



# MAULANA ABUL KALAM AZAD UNIVERSITY OF TECHNOLOGY, WEST BENGAL

Paper Code : CE(BS)302 Mathematics-III (Transform & Discrete Mathematics)  
UPID : 003503

Full Marks : 70

Time Allotted : 3 Hours

The Figures in the margin indicate full marks.  
Candidate are required to give their answers in their own words as far as practicable

## Group-A (Very Short Answer Type Question)

1. Answer any ten of the following :

[ 1 x 10 = 10 ]

- (i) If  $x = \{1, 2, 3, 4, 5, 6\}$  &  $y = \{1, 3, 5, 7, 9\}$ , then find out  $(x-y) \cap (y-x) \cup (x-y)$
- (ii) Which of the following is/are tautology?  
(a)  $p \rightarrow q$  (b)  $\neg p \vee q$  (c)  $(p \rightarrow q) \wedge (q \rightarrow p)$  (d)  $p \rightarrow (q \rightarrow p)$
- (iii) An algebraic structure \_\_\_\_\_ is called a semigroup.  
a)  $(P, *)$   
b)  $(Q, +, *)$   
c)  $(P, +)$   
d)  $(+, *)$
- (iv) What is the number of edges in a null graph with 3 vertices?  
(a) 0 (b) 1 (c) 2 (d) 3
- (v) The statement  $p \wedge (\neg p \vee q)$  is-  
(a) a tautology (b) logically equivalent to  $p \wedge q$  (c) logically equivalent to  $p \vee q$   
(d) contradiction
- (vi) The number of subsets of a set with  $n$  elements is ----  
a)  $2n$  b)  $2^n$  c)  $n$  d) none of these
- (vii)  $p \wedge (q \vee r) \leftrightarrow$   
(a)  $(p \wedge q) \wedge (q \vee r)$  (b)  $p \wedge q \vee r$  (c)  $(p \wedge q) \vee (p \wedge r)$  (d) none of these
- (viii) What is the minimum number of pendant vertices in a tree with 7 vertices?  
(a) 0 (b) 1 (c) 2 (d) 3
- (ix)  $X = \{x : x^2 = 9, 2x = 4\}$ . Is  $X$  the empty set?
- (x) If  $A$  &  $B$  are subsets of a set  $X$  such that  $n(X) = 900, n(A) = 300, n(B) = 400, n(A \cap B) = 130$ , find  $n(A \cup B)$
- (xi) The range of the function  $f : \mathbb{R} \rightarrow \mathbb{R}$  defined by  $f(x) = \begin{cases} x^2, & \text{if } x \geq 0 \\ 2x+3, & \text{if } x < 0 \end{cases}$  is ----  
a)  $\mathbb{R}$  (the set of real numbers),  
b)  $\mathbb{Q}$  (the set of rational numbers),  
c) the set of all positive real numbers,  
d) none of these
- (xii)  $P(n)$  be the statement " $P(n) : n = n+1$ ", then  
a)  $P(n) \rightarrow P(n+1)$   
b)  $P(n)$  is true for all  $n$   
c)  $P(n)$  does not tends  $P(n+1)$   
d) none of these

## Group-B (Short Answer Type Question)

Answer any three of the following

[ 5 x 3 = 15 ]

2. Find  $L^{-1} \{1/(s+2)^2(s-2)\} = ?$  [ 5 ]
3. Find  $L^{-1} \{3s/(4s^2 + 16)\} = ?$  [ 5 ]
4. Solve  $(D^2 - 1)y = a \cosh nt$  where  $y(0) = 0, y'(0) = 2$  [ 5 ]
5. The centre  $Z(G)$  of a group  $G$  is a normal subgroup of  $G$ . [ 5 ]
6. If  $X(z) = (2z^2 + 5z + 14)/(z-1)^4$  be generating function of the sequence  $\{x_n\}_{n=0}^{\infty}$  then find  $x_0, x_1, x_2$  &  $x_3$  [ 5 ]

## Group-C (Long Answer Type Question)

Answer any three of the following

[ 15 x 3 = 45 ]

1/2

7. (a) 1) Prove by mathematical induction that  $6^{n+2} + 7^{2n+1}$  is divisible by 43 for each positive integer  $n$ . [8]  
 (b) 2) Using set theory, find the L.C.M & H.C.F of 48, 72, 24 [5]
8. (a) Let  $G$  be a group &  $a \in G$ . Prove that the mapping  $f_a: G \rightarrow G$  defined by  $f_a(x) = ax \forall x \in G$  is a bijection. [2]  
 (b) Give an example of an abelian group which is not cyclic [2]  
 (c) Find the generators of the group  $(\mathbb{Z}_8, +)$  [6]  
 (d) Let  $H$  be a subgroup of a group  $G$ . Define a relation  $\rho$  on  $G$  by  $a \rho b$  iff  $a^{-1}b \in H$ . Prove that  $\rho$  is an equivalence relation on  $G$ . Also prove that for  $a \in G$ , the equivalence class of  $a$  is the left coset  $aH$  [7]
9. (a) Solve the following recurrence relations by substitution: [7]  
 i)  $a_n = a_{n-1} + n^2, n \geq 1$  Where  $a_0 = 7$   
 ii)  $a_n = a_{n-1} + 2n + 1$  where  $a_0 = 1$
- (b) Solve the following recurrence relations together with the given initial conditions: [8]  
 i)  $a_n = a_{n-1} + 2a_{n-2}, n \geq 2$  where  $a_0 = 0, a_1 = 1$   
 ii)  $a_n - 8a_{n-1} + 16a_{n-2} = 0$  for  $n \geq 2$ , where  $a_2 = 16, a_3 = 80$
10. (a) A relation  $\rho$  on the set  $N$  is given by " $\rho = \{(a, b) \in N \times N : a/b\}$ " Examine if  $\rho$  is i) reflexive ii) symmetric iii) transitive [8]
- (b) A relation  $\rho$  is defined on the set  $Z$  by " $a \rho b$  if and only if  $ab > 0$ " for  $a, b \in Z$ . Examine if  $\rho$  is i) reflexive ii) symmetric iii) transitive [7]
11. (a) Solve the following equation using generating function: [8]  
 $a_n - 5a_{n-1} + 6a_{n-2} = 0, n \geq 2$  with the boundary conditions  $a_0 = 1, a_1 = 1$
- (b) Using generating function solve the following recurrence relation: [7]  
 $a_n - 7a_{n-1} + 10a_{n-2} = 0$  for all  $n \geq 2$  &  $a_0 = 3, a_1 = 3$