Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/B.TECH(NEW)/SEM-2/M-201/2012

2012

MATHEMATICS – II

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) The integrating factor of

$(2xy - 3y^3)\,dx + (4x^2 + 6xy^2)\,dy = 0$ is

a) x^2y b) x^2y^2

c) xy^2 d) xy^3.

ii) The substitution $x = e^z$ transforms the differential equation $x^2 \dfrac{d^2y}{dx^2} - 5y = \log_e x$ to

a) $\dfrac{d^2y}{dz^2} + \dfrac{dy}{dz} - 5y = z$ b) $\dfrac{d^2y}{dz^2} - \dfrac{dy}{dz} + 5y = z$

c) $\dfrac{d^2y}{dz^2} - \dfrac{dy}{dz} + 3y = 0$ d) $\dfrac{d^2y}{dz^2} - \dfrac{dy}{dz} - 5y = z$.

iii) If the differential equation

$$\left(y+\frac{1}{x}+\frac{1}{x^2y}\right)dx+\left(x-\frac{1}{y}+\frac{A}{xy^2}\right)dy=0$$

is exact, then the value of A is

a) 2 b) 1

c) -1 d) 0.

iv) The value of $\int_0^\infty e^{-x}x^{\frac{3}{2}}\,dx$ is

a) $\frac{3}{4}\sqrt{\pi}$ b) $\frac{5}{4}\sqrt{5}$

c) $\frac{3}{5}\sqrt{\pi}$ d) $\frac{1}{4}\sqrt{\pi}$.

v) The value of $\Gamma(6)$ is

a) 720 b) 5

c) 6 d) 120.

vi) The Laplase transform of $e^{-3t}\sin 4t$ is

a) $\frac{4}{s^2+6s-7}$ b) $\frac{s}{s^2+6s-7}$

c) $\frac{1}{s^2+6s-7}$ d) $\frac{s}{s^2+6s+24}$.

vii) The maximum number of edges in a connected graph of 7 vertices is

a) 6 b) 7

c) 21 d) 14.

viii) The maximum degree of any vertex in a simple graph with 10 vertices is

a) 10 b) 5

c) 20 d) 9.

ix) Tree is a connected graph without any

a) odd vertex b) even vertex

c) circuit d) pendent vertex.

x) The improper integral $\int_0^1 \frac{dx}{(x-a)^n}$ converges for

a) $n < 1$ b) $n \geq 1$

c) $n > 1$ d) none of these.

xi) The particular integral of $(D^2 - 4D + 4)\, y = x^3 e^{2x}$ is

a) $\frac{e^{2x}x^4}{20}$ b) $\frac{e^{2x}x^5}{20}$

c) $\frac{e^{2x}x^4}{60}$ d) $\frac{e^{x}x^5}{20}$.

xii) The inverse Laplace transform of $\left(\frac{4}{s^2-7} + \frac{2}{s^2+7}\right)$ is

a) $\frac{1}{7}\left\{4\cos\left(\sqrt{7}t\right) - 2\sin\left(\sqrt{7}t\right)\right\}$

b) $\frac{1}{7}\left\{4\cos\left(7t\right) + 2\sin\left(7t\right)\right\}$

c) $\frac{1}{\sqrt{7}}\left\{4\sin h\left(\sqrt{7}t\right) + 2\sin h\left(\sqrt{7}t\right)\right\}$

d) $\frac{1}{7}\left\{4\sin h\left(7t\right) - 2\sin\left(7t\right)\right\}$.

xiii) The general solution of $p = \log_e (px - y)$ is

a) $y = cx - c$ b) $y = cx - e^c$

c) $y = c^2x - e^{-c}$ d) none of these.

xiv) The differential equation $\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$ can be reduced to the linear equation

a) $\frac{dz}{dx} + x \sin 2y = x^3$ b) $\frac{dz}{dx} + 2xz = x^3$

c) $\frac{dz}{dx} - 2xz = x^3$ d) none of these.

