ENGINEERING & MANAGEMENT EXAMINATIONS, DECEMBER – 2008

DIGITAL SIGNAL PROCESSING

SEMESTER – 5

Time : 3 Hours] [Full Marks : 70

Graph sheet is provided on Page 31.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) The linear time invariant system described by the impulse response $h(n) = a^n u(n)$ is stable if

a) $|a| > 1$ b) $|a| \geq 1$

c) $|a| \leq 1$ d) $|a| < 1$.

ii) The digital system in $y[n] = x[n^2]$ is

a) linear and causal b) linear and non-causal

c) non-linear and causal d) non-linear and non-causal.

iii) FIR filters have a symmetrical impulse response when

a) phase delay is constant

b) group delay is constant

c) both phase and group delays are constant

d) none of these.

iv) The value of the sinc function $\left(\frac{\sin(x)}{x}\right)$ at $x = 0$ is

a) infinity b) 1

c) 0 d) -1.

v) Discrete time sinusoids whose frequencies are separated by an integer multiple of 2π are

a) identical

b) different

c) identical with some restriction in phase

d) none of these.

vi) If the frequency of a discrete time sinusoidal is a rational number, then the signal is

a) aperiodic

b) periodic

c) periodic with period 2π only

d) none of these.

vii) If $x_1(t) = \sin 15\pi t$, $x_2(t) = \sin 20\pi t$ and $x_5(t) = x_1(t) + x_2(t)$, then $x_5(t)$ is

a) periodic and singular

b) aperiodic and singular

c) periodic and non-singular

d) none of these.

viii) Parseval's identity is

a) $1/T \int_{-T/2}^{3T} [f(t)]^2 \, dt = (a_0)^2 + \frac{1}{2} \sum_{n=1}^{\infty} (a_n^2 - b_n^2)$

b) $1/T \int_{-T/2}^{2T} [f(t)]^2 \, dt = (a_0/2)^2 - \frac{1}{2} \sum_{n=1}^{\infty} (a_n^2 - b_n^2)$

c) $1/T \int_{-T/2}^{2T} [f(t)]^2 \, dt = (a_0/2)^2 + \frac{1}{2} \sum_{n=1}^{\infty} (a_n^2 + b_n^2)$

d) none of these.

ix) N-point DFT of the sequence $x(n)$ is expressed by

a) $X(k) = \sum_{n=0}^{N-1} x(n)\, e^{j 2\pi nk/N}$, $k = 0, 1, \dots, N-1$

b) $X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j 2\pi nk/N}$, $k = 0, 1, \dots, N-1$

c) $X(k) = \sum_{n=0}^{N+1} x(n)\, e^{j 2\pi nk/N}$, $k = 0, 1, \dots, N+1$

d) $X(k) = \sum_{n=0}^{N+1!} x(n)\, e^{-2\pi nk/2N}$, $k = 0, 1, \dots, N+1!$

x) An infinite memory system is also known as

a) Analog system b) Digital system

c) FIR system d) IIR system.

xi) If $X(z)$ is the z-transform of $x(n)$, then z-transform of $x(n+k)$ is

a) $z^k x(k)$ b) $z^k x(z)$

c) $z^{-k} x(k)$ d) $z^{-k} x(z)$.

xii) $Z[x(n)] = X(Z) = \sum_{n=-\infty}^{\infty} x(n) Z^{-n}$, Z = complex variable, this mathematical expression indicates

a) one-sided Z-transform

b) three-sided Z-transform

c) two-sided Z-transform

d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. a) What are singularity functions ?

   b) With an example, explain casual system. Why are non-casual systems unrealizable ? $1 + (2 \times 2)$

3. Determine the Fourier transform of $f(t) = e^{-a|t|}\, sgn(t)$.

4. Obain the cascade realization for the system function given by

$$H(z) = \frac{1 + \frac{z^{-1}}{4}}{\left(1 + \frac{1}{2} z^{-1}\right)\left(1 + \frac{1}{2} z^{-1} + \frac{1}{4} z^{-2}\right)}.$$

5. Using z-transform find out the solution of difference equation given by

$$Y(n) = \frac{1}{2} y(n-1) + x(n); \quad n > 0.$$

Given that $Y(-1) = 1$ and $x(n) = \left(\frac{1}{3}\right)^n$.

6. Define energy signal and power signal. Calculate the power of the signal sequence

$$x(n) = 2(-1)^n \text{ for } n \geq 0$$
$$= 0 \text{ for } n < 0$$

$2 + 3$

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following questions. $3 \times 15 = 45$

7. a) Define LTI system.

   b) Check whether the following system is linear :

$$y(n) = \frac{1}{N} \sum_{m=0}^{N-1} x(n-m).$$

   c) Find the impulse response for the casual system

$$y(n) - y(n-1) = x(n) + x(n-1).$$

   d) Show that the system described by the differential equation

$$dy(t)/dt + 10y(t) + 5 = x(t) \text{ is non-linear.}$$

$2 + 5 + 4 + 4$

8. a) Distinguish between FIR and IIR filters.

   b) What is warping effect ?

   c) Convert the analog filter with system function $G(s) = \frac{s + 0 \cdot 1}{(s + 0 \cdot 1)^2 + 16}$ into a digital filter using bilinear transformation. The digital filter should have a resonant frequency of $\omega_r = \pi/4$ rad. 5 + 2 + 8

9. a) Determine the direct form of realization of a linear phase FIR filter specified by the impulse response $h(n) = \{2, 4, 6, 6, 6, 4, 2\}$.

   b) Draw

      i) direct form I

      ii) direct form II

      iii) cascade

      iv) parallel structures

   for the system described by the difference equation

$$y(n) = \frac{3}{4} y(n-1) - \frac{1}{8} y(n-2) + x(n) + \frac{1}{3} x(n-1).$$

5 + 10

10. a) Find the 8 point DFT using decimation in time FFT algorithm for a sequence

   $x(n) = \{1, 3, 5, 7, 2, 4, 6, 8\}$.

   b) What do you mean by zero padding ?

   c) Find the convolution sum of the following discrete time sequences :

   $X_1(n) = \{1, 3, 5, 7\}$ and $X_2(n) = \{2, 4, 6, 8\}$ using graphical method.

   Check the result by matrix method of convolution. 5 + 2 + 5 + 3

11\. a) In this problem you will be comparing some of the characteristics of analog and digital implementation of the single-pole low-pass analog system :

$$H_a(s) = \frac{\alpha}{s+\alpha} \rightarrow h_a(t) = e^{-\alpha t}$$

i) What is the gain at d.c. ? At what radian frequency is the analog frequency response 3 dB down from its d.c. value ? At what frequency is the analog frequency response zero ? At what time has the analog impulse response decayed to $1/e$ of its initial value ?

ii) 'Prewarp' the parameter α and perform the bilinear transformation to obtain the digital system function $H(z)$ from the analog design. What is the gain at d.c. ? At what (real valued) frequency is the response zero ? Give an expression for the 3 dB radian frequency. How many samples in the unit sample time-domain response before it have decayed to $1/e$ of their initial value ?

b) An ideal discrete-time high-pass filter with cut-off frequency $w_c = \frac{\pi}{2}$ was designed using the bilinear transformation with $T = 1$ ms. What was the cut-off frequency Ω_c for the prototype continuous time ideal high-pass filter ?

6 + 6 + 3

END