



Name :

Roll No. :

Invigilator's Signature :

CS/B.Tech (OLD)/SEM-2/M-201/2013
2013
MATHEMATICS

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

GROUP - A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :
 $10 \times 1 = 10$

i) $\frac{1}{1-D} x^2 =$

a) $x^2 + 2x + 1$

b) $x^2 + 2x$

c) $x^2 - 2x + 1$

d) $x^2 + 2x + 2.$

ii) The value of $\begin{bmatrix} 1 & 1 & -ac & bc \\ 1 & 1 & +ad & bd \\ 1 & 1 & +ae & be \end{bmatrix} =$

a) 1

b) $abcd$

c) 0

d) none of these.

iii) If $A = \begin{bmatrix} 3 & -2 & 4 \\ 1 & 2 & -1 \\ 0 & 1 & 1 \end{bmatrix}$ and $A^{-1} = \frac{\text{adj}(A)}{k}$ then $k =$

- a) 15 b) 16
- c) 0 d) 1.

iv) The eigenvalues of $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ are

- a) 1, 1 b) 6, 1
- c) 1, 0 d) 0, 6.

v) If A is an orthogonal matrix then $\det (A) =$

- a) 1
- b) -1
- c) ± 1
- d) $0.$

vi) Integrating factor of $x \frac{dy}{dx} + y = \log x$ is

- a) e^x
- b) x
- c) $\log x$
- d) none of these.



vii) Order and degree of the differential equation $\frac{d^2 y}{dx^2} = \sqrt{1 + \frac{dy}{dx}}$ are

- a) 2, 2 b) 2, 1
c) 1, 2 d) 1, 1.

viii) The Wronskian of the functions $\cos 2x$ and $\sin 2x$ is

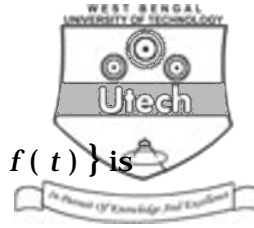
- a) 1 b) 2
c) -2 d) none of these.

ix) The vectors $(1, 1, 0, 0)$, $(1, 0, 0, 1)$, $(1, 0, a, 0)$, $(0, 1, a, b)$ are linearly independent if

- a) $a \neq 0$, $b \neq 2$ b) $a \neq 2$, $b \neq 0$
c) $a \neq 0$, $b \neq -2$ d) $a \neq -2$, $b \neq 0$.

x) $T : R^2$ is defined by $T(x, y) = (2x - y, x + y)$ then kernel of T is

- a) $\{(1, 2)\}$ b) $\{(1, -1)\}$
c) $\{(0, 0)\}$ d) $\{(1, 2)\}, \{(1, -1)\}$.



xi) If $L\{f(t)\} = \tan^{-1}\left(\frac{1}{p}\right)$ then $L\{t, f(t)\}$ is

- a) $\tan^{-1}\left(\frac{1}{p^2}\right)$ b) $\frac{1}{1+p^2}$
 c) $\frac{1}{1+p}$ d) $\tan^{-1}\left(\frac{2}{\pi p}\right)$.

xii) $(\Delta - \nabla) x^2$ is equal to

- a) h^2 b) $-2h^2$
 c) $2h^2$ d) none of these,

where h is equal interval.

xiii) If E_a is the absolute error in a numerical calculation whose true and approximate values are X_t and X_a then the relative error is given by

- a) $\left| \frac{E_a}{X_a} \right|$ b) $\left| \frac{E_a}{X_t} \right|$
 c) $\left| \frac{E_a}{X_t - X_a} \right|$ d) $|E_a|$.

GROUP - B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Examine whether the transformation $T : R^2 \rightarrow R^2$ defined by $T(x, y) = (2x - y, x)$ is linear or not.
3. Apply convolution theorem to find Inverse Laplace Transform of $\frac{s}{(s^2 + 9)^2}$.



