

Name :

Roll No. :

Invigilator's Signature :

CS / B.Tech(ECE) / SEM-7 / EC-704B / 2012-13

2012

**ADVANCED ENGINEERING MATHEMATICS FOR
ELECTRONIC ENGINEERING**

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the
following : 10 × 1 = 10

i) If A be an orthogonal matrix then

a) $A^{-1} = A + A^T$

b) $A^{-1} = A^T$

c) A^{-1} does not exist

d) none of these.

$$\text{determinant} \left| \begin{array}{c|c|c} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & \omega & 1 \end{array} \right| \text{ is}$$

- a) $z \operatorname{Im}(z)$
 b) $(x^3 - 3xy^2) + i(3x^2y - y^3)$
 c) $iz + 2$
 d) $\operatorname{Re}(z)$.

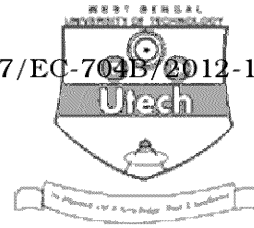
a) $2 \pi i e^3$
b) $-2 \pi i e^{-3}$
c) 0
d) none of these.

a) e^{at}

b) e^{-at}

c) $t e^{at}$

d) $t e^{-at}$.



vi) $L \left\{ e^{-2t} \sin t \right\}$ is

a) $\frac{1}{s^2 - 4s + 5}$

b) $\frac{s}{s^2 - 4s + 5}$

c) $\frac{1}{s^2 + 4s + 5}$

d) none of these.

vii) If Fourier transform of $f(x, y)$, $\mathcal{F}\{f(x, y) : x \rightarrow s\} = F(s, y)$ and $f(x, y) \rightarrow 0$ as $x \rightarrow \pm \infty$ then

a) $\mathcal{F} \left\{ \frac{\partial^n f}{\partial x^n} \right\} = (-is)^{-n} F(s, y)$

b) $\mathcal{F} \left\{ \frac{\partial^n f}{\partial x^n} \right\} = (-is)^n F(s, y)$

c) $\mathcal{F} \left\{ \frac{\partial^n f}{\partial x^n} \right\} = (-is)^{\frac{n}{2}} F(s, y)$

d) $\mathcal{F} \left\{ \frac{\partial^n f}{\partial x^n} \right\} = (-is)^{\frac{1}{n}} F(s, y).$

viii) If the trace of a 3×3 matrix is 12, and two of its eigenvalues are 4 and 6 then the third eigenvalue is

a) 2

b) 1

c) 8

d) 3.

ix) The value of a for which the rank of the matrix

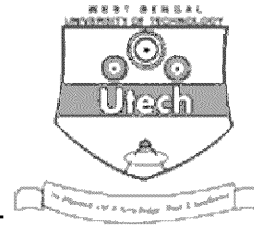
$$\begin{pmatrix} 2 & 0 & 1 \\ 5 & a & 3 \\ 0 & 3 & 1 \end{pmatrix} \text{ is less than 3 is}$$

a) $\frac{3}{4}$

b) $\frac{3}{5}$

c) $\frac{3}{2}$

d) none of these.



x) $J_{\frac{1}{2}}(x)$ is given by

a) $\sqrt{\frac{2\pi}{x}} \sin x$

b) $\sqrt{\frac{2\pi}{x}} \cos x$

c) $\sqrt{\frac{\pi}{2x}} \cos x$

d) $\sqrt{\frac{2}{\pi x}} \sin x$.

xi) The pde $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ is of type

a) elliptic

b) parabolic

c) hyperbolic

d) none of these.

xii) The order of the differential equation

$$\frac{\partial^2 x}{\partial t^2} + 2\left(\frac{\partial x}{\partial t}\right)^3 \left(\frac{\partial y}{\partial t}\right)^2 + t^3 \left(\frac{\partial y}{\partial t}\right)^3 = (\log t)^4$$

is

a) 0

b) 2

c) 3

d) None of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following.

