a set of n points P in the plane such that exactly h of them lies on the convex hull. How many triangles are there in any triangulation of P ? Give your answer in terms of n and h.

3. Prove the theorem : Let P be a convex polytope with n vertices. The number of edges of P is at most $3n - 6$ and number of facets is at most $2n - 4$.

4. Let $P1$ and $P2$ be two disjoint convex polygons with n vertices in total. Describe an O (n log n)-time algorithm that computes the convex hull of $P1$, $P2$.

5. Given a set S on n sites, Vor (S) is the Voronoi diagram on S. Let V, E, F be the numbers of vertices, edges and faces respectively, of Vos (S). Then prove that, $F = n$ and $V \leq 2n - 5$.

6. Prove that : Every simple polygon admits a triangulation, and any triangulation of a simple polygon with n vertices consists of exactly $n - 2$ triangles.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. 3 × 15 = 45

7. a) Describe the beach line algorithm to construct a Voronoi diagram. 5

   b) Prove that the complexity of the beach line algorithm is $O(n)$. 5

   c) Discuss any two important applications of Voronoi diagram. 5

8. a) Give a set of data : 1, 3, 4, 7, 9, 12, 14, 15, 17, 20, 22, 24, 25, 27, 29 and 31.

      i) Construct a balanced binary tree 4

      ii) Describe how you search for a range [2 : 23]. 5

   b) "Let P be the set of n points in a 1-dimensional space. The set P can be stored in balanced binary search tree, which uses $O(n)$ storage and has $O(n \log n)$ construction time, such that the points in the query range can be reported in time $O(k + \log n)$, where k is the number of reported points." Justify the statement. 6

9. a) Write an algorithm that can construct a *kd*-tree in two dimensions. 5

b) Prove that a *kd*-tree for a set of *n*-points in two dimensions uses $O(n)$ storage and can be constructed in $O(n \log n)$ time. 5

c) Given the root of the *kd*-tree and a search range *R*. Write an algorithm to search the *kd*-tree. 5

10. a) In case of non-convex polygon, discuss the following methods of finding out whether a point lies within a polygon or not :

i) Computing winding numbers. 2

ii) Ray crossing. 3

b) Let *S* be a plan, subdivided by *n* edges. Describe a method to answer queries like : Given a query point *q*, report the face *f* of S that contains *q*. If *q* lies on an edge or coincides with a vertex, the query algorithm should return this information. 10

11. Write short notes on any *three* of the following : 3 × 5

a) Delaunay triangulations

b) Triangular approximations

c) Minimum spanning trees

d) *kd*-tree

e) Voronoi diagram

f) Convex hull.