2013
DISCRETE MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

   i) Let G be a connected simple graph with 8 vertices such that no vertex in G has degree < 4. Let n be the number of edges in G. Then which one of the following statements must be FALSE ?

   a) $n = 15$ b) $n = 16$

   c) $n = 17$ d) $n = 19$.

   ii) The number of permutations of a set with k elements is

   a) $k!$ b) $(k-1)!$

   c) $(k+1)!$ d) none of these

iii) The generating function for the sequence $\{3, -3, 3, -3, \ldots\}$ is

a) $\frac{1}{1+x}$ b) $\frac{3}{1-x}$

c) $\frac{3}{(1-x)^2}$ d) $\frac{3}{1+x}$.

iv) In the Lattice { 1, 5, 25, 125 } with respect to the order relation "divisibility" the complement of 1 is

a) 1 b) 5

c) 25 d) 125.

v) The total number of ways in which a null graph with 5 vertices can be properly coloured is

a) 225 b) 243

c) 125 d) none of these.

vi) The non-empty finite poset has

a) at most one greatest element

b) either (a) or (b)

c) at most one least element

d) both (a) and (b).

http://www.makaut.com

vii) Let A be the set of all positive even integers. Let R be a relation defined on A as follows :

(x, y) is in R if and only if x does not divide y. Then which of the following is *True* ?

a) R is a partial ordering

b) R is neither symmetric nor anti-symmetric

c) R is symmetric but not transitive

d) R is transitive but not symmetric.

viii) For a perfect matching, the corresponding graph in a matching problem should be

a) a bipartite graph

b) a cycle having even number of vertices

c) a complete graph

d) none of these.

ix) $\sim(p \vee q) \vee (p \wedge \sim q) \equiv$

a) $\sim p$ b) p

c) $\sim q$ d) none of these.

x) A non-empty finite poset has

a) at most one greatest element

b) at most one least element

c) either (a) or (b)

d) both (a) and (b).

xii) In how many ways 7 different beads can be arranged to form a necklace ?

a) 250 b) 300

c) 360 d) 350.

xiii) The solution of the recurrence relation $a_n = 2a_{n-1} - 1$ with $a_0 = 0$ is

a) $1 - 2^n$ b) $2^n - 2$

c) $2^{n-1} - 1$ d) $2^n - 1$.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Write an equivalent formula for $p \wedge (q \leftrightarrow r) \vee (r \leftrightarrow p)$ which involves only the connectives (~, ∨).

3. Show that every tree with two or more vertices is 2-chromatic.

4. Find the remainder when the sum $1! + 2! + 3! + \ldots + 100!$ is divided by 5.

5. Assume that in a group of six people, each pair of individuals are either friends or enemies. Show that there are either three mutual friends or three mutual enemies.

6. Show that the following pair of propositions are logically equivalent :

i) $\sim ((\sim p \wedge q) \vee (\sim p \wedge \sim q) \vee (p \wedge q)$ and p.

ii) $p \rightarrow (q \rightarrow r)$ and $(p \wedge q) \rightarrow r$.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Using principle of inclusion and exclusion show that for any three sets A, B and C

$n(A \cup B \cup C) = n(A) + n(B) + n(C)$ if they are pair wise mutually disjoint.

b) Prove that the chromatic number of complete graph with n vertices ((K_n)) is n.

c) Solve, by characteristic root method, the recurrence relation $a_n = 3a_{n-1} + 4a_{n-2}$, $a_0 = 0$, $a_1 = 5$. Hence find a_8. 5 + 5 + 5

8. a) If a and b be integers, not both zero, then a and b are prime to each other if and only if there exist integers u and v such that $gcd(a, b) = 1 \Leftrightarrow au + bv = 1$ for some integers u, v.

b) Define sub-lattice of a lattice. Show with an example union of two sub-lattice may not be sub-lattice.

c) Let m ≥ 2 be a prime number. Let x be any integer that is not congruent to 0 (mod m). Show that there is a unique integer y < m such that $x.y \equiv 1$ (mod m).

5 + 5 + 5

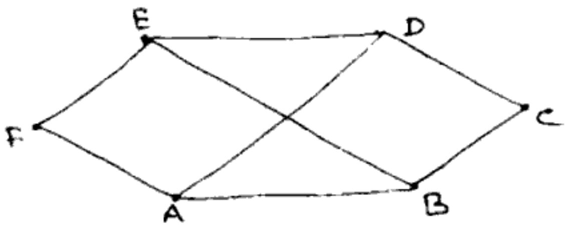
9. a) Let D be a square drawn in the plane with sides of length $\sqrt{2}$. Prove that in every set of 5 distinct points in D, there exist two points whose distance from one another is at most 1.

b) By mathematical induction, prove that $6^{n+2} + 7^{2n+1}$ is divisible by 43, for each natural number n.

c) Find the minimum number of students in a class to be sure that six of them are born in the same month.

5 + 5 + 5

10. a) Find the chromatic number of the following graph.



b) Applicant a_1, a_2, a_3 and a_4 apply for five posts p_1, p_2, p_3, p_4 and p_5. The applications are done as follows :

$a_1 \to \{p_1, p_2\}, a_2 \to \{p_1, p_3, p_5\}, a_3 \to \{p_1, p_2, p_3, p_5\}$ and $a_4 \to \{p_3, p_4\}$. Using graph theory find (i) whether there is any perfect matching of the set of applicants into the set of posts. If yes, find matching (ii) whether every applicant can be offered a single post.

c) Find the chromatic polynomial of a connected graph on three vertices.

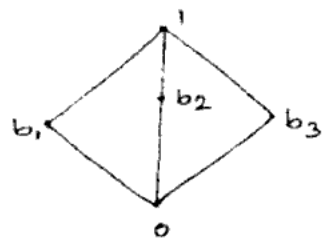
4 + 6 + 5

11. a) Let L_1 be a lattice D_6 (Divisor of 6) = { 1, 2, 3, 6 } and L_2 be a lattice (P (S), ≤) where S { a, b }. Show that two lattices are isomorphic.

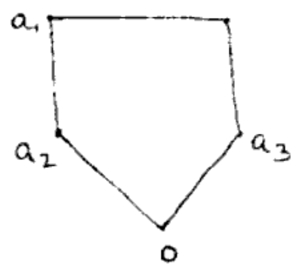
b) Prove that a poset has at most one greatest element and at most one least element.

c) Show that the lattice given in the diagrams are not distributive.

5 + 5 + 5



(a)



(b)