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Invigilator's Signature:.....

CS/B.Tech (OLD)/SEM-2/M-201/2013 **2013 MATHEMATICS**

Time Allotted: 3 Hours Full Marks: 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP - A (Multiple Choice Type Questions)

1. Choose the correct alternatives for any ten of the following: $10 \times 1 = 10$

i)
$$\frac{1}{1-D} x^2 =$$

a)
$$x^2 + 2x + 1$$
 b) $x^2 + 2x$

b)
$$x^2 + 2x$$

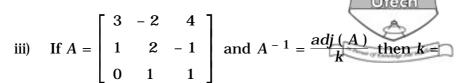
c)
$$x^2 - 2x + 1$$

c)
$$x^2 - 2x + 1$$
 d) $x^2 + 2x + 2$.

ii) The value of
$$\begin{bmatrix} 1 & 1 & -ac & bc \\ 1 & 1 & +ad & bd \\ 1 & 1 & +ae & be \end{bmatrix} =$$

d) none of these.

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a) 15

b) 16

c) 0

- d) 1.
- iv) The eigenvalues of $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ are
 - a) 1, 1

b) 6, 1

c) 1, 0

- d) 0, 6.
- v) If A is an orthogonal matrix then det (A) =
 - a) 1

b) - 1

c) ± 1

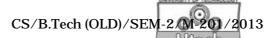
- d) 0.
- vi) Integrating factor of $x \frac{dy}{dx} + y = \log x$ is
 - a) e^{x}

b) *x*

c) $\log x$

d) none of these.

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- vii) Order and degree of the differential equation $\frac{d^2y}{dx^2} = \sqrt{1 + \frac{dy}{dx}} \text{ are }$
 - a) 2, 2

b) 2, 1

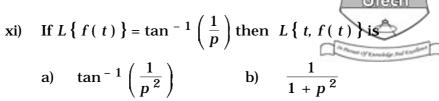
c) 1, 2

- d) 1, 1.
- viii) The Wronskian of the functions $\cos 2x$ and $\sin 2x$ is
 - a) 1

b) 2

c) - 2

- d) none of these.
- ix) The vectors (1, 1, 0, 0), (1, 0, 0, 1), (1, 0, a, 0),(0, 1, a, b) are linearly independent if
 - a) $a \neq 0$, $b \neq 2$
- b) $a \neq 2$, $b \neq 0$
- c) $a \neq 0, b \neq -2$
- d) $a \neq -2, b \neq 0.$
- x) $T: \mathbb{R}^2$ is defined by T(x, y) = (2x y, x + y) then kernel of T is
 - a) { (1, 2)}
- b) { (1, -1)}
- c) $\{(0,0)\}$
- d) $\{(1,2)\},\{(1,-1)\}.$



a)
$$\tan^{-1}\left(\frac{1}{p^2}\right)$$

b)
$$\frac{1}{1+p^2}$$

c)
$$\frac{1}{1+p}$$

c)
$$\frac{1}{1+p}$$
 d) $\tan^{-1}\left(\frac{2}{\pi p}\right)$.

xii) $(\Delta - \nabla) x^2$ is equal to

a)
$$h^2$$

b)
$$-2h^2$$

c)
$$2h^2$$

d) none of these,

where h is equal interval.

xiii) If E_a is the absolute error in a numerical calculation whose true and approximate values are \boldsymbol{X}_t and \boldsymbol{X}_a then the relative error is given by

a)
$$\left| \frac{E_a}{X_a} \right|$$
 b) $\left| \frac{E_a}{X_t} \right|$

b)
$$\frac{E_a}{X_t}$$

c)
$$\left| \frac{E_a}{X_t - X_a} \right|$$
 d) $\left| E_a \right|$.

d)
$$\mid E_a \mid$$

GROUP - B

(Short Answer Type Questions)

Answer any three of the following.

 $3 \times 5 = 15$

- Examine whether the transformation $T: \mathbb{R}^{2} \to \mathbb{R}^{2}$ defined 2. by T(x, y) = (2x - y, x) is linear or not.
- Apply convolution theorem to find Inverse Laplace Transform 3. of $\frac{s}{(s^2+9)^2}$.

