Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS / B.TECH (CSE/IT) / SEM-3 / M-301 / 2010-11

2010-11

MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :

$10 \times 1 = 10$

i) If $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$, $P(A \cup B) = \frac{1}{2}$ then $P\left(\frac{B}{A}\right)$ is

a) $\frac{3}{4}$ b) $\frac{4}{3}$

c) $\frac{1}{4}$ d) $\frac{1}{3}$.

ii) The random variable X has distribution function F. If $F(0) = \frac{1}{6}$, $F(1) = \frac{1}{2}$, $F(3) = \frac{3}{4}$ then $p(0 < X \leq 3)$ is

a) $\frac{3}{4}$ b) $\frac{1}{3}$

c) $\frac{7}{12}$ d) none of these.

iii) A box contains 6 white and 4 black balls. One ball is drawn. What is the probability that it is white ?

a) $\frac{2}{5}$ b) $\frac{3}{5}$

c) $\frac{1}{5}$ d) none of these.

iv) If $\overline{A}$ is the complement of the event A, then

a) $P(\overline{A}) = 1 - P(A)$ b) $P(\overline{A}) = P(A)$

c) $P(\overline{A}) = 1 + P(A)$ d) none of these.

v) If the random variable X has Binomial distribution with parameters n and p , then the mean and variance are respectively

a) np & $np(1-p)$ b) n/p & $2n(1-p)$

c) $np(1-p)$ & np d) none of these.

vi) In random sampling with replacement from a population with S.D. σ, if the sample size is equal to population size ($=N$), then the standard error of sample mean will be

a) 0 b) σ

c) $\sigma/\sqrt{N}$ d) none of these.

vii) $H_1(\mu > 60)$ be an alternative hypothesis then the null hypothesis is

a) $H_0(\mu < 60)$ b) $H_0(\mu \geq 60)$

c) $H_0(\mu \leq 60)$ d) none of these.

viii) For two random variables X and Y $E(XY) = E(X).E(Y)$ hold if X and Y are

a) independent b) uncorrelated

c) continuous variate d) none of these.

ix) The distribution function $F(X)$ of a random variable X is given by where $(-\infty < X \leq \infty)$

a) $P(-\infty < X < \infty)$ b) $P(-\infty < X \leq x)$

c) $P(-\infty \leq X < \infty)$ d) none of these.

x) In a testing hypothesis corresponding to a partial level of significance, among all critical region has

a) least Type I error b) least Type II error

c) greater Type I error d) greater Type II error.

xi) If a population has Poission distribution with parameter m, then the maximum likelihood estimate of m is

a) $n\tilde{x}$ b) $\tilde{x}^2$

c) $\tilde{x}^{1/2}$ d) $\tilde{x}$.

xii) Let S^2 be the sample variance of sample of size n. Then the statistics s^2 is an unbiased estimator of σ^2 where s^2 is equal to

a) $\frac{n}{n+1}s^2$ b) $\frac{n-1}{n}s^2$

c) $\frac{n}{n-1}s^2$ d) $\frac{n+1}{n}s^2$.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Urn I has 2 white and 3 black balls. Urn II has 4 white and 1 black and Urn III 3 has white and 4 black balls. An Urn is selected at random and ball drawn at random is found to be white. Find the probability that Urn I was selected.

3. Show that $f(x) = x \qquad 0 < x < 1$

$= k - x, \qquad 1 < x < 2$

$= 0, \qquad$ otherwise

is a density function for a suitable value of k. Calculate $P(1/3 < x < 1/2)$.

4. In 60 throws of a die, the face one turned up 6 times, face two or three 18 times, face four or five 24 times and face six 12 times. Test at 10% level of significance if the die is fair i.e., whether each side is equally likely to occur.

5. Find the regression coefficient of y on x, x on y and the correlation coefficient between x and y from the following values :

$\sum_i x_i y_i = 1500^{-}, x = 15^{-}, y = 12, \sigma_x = 6 \cdot 4, \sigma_y = 9$ and the number of observations is 10.

