Name : ....

Roll No. : ....

Invigilator's Signature : ....

**CS/B.TECH(NEW)/AUE/APM/CE/CSE/IT/MRE/
PE/ME/TT/SEM-3/PH-301/2011-12**

2011

PHYSICS – II

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) Which of the following functions is acceptable as a wave function of a one-dimensional quantum mechanical system ?

a) $\psi = \frac{A}{x}$ b) $\psi = Ae^{-x^2}$

c) $\psi = Ax^2$ d) $\psi = A \sec x$.

ii) A system with time independent potential is in an energy state *E*. The wave function of this state is

a) independent of time

b) an exponentially decaying function of time

c) a periodic function of time with time period proportional to *E*

d) a periodic function of time with time period inversely proportional to *E*.

iii) For a free particle in a cubical box the degeneracy of the first excited state is

a) two-fold b) three-fold
c) six-fold d) nine-fold.

iv) Wave function of a certain particle is given by

$\psi = A\cos x$ for $|x| \leq \frac{\pi}{2}$

$= 0$ otherwise

Then the value of A is

a) $\frac{2}{\pi}$ b) $\sqrt{\frac{2}{\pi}}$
c) 1 d) zero.

v) The physical interpretation of $\vec{\nabla} \cdot \vec{B} = 0$ is ($\vec{B}$ is the magnetic field)

a) magnetic monopole cannot exist
b) magnetic field is irrotational
c) magnetic field is conservative
d) magnetic lines of force are open curves.

vi) If a vector field, F is given by $\vec{F} = \frac{\sin h\,(r)}{1+2r^3}\vec{r}$ then $\vec{\nabla} \times \vec{F}$ is given by

a) $3\vec{r}$
b) zero
c) $\left(\frac{\cos h\,(r)}{1+2r^3} - 3r\right)\hat{\theta}$, $\hat{\theta}$ is the unit vector along θ-line
d) $\left(\frac{\cos h\,(r)}{1+2r^3} - 3r\right)\vec{F}$.

vii) A proton travelling vertically downwards experiences a southward force due to a magnetic field directed at right angles to its path. An electron travelling northward in the same magnetic field will experience a magnetic force directed

a) upwards b) downwards
c) towards east d) towards west.

viii) Ampere's circuital law is applicable when the current density is

a) constant over space b) time independent

c) solenoidal d) irrotational.

ix) Dimension of $\mu_0 \in_0$ (symbols with their usual significance) is

a) $L^{-2}T^{-2}$ b) LT^{-1}

c) $L^{-2}T^{-1}$ d) $L^{-2}T^{2}$.

x) Electrostatic field may be obtained as the gradient of a scalar potential

a) because it is a conservative field

b) because it is always solenoidal

c) when the field is linear in x, y, z

d) where there is no magnetic field in the same location.

xi) If $\vec{P}$ represents the polarization vector then $\vec{\nabla} \cdot \vec{P} = -\rho$, where ρ is the density of

a) free charge

b) bound charge

c) free charge at the boundary of the dielectric

d) sum of free and bound charge.

xii) Two conducting spheres of radii r and $2r$ contain charges q and $2q$. Electric field just outside the surface of these two spheres are E_1 and E_2 respectively. Then

a) $E_1 = E_2$ b) $E_1 = 2E_2$

c) $2E_1 = E_2$ d) $E_1 = 4E_2$.

xiii) Which of the following particles is not a Fermion ?

a) Proton b) Neutron

c) Alpha-particle d) Electron.

xiv) For $T > 0$ the probability of occupancy of a state of a Fermion with energy equal to Fermi energy is

a) 1

b) decreases linearly with T

c) $\frac{1}{2}$

d) decreases exponentially with T.

xv) Number of ways a total of N distinguishable particles may be placed in different energy levels (N_i particles in energy level E_i with degeneracy g_i) is

a) $\frac{N!}{\sum_i N_i g_i}$

b) $N! \sum_i \frac{g_i^{N_i}}{N_i!}$

c) $\frac{\sum_i (N_i!)}{\sum_i g_i^{N_i}}$

d) $\frac{N!}{\sum_i (N_i + g_i - 1)!}$.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. a) Show that $\nabla^2 \left(\frac{1}{r} \right) = 0$ when $r \neq 0$. 3

   b) Find the angle between the normal to the x–y plane and the normal to surface $x^2 + y^2 = z^2$. 2

3. Starting from Maxwell's equation in a charge free conducting media arrive at the concept of skin depth and express it in terms of signal frequency and conducting for a very good conductor.

4. A capacitor is made with two infinitely long cylindrical conductors of radii a and b ($a > b$), with vacuum in the intervening space. If the internal cylinder is kept at ground and the outer cylinder has a charge density σ then solve Laplace equation to find the electrostatic potential in the space between the cylinders.

