Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/B.Tech (CE-OLD)/SEM-3/CE-301/2011-12

2011

MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

   i) If A_1 and A_2 are mutually exclusive events then

      a) $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1) \cap P(A_2)$

      b) $P(A_1 \cup A_2) = P(A_1) + P(A_2)$

      c) $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$

      d) $P(A_1 \cup A_2) = P(A_1)\ P(A_2)$.

   ii) cos ($5x$) is a periodic function with the period

      a) 2π b) π

      c) $\frac{2\pi}{5}$ d) none of these.

iii) If $F(p)$ is the Fourier transform of $f(x)$, then the Fourier transform of $f(x-a)$ is

a) $F(p-a)$

b) $F(p+a)$

c) $F(p).e^{ipa}$

d) $F(p).e^{-ipa}$.

iv) Fourier cosine transform of e^{-x} is

a) $\frac{1}{1+s^2}$

b) $\frac{1}{1-s^2}$

c) $\frac{s}{1+s^2}$

d) $\frac{s}{1-s^2}$.

v) If $P(A)=\frac{1}{3}$, $P(B)=\frac{3}{5}$ and A and B are mutually exclusive events, then $P(A \cup B)$ is equal to

a) $\frac{14}{15}$ b) $\frac{11}{15}$

c) 0 d) $\frac{1}{5}$.

vi) If $\overline{A}$ is the complementary event of A, then $P(A)+P(\overline{A})$ is equal to

a) 0 b) 1

c) 2 d) none of these.

vii) The partial differential equation for $Z = f\left(\frac{xy}{z}\right)$ is

(Notations have their usual meanings)

a) $\frac{p}{q} = \frac{y}{x}$ b) $xyz = pq$

c) $pq = z$ d) none of these.

viii) If A and B are independent events, $P(B) = 0{\cdot}14$ and $P(A|B) = 0{\cdot}24$, then the value of $P(A)$ is

a) 0·14 b) 0·0336

c) 0·38 d) 0·24.

ix) The mean of an exponential distribution with parameter α ($\alpha > 0$) is

a) $\frac{1}{\alpha^3}$ b) $\frac{1}{\alpha^2}$

c) $\frac{1}{\alpha}$ d) α.

x) If F be a distribution function of a random variable X, then

a) $\lim_{x \to \infty} F(x) = 0$

b) $\lim_{x \to \infty} F(x) = 1$

c) $\lim_{x \to \infty} F(x) = \infty$

d) $\lim_{x \to \infty} F(x) = -\infty$.

xi) The partial differential equation $\frac{\partial u}{\partial t} = c^2 . \frac{\partial^2 u}{\partial x^2}$, c^2 being constant, is known as

a) one dimensional wave equation

b) one dimensional heat-flow

c) two dimensional heat-flow equation

d) none of these.

xii) For the random experiment of tossing two coins if X be the random variable such that X (an outcome) = "the number of heads", then the spectrum of X is

a) $\{ 0, 1, 2 \}$ b) $\{ 1, 2, 3 \}$

c) $\{ 0, 1 \}$ d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Using Parseval's identities, prove that $\int_0^\infty \frac{dt}{(a^2+t^2)(b^2+t^2)} = \frac{\pi}{2ab(a+b)}$.

3. Form the partial differential equation by eliminating the arbitrary functions from $f(x^2+y^2, z-xy) = 0$.

4. Solve two dimensional Laplace's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ by separation of variables.

5. An integer is chosen at random from the first 100 positive integers. What is the probability that the integer is divisible by 6 or 8 ?

6. Expand the function $f(x) = x \sin(x)$ as a Fourier Series in $[-\pi, \pi]$.

7. Find the Fourier integral representation of the function $f(x) = e^{-x}$ when $x > 0$ with $f(-x) = f(x)$. Hence evaluate $\int_0^\infty \frac{\cos(\lambda x)}{1+\lambda^2} d\lambda$.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

8. a) What is the Fourier expansion of the periodic function

$$f(x) = \begin{cases} 0 & \text{when } -\pi < x < 0 \\ \sin(x) & \text{when } 0 \le x < \pi \end{cases}$$

Hence evaluate $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \dots\dots\dots$

b) State giving reasons whether the following functions can be expanded in Fourier Series in the interval $-\pi \le x < \pi$:

i) cosec (x)

ii) $\sin\left(\frac{1}{x}\right)$. 10 + 5

9. Derive one dimensional wave equation for vibrating string and solve it using the method of separation of variables.

10 + 5

10. a) From the Fourier Series expansion of $f(x) = x^2$ in $-\pi < x < \pi$ prove that $\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$.

b) Find the Fourier sine transform of $f(x) = \frac{1}{x} . e^{-ax}$.

10 + 5

11. a) The distribution function $F(x)$ of a variate X is defined as follows :

$F(x) = A, -\infty < x < -1$

$= B, -1 \le x < 0$

$= C, 0 \le x < 2$

$= D, 2 \le x < \infty$.

b) Solve the two dimensional heat equation, using Fourier transform :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \; 0 < x < \pi, \; 0 < y < \pi .$$

$u(0, y) = u(\pi, y) = 0$ for $0 < y < \pi$ and $u(x, 0) = 0$ and $u(x, \pi) = u_0$ for $0 < x < \pi$. 10 + 5

12. a) If A and B are independent events then show that the following pairs are independent :

i) $\overline{A}$ and $\overline{B}$

ii) $\overline{A}$ and B.

b) Show by Tchebycheff's inequality that in 2000 throws with a coin the probability that the number of heads lies between 900 and 1100 is at least $\frac{19}{20}$. 10 + 5