Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/B. Tech (CSE)/SEM-7/CS-704E/2011-12

2011
COMPUTATIONAL GEOMETRY

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives of the following : $10 \times 1 = 10$

   i) Which of the following convex hull algorithms is output-sensitive ?

      a) Graham's Scan

      b) Chan's Algorithm

      c) Quickhull

      d) Divide-and-conquer algorithm.

   ii) Graham's scan to find the convex hull of a simple polygon with n vertices takes

      a) O(nlog n) time b) $O(n^2)$ time

      c) O(nloglog n) time d) O(n) time.

iii) In the wrost case, the number of edge-edge intersections between edges of two convex polygons with a total of n vertices is

a) $O(\text{nlog } n)$ b) $O(n)$

c) $O(n^2)$ d) $O(n^{3/2})$.

iv) Give a point set in the plane, a dynamic data structure D is maintained as follows : whenever a new point p is inserted into D, all points in D dominated by p are deleted. An insert operation takes $O(\log n)$ time, where n is the number of points in D. A delete operation takes $O(k \log n)$ time where n is the number of points in D and k is the number of points deleted. Starting with an empty data structure, what is the total time taken by n operations in the worst case ?

a) $O(n^2)$ b) $O(\text{nlog } n)$

c) $O(n)$ d) $O(\text{nlog}^2 n)$.

v) For a set S of n points in the plane whose convex hull has h points, the number of triangles in the triangulation of S is

a) $n-2h+2$ b) $2n-h-2$

c) $2n-h+2$ d) $2n-2h+1$.

vi) In the delaunay triangulation of a set of points in the plane, the number of points in the circumcircle of any triangle of the triangulation is

a) 0,1, or 2 b) may be more than 4

c) always 0 d) 0 or 1.

vii) If the closest pair of points in a set of points S in the plane are at a distance δ apart, then the number of points in any $\delta \times \delta$ square is at most

a) 4 b) 3

c) 2 d) 5.

viii) How many points are there in the convex hull of S = {(9,5900), (13, 770), (9, 40), (573, 4444), (666, 5555), (777, 3333), (1900, 40), (1900, 5900)} ?

a) 3 b) 5

c) 6 d) 4.

ix) Consider the duality transform D which maps the line $ax + by = 1$ to the point (a, b) and vice versa. Then the dual of a point inside the unit circle around the origin is

a) a line that does not intersect the unit circle

b) a line that intersects the unit circle

c) a line that is tangent to the unit circle

d) another circle.

x) In an art gallery shaped as a simple polygon with 59 vertices how many guards are sufficient ?

a) 20 b) 30

c) 29 d) 19.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Let R be a set of (possible intersecting) n axes-parallel rectangles in the plane. We wish to preprocess R into a data structure D such that the rectangles containing a query point q can be reported efficiently. Give the outline of the data structure D that uses a segment tree. What is the space complexity of the data structure ? What is the query time ?

5

3. Given a set S of n points in general position in R^2, we wish to find the convex hull $CH(S)$. We have an algorithm $A(P, T)$ which given a set of points P and a set of triangles T both in R^2, reports all points in P which are on or outside all triangles in T. Briefly outline how to find $CH(S)$ using algorithm A. 5

4. Given a set S of orthogonal rectangles in R^2, there exists an algorithm A which finds all pairs of rectangles (a, b) such that a encloses b in O (nlognloglog $n + k$) time where k is the output size. Given a set R of orthogonal rectangles and a set P of convex polygons, outline an efficient algorithm which uses algorithm A to find all pairs (r, p), $r \varepsilon R$, $p \varepsilon P$ such that r encloses p. What is its time complexity ? 5

5. Explain point-line duality with an example. 5

6. Describe an O (nlog h) output-sensitive algorithm for finding convex hulls in 2-dimensions where n is the total number of points and h is the number of points in the hull. 5

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) We wish to preprocess a set S of n points in the plane into a data structure D such that given a query point q, all points in S within a L_1 distance d from q can be reported efficiently. Give an efficient solution indicating the space requirement of D and the running time of the query algorithm. (The L_1 distance between $p = (c,d)$ and $q = (c,d)$ is $|c-a| + |d-b|$). 5

   b) Given a set S of n points in the plane, already sorted by x-coordinates, give the outline of an efficient algorithm to find all points in S which are not dominated by any other point. Indicate the running time of your algorithm. 5

c) Given a set S of n points in the plane, give the outline of an efficient algorithm to decide if the convex hull $CH(S)$ of S is a triangle. (You are given the point set and not the convex hull). Indicate the running time of your algorithm. 5

8. a) We using the fact that the centre of gravity of a triangle is at its centroid, design and analyze an algorithm to compute the centre of gravity of a metallic plate in the shape of a simple polygon, assuming the thickness of the metal is uniform. Assume the polygon has n vertices. 5

b) We are given a set G of k simple polygons P_1, P_2 ,....., P_k with a total of n vertices. Give the outline of an $O(n \log k)$ algorithm to compute the convex hull of the vertices of the polygons in G. 5

c) i) Give an algorithm to determine whether a point is inside a polygon or not. 3

ii) What improvements can you suggest if it is known in advance that the polygon in convex ? 2

9. a) Describe briefly the Bentley-Ottman algorithm for finding segment intersections. 5

b) Describe briefly the segment tree data structure. 5

c) Describe brielfy the Graham's Scan algorithm for finding the convex hulls of a set of points in the plane. 5

10. a) Give an $O(n)$ algorithm to test if a given point q is in the convex hull of a set S of n points in the plane. The point q is not in S. Only the points in set S are given (in no particular order), not its convex hull. Obviously you cannot compute the hull and then test if q is in it. 5

    b) We are given two sets of points S and A with n points each. We wish to find for each point in A, the second nearest neighbour amongst points in S. A Voronoi diagram for S VOR (S) is given and also an associated planar point location data structure. Design and analyze an algorithm to show how the task can be achieved on $O(\text{nlog } n)$ time. 5

    c) We wish to preprocess a set S of horizontal line segments in the plane such that given a query vertical line segment q, the segments in S interecting q can be reported efficiently. Give the outline of a data structure for this problem that uses an interval tree. What is the space complexity of the data structure ? 5

11. a) Write a short note on Voronoi diagrams. 5

    b) Describe an $O(\text{nlog } n)$ sweep line algorithm for finding the closest pair of points for a set of points in the plane. 5

    c) Describe briefly the dynamic programming algorithm for minimum weighted triangulation in the plane. 5