Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/B.TECH(N)/SEM-2/M-201/2013

2013

MATHEMATICS – II

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) The general solution of $y = px - \log p$ is

a) $y = cx - \log c$ b) $y = 1 + \log x$

c) $y = 1 + \log x + c$ d) none of these.

ii) The particular integral of $\frac{d^2y}{dx^2} + y = \cos x$ is

a) $\frac{1}{2}\sin x$ b) $\frac{1}{2}\cos x$

c) $\frac{1}{2}x\sin x$ d) $\frac{1}{2}x\cos x$.

iii) $\frac{1}{D-1}x^2$ is equal to

a) x^2+2x+2 b) $-(x^2+2x+2)$

c) $2x-x^2$ d) $-(2x-x^2)$.

iv) The general solution of $\frac{d^2y}{dx^2}+y=0$ is

a) Ae^x+Be^{-x} b) $(A+Bx)e^x$

c) $(A+Bx)\cos x$ d) $A\cos x+B\sin x$.

v) A simple graph can have

a) no pendant vertex b) no isolated vertex

c) no circuit d) none of these.

vi) A simple graph with 20 vertices and 5 components has at least

a) 15 edges b) 10 edges

c) 190 edges d) 120 edges.

vii) Which of the following is incorrect about a tree T with n vertices ?

a) There exist multiple paths between every pair of vertices in T

b) T is minimally connected

c) T is connected and circuitless

d) T has ($n-1$) edges.

viii) If the incidence matrix of a graph has five identical columns, the G has

a) five loops b) five isolated vertices

c) five parallel edges d) five edges in series.

ix) $L^{-1}\left(\frac{s}{s^2-a^2}\right) =$

a) $\sin at$ b) $\sin hat$

c) $\cos at$ d) $\cos hat$.

x) $L\{H(t-a)\}$, H being Heavyside unit step function, is

a) e^{-as} b) se^{-as}

c) $\frac{e^{-as}}{s}$ d) none of these.

xi) Laplace transform of $\frac{\sin 2t}{t}$ is

a) $\cot^{-1}\frac{s}{2}$ b) $\cot^{-1}\frac{2}{s}$

c) $\frac{2}{s^2+4}$ d) $\frac{s}{s^2-4}$.

xii) $\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{2}{3}\right)$ equals to

a) $\frac{2\pi}{\sqrt{3}}$ b) $\frac{3\pi}{\sqrt{2}}$

c) $\frac{\pi}{\sqrt{3}}$ d) $\frac{\pi}{\sqrt{2}}$.

xiii) $\int_{-\infty}^{\infty} xe^{-x^2}\,dx =$

a) -1 b) 0

c) 1 d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following $3 \times 5 = 15$

2. Solve : $(x^2y - 2xy^2)\,dx + (3x^2y - x^3)\,dy = 0$.

3. Solve the following simultaneous ODE :

$$\frac{dx}{dt} - 7x + y = 0,\ \frac{dy}{dt} - 2x + 5y = 0$$

4. Prove that the number of edges in a simple graph cannot exceed $\frac{n(n-1)}{2}$.

5. Prove that a graph is a tree if and only if it is minimally connected.

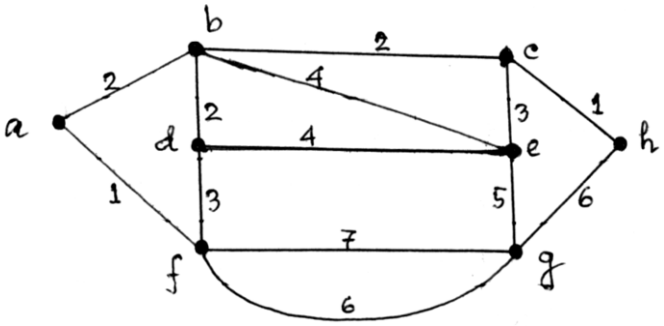
6. Define Gamma function. Show that $\Gamma(n+1) = n\Gamma(n)$. 2 + 3

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Apply Dijkstra's algorithm to find shortest path between the vertices a and h in the following graph : 7



b) Solve : $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x$ 5

c) Construct a diagraph from the following incidence matrix : 3

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & -1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \end{bmatrix}$$

8. a) Prove that a tree with n vertices has ($n - 1$) edges. 6

b) Solve the following by the method of variation of parameters :

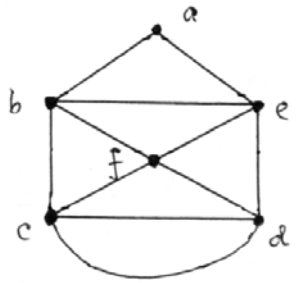
$$\frac{d^2y}{dx^2} + y = \tan x$$ 5

c) Solve the following differential equation by Laplace Transform :

$(D^2 + 6D + 9)y = 0,\ y(0) = y'(0) = 1$ 4

9. a) i) Define Euler circuit. Write the necessary and sufficient condition for a graph to contain an Euler circuit.

ii) Find, if possible, an Euler circuit in the following graph :



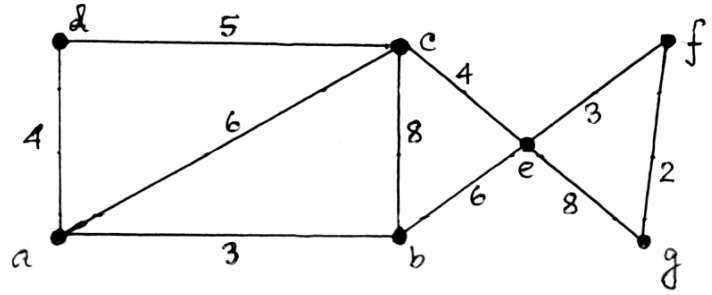
(2 + 1) + 3

b) Using convolution theorem prove that

$$L^{-1}\left(\frac{s}{(s^2+a^2)^2}\right)=\frac{t\sin t}{2a}$$ 5

c) Prove that : $\int_0^{\infty} e^{-x^2}\,dx=\frac{\sqrt{\pi}}{2}$ 4

10. a) By Kruskal's algorithm find a minimal spanning tree in the following graph : 5



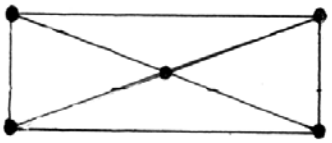
b) Find the Laplace Transform of $f(t)$ defined as :

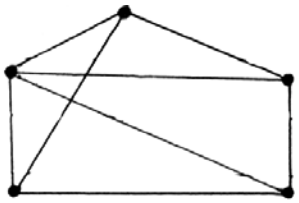
$$f(t)=\begin{cases}\frac{t}{k}, \text{when } 0<t<k\\ 1, \text{when } t>k\end{cases}$$ 4

c) Solve : $x^2\frac{d^2y}{dx^2}-3x\frac{dy}{dx}+4y=2x^2$ 6

11. a) Evaluate : $L^{-1}\left[\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right]$ 5

b) Examine whether the following graphs are isomorphic or not : 5





c) Solve : $y = px + \sqrt{a^2p^2 + b^2}$ 5

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