2013

NUMERICAL METHODS

Time Allotted : 3 Hours | Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP - A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) The ratio of absolute error of the true value is

a) Relative error

b) Absolute error

c) Truncation error

d) Inherent error.

ii) The significant digit of 0·0001234 is

a) 7 b) 4

c) 8 d) 6

iii) The percentage error in approximation of 4/3 to 1·3333 is

a) 0·0025% b) 25%

c) 0·000025% d) 0·25%.

iv) If the interval of differencing is unity and $f(x) = ax^2$ (a is constant), which one of the following choices is wrong ?

a) $\Delta f(x) = a(2x + 1)$

b) $\Delta^2 f(x) = 2a$

c) $\Delta^3 f(x) = 2$

d) $\Delta^4 f(x) = 0$.

v) In Simpson's 1/3 rule of finding $\int_a^b f(x)\,dx$, $f(x)$ is approximated by

a) line segment

b) parabola

c) circular sector

d) part of ellipse.

vi) Runge-Kutta formula has a truncation error which is of the order of

a) h^2 b) h^3

c) h^4 d) h^5.

vii) If $f(x) = \frac{1}{x^2}$, then the dividend difference $f(a, b)$ is

a) $\frac{(a+b)}{(ab)^2}$ b) $\frac{(-a-b)}{(ab)^2}$

c) $\frac{1}{(a^2-b^2)}$ d) $\frac{1}{a^2} - \frac{1}{b^2}$.

viii) The method of Iteration formula $\varphi(x)$ must satisfy

a) $|\phi'(x)| < 1$ b) $|\phi'(x)| > 1$

c) $|\phi'(x)| = 1$ d) $|\phi'(x)| = 2$.

ix) Which of the following methods is an iterative method ?

a) Gauss-elimination method

b) Gauss-Seidel method

c) LU-factorization method

d) Matrix-inversion method.

x) Regula-falsi method is

a) conditionally convergent

b) linearly convergent

c) divergent

d) none of these.

xi) Simpson's one-third rule is applicable only when the number of sub-intervals is

a) even b) odd

c) both even & odd d) none of these.

xii) In LU-factorization method, the given system equation represented by $AX = B$ is converted to another system $LUX = B$ where U is

a) lower triangular matrix

b) upper triangular matrix

c) identity matrix

d) null matrix.

GROUP - B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Show that if Δ operates on n, then $\Delta\binom{n}{x+1} = \binom{n}{x}$ and hence $\sum_{n=1}^{N} \binom{n}{x} = \binom{n+1}{x+1} - \binom{1}{x+1}$.

3. Evaluate $\int_0^1 \cos x \, dx$, taking five equal intervals. Explain the reason behind your choice of integration formula used.

4. Apply Lagrange's interpolation formula to find $f(x)$ using following table :

x :	1	2	3	4	7
$f(x)$:	2	4	8	16	128

5. Solve by using Euler's method the following differential equation for $x = 1$ by taking $h = 0{\cdot}2$. $\frac{dy}{dx} = xy$, $y = 1$ when $x = 0$.

6. Solve the system of linear equations by Gauss-Jordan method :

$$2x + y + z = 0$$

$$3x + 2y + 3z = 0$$

$$x + 4y + 9z = 16.$$

GROUP - C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Compute $f(0{\cdot}23)$ and $f(0{\cdot}29)$ using suitable formula from the table given below : 7

x :	0·20	0·22	0·24	0·26	0·28	0·30
$f(x)$:	1·6596	1·6698	1·6804	1·6912	1·7024	1·7139

b) Describe geometric significance of Simpson's $\frac{1}{3}$ rule. 5

c) Determine the absolute error E_A of the following approximate number given their relative error $x_A = 67{\cdot}84$, $E_R = 1\%$. 3

8. a) Using Gauss-Seidel method find the solution of the following system of linear equations correct up to two decimal places :

$3x + y + 5z = 13$, $5x - 2y + z = 4$, $x + 6y - 2z = -1$. 7

b) Solve the equation $\frac{dy}{dx} = \frac{1}{x+y}$, $y(0) = 1$, for $y(0{\cdot}1)$ and $y(0{\cdot}2)$. using Runge-Kutta method of the fourth order. 8

9. a) Round off 35·7218 to four significant figures. 1

b) What is interpolation ? Prove that

$$f(x) = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \ldots + \frac{u(u-1)\ldots(u-n+1)}{n!}\Delta^n y_0 .$$

1 + 5

c) Prove that $\nabla^r y_k = \nabla^r y_{k+r}$. 3

d) Find the mising term : 5

x :	1	2	3	4	5	6	7
$f(x)$:	2	4	8	?	32	64	128

10. a) Prove the convergence of Newton-Raphson method. Hence find the cube root of 10 up to 5 significant figures by Newton Raphson method. 5 + 5

b) Evaluate $\int_0^{0.6} \frac{dx}{\sqrt{1-x^2}}$, using Weddle's rule taking 12 equal sub-intervals. 5

11. a) Find the polynomial $f(x)$ and hence calculate $f(5.5)$ for the given data : 4

x:	0	2	3	5	7
$f(x)$:	1	47	97	251	477

b) Given $\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2}$, $y(1) = 1$. Evaluate $y(1.2)$ by modified Euler's method correct up to 4 decimal places. 6

c) Solve the following system of equations by L-U decomposition method : 5

$x + y - z = 2$, $2x + 3y + 5z = -3$, $3x + 2y - 3z = 6$.