Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/B.TECH(NEW)/SEM-2/M-201/2012

2012

MATHEMATICS – II

Time Allotted : 3 Hours   *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words

as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1.   Choose the correct alternatives for any *ten* of the following :

$$10 \times 1 = 10$$

i)   The integrating factor of

$$(2xy - 3y^3)\, dx + (4x^2 + 6xy^2)\, dy = 0 \text{ is}$$

a) $x^2 y$ 　　　　　　　　b) $x^2 y^2$

c) xy^2 　　　　　　　　d) xy^3.

ii)   The   substitution $x = e^z$ transforms   the   differential

equation $x^2 \dfrac{d^2 y}{dx^2} - 5y = \log_e x$ to

a) $\dfrac{d^2 y}{dz^2} + \dfrac{dy}{dz} - 5y = z$ 　　　b) $\dfrac{d^2 y}{dz^2} - \dfrac{dy}{dz} + 5y = z$

c) $\dfrac{d^2 y}{dz^2} - \dfrac{dy}{dz} + 3y = 0$ 　　　d) $\dfrac{d^2 y}{dz^2} - \dfrac{dy}{dz} - 5y = z$.

2073   [Turn over

iii) If the differential equation

$$\left(y + \frac{1}{x} + \frac{1}{x^2 y} \right) dx + \left(x - \frac{1}{y} + \frac{A}{xy^2} \right) dy = 0$$

is exact, then the value of A is

a) 2   b) 1

c) -1 d) 0.

iv) The value of $\displaystyle\int_0^\infty e^{-x} x^{\frac{3}{2}}\, dx$ is

a) $\dfrac{3}{4}\sqrt{\pi}$ b) $\dfrac{5}{4}\sqrt{5}$

c) $\dfrac{3}{5}\sqrt{\pi}$ d) $\dfrac{1}{4}\sqrt{\pi}$.

v) The value of $\Gamma(6)$ is

a) 720   b) 5

c) 6   d) 120.

vi) The Laplase transform of $e^{-3t}\sin 4t$ is

a) $\dfrac{4}{s^2 + 6s - 7}$ b) $\dfrac{s}{s^2 + 6s - 7}$

c) $\dfrac{1}{s^2 + 6s - 7}$ d) $\dfrac{s}{s^2 + 6s + 24}$.

vii) The maximum number of edges in a connected graph of 7 vertices is

a) 6   b) 7

c) 21   d) 14.

viii) The maximum degree of any vertex in a simple graph with 10 vertices is

a) 10   b) 5

c) 20   d) 9.

ix) Tree is a connected graph without any

    a)   odd vertex   b)   even vertex

    c)   circuit   d)   pendent vertex.

x)   The improper integral $\displaystyle\int_0^1 \frac{dx}{(x-a)^n}$ converges for

    a) $n < 1$ b) $n \geq 1$

    c) $n > 1$ d)   none of these.

xi)   The particular integral of $(D^2 - 4D + 4)\, y = x^3 e^{2x}$ is

    a) $\dfrac{e^{2x} x^4}{20}$ b) $\dfrac{e^{2x} x^5}{20}$

    c) $\dfrac{e^{2x} x^4}{60}$ d) $\dfrac{e^{x} x^5}{20}$.

xii)   The inverse Laplace transform of $\left(\dfrac{4}{s^2 - 7} + \dfrac{2}{s^2 + 7}\right)$ is

    a) $\dfrac{1}{7}\left\{4 \cos\left(\sqrt{7}\,t\right) - 2 \sin\left(\sqrt{7}\,t\right)\right\}$

    b) $\dfrac{1}{7}\left\{4 \cos\left(7t\right) + 2 \sin\left(7t\right)\right\}$

    c) $\dfrac{1}{\sqrt{7}}\left\{4 \sin h\left(\sqrt{7}\,t\right) + 2 \sin h\left(\sqrt{7}\,t\right)\right\}$

    d) $\dfrac{1}{7}\left\{4 \sin h\left(7t\right) - 2 \sin\left(7t\right)\right\}$.

xiii)   The general solution of $p = \log_e (px - y)$ is

    a) $y = cx - c$ b) $y = cx - e^c$

    c) $y = c^2 x - e^{-c}$ d)   none of these.
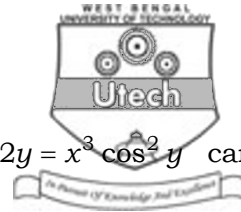
xiv) The differential equation $\dfrac{dy}{dx} + x\sin 2y = x^3 \cos^2 y$ can be reduced to the linear equation

a) $\dfrac{dz}{dx} + x\sin 2y = x^3$

b) $\dfrac{dz}{dx} + 2xz = x^3$

c) $\dfrac{dz}{dx} - 2xz = x^3$

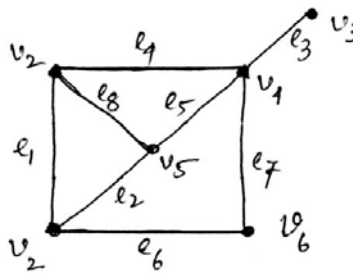d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. 3 × 5 = 15

