Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS / B.TECH (NEW) / SEM-2 / M-201 / 2011

2011

MATHEMATICS – II

Time Allotted : 3 Hours   *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words

as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :

$$10 \times 1 = 10$$

   i)   The order and degree of the differential equation
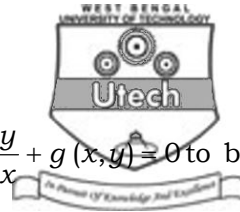
$$\frac{d^2 y}{dx^2} = \left\{ y + \left(\frac{dy}{dx} \right)^2 \right\}^{\frac{1}{4}} \text{ is}$$

       a)   2, 4   b)   4, 2

       c)   1, 4   d)   none of these.

  ii)   The integrating factor of the differential equation

$$\frac{dy}{dx} - 3y = \sin 2x \text{ is}$$

       a) e^{3x} b) e^{-3x}

       c) e^{x} d)   none of these.

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iii) For the differential equation $f(x,y)\dfrac{dy}{dx} + g(x,y) = 0$ to be exact if

   a) $\dfrac{\partial f}{\partial y} = \dfrac{\partial g}{\partial x}$ b) $\dfrac{\partial f}{\partial x} = \dfrac{\partial g}{\partial y}$

   c) $\dfrac{\partial^2 f}{\partial x^2} = \dfrac{\partial^2 g}{\partial y^2}$ d) none of these.

iv) The auxiliary equation of $\dfrac{d^2 y}{dx^2} + a^2 y = \sec ax\ (a \neq 0)$ is

   a) $m^2 + a^2 = 0$ b) $m^2 + 2a^2 = 0$

   c) $m^2 + a = 0$ d) none of these.

v) The general solution of $y = px + \sqrt{a^2 p^2 + b^2}$, where $p = \dfrac{dy}{dx}$ is

   a) $y = cx + \sqrt{a^2 c^2 + b^2}$ b) $y = cx - \sqrt{a^2 c^2 + b^2}$

   c) $y = c - x\sqrt{a^2 c^2 + b^2}$ d) none of these.

vi) The maximum number of edges in a simple graph with n vertices is

   a) n b) $\dfrac{n-1}{2}$

   c) $\dfrac{n(n-1)}{2}$ d) none of these.

vii) A binary tree has exactly

    a) two vertices of degree two

    b) one vertex of degree 2

    c) one vertex of degree one

    d) none of these.

viii) If a graph G has 7 vertices and 9 edges, then the size of the adjacency matrix is

    a) 7×7 b) 7×9

    c) 9×9 d) none of these.

ix) Dijkstra's algorithm is used to

    a) find maximum flow in a network

    b) scan all vertices of a graph

    c) find the shortest path from a specific vertex to another one

    d) none of these.

x) The singularities of the integral $\int_{-1}^{2} \dfrac{dx}{x(x-1)}$ are

    a) 0, 1   b) 1, 2

    c) -1, 2   d) 0, 2.

xi) The value of $\Gamma\left(\dfrac{1}{2}\right)$ is

    a) 2π b) $\sqrt{\pi}$

    c) $\dfrac{\pi}{2}$ d) none of these.

xii) Laplace transform of the function $\sin at$ is

    a) $\dfrac{s}{s^2 + a^2}$ b) $\dfrac{s}{s^2 - a^2}$

    c) $\dfrac{a}{s^2 + a^2}$ d) $\dfrac{a}{s^2 - a^2}$.

xiii) The value of $\Gamma(m)\,\Gamma(1-m)$ is

    a) $\dfrac{2\pi}{\sin \pi}$ b) $\dfrac{3\pi}{\sin m\pi}$

    c) $\dfrac{\pi}{\sin m\pi}$ d)   none of these.

xiv) The value of $\beta\left(\dfrac{1}{2},\dfrac{1}{2}\right)$ is

    a) $\sqrt{\pi}$ b) π

    c) $\dfrac{\pi}{2}$ d)   none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following   3 × 5 = 15

2.   Solve $(D^2 - 5D + 6)\, y = e^x \cos x$ where $D = \dfrac{d}{dx}$.

3.   Find the general and singular solution of $(y - px)(p - 1) = p$

     where $p = \dfrac{dy}{dx}$.

