2013

NUMERICAL METHODS

Time Allotted : 3 Hours | *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

   i) The number of significant figures in 0·03409 is

      a) five b) six

      c) seven d) four.

   ii) The kind of error occurs when π approximated by 3·14 is

      a) truncation error b) round-off error

      c) inherent error d) relative error.

iii) If $f(0) = 12$, $f(3) = 6$ and $f(4) = 8$, then the interpolation function $f(x)$ is

a) $x^3 - 3x + 12$ b) $x^2 - 5x$

c) $x^3 - x^2 - 5x$ d) $x^2 - 5x + 12$.

iv) Newton-Raphson method for solution of the equation $f(x) = 0$ fails when

a) $f'(x) = 1$ b) $f'(x) = 0$

c) $f'(x) = -1$ d) none of these.

v) In Gaussian elimination method, the given system of equation represented by $Ax = B$ is converted to another system $Ux = Y$ where U is

a) diagonal matrix

b) null matrix

c) identity matrix

d) upper triangular matrix.

vi) Error in Weddle method of integration is

a) 0 b) $-\frac{h^4}{180}(b-a)M_4$

c) $-\frac{h^2}{12}(b-a)M_2$ d) $-\frac{h^6}{840}(b-a)M_6$.

vii) In Trapezoidal rule, the portion of curve is replaced by

a) straight line b) circular path

c) parabolic path d) none of these.

viii) Which of the following is an iterative method ?

a) Gauss Elimination Method

b) Gauss Jordan Method

c) LU decomposition Method

d) Gauss-Seidel Method.

ix) The number 9·6506531 when rounded-off to 4 places of decimal will give

a) 9·6506 b) 9·6507

c) 9·6505 d) none of these.

x) $\Delta^3 y_0$ may be expressed as

a) $y_3 - 3y_2 + 3y_1 - y_0$ b) $y_2 - 2y_1 + y_0$

c) $y_3 - 3y_2 + 3y_1 + y_0$ d) none of these.

xi) Which of the following statements applies to the bisection method used for finding roots of functions ?

a) Convergence within a few iteration

b) Guaranteed to work for all continuous functions

c) Is faster than the Newton-Raphson method

d) Requires that there be no error in determining the sign of the function.

xii) Runge-Kutta formula has a truncation error, which is of the order

a) h^2 b) h^4

c) h^5 d) none of these.

xiii) In finite difference method, $\frac{d^2y}{dx^2}$ is replaced by

a) $\frac{y_{n+1} - 2y_{n-1} + y_n}{2h^2}$ b) $\frac{y_{n+1} - 2y_n + y_{n-1}}{h^2}$

c) $\frac{y_{n+1} - 2y_{n-1} + y_n}{2h}$ d) $\frac{y_{n+1} - 2y_{n-1} + y_n}{4h^2}$.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. a) Show that $\Delta \log f(x) = \log\left[1 + \frac{\Delta f(x)}{f(x)}\right]$.

b) Define forward difference operator Δ and shift operator E. Prove that $E.\Delta = \Delta.E$.

Find the missing terms in the following table :

x	0	5	10	15	20	25
y	6	10	?	17	?	31

Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Simpson's $\frac{1}{3}$rd rule taking $n = 6$.

Hence find the value of π.

Using Runge-Kutta method of 4th order solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$

with $y(0) = 1$ at $x = 0.2$.

Solve the following system of linear equations by Gaussian Elimination method :

$3x + 4y + 5z = 18$, $2x - y + 8z = 13$, $5x - 2y + 7z = 20$.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

a) What do you mean by interpolation ? Derive Newton's backward interpolation formula. Can you apply this formula for unequispaced interpolating points ? 7

b) Using Trapezoidal and Simpson's $\frac{1}{3}$rd rule compute $\int_4^{5.2} \log_e x\,dx$ by taking seven ordinates correct up to four decimal places. 8

a) Find the value of $\sqrt{2}$ from the following table : 7

x	1·9	2·1	2·3	2·5	2·7
$f(x) = \sqrt{x}$	1·3784	1·4491	1·5166	1·5811	1·6432

b) Solve the following system of equations by LU-factorization method : 8

$$3x + 4y + 2z = 15$$

$$5x + 2y + z = 18$$

$$2x + 3y + 2z = 10$$

9. a) Find a root of the equation $x \log_{10} x = 1 \cdot 2$ by the method of false position correct to three decimal places. 7

b) Find the inverse of the matrix $A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix}$ by using Gaussian elimination method. 8

10. a) Apply Milne's method to find $y(0 \cdot 8)$ for the equation $\frac{dy}{dx} = x + y^2$, given that $y(0) = 0$, $y(0 \cdot 2) = 0 \cdot 02$, $y(0 \cdot 4) = 0 \cdot 0805$, $y(0 \cdot 6) = 0 \cdot 1839$. 8

b) Evaluate $\int_0^{0 \cdot 6} \frac{dx}{\sqrt{1 - x^2}}$, using Weddle's rule taking 12 equal subintervals. 7

1. a) Using Gauss-Seidel method find the solution of the following system of linear equations correct up to two decimal places :

$$3x + y + 5z = 13$$

$$5x - 2y + z = 4$$

$$x + 6y - 2z = -1$$ 7

b) Using finite difference method solve the boundary value problem :

$\frac{d^2y}{dx^2} + y + 1 = 0$ with $y(0) = 0$, $y(1) = 0$. 8

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