Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/B.Tech(CSE/IT)(O)/SEM-3/M-301/2012-13

2012
MATHEMATICS

Time Allotted : 3 Hours *Full Marks :* 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

   i) For any two events A and B,

   $P(AB) \leq P(A) \leq P(A+B) \leq P(A) + P(B)$

   a) True b) False.

   ii) A random sample of two individuals is to be drawn from a population of size 43. The possible numbers of distinct samples under SRSWR is

   a) $43 \infty 43$ b) ${}^{43}C_2$

   c) ${}^{43}P_2$ d) none of these.

   iii) In random sampling with replacment from a population with s.d. σ, if the sample size is equal to the population size ($= N$), then the standard error of sample mean will be

   a) 0 b) σ

   c) $\frac{\sigma}{\sqrt{n}}$ d) none of these.

iv) The moment generating function of the normal distribution with parameters m and σ is

a) $e^{mt + t^2}$ b) $e^{mt + \frac{1}{2}\sigma^2 t^2}$

c) $e^{mt - \frac{1}{2}\sigma^2 t^2}$ d) none of these.

v) The expected value of the sample variance of size n drawn from a population with mean m and standard deviation σ is

a) σ^2 b) $n\sigma^2$

c) $\frac{n-1}{n}\sigma^2$ d) $\frac{\sigma^2}{n}$.

vi) If $F(x)$ is the distribution function of a random variable, then

a) $F(x)$ is continuous at all points

b) $F(x)$ is monotonic decreasing function

c) $F(-\bullet) = 1$

d) $F(+\bullet) = 1$.

vii) If $cov(x, y) = 12$, $\sigma_x = 5$ and $r_{xy} = 0{\cdot}6$, then the value of σ_y is

a) 16 b) 8

c) 2 d) 4.

viii) Three unbiased coins are tossed simultaneously. This is repeated four times. The probability of getting at least one head each time is

a) $\left(\frac{1}{8}\right)^4$ b) $\left(\frac{2}{8}\right)^4$

c) $\left(\frac{3}{8}\right)^4$ d) $\left(\frac{7}{8}\right)^4$.

ix) Let A and B the two events, the $P(A \cap B) \geq$

a) $P(A) + P(B)$ b) $P(A)$

c) $P(B)$ d) $P(A) + P(B) - 1$.

x) If X is a discrete random variable, then

a) $E(|X|) \leq |E(X)|$ b) $E(|X|) \geq |E(X)|$

c) $E(|X|) = |E(X)|$ d) none of these.

xi) If $A \subset B$ then $P(B - A)$ is equal to

a) $P(A) - P(B)$ b) $P(B) - P(A)$

c) $P(A) - P(AB)$ d) none of these.

xii) The value of k for which $f(x) = kx(1 - x)$, $0 < x < 1$

$= 0$, otherwise

will be the *p.d.f.* of a random variable X is

a) 6 b) 2

c) 1 d) 3.

xiii) The mean of the Poisson distributin is μ, then its standard deviation is

a) $\frac{1}{\sqrt{\mu}}$ b) $\sqrt{\mu}$

c) μ d) $\frac{1}{\mu}$.

xiv) The area under the standard normal curve beyond the lines $z = \pm 1 \cdot 96$ is

a) 95 per cent b) 90 per cent

c) 5 per cent d) 10 per cent.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. State and prove Bayes theorem.

3. If $r_{xy} = \pm 1$, then show that y is a linear function of x.

4. The following table gives the number of aircraft accidents that occurred during various days of by the week. Test whether the accidents are uniformly distributed over the week.

Day	**Sun**	**Mon**	**Tues**	**Wed.**	**Thurs**	**Fri**	**Sat**
No. of accidents	13	14	19	12	11	15	14

Given $\chi^2_{0\cdot5,\,6} = 12\cdot59$.

5. A box contains 5 defective and 10 non-defective lamps. 8 lamps are drawn at random in succession without replacement. What is the probability that the 8^{th} lamp is the 5^{th} defective ?

