Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/B.TECH (CSE)/SEM-7/CS-704E/2012-13

2012

COMPUTATIONAL GEOMETRY

Time Allotted : 3 Hours Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for the following :

$10 \times 1 = 10$

i) Let *D* to be a square drawn in the plane with sides of length $\sqrt{2}$. What is the minimum number of points that can be placed in *D*, such that the distance between closest pair of points is at most 1 ?

a) 2 b) 3

c) 4 d) 5.

ii) A set *S* of 64 points are located on a circle of radius 1 around the origin. Then how many points does the convex hull of *S* have ?

a) 4 b) 8

c) 32 d) 64.

iii) Given a set S of n points in the plane such that the convex hull of S has h points, how many edges does a triangulation of S have ?

a) $\frac{(3n - h - 3)}{2}$ b) $(3n - h - 3)$

c) $(n + h - 2)$ d) $n \log h$.

iv) We are given a set S of intervals on the real line. Which of the following data structures does not have $O(n)$ space complexity ?

a) A binary search tree built on the endpoints of intervals in S.

b) A segment tree built on the intervals in S.

c) An interval tree built on the intervals in S.

d) A range tree built on the endpoints of intervals in S.

v) A data structure D built on a set S of intervals on the real line has an interval tree as an outer structure. Each internal node v has a secondary data structure $H(v)$ built on the endpoints of the intervals allocated on it. An instance of H built on n points occupies $O(n \log n)$ space. What is the total space complexity of the data structure D ?

a) $O(n \log n)$ b) $O(n)$

c) $O(n \log^2 n)$ d) $O(n^2)$.

vi) A data structure D built on a set S of points on the real line has a 1-dimensional range tree as an outer structure. Each internal node ν has a secondary data structure $H(\nu)$ built on the points of the subtree rooted at ν. An instance of H built on n points occupies $O(n \log n)$ space. What is the total space complexity of the data structure D ?

a) $O(n \log n)$ b) $O(n)$

c) $O(n \log^2 n)$ d) $O(n^2)$.

vii) Which of the following are true about the Delaunay triangulation for a set of sites S in the plane ?

I. The number of sites inside the circumcircle of any triangle in the Delaunay triangulation is zero.

II. The closest pair of sites are neighbours in the Delaunay triangulation.

III. The minimum spanning tree of S is a subgraph of the Delaunay triangulation.

a) I only b) II only

c) III only d) All of these.

viii) Consider the point-line duality transform

$L : ax + by = 1 \Leftrightarrow p : (a, b)$. What is the dual of a point outside the unit circle around the origin ?

a) A line that intersects the unit circle.

b) A line that is tangential to the unit circle.

c) A line that does not intersect the unit circle.

d) None of these.

ix) Consider the point-line duality transform

$L : y = ax + b \Leftrightarrow p : (a, - b)$. What is the dual of a line segment ?

a) A point

b) A line

c) A pair of lines

d) A double wedge bounded by a pair of lines.

x) Consider the problem of guarding an art gallery shaped as a simple polygon with 40 vertices. Which of the following is true ?

a) 13 guards are always necessary and 14 guards are always sufficient.

b) 13 guards are always sufficient and sometimes necessary.

c) 13 guards are always necessary and sufficient.

d) 14 guards are always necessary and sufficient.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Give an $O(n)$ algorithm which takes a set S of n points in the plane and reports a point p that lies inside the convex hull of S. Note that p need not be in S.

3. Consider the duality transform D which maps the line $ax + by = 1$ to the point (a, b) and vice versa. Prove that the dual of a point inside the unit circle around the origin is a line that does not intersect the unit circle.

4. Describe an $O(n \log n)$ sweep line algorithm for finding the closest pair of points for a set on n points in the plane.

5. Describe Timothy Chan's $O(n \log h)$ convex hull algorithm for a set of n points in the plane, where h is the number of points in the convex hull.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

6. a) We are given a set G of k *simple polygons* P (1), $P(2)$, ... , $P(k)$ with a total of n vertices. Give the outline of an $O(n \log k)$ algorithm to compute the convex hull of the vertices of the polygons in G. 10

   b) Explain why in the interval tree used for the 1-dimensional point enclosure problem, the secondary data structures are built by sorting right end points in the right to left order and the left end points in the left to right order. 5

7. a) We wish to preprocess a set S of horizontal line segments in the plane such that given a query vertical line segment q, the segments in S intersecting q can be reported efficiently. Give a data structure for this problem using an interval tree. What is the time complexity of the query algorithm and the space complexity of the data structure ? 10

   b) Explain the standard point-line $m - c$ duality with an example. 5

8. a) Let R be a set of (possibly intersecting) n axes-parallel rectangles in the plane. We wish to preprocess R into a data structure D such that the rectangles containing a query point q can be reported efficiently. Give the outline of the data structure D that uses a segment tree. What is the time complexity of the query algorithm and the space complexity of the data structure ? 10

   b) We wish to preprocess a set S of n points in the plane into a data strucgture D such that given a query point q, all points in S within an $L1$ distance d from q can be reported efficiently. Show how this problem can be reduced to the 2-dimensional orthogonal range searching problem.

   (The $L1$ distance between $p = (a, b)$ and $q = (c, d)$ is $| c - a | + | d - b |$) 5

9. a) We are given two sets of polygons red and blue. No two polygons in the same set intersect except possibly at end points. Given the outline of an $O(n \log n)$ algorithm which removes each blue polygon that intersects one or more red polygons, where n is the total number of polygon vertices in the red and blue sets combined. 10

b) Discuss how the algorithm in (*a*) above needs to be modified if we wish to remove each red polygon that intersects one or more blue polygons besides removing each blue polygon that intersects one or more red polygons. The time complexity of the modified algorithm should still be $O(n \log n)$. 5