Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/B.Tech (ECE-NEW)/SEM-7/EC-704B/2010-11

2010-11

ADVANCED ENGINEERING MATHEMATICS FOR ELECTRONICS ENGINEERING

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

   i) If $A = (a_{ij})$ be an $n \times n$ matrix and A_{rs} be the cofactor a_{rs} in det A, then

      a) $\det (A_{ij}) = [\det (a_{ij})]^{n-1}$

      b) $\det (A_{ij}) = [\det (a_{ij})]^{n}$

      c) $\det (A_{ij}) = [\det (a_{ij})]^{1-n}$

      d) none of these.

ii) The value of $J_{1/2}(x)$ is equal to

a) $\sqrt{\frac{2}{\pi x}}\sin x$

b) $\sqrt{\frac{b}{\pi x}}\tan x$

c) $\sqrt{\frac{2}{\pi x}}\cos x$

d) $\sqrt{\frac{2}{\pi x}}\sec x$.

iii) The Fourier sine transform of $\frac{1}{x}$ is

a) 1 b) $\frac{\pi}{2}$

c) 0 d) π.

iv) Let A be an orthogonal matrix of order 2 having two eigenvalues λ_1 and λ_2. Then the eigenvalues of A^{-1} are

a) $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}$

b) λ_1, λ_2

c) $\lambda_1 + \lambda_2, \lambda_1 - \lambda_2$

d) λ_1^2, λ_2^2.

v) The eigenvalues of a real skew-symmetric matrix are

a) purely imaginary or zero

b) real

c) 0

d) none of these.

vi) The general solution of $\left(D^2 - 3DD' + 2D'^2\right)z = e^{2x-y}$ is given by

a) $z(x,y) = f(x+y) + g(2x+y) + \frac{1}{12}e^{2x-y}$

b) $z(x,y) = f(x+y) + g(2x+y) - \frac{1}{12}e^{2x-y}$

c) $z(x,y) = f(x-y) + g(2x-y) - \frac{1}{6}e^{2x-y}$

d) $z(x,y) = f(x-y) + g(2x+y) + \frac{1}{6}e^{2x-y}$.

vii) The function $f(z) = |z|^2$ is

a) continuous nowhere

b) continuous everywhere but nowhere differentiable

c) continuous everywhere but nowhere differentiable except at the origin

d) continuous at origin only.

viii) The value of the integral $\int_C \frac{1}{z-a} dz$ where C is the circle $|z-a| = r$ is

a) 0 b) $2\pi i$

c) πi d) $-\pi i$.

ix) The residue of a function can be evaluated only if the pole is an isolated singularity.

a) True

b) False.

x) The residue of $\frac{2+3\sin z}{z(z-1)}$ at $z = 0$ is

a) -2 b) 3

c) i d) 2.

xi) The sum of eigenvalues of matrix A is equal to

a) the trace of A b) the determinant of A

c) 1 d) 0.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Show that $\frac{d}{dx}\left[x^n J_n(x)\right] = x^n J_{n-1}(x)$.

3. Solve : $z^2\left(x^2p^2 + y^2q^2\right) = 1$

4. Evaluate $L^{-1}\left\{\frac{1}{s^2(s+1)^2}\right\}$ using convolution theorem.

5. Show that for a matrix $A=\begin{pmatrix}1 & -3 & 3\\ 3 & -5 & 3\\ 6 & -6 & 4\end{pmatrix}$, product of eigenvalues of $A = -|A|$.

6. Using contour integration evaluate $\int_0^{\pi}\frac{1+2\cos\theta}{5+4\cos\theta}\,d\theta$.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Find the Fourier cosine transform of e^{-x^2}.

b) Solve completely the equation $\frac{\partial^2 y}{\partial t^2}=e^2\frac{\partial^2 y}{\partial x^2}$, representing the vibration of a string of length l, fixed at both ends, subject to the condition

i) $y(0, t) = 0$

ii) $y(l, t) = 0$

iii) $y(x, 0) = f(x)$

iv) $\frac{\partial y(x,0)}{\partial t}=0,\ 0<x<l$. 7 + 8

8. a) Using residue theorem show that

$$\int_0^{2\pi} \frac{d\theta}{1 + a\sin\theta} = \frac{2\pi}{\sqrt{1-a^2}}, \; (-1 < a < 1)$$

b) Using convolution theorem evaluate :

$$L^{-1}\left\{\frac{s}{\left(s^2 + a^2\right)^2}\right\}.$$

9. a) Expand $f(z) = \sin z$ in a Taylor series about $z = \pi/4$ and determine the region of convergence of this series.

b) Determine the region of w-plane into which the region bounded by $x = 1$, $y = 1$ and $x + y = 1$ is mapped by the transformation $w = z^2$. 10 + 5

10. a) State and prove Rodrigue's formula.

b) Prove that $\int P_m(x) P_n(x) = 0$, where $m \neq n$, $P_m(x), P_n(x)$ being Legendre polynomials of order m & n respectively.

c) Prove that $J_n(x) = \frac{x}{2n}\left[J_{n-1}(x) + J_{n+1}(x)\right], J_n(x)$ being Bessel's function of order n. 6 + 4 + 5

11. a) Applying elementary row operation find the rank of the matrix $\begin{pmatrix} 1 & 2 & 1 & 0 \\ 2 & 4 & 8 & 6 \\ 0 & 0 & 5 & 8 \\ 3 & 6 & 6 & 3 \end{pmatrix}$.

b) Diagonalize the matrix $\begin{pmatrix} 3 & 2 & 1 \\ 2 & 3 & 1 \\ 0 & 0 & 1 \end{pmatrix}$.

c) Determine α, β, γ so that the matrix $\begin{pmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{pmatrix}$ is orthogonal. 5 + 5 + 5