



ENGINEERING & MANAGEMENT EXAMINATIONS, JUNE - 2009

MATHEMATICS**SEMESTER - 2**

Time : 3 Hours]

[Full Marks : 70

GROUP - A**(Multiple Choice Type Questions)**

1. Choose the correct alternatives for any ten of the following :

10 × 1 = 10

i) If $A = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$, then A^{100} is

a) $\begin{bmatrix} 1 & 0 \\ -150 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 \\ -50 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 0 \\ -100 & 1 \end{bmatrix}$

d) None of these.

ii) The set of vectors $\{ (2, 1, 1), (1, 2, 2), (1, 1, 1) \}$ in R^3 is

a) linearly dependent

b) linearly independent

c) basis of R^3

d) none of these.

iii) The matrix $A = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$ is

a) an orthogonal matrix

b) a symmetric matrix

c) an idempotent matrix

d) a null matrix.



ix) $\Delta^2 e^x$ is equal to ($h = 1$)

a) $(e-1)^2 e^x$

b) $(e-1) e^x$

c) $e^{2x} (e-1)$

d) e^{2x}

x) The value of $\int_0^{\infty} \frac{\sin t}{t} dt$ is equal to

a) $\frac{\pi}{3}$

b) $\frac{\pi}{6}$

c) $\frac{\pi}{4}$

d) $\frac{\pi}{2}$

xi) If S and T are two subspaces of a vector space V , then which one of the following is a subspace of V also?

a) $S \cup T$

b) $S \cap T$

c) $S - T$

d) $T - S$

xii) If $\lambda^3 - 6\lambda^2 + 9\lambda - 4$ is the characteristic equation of a square matrix A , then A^{-1} is equal to

a) $A^2 - 6A + 9I$

b) $\frac{1}{4} A^2 - \frac{3}{2} A + \frac{9}{4} I$

c) $\frac{1}{4} A^2 - \frac{3}{2} A + \frac{9}{4}$

d) $A^2 - 6A + 9$

xiii) Co-factor of -3 in the determinant $\begin{vmatrix} -2 & -3 & 4 \\ 1 & 0 & 1 \\ 0 & -1 & 4 \end{vmatrix}$ is

a) 4

b) -4

c) 0

d) none of these.

**GROUP - B****(Short Answer Type Questions)**Answer any *three* of the following.

3 × 5 = 15

2. If A be a skew symmetric and $(I + A)$ be a non-singular matrix, then show that $B = (I - A)(I + A)^{-1}$ is orthogonal.

3. Evaluate $L^{-1} \left\{ \frac{1}{(s-1)^2(s-2)^3} \right\}$.

4. Solve the differential equation

$$\frac{dy}{dx} + y = y^3 (\cos x - \sin x).$$

5. Evaluate the definite integral $\int_1^4 (x + x^3) dx$ by using Trapezoidal rule, taking five ordinates and calculate the error.

6. If $A(\theta) = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$, then show that

$$A(\theta)A(\phi) = A(\phi)A(\theta) = A(\theta + \phi).$$

GROUP - C**(Long Answer Type Questions)**Answer any *three* of the following.

3 × 15 = 45

7. a) If $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & -4 & 1 \\ 3 & 0 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -1 & 0 \\ 2 & 1 & -1 \end{bmatrix}$, show that $AB = 6I$.

Utilise this result to solve the following system of equations :

$$2x + y + z = 5$$

$$x - y = 0$$

$$2x + y - z = 1$$



- b) Solve : $(y - px)(p - 1) = p$ and obtain the singular solution. Here $p = \frac{dy}{dx}$.
- c) Construct the interpolation polynomial for the function $y = \sin \pi x$, taking the points $x_0 = 0, x_1 = \frac{1}{6}, x_2 = \frac{1}{2}$.

Hence find $f\left(\frac{1}{3}\right)$ where $y = f(x)$.

8. a) Solve the differential equation

$$\frac{d^3y}{dx^3} - 6 \frac{dy}{dx} + 6y = x^2 e^{3x}.$$

- b) Apply suitable interpolation formula to calculate $f(9)$ correct up to two significant figures from the following data :

$x :$	2	4	6	8	10
$f(x) :$	5	10	17	29	49

- c) Determine the conditions under which the system of equations

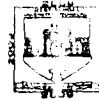
$$x + y + z = 1$$

$$x + 2y - z = b$$

$$6x + 7y + az = b^2$$

admits of

- i) only one solution
- ii) no solution
- iii) many solutions.



9. a) Prove that $P^T A P$ is a symmetric or a skew-symmetric matrix according as A is symmetric or skew-symmetric.

b) Find the eigenvalues and the eigenvectors of the matrix $\begin{bmatrix} 4 & 6 \\ 2 & 9 \end{bmatrix}$.

c) Solve by Cramer's rule the following system of equations :

$$3x + y + z = 4$$

$$x - y + 2z = 6$$

$$x + 2y - z = -3.$$

10. a) What is meant by linear independence of a set of n -vectors ?

b) Solve by the method of variation of parameters the equation

$$\frac{d^2 y}{dx^2} + 9y = \sec 3x.$$

c) Prove that $\Delta = \begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$.

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