

Name :

Roll No. :

Invigilator's Signature :

CS/B.Tech (CE-OLD)/SEM-3/CE-301/2011-12

2011

MATHEMATICS

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the
following : 10 × 1 = 10

i) If A_1 and A_2 are mutually exclusive events then

a) $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1) \cap P(A_2)$

b) $P(A_1 \cup A_2) = P(A_1) + P(A_2)$

c) $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$

d) $P(A_1 \cup A_2) = P(A_1) P(A_2)$.

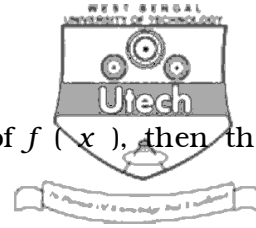
ii) $\cos (5x)$ is a periodic function with the period

a) 2π

b) π

c) $\frac{2\pi}{5}$

d) none of these.



iii) If $F(p)$ is the Fourier transform of $f(x)$, then the Fourier transform of $f(x-a)$ is

- a) $F(p-a)$
- b) $F(p+a)$
- c) $F(p).e^{ipa}$
- d) $F(p).e^{-ipa}$.

iv) Fourier cosine transform of e^{-x} is

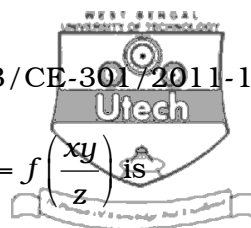
- a) $\frac{1}{1+s^2}$
- b) $\frac{1}{1-s^2}$
- c) $\frac{s}{1+s^2}$
- d) $\frac{s}{1-s^2}$.

v) If $P(A) = \frac{1}{3}$, $P(B) = \frac{3}{5}$ and A and B are mutually exclusive events, then $P(A \cup B)$ is equal to

- a) $\frac{14}{15}$
- b) $\frac{11}{15}$
- c) 0
- d) $\frac{1}{5}$.

vi) If \bar{A} is the complementary event of A , then $P(A) + P(\bar{A})$ is equal to

- a) 0
- b) 1
- c) 2
- d) none of these.



vii) The partial differential equation for $Z = f\left(\frac{xy}{z}\right)$ is

(Notations have their usual meanings)

a) $\frac{p}{q} = \frac{y}{x}$

b) $xyz = pq$

c) $pq = z$

d) none of these.

viii) If A and B are independent events, $P (B) = 0.14$ and

$P (A | B) = 0.24$, then the value of $P (A)$ is

a) 0.14

b) 0.0336

c) 0.38

d) 0.24.

ix) The mean of an exponential distribution with parameter

α ($\alpha > 0$) is

a) $\frac{1}{\alpha^3}$

b) $\frac{1}{\alpha^2}$

c) $\frac{1}{\alpha}$

d) α .

x) If F be a distribution function of a random variable X ,

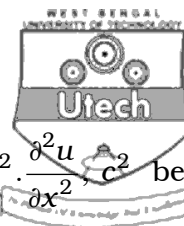
then

a) $\lim_{x \rightarrow \infty} F(x) = 0$

b) $\lim_{x \rightarrow \infty} F(x) = 1$

c) $\lim_{x \rightarrow \infty} F(x) = \infty$

d) $\lim_{x \rightarrow \infty} F(x) = -\infty$.



xi) The partial differential equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$, c^2 being constant, is known as

- a) one dimensional wave equation
 - b) one dimensional heat-flow
 - c) two dimensional heat-flow equation
 - d) none of these.
- xii) For the random experiment of tossing two coins if X be the random variable such that X (an outcome) = “the number of heads”, then the spectrum of X is
- a) $\{ 0, 1, 2 \}$
 - b) $\{ 1, 2, 3 \}$
 - c) $\{ 0, 1 \}$
 - d) none of these.

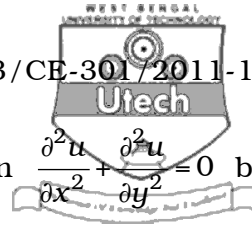
GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Using Parseval's identities, prove that

$$\int_0^{\infty} \frac{dt}{(a^2 + t^2)(b^2 + t^2)} = \frac{\pi}{2ab(a+b)}.$$
3. Form the partial differential equation by eliminating the arbitrary functions from $f(x^2 + y^2, z - xy) = 0$.



4. Solve two dimensional Laplace's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ by separation of variables.
5. An integer is chosen at random from the first 100 positive integers. What is the probability that the integer is divisible by 6 or 8 ?
6. Expand the function $f(x) = x \sin(x)$ as a Fourier Series in $[-\pi, \pi]$.
7. Find the Fourier integral representation of the function $f(x) = e^{-x}$ when $x > 0$ with $f(-x) = f(x)$. Hence evaluate $\int_0^\infty \frac{\cos(\lambda x)}{1+\lambda^2} d\lambda$.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

8. a) What is the Fourier expansion of the periodic function

$$f(x) = \begin{cases} 0 & \text{when } -\pi < x < 0 \\ \sin(x) & \text{when } 0 \leq x < \pi \end{cases}$$

Hence evaluate $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \dots$



- b) State giving reasons whether the following functions can be expanded in Fourier Series in the interval $-\pi \leq x < \pi$:

i) $\operatorname{cosec}(x)$

ii) $\sin\left(\frac{1}{x}\right)$. 10 + 5

9. Derive one dimensional wave equation for vibrating string and solve it using the method of separation of variables.

10 + 5

10. a) From the Fourier Series expansion of $f(x) = x^2$ in $-\pi < x < \pi$ prove that $\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$.

- b) Find the Fourier sine transform of $f(x) = \frac{1}{x} \cdot e^{-ax}$.

10 + 5

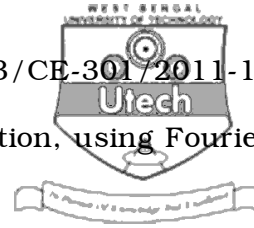
11. a) The distribution function $F(x)$ of a variate X is defined as follows :

$$F(x) = A, -\infty < x < -1$$

$$= B, -1 \leq x < 0$$

$$= C, 0 \leq x < 2$$

$$= D, 2 \leq x < \infty.$$



- b) Solve the two dimensional heat equation, using Fourier transform :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 < x < \pi, \quad 0 < y < \pi.$$

$$u(0, y) = u(\pi, y) = 0 \text{ for } 0 < y < \pi \text{ and } u(x, 0) = 0 \\ \text{and } u(x, \pi) = u_0 \text{ for } 0 < x < \pi. \quad 10 + 5$$

12. a) If A and B are independent events then show that the following pairs are independent :

i) \bar{A} and \bar{B}

ii) \bar{A} and B .

- b) Show by Tchebycheff's inequality that in 2000 throws with a coin the probability that the number of heads lies between 900 and 1100 is at least $\frac{19}{20}$. 10 + 5

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