(a) State D'Alembert's Ratio test. Applying this test, examine the convergence of the following series 2+3

$$1+\frac{2^a}{2!}+\frac{3^a}{3!}+\frac{4^a}{4!}+\dots\quad (a>0)$$

(b) Show that $\left[\vec a+\vec b,\ \vec b+\vec c,\ \vec c+\vec a\right]=2\left[\vec a,\vec b,\vec c\right]$. 4

(c) If $f(v^2-x^2, v^2-y^2, v^2-z^2)=0$, where v is a function of x, y, z then show that 6

$$\frac{1}{x}\frac{\partial v}{\partial x}+\frac{1}{y}\frac{\partial v}{\partial y}+\frac{1}{z}\frac{\partial v}{\partial z}=\frac{1}{v}$$

(a) Determine the conditions under which the system of equations 5

$x+y+z=1$ $\quad$ $x+2y-z=k$ $\quad$ $5x+7y+az=k^2$

admits (i) only one solution, (ii) no solution, (iii) many solutions

(b) Verify the divergence theorem for the vector function $\vec F=4xz\hat i-y^2\hat j+yz\hat k$ taken over a cube bounded by $x=0$, $x=1$; $y=0$, $y=1$; $z=0$, $z=1$. 6

(c) If $I_n=\int_0^{\pi/2} x^n\sin x\,dx,\ (n>1)$ then prove that $I_n+n(n-1)I_{n-2}=n\left(\frac{\pi}{2}\right)^{n-1}$ 4

(a) Verify Lagrange's Mean Value Theorem at $[-1, 1]$ for 5

$$f(x)=x\sin\frac{1}{x},\quad x\neq 0$$
$$=0,\quad x=0$$

(b) If $u=xf\left(\frac{y}{x}\right)+g\left(\frac{y}{x}\right)$ then show that 5

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=xf\left(\frac{y}{x}\right) \text{ and } x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=0$$

(c) Find the rank of the following matrix $\begin{bmatrix}2&3&16&5\\4&5&6&7\\2&0&1&3\\8&8&23&15\end{bmatrix}$ 5

(a) Find the extremum of the following function x^3+y^3-3axy. 5

(b) Show that $\nabla\phi$ is irrotational, where $\phi=x^2y+2xy+z^2$ 5

(c) Evaluate $\iiint x^3y^2z\,dz\,dy\,dx$ 5

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M-101

MATHEMATICS-I

Time Allotted : 3 Hours Full Marks : 70

The questions are of equal value
The figures in the margin indicate full marks
Candidates are required to give their answers in their own words as far as practicable

GROUP A
(Multiple Choice Type Questions)

1. Answer any *ten* questions. 10 × 1 = 10

(i) $\int_0^{\pi/2}\sin^5\theta\,d\theta=$

(A) $\frac{8}{15}$ (B) $\frac{8\pi}{15}$ (C) $\frac{8}{15}$ (D) $\frac{4}{15}$

(ii) If $u(x,y)=yf\left(\frac{x^2}{y}\right)$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=$

(A) 0 (B) $2u(x,y)$ (C) $u(x,y)$ (D) 2

(iii) The value of $\int_c (x\,dx - dy)$, where c is the line joining (0,1) to (,0) is

(A) $\frac{3}{2}$ (B) $\frac{1}{2}$ (C) 0 (D) $\frac{2}{3}$

(iv) Component of the vector $2\hat i+5\hat j+7\hat k$ on $\hat i+2\hat j+2\hat k$ is

(A) $\sqrt{78}$ (B) 3 (C) 6 (D) 2

(v) The value of t for which $(x+3y)\hat i+(y-2z)\hat j+(x+tz)\hat k$ is solenoidal is

(A) 2 (B) −2 (C) 0 (D) 1

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vi) If $x = r\cos\theta$ and $y = r\sin\theta$ then $\dfrac{\partial(r,\theta)}{\partial(x,y)} =$

(A) r (B) 1 (C) $\dfrac{1}{r}$ (D) 0

vii) $f(x,y) = \dfrac{x^3 + y^3}{\sqrt{x^2 + y^2}}$ is a homogeneous function of degree

(A) 0 (B) 2 (C) 3 (D) $\dfrac{1}{2}$

viii) If $A = \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & -5 & 5 \end{bmatrix}$ then A is

(A) idempotent (B) nilpotent (C) involutary (D) none of these

ix) If $y = \tan^{-1} x$ then

(A) $(1+x^2)y_1 = 1$ (B) $(1+x^2)y_2 = 1$ (C) $(1+x^2)y_1 = 0$ (D) $(1+x^2)y_1 = 2$

(x) If A is real skew-symmetric matrix such that $A^2 + I = 0$, then A is

(A) singular (B) unit matrix (C) orthogonal (D) none of these

(xi) The Sequence $\left\{1, \frac{1}{3}, \frac{1}{3^2}, \ldots, \frac{1}{3^n}, \ldots \infty\right\}$ is

(A) divergent (B) oscillatory (C) convergent (D) none of these

(xii) For a function $f(x)$ the expression $\dfrac{h^n(1-\theta)^{n-1}}{(n-1)!} f^n(a+\theta h)$ is known as

(A) Lagrange's remainder (B) Cauchy's remainder
(C) Maclaurin's remainder (D) Taylor's remainder

GROUP B
(Short Answer Type Questions)

Answer any *three* questions $3 \times 5 = 15$

Using Laplace's method of expansion, prove that

$$\begin{vmatrix} x & y & -u & v \\ y & x & v & u \\ u & v & x & y \\ -v & -u & y & x \end{vmatrix} = (x^2 + y^2 - v^2 - u^2)^2$$

For what values of x is the following infinite series convergent?

$$\sum_{n=1}^{\infty} \frac{(n+1)^n x^n}{n^{(n+1)}} \quad (x > 0).$$

If α, β, γ are the angles which a vector makes with the co-ordinate axes, prove that $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

If $y = x^{n-1}\log x$, using Leibnitz's theorem show that $y_n = \dfrac{(n-1)!}{x}$

Using Green's theorem evaluate $\oint_c \{(\cos x\sin y - xy)dx + \sin x\cos y\,dy\}$ where c is the circle $x^2 + y^2 = 1$

GROUP C
(Long Answer Type Questions)

Answer any *three* questions $3 \times 15 = 45$

If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ then show that $A^2 - 4A - 5I_3 = 0$. Hence find A^{-1} 5

If $y = e^{m\sin^{-1}x}$, then show that 5

(i) $(1-x^2)y_2 - xy_1 - m^2y = 0$ (ii) $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$

Also find $(y_n)_0$.

Is Rolle's Theorem applicable to the function $f(x) = (x-p)^m(x-q)^n$, $x \in [p,q]$, where m, n are positive integers? If so, find the constant c of Rolle's Theorem, where c has its usual meaning 5

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