5. Write down the Lagrangian of a simple harmonic oscillator moving in one dimension. Calculate the generalized momentum. Write down Lagrange's equation of motion. Find out the Hamiltonian of this system. Write down Hamilton's equations for this system. 1 + 1 + 1 + 1 + 1

6. In how many ways 2 indistinguishable particles can be distributed in three distinct nondegenerate states, if the particles obey (i) *F-D* statistics, (ii) *B-E* statistics ?
   Write down the expression of occupation probability of photon in the frequency interval ν to $\nu + d\nu$. (2 + 2) + 1

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. 3 × 15 = 45

7. a) Prove that for any closed curve $\oint_s \vec{B} \cdot d\vec{s} = 0$ where $\vec{B}$ is the magnetic field. 3

   b) Given $\vec{v} = \vec{w} \times \vec{r}$, where $\vec{r}$ is the position vector and $\vec{w}$ is a constant angular velocity vector, then find out $\vec{\nabla} \times \vec{v}$. 4

   c) Use the expression of gradient in the spherical coordinate system to find the normal to the surface $r \sin\theta = 1$. 3

   d) If $\vec{A}$ is a constant vector, show that $\vec{\nabla}(\vec{r} \cdot \vec{A}) = \vec{A}$, where $\vec{r}$ is the position vector. 3

   e) If the electric field is given by $E = \frac{1}{\epsilon_0}\left(x\,\hat{i} + y\,\hat{j} - 2\,z\,\hat{k}\right)$, then find the charge density. 2

8. a) Define polarization vector and displacement vector. Rewrite differential form of Coulomb's law in terms of displacement vector. 1 + 1 + 2

   b) Define electronic polarizibility. Discuss how monatomic gases can be polarized. 1 + 1

   c) State Gauss's law in electrostatics and derive Poisson's equation from that. 2 + 2

   d) Starting from the principle of charge conservation establish the equation of continuity. 5

9. a) Find magnetic field at a point (1, 1, 1) if vector potential at that position is $\vec{A} = \left(10x^2 + y^2 = z^2\right)\hat{j}$. 2

   b) A very long cylinder of radius a carries current I, which is uniformly distributed over the cross-section of the cylinder. If the current flows along the axis of the cylinder then find out the electric field in the conductor, the magnetic field just outside the conductor and the Poynting vector at the surface of the cylinder. You may assume that the conductivity of the material is σ.

   2 + 2 + 2

   c) Calculate the magnetic field at the centre of a circular lamp carrying current I. 5

   d) If $\vec{E} = \vec{E}_0 e^{i(kx-\omega t)}$ denotes the electric vector of an electromagnetic field in vacuum then find out the magnetic vector. 2

10. a) What do you mean by cyclic coordinate ? Explain with an example. 2

    b) Show that if generalized force for a conservative system is zero then the generalized momentum will be conserved. 3

c) The wave function in a one-dimensional potential box with rigid walls is given by

$$\psi_n(x) = \sqrt{\frac{2}{l}} \sin \frac{n\pi x}{l}, \quad |x| \le l,$$

$$= 0 \qquad \text{otherwise}$$

Find the expectation value of $\hat{x}$ and $\hat{p}$. 3 + 3

d) Consider $\varphi = c_1\varphi_1 + c_2\varphi_2$, where φ_1 and φ_2 are orthonormal energy eigenstates of a system corresponding to energy E_1 and E_2 at t = 0. If φ is normalized and $c_1 = \frac{1}{\sqrt{3}}$ then what is the value of c_2 ?

Find the expectation value of E^2. Write the wave function at a subsequent time. 1 + 2 + 1

11. a) Calculate the total number of particles in a fermionic gas in terms of Fermi level at absolute zero. 6

b) Calculate the total energy of particles in a fermionic gas at absolute zero. 3

c) Show that both *F-D* statistics and *B-E* statistics approach *MB* statistics at a certain limit. When does that happen ? 3

d) 2 distinguishable particles are distributed in 2 non-degenerate states of energy 0 and $\in$. List all the microstates and find the total energy corresponding to the state with maximum number of microstates. 3

12. a) Use separation of variables technique to calculate the eigenfunctions and eigenvalues of a rigid box of side *L*. Calculate the wavelength of the photon that must be absorbed for a transition from the lowest to the second excited state. 4 + 1

b) Starting from Maxwell's equation show that the electric field can be written in terms of a scalar potential and the magnetic vector potential as

$$\vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{\partial t}$$ 5

c) Using Gauss's law in integral form obtain the electric field due to the following charge distribution :

$$\rho = \rho_0 \left(1 - \frac{r^2}{a^2}\right), \quad 0 < r \le a$$

$$= 0 \qquad a < r < \alpha$$

at a point outside the distribution as well as at a point inside the distribution. Sketch the field as a function of r. 4 + 1

=============