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# CS/B.Tech(CE)/SEM-3/CE-301/2009-10 2009

### **MATHEMATICS**

Time Allotted: 3 Hours Full Marks: 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

#### **GROUP - A**

## ( Multiple Choice Type Questions )

- 1. Choose the correct alternatives for any ten of the following :  $10 \times 1 = 10$ 
  - i) When two dice are thrown, the probability of getting a sum of 10 or 11 points is
    - a)  $\frac{3}{36}$

b)  $\frac{7}{36}$ 

c)  $\frac{6}{36}$ 

- d)  $\frac{5}{36}$
- ii) The distribution for which mean and variance are equal is
  - a) Poisson
- b) Normal
- c) Binomial
- d) Exponential.

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- iii)  $y = e^x$  is an
  - a) odd function
- b) even function
- c) both of these
- d) none of these.
- iv) If X and Y are independent random variables, then

a) 
$$E(XY) = E(X) + E(Y)$$

b) 
$$E(XY) = E(X) \cdot E(Y)$$

c) 
$$E(XY) = E(X) - E(Y)$$

d) 
$$E(XY) = E(X) / E(Y)$$
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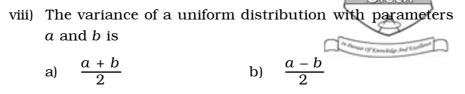
- v) Standard deviation
  - a) varies between 0 and 1
  - b) is a positive quantity
  - c) is a negative quantity
  - d) none of these.
- vi) The probability of having at least one 'six' in 3 throws of a perfect die is
  - a)  $\left(\frac{5}{6}\right)^3$

- b)  $\left(\frac{1}{6}\right)^3$
- c)  $1 \left(\frac{5}{6}\right)^3$
- d) none of these.
- vii) The probability that A passes a test is  $\frac{2}{3}$  and that probability that B passes the same test is  $\frac{3}{5}$ . The probability that only one of them passes is
  - a)  $\frac{4}{5}$

b)  $\frac{7}{15}$ 

c)  $\frac{3}{5}$ 

d)  $\frac{5}{9}$ .



a) 
$$\frac{a+b}{2}$$

b) 
$$\frac{a-b}{2}$$

c) 
$$\frac{(b-a)^2}{12}$$

d) 
$$\frac{(b+a)^2}{12}$$
.

Which of the following is true for random variable X, ix) where a and b are arbitrary constants?

a) 
$$Var(aX + b) = b^{2} Var(X)$$

b) 
$$E(aX + b) = aE(X)$$

c) 
$$Var(aX + b) = a^{2} Var(X)$$

d) 
$$E(aX + b) = b$$
.

For any two events  $A_1 \ \ {\rm and} \ A_2 \ \ {\rm where} \ A_1 \subseteq A_2$  , then X)

a) 
$$P(A_1) > P(A_2)$$

a) 
$$P(A_1) > P(A_2)$$
 b)  $P(A_1) \ge P(A_2)$ 

c) 
$$P(A_1) \le P(A_2)$$
 d) None of these.

The solution of the partial differential equation xi)

$$Z = px + qy + p^2 + pq + q^2$$
, where  $p = \frac{\partial z}{\partial x}$ ,  $q = \frac{\partial z}{\partial y}$ 

is

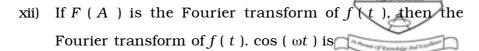
a) 
$$z = ax + by + a^2 + ab + b^2$$

b) 
$$z = ax + by$$

c) 
$$z = a^2 + ab + b^2$$

d) none of these.

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a) 
$$F(s-\omega)+F(s+\omega)$$

b) 
$$\frac{1}{2} [F(s-\omega) + F(s+\omega)]$$

c) 
$$F(s-\omega)-F(s+\omega)$$

d) none of these.

#### **GROUP - B**

## (Short Answer Type Questions)

Answer any *three* of the following.  $3 \times 5 = 15$ 

2. a) Define probability density function of a random variable X.

b) Show that 
$$f(x) = x$$
,  $0 \le x < 1$   
=  $k - 1 \le x \le 2$   
= 0, elsewhere

is a p.d.f. of a random variable x for a suitable value of k. Determine the value of k and then find the distribution function of the random variable X.

- 3. Find the standard deviation of the binomial distribution with parameters n, p.
- 4. If x is normally distributed with mean 3 and s.d. 2, find c such that  $P(X > c) = 2 P(X \le c)$ .

Given that 
$$\int_{0}^{0.43} \phi(t) dt = 0.6666.$$

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Find the deflection of a vibrating string of unit length having 5. fixed ends with initial velocity zero and initial deflection  $f(x) = k (\sin x - \sin 2x)$  where k is a constant.

Expand the function  $f(x) = x \sin(x)$  as a Fourier series in

#### **GROUP - C**

## (Long Answer Type Questions)

Answer any three of the following.  $3 \times 15 = 45$ 

- Find Fourier sine transform of  $e^{-x}$  and using the 7. inversion formula, recover the original function.
  - Find the Fourier transform of  $f(x) = \frac{1}{x} e^{-ax}$ . 10 + 5 b)
- 8. Derive one dimensional wave equation for vibrating string and solve it using the method of separation of variables.

10 + 5

9. Solve the following one dimensional heat conduction a) equation:

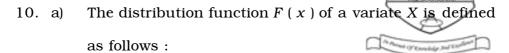
$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$
,  $0 < x < \infty$ ,  $t > 0$  with

u(x, 0) = f(x), u(0, t) = 0, t > 0 using Fourier transform.

b) Solve  $x^2 \frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial x} = 0$  by the method of separation of variables.

10 + 5

 $[-\pi, \pi].$ 



$$F(x) = A, -\infty < x < -1$$

$$= B, -1 \le x < 0$$

$$= C, 0 \le x < 2$$

$$= D, 2 \le x < \infty$$

where A, B, C, D are constants, Determine the values of A, B, C, D, given that  $P(X=0) = \frac{1}{6}$  and  $P(X>1) = \frac{2}{3}$ .

b) Classify the following partial differential equation :

$$y\frac{\partial^{2}u}{\partial x^{2}} + 2x\frac{\partial^{2}u}{\partial x\partial y} + y\frac{\partial^{2}u}{\partial y^{2}} = 0.$$
 10 + 5

- 11. a) If A and B are independent events then show that the following pairs are independent:
  - i)  $\bar{A}$  and  $\bar{B}$
  - ii)  $\bar{A}$  and B.
  - b) Show by Tchebycheff's Inequality that in 2000 throws with a coin the probability that the number of heads lies between 900 and 1100 is at least  $\frac{19}{20}$ . 10 + 5

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12. a) Find the Fourier transform of  $f(x) = \begin{cases} 1 & \text{if } |x| < a \\ 0 & \text{if } |x| > a \end{cases}$ 

where a is a positive real number. Hence deduce that

$$\int_{0}^{\infty} \frac{\sin(t)}{t} dt = \frac{\pi}{2} .$$

b) What is the Fourier expansion of the periodic function

$$f(x) = \begin{cases} 0 & \text{when } -\pi < x < 0 \\ \sin(x) & \text{when } 0 \le x < \pi \end{cases}$$

Hence evaluate  $\frac{1}{1.3}$   $-\frac{1}{3.5}$   $+\frac{1}{5.7}$   $+\dots \infty$ .

(4+4)+(4+3)