



Name :

Roll No. :

Invigilator's Signature :

CS/B.Tech (CSE)/SEM-7/CS-704E/2010-11
2010-11
COMPUTATIONAL GEOMETRY

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for the following : $10 \times 1 = 10$

i) A conflict graph is a

- | | |
|-------------------|--------------------|
| a) line graph | b) bipartite graph |
| c) complete graph | d) directed graph. |

ii) Let V , E and F are the number of vertices, edges and faces (including the infinite faces) respectively. Euler's formula says

- | | |
|--------------------|----------------------|
| a) $V - E + F = 2$ | b) $V + E = F$ |
| c) $V + E - F = 2$ | d) $V - E - F = 2$. |



- iii) A simple polygon is a polygon
- a) whose vertices are finite
 - b) with only one reflex angle
 - c) with only one convex angle
 - d) whose boundaries do not intersect.
- iv) Triangulation means to
- a) divide a region into triangles in respect to planner point location problem
 - b) prepare a triangular search window
 - c) partition a polygon into non-overlapping triangles using diagonals only
 - d) generate triangles in incremental hall algorithm.
- v) Every triangulation of an n -gon has exactly triangles.
- a) $n - 2$
 - b) $n^2 - 2$
 - c) n
 - d) n^2 .
- vi) In an art gallery with n vertices, necessary numbers of cameras are the
- a) smallest integer not less than $(n/3)$
 - b) largest integer not greater than $(n/3)$
 - c) smallest integer not less than $(n/2)$
 - d) largest integer not greater than $(n/2)$.



vii) *kd* tree is used in

- a) line segment searching
- b) point location searching
- c) range searching
- d) searching indexes.

viii) A Voronoi cell is the intersection of $n - 1$
(n is the number of sites).

- a) polylines
- b) contours
- c) half-planes
- d) parabolas.

ix) In Voronoi diagram, equi-distant sites are

- a) co-circular
- b) co-linear
- c) co-triangular
- d) impossible to exist.

x) Chan's algorithm for computing a convex hull is

- a) an output-sensitive algorithm
- b) an input-sensitive algorithm
- c) a neutral algorithm
- d) a computation-sensitive algorithm.



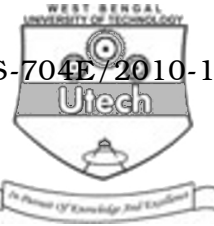
GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following.

$3 \times 5 = 15$

2. Suppose we have a set of n points P in the plane such that exactly h of them lies on the convex hull. How many triangles are there in any triangulation of P ? Give your answer in terms of n and h .
3. Prove the theorem : Let P be a convex polytope with n vertices. The number of edges of P is at most $3n - 6$ and number of facets is at most $2n - 4$.
4. Let P_1 and P_2 be two disjoint convex polygons with n vertices in total. Describe an $O(n \log n)$ -time algorithm that computes the convex hull of P_1, P_2 .
5. Given a set S on n sites, $\text{Vor}(S)$ is the Voronoi diagram on S . Let V, E, F be the numbers of vertices, edges and faces respectively, of $\text{Vor}(S)$. Then prove that, $F = n$ and $V \leq 2n - 5$.
6. Prove that : Every simple polygon admits a triangulation, and any triangulation of a simple polygon with n vertices consists of exactly $n - 2$ triangles.



GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Describe the beach line algorithm to construct a Voronoi diagram. 5
- b) Prove that the complexity of the beach line algorithm is $O(n)$. 5
- c) Discuss any two important applications of Voronoi diagram. 5
8. a) Give a set of data : 1, 3, 4, 7, 9, 12, 14, 15, 17, 20, 22, 24, 25, 27, 29 and 31.
- i) Construct a balanced binary tree 4
- ii) Describe how you search for a range [2 : 23]. 5
- b) “Let P be the set of n points in a 1-dimensional space. The set P can be stored in balanced binary search tree, which uses $O(n)$ storage and has $O(n \log n)$ construction time, such that the points in the query range can be reported in time $O(k + \log n)$, where k is the number of reported points.” Justify the statement. 6



9. a) Write an algorithm that can construct a *kd-tree* in two dimensions. 5
- b) Prove that a *kd-tree* for a set of n -points in two dimensions uses $O(n)$ storage and can be constructed in $O(n \log n)$ time. 5
- c) Given the root of the *kd-tree* and a search range R . Write an algorithm to search the *kd-tree*. 5
10. a) In case of non-convex polygon, discuss the following methods of finding out whether a point lies within a polygon or not :
- i) Computing winding numbers. 2
- ii) Ray crossing. 3
- b) Let S be a planar subdivision by n edges. Describe a method to answer queries like : Given a query point q , report the face f of S that contains q . If q lies on an edge or coincides with a vertex, the query algorithm should return this information. 10



11. Write short notes on any *three* of the following : 3 × 5

- a) Delaunay triangulations
- b) Triangular approximations
- c) Minimum spanning trees
- d) *kd*-tree
- e) Voronoi diagram
- f) Convex hull.
