Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/B.Tech (EE-N)/SEM-6/EC-611/2011

2011
DIGITAL SIGNAL PROCESSING

Time Allotted : 3 Hours *Full Marks :* 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

   i) The system described by $y(n) = x(n) + 2x(n-2) + 3x(n-3)$ is
      a) causal and stable
      b) causal and unstable
      c) noncausal and stable
      d) noncausal and unstable.

   ii) If $x(n) = \{2, 1, 3, \underset{\uparrow}{0}, 1, 2, 4\}$, then $x(-n+2)$ is given by
      a) $\{2, 1, 3, 0, 1, \underset{\uparrow}{2}, 4\}$
      b) $\{2, \underset{\uparrow}{1}, 3, 0, 1, 2, 4\}$
      c) $\{4, \underset{\uparrow}{2}, 1, 0, 3, 1, 2\}$
      d) $\{4, 2, 1, 0, 3, \underset{\uparrow}{1}, 2\}$.

iii) If $x(n)$ and $h(n)$ are two finite length sequences, then their convolution has length

a) 8 b) 10

c) 11 d) 9.

iv) The overall impulse response of a cascade connection of two systems with impulse responses $h_1(n)$ and $h_2(n)$ is

a) $h_1(n) + h_2(n)$ b) $h_1(n)h_2(n)$

c) $h_1(n) * h_2(n)$ d) $h_1(n) - h_2(n)$.

v) A discrete-time LTI system is causal if

a) impulse response $h(n) > 0$, $n > 0$

b) impulse response $h(n) < 0$, $n > 0$

c) impulse response $h(n) = 0$, $n > 0$

d) impulse response $h(n) = 0$, $n < 0$.

vi) The ROC of an infinite causal sequence is the

a) interior of a circle

b) exterior of a circle

c) entire z-plane except $z = 0$

d) entire z-plane except $z = \infty$.

vii) The Z-transform of $u[n-1]$ is

a) $1/(1 - Z^{-1})$ b) $Z/(1 - Z^{-1})$

c) $1/[Z(1 - Z^{-1})]$ d) $(1 + Z^{-1})$.

viii) If $x(K)$ represents the 8 point DFT of $x(n) = \{1, 1, 1, 1, 1, 1, 0, 0\}$ then $x(0)$ is

a) 3 b) 6

c) 1 d) 0.

ix) The mapping from analog to digital domain in impulse invariant method is

a) one to many b) many to one

c) one to one d) none of these.

x) Overlap save method is used to find

a) circular convolution b) linear convolution

c) DFT d) Z-transform.

xi) Number of multiplications is FFT algorithm is

a) $n \log(n)$ b) $(n/2) * \log(n)$

c) $(n/2) * \log(n/2)$ d) $n \log(n/2)$.

xii) FIR filter is

a) recursive and linear

b) non-recursive linear

c) recursive and non-linear

d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. The impulse response of an LTI system is $h(n) = \{1, 2, 1, -1\}$. Determine the response of the system to the input signal $x(n = \{1, 2, 3, 1\}$.

3. If a discrete-time LTI system is BIBO stable, show that the ROC of its system function $H(z)$ must contain the unit circle, i.e., $|z|=1$.

4. Explain the relationship between *S*-plane and *Z*-plane.

5. a) Find the DTFT of the sequence $x(n) = \{1, -1, 1, -1\}$.

   b) Find the IDTFT of $X\left(e^{i\omega}\right) = e^{-j\omega}\left(\frac{1}{2} + \frac{1}{2}\cos\omega\right)$. 2 + 3

6. Determine the convolution of the two following sequences using overlap add method :

   $x(n) = \{3, 2, 1, 2\}$ $h(n) = \{1, 2, 1, 1\}$.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. 3 × 15 = 45

7. a) Justify whether the system is LTI or not.

$$y(n) = y(n-1) + \sum_{k=0}^{2} x(n-k).$$

b) Compute the circular convolution of the two sequences given below.

$x(n) = \{2 \;\; -1 \;\; 0 \;\; 1 \;\; -2 \;\; 3 \;\; 0 \;\; 1\}$.

$h(n) = \{1 \;\; 2 \;\; -1 \;\; 1\}$.

c) Determine the linear convolution of the above sequences using over-lap save method.

8. a) What is ROC ? State its properties.

b) Find the system function & impulse response of the system described by $y(n) = x(n) + 2x(n-1) - 4x(n-2) + x(n-3)$

c) Find the Inverse *Z*-transform of

$$X(Z) = Z\left(Z^2 - 4Z + 5\right)/(Z-3)(Z-2)(Z-1)$$

d) Prove that an LTI system is BIBO stable if the ROC system function includes the unit circle. 2 + 5 + 5 + 3

9. a) Find the 8-point DFT using decimation in time FFT algorithm for a sequence $x(n) = \{1,\ 3,\ 5,\ 7,\ 2,\ 4,\ 6,\ 8\}$.

b) What do you mean by zero padding ?

c) Using linear convolution find $y(n) = x(n) * h(n)$ for the sequence $x(n) = \{1,\ 2,\ -1,\ -2,\ 0,\ 1,\ 3,\ -1\}$. Compare the result by solving the problem using

i) overlap save method

ii) overlap add method. 5 + 2 + 8

10. Following specifications are given for a filter function :

$\alpha_{pass} = 4$ dB, $\alpha_{stop} = 48$ dB, $f_{stop} = 7$ kHz, $f_{pass} = 2$ kHz, $f_{sampling} = 20$ kHz

Determine an IIR filter using Butterworth approximation and impulse invariant method.

11. a) Design a digital Butteworth IIR filter for the given frequency response :

$0 \cdot 85 \le \left| H\left(e^{j\omega}\right) \right| \le 1, \quad$ for $0 \le \omega \le 0 \cdot 2\ \pi$

$\left| H\left(e^{j\omega}\right) \right| \le 0 \cdot 02, \quad$ for $0 \cdot 45\ \pi \le \omega \le \pi$

Use impulse invariant method.

b) Convert the analog filter with system function $G(s) = \dfrac{s + 0 \cdot 1}{(s + 0 \cdot 1)^2 + 16}$ into a digital filter using bilinear transformation. The digital filter should have a resonant frequency of $\omega_r = \dfrac{\pi}{4}$ radian. 8 + 7

12. Write short notes on any *three* of the following : 3 × 5

a) Causal and non-causal system

b) Circular convolution and linear convolution

c) DIT-FFT algorithm

d) Difference between DTFT and DFT

e) Bilinear transformation.