GROUP – B

(Short Answer Type Questions)

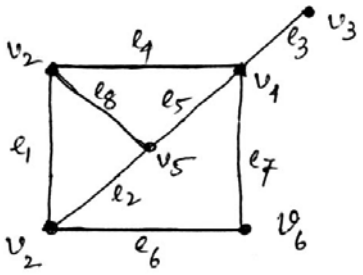
Answer any *three* of the following. 3 × 5 = 15

2. Solve : $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = \log_e x \sin(\log_e x)$

3. Evaluate : $L^{-1}\left(\frac{s+4}{s(s-1)(s^2+4)}\right)$

4. Use Beta and Gamma functions to evaluate $\int_0^{\frac{\pi}{2}} \sqrt{\tan x}\, dx$.

5. Determine adjacency matrix of the following graph :



6. Solve : $\frac{dx}{dt} + 3x + y = e^t$, $\frac{dy}{dt} - x + y = e^{2t}$.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Solve the following differential equation using Laplace transform :

$(D^2 + 2D + 5)\, y = e^{-t} \sin t,\ y(0) = 0,\ y'(0) = 1$

b) Apply the variation of parameters to solve

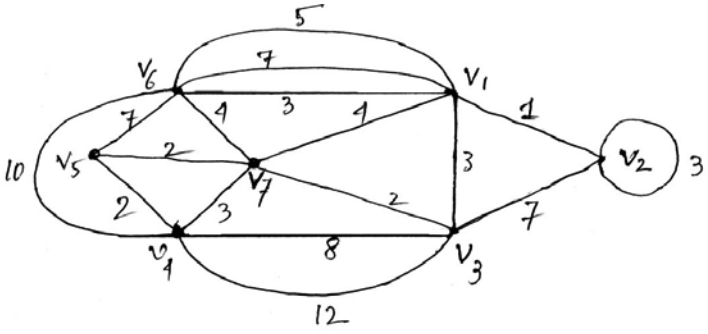
$$\frac{d^2y}{dx^2} + y = \sec^3 x . \tan x$$

c) Show that $\int_0^\infty e^{-4x} x^{\frac{3}{2}}\, dx = \frac{3}{128}\sqrt{\pi}$ 5 + 5 + 5

8. a) Draw the graph whose incidence matrix is,

$$\begin{pmatrix} 0 & 0 & 1 & -1 & 1 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 & 0 \end{pmatrix}$$

b) By Dijkstra's procedure, find the shortest path and the length of the shortest path from the vertex V_2 to V_5 in the following graph :



c) Solve : $y = 2px - p^2$ 5 + 5 + 5

9. a) Examine whether the differential equation

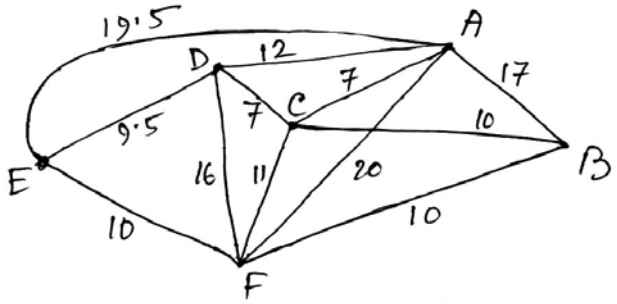
$$\left(xy^2 - e^{\frac{1}{x^3}} \right) dx - x^2 y \, dy = 0$$

is exact or not and then solve it.

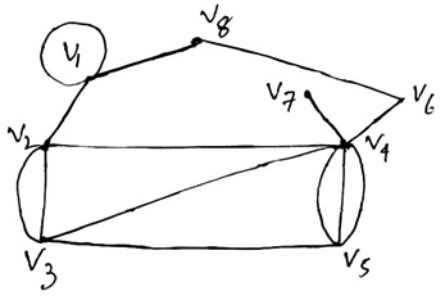
b) Prove that a complete graph of n vertices has $\frac{n(n-1)}{2}$ number of edges.

c) Apply convolution theorem to evaluate $L^{-1}\left(\frac{1}{(s^2+2s+5)^2} \right)$. 5 + 5 + 5

10. a) By Kruskal's Algorithm, find a minimal (or shortest) spanning tree and the corresponding weight of the spanning tree in the following graph :



b) Find by BFS algorithm a spanning tree in the following graph :

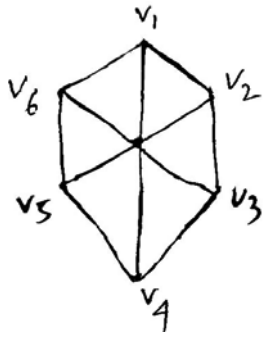


c) Examine the convergence of the improper integral

$$\int_0^2 \frac{dx}{x(2-x)}$$

5 + 5 + 5

11. a) Define complement of a graph. Find the complement of the graph.



b) Solve : $x^2 \frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = e^x$

c) Prove that in a binary tree with n vertices, the number of internal vertices is one less than the number of pendant vertices.

5 + 5 + 5

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