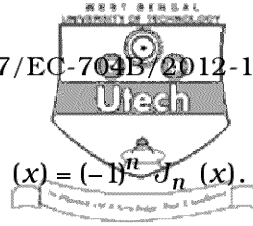
3 × 5 = 15

2. If $A(\theta) = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$, prove that

$$A(\theta) A(\varphi) = A(\varphi) A(\theta) = A(\theta + \varphi).$$

3. Using Cauchy's integral formula evaluate $\int_C \frac{z}{z^2 - 1} dz$,

where C is the circle $|z| = 2$.



4. When n is a positive integer, show that $J_{-n}(x) = (-1)^n J_n(x)$.

5. Evaluate $L^{-1}\left\{\frac{1}{s^2(s^2 + a^2)}\right\}$ using convolution theorem.

6. Obtain the general solution of the following linear PDE :

$$(mz - nz) \frac{\partial z}{\partial x} + (nx - lz) \frac{\partial z}{\partial y} = ly - mx.$$

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Show that the function $f(x) = e^{-\frac{x^2}{2}}$ is self reciprocal w.r.t. symmetric definition of Fourier transform.

b) Determine the temperature $u(x, t)$ in a bar of infinite length $(-\infty < x < \infty)$ from the governing equation

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}, \quad x > 0, \quad t > 0$$

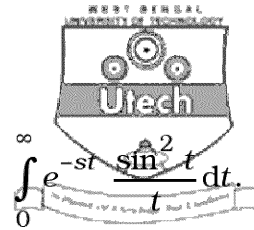
subject to the conditions :

(i) $u(0, t) = 0$, for $t > 0$

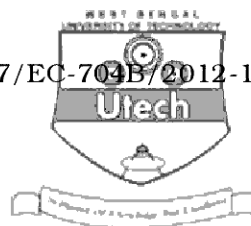
(ii) $u(x, 0) = \begin{cases} 1, & 0 < x < 1 \\ 0, & x \geq 1 \end{cases}$

(iii) $u(x, t)$ is bounded.

7 + 8



8. a) Find $L \left\{ \frac{\sin^2 t}{t} \right\}$ and hence evaluate $\int_0^{\infty} e^{-st} \frac{\sin^2 t}{t} dt$.
- b) Solve the PDE : $(D^3 + D^2 D' - D D'^2 - D'^3) Z = e^x \cos 2y$,
where $\frac{\partial^r}{\partial x^r} = D^r$ and $\frac{\partial^r}{\partial y^r} = D'^r$
- c) Find the bilinear transformation which maps the points $z = 1, 0, -1$ on the points $w = i, 0, -i$ and find the fixed points of transformation. (3 + 2) + 5 + (4 + 1)
9. a) State Cauchy's Residue theorem. Use it to evaluate $\int_0^{2\pi} \frac{d\theta}{5 + 4 \cos \theta}$
- b) Expand $f(z) = \frac{z+1}{(z-3)(z-4)}$ in a Laurent's series for the region $2 < |z| < 4$. Find also the region of convergence. (2 + 5) + (5 + 3)
10. a) Prove that $J_n'(x) = \frac{1}{2} [J_{n-1}(x) - J_{n+1}(x)]$, where $J_n(x)$ is Bessel's Function of order n .
- b) Prove that $\int_{-1}^1 [P_n(x)]^2 dx = \frac{2}{2n+1}$, where $P_n(x)$ is Legendre's polynomial of order n .
- c) Express $P(x) = x^4 + 2x^3 + 2x^2 - x - 3$ in terms of Legendre's polynomials. 4 + 6 + 5



11. a) Diagonalise the matrix $\begin{pmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{pmatrix}$.

b) Find the rank of the following matrix :

$$\begin{pmatrix} 1 & 3 & 2 & 4 & 1 \\ 0 & 0 & 2 & 2 & 0 \\ 2 & 6 & 2 & 6 & 2 \\ 3 & 9 & 1 & 10 & 6 \end{pmatrix}.$$

c) Solve the following partial differential equation :

$$(p^2 + q^2) y = qu.$$

7 + 4 + 4

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