6. A sample of 600 screws is taken from a large consignment and 75 are found to be defective. Set up a 99% confidence interval for the proportion of the defectives in the population.

7. Use Tchebycheff's inequality to show that for $n \geq 36$, the probability that in n throws of a fair die the number of sixes lies between $\frac{1}{6}\ n - \sqrt{n}$ and $\frac{1}{6}\ n + \sqrt{n}$ is at least $\frac{31}{36}$.

8. If X is uniformly distributed over [1, 2], find K so that $P\,(\,X > K + m\,) = \frac{1}{6}$,

   where $m = E\,(\,X\,)$.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

9. a) If the random variable X has the *p.d.f.*

$f(x) = 3x,\ 0 < x < 1$

$= 0,$ elsewhere

then find the *p.d.f.* of $y = 4x + 3$.

b) A normal population has a mean $0 \cdot 1$ and standard deviation $2 \cdot 1$. Find the probability that the mean of a sample of size 900 will be negative. Given that $P(|Z| < 1 \cdot 43) = 0 \cdot 847$.

c) 100 unbiased coins are tossed. Using normal approximation to binomial distribution calculate the probability to get

i) exactly 40 heads

ii) 55 heads or more.

Given $\phi(2 \cdot 1) = 0 \cdot 9821$, $\phi(1 \cdot 9) = 0 \cdot 9713$, $\phi(0 \cdot 9) = 0 \cdot 8159$.

10. a) In a certain city 100 men in a sample of 400 were found to be smokes. In another city the number of smokes was 300 in a random sample of 800. Does this indicate that there is a greater proportion of smokers in the second city than in the first city ? 5

b) The marks obtained by 17 students is an examination have a mean 57 and variance 64. Find 99% confidence interval for the mean of the population of marks assuming it to be normal. [Given that $P(t > 3 \cdot 250) = 0 \cdot 005$ for 16 degree of freedom]. 5

c) The proportion of defective items in a large lot of items is p. To test the hypothesis $p = 0{\cdot}2$, we take a random sample of 3 items and accept the hypothesis. If the number of defectives in the sample is 6 or less, find the probability of Type-I error of the test. Find the probability of Type-II error if $p = 0{\cdot}3$. 5

11. a) The probability density function of a random variable X is given by

$$f(x) = \frac{3}{4}(2 - x)$$

Compute the mean and the variance of X. 5

b) The mean weight of 500 male students at a certain college is 150 lbs and the standard deviation is 15 lbs. Assuming that the weight are normally distributed find how many students weigh

i) between 120 and 155 lbs

ii) more than 155 lbs

[Given $\varphi(2) = 0{\cdot}9772$; $\varphi(0{\cdot}33) = 0{\cdot}6293$] 10

12. a) Find the regression coefficient of y on x, of x on y and the correlation coefficient between x and y from the following values :

$\sum xy = 1500$, $\bar{x} = 15$, $\bar{y} = 12$, $\sigma_x = 6{\cdot}4$, $\sigma_y = 9$ and number of observations is 10 where the notations have their usual meanings. 5

b) From the moment generating function of Normal Distribution $N(\mu, \sigma)$ with parameters μ and σ determine its mean and variance. 5

c) If X_1, X_2, X_3, X_4, X_5, X_6 be an independent simple random sample from a normal population with unknown variance σ^2 find K so that

$K\left[(X_1 - X_2)^2 + (X_3 - X_4)^2 + (X_5 - X_6)^2\right]$ is an unbiased estimation of σ^2. 5

13. a) Find the standard deviation from the following frequency distribution : 5

Salary	***No. of Workers***
up to Rs. 10	8
up to Rs. 20	24
up to Rs. 30	56
up to Rs. 40	95
up to Rs. 50	136
up to Rs. 60	178
up to Rs. 70	192
up to Rs. 80	200

b) Let X be a standard normal variate. Find the probability density function of y where $y = \frac{1}{2} X^2$. 5

c) If T is an unbiased estimator of the parameter θ, show by means of an example that $\sqrt{T}$ is a biased estimator of $\sqrt{\theta}$. 5