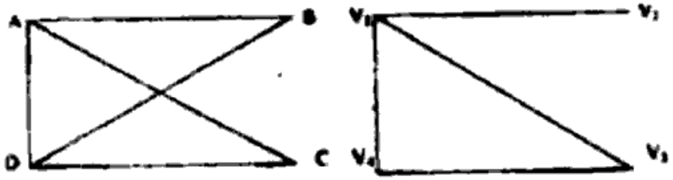
c) Solve the following differential equation using Laplac[e] transform method $\left(D^2 - 3D + 2\right) y = 4t + e^{3t}$, where $y(0) = 0, y'(0) = -1$ 6

11. a) Discuss the convergence of the improper integral: $\int \frac{dx}{x(2-x)}$

b) Solve that $\int_0^{\pi/2} \sin^4 x \cos^5 x dx = \frac{8}{315}$

c) Examine whether the following two graphs are isomorphic or not



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The figure in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable

GROUP - A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any ten of the following: 10x1=10

i) The general solution of the ordinary differential equation $\frac{d^2y}{dx^2} + 4y = 0$ where A & B are arbitrary constants is

a) $Ae^{2x} + Be^{-2x}$

b) $(A+B)e^{2x}$

c) $A\cos 2x + B\sin 2x$

d) $(A+Bx)\cos 2x$

ii) If the differential equation

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$\left(y+\frac{1}{x}+\frac{1}{x^{2}y}\right)dx+\left(x+\frac{1}{y}+\frac{A}{xy^{2}}\right)dy=0$ is exact, then the value of A is

a) 2
b) 1
c) -1
d) 0

iii) The number of edges in a tree with n vertices is

a) n
b) n-1
c) n+1
d) none of these

iv) A binary tree has exactly

a) two vertices of degree two
b) one vertex of degree two
c) one vertex of degree one
d) none of these.

v) $L^{1}\left\{\frac{1}{s(s+1)}\right\}$ is equal

a) $1+e^{t}$
b) $1-e^{t}$
c) $1+e^{-t}$
d) $1-e^{-t}$

vi) The value of $\Gamma\left(\frac{1}{2}\right)$ is

a) π
b) $\sqrt{\pi}$
c) $1/\pi$
d) $1/\sqrt{\pi}$

vii) The general solution of the differential equation y=px+f(p) is

a) $y=c^{2}x+f(c)$
b) $y=cx+f(c^{2})$
c) $y=cx+f(c)$
d) none of these.

viii) The improper integral $\int_{a}^{b}\frac{dx}{(b-x)^{n}}$ converges for

a) n>1
b) n<1
c) n≥1
d) none of these.

ix) The sum of the degrees of all vertices of a graph is 40, the number of edges is

a) 20
b) 25
c) 40
d) none of these

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x) $\frac{1}{(D-3)}e^{3x}$ is equal to

a) xe^{3x}

b) $3e^{3x}$

c) x^2e^{3x}

d) none

xi) The value of the $\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{5}{2}\right)$ is

a) $3\frac{\sqrt{\pi}}{4}$

b) $\frac{3}{2}\pi$

c) $\frac{3}{4}\pi$

d) none of these .

xii) L{t cost} =

a) $\frac{s}{s^2+1}$

b) $\frac{s+1}{s^2+1}$

c) $\frac{2s}{s^2+1}$

d) $\frac{s^2-1}{(s^2+1)^2}$

xiii) The integrating factor of the differential equation

$$\frac{dx}{dy}+\frac{x}{1+y^2}=\frac{e^{-\tan^{-1}y}}{1+y^2}$$ is

a) $\tan^{-1}y$

b) $e^{\tan^{-1}y}$

c) $e^{\cot^{-1}y}$

d) e^y

GROUP - B

(Short Answer Type Questions) 3x5=15

Answer any *three* of the following.

Solve: $(3x^2y^4 + 2xy)\,dx + (2x^3y^3 - x^2)\,dy = 0$

Solve: $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 3y = x^2\log x$

Show that $\int_{-1}^{1}\frac{1}{x^3}dx$ exists in the Cauchy's principal value sense but not in the general sense

Prove that the maximum number of edges in a graph with n

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vertices and k components is (n-k)(n-k+1)/2.

6. State the Convolution theorem for Laplace transform. Use theorem to find $L^{-1}\left\{\dfrac{1}{(s-2)(s^2+1)}\right\}$

GROUP - C
(Long Answer Type Questions)

Answer any *three* of the following. 3x15

7. a) Solve: $\left(xy^2 - e^{\frac{1}{x^3}}\right)dx - x^2 y\,dy = 0$

b) Prove that $B(m,n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

c) Show that $\int_0^{\infty} \sqrt{\tan x}\,dx = \dfrac{\pi}{\sqrt{2}}$

8. a) Solve the following simultaneous equation:

$$\frac{dx}{dt} + 3x + y = e^t$$
$$\frac{dy}{dt} - x + y = e^{2t}$$

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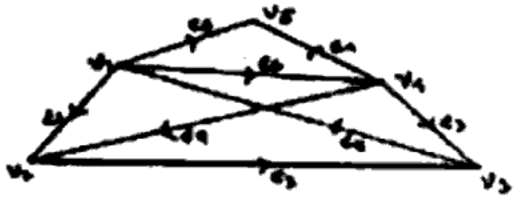
Find the inverse Laplace transform of $\dfrac{s^2+s-2}{s(s+3)(s-2)}$ 4

Find the incidence matrix for the graph given below:



5

a) Prove that the number of vertices in a binary tree is always odd.

5

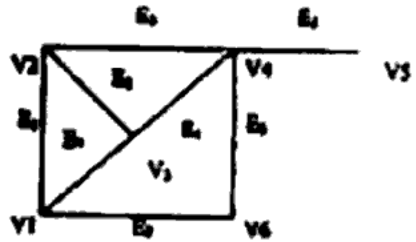
b) Solve $\dfrac{d^2y}{dx^2} - 5\dfrac{dy}{dx} + 6y = x^2 e^{3x}$

6

c) Use Laplace Transform to find the integral $\int_0^{\infty} e^{-4t}\, t\, \sin t\, dt$.

4

a) Determine the adjacency matrix of the given graph



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b) Evaluate: $L^{-1}\left\{\log_e \dfrac{(s+2)}{(s+1)}\right\}$

4