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CS/B.TECH(OLD)/PWE/BME/ICE/EE/ECE/EIE/EEE /SEM-3/M-302/2011-12

2011

MATHEMATICS

Time Allotted: 3 Hours Full Marks: 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP - A (Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following:

 $10 \times 1 = 10$

- i) Let C be a circle |z-i|=2. Then $\int_C \frac{dz}{(z^2+4)^2} =$
 - a) $\frac{\pi}{16}$

b) $\frac{\pi}{4}$

c) $\frac{\pi}{8}$

- d) $\frac{\pi}{32}$
- ii) The minimum number of edges in a simple graph with n vertices and k connected components is
 - a) n

b) *k*

c) n-k

d) n + k.

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- iii) If x = 4y + 5 be a regression line of x on y then bx
 - a) 4

b) $\frac{1}{4}$

c) (

- d) 1.
- iv) The function $f(x) = x^2$, $-\pi \le x \le \pi$ is represented by a Fourier series as

 $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$. Then a_0 is

a) $\frac{2}{3}\pi^2$

b) $\frac{\pi^2}{3}$

c) π^2

d) $2\pi^2$

v) If $f(z) = \frac{1}{z^4 - 2z^3}$, then z = 0 is a pole of order

a) 3

b) 2

c) 1

d) 4.

vi) If a simple graph has 15 edges then sum of the degrees of all the vertices is

a) 25

b) 24

c) 50

d) 30.

vii) The function $f(z) = e^{1/z}$ has

- a) Removable singularity
- b) Isolated singularity
- c) Essential singularity
- d) None of these.



d)

20.

viii)	Two	variables x and y a	re rela	ated by	x = 2y + 5. The
	med	ian of x is 25, the med	lian of	y is	As Against 15' Knowledge End Explained
	a)	9	b)	10	

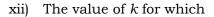
ix) Fourier coefficients are given by

c)

8

- a) Dirichlet's formula b) Euler's formula
- c) Cauchy's formula d) Parseval's formula.
- x) If F(x) is the distribution function of a random variable, then
 - a) F(x) is continuous at all points
 - b) F(x) is strictly monotonic increasing at all points
 - c) $F(\infty)=1$ and F(x) is right continuous at all points
 - d) F(x) is monotone non-decreasing with F(0) = 1 always.
- xi) A path P in a graph G, includes every edge exactly once. Then P is a/an
 - a) Hamiltonian b) Circuit
 - c) Eulerian path d) Eulerian Circuit.

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$$f(x) = kx(1-x)$$
, $0 \le x \le 1$

= 0, otherwise

will be a p.d.f. of a random variable X is

a) 6

b) 2

c) 1

d) 3.

GROUP - B

(Short Answer Type Questions)

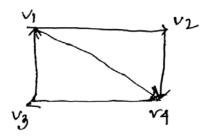
Answer any three of the following

 $3 \times 5 = 15$

2. Find the Fourier series to represent $x - x^2$ from $x = -\pi$ to $x = \pi$. Hence find the value of the series

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$$

- 3. Define a binary tree. Prove that the number of vertices in a binary tree is always odd.
- 4. Define digraphs. Find the indegree and outdegree of the vertices of the following digraph. Write the adjacency matrix.



- 5. State and prove Baye's theorem.
- 6. Find the bilinear transformation which maps z = 1, 1, -1 respectively to $\omega = i, 0, -1$. Also find the image of $|z| \le 1$ under the transformation.

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GROUP - C

(Long Answer Type Questions)

Answer any three of the following.

 $3 \vee 15 = 45$

7. a) Examine the nature of the function

$$f(z) = \begin{cases} \frac{x^2 y^5 (x + iy)}{x^4 + y^{10}}, & z \neq 0 \\ 0, & z = 0 \end{cases}$$

in the region including the origin.

b) Find the Taylor's series expansion of $f(z) = \frac{(z+1)}{(z-3)(z-4)} \text{ about } z=2.$

Find the region of convergence.

c) Evaluate $\int_C \frac{z^2 e^{zt}}{z^2 + 1} dz$, where C is the circle |z| = 2 and t

is a quantity independent of z.

5 + 5 +

- 8. a) Prove that mean and variance of Binomial distribution is *np* and *npq* respectively.
 - b) If x = 4y + 5 and y = kx + 4 are two regression equations of x on y and y on x respectively, obtain the interval in which k lies.
 - c) A batch of 1000 lamps is known to have 10% defectives. If 9 lamps are chosen at random and are tested, what is the probability that none of them will be defective? What is the probability that exactly 5 defectives will be found?

 5 + 5 + 5
- 9. a) Find the Fourier transform of the function

$$f(x) = 1, |x| < a$$

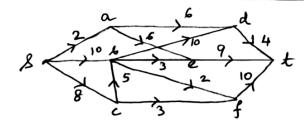
= 0, |x| > a

and hence evaluate $\int_{-\infty}^{\infty} \frac{\sin \alpha a \cos \alpha x}{\alpha} d\alpha$.

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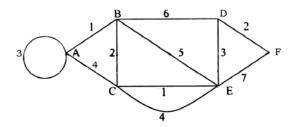


b) Find a maximum flow in the directed network shown in the following figure and prove that it is maximum.

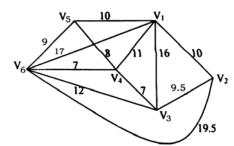


7 + 8

- 10. a) Prove that a tree with n vertices contains exactly (n-1) edges.
 - b) Find the shortest path from *A* to *F* applying Dijkstra's algorithm to the graph given below:



c) Describe Kruskal's algorithm to find the shortest spanning tree in a graph *G*. Use this algorithm to find the shortest spanning tree for the following graph:



3 + 6 + 6



11. a) Calculate Pearson's correlation coefficient from the following data:

Height (cm): 162 165 167 168 170 175 Weight (kg): 58 60 65 67 72 75

b) A random variable follows Normal distribution such that $P(9 \cdot 6 \le X \le 13 \cdot 8) = 0 \cdot 7008$, $P(X \ge 9 \cdot 6) = 0 \cdot 8504$, where

 $1/\sqrt{(2\pi)}\int_{-\alpha}^{0.9} e^{-t^2/2} dt = 0.8159, \ 1/\sqrt{(2\pi)}\int_{-\alpha}^{1.2} e^{-t^2/2} dt = 0.8849.$

Find the mean & variance of X.

6 + 9

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