4.   Evaluate $L^{-1}\left(\dfrac{s^2}{(s^2 + a^2)(s^2 + b^2)}\right)$.

5. Prove that a tree with n vertices has $(n-1)$ edges.

6. Find the value of the improper integral $\int\limits_{0}^{\infty}\dfrac{dx}{(1+x)\sqrt{x}}$.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $\qquad$ 3 × 15 = 45

7. a) Apply the variation of parameters to solve $\dfrac{d^2 y}{dx^2}+4y=\sin 2x$.

   b) Solve : $(x^2 D^2 - xD + 4)\,y = x\sin(\log x)$ where $D = \dfrac{d}{dx}$.

   c) Show that $\int\limits_{-\infty}^{\infty} x e^{-x^2}\,dx = 0$. $\qquad$ 5 + 5 + 5

8. a) State convolution theorem. Using convolution theorem prove that $L^{-1}\left(\dfrac{s}{(s^2+a^2)^2}\right) = \dfrac{t\sin at}{2a}$.

   b) Solve the following differential equation using Laplace transform $(D^2 + 6D + 9)\,y = 1$:

   $y\,(0) = 0$, $y'(0) = 1$ $\left[D = \dfrac{d}{dx}\right]$.

   c) Evaluate $\int\limits_{0}^{\infty} e^{-3t}\,\sin t\,\cos t\,dt$ using Laplace transform.

   $\qquad$ 6 + 5 + 4

9. a) Prove that $\beta(m,n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.
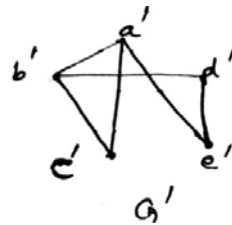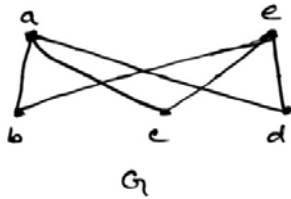
   b) Evaluate $\displaystyle\int_0^1 x^2\left(1-x^2\right)^{\frac{7}{2}}dx$.

   c) Prove that the number of odd degree vertices in a graph is always even.   5 + 5 + 5

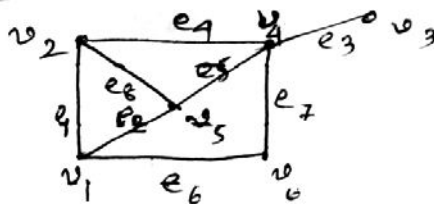10. a) Draw the graph whose incidence matrix is

   b) Define isomorphism. Examine whether the following two graphs are isomorphic or not.
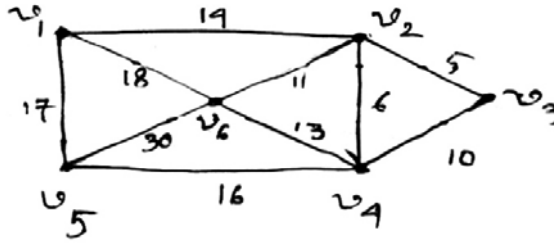

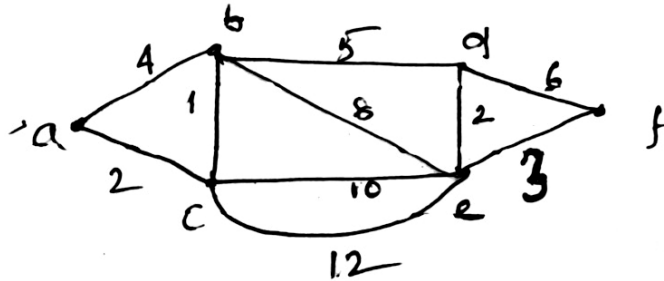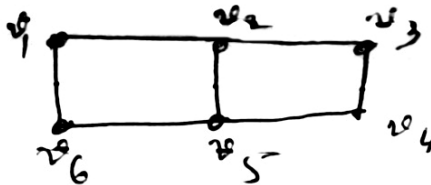   c) Determine the adjacency matrix of the given graph :

     5 + 5 + 5

11. a) Apply Prim's algorithm to find the shortest spanning tree of the following graph and find the corresponding minimum weight.

b) Find the shortest path from the vertex a to f in the following graph using Dijkstra's algorithm.

c) Construct the spanning tree of the following graph by BFS algorithm.

6 + 5 + 4

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