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Paper Code : BSM201 Mathematics - IIA

UPID : 002005

Time Allotted : 3 Hours   *Full Marks :70*

The Figures in the margin indicate full marks.

Candidate are required to give their answers in their own words as far as practicable

Group-A (Very Short Answer Type Question)

1. Answer any ten of the following :   [1 x 10 = 10]

   (I) In a scatter diagram, if all the points lie on a rising straight line, then (x,y) is said to have a _____ correlation.

   (II) The rejection probability of Null Hypothesis when it is true is called _____.

   (III) Two dice are rolled. If X and Y denote the number appeared on first and second die, then P(X+Y=6) = _____.

   (IV) Let X be a random variable with probability distribution function f (x)=0.2 for |x|<1
   = 0.1 for 1 < |x| < 4
   = 0 otherwise
   The probability P (0.5 < x < 5) is _____.

   (V) If X and Y are two normal variates with standard deviations s_x and s_y respectively, then the standard deviation of X+Y is _____.

   (VI) Runs scored by batsman in 5 one day matches are 50, 70, 82, 93, and 20. The standard deviation is _____.

   (VII) The first moment about zero is the _____ of the random variable.

   (VIII) Mean of an exponential distribution with parameter λ is _____.

   (IX) If X and Y are independent, then their covariance is _____.

   (X) Suppose for 40 observations, the variance is 50. If all the observations are increased by 20, the variance of these increased observations will be _____.

   (XI) For a continuous random variable X, P(X=a), where a is a finite number, is _____.

   (XII) If Cov(X,Y) = 0, the X and Y _____ independent.

Group-B (Short Answer Type Question)

Answer any three of the following :   [5 x 3 = 15]

2. Fit a straight line y = a + bx to the following data: [5]
    X : 2   5   6   8   9
    Y: 8   14   19   20   31
    Predict Y when X = 9.6.

3. A random variable X has the following pdf: f(x) = k(x-1)(2-x), for 1<x<2. Find   [5]
    (i) k   (ii) The distribution function F(x).

4. If x = 4y + 5 and y = kx + 4 be two regression equations of 'x on y' and 'y on x' respectively, then find the   [5]
    interval in which k lies.

5. From a large population, a sample of size 400 is drawn with mean 171.38. Can it be reasonably regarded   [5]
    that the mean of the population is 171.17? The s.d. of the population is 3.3. Test 5% level of significance.

6. The coefficient of rank correlation coefficient of marks obtained by 10 students in English and Economics   [5]
    was found to be 0.5. It was later discovered that the difference in ranks in the two subjects obtained by
    one of the students was wrongly taken as 3 instead of 7. Find the correct rank correlation coefficient.

Group-C (Long Answer Type Question)

Answer any three of the following :   [15 x 3 = 45]

7. (a) Show that a function which is |x| in (-1,1) and zero elsewhere is a possible p.d.f. and find the corresponding   [10]
    distribution function.

    (b) Find the mean and variance of a continuous random variable having p.d.f.   [5]
    f(x) = 1 − |1 − x|, 0<x<2.

8. (a) Prove that Var(X + Y) = Var(X) + Var(Y) + 2Cov(X,Y).   [5]

(b) In a joint distribution of X,Y, the marginal distributions of X and Y are as follows: [10]

$$X : \quad 5 \qquad 7$$

$$p_{Xi} : \quad \frac{1}{2} \qquad \frac{1}{2}$$

and $\quad Y : \quad 3 \qquad 6$

$$p_{Yj} : \quad \frac{1}{3} \qquad \frac{2}{3}$$

If $Cov(X,Y) = -\frac{1}{2}$,

(i) Construct the joint distribution

(ii) Are X and Y independent ?

(iii) Find $P(X = 5, Y = 6)$ and $P(X > Y)$

9. Fit an exponential curve of the form $y = ab^x$ to the following data: [15]
x: 2   3   4   5   6   7
y: 640   512   410   328   262   210

10. (a) From two cities, two random samples of 600 and 1000 men are drawn respectively. It is found that 400 and 600 men are illiterate among the men in the two samples respectively. Test at 5% level whether the population of the two cities have same percentage of literacy. Given $z_{0.025}$ = 1.96. [5]

(b) A company claims that the consistency regarding life time of electric bulbs produced by them is superior to those of a competitor on the basis of a study which showed that a sample of 30 bulbs made by them has s.d. 25 hours while a sample of 40 bulbs of the competitor has s.d. 27 hours. Test at 5% level whether the claim of the company is justified. Given $z_{0.05}$ = 1.645. [5]

(c) There are two normal populations, I and II, having s.d. 8 and 6 respectively. Two independent samples of size 10 and 12 are drawn from the two populations respectively with sample mean 20 and 27. Test at 1% level whether the two population means are equal. Given $z_{0.005}$ = 2.58. [5]

11. (a) The length of bolts produced by a machine is normally distributed with mean 4 and s.d. 0.5. A bolt is defective if its length does not lie in the interval (3.8, 4.3). Find the percentage of defective bolts produced by the machine. Given. [6]

$$\left[\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0.6} e^{-\frac{t^2}{2}} dt = 0.7257, \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0.4} e^{-\frac{t^2}{2}} dt = 0.6554 \right]$$

(b) If the weekly wages of 10,000 workers in a factory follows normal distribution with mean and standard deviation Rs. 70 and Rs. 5 respectively then find the expected number of workers whose weekly wages are   https://www.makaut.com [9]
(i) between Rs. 66 and Rs. 72. (ii) less than Rs. 66. (iii) more than Rs. 72.
Given that the area under the standard normal curve between z = 0 and z = 0.4 is 0.1554 and z = 0 and z = 0.8 is 0.2881.

*** END OF PAPER ***