Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/B.Tech(CSE/IT)/SEM-3/M-301/2009-10

2009

MATHEMATICS

Time Allotted : 3 Hours   Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1.   Choose the correct alternatives for any *ten* of the following :

$10 \times 1 = 10$

i)   If A and B are any two events such that $P(A \cap B) = \dfrac{1}{2}$, $P(A^C \cap B^C) = \dfrac{1}{3}$ and $P(A) = P(B) = p$, then the value of p is

a) $\dfrac{7}{12}$ b) $\dfrac{5}{6}$

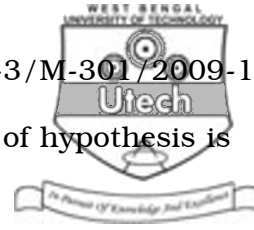c) $\dfrac{1}{3}$ d) $\dfrac{1}{2}$.

ii)   If the exponential distribution is given by the probability density function

$f(x) = e^{-x}$, $0 < x < \infty$,

then the mean of the distribution is

a)   1   b)   3

c) $\dfrac{1}{3}$ d)   4.

33801   [Turn over

iii) The variance of the binomial distribution $B(n, p)$ is

    a) np b) npq

    c) $(n-1)p$ d) $n(p-1)$.

iv) The mean of a Poisson distribution with parameter μ is

    a) μ b) μ^2

    c) $-\mu$ d) $-\mu^2$.

v) If A and B are any two events, then

    a) $P(A+B) = P(A) + P(B)$

    b) $P(A+B) = P(A) + P(B) - P(AB)$

    c) $P(A+B) = P(A) + 1$

    d) $P(A+B) = 1 - P(B)$.

vi) If A and B are any two events such that A is the subset of B, then

    a) $P(A) = P(B)$ b) $P(A) \le P(B)$

    c) $P(A) \ge P(B)$ d) $P(A) = P(B) = 1$.

vii) The value of $\lim\limits_{z \to \pi/2} \left(\dfrac{\sin z}{z} \right)$ is

    a) $\pi/2$ b) $2/\pi$

    c) π d) 1.

viii) If b_{yx}, b_{xy} be the regression coefficients and r be the correlation coefficient, then $b_{yx} \times b_{xy}$ is equal to

    a) r b) r^2

    c) $\dfrac{1}{r}$ d) $\dfrac{1}{r^2}$

ix) The power of a test in case of testing of hypothesis is

    a) $1 - P$ (Type I Error)

    b) $1 - P$ (Type II Error)

    c) $1 - P$ (Type I Error) P (Type II Error)

    d) P (Type I Error) P (Type II Error) .

x) The standard deviation of a sample mean for SRSWR is

    a) σ^2 / n b) $\sigma / \sqrt{n}$

    c) σ / n d) n.

xi) The maximum likelihood estimate is a solution of the equation

    a) $\dfrac{\partial\, L(\theta)}{\partial\theta} = 0$ b) $\dfrac{\partial\, L(\theta)}{\partial\theta} = \text{constant}$

    c) $\dfrac{\partial\, L(\theta)}{\partial\theta} = \theta$ d)   none of these.

xii) A statistic t is said to be an unbiased estimator of a population parameter θ when

    a) $E(t) = \theta$ b) $E(t^2) = \theta$

    c) $E(t^2) = [E(\theta)]^2$ d) $[E(t)]^2 = E(t^2)$.

xiii) The probability $P(a < x \le b)$ (where $F(x)$ is the distribution function of the random variable x) is given by

    a) $F(b) - F(a)$ b) $F(b) + F(a)$

    c) $F(a) - F(b)$ d) $F(a) F(b)$.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following.   3 × 5 = 15

2. If A and B are two events such that $P (A^C \cup B^C) = 5/6$, $P (A) = 1/2$ and $P (B^C) = 2/3$, show that A and B are independent.

3. A random variable X has the following probability function :

$X =$	– 2	– 1	0	1	2	3
$P (X) =$	0·1	k	0·2	$2k$	0·3	$3k$

   i)   Calculate k

   ii) Find $P (X < 2)$, $P (X \geq 2)$, $P (– 2 < X \geq 2)$.

4. If the chance of being killed by flood during a year is 1/3000, use Poisson distribution to calculate probability that out of 3000 persons living in a village, at least one will die in flood in a year.

5. A random sample with observations 65, 71, 64, 71, 70, 69, 64, 63, 67, 68 is drawn from a normal population with standard deviation $\sqrt{7\cdot056}$. Test the hypothesis that the population mean is 69 at 1% level of significance.

   [Given : $P (0 < Z < 2\cdot58) = 0\cdot495$].

6. If x follows a Normal Distribution with mean 12 and variance 16, find $P (x \geq 20)$.

   [Given : $\int_{-\infty}^{2} 1/\sqrt{2\pi} \, e^{-1/2 \, t^2} \, \mathrm{d}t = 0\cdot977725$]

7. Find the maximum likelihood estimate for the parameter λ of a Poisson distribution on the basis of a sample of size n.

GROUP – C
(Long Answer Type Questions)
Answer any *three* of the following.   3 × 15 = 45

8. a) A random variable follows binomial distribution with mean 4 and standard deviation $\sqrt{2}$. Find the probability of assuming non-zero value of the variable.   7

   b) Find the mathematical expectation of the number of the points obtained in a single throw of an unbiased die.   8

9. a) A bag contains 7 red and 5 white balls. 4 balls are drawn at random. What is the probability that (i) all of them are red (ii) two of them be white and two red ?   7

   b) If a random variable follows Poisson Distribution such that $P(1) = P(2)$,

   find   (i)   mean of the distribution

          (ii) $P(4)$.   8

10. a) State "Central Limit Theorem".

   A random variable x has the function e^{-x}, $x \geq 0$.

   Show that Tchebycheff's inequality gives $P[\ |X-1|\ > 2\ ] < \dfrac{1}{4}$ and show that actual probability is e^{-3}.   8

   b) If T is an unbiased estimator of θ, show that $\sqrt{T}$ is biased estimator of θ.   7

11. a) A pair of dice is thrown. Find the probability of getting a sum of 7, when it is known that the digit in the first die is greater than that of the second. 5

b) The manufacturing process of an article consists of two parts x and y. The probabilities of defect in parts x and y are 10% and 15% respectively. What is the probability that the assembled product will not have any defect ? 5

c) The probabilities of solving a problem by three students A, B and C are $\frac{2}{7}$, $\frac{3}{8}$ and $\frac{1}{2}$ respectively. If all of them try independently, find the probability that the problem is solved. 5

12. a) The probability density function of a continuous distribution is given by $f(x) = \frac{3}{4} x(2 - x)$. Compute mean and variance of the distribution. 5

b) The mean weight of 500 male students at a certain college is 150 lbs and the standard deviation is 15 lbs. Assuming that the weight is normally distributed find how many students weigh

i) between 120 and 155 lbs

ii) more than 155 lbs.

[Given $\phi(2) = 0.9772$; $\phi(0.33) = 0.6293$] 10

13. a) A survey of 320 families with 5 children each revealed the following distribution :

No. of boys	5	4	3	2	1	0
No. of girls	0	1	2	3	4	5
No. of families	14	56	110	88	40	12

Is the result consistent wtih the hypothesis that male and female births are equally probable ?

[Given : $X^2_{Tab,5\%} = 11·07$ at 5 degrees of freedom]   8

b) Intelligence tests on two groups of boys and girls gave the following results :

	Mean	SD	N
Boys	70	20	250
Girls	75	15	150

Is there any significant difference in the mean scores obtained by boys and girls ?   7