



Name : .....

Roll No. : .....

Invigilator's Signature : .....

**CS/B.Tech (CSE/IT)/SEM-4/M-401/2010**

**2010**

**MATHEMATICS**

Time Allotted : 3 Hours

Full Marks : 70

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words  
as far as practicable.*

**GROUP – A**

**( Multiple Choice Type Questions )**

1. Choose the correct alternatives for any *ten* of the following :

$$10 \times 1 = 10$$

i) The generators of the cyclic group  $(\mathbb{Z}, +)$  are

- |            |              |
|------------|--------------|
| a) $1, -1$ | b) $0, 1$    |
| c) $0, -1$ | d) $2, -2$ . |

ii) The mapping  $f: \mathbb{R} \rightarrow \mathbb{R}$  given by  $f(x) = |x|$ ,  $x \in \mathbb{R}$  is

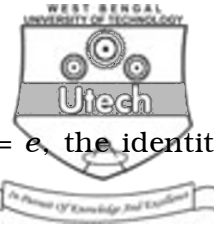
- |              |                   |
|--------------|-------------------|
| a) Injective | b) Surjective     |
| c) Bijective | d) None of these. |

iii) Let  $S$  be a finite set of  $n$  distinct elements. The number of bijective mapping from  $S$  to  $S$  is

- |          |                   |
|----------|-------------------|
| a) $n^2$ | b) $n!$           |
| c) $2^n$ | d) None of these. |

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4. Let  $G$  be a group, if  $a, b \in G$  such that  $a^4 = e$ , the identity element of  $G$  and  $ab = ba^2$ , prove that  $a = e$ .
5. Prove that every cyclic group is an Abelian group.
6. Show that the mapping  $F : (Z, \bullet) \rightarrow (R, \bullet)$  defined by  $f(x) = x^2 \forall x \in Z$  is a monomorphism but not isomorphism.
7. If in a ring  $R$  with unity,  $(xy)^2 = x^2 y^2 \forall x, y \in R$ , then show that  $R$  is a commutative.

**GROUP – C**

**( Long Answer Type Questions )**

Answer any *three* of the following.  $3 \times 15 = 45$

8. a) Examine whether the following two graphs are isomorphic.

Dia.

5

- b) Draw the dual of the graph.

Dia.

5



- c) Determine the adjacency matrix of the following di-graph :

Dia.

5

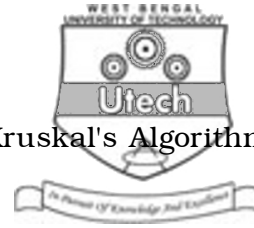
9. a) Construct a simple logic circuit which would satisfy the truth table.

$x$	$y$	$f$
1	1	1
0	1	1
1	0	0
0	0	1

5

- b) Prove that a graph  $G$  has a spanning tree if and only if  $G$  is connected.

5



- c) Find the minimal spanning tree by Kruskal's Algorithm from the following graph G :

Dia.

5

10. a) Consider the lattice  $L = \{ 1, 2, 3, 4, 6, 12 \}$ , the divisors of 12 ordered by divisibility. Find the lower and upper bound of  $L$ . Is  $L$  a complemented lattice ?

5

- b) For any Boolean Algebra, show that.

$$(xy' + xz') + x' = (x' + y + z)(x' + y + z')(x' + y' + z').$$

5

- c) Using generating function solve the recurrence relation,

$$a_n - 7a_{n-1} + 10a_{n-2} = 0, \text{ for } n > 1 \text{ and } a_0 = 3,$$

$$a_1 = 3.$$

5



11. a) Prove that the number of vertices in a binary tree is always odd. 5
- b) Find the truth table of the Boolean function
- $$f = z'xy + xy' + y. \quad 5$$
- c) Prove that a complete graph with  $n$  vertices consist of  $\frac{n(n-1)}{2}$  number of edges. 5
12. a) Prove that the identity elements and the inverse of an element in a group is unique. 5
- b) Prove that in a group  $(G, *)$ ,  $(a * b)^{-1} = b^{-1} * a^{-1}$ . 5
- c) Prove that the set of matrices
- $$H = \left\{ \begin{pmatrix} x & 0 \\ 0 & x \end{pmatrix} : x \in R, x \neq 0 \right\} \text{ forms a normal subgroup of } GL(2, R), \text{ the group of all real non-singular } 2 \times 2 \text{ matrices under multiplication.} \quad 5$$
13. a) Using Ford-Fulkerson's algorithm, find the maximum flow in the following network :

Dia.

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b) Using Floyd's algorithm, find the shortest path between

i)  $w_2$  and  $w_6$

ii)  $w_1$  and  $w_6$  .

Dia.

7

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