Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS / B.TECH(NEW)BME / ECE / EE / EIE / PWE / ICE / EEE / SEM-3 / M-302 / 2012-13

2012

MATHEMATICS - III

Time Allotted : 3 Hours | *Full Marks :* 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

1. Answer any *ten* from the following : 10 × 2 = 20

   i) If $f(x) = |x|, -2 < x < 2$ is a periodic of period 4, be represented in a Fourier series, then find the value of a_0.

   ii) Using Fourier transform evaluate $\int_0^\infty \frac{\cos t}{1+s^2}\, ds$.

   iii) Find the poles of the function $f(z) = \frac{z^2}{(z-1)^2(z+2)}$.

   iv) Evaluate $\int_C \frac{e^z}{z-2}\, dz$, where $c : |z-2| = 4$.

   v) Find the probability that a leap year selected at random contain 53 Tuesdays.

vi) A random variable X has the *p.d.f.*

$$f(x) = 3x,\ 0 < x < 1$$
$$= 0, \text{ otherwise.}$$

Then find $P(2x + 3 > 2)$.

vii) Express $J_3(x)$ in terms of $J_0(x)$ and $J_1(x)$.

viii) Obtain the partial differential equation for $z = (x + a)(x + b)$, where a and b are arbitrary constants.

ix) Determine the singular point of the following equation :

$$x(x-1)\frac{d^2y}{dx^2} + (3x-1)\frac{dy}{dx} + y = 0$$

x) If A and B are independent events then show that A^C and B^C are independent.

xi) If X is normally distributed with zero mean and unit variance, find $E(X^2)$.

xii) Check whether the function

$u(x, y) = 2xy + 3xy^2 - 2y^3$ is harmonic or not.

xiii) Write the Convolution Theorem of Fourier Transform.

xiv) Prove that $P_n(-1) = (-1)^n$.

xv) The mean and standard deviation of a Binomial distribution are 4 and $\sqrt{3}/3$ respectively. Find n and p.

GROUP – B

(Answer any *five* questions taking at least one question from each Module) $5 \times 10 = 50$

Module - I

2. a) Obtain the Fourier series to represent : 6

$f(x) = x^2$ in $-\pi \le x \le \pi$.

Hence show that $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\ldots\ldots = \frac{\pi^2}{6}$.

b) Find the Fourier inverse transform of the function $F(p) = \frac{1}{p^2 + 4p + 13}$. 4

3. a) State Parseval's Identify on Fourier cosine transform. Find the Fourier cosine transform of e^{-ax}, $a > 0$. 5

b) Using the Parseval's identify of Fourier cosine transform, show that

$$\int_0^\infty \frac{dx}{(a^2 + x^2)(b^2 + x^2)} = \frac{\pi}{2ab(a+b)},$$

where $a > 0$, $b > 0$. 5

Module - II

4. a) Show that by considering the function $f(z)$ defined as

$$f(z) = \frac{xy(y - ix)}{x^2 + y^2}, \text{ for } z \neq 0$$
$$= 0 \quad \text{for } z = 0,$$

the C-R equations are not the sufficient conditions for a function to be analytic. 5

b) Expand the function $f(z) = \dfrac{1}{(z+1)(z+3)}$ in a Laurent's series valid for $0 < |z+1| < 2$. 5

5. a) Evaluate $\displaystyle\int_0^{2\pi} \frac{d\theta}{2 + \cos\theta}$ 5

b) Use Cauchy residue theorem to evaluate

$$\int_C \frac{3z^2 + z - 1}{(z^2 - 1)(z - 3)}\, dz \text{ around the circle } C : |z| = 2.$$

5

Module – III

6. a) A random variable X has the following probability function :

X	0	1	2	3	4	5	6	7
$P(x)$	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2+K$

Obtain the value of K and estimate P (X < 6) and P (0 < X < 5). 5

b) In a bolt factory, the machines MI, M2 and M3 manufacture respectively 25, 35 and 40 per cent of the total product. Of their output 5, 4 and 2 per cent respectively are defective bolts. One bolt is drawn at random from the product and is found to be defective. What is the probability that it was manufactured by machine M3 ? 5

7. a) In a shooting competition the probability of a man hitting a target is $\frac{1}{5}$. If he fires 5 times, what is the probability of hitting the target at least twice ? 5

b) Assuming that the height distribution of a group is normally, find the mean and *s.d.* if 84% of the men have heights less than 65·2 inches and 68% have heights lying between 65·2 and 62·8 inches.

$$\left[\text{given} \int_{-\infty}^{0\cdot 9} \phi(t)\, dt = 0\cdot 84, \int_{-\infty}^{-0\cdot 9} \phi(t)\, dt = 0\cdot 16 \right]$$

5

Modulue - IV

8. Use Laplace Transform to solve the one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ $(x > 0, t > 0)$

where $u(x, 0) = 0$, $\frac{\partial u}{\partial t}(x, 0) = 0$, $x > 0$

$$u(0, t) = F(t),\ u(\infty, t) = 0,\ t \geq 0.$$ 10

9. a) Solve in series the equation

$$(1 + x^2)\frac{d^2 y}{dx^2} + x\frac{dy}{dx} - y = 0.$$ 5

b) Establish the recurrence formula for Legendre Polynomials

$$(2n + 1)\, x P_n = (n + 1) P_{n+1} + n P_{n-1}.$$ 5