



Name :

Roll No. :

Invigilator's Signature :

CS/B.TECH(N)/SEM-2/M-201/2013

2013

MATHEMATICS – II

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :

$$10 \times 1 = 10$$

i) The general solution of $y = px - \log p$ is

- | | |
|-------------------------|---------------------|
| a) $y = cx - \log c$ | b) $y = 1 + \log x$ |
| c) $y = 1 + \log x + c$ | d) none of these. |

ii) The particular integral of $\frac{d^2y}{dx^2} + y = \cos x$ is

- | | |
|---------------------------|-----------------------------|
| a) $\frac{1}{2} \sin x$ | b) $\frac{1}{2} \cos x$ |
| c) $\frac{1}{2} x \sin x$ | d) $\frac{1}{2} x \cos x$. |



iii) $\frac{1}{D-1}x^2$ is equal to

- a) $x^2 + 2x + 2$ b) $-(x^2 + 2x + 2)$
c) $2x - x^2$ d) $-(2x - x^2)$.

iv) The general solution of $\frac{d^2y}{dx^2} + y = 0$ is

- a) $Ae^x + Be^{-x}$ b) $(A + Bx)e^x$
c) $(A + Bx)\cos x$ d) $A\cos x + B\sin x$.

v) A simple graph can have

- a) no pendant vertex b) no isolated vertex
c) no circuit d) none of these.

vi) A simple graph with 20 vertices and 5 components has at least

- a) 15 edges b) 10 edges
c) 190 edges d) 120 edges.

vii) Which of the following is incorrect about a tree T with n vertices ?

- a) There exist multiple paths between every pair of vertices in T
b) T is minimally connected
c) T is connected and circuitless
d) T has $(n - 1)$ edges.

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xii) $\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{2}{3}\right)$ equals to

a) $\frac{2\pi}{\sqrt{3}}$

b) $\frac{3\pi}{\sqrt{2}}$

c) $\frac{\pi}{\sqrt{3}}$

d) $\frac{\pi}{\sqrt{2}}$.

xiii) $\int_{-\infty}^{\infty} xe^{-x^2} dx =$

a) -1

b) 0

c) 1

d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following

$$3 \times 5 = 15$$

2. Solve : $(x^2y - 2xy^2)dx + (3x^2y - x^3)dy = 0$.

3. Solve the following simultaneous ODE :

$$\frac{dx}{dt} - 7x + y = 0, \quad \frac{dy}{dt} - 2x + 5y = 0$$

4. Prove that the number of edges in a simple graph cannot exceed $\frac{n(n-1)}{2}$.

5. Prove that a graph is a tree if and only if it is minimally connected.

6. Define Gamma function. Show that $\Gamma(n+1) = n\Gamma(n)$. $2 + 3$



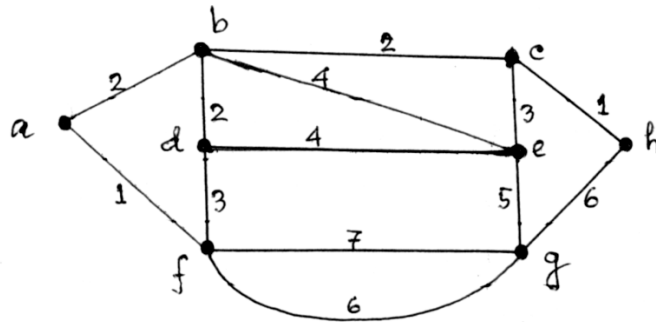
GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following.

3 × 15 = 45

7. a) Apply Dijkstra's algorithm to find shortest path between the vertices *a* and *h* in the following graph : 7



- b) Solve : $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x$ 5

- c) Construct a diagraph from the following incidence matrix : 3

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & -1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \end{bmatrix}$$

8. a) Prove that a tree with *n* vertices has (*n* – 1) edges. 6
b) Solve the following by the method of variation of parameters :

$$\frac{d^2y}{dx^2} + y = \tan x \quad 5$$

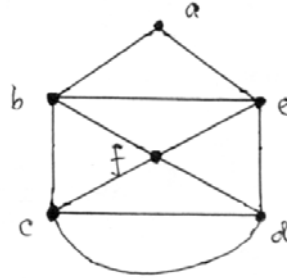
- c) Solve the following differential equation by Laplace Transform :

$$(D^2 + 6D + 9)y = 0, \quad y(0) = y'(0) = 1 \quad 4$$

9. a) i) Define Euler circuit. Write the necessary and sufficient condition for a graph to contain an Euler circuit.



- ii) Find, if possible, an Euler circuit in the following graph :



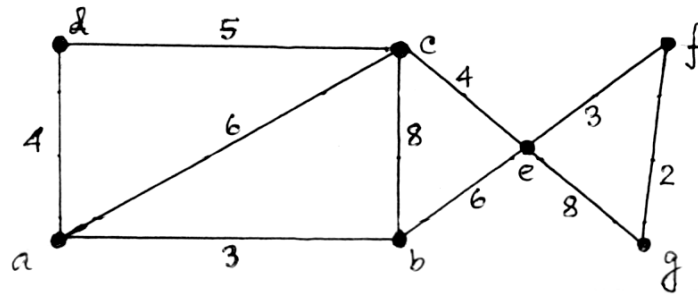
(2 + 1) + 3

- b) Using convolution theorem prove that

$$L^{-1}\left(\frac{s}{(s^2 + a^2)^2}\right) = \frac{t \sin t}{2a} \quad 5$$

- c) Prove that : $\int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$ 4

10. a) By Kruskal's algorithm find a minimal spanning tree in the following graph : 5



- b) Find the Laplace Transform of $f(t)$ defined as :

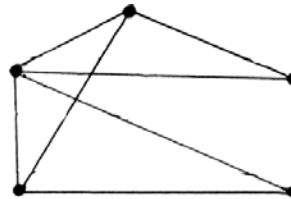
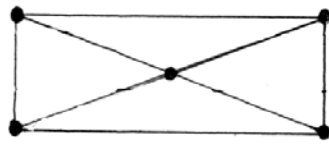
$$f(t) = \begin{cases} \frac{t}{k}, & \text{when } 0 < t < k \\ 1, & \text{when } t > k \end{cases} \quad 4$$

- c) Solve : $x^2 \frac{d^2 y}{dx^2} - 3x \frac{dy}{dx} + 4y = 2x^2$ 6



11. a) Evaluate : $L^{-1} \left[\frac{s^2}{(s^2 + a^2)(s^2 + b^2)} \right]$ 5

b) Examine whether the following graphs are isomorphic or not : 5



c) Solve : $y = px + \sqrt{a^2 p^2 + b^2}$ 5

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