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Paper Code : BSM-201(N)

MATHEMATICS-IIA

Full Marks : 70

Time Allotted : 3 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

   i) If $P(A) = 1/3$, $P(B) = 1/4$, $P(AUB) = 1/2$ then $P(B/A)$ is

   a) $\frac{3}{4}$ b) $\frac{4}{3}$

   c) $\frac{1}{4}$ d) $\frac{1}{3}$.

   ii) The random variable X has distribution function F. If $F(0) = 1/6$, $F(1) = \frac{1}{2}$, $F(3) = \frac{3}{4}$ then $P(0 < X \le 3)$ is

   a) $\frac{3}{4}$ b) $\frac{1}{3}$

   c) $\frac{7}{12}$ d) none of these.

[Turn over

iii) If the random variable X has Binomial distribution with parameters n & p then the mean and variance are respectively

   a) np & $np(1-p)$ b) n/p & $2n(1-p)$

   c) $np(1-p)$ & np d)   none of these.

iv) The distribution function $F(x)$ of a random variable X is given by

   a) $P(-\infty < X < \infty)$ b) $P(-\infty < X \le x)$

   c) $P(-\infty \le X < \infty)$ d)   none of these.

v) For two random variable X and Y are independent, then

   a) $E(XY) = E(X)E(Y)$

   b) $E(XY) = E(X) + E(Y)$

   c) $E(XY) = E(X) / E(Y)$

   d)   none of these.

vi) The maximum likelihood estimate is a solution of the equation

a) $\dfrac{\partial L(\theta)}{\partial \theta} = 0$

b) $\dfrac{\partial L(\theta)}{\partial \theta} = $ const.

c) $\dfrac{\partial L(0)}{\partial \theta} = \theta$

d) none of these.

vii) A statistic t is said to be an unbiased estimator of a population parameter θ when

a) $E(t) = \theta$

b) $E(t^2) = \theta$

c) $E(t^2) = [E(\theta)]^2$

d) $E(t^2) = [E(t)]^2$.

viii) The probability of sample space is

a) 1

b) $\dfrac{1}{2}$

c) 0

d) none of these.

ix) For two events, $P(A) = 0.4$, $P(B) = x$ and $P(A + B) = 0.7$. The value of x for which A and B will be mutually exclusive is

a) 2/10

b) 3/10

c) 2/5

d) 1/5.

x) The number of ways 10 persons can occupy 10 chairs arranged at a round table is

a) 10! – 1

b) 9! – 1

c) 10!

d) 9!.

xi) The probability $P (a \leq x \leq b)$ is defined by $F (x)$ (where $F (x)$ is the distribution function of the random variable X) http://www.makaut.com

a) $F (b) - F (a)$

b) $F (b) + F (a)$

c) $F (a) - F (b)$

d) $F (a) F (b)$.

xii) A random variable X has the following p.m.f :

$x :$	1	2	3
$f (x) :$	$\frac{1}{2}$	$\frac{1}{3}$	k

Then the value of k is

a) 1

b) 0

c) $\frac{5}{6}$

d) $\frac{1}{6}$.

xiii) A random variable has a Poisson distribution such that $P (1) = P (2)$. Then the s. d. of X is

a) 0

b) 2

c) $\sqrt{2}$

d) – 2.

GROUP – B
(Short Answer Type Questions)
Answer any *three* of the following. $3 \times 5 = 15$

2. In a bolt factory, machines A, B and C manufacture respectively 25%, 35% and 40% of the total output. 5%, 4% and 2% are defective bolts. A bolt is drawn at random from the product and is found to be defective. What are the probabilities that it was manufactured by machines A, B and C?

3. Find mean and variance of binomial distribution.

4. Prove that $-1 \le \rho_{xy} \le 1$, where ρ_{xy} denotes correlation coefficient between x and y.

5. The distribution function $F(x)$ of a variable X is defined as follows :

$$F(x) = A, \quad -\infty < x < -1$$
$$= B, \quad -1 \le x < 0$$
$$= C, \quad 0 \le x < 2$$
$$= D, \quad 2 \le x < \infty$$

where A, B, C, D are constant. Determine the values of A, B, C and D given that $P(X = 0) = \frac{1}{6}$ and $P(X > 1) = \frac{2}{3}$.

