	Utech
Name:	
Roll No.:	A Grant of Sampling and College
Invigilator's Signature :	

$\frac{\text{CS/B.Tech(O)/SEM-1/M-101/2012-13}}{2012}$ MATHEMATICS

Time Allotted: 3 Hours Full Marks: 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP - A

(Multiple Choice Type Questions)

- 1. Choose the correct alternatives for any ten of the following: $10 \times 1 = 10$
 - i) $\lim_{n \to \frac{\pi}{2}} (1 \sin x) \tan x =$
 - a) 1

b) 0

c) $\frac{1}{2}$

- d) e.
- ii) $\frac{dt}{dx}$ of $y = \sin^{-1} x + \sin^{-1} \sqrt{(1 x^2)}$ is
 - a) 0

- b) $\frac{1}{2}$
- c) $\frac{x}{\sqrt{1-x^2}}$
- d) none of these.

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iii)
$$\sum_{n=1}^{\infty} \frac{n}{2n+1}$$
 is

- a) convergent
- b) divergent
- c) neither convergent nor divergent
- d) none of these.

iv) If
$$u + v = x$$
, $uv = y$ then $\frac{\partial (x, y)}{\partial (u, v)} =$

a) u - v

b) uv

c) u + v

d) $\frac{u}{v}$.

v) If
$$u(x, y) = \tan^{-1}\left(\frac{y}{x}\right)$$
, then the value of $x\frac{\partial u}{\partial x} + y\frac{\partial}{\partial y}$ is

a) 0

- b) 2 u (x, y)
- c) u(x, y)
- d) none of these.
- vi) If f(x) is continuous in [a, a + h], derivable in (a, a + h) then $f(a + h) f(a) = hf(a + \theta h)$, where
 - a) θ is any real
- b) $0 < \theta < 1$

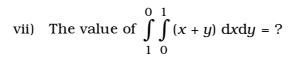
c) $\theta > 1$

d) θ is an integer.

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viii) The series $\sum \frac{1}{n^p}$ is convergent if

a)
$$p \ge 1$$

b)
$$p > 1$$

c)
$$p < 1$$

d)
$$p \le 1$$
.

ix) The reduction formula of $I_n = \int_0^{\pi/2} \cos^n x \, dx$ is

a)
$$I_n = \left(\begin{array}{c} n-1 \\ n \end{array}\right) I_{n-1}$$

b)
$$I_n = \left(\frac{n}{n-1}\right)I_n$$

c)
$$I_n = \left(\begin{array}{c} n-1 \\ n \end{array}\right) I_{n-2}$$

d) none of these.

x) The equation of the straight line passing through (1, 1, 1) and (2, 2, 2) is (x-1) = (y-1) = (z-2)

- a) True
- b) False.

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xi) If
$$f(x, y) = x^3 + 3xy^2 + y^3 + x^2$$
, then
$$x \frac{df}{\partial x} + y \frac{df}{\partial y} = 3f$$



- a) True
- b) False.

GROUP - B

(Short Answer Type Questions)

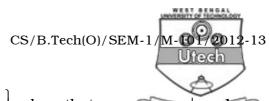
Answer any *three* of the following. $3 \times 5 = 15$

2. For what values of x the following series is convergent?

$$\frac{x}{1.3} + \frac{x^2}{3.5} + \frac{x^3}{5.7} + \dots$$

- 3. Find the moment of inertia of a thin uniform rectangular lamina of adjacent sides of lengths 2a and 2b and of mass M about an axis of symmetry through its centre.
- 4. Evaluate $\int_C (3xy \, dx y^2 \, dy)$ where C is the arc of the parabola $y = 2x^2$ from (0,0) to (1,2).

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5. If $y_n = \frac{d^n}{dx^n} \left\{ x^n \log_e x \right\}$, show that $y_n = ny_{n-1} + \lfloor n-1 \rfloor$

GROUP - C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

- 6. a) Verify Gauss divergence theorem for the vector field $F = y i + x j + z^2 k$ over the cylindrical region bounded by $x^2 + y^2 = 9$, z = 0, z = 2.
 - b) Verify Stokes' theorem for $\overrightarrow{A} = (y-z+2) \overrightarrow{i} + (yz+4) \overrightarrow{j} xz \overrightarrow{k}$ over the surface of the cube x = y = z = 0 and x = y = z = 2 above xy plane.

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7. a) Using mean value theorem prove that

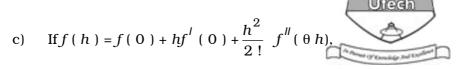
$$\frac{x}{1+x^2} < \tan^{-1} x < x, \quad 0 < x < \frac{\pi}{2}.$$

b) If z is a function of x and y and $x = r \cos \theta$, $y = r \sin \theta$ then prove that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{\partial^2 z}{\partial r^2} + \frac{1}{r} \frac{\partial z}{\partial r} + \frac{1\partial^2 z}{r^2 \partial \theta^2}$.

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$$0 < \theta < 1, f(x) = 1 / (1 + x)$$
 and $h = 7$, find θ .

8. a) Show that for the function

$$f(x, y) = \begin{cases} \frac{x^2 y^2}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

$$f_{xy}(0, 0) = f_{yx}(0, 0).$$
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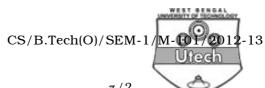
- b) State comparison test for convergence of an infinite series. Test the convergence of any *one* of the following series:
 - (i) $\frac{6}{1.3.5} + \frac{8}{4.5.7} + \frac{10}{5.7.9} + \dots$

(ii)
$$1 + \frac{2^p}{2!} + \frac{3^p}{3!} + \frac{4^p}{4!} + \dots (p > 0)$$
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c) Find the extreme values, if any, of the following function:

$$f(x, y) = x^3 + y^3 - 3axy.$$
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- 9. a) Obtain the reduction formula for $\int_{0}^{\pi/2} \cos^{n} x \, dx$. Hence evaluate $\int_{0}^{\pi/2} \cos^{5} x \, dx$.
 - b) Compute the value of $\bigvee_R\bigvee_R$ where R is the region in the first quadrant bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$
 - c) Obtain the reduction formula for $\int_{0}^{\pi/2} \sin^{m} x \cos^{n} x dx$, where m, n are positive integers (m > 1, n > 1). Hence evaluate $\int_{0}^{\pi/2} \sin^{4} x \cos^{8} x dx$.