



Name :

Roll No. :

Invigilator's Signature :

CS/B.Tech(ECE,EE,EIE,EEE,PWE,BME,ICE)/SEM-3/M-302/2009-10

2009

MATHEMATICS

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :

$$10 \times 1 = 10$$

i) The probability that a leap-year selected at random will contain 53 sundays is

a) $\frac{3}{7}$

b) $\frac{2}{7}$

c) $\frac{5}{7}$

d) $\frac{4}{9}$.

ii) If a coin is tossed 6 times in succession, the probability of getting at least one head is

a) $\frac{63}{64}$

b) $\frac{3}{64}$

c) $\frac{7}{63}$

d) None of these.



- iii) The probability that the 4 children of a family have different birthdays is
- a) 0.9836 b) 0.4735
- c) 0.9 d) 0.757.
- iv) A tree has n vertices. The number of its edges is
- a) $n + 1$ b) $n - 1$
- c) $2n$ d) none of these.
- v) The value of m such that $3y - 5x^2 + my^2$ is a harmonic function is
- a) 5 b) -5
- c) 0 d) 3.
- vi) Let X and Y be two random variables such that $Y = a + bx$ where a and b are constants. Then, $\text{Var}(y)$ is
- a) $b^2 \text{Var}(X)$ b) $\text{Var}(X)$
- c) $a^2 \text{Var}(X)$ d) $(b/a) \text{Var}(X)$.
- vii) The value of $\int_C \frac{dz}{z+3}$ where C is a circle $|z| = 1$ is
- a) 0 b) 1
- c) 2 d) -1.
- viii) If $f(z) = \frac{1}{z^4 - 2z^3}$, then $z = 0$ is a pole of order
- a) 3 b) 2
- c) 1 d) 4.

- [Turn over



GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following.

$$3 \times 5 = 15$$

2. Show that $f(x)$ given by

$$f(x) = x; \quad 0 < x < 1$$

$$= k - x; \quad 1 < x < 2$$

$$= 0; \text{ elsewhere,}$$

is a probability density function for a suitable value of k .

Calculate the probability that the random variable lies between $\frac{1}{2}$ and $\frac{3}{2}$.

3. Find the Fourier sine transform of $\frac{e^{-ax}}{x}$.

4. Evaluate $\int_C \frac{3z^2 - 2}{z - 1} dz$, where C is the circle $|z| = \frac{1}{2}$.

5. An urn contains 3 white and 5 black balls. One ball is drawn and its colour is unnoted, kept aside and then another ball is drawn. What is the probability that it is (i) black (ii) white?

6. Find the mean and standard deviation of a binomial distribution.

**GROUP – C****(Long Answer Type Questions)**Answer any *three* of the following. $3 \times 15 = 45$

7. a) If A and B are mutually independent events, prove that A^C and B^C are also mutually independent events.
- b) There are three identical urns containing white and black balls. The first urn contains 3 white and 4 black balls, the 2nd urn contains 4 white and 5 black balls and the 3rd urn contains 2 white and 3 black balls. An urn is chosen at random and a ball is drawn from it. If the drawn ball is white, what is the probability that the 2nd urn chosen ?
- c) A random variable X has the following p.d.f.

$$f(x) = cx^2 \quad 0 \leq x \leq 1$$

$$= 0, \text{ otherwise.}$$

$$\text{Find (i) } c \text{ (ii) } P\left(0 \leq X \leq \frac{1}{2}\right).$$

$$5 + 5 + 5$$

8. a) Find the Fourier series expansion of the periodic function of period 2π ,

$$f(x) = x^2, \quad -\pi \leq x \leq \pi. \text{ Hence deduce}$$

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots = \frac{\pi^2}{12}.$$

- b) The following marks have been obtained by students in Mathematics and Statistics (out of 100) :

Maths	45	55	56	58	60	65	68	70	75	80	85
Stats	56	50	48	60	62	64	65	70	74	82	90

Compute the co-efficient of correlation for the above data. Find also the equations of the lines of regression.

$$7 + 8$$



9. a) Solve

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}, \quad x > 0, t > 0,$$

if $u(0, t) = 0$, $u(x, 0) = e^{-x}$, $x > 0$, $u(x, t)$ is unbounded.

b) If $f(z)$ is a regular function of z , then prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) |f(z)|^2 = 4 |f'(z)|^2. \quad 8 + 7$$

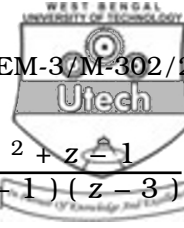
10. a) Apply *Dijkstra's algorithm* to determine a shortest path between s to z in the following graph :

Dia.

b) Define isomorphism of two graphs. Examine whether the following graph G and G' are isomorphic. Give reasons.

Dia.

8 + 7



11. a) Use residue theorem to evaluate $\int_C \frac{3z^2 + z - 1}{(z^2 + 1)(z - 3)} dz$

around the circle $|z| = 2$.

b) Expand the function $f(z) = \frac{1}{(z^2 + 1)(z + 2)}$ in the region $|z| < 1$.

c) Show that the function $f(z) = \begin{cases} \frac{3xy^2}{x^2 + y^2} & \text{for } z \neq 0 \\ 0 & \text{for } z = 0 \end{cases}$

is continuous at $z = 0$.

5 + 7 + 3

12. a) Show that a simple graph with n vertices and k -components can have at most $\frac{(n - k)(n - k + 1)}{2}$ edges.

b) Find the incidence matrix of the following graph.

Dia.

c) Find the Fourier sine transform of the function

$$f(x) = \begin{cases} 1 & \text{for } 0 < x \leq \pi \\ 0 & \text{for } x > \pi \end{cases}$$

and hence evaluate the integral

$$\int_0^\infty \frac{1 - \cos p\pi}{p} \sin pxdp.$$

5 + 5 + 5