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viii) The law of mean is given by

a)  $\frac{f(b) + f(a)}{b - a} = f'(c)$

b)  $\frac{f(b) + f(a)}{b + a} = f'(c)$

c)  $\frac{f(b) - f(a)}{b - a} = f'(c)$

d)  $\frac{f(b) - f(a)}{b - a} = f(c)$ .

ix) If  $x = r \cos \theta$  and  $y = r \sin \theta$ , then the value of  $\frac{\partial (r, \theta)}{\partial (x, y)}$  is

a) 0

b)  $r$

c)  $\frac{1}{r}$

d)  $-r$ .

x) The series  $\sum_{n=1}^{\infty} \frac{2}{e^n}$  is

a) convergent

b) divergent

c) oscillatory

d) none of these.

xi) The function  $f(x) = \begin{cases} x \sin \frac{1}{x} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$  is

a) continuous and differentiable at  $x = 0$

b) continuous but not differentiable at  $x = 0$

c) neither continuous nor differentiable at  $x = 0$

d) none of these.



xii) If  $f(x, y) = \tan(x/y)$ , then  $x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y}$  is

- a)  $\tan(x/y)$                       b)  $\cot(x/y)$   
c) 0                                      d) none of these.

xiii) The moment of inertia of a thin uniform rod of mass  $M$  and length  $2a$  about an axis perpendicular to the rod at its centre is

- a)  $\frac{Ma^2}{3}$                                       b)  $\frac{Ma^2}{2}$   
c)  $Ma^2$                                       d)  $\frac{Ma^2}{4}$ .

xiv) The point of intersection of the line  $\frac{x-1}{2} = \frac{y}{3} = \frac{z+1}{-1}$  with the plane  $x + 2y - z = 5$  is

- a)  $(1, 1, 1)$                               b)  $(0, 1, 3)$   
c)  $\left(\frac{5}{3}, 1, \frac{-4}{3}\right)$                               d) none of these.

xv) The reduction formula of  $I_n = \int_0^{\pi/2} \cos^n x \, dx$  is

- a)  $I_n = \frac{n-1}{n} I_{n-1}$                               b)  $I_n = \frac{n}{n-1} I_{n-1}$   
c)  $I_n = \frac{n-1}{n} I_{n-2}$                               d) none of these.



**GROUP – B**

**( Short Answer Type Questions )**

Answer any *three* of the following.

$$3 \times 5 = 15$$

2. If  $y = (x^2 - 1)^n$ , then show that

$$(x^2 - 1) y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0.$$

3. If  $\vec{a}, \vec{b}, \vec{c}$  are three vectors, then show the

$$[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}] = [\vec{a}, \vec{b}, \vec{c}]^2, \text{ where symbols}$$

have their usual meanings.

4. Test the convergence of the series

5. A, B, C and D are points  $(\alpha, 3, -1), (3, 5, -3), (1, 2, 3)$  and  $(3, 5, 7)$  respectively. If AB is perpendicular to CD, then find the value of  $\alpha$ .

6. If \_\_\_\_\_, then prove that

7. Verify Rolle's theorem for the function

$$f(x) = |x|, -1 \leq x \leq 1.$$



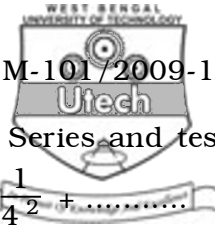
**GROUP – C**

**( Long Answer Type Questions )**

Answer any *three* of the following.

$$3 \times 15 = 45$$

8. a) Examine continuity and differentiability of  $f(x)$  at  $x = 0$ , when  $f(x) = x \sin\left(\frac{1}{x}\right)$ ; ( $x \neq 0$ ) and  $f(0) = 0$ .
- b) Show that
- is not continuous at  $(0, 0)$
- c) Find the extrema of the function
- $$f(x, y) = x^3 + 3xy^2 - 3y^2 - 3x^2 + 4. \quad 5 + 5 + 5$$
9. a) Obtain a reduction formula for  $\int_0^{\pi/2} \sin^n x \, dx$  and evaluate  $\int_0^{\pi/2} \sin^5 x \, dx$ .
- b) If  $z = f(x, y)$  where  $x = e^u \cos v$ ,  $y = e^u \sin v$  then show that
- c) Prove that the function  $f(x) = |x - 1|$ ,  $0 < x < 2$ , is continuous at  $x = 1$ , but not differentiable there. Is it continuous and derivable at  $x = 0$ ? 5 + 5 + 5



10. a) State Leibnitz's theorem for Alternating Series and test convergence of the series  $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$

- b) Define absolute and conditional convergence of Series. Also show that the series  $\sum_{n=1}^{\infty} \frac{\cos nx}{n^2}$  is absolutely

convergent.

6 + 9

11. a) A particle moves on the curve  $x = 2t^2$ ,  $y = t^2 - 4t$ ,  $z = 3t - 5$ , where  $t$  is the time. Find the components of velocity and acceleration at time  $t = 1$  in the direction  $\hat{i} - 3\hat{j} + 2\hat{k}$ .

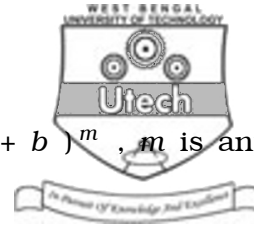
- b) Find the angles between the lines whose direction cosines are given by the equations  $l + m + n = 0$  and  $l^2 + m^2 - n^2 = 0$ .

- c) Find the shortest distance between the lines

$$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1} \text{ and } \frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}.$$

5 + 5 + 5

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12. a) Find the  $n$ -th derivative of  $y = (ax + b)^m$ ,  $m$  is any number.

b) Test the convergence of the series

$$1 + \frac{x}{2} + \frac{x^2}{5} + \frac{x^3}{10} + \dots$$

c) Find :

$\text{div } \vec{F}$  and  $\text{curl } \vec{F}$ , where  $\vec{F} = \text{grad } (x^3 + y^3 + z^3 - 3xyz)$ .

5 + 5 + 5

13. a) Find the whole length of the loop of the curve

$$9y^2 = (x - 2)(x - 5)^2.$$

b) Evaluate

c) State Green's Theorem.

6 + 6 + 3

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