Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/B.Tech (OLD)/SEM-1/M-101/2011-12

2011

MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

   i) The series $\sum x^n = 1 + x + x^2 + ... \infty$ is convergent if

      a) $x \geq 1$ b) $x \leq -1$

      c) $-1 < x < 1$ d) none of these.

   ii) A positive term series cannot be oscillatory.

      a) True

      b) False.

   iii) The limit $\underset{x \to 0}{Lt} \; |x|/x$ does not exist.

      a) True

      b) False.

iv) If $y = x^n$, then y_n is equal to

a) 0 b) 1

c) $n\,!$ d) none of these.

v) The function $\log x$ is defined for

a) all values of x b) $x > 0$

c) $x < 0$ d) $x = 0$.

vi) If $I_n = \int_0^{\pi/2} \sin^n x \, dx = \{(n-1)/n\} I_n - 2$, the value of $\int_0^{\pi/2} \sin^6 x \, dx$ is

a) 0 b) $\frac{5\pi}{32}$

c) $\frac{5}{32}$ d) 1.

vii) If the arc is symmetrical about the x-axis, then for C.G. of the arc

a) $x = 0$ b) $y = 0$

c) $x = y$ d) none of these.

viii) $f(x, y) = \left(x^2 + y^2\right)^{1/3}$ is a homogeneous function of degree

a) $\frac{1}{3}$ b) $\frac{2}{3}$

c) 2 d) none of these.

ix) If $u = f(x, y)$, where $x = \varphi(t)$, $y = (\Psi)\, t$ then the total derivative is denoted by

a) $\frac{\partial u}{\partial t}$ b) $\frac{\partial x}{\partial t}$

c) $\frac{du}{dt}$ d) $\frac{\partial y}{\partial t}$.

x) If $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$, then the angle between the vectors $\vec{a}$ and $\vec{b}$ is

a) 90° b) 45°

c) 60° d) none of these.

xi) The centre and radius of the sphere $2x^2 + 2y^2 + 2z^2 - 2x + 4y + 2z + 3 = 0$ are

a) $(-1, 2, 1)$; 2 b) $(1, -2, -1)$; 1

c) $\left(\frac{1}{2}, -1, -\frac{1}{2}\right)$; 0 d) $\left(-\frac{1}{2}, 1, \frac{1}{2}\right)$; 3.

xii) The normal to the planes $x - y + z = 1$, $3x + 2y - z + 2 = 0$ are

a) parallel

b) inclined at angle $\frac{\pi}{3}$

c) passing through origin

d) perpendicular.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. If $y = \tan^{-1} x$, then prove that $\left(1+x^2\right) y_{n+1} + 2nx\, y_n + n\,(n-1)\, y_{n-1} = 0$. Find also the value of $(y_n)_0$.

3. Test the convergence of the series $1 + \frac{1}{2^2} + \frac{2^2}{3^3} + \frac{3^3}{4^4} + \frac{4^4}{5^5} + \ldots\ldots$.

4. Obtain the reduction formula for $\int_0^{\pi/2} \cos^n x \, dx$ and hence evaluate $\int_0^{\pi/2} \cos^7 x \, dx$.

5. Find the directional derivative of $p = 4e^{2x-y+z}$ at the point (1, 1, – 1) in a direction towards the point (– 3, 5, 6).

6. Find the points of intersection of the line $x + y + z + 1 = 0 = 14x + 9y - 7z - 1$ with XY and YZ planes and hence put down the symmetrical form of its equations.

7. Find a point in the plane $x + 2y + 3z = 13$ nearest to the point (1, 1, 1) using the method of Lagrange's multipliers.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

8. a) Examine if the function

$$f(x) = x \tan^{-1}\left(\frac{1}{x}\right), \quad x \neq 0$$

$$= 0 \qquad , x = 0$$

is derivable at $x = 0$.

b) State Leibnitz's theorem. If $y = e^{m \cos^{-1} x}$, show that

$$\left(1 - x^2\right) y_{n+2} - (2n+1)\, x\, y_{n+1} - \left(n^2 + m^2\right) y_n = 0 .$$

c) If $y^{1/m} + y^{-1/m} = 2x$, prove that

$$\left(x^2 - 1\right) y_{n+2} + (2n+1)\, x\, y_{n+1} + \left(n^2 - m^2\right) y_n = 0 .$$

9. a) State Rolle's thorem. Verify Rolle's theorem for the function $f(x) = \sin x \cos x$ in $[0, \pi/2]$.

b) Prove that

$$(\text{if } 0 < a < b < 1),\ (b-a)/\left(1+b^2\right) < \tan^{-1} b - \tan^{-1} a < (b-a)/\left(1+a^2\right)$$

c) Expand $\sin x$ in power of x in infinite series stating the condition under which the expansion is valid.

10. a) State divergence theorem of Gauss. Verify divergence theorem for $\vec{F} = y\vec{i} + x\vec{j} + z\vec{k}$ over the cylindrical region bounded by $x^2 + y^2 = 9,\ z = 0,\ z = 2$.

b) State Green's theorem in plane. Verify Green's theorem in plane for $\int_C \left(3x^2 - 8y^2\right) dx + (4y - 6xy)\, dy$, where C is the boundary of the region defined $y = x^{1/2},\ y = x^2$.

c) Find the work done in moving a particle in the force field $\vec{F} = 3x^2\vec{i} + (2xz - y)\vec{j} + z\vec{k}$ along the curve defined by $x^2 = 4y,\ 3x^3 = 4z$ from x = 0 to x = 2.

11. a) Show that $\vec{\nabla} r^n = nr^{n-2}\vec{r}$ where $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$.

b) Prove that *curl* [φ *grad* φ] = 0.

c) Find the co-ordinate of the foot of the perpendicular from (1, 2, 3)on the line (x – 2)/1 = (y – 1)/2 = z/3. Find also the length of the perpendicular and its equation.

12. a) Evaluate $\iint \left(4x^2 - y^2\right)^{1/2}\, dx\, dy$ over the triangle formed by the straight lines $y = 0$, $x = 1$ and $y = x$.

b) Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z}\, dz\, dy\, dx$.

c) If $f(x, y) = \left(x^2 + y^2\right)^{1/3}$, use Euler's theorem to find the value of $x\,(\partial f/\partial x) + y\,(\partial f/\partial x)$ and hence prove that,

$$x^2\left(\partial^2 f/\partial x^2\right) + 2xy\left(\partial^2 f/\partial x \partial y\right) + y^2\left(\partial^2 f/\partial y^2\right) + (2/9)\, f = 0.$$