[Turn over

6. A box contains five defective and 10 non-defective lamps. Eight lamps are drawn at random in succession without replacement. What is the probability that eighth lamp is the fifth defective ?

GROUP – C
(Long Answer Type Questions)
Answer any *three* of the following. $3 \times 15 = 45$

7. a) What is the axiomatic definition of probability ?

   b) For two vents A and B which are not necessarily exclusive, prove that

   $$P (A \cup B) = P (A) + P (B) - P (A \cap B).$$

   c) Two bags contains respectively 3 white and 2 red balls, 2 white and 4 red balls. One ball is drawn at random from the first bag and is put into the second; then a ball is drawn from the second bag. What is the probability that the ball drawn from the second bag is white ?

   d) If A and B are two events, prove that

   $$P (A/B) = 1 - P (A^o/B), P (B) > 0.$$

   $$2 + 5 + 5 + 3$$

8. a) Two discrete random variable X and Y are connected by the relation $2X + 3Y + 4 = 0$. Prove that the correlation coefficient between X and Y is -1.

b) Suppose that during rainy season, on a tropical island, the length of shower has an exponential distribution with average length of shower $\frac{1}{2}$ mins.

What is the probability that a shower will last more than three times ? If a shower has already lasted for 2 minutes, what is the probability that it will last for at least one more minute ?

c) The joint probability density function (pdf) of a bivariate (X, Y) is

$$f(x, y) = C(x + y), \quad x < 0,\ y > 0,\ x + y < 2$$

$$= 0 \qquad\qquad \text{, elsewhere}$$

Find C and $P(X < 1, Y > 1/2)$.   5 + 5 + 5

9 a) The median and mode of the following frequency distribution are known to be 27 and 26 respectvely. Find the values of *a* and *b*.

Class interval :	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency :	3	*a*	20	12	*b*

b) The scores of two batsman *A* and *B* in 10 innings are :

A :	19	31	48	53	67	90	10	62	40	80
B :	32	28	47	63	71	39	10	60	96	14

Find which batsman is more consistent in scoring.

c) The radius of a circle has distribution given by the probability density function :

$$f(x) = 1,\ 1 < x < 2$$

$$= 0,\ \text{elsewhere}$$

Find the mean and variance of the area of the circle.

5 + 5 + 5

10. a) Fit a second degree parabola, $y = a + bx + cx^2$, to the following data :

x:	0	1	2	3	4
y:	1	5	10	22	38

Predict y when $x = 4.8$.

b) A normal population has a mean 0.1 and standard deviation 2.1. Find the probability that the mean of a sample of size 900 will be negative. Given that $P(|x| < 1.43) = 0.847$.

c) In order to test whether a coin is perfect the coin tossed 5 times. The null hypothesis of perfectness is rejected if more than 4 heads are obtained. What is the probability of Type I Error ? Find the probability of Type II Error when the corresponding probability of head is 0.2.

5 + 5 + 5

11. a) A machine part was designed to withstand an average pressure of 120 units. A random sample of size 100 from a large batch was tested and it was found that the average pressure which these parts can withstand is 105 units with a s.d of 20 units. Test at 5% level wheter the batch meet the specification. Suppose the population has normal distribution.

b) Survey of 320 families with 5 children each revealed the following distribution :

No. of boys :	5	4	3	2	1	0
No. of girls :	0	1	2	3	4	5
No. of family :	14	56	110	88	40	12

Is the result consistent with the hypothesis that male and female births are equally probable ? The 5% value of χ^2 with 5 d.o.f is 11.07.

c) Two scanners are needed for an experiment of the five available, two have electronic defects, one has defect in memory and two are in good working condition. Two units are selected at random. Let X_1 = Number of units with electronic defects, X_2 = Number of units with defect in memory.

i) Find the joint distribution of X_1 and X_2.

ii) Find the probability of no or one defects among the selected two items.

iii) Find the marginal distribution of X_1.

5 + 5 + 5