MAULANA ABUL KALAM AZAD UNIVERSITY OF TECHNOLOGY WEST BENGAL

Utech

MAULANA ABUL KALAM AZAD UNIVERSITY OF TECHNOLOGY, WEST BENGAL

Paper Code : BS-M-102

MATHEMATICS-1B

Time Allotted: 3 Hours

Full Marks: 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group – A

(Multiple Choice Type Questions)

1. Choose the correct alternative for *any ten* of the following: 1×10=10

(i) If eigenvalues of a 3 × 3 matrix A are –1, 1 and 0, then what is the trace of $[A^{100} + I]$?

(a) 2
(b) 5
(c) 4
(d) 0

(ii) The series $1 - \frac{1}{3} + \frac{1}{3^2} - \frac{1}{3^3} + \ldots\ldots$ is

(a) convergent
(b) divergent
(c) oscillatory
(d) None of these

(iii) If $y = \sin(ax + b)$, then $y_n =$

(a) $a^{n-1}\sin\left(n\frac{\pi}{2} + ax + b\right)$
(b) $a^n\sin(n\pi + ax + b)$
(c) $a^n\sin\left(n\frac{\pi}{2} + ax + b\right)$
(d) $a^n\sin\left(n\frac{\pi}{2} - ax - b\right)$

(iv) $\lim\limits_{x\to\infty}\frac{1-\cos x}{\sin x} = ?$

(a) 0
(b) 1
(c) –1
(d) None of these

Turn Over

(v) $f(x,y) = \frac{\sqrt{y}+\sqrt{x}}{y+x}$ is a homogeneous function of degree

(a) 1/2 (b) -1/2

(c) 1 (d) 2

(vi) The sequence $\left\{\frac{1}{3^n}\right\}$ is

(a) monotonic increasing (b) oscillatory

(c) divergent (d) monotonic decreasing

(vii) The law of mean is given by

(a) $\frac{f(b)+f(a)}{b-a} = f'(c)$ (b) $\frac{f(b)+f(a)}{b+a} = f'(c)$

(c) $\frac{f(b)-f(a)}{b-a} = f'(c)$ (d) $\frac{f(b)-f(a)}{b-a} = f(c)$

(viii) The critical point of the function $f(x,y) = xy$ is

(a) (1, 1) (b) (1, –1)

(c) (–1, 1) (d) (0, 0)

(ix) 5 is an eigenvalue of the matrix A then 0 is an eigenvalue of the matrix

(a) A (b) $A-5I$

(c) $A-I$ (d) None of these

(x) The greatest value of the function $f(x) = x(x-1)^2$ in the interval $0 \leq x \leq 2$ is

(a) 1 (b) 2

(c) –1 (d) –2

(xi) The value of $\Gamma\left(\frac{1}{2}\right)$ is

(a) $\sqrt{\pi}$ (b) $\frac{\sqrt{\pi}}{2}$

(c) $\frac{\sqrt{\pi}}{4}$ (d) π

Group – B

(Short Answer Type Questions)

Answer *any three* of the following. 5×3=15

2. Expanding the determinant by Laplace's method in terms of minors of 2nd order formed from the fitst two rows, prove that $\begin{vmatrix} a & -b & -a & b \\ b & a & -b & -a \\ c & -d & c & -d \\ d & c & d & c \end{vmatrix} = 4(a^2+b^2)(c^2+d^2)$ 5

3. Test the convergence of the series $\frac{6}{1.3.5}+\frac{8}{3.5.7}+\frac{10}{5.7.9}+\cdots$ 5

4. Given the function $f(x,y) = \left.\begin{matrix} \frac{xy(x^2-y^2)}{x^2+y^2} & ,(x,y)\neq(0,0) \\ 0 & ,(x,y)=(0,0) \end{matrix}\right\}$, find from definition $f_{xy}(0,0)$ and $f_{yx}(0,0)$. Verify whether $f_{xy}(0,0)=f_{yx}(0,0)$. 5

5. If $xyz = abc$ by using Lagrange's undetermined multipliers prove that the minimum value of $bcx + cay + abz$ is $3abc$, where $a,b,c>0$. 5

6. Define Gamma function. Using it or otherwise prove that $\int_0^\infty a^{-x^2}dx = \frac{1}{2}\sqrt{\frac{\pi}{\log a}}$. 1+4=5

Group – C

(Long Answer Type Questions)

Answer *any three* of the following. 15×3=45

7. (a) Prove that $\int_0^\infty e^{-x^4}dx \times \int_0^\infty x^2 e^{-x^4}dx = \frac{\pi}{8\sqrt{2}}$

(b) Prove that $\int_a^b (x-a)^{m-1}(b-x)^{n-1}dx = (b-a)^{m+n-1}B(m,n)$, where $m,n>0$.

(c) Find the area of surface of revolution generated by the region about x–axis enclosed by $x = a\cos^3\theta$ and $y = a\sin^3\theta$ within first quadrant. 5+5+5=15

8. (a) If $z = f(x,y)$ and $x = e^u\cos v, y = e^u\sin v$ then show that $y\frac{\partial z}{\partial u} + x\frac{\partial z}{\partial v} = e^{2u}\frac{\partial z}{\partial y}$.

(b) Find the maximum and the minimum values of the following function: $f(x,y) = x^3 + y^3 - 3axy$.

(c) Expand $f(x) = x, -\pi \le x \le \pi$ in Fourier series. Hence deduce that $1 - \frac{1}{3} - \frac{1}{5} \dots = \frac{\pi}{4}$. 4+5+6=15

9. (a) Expand a^x in a finite series with Lagrange's form of remainder.

(b) Verify Rolle's theorem for the function $f(x) = (x-a)^m(x-b)^n$ in $[a,b]$ where m, n are integers.

(c) Using L'Hospital rule find the value of $\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{\frac{1}{x}}$. 5+5+5=15

10. (a) Using D'Alembert's ratio test check the convergence of the infinite series $1 + \frac{x}{2} + \frac{x^2}{5} + \frac{x^3}{10} + \frac{x^4}{17} + \cdots$

(b) If $r = |\vec{r}|$ where $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ prove that $\vec{\nabla}\left(\frac{1}{r}\right) = -\frac{\vec{r}}{r^3}$.

(c) Determine the conditions under which the system of equations

$$x + y + z = 1$$
$$x + 2y - z = k$$
$$5x + 7y + az = k^2$$

admits (I) only one solution, (II) no solution, (III) many solutions. 5+5+5=15

11. (a) Prove that $\begin{vmatrix} bc-a^2 & ca-b^2 & ab-c^2 \\ ca-b^2 & ab-c^2 & bc-a^2 \\ ab-c^2 & bc-a^2 & ca-b^2 \end{vmatrix} = (a^3+b^3+c^3-3abc)^2$.

(b) Find the eigenvalues and the corresponding eigenvectors of the matrix $A = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$.

(c) Use Mean-value theorem to prove the inequality $0 < \frac{1}{x}\log\frac{e^x - 1}{x} < 1$.
