Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/B.TECH(OLD)/CSE,IT/SEM-3/M-301/2011-12

2011

MATHEMATICS

Time Allotted : 3 Hours *Full Marks :* 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

   i) If the R.V. X has *p.d.f.* $f(x) = \frac{1}{2}x,\ 0 \le x \le 2$, find the mean value of X .

      a) $\frac{4}{3}$ b) $\frac{7}{8}$

      c) $\frac{3}{4}$ d) $\frac{2}{3}$.

   ii) If $x = 4y + 5$ and $y = kx + 4$ be two regression equations of x on y and y on x respectively, then the value of k lies in the interval

      a) [4, 5] b) [0, 4]

      c) [0, 5] d) none of these.

iii) The variable of a uniform distribution with parameters a and b is

a) $\frac{a+b}{2}$ b) $\frac{a-b}{2}$

c) $\frac{(b-a)^2}{12}$ d) $\frac{(b+a)^2}{12}$.

iv) Probability of type II Error in the case of testing hypothesis, is (notations have their usual meanings)

a) Probability of rejection of $H_0(\theta \neq \theta_0)$ on the hypothesis that H_0 is true

b) Probability of acceptance of $H_0(\theta = \theta_0)$ on the hypothesis that $(\theta \neq \theta_0)$

c) Level of significance of the test

d) None of these.

v) In Tchebycheff's Inequality, where X is an R.V. having mean m and S.D. σ, then for any $\in > 0, P(|X-m| \geq \in) \leq$

a) $\sigma/\in$ b) $\sigma^2/\in$

c) $\sigma^2/\in^2$ d) $\sigma/\in^2$

vi) Under SRSWOR total possible samples of size two from the Population {1, 2, 3} are

a) {1, 1}, {2, 2} b) {1, 1}, {2, 2}, {3, 3}

c) {1, 2}, {1, 3}. {2, 3} d) {2, 2}, {3, 3}

vii) 50 tickets are serially numbered 1 to 50. One ticket is drawn from these at random. The probability of it being a multiple of 3 or 4 is

a) $\frac{12}{25}$ b) $\frac{6}{25}$

c) $\frac{18}{25}$ d) none of these.

viii) A random variable X has the following probability density function

$$f(x)=\begin{cases}1 & for\ 0\le x\le 1\\ 0 & otherwise\end{cases}$$

The mean of X is

a) $\frac{1}{2}$ b) $\frac{1}{3}$

c) $\frac{1}{4}$ d) 1.

ix) The moment generating function on the Poisson distribution with parameters λ is

a) $e^{\lambda t}$ b) $e^{(e^t+1)}$

c) $e^{\lambda(e^t-1)}$ d) none of these.

x) If S^2 be the sample variance of a sample of size 10 then an unbiased estimate of the population variance will be

a) $\frac{10}{9}S^2$ b) S^2

c) $\frac{9}{10}S^2$ d) $\frac{11}{9}S^2$

xi) A statistic t_n is called a consistent estimator of θ if for arbitrary $\varepsilon > 0$

a) $\lim_{n \to \infty} \{|t_n - \theta| < \varepsilon\} = 0$

b) $\lim_{n \to \infty} \{|t_n - \theta| > \varepsilon\} = 0$

c) $\lim_{n \to \infty} \{|t_n - \theta| = \varepsilon\} = 0$

d) none of these.

xii) If $-4 < t < 4$ be a region of acceptance for a statistical test of a hypothesis then the critical region of the test is given by

a) $t < -4$ b) $t > 4$

c) $-4 < t < 4$ d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. The distribution function $F(x)$ of a random variable x is defined as follows :

$$F(x) = A, \quad -\infty < x < -1$$
$$= B, \quad -1 \le x < 0$$
$$= C, \quad 0 \le x < 2$$
$$= D, \quad 2 \le x < \infty$$

where A, B, C, D are constants. Determine the value of A, B, C, D; given $P(X = 0) = \frac{1}{6}$ and $P(X > 1) = \frac{2}{3}$.

3. Show that if S^2 be sample variance of a sample of size n drawn from a population with mean μ and $S.D.$ σ then $E(S^2) = ((n-1)/n)\sigma^2$ (Where population size is infinite or the sample is drawn with replacement).

