	Utech
Name:	
Roll No.:	To Disease of Exemplify 2nd Explane
Invigilator's Signature :	

CS/B.Tech (OLD)/SEM-1/M-101/2011-12 2011 MATHEMATICS

Time Allotted: 3 Hours Full Marks: 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP - A

(Multiple Choice Type Questions)

- 1. Choose the correct alternatives for any ten of the following: $10 \times 1 = 10$
 - i) The series $\sum x^n = 1 + x + x^2 + \dots \infty$ is convergent if
 - a) $x \ge 1$

- b) $x \le -1$
- c) -1 < x < 1
- d) none of these.
- ii) A positive term series cannot be oscillatory.
 - a) True
 - b) False.
- iii) The limit $\lim_{x\to 0} |x|/x$ does not exist.
 - a) True
 - b) False.

1151 (O) [Turn over

CS/B.Tech (OLD)/SEM-1/M-101/2011-12



- iv) If $y = x^n$, then y_n is equal to
 - a) 0

b) 1

c) n!

- d) none of these.
- v) The function $\log x$ is defined for
 - a) all values of x
- b) x > 0

c) x < 0

- d) x = 0.
- vi) If $I_n = \int_0^{\pi/2} \sin^n x \, dx = \{(n-1)/n\} I_n 2$, the value of $\int_0^{\pi/2} \sin^6 x \, dx$ is
 - a) 0

b) $\frac{5\pi}{32}$

c) $\frac{5}{32}$

- d) 1
- vii) If the arc is symmetrical about the x-axis, then for C.G. of the arc
 - a) x = 0

b) u = 0

c) x = y

- d) none of these.
- viii) $f(x, y) = (x^2 + y^2)^{1/3}$ is a homogeneous function of degree
 - a) $\frac{1}{3}$

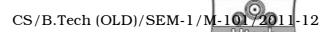
b) $\frac{2}{3}$

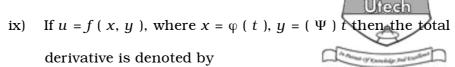
c) 2

d) none of these.

1151 (O)

2





a) $\frac{1}{\partial t}$ b) $\frac{\partial x}{\partial t}$

c)

d) $\frac{\partial y}{\partial t}$.

If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then the angle between the vectors \vec{a} x) and \vec{b} is

90° a)

b) 45°

c) 60° d) none of these.

The centre and radius the sphere xi) $2x^2 + 2y^2 + 2z^2 - 2x + 4y + 2z + 3 = 0$ are

- a) (-1, 2, 1); 2
- b) (1, -2, -1); 1
- c) $\left(\frac{1}{2}, -1, -\frac{1}{2}\right)$; 0 d) $\left(-\frac{1}{2}, 1, \frac{1}{2}\right)$; 3.

The normal to the planes x-y+z=1, 3x+2y-z+2=0are

- a) parallel
- inclined at angle $\frac{\pi}{3}$ b)
- passing through origin c)
- perpendicular. d)

1151 (O)

3

[Turn over





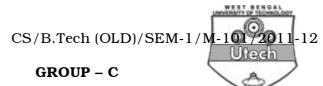
(Short Answer Type Questions)

Answer any three of the following.

- $y = \tan^{-1} x$. 2. If then prove that $\left(1+x^{2}\right)y_{n+1}+2nx\,y_{n}+n\left(n-1\right)y_{n-1}=0\,.$ Find also the value of $(y_n)_0$.
- Test the convergence of the series $1 + \frac{1}{2^2} + \frac{2^2}{3^3} + \frac{3^3}{4^4} + \frac{4^4}{5^5} + \dots$.
- Obtain the reduction formula for $\int_{0}^{\pi/2} \cos^{n} x \, dx$ and hence 4. evaluate $\int_{0}^{\pi/2} \cos^{7} x \, dx$.
- Find the directional derivative of $p = 4e^{2x-y+z}$ at the point 5. (1, 1, -1) in a direction towards the point (-3, 5, 6).
- 6. Find the points of intersection the line x + y + z + 1 = 0 = 14x + 9y - 7z - 1 with XY and YZ planes and hence put down the symmetrical form of its equations.
- Find a point in the plane x + 2y + 3z = 13 nearest to the 7. point (1, 1, 1) using the method of Lagrange's multipliers.

1151 (O)

4



(Long Answer Type Questions

Answer any *three* of the following. $3 \times 15 = 45$

8. a) Examine if the function

$$f(x) = x \tan^{-1}\left(\frac{1}{x}\right), \ x \neq 0$$
$$= 0 \qquad , x = 0$$

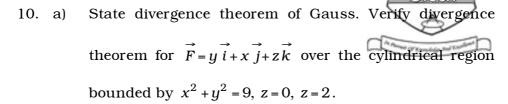
is derivable at x = 0.

- b) State Leibnitz's theorem. If $y = e^{m \cos^{-1} x}$, show that $(1-x^2)y_{n+2} (2n+1)xy_{n+1} (n^2 + m^2)y_n = 0.$
- c) If $y^{1/m} + y^{-1/m} = 2x$, prove that $(x^2 1)y_{n+2} + (2n+1)xy_{n+1} + (n^2 m^2)y_n = 0.$
- 9. a) State Rolle's thorem. Verify Rolle's theorem for the $\operatorname{function} f(x) = \sin x \cos x \text{ in } [0, \pi/2].$
 - b) Prove that

$$(if 0 < a < b < 1), (b-a)/(1+b^2) < tan^{-1}b - tan^{-1}a < (b-a)/(1+a^2)$$

- c) Expand sin *x* in power of *x* in infinite series stating the condition under which the expansion is valid.
- 1151 (O) 5 [Turn over

CS/B.Tech (OLD)/SEM-1/M-101/2011-12



- b) State Green's theorem in plane. Verify Green's theorem in plane for $\int_C (3x^2 8y^2) dx + (4y 6xy) dy$, where C is the boundary of the region defined $y = x^{1/2}$, $y = x^2$.
- c) Find the work done in moving a particle in the force field $\vec{F} = 3x^2 \vec{i} + (2xz y) \vec{j} + z \vec{k}$ along the curve defined by $x^2 = 4y$, $3x^3 = 4z$ from x = 0 to x = 2.
- 11. a) Show that $\overrightarrow{\nabla} r^n = nr^{n-2} \overrightarrow{r}$ where $\overrightarrow{r} = x \overrightarrow{i} + y \overrightarrow{j} + z \overrightarrow{k}$.
 - b) Prove that *curl* [φ *grad* φ] = 0.
 - c) Find the co-ordinate of the foot of the perpendicular from (1, 2, 3) on the line (x-2)/1 = (y-1)/2 = z/3. Find also the length of the perpendicular and its equation.

1151 (O)



- 12. a) Evaluate $\iint (4x^2 y^2)^{1/2} dx dy$ over the triangle formed by the straight lines y = 0, x = 1 and y = x.
 - b) Evaluate $\int_{0}^{a} \int_{0}^{x} \int_{0}^{x+y} e^{x+y+z} dz dy dx.$
 - c) If $f(x, y) = (x^2 + y^2)^{1/3}$, use Euler's theorem to find the value of $x(\partial f/\partial x) + y(\partial f/\partial x)$ and hence prove that, $x^2(\partial^2 f/\partial x^2) + 2xy(\partial^2 f/\partial x\partial y) + y^2(\partial^2 f/\partial y^2) + (2/9)f = 0.$

1151 (O)

7

[Turn over