



Name :

Roll No. :

Invigilator's Signature :

CS/B.TECH(OLD)/SEM-2/M-201/2011

2011

MATHEMATICS

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :

$$10 \times 1 = 10$$

i) The rank of the matrix $\begin{bmatrix} 2 & 3 & 5 \\ 1 & -1 & 2 \\ 1 & 4 & 3 \end{bmatrix}$ is

- a) 3 b) $5/2$
c) 2 d) 1.

ii) In the Newton's forward interpolation formula the value of $u = (x - x_0) / h$ lies between

- a) 1 and 2 b) -1 and 1
c) 0 and ∞ d) 0 and 1.



iii) The eigenvalues of a matrix A are 2 and 4. Then the eigenvalues of A^{-1} are

- a) 2, 4 b) 4, 4
c) 0.25, 0.5 d) 0.5, 0.25.

iv) The value of a for which the vectors $(1, 2, 1)$, $(a, 1, 1)$ and $(1, 1, 2)$ are linearly dependent is

- a) 2 b) $\frac{2}{3}$
c) -1 d) $\frac{3}{2}$.

v) The value of the determinant $\begin{vmatrix} 0 & -a & -b \\ a & 0 & c \\ b & -c & 0 \end{vmatrix}$ is

- a) 0 b) abc
c) $-abc$ d) $2abc$.

vi) If the system of equations $4x + 2y - 5z = 0$, $x + \lambda y + 2z = 0$, $2x + y - z = 0$ has a non-zero solution, then λ is

- a) $\frac{1}{3}$ b) $\frac{1}{2}$
c) 1 d) none of these.

vii) The integrating factor of $\frac{dy}{dx} + y = 1$ is

- a) e^x b) x
c) e^2 d) 2.

dimension of

- of linear equations

linear equation

- $$[m, r] - r.$$



5. Solve :

$$\begin{cases} \frac{dx}{dt} + y = e^t \\ \frac{dy}{dt} - x = e^{-t} \end{cases}$$

6. Evaluate : $\int_0^2 (x^3 + 1) dx$ by Simpson's one-third rule

taking 4 intervals.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Investigate for what values of λ and μ the following equations

$$x + y + z = 6$$

$$x + 2y + 3z = 10$$

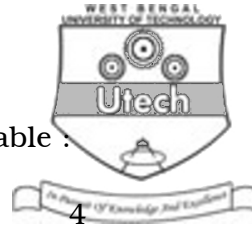
$$x + 2y + \lambda z = \mu$$

have (i) no solution (ii) a unique solution and (iii) an infinite number of solutions.

b) Prove that

$$\begin{vmatrix} bc - a^2 & ca - b^2 & ab - c^2 \\ ca - b^2 & ab - c^2 & bc - a^2 \\ ab - c^2 & bc - a^2 & ca - b^2 \end{vmatrix} = (a^3 + b^3 + c^3 - 3abc)^2$$

c) Show that every square matrix can be expressed as a sum of a symmetric and skew-symmetric matrix.



8. a) Compute $f(1 \cdot 3)$ from the following table :

$x :$	0	1	2	3	4
$f(x) :$	1	1.5	2.2	3.1	4.3.

- b) Solve by the method of variation of parameters

$$\frac{d^2 y}{dx^2} + 3 \frac{dy}{dx} + 2y = 3e^{-x} + x$$

- c) If α , β and γ form a basis of a vector space V , then prove that $\alpha + \beta + \gamma$, $\beta + \gamma$ and γ also form a basis of V .

9. a) Show that the matrix

$$A = \frac{1}{3} \begin{pmatrix} -1 & 2 & -2 \\ -2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix} \text{ is orthogonal and hence obtain } A^{-1}.$$

- b) Using the method of separation of symbols, prove that

$$u_0 + u_1 + u_2 + \dots + u_n = {}^{n+1}C_1 u_0 + {}^{n+1}C_2 \Delta u_0 + {}^{n+1}C_3 \Delta^2 u_0 + \dots + {}^{n+1}C_{n+1} \Delta^n u_0.$$

- c) Show that

$$M = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} / \begin{matrix} a, b, c, d \in \mathbb{R} \\ a = d \end{matrix} \right\} \text{ is a subspace of the vector space of } 2 \times 2 \text{ real matrices. Obtain a basis and dimension of } M.$$



10. a) If possible diagonalise the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 2 & 1 & 2 \end{pmatrix},$$

Specify the matrix which diagonalises A and the diagonal matrix to which A is changed after being diagonalised.

- b) Find $L \left(\frac{1 - e^t}{t} \right)$.
- c) Assuming orthogonal property of Legendre function, prove that

$$\int_{-1}^1 [P_n(x)]^2 dx = \frac{2}{2n+1}.$$

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