2. Solve : $x^2 \dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + y = \log_e x \sin(\log_e x)$

3. Evaluate : $L^{-1}\left(\dfrac{s+4}{s\,(s-1)\,(s^2+4)}\right)$

4. Use Beta and Gamma functions to evaluate $\displaystyle\int_0^{\frac{\pi}{2}} \sqrt{\tan x}\ dx$.

5. Determine adjacency matrix of the following graph :

6. Solve : $\dfrac{dx}{dt} + 3x + y = e^t$, $\dfrac{dy}{dt} - x + y = e^{2t}$.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Solve the following differential equation using Laplace transform :

$(D^2 + 2D + 5)y = e^{-t}\sin t$, $y(0) = 0$, $y'(0) = 1$

b) Apply the variation of parameters to solve

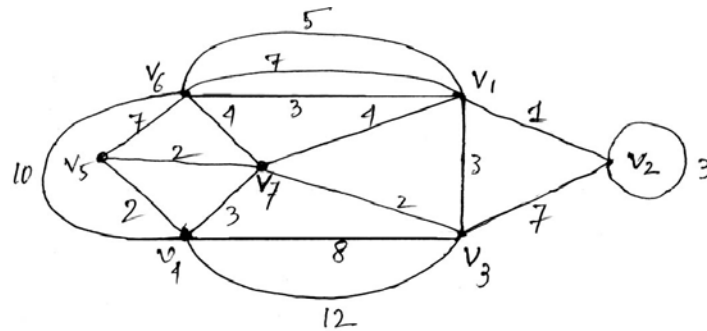$$\frac{d^2y}{dx^2} + y = \sec^3 x . \tan x$$

c) Show that $\displaystyle\int_0^\infty e^{-4x}x^{\frac{3}{2}}\, dx = \frac{3}{128}\sqrt{\pi}$ $5 + 5 + 5$

8. a) Draw the graph whose incidence matrix is,

$$\begin{pmatrix} 0 & 0 & 1 & -1 & 1 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 & 0 \end{pmatrix}$$

b) By Dijkstra's procedure, find the shortest path and the length of the shortest path from the vertex V_2 to V_5 in the following graph :

c) Solve : $y = 2px - p^2$ $5 + 5 + 5$
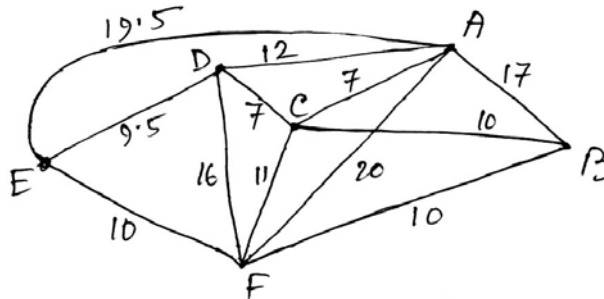
9. a) Examine whether the differential equation

$$\left(xy^2 - e^{\frac{1}{x^3}} \right) dx - x^2 y \, dy = 0$$
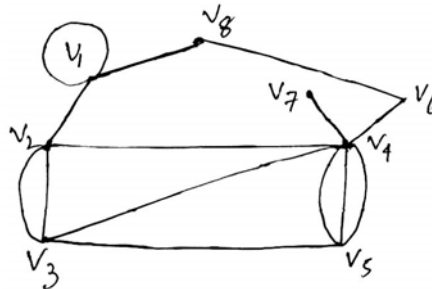
is exact or not and then solve it.

b) Prove that a complete graph of n vertices has $\dfrac{n(n-1)}{2}$ number of edges.

c) Apply convolution theorem to evaluate

$$L^{-1}\left(\frac{1}{(s^2 + 2s + 5)^2} \right).$$

5 + 5 + 5

10. a) By Kruskal's Algorithm, find a minimal (or shortest) spanning tree and the corresponding weight of the spanning tree in the following graph :

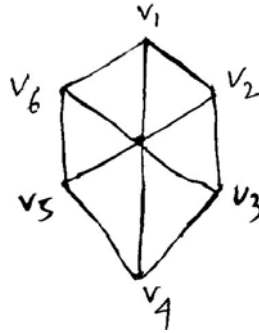b) Find by BFS algorithm a spanning tree in the following graph :

c) Examine the convergence of the improper integral

$$\int_0^2 \frac{dx}{x(2-x)}$$

5 + 5 + 5

11. a) Define complement of a graph. Find the complement of the graph.

b) Solve : $x^2 \dfrac{d^2 y}{dx^2} + 4x \dfrac{dy}{dx} + 2y = e^x$

c) Prove that in a binary tree with n vertices, the number of internal vertices is one less than the number of pendant vertices.   5 + 5 + 5

=============