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4. Prove (without expanding) that

$$\begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1+b & 1 & 1 \\ 1 & 1 & 1+c & 1 \\ 1 & 1 & 1 & 1+d \end{vmatrix} = abcd \left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right).$$

5. If P_n (x) is the Legendre's polynomial or Legendre's function of 1st kind then prove the following :

a)
$$(2n+1) xP_n = (n+1) P_{n+1} + nP_{n-1}$$

- b) $(2n+1)P_n = P_{n+1}^{\dagger} P_{n-1}^{\dagger}$ where P_{n+1}^{\dagger} denotes the derivative of P_{n+1}
- 6. Find the inverse of the matrix $A = \begin{bmatrix} 2 & -3 & 4 \\ 1 & 0 & 1 \\ 0 & -1 & 4 \end{bmatrix}$

and hence solve the system : 2x - 2y + 4z = -4

$$x + z = 0$$

$$4z - y = 2$$

7. Evaluate $\int_{0}^{1} \frac{dx}{1+x}$ using Trapezoidal Rule, taking four equal sub-intervals.

GROUP - C

(Long Answer Type Questions)

Answer any three of the following.



8. a) Solve the following differential equation by Variation of Parameter Method :

$$(D^2 + 1) y = \sec x \tan x$$

b) Solve the following system using Cramer's rule :

$$x + y + z = 1$$
$$ax + by + cz = r$$

$$a^2x + b^2y + c^2z = r^2$$

where $a \neq b \neq c$

c) Find the missing data from the following table :

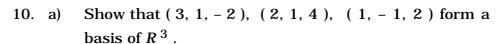
x:	- 2	- 1	0	1	2
y :	6	0	?	0	6

- 9. a) A linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^2$ is defined by T(x, y, z) = (x + y, x z). Find the Rank and Nullity.
 - b) From the following table, construct the difference table and compute f (19) by Newton's Backward Interpolation formula.

x:	0	5	10	15	20
f(x):	1.0	1.6	3.8	8.2	15.4

c)

Define the rank of a matrix. Find the rank of the matrix
$$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 6 & 7 & 8 \\ 2 & 6 & 9 & 12 & 15 \\ 4 & 8 & 12 & 14 & 16 \end{pmatrix}.$$



- Find the Laplace Transform of $f(t) = \sin t$, $0 < t < \pi$ b)
- $= 0 , t > \pi$ c) Apply Simpson's $\frac{1}{3}$ rd rule to evaluate $\int_{0}^{6} \frac{1}{(1+x)^{2}} dx$ taking six equal sub-intervals from [0, 6] and correct up to three decimal places.

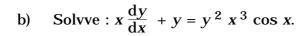
Sove $(D^3 - 2D^2 + D - 2)$ $y = e^x + e^{-3x}$ where 11. a) $D=\frac{\mathrm{d}}{\mathrm{d}\mathbf{v}}\,.$

- Solve: $\frac{dx}{dt} + 2x 3y = t$ b) $\frac{\mathrm{d}y}{\mathrm{d}t} - 3x + 2y = e^{2t}$
- c) Find the general solution and singular solution of

$$y = px + \sin^{-1} p$$
 where $p = \frac{dy}{dx}$.

following Cauchy-Euler 12. a) **Solve** the homogeneous differential equation:

$$(x^2 D^2 - 3xD + 4)$$
 $y = x^2$ given that $y(1) = 1$, $y'(1) = 0$ where $D = \frac{d}{dx}$.





c) Compute f(0.5) and f(0.9) from the following table :

x:	0	1	2	3
f(x):	1	2	11	34

13. a) Find the eigenvalues and eigenvectors of the matrix

$$\left(\begin{array}{ccc}
1 & -3 & 3 \\
3 & -5 & 3 \\
6 & -6 & 4
\end{array}\right).$$

Hence diagonalise the above matrix.

b) If $P_n(x)$ is Legendre's Polynomial then prove that

$$\int_{0}^{6} P_{n}(x) P_{m}(x) dx = 0 , m \neq n$$

$$= \frac{2}{2n+1} , m = n$$

OR

Find the Bessel's function, $J_n(x)$ of 1st kind from the Bessel's equation $x^2 \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \frac{\mathrm{d}y}{\mathrm{d}x} + \left(x^2 - n^2\right) y = 0$.

Hence prove that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x$.

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