	Utech
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# CS/B.Tech (OLD)/SEM-1/M-101/2011-12 2011 MATHEMATICS

Time Allotted: 3 Hours Full Marks: 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

#### **GROUP - A**

### ( Multiple Choice Type Questions )

- 1. Choose the correct alternatives for any ten of the following:  $10 \times 1 = 10$ 
  - i) The series  $\sum x^n = 1 + x + x^2 + \dots \infty$  is convergent if
    - a)  $x \ge 1$

- b)  $x \le -1$
- c) -1 < x < 1
- d) none of these.
- ii) A positive term series cannot be oscillatory.
  - a) True
  - b) False.
- iii) The limit  $\lim_{x\to 0} |x|/x$  does not exist.
  - a) True
  - b) False.

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- iv) If  $y = x^n$ , then  $y_n$  is equal to
  - a) 0

b) 1

c) n!

- d) none of these.
- v) The function  $\log x$  is defined for
  - a) all values of x
- b) x > 0

c) x < 0

- d) x = 0.
- vi) If  $I_n = \int_0^{\pi/2} \sin^n x \, dx = \{(n-1)/n\} I_n 2$ , the value of  $\int_0^{\pi/2} \sin^6 x \, dx$  is
  - a) 0

b)  $\frac{5\pi}{32}$ 

c)  $\frac{5}{32}$ 

- d) 1
- vii) If the arc is symmetrical about the x-axis, then for C.G. of the arc
  - a) x = 0

b) y = 0

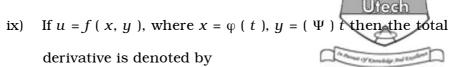
c) x = y

- d) none of these.
- viii)  $f(x, y) = (x^2 + y^2)^{1/3}$  is a homogeneous function of degree
  - a)  $\frac{1}{3}$

b)  $\frac{2}{3}$ 

c) 2

d) none of these.



a)  $\frac{1}{\partial t}$  b)  $\frac{\partial x}{\partial t}$ 

c)

d)  $\frac{\partial y}{\partial t}$ .

If  $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ , then the angle between the vectors  $\vec{a}$ x) and  $\vec{b}$  is

a) 90° b) 45°

 $60^{\circ}$ c)

none of these. d)

radius The centre and of the sphere xi)  $2x^2 + 2y^2 + 2z^2 - 2x + 4y + 2z + 3 = 0$  are

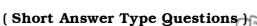
- a) (-1, 2, 1); 2
- b) (1, -2, -1); 1
- c)  $\left(\frac{1}{2}, -1, -\frac{1}{2}\right)$ ; 0 d)  $\left(-\frac{1}{2}, 1, \frac{1}{2}\right)$ ; 3.

xii) The normal to the planes x-y+z=1, 3x+2y-z+2=0are

- a) parallel
- inclined at angle  $\frac{\pi}{3}$ b)
- passing through origin c)
- perpendicular. d)

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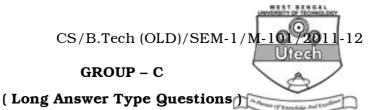


Answer any three of the following.



- 2. If  $y = \tan^{-1}x$ , then prove that  $(1+x^2)y_{n+1} + 2nx y_n + n(n-1)y_{n-1} = 0.$  Find also the value of  $(y_n)_0.$
- 3. Test the convergence of the series  $1 + \frac{1}{2^2} + \frac{2^2}{3^3} + \frac{3^3}{4^4} + \frac{4^4}{5^5} + \dots$ .
- 4. Obtain the reduction formula for  $\int_{0}^{\pi/2} \cos^{n} x \, dx$  and hence evaluate  $\int_{0}^{\pi/2} \cos^{7} x \, dx$ .
- 5. Find the directional derivative of  $p = 4e^{2x-y+z}$  at the point (1, 1, -1) in a direction towards the point (-3, 5, 6).
- 6. Find the points of intersection of the line x + y + z + 1 = 0 = 14x + 9y 7z 1 with XY and YZ planes and hence put down the symmetrical form of its equations.
- 7. Find a point in the plane x + 2y + 3z = 13 nearest to the point (1, 1, 1) using the method of Lagrange's multipliers.

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Answer any three of the following.

 $3 \times 15 = 45$ 

8. a) Examine if the function

$$f(x) = x \tan^{-1}\left(\frac{1}{x}\right), x \neq 0$$
$$= 0 \qquad x = 0$$

is derivable at x = 0.

- b) State Leibnitz's theorem. If  $y = e^{m \cos^{-1} x}$ , show that  $\left(1 x^2\right) y_{n+2} \left(2n+1\right) x \, y_{n+1} \left(n^2 + m^2\right) y_n = 0 \, .$
- c) If  $y^{1/m} + y^{-1/m} = 2x$ , prove that  $(x^2 1)y_{n+2} + (2n+1)xy_{n+1} + (n^2 m^2)y_n = 0.$
- 9. a) State Rolle's thorem. Verify Rolle's theorem for the function  $f(x) = \sin x \cos x$  in  $[0, \pi/2]$ .
  - b) Prove that

$$(if 0 < a < b < 1), (b-a)/(1+b^2) < tan^{-1}b - tan^{-1}a < (b-a)/(1+a^2)$$

c) Expand sin *x* in power of *x* in infinite series stating the condition under which the expansion is valid.

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- 10. a) State divergence theorem of Gauss. Verify divergence theorem for  $\vec{F} = y \vec{i} + x \vec{j} + z \vec{k}$  over the cylindrical region bounded by  $x^2 + y^2 = 9$ , z = 0, z = 2.
  - b) State Green's theorem in plane. Verify Green's theorem in plane for  $\int_C (3x^2 8y^2) dx + (4y 6xy) dy$ , where C is the boundary of the region defined  $y = x^{1/2}$ ,  $y = x^2$ .
  - c) Find the work done in moving a particle in the force field  $\vec{F} = 3x^2 \vec{i} + (2xz y) \vec{j} + z \vec{k}$  along the curve defined by  $x^2 = 4y$ ,  $3x^3 = 4z$  from x = 0 to x = 2.
- 11. a) Show that  $\overrightarrow{\nabla} r^n = nr^{n-2} \overrightarrow{r}$  where  $\overrightarrow{r} = x \overrightarrow{i} + y \overrightarrow{j} + z \overrightarrow{k}$ .
  - b) Prove that *curl* [  $\varphi$  *grad*  $\varphi$  ] = 0.
  - c) Find the co-ordinate of the foot of the perpendicular from (1, 2, 3) on the line (x-2)/1 = (y-1)/2 = z/3. Find also the length of the perpendicular and its equation.

- 12. a) Evaluate  $\iint (4x^2 y^2)^{1/2} dx dy$  over the triangle formed by the straight lines y = 0, x = 1 and y = x.
  - b) Evaluate  $\int_{0}^{a} \int_{0}^{x} \int_{0}^{x+y} e^{x+y+z} dz dy dx.$
  - c) If  $f(x, y) = (x^2 + y^2)^{1/3}$ , use Euler's theorem to find the value of  $x(\partial f/\partial x) + y(\partial f/\partial x)$  and hence prove that,  $x^2(\partial^2 f/\partial x^2) + 2xy(\partial^2 f/\partial x\partial y) + y^2(\partial^2 f/\partial y^2) + (2/9)f = 0.$