4. If the weekly wage of 10,000 workers in a factory follows normal distribution with mean and S.D. Rs. 70 and Rs. 5 respectively, find the expected number of workers whose weekly wages are (i) between Rs. 66 and Rs. 72, (ii) less than Rs. 66 and (iii) More than Rs. 72. [Given that $\frac{1}{\sqrt{2\pi}}\int_0^z e^{-t^2/2}dt = 0.1554$ and 0.2881 according as z = 0.4 and z = 0.8].

5. Ten individuals are choosen at random from a normal ((m, σ) population and their heights in inches are found to be 63, 66, 63, 67, 68, 69. 70,71, 72, 71. On the basis of the above data, obtain 95% confidence interval for the parameter m when σ is unknown. [Given $P(\chi^2 > 2.7) = 0.975$ and $P(t > 2.262) = 0.025$ for 9 d.o.f]

6. A random sample with observations 65, 71, 64, 71, 70, 69, 64, 63, 67, 68 is drawn from a normal population with S.D. $\sqrt{7.056}$. Test the hypothesis that the population mean is 69 at 1% level of significance. [Given that $P(0 < z < 2.58) = 0.495$].

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) A random variable X has a density function $f(x)$ is given by $f(x) = e^{-x}, x \geq 0$

$= 0$, elsewhere.

Show that Techebycheff's inequality gives

$P(|X-1| \geq 2) \leq \frac{1}{4}$ and show that actual probability is e^{-3}.

6

b) If $x_1, x_2, x_3, x_4, x_5, x_6$ be an independent simple random sample from a normal population with unknown variable σ^2, Find k so that $k\left[(x_1-x_2)^2+(x_3-x_4)^2+(x_5-x_6)^2\right]$ is an unbiased estimator. 5

c) If X is uniformly distributed in $(-1,1)$, find the density of $|X|$. 4

8. a) Show by Techebycheff's inequality that if a die is thrown 3,600 times, the probability of number of sixes lies between 550 and 650 is at least $\frac{4}{5}$.

b) If X_n is mutually independent and identically distributed random variable with mean m and finite variance σ^2, and $S_n = X_1 + X_2 + \ldots + X_n$, then prove that WLLN (Weak Law of Large Numbers) holds for the sequence $\{S_n\}$.

c) If T is an unbiased estimator of θ, show that $\sqrt{T}$ is biased estimate of $\sqrt{\theta}$.

9. a) Find the regression coefficient of y on x, of x on y and the correlation coefficient between x and y from the following values :

$\sum xy = 1500, \overline{x} = 15, \overline{y} = 12, \sigma_x = 6.4, \sigma_y = 9$ and number of observations is 10 where the notation have their usual meanings.

b) Find the moment generating function of Normal Distribution $N(\mu, \sigma)$ with parameters μ and σ and from it determine its mean and variance. 5 + 10

10. a) X follows $Bi(3, p)$ i.e. Binomial distribution with parameters 3 and p where $0 < p < 1$.

$H_0 : p = \frac{1}{4}, H_1 : P = \frac{3}{4}$

A test rejects H_0 *if* $X \geq 2$. Find the Type I and Type II errors of this test.

b) A company claims that its light bulbs are superior to those of a competitor on the basis of a study which showed that a sample of 40 of its bulbs had an average life time of 628 hours of continuous use with a standard deviation of 27 hours, while a sample of 30 bulbs made by the competitor had an average life time of 619 hours of continuous use with a standard deviation of 25 hours. Check at 5% level of significance whether this claim is justified. 8 + 7

11. a) Show that the weak law of large Nos. does not hold for the sequence of independent random variables $\{X_n\}$ with the distribution given as $P(X_n = \pm n) = \frac{1}{2}$.

b) For the variables x and y the equations of the regression lines are $4x - 5y + 33 = 0$ and $20x - 9y = 107$. Identify the regression line of y on x and that of x on y. Find the standard deviation of y.

c) The probability of a missile hitting a target is $\frac{1}{4}$. How many missiles must be sent so that the probability of hitting the Target at least once is greater than $\frac{2}{3}$?

5 + 5 + 5