Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/B.TECH (OLD)/SEM-2/M-201/2012

2012
MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for the following : $10 \times 1 = 10$

i) Rank of the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 3 & 1 & 1 \end{pmatrix}$ is

a) 4 b) 3

c) 2 d) 1

ii) The value of t for which the matrix $A = \begin{pmatrix} 2 & 0 & 1 \\ 5 & t & 3 \\ 0 & 3 & 1 \end{pmatrix}$ is singular is

a) $\frac{3}{2}$ b) 2

c) 1 d) $\frac{1}{3}$.

iii) The equation $x + y + z = 0$ has

a) infinite number of solutions

b) no solution

c) unique solution

d) two solutions.

iv) The value of k for which the vectors (1, 2, 1), (k, 1, 1) and (1, 12) are linearly dependent is

a) 2 b) $\frac{2}{3}$

c) -1 d) $\frac{3}{2}$.

v) The eigenvalues of the matrix A and B are a and b. then The eigenvalues of A^2 are

a) ab, b^2 b) a^2, b

c) a^2, b^2 d) a, b.

vi) If a linear transformation $T : R^2 \to R^2$ be defined by $T(x_1, x_2) = (x_1, x_2)$, then Ker (T) is

a) $\{(-1, -1), (1, 1)\}$ b) $\{(1, 2), (1, \frac{1}{2})\}$

c) $\{(1, 0), (0, 1)\}$ d) $\{(0, 0)\}$.

vii) The interpolation formula which can be used to find a polynomial from the following data

X : 0 1 2 4

Y : 3 9 17 22

is

a) Newton's forward interpolation formula

b) Gaussian interpolation formula

c) Lagrange's interpolation formula

d) Newton's backward interpolation formula.

viii) Which of the following is not true ?

a) $\Delta = E - 1$

b) $\Delta . \nabla = \Delta - \nabla$

c) $\frac{\Delta}{\nabla} = \Delta + \nabla$

d) $\nabla = 1 - E^{-1}$.

ix) The degree and order of the differential equation

$$\left(\frac{d^2 y}{dx_2} \right)^{\frac{3}{2}} = x \frac{dy}{dx} \text{ are}$$

a) $\left(\frac{3}{2}, 2 \right)$

b) (2, 3)

c) (3, 2)

d) (2, 1).

x) $L \left\{ \frac{\sin t}{t} \right\} = \tan^{-1} \left(\frac{1}{s} \right)$. Then $L \left\{ \frac{\sin at}{t} \right\}$ is

a) $\tan^{-1} \left(\frac{1}{s^2} \right)$

b) $\tan^{-1} \left(\frac{a}{s} \right)$

c) $\tan^{-1} \left(\frac{1}{as} \right)$

d) $\tan^{-1} \left(\frac{1}{a^2 + s^2} \right)$.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Solve the differential equation by Laplace Transformation :

$$\frac{d^2 y}{dt^2} + 9y = 1, \; y(0) = 1, \; y\left(\frac{\pi}{2}\right) = -1$$

3. Solve by the method of variation of parameter :

$$\frac{d^2 y}{dx^2} + 9y = \sec 3x$$

4. Solve the system of equations if possible :

$x + y + z = 1$

$2x + y + 2z = 2$

$3x + 2y + 3z = 5$

5. If $W = \{ (x, y, z) \in R^3 : x + y + z = 0 \}$, show that W is a subspace of R^3, and find a basis of W.

6. Examine whether the mapping $T : R^2 \to R^2$ defined by $T(x, y) = (2x - y, x)$ is linear.

7. Evaluate $\int_0^1 \frac{dx}{1+x}$ using Trapezoidal rule, taking four equal subintervals.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

8. a) Evaluate $\left(\frac{\Delta^2}{E}\right) x^3$.

b) Find the missing data in the following table :

x	-2	-1	0	1	2
$f(x)$	6	0	?	0	6

c) Show that $\int_0^{\infty} \frac{\sin t}{t} \, dt = \frac{\pi}{2}$, by Laplace transform.

9. a) Show that (3, 1, –2), (2, 1, 4) and (1, –1, 2) form a basis of R^3.

b) Apply convolution theorem to prove that $\int_0^t \sin u \cos (t - u) \, dt = \frac{t}{2} \sin t$.

c) Solve $\frac{dy}{dx} - \frac{\tan y}{(1 + x)} = (1 + x) e^x \sec y$.

10. a) Solve by Cramer's rule :

$x + y + z = 7$

$x + 2y + 3z = 15$

$x - y + z = 3$

b) Find general solution of $p = \cos (y - px)$, where $p = \frac{dy}{dx}$.

c) Solve $\frac{dy}{dx} + \frac{y \log y}{x} = \frac{y (\log y)^2}{x^2}$

11. a) Solve $(D^2 - 2D) y = e^x \sin x$, where $D \equiv \frac{d}{dx}$.

b) Solve $\frac{dx}{dt} - 7x + y = 0$ and $\frac{dy}{dt} - 2x - 5y = 0$

c) Solve by Gauss-elimination method :

$2x + 2y + z = 12$

$3x + 2y + 2z = 8$

$5x + 10y - 8z = 10$

12. a) Use Lagrange's interpolation formula to find $f(x)$, where $f(0) = -18$, $f(3) = 0$, $f(5) = -248$, $f(6) = 0$ and $f(9) = 13104$.

b) Apply appropriate interpolation formula to calculate $f(2.1)$, correct up to two significant figures from the following data :

x	0	2	4	6	8	10
$f(x)$	1	5	17	37	45	51

c) Apply method of variation of parameter to solve the equation

$$\frac{d^2 y}{dx^2} + y = \sec^3 x . \tan x$$

13. a) Apply Simpson's $\frac{1}{3}$rd rule to evaluate $\int_0^6 \frac{dx}{(1+x)^2}$ taking six equal intervals from (0, 6) and correct up to three decimal places. 5 + 10

b) Expand by Laplace method

$$\begin{vmatrix} 0 & a & b & c \\ -a & 0 & d & e \\ -b & -d & 0 & f \\ -c & -e & -f & 0 \end{vmatrix} = (af - be + cd)^2$$

c) Given that

$$L\left\{\frac{\sin t}{t}\right\} = \tan^{-1}\left(\frac{1}{s}\right), \text{ Find } L\left\{\frac{\sin at}{t}\right\}.$$

14. a) Assuming the orthogonal properties of Legendre function, prove that

$$\int_{-1}^{+1} [P_n(x)]^2 \mathrm{d}x = \frac{2}{2n+1}$$

b) Show that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x.$

e) State Cayley-Hamilton theorem and show that the matrix

$A = \begin{pmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{pmatrix}$ satisfies the above theorem.