Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/B.Tech/SEM-2/M-201/2010

2010

MATHEMATICS

Time Allotted : 3 Hours   Full Marks : 70

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :

    $10 \times 1 = 10$

    i)   If $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, then A^{100} is

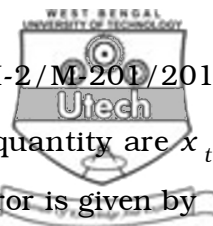   a) $2^{99} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

         b) $2^{101} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

         c) $2^{100} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

         d)   none of these.

    ii) If λ is a eigenvalue of A, then λ^4 is an eigenvalue of

    a) A^4 b) A^3

    c) A d)   none of these.

iii)   The rank of $\begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$ is

    a)   0   b)   2

    c)   1   d)   none of these.

iv)   Which of the following sets is linearly independent ?

    a) $\{(\,1,\,2\,),\,(\,2,\,4\,)\}$

    b) $\{(\,1,\,2,\,3\,),\,(\,2,\,4,\,6\,),\,(\,1,\,1,\,1)\}$

    c) $\{(\,2,\,0,\,0\,),\,(\,0,\,3,\,0\,),\,0,\,0,\,4\,)\}$

    d)   none of these.

v) $L\left\{e^{-2t}\cos t\right\}$ is equal to

    a) $\dfrac{p}{p^2 + 4p + 5}$

    b) $\dfrac{p-1}{p^2 + 4p + 5}$

    c) $\dfrac{p+1}{p^2 + 4p + 5}$

    d) $\dfrac{p+2}{p^2 + 4p + 5}$.

vi)   Which of the following is not true ?

    a) $\Delta \int E - 1$ b) $\Delta \quad \int \Delta -$

    c) $\Delta \dfrac{1}{} \int \Delta +$ d) $\Delta \int 1 - E^{-1}$.

vii) If the true and approximate value of a quantity are x_t and x_a respectively, then the relative error is given by

a) $\left| \dfrac{x_t - x_a}{x_t} \right|$

b) $\left| \dfrac{x_a - x_t}{x_a} \right|$

c) $\left| \dfrac{x_a - x_t}{x_t - x_a} \right|$

d) $\left| x_t - x_a \right|$.

viii) The sum of the eigenvalues of

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{bmatrix} \text{ is}$$

a) 4

b) 5

c) 2

d) none of these.

ix) The value of the determinant $\begin{vmatrix} 17 & 58 & 97 \\ 19 & 60 & 99 \\ 18 & 59 & 98 \end{vmatrix}$ is

a) 1

b) 2

c) 3

d) 0.

x) The value of λ for which the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 2 & -3 & 1 \\ 3 & -2 & \lambda \end{bmatrix}$ is singular, is

a) 3/2

b) 2

c) 1

d) 1/3.

xi) $\dfrac{1}{D-1} x^2$ is equal to

    a) $x^2 + 2x + 2$

    b) $-(x^2 + 2x + 2)$

    c) $2x - x^2$

    d) $-(2x - x^2)$.

xii) The norm of the vector $\alpha = (-1, 2, 3)$ in R^3 with standard inner product is

    a) $\sqrt{12}$ b) $\sqrt{14}$

    c) $\sqrt{3}$ d) $\sqrt{2}$.

xiii) The degree and order of the differential equation $\left(\dfrac{d^2y}{dx^2} + 2\right)^{3/2} = x \dfrac{dy}{dx}$ are respectively

    a) $\dfrac{3}{2}$, 2   b)   2, 3

    c)   3, 2   d)   1, 4.

GROUP – B
(Short Answer Type Questions)
Answer any *three* of the following.   3 × 5 = 15

2. Solve the following system of equations with the help of Gauss' Elimination method :

$x_1 + x_2 + 4x_3 = 6$

$3x_1 + 2x_2 - 2x_3 = 9$

$5x_1 + x_2 + 2x_3 = 13.$

3. Prove that $\Delta \int e^{hD} - 1$.

(The notations have their usual meanings).

4. Expand by Laplace's method to prove that

$$\begin{vmatrix} o & a & b & c \\ -a & o & d & e \\ -b & -d & o & f \\ -c & -e & -f & o \end{vmatrix} = (af - be + cd)^2 .$$

5. Solve $\dfrac{dy}{dx} + y = y^3 (\cos x - \sin x)$.

6. Evaluate $\displaystyle\int_{0}^{\pi/6} \sqrt{1 + \sin x}\ dx$ using Simpson's one-third rule by

taking five ordinates.

GROUP – C
(Long Answer Type Questions)
Answer any *three* of the following. $3 \times 15 = 45$

7. a) Show that (3, 1, – 2), (2, 1, 4) and (1, – 1, 2) form a basis of R^3 .

   b) Find the eigenvalues and eigenvectors of the matrix
   $$\begin{bmatrix} 1 & -1 & 2 \\ 2 & -2 & 4 \\ 3 & -3 & 6 \end{bmatrix}$$

   c) Solve by Cramer's rule :

   $x + y + z = 6$

   $x + 2y + 3z = 14$

   $x - y + z = 2.$

8. a) Solve the differential equaion by Laplace Transformation :

$$\frac{d^2 y}{dt^2} - 2 \frac{dy}{dt} - 3y = t\cos t$$

$y(0) = 0, y'(0) = 0.$

b) Solve by the method of variation of parameters :

$$\frac{d^2 y}{dx^2} + a^2 y = \sec ax .$$

c) Find the particular integral of

$$(D^2 + 4) y = x \sin^2 x .$$

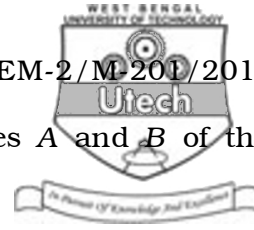9. a) Estimate the missing term from the table :

x	2	4	6	8	10
y	5	13	*	53	85

b) The values of a function $f(x)$ are given for certain values of x as follows :

x :	4	5	6	8
f (x) :	3·11	2·96	2·85	2·7

Obtain the value of $f(5·5)$ using Lagrange's interpolation formula.

c) Compute $\int_{1}^{2} \frac{\sin x}{\sqrt{x}} \, dx$ using Simpson's one-third rule taking $h = \frac{1}{6}$.

6

10. a) Prove that for two invertible matrices A and B of the same order $(AB)^{-1} = B^{-1} A^{-1}$.

b) Reduce the following matrix to a row–reduced echelon form and hence find its rank :

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ 2 & 4 & 8 & 6 \\ 0 & 0 & 5 & 8 \\ 3 & 6 & 6 & 3 \end{bmatrix}$$

c) Solve $(D^2 - 5D + 6) y = x^2 \ e^{3x}$, $D \int \dfrac{d}{dx}$.