6. If X is the number in a throw of fair die, show that the Chebychev's inequality gives $P\left[\,|X - \mu| > 2 \cdot 5\,\right] < 0 \cdot 47$ where μ is the mean of X, while the actual probability is zero.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Show that

$$f(x) = \begin{cases} x, 0 \le x < 1 \\ k - x, 1 \le x \le 2 \\ 0, otherwise \end{cases}$$

is a *p.d.f.* fo a ramdom variable x for a suitable value k. Find the value of k. Then find the distribution function of the random variable x. Calculate the probability that the random variable lie between $\frac{1}{2}$ and $\frac{3}{2}$

b) A fair coin is tossed 400 times, Using normal approximation to binomial distribution. Find the probability of obtaining (i) exactly 200 heads, (ii) between 190 and 210 heads, both inclusive, given that the area under standard normal curve between $z = 0$ and $z = 5{\cdot}05$ is $= 0{\cdot}0199$ and between $z = 0$ and $z = 1.05$ is $0{\cdot}3531$. 8 + 7

8. a) Calculate the Rank Correlation R between the marks in Physics and Chemistry obtained by 10 students

Roll No.	1	2	3	4	5	6	7	8	9	10
Marks in Phys.	78	36	98	25	75	82	90	62	65	39
Marks in Chem.	84	51	91	60	68	62	86	58	53	47

b) A dice is thrown 120 times with the following result :

Face :	1	2	3	4	5	6
Frequency :	20	18	22	30	16	14

Test the hypothesis that the die is unbaised at 5% level.

[Given $\lambda^2 = 11{\cdot}07$ at 5% level of 5 d.f.] 8 + 7

9. a) Let F be a distribution function of a random variable X, then show that

i) F is monotonically increasing on the real line.

ii) F is continuous from the right at each point.

iii) $\lim_{x\to\infty} F(x) = 1$ i.e. , $F(\infty) = 1$ and $\lim_{x\to\infty} F(x) = 0$

i.e. , $F(-\infty) = 0$.

b) The number of students who enroll in a course conducted by an institute in a Poisson random variable with mean 100. The institute decided that if the number enrolling is 120 or more the institute will run the course in two separate section, whereas if fewer than 120 students enroll it will teach all of the students in a single section. Find the probability that the institute would make two sections. (Use central limit theorem to avoid difficult calculations).

Given $\frac{1}{\sqrt{2\pi}} \int_{1.95}^{\infty} e^{-\frac{z^2}{2}} dz = 0 \cdot 0256.$ 5 + 10

10. a) Survey of 320 families with 5 children each revealed the following distribution :

No. of boys :	5	4	3	2	1	0
No. of girls :	0	1	2	3	4	5
No. of family :	14	56	110	88	40	12

Is the results consistent with the hypothesis that male and female births are equally probable ? The 5% value of χ^2 with 5 d.o.f. is 11·07.

b) If the equations of two regression lines obtained in a correlation analysis are $3x + 12y = 19$ and $3y + 9x = 46$, determine which one of these is regression equations of y on x and which one of these is the regression equations of x on y. Find the means, correlation coefficients and ratio of standard deviations of x and y.

8 + 7

11. a) Let P denotes the probability of getting a head when a given coin is tossed once. Suppose that the hypothesis $H_0 : P = 0 \cdot 5$ is rejected in favour of $H_1 : P = 0 \cdot 6$, if 10 trials result in 7 or more heads. Calculate the probability of Type I error and Type II error.

b) In a normal distribution, 41% of the items are under 55 and 7% are above 64. Find the mean and standard deviation. Given $P(-0 \cdot 22 < z < 0) = 0 \cdot 09$ and $P(0 < z < 1 \cdot 34) = 0 \cdot 43$.

8 + 7

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