Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/B.TECH(CSE)/SEM-4/M(CS)-402/2010

2010

OPERATION RESEARCH & OPTIMIZATION TECHNIQUES

Time Allotted : 3 Hours Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Graph sheet(s) to be supplied by the Institute.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

   i) An activity is said to be ..... if a delay in the start of the course makes a delay in the completion time of the entire project.

   a) non-critical b) critical

   c) delay d) all of these.

   ii) The set $S = \{ x^2 + y^2 < 25 : (x, y) \varepsilon R \}$ is a

   a) Convex set b) Concave set

   c) Non-convex set d) Non-concave set.

   iii) In a transportation problem ..... method gives the closest initial basic feasible solution to the optimal basic feasible solution

   a) NWC b) MMM

   c) VAM d) MODI.

iv) In an $m \times n$ transportation problem we get a degenerate basic feasible solution if the number of occupied cells are

a) $m + n - 1$ b) less than $m + n - 1$

c) more than $m + n - 1$ d) $m - n$.

v) Assignment problem is solved by

a) Stepping stone method

b) Two phase method

c) Hungarian method

d) Karmakar's Algorithm.

vi) If the primal problem has an unbounded solution then the solution to the dual problem will be

a) unbounded b) unique

c) multiple d) infeasible.

vii) A queuing model is generally expressed using

a) Fulkerson's Notation

b) Newton's Notation

c) Dantzing's Notation

d) Kendall's Notation.

viii) Which of the following set is not convex set ?

a) $x = \{ (x_1 \; y) : | x | \leq 3, | y | \leq 2 \}$

b) $x = \{ (x_1 \; y) ; 5x - y = 4 \}$

c) $x = \{ (x_1 \; y) : 2x - y \leq 3 \}$

d) $x = \{ (x_1 \; y) ; x^2 + y^2 = 5 \}$.

ix) A basic solution of the system of equations $2x_1 + x_2 - x_3 = 2,\ 3x_1 + 2x_2 - x_3 = 3$ is

a) (1, 1, 1) b) (1, 1, 0)

c) (1, 0, 0) d) none of these.

x) The Canonical form of L.P.P. can be written in matrix form as

a) Max $Z = c\,x$
Sub to, $Ax \le b$
$x \le 0$

b) Max $Z = c\,x$
Sub to, $Ax \le b$
$x \le 0$

c) Min $Z = c\,x$
Sub to, $Ax \le b$
$x \ge 0$

d) Min $Z = c\,x$
Sub to, $Ax \ge b$
$x \ge 0$.

xi) For a $m \times n$ degenerate Transportation problem the number of occupied cells is

a) less than ($m + n - 1$)

b) greater than ($m + n - 1$)

c) less than equal to ($m + n - 1$)

d) equal to ($m + n - 1$).

xii) A game is solved graphically when the pay off matrix is of the form

a) $m \times 1$ b) $m \times n$

c) $m \times 2$ d) $n \times m$.

xiii) The formula for probability of n units in the system under single server, FCFS discipline is

a) $P_n = \left(\frac{\lambda}{\mu}\right)^n P_o$, $P_o = 1 - \lambda/\mu$

b) $P_n = \left(\frac{\lambda}{\mu}\right)^{n-1} P_o$, $P_o = 1$

c) $P_n = \left(\frac{\lambda}{\mu}\right)^{n+1} P_o$, $P_o = 1$

d) $P_n = \left(\frac{\lambda}{\mu}\right)^n P_o$, $P_o \neq 1$.

xiv) The state is referred as explosive state if

a) $\left(\frac{\lambda}{\mu}\right) > 1$ b) $\left(\frac{\lambda}{\mu}\right) < 1$

c) $\left(\frac{\lambda}{\mu}\right) = 1$ d) $\left(\frac{\lambda}{\mu}\right) > 0$.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Solve the game whose pay-off matrix is given by

Player B

$$\textbf{Player } A \quad \begin{matrix} A_1 \\ A_2 \\ A_3 \end{matrix} \begin{bmatrix} B_1 & B_2 & B_3 \\ 1 & 3 & 1 \\ 0 & -4 & -3 \\ 1 & 5 & -1 \end{bmatrix}$$

3. Solve the following by graphical method

Maximize $Z = 4x_1 + 7x_2$

subject to

$2x_1 + 5x_2 \leq 40$,

$x_1 + x_2 = 11$,

$x_2 \geq 4$,

$x_1 , x_2 \geq 0$.

