

Name :

Roll No. :

Invigilator's Signature :

CS/B.Tech/SEM-2/M-201/2010**2010****MATHEMATICS**

Time Allotted : 3 Hours

Full Marks : 70

*The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words as far as practicable.***GROUP - A****(Multiple Choice Type Questions)**

1. Choose the correct alternatives for any ten of the following :

$$10 \times 1 = 10$$

i) If $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, then A^{100} is

a) $2^{99} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

b) $2^{101} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

c) $2^{100} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

d) none of these.

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vii) If the true and approximate value of a quantity are x_t and x_a respectively, then the relative error is given by

a) $\left| \frac{x_t - x_a}{x_t} \right|$

b) $\left| \frac{x_a - x_t}{x_a} \right|$

c) $\left| \frac{x_a - x_t}{x_t - x_a} \right|$

d) $|x_t - x_a|$

viii) The sum of the eigenvalues of

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{bmatrix} \text{ is}$$

a) 4

b) 5

c) 2

d) none of these.

ix) The value of the determinant

$$\begin{vmatrix} 17 & 58 & 97 \\ 19 & 60 & 99 \\ 18 & 59 & 98 \end{vmatrix} \text{ is}$$

a) 1

b) 2

c) 3

d) 0.

x) The value of λ for which the matrix

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & -3 & 1 \\ 3 & -2 & \lambda \end{bmatrix}$$

is singular, is

a) $3/2$

b) 2

c) 1

d) $1/3$.

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xi) $\frac{1}{D-1} x^2$ is equal to

a) $x^2 + 2x + 2$

b) $-(x^2 + 2x + 2)$

c) $2x - x^2$

d) $-(2x - x^2)$.

xii) The norm of the vector $\alpha = (-1, 2, 3)$ in R^3 with standard inner product is

a) $\sqrt{12}$

b) $\sqrt{14}$

c) $\sqrt{3}$

d) $\sqrt{2}$.

xiii) The degree and order of the differential equation $\left(\frac{d^2y}{dx^2} + 2\right)^{3/2} = x \frac{dy}{dx}$ are respectively

a) $\frac{3}{2}, 2$

b) $2, 3$

c) $3, 2$

d) $1, 4$.

GROUP - B**(Short Answer Type Questions)**Answer any *three* of the following. $3 \times 5 = 15$

2. Solve the following system of equations with the help of Gauss' Elimination method :

$$x_1 + x_2 + 4x_3 = 6$$

$$3x_1 + 2x_2 - 2x_3 = 9$$

$$5x_1 + x_2 + 2x_3 = 13.$$

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3. Prove that $\Delta \equiv e^{hD} - 1$.

(The notations have their usual meanings).

4. Expand by Laplace's method to prove that

$$\begin{vmatrix} 0 & a & b & c \\ -a & 0 & d & e \\ -b & -d & 0 & f \\ -c & -e & -f & 0 \end{vmatrix} = (af - be + cd)^2.$$

5. Solve $\frac{dy}{dx} + y = y^3 (\cos x - \sin x)$.6. Evaluate $\int_0^{\pi/6} \sqrt{1 + \sin x} dx$ using Simpson's one-third rule by taking five ordinates.**GROUP - C****(Long Answer Type Questions)**Answer any three of the following. $3 \times 15 = 45$ 7. a) Show that $(3, 1, -2)$, $(2, 1, 4)$ and $(1, -1, 2)$ form a basis of R^3 .

b) Find the eigenvalues and eigenvectors of the matrix

$$\begin{bmatrix} 1 & -1 & 2 \\ 2 & -2 & 4 \\ 3 & -3 & 6 \end{bmatrix}$$

c) Solve by Cramer's rule :

$$x + y + z = 6$$

$$x + 2y + 3z = 14$$

$$x - y + z = 2.$$

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8. a) Solve the differential equation by Laplace Transformation :

$$\frac{d^2 y}{dt^2} - 2 \frac{dy}{dt} - 3y = t \cos t$$

$$y(0) = 0, y'(0) = 0.$$

- b) Solve by the method of variation of parameters :

$$\frac{d^2 y}{dx^2} + a^2 y = \sec ax.$$

- c) Find the particular integral of

$$(D^2 + 4)y = x \sin^2 x.$$

9. a) Estimate the missing term from the table :

x	2	4	6	8	10
y	5	13	*	53	85

- b) The values of a function $f(x)$ are given for certain values of x as follows :

x :	4	5	6	8
$f(x)$:	3.11	2.96	2.85	2.7

Obtain the value of $f(5.5)$ using Lagrange's interpolation formula.

- c) Compute $\int_1^2 \frac{\sin x}{\sqrt{x}} dx$ using Simpson's one-third rule taking $h = \frac{1}{6}$.

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10. a) Prove that for two invertible matrices A and B of the same order $(AB)^{-1} = B^{-1}A^{-1}$.

b) Reduce the following matrix to a row-reduced echelon form and hence find its rank :

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ 2 & 4 & 8 & 6 \\ 0 & 0 & 5 & 8 \\ 3 & 6 & 6 & 3 \end{bmatrix}$$

c) Solve $(D^2 - 5D + 6)y = x^2 e^{3x}$, $D \equiv \frac{d}{dx}$.
