Name : ....
Roll No. : ....
Invigilator's Signature : ....

CS / B.Tech(ECE) / SEM-7 / EC-704B / 2012-13

2012

ADVANCED ENGINEERING MATHEMATICS FOR ELECTRONIC ENGINEERING

Time Allotted : 3 Hours *Full Marks :* 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

   i) If A be an orthogonal matrix then

      a) $A^{-1} = A + A^T$

      b) $A^{-1} = A^T$

      c) A^{-1} does not exist

      d) none of these.

ii) If ω is the cube root of unity then the value of the determinant $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & \omega & 1 \end{vmatrix}$ is

a) 0 b) ω^2

c) $1 + \omega$ d) none of these.

iii) Which of the following functions is nowhere differentiable ?

a) $z \, Im \, (z)$

b) $(x^3 - 3xy^2) + i \, (3x^2 y - y^3)$

c) $iz + 2$

d) $Re \, (\, z \,)$.

iv) The value of the integral $\int_c \frac{e^z}{(z-3)^2} \, dz$, c being $| \, z \, | = 2$, is

a) $2 \, \pi \, i \, e^3$

b) $-2 \, \pi \, i \, e^{-3}$

c) 0

d) none of these.

v) $L^{-1} \left\{ \frac{1}{(s+a)^2} \right\}$ is

a) e^{at} b) e^{-at}

c) $t \, e^{at}$ d) $t \, e^{-at}$.

vi) $L\left\{e^{-2t}\ \sin t\right\}$ is

a) $\dfrac{1}{s^2-4s+5}$ b) $\dfrac{s}{s^2-4s+5}$

c) $\dfrac{1}{s^2+4s+5}$ d) none of these.

vii) If Fourier transform of $f(x, y)$, $\mathcal{F}\{f(x, y) : x \to s\} = F(s, y)$ and $f(x, y) \to 0$ as $x \to \pm\infty$ then

a) $\mathcal{F}\left\{\dfrac{\partial^n f}{\partial x^n}\right\} = (-is)^{-n} F(s, y)$

b) $\mathcal{F}\left\{\dfrac{\partial^n f}{\partial x^n}\right\} = (-is)^{n} F(s, y)$

c) $\mathcal{F}\left\{\dfrac{\partial^n f}{\partial x^n}\right\} = (-is)^{\frac{n}{2}} F(s, y)$

d) $\mathcal{F}\left\{\dfrac{\partial^n f}{\partial x^n}\right\} = (-is)^{\frac{1}{n}} F(s, y)$.

viii) If the trace of a 3×3 matrix is 12, and two of its eigenvalues are 4 and 6 then the third eigenvalue is

a) 2 b) 1

c) 8 d) 3.

ix) The value of a for which the rank of the matrix $\begin{pmatrix} 2 & 0 & 1 \\ 5 & a & 3 \\ 0 & 3 & 1 \end{pmatrix}$ is less than 3 is

a) $\dfrac{3}{4}$ b) $\dfrac{3}{5}$

c) $\dfrac{3}{2}$ d) none of these.

x) $J_{\frac{1}{2}}(x)$ is given by

a) $\sqrt{\frac{2\pi}{x}}\ \sin x$ b) $\sqrt{\frac{2\pi}{x}}\ \cos x$

c) $\sqrt{\frac{\pi}{2x}}\ \cos x$ d) $\sqrt{\frac{2}{\pi x}}\ \sin x$.

xi) The *pde* $\frac{\partial u}{\partial t} = c^2\ \frac{\partial^2 u}{\partial^2 x}$ is of type

a) elliptic b) parabolic

c) hyperbolic d) none of these.

xii) The order of the differential equation

$$\frac{\partial^2 x}{\partial t^2} + 2\left(\frac{\partial x}{\partial t}\right)^3\left(\frac{\partial y}{\partial t}\right)^2 + t^3\left(\frac{\partial y}{\partial t}\right)^3 = (\log\ t)^4$$

is

a) 0 b) 2

c) 3 d) None of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. If $A(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$, prove that

$A(\theta)\ A(\varphi) = A(\varphi)\ A(\theta) = A(\theta + \varphi)$.

3. Using Cauchy's integral formula evaluate $\int_c \frac{z}{z^2 - 1}\ dz$, where C is the circle $|\ z\ | = 2$.

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4. When n is a positive integer, show that $J_{-n}(x) = (-1)^n J_n(x)$.

5. Evaluate $L^{-1}\left\{ \frac{1}{s^2(s^2 + a^2)} \right\}$ using convolution theorem.

6. Obtain the general solution of the following linear PDE :

$$(mz - nz)\frac{\partial z}{\partial x} + (nx - lz)\frac{\partial z}{\partial y} = ly - mx.$$

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Show that the function $f(x) = e^{-\frac{x^2}{2}}$ is self reciprocal w.r.t. symmetric definition of Fourier transform.

b) Determine the temperature $u(x, t)$ in a bar of infinite length $(-\infty < x < \infty)$ from the governing equation

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}, \quad x > 0, \ t > 0$$

subject to the conditions :

(i) $u(0, t) = 0$, for $t > 0$

(ii) $u(x, 0) = \begin{cases} 1, & 0 < x < 1 \\ 0, & x \geq 1 \end{cases}$

(iii) $u(x, t)$ is bounded. 7 + 8

8. a) Find $L\left\{ \dfrac{\sin^2 t}{t} \right\}$ and hence evaluate $\displaystyle\int_0^\infty e^{-st} \frac{\sin^2 t}{t} \, dt$.

b) Solve the PDE : $(D^3 + D^2 D' - DD'^2 - D'^3)\, Z = e^x \cos 2y$,

where $\dfrac{\partial^r}{\partial x^r} = D^r$ and $\dfrac{\partial^r}{\partial y^r} = D'^r$

c) Find the bilinear transformation which maps the points $z = 1, 0, -1$ on the points $w = i, 0, -i$ and find the fixed points of transformation. (3 + 2) + 5 + (4 + 1)

9. a) State Cauchy's Residue theorem. Use it to evaluate

$$\int_0^{2\pi} \frac{d\theta}{5 + 4\cos\theta}$$

b) Expand $f(z) = \dfrac{z+1}{(z-3)(z-4)}$ in a Laurent's series for the region $2 < | z | < 4$. Find also the region of convergence.

(2 + 5) + (5 + 3)

10. a) Prove that $J_n'(x) = \dfrac{1}{2}\left[J_{n-1}(x) - J_{n+1}(x) \right]$, where $J_n(x)$ is Bessel's Function of order n.

b) Prove that $\displaystyle\int_{-1}^{1} \left[P_n(x) \right]^2 dx = \frac{2}{2n+1}$, where $P_n(x)$ is Legendre's polynomial of order n.

c) Express $P(x) = x^4 + 2x^3 + 2x^2 - x - 3$ in terms of Legendre's polynomials. 4 + 6 + 5

11. a) Diagonalise the matrix $\begin{pmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{pmatrix}$.

b) Find the rank of the following matrix :

$$\begin{pmatrix} 1 & 3 & 2 & 4 & 1 \\ 0 & 0 & 2 & 2 & 0 \\ 2 & 6 & 2 & 6 & 2 \\ 3 & 9 & 1 & 10 & 6 \end{pmatrix}.$$

c) Solve the following partial differential equation :

$(p^2 + q^2)\, y = qu.$ 7 + 4 + 4