Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/B.Tech(CE)/SEM-3/CE-301/2009-10

2009

MATHEMATICS

Time Allotted : 3 Hours | *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

   i) When two dice are thrown, the probability of getting a sum of 10 or 11 points is

   a) $\frac{3}{36}$ b) $\frac{7}{36}$

   c) $\frac{6}{36}$ d) $\frac{5}{36}$.

   ii) The distribution for which mean and variance are equal is

   a) Poisson   b) Normal

   c) Binomial   d) Exponential.

iii) $y = e^{x}$ is an

a) odd function b) even function

c) both of these d) none of these.

iv) If X and Y are independent random variables, then

a) $E(XY) = E(X) + E(Y)$

b) $E(XY) = E(X) \cdot E(Y)$

c) $E(XY) = E(X) - E(Y)$

d) $E(XY) = E(X) / E(Y)$.

v) Standard deviation

a) varies between 0 and 1

b) is a positive quantity

c) is a negative quantity

d) none of these.

vi) The probability of having at least one 'six' in 3 throws of a perfect die is

a) $\left(\frac{5}{6}\right)^3$ b) $\left(\frac{1}{6}\right)^3$

c) $1 - \left(\frac{5}{6}\right)^3$ d) none of these.

vii) The probability that A passes a test is $\frac{2}{3}$ and that probability that B passes the same test is $\frac{3}{5}$. The probability that only one of them passes is

a) $\frac{4}{5}$ b) $\frac{7}{15}$

c) $\frac{3}{5}$ d) $\frac{5}{9}$.

viii) The variance of a uniform distribution with parameters a and b is

a) $\dfrac{a+b}{2}$ b) $\dfrac{a-b}{2}$

c) $\dfrac{(b-a)^2}{12}$ d) $\dfrac{(b+a)^2}{12}$.

ix) Which of the following is true for random variable X, where a and b are arbitrary constants ?

a) $\text{Var}(aX+b) = b^2 \text{ Var}(X)$

b) $E(aX+b) = aE(X)$

c) $\text{Var}(aX+b) = a^2 \text{ Var}(X)$

d) $E(aX+b) = b$.

x) For any two events A_1 and A_2 where $A_1 \subseteq A_2$, then

a) $P(A_1) > P(A_2)$ b) $P(A_1) \geq P(A_2)$

c) $P(A_1) \leq P(A_2)$ d) None of these.

xi) The solution of the partial differential equation

$Z = px + qy + p^2 + pq + q^2$, where $p = \dfrac{\partial z}{\partial x}$, $q = \dfrac{\partial z}{\partial y}$

is

a) $z = ax + by + a^2 + ab + b^2$

b) $z = ax + by$

c) $z = a^2 + ab + b^2$

d) none of these.

xii) If $F(A)$ is the Fourier transform of $f(t)$, then the Fourier transform of $f(t).\cos(\omega t)$ is

a) $F(s-\omega)+F(s+\omega)$

b) $\frac{1}{2}[F(s-\omega)+F(s+\omega)]$

c) $F(s-\omega)-F(s+\omega)$

d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. a) Define probability density function of a random variable X.

b) Show that
$$f(x) = \left.\begin{array}{ll} = x, & 0 \le x < 1 \\ = k - , & 1 \le x \le 2 \\ = 0, & \text{elsewhere} \end{array}\right\}$$

is a p.d.f. of a random variable x for a suitable value of k. Determine the value of k and then find the distribution function of the random variable X.

3. Find the standard deviation of the binomial distribution with parameters n, p.

4. If x is normally distributed with mean 3 and s.d. 2, find c such that $P(X > c) = 2P(X \le c)$.

Given that $\int_{-\infty}^{0\cdot 43} \phi(t)\, dt = 0\cdot 6666$.

5. Find the deflection of a vibrating string of unit length having fixed ends with initial velocity zero and initial deflection

$f(x) = k(\sin x - \sin 2x)$ where k is a constant.

6. Expand the function $f(x) = x \sin(x)$ as a Fourier series in $[-\pi, \pi]$.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Find Fourier sine transform of e^{-x} and using the inversion formula, recover the original function.

b) Find the Fourier transform of $f(x) = \dfrac{1}{x} e^{-ax}$. 10 + 5

8. Derive one dimensional wave equation for vibrating string and solve it using the method of separation of variables.

10 + 5

9. a) Solve the following one dimensional heat conduction equation :

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < \infty, \; t > 0 \text{ with}$$

$u(x, 0) = f(x)$, $u(0, t) = 0$, $t > 0$ using Fourier transform.

b) Solve $x^2 \dfrac{\partial^2 u}{\partial y^2} + \dfrac{\partial u}{\partial x} = 0$ by the method of separation of variables. 10 + 5

10. a) The distribution function $F(x)$ of a variate X is defined as follows :

$$F(x) = A, \quad -\infty < x < -1$$
$$= B, \quad -1 \le x < 0$$
$$= C, \quad 0 \le x < 2$$
$$= D, \quad 2 \le x < \infty$$

where A, B, C, D are constants, Determine the values of A, B, C, D, given that $P(X = 0) = \frac{1}{6}$ and $P(X > 1) = \frac{2}{3}$.

b) Classify the following partial differential equation :

$$y \frac{\partial^2 u}{\partial x^2} + 2x \frac{\partial^2 u}{\partial x \partial y} + y \frac{\partial^2 u}{\partial y^2} = 0.$$ 10 + 5

11. a) If A and B are independent events then show that the following pairs are independent :

i) $\bar{A}$ and $\bar{B}$

ii) $\bar{A}$ and B.

b) Show by Tchebycheff's Inequality that in 2000 throws with a coin the probability that the number of heads lies between 900 and 1100 is at least $\frac{19}{20}$. 10 + 5

12. a) Find the Fourier transform of $f(x) = \begin{cases} 1 \text{ if } |x| < a \\ 0 \text{ if } |x| \geq a \end{cases}$

where a is a positive real number. Hence deduce that

$$\int_0^\infty \frac{\sin(t)}{t}\, dt = \frac{\pi}{2}.$$

b) What is the Fourier expansion of the periodic function

$$f(x) = \begin{cases} 0 & \text{when } -\pi < x < 0 \\ \sin(x) & \text{when } 0 \leq x < \pi \end{cases}$$

Hence evaluate $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} + \ldots\ldots\ldots\ldots \infty$.

(4 + 4) + (4 + 3)