4. Obtain the dual of the following L.P.P :

Minimize $Z = x_1 - x_2 + 2x_3$

subject to

$$x_1 + x_2 + 4x_3 \geq 7,$$

$$x_2 - 2x_3 \geq 10,$$

$$3x_1 + x_2 + x_3 \geq 3,$$

$$x_1 , x_2 , x_3 \geq 0.$$

5. Find the optimal assignment for the assignment problem with the given cost matrix

	I	II	III	IV
A	10	9	7	8
B	5	8	7	7
C	5	4	6	5
D	2	3	4	3

6. Determine EOQ in an inventory control problem having

i) Constant rate of demand

ii) Instantaneous replenishment and

iii) Finite rate of production.

7. Food *X* contains 6 unit of vitamin *A* & 7 unit of vitamin *B* per gram and costs 12p/gm. Food *Y* contains 8 units and 12 units of *A* and *B* per gram respectively and costs 20 p/gm. The daily requirement of vitamin *A* & *B* are at least 100 units & 120 units respectively. Formulate the above as a L.P.P. to minimize the costs.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. 3 × 15 = 45

8. a) Use duality to solve the LLP,

Min $Z = 3x_1 + x_2$

Subject to

$2x_1 + 3x_2 \geq 2$,

$x_1 + x_2 \geq 1$,

$x_1 , x_2 \geq 0$.

b) Solve graphically the game having the following pay of matrix

	B1	B2	B3	B4
A1	2	2	3	– 1
A2	4	3	2	6

7 + 8

9. a) A television repairman find that the time spent on his jobs has an exponential distribution with a mean of 30 minutes. If he repairs the sets in the order in which they came in and if the arrival of sets follows a Poisson distribution with an approximate average rate of 10 per 8-hours day, what is the repairman's expected idle time each day ? How many jobs are ahead of the average set just brought ?

b) Solve the following LLP :

Max $X = x_1 + x_2$

Subject to

$3x_1 + 2x_2 \leq 5$,

$x_2 \leq 2$,

$x_1 , x_2 \geq 0$. 6 + 9

10. a) Solve the transportation problem by VAM :

	I	II	III	a_i
1	8	7	3	60
2	3	8	9	70
3	11	3	5	80
b_j	50	80	80	

Also verify whether the solution obtained by VAM is optimal or not ? Find optimal solution, if the above solution is not optimal.

b) Solve the following LPP,

Maximize $Z = 6x_1 + 10x_2$

Subject to $3x_1 + 5x_2 \leq 10$,

$5x_1 + 3x_2 \leq 15$

and $x_1 , x_2 \geq 0$. 8 + 7

11. a) Construct a network for the project whose activities and precedence relationship are as given below :

Activities	A	B	C	D	E	F	G	H
Immediate Predecessor	—	A	A	B	B, C	E	D, F	G

b) The time of each activity of the network is given in $t_o - t_m - t_p$ form, where t_o is the optimistic time, t_p is the pessimistic time and t_m is most likely time.

i) Determine the expected project length (T_{cp})

ii) Calculate s.d. of the project length (σ)

iii) What is the percentage of confidence that the project will complete

a) at least 4 weeks earlier than expected time

b) not more than 4 weeks than the expected time ?

iv) What should be the scheduled complication times for the probability of complication are 90% confidence and 100% confidence ?

Dia.

$P (Z \leq 1.33) = 0.9082$

Given data $P (Z \leq 1.28) = 0.9$

$P (Z \leq 5) = 0.99999.$ 5 + 10

12. a) Find the shortest path from Node 1 to Node 9 of the distance network shown in the following *figure* using Dijkstra's algorithm.

Dia.

b) Using Floyd's algorithm, find the shortest distances & routes between every two nodes from the network given below. The distances (in kms.) are given on the arcs.

Dia.

8 + 7