4. Prove (without expanding) that

$$\begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1+b & 1 & 1 \\ 1 & 1 & 1+c & 1 \\ 1 & 1 & 1 & 1+d \end{vmatrix} = abcd \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} \right).$$

5. If $P_n (x)$ is the Legendre's polynomial or Legendre's function of 1st kind then prove the following :

a) $(2n + 1) x P_n = (n + 1) P_{n+1} + n P_{n-1}$

b) $(2n + 1) P_n = P'_{n+1} - P'_{n-1}$ where P'_{n+1} denotes the derivative of P_{n+1}

6. Find the inverse of the matrix $A = \begin{bmatrix} 2 & -3 & 4 \\ 1 & 0 & 1 \\ 0 & -1 & 4 \end{bmatrix}$

and hence solve the system : $2x - 2y + 4z = -4$

$$x + z = 0$$

$$4z - y = 2$$

7. Evaluate $\int_0^1 \frac{dx}{1+x}$ using Trapezoidal Rule, taking four equal sub-intervals.

**GROUP – C****(Long Answer Type Questions)**Answer any *three* of the following. $3 \times 15 = 45$

8. a) Solve the following differential equation by Variation of Parameter Method :

$$(D^2 + 1)y = \sec x \tan x$$

- b) Solve the following system using Cramer's rule :

$$x + y + z = 1$$

$$ax + by + cz = r$$

$$a^2x + b^2y + c^2z = r^2$$

where $a \neq b \neq c$

- c) Find the missing data from the following table :

x :	- 2	- 1	0	1	2
y :	6	0	?	0	6

9. a) A linear transformation $T : R^3 \rightarrow R^2$ is defined by $T(x, y, z) = (x + y, x - z)$. Find the Rank and Nullity.

- b) From the following table, construct the difference table and compute $f(19)$ by Newton's Backward Interpolation formula.

x :	0	5	10	15	20
f(x) :	1.0	1.6	3.8	8.2	15.4



- c) Define the rank of a matrix. Find the rank of the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 6 & 7 & 8 \\ 2 & 6 & 9 & 12 & 15 \\ 4 & 8 & 12 & 14 & 16 \end{pmatrix}.$$

10. a) Show that $(3, 1, -2)$, $(2, 1, 4)$, $(1, -1, 2)$ form a basis of R^3 .

- b) Find the Laplace Transform of $f(t) = \sin t$, $0 < t < \pi$
 $= 0$, $t > \pi$

- c) Apply Simpson's $\frac{1}{3}$ rd rule to evaluate $\int_0^6 \frac{1}{(1+x)^2} dx$

taking six equal sub-intervals from $[0, 6]$ and correct up to three decimal places.

11. a) Solve $(D^3 - 2D^2 + D - 2)y = e^x + e^{-3x}$ where $D = \frac{d}{dx}$.

- b) Solve : $\frac{dx}{dt} + 2x - 3y = t$
 $\frac{dy}{dt} - 3x + 2y = e^{2t}$

- c) Find the general solution and singular solution of

$$y = px + \sin^{-1} p \text{ where } p = \frac{dy}{dx}.$$

12. a) Solve the following Cauchy-Euler homogeneous differential equation :

$$(x^2 D^2 - 3xD + 4)y = x^2 \text{ given that } y(1) = 1, y'(1) = 0 \text{ where } D = \frac{d}{dx}.$$



b) Solve : $x \frac{dy}{dx} + y = y^2 x^3 \cos x$.

c) Compute $f(0.5)$ and $f(0.9)$ from the following table :

x :	0	1	2	3
f (x) :	1	2	11	34

13. a) Find the eigenvalues and eigenvectors of the matrix

$$\begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}.$$

Hence diagonalise the above matrix.

b) If $P_n(x)$ is Legendre's Polynomial then prove that

$$\int_0^6 P_n(x) P_m(x) dx = 0, \quad m \neq n$$

$$= \frac{2}{2n+1}, \quad m = n$$

OR

Find the Bessel's function, $J_n(x)$ of 1st kind from the

Bessel's equation $x^2 \frac{d^2 y}{dx^2} + \frac{dy}{dx} + (x^2 - n^2) y = 0$.

Hence prove that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x$.

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