



Name : .....

Roll No. : .....

Invigilator's Signature : .....

**CS/B.TECH(ECE)/SEP.SUPPLE/SEM-7/EC-704B/2012**

**2012**

**ADVANCED MATHEMATICS FOR  
ELECTRONICS ENGINEERING**

Time Allotted : 3 Hours

Full Marks : 70

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words  
as far as practicable.*

**GROUP – A**

**( Multiple Choice Type Questions )**

1. Choose the correct alternatives for any *ten* of the following :

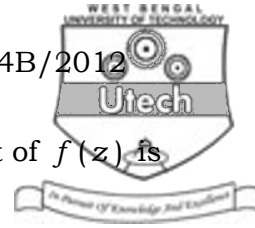
$$10 \times 1 = 10$$

i) The order of pole at  $z = 0$  of the function  $\left[ \frac{\sin z}{z^3} \right]$  is

- |      |       |
|------|-------|
| a) 1 | b) 2  |
| c) 3 | d) 4. |

ii) The relation  $|3 - z| + |3 + z| = 5$  represents

- |               |                 |
|---------------|-----------------|
| a) a circle   | b) a parabola   |
| c) an ellipse | d) a hyperbola. |



iii) If  $f(z) = e^{2z}$ , then the imaginary part of  $f(z)$  is

- a)  $e^y \sin x$
- b)  $e^x \cos y$
- c)  $e^{2x} \cos 2y$
- d) none of these.

iv) If  $A$  is a matrix and  $A^2 = A$  then  $A$  is

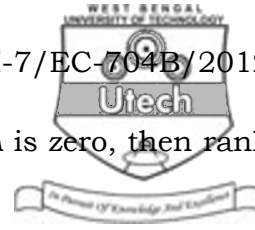
- a) Idempotent
- b) Nipotent
- c) Involutory
- d) none of these.

v) Matrix has a value. This is

- a) always true
- b) false
- c) depends upon the value of the matrices
- d) none of these.

vi) To multiply a matrix by scalar  $k$ , multiply

- a) any row by  $k$
- b) every element by  $k$
- c) any column by  $k$
- d) none of these.



vii) If every minor of order  $r$  of a matrix  $A$  is zero, then rank of matrix is

- a) greater than  $r$
- b) equal to  $r$
- c) less than or equal to  $r$
- d) less than  $r$ .

viii) The number  $i^i$  is

- a) a purely imaginary number
- b) an irrational number
- c) a rational number
- d) an integer.

ix) If  $x + iy = \sqrt{2} + 3i$ , then  $x^2 + y$  is

- a) 1
- b) 5
- c) 13
- d)  $\sqrt{2} + 3$ .

x) If  $A$  is a square matrix such that  $AA' = I$ , then value of  $A'A$  is

- a)  $A^2$
- b)  $I$
- c)  $A^{-1}$
- d) none of these.



xi) A square matrix  $A$  is called orthogonal if

- a)  $A = A^2$
- b)  $A' = A^{-1}$
- c)  $AA^{-1} = I$
- d) none of these.

### GROUP – B

#### ( Short Answer Type Questions )

Answer any *three* of the following.  $3 \times 5 = 15$

2. If  $\lambda$  is an eigenvalue of a nonsingular matrix  $A$ , then prove that  $\lambda^{-1}$  is the eigenvalue of  $A^{-1}$ . Hence find the eigenvalues of  $A^{-1}$ , where

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{pmatrix}.$$

3. Determine the row rank and the column rank of the matrix  $A$  and verify that these are equal, where

$$A = \begin{pmatrix} 2 & 1 & 4 & 3 \\ 3 & 2 & 6 & 9 \\ 1 & 1 & 2 & 6 \end{pmatrix}.$$

4. Use Cauchy's Residue theorem to evaluate

$$\oint_C \frac{3z^2 + z - 1}{(z^2 - 1)(z - 3)} dz \text{ around the circle } C: |z| = 2.$$



5. Given  $f(x) = \frac{\pi x}{4}$ ,  $0 \leq x \leq \pi/2$   
 $= \frac{\pi(\pi - x)}{4}$   $\pi \leq x \leq 2\pi$

Expand  $f(x)$  in an infinite series of sines multiples of  $x$  in  $[0, \pi]$ .

6. Find  $f(x)$  whose Fourier Sine Transform is  $\frac{e^{-as}}{s}$  where  $a > 0$ .

7. Find  $L^{-1} \left\{ \frac{s^2}{(s+1)^5} \right\}$ .

### GROUP – C

#### ( Long Answer Type Questions )

Answer any *three* of the following.  $3 \times 15 = 45$

8.  $A = \begin{bmatrix} 9 & 2 & 8 \\ 2 & 18 & 10 \\ 8 & 10 & 15 \end{bmatrix}$ ,  $B = \begin{bmatrix} 0 & 2 & 6 \\ -2 & 0 & -3 \\ -6 & 3 & 0 \end{bmatrix}$

Find (i) inverse of  $A$ , (ii) inverse of  $B$ , (iii) inverse of  $(AB)$ .

5 + 5 + 5

9. a) Show that  $J_{-n}(x) = (-1)^n J_n(x)$  where  $n$  is a positive integer. 5

b) Evaluate  $J_{-\frac{1}{2}}(x)$  5



c) Show that  $\frac{d}{dx} [J_n^2(x) + J_{n+1}^2(x)]$

$$= 2 \left[ \frac{n}{x} J_n^2(x) - \left( \frac{n+1}{x} \right) J_{n+1}^2(x) \right]. \quad 5$$

10. a) Prove that, the function  $u = \frac{1}{2} \log(x^2 - y^2)$  is harmonic. Find its harmonic conjugate and the corresponding analytic function  $f(z)$  in terms of  $z$ . 8

b) Express  $4x^3 - 2x^2 - 3x + 8$  in terms of Legendre polynomials. 7

11. a) Prove that for  $0 < x < 2$

$$x = 1 + \sum_{n=1}^{\infty} \frac{4}{n^2 \pi^2} (\cos n\pi - 1) \cos \frac{n\pi x}{2}. \quad 5$$

b) If  $f(x) = 1, 0 \leq x < 1$   
 $= 0, x \geq 1$

Find

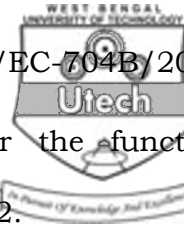
i) Fourier Sine Transform of  $f(x)$

ii) Fourier Cosine Transform of  $f(x)$ .

Use this to show that

x)  $\int_0^{\infty} \left( \frac{1 - \cos x}{x} \right)^2 dx = \frac{\pi}{2}$

y)  $\int_0^{\infty} \frac{\sin^4 x}{x^2} dx = \frac{\pi}{2}. \quad 10$



12. a) Write all possible Laurent series for the function  $f(z) = \frac{1}{z(z+2)^3}$  about the pole  $z = -2$ . 5

- b) Using contour integration, evaluate the real integral

$$\int_0^{\pi} \frac{1 + 2 \cos \theta}{5 + 4 \cos \theta} d\theta . \quad 5$$

- c) Find the mapping of  $x$ -axis under the transformation

$$w = \frac{i-z}{i+z} \text{ onto the } w\text{-plane.}$$

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