



Name :

Roll No. :

Invigilator's Signature :

CS/B. Tech (CSE)/SEM-7/CS-704E/2011-12

2011

COMPUTATIONAL GEOMETRY

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

GROUP – A

(Multiple Choice Type Questions)

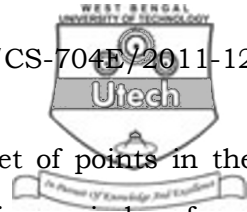
1. Choose the correct alternatives of the following :

$$10 \times 1 = 10$$

- i) Which of the following convex hull algorithms is output-sensitive ?
- a) Graham's Scan
 - b) Chan's Algorithm
 - c) Quickhull
 - d) Divide-and-conquer algorithm.
- ii) Graham's scan to find the convex hull of a simple polygon with n vertices takes
- a) $O(n \log n)$ time
 - b) $O(n^2)$ time
 - c) $O(n \log \log n)$ time
 - d) $O(n)$ time.



- iii) In the worst case, the number of edge-edge intersections between edges of two convex polygons with a total of n vertices is
- a) $O(n \log n)$ b) $O(n)$
 c) $O(n^2)$ d) $O(n^{3/2})$.
- iv) Give a point set in the plane, a dynamic data structure D is maintained as follows : whenever a new point p is inserted into D , all points in D dominated by p are deleted. An insert operation takes $O(\log n)$ time, where n is the number of points in D . A delete operation takes $O(k \log n)$ time where n is the number of points in D and k is the number of points deleted. Starting with an empty data structure, what is the total time taken by n operations in the worst case ?
- a) $O(n^2)$ b) $O(n \log n)$
 c) $O(n)$ d) $O(n \log^2 n)$.
- v) For a set S of n points in the plane whose convex hull has h points, the number of triangles in the triangulation of S is
- a) $n-2h+2$ b) $2n-h-2$
 c) $2n-h+2$ d) $2n-2h+1$.



- vi) In the delaunay triangulation of a set of points in the plane, the number of points in the circumcircle of any triangle of the triangulation is
- a) 0, 1, or 2 b) may be more than 4
- c) always 0 d) 0 or 1.
- vii) If the closest pair of points in a set of points S in the plane are at a distance δ apart, then the number of points in any $\delta \times \delta$ square is at most
- a) 4 b) 3
- c) 2 d) 5.
- viii) How many points are there in the convex hull of $S = \{(9, 5900), (13, 770), (9, 40), (573, 4444), (666, 5555), (777, 3333), (1900, 40), (1900, 5900)\}$?
- a) 3 b) 5
- c) 6 d) 4.
- ix) Consider the duality transform D which maps the line $ax + by = 1$ to the point (a, b) and vice versa. Then the dual of a point inside the unit circle around the origin is
- a) a line that does not intersect the unit circle
- b) a line that intersects the unit circle
- c) a line that is tangent to the unit circle
- d) another circle.



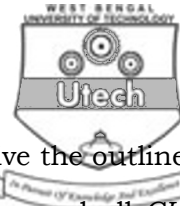
4. Given a set S of orthogonal rectangles in R^2 , there exists an algorithm A which finds all pairs of rectangles (a, b) such that a encloses b in $O(n \log \log n + k)$ time where k is the output size. Given a set R of orthogonal rectangles and a set P of convex polygons, outline an efficient algorithm which uses algorithm A to find all pairs (r, p) , $r \in R$, $p \in P$ such that r encloses p . What is its time complexity ? 5
5. Explain point-line duality with an example. 5
6. Describe an $O(n \log h)$ output-sensitive algorithm for finding convex hulls in 2-dimensions where n is the total number of points and h is the number of points in the hull. 5

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) We wish to preprocess a set S of n points in the plane into a data structure D such that given a query point q , all points in S within a L_1 distance d from q can be reported efficiently. Give an efficient solution indicating the space requirement of D and the running time of the query algorithm. (The L_1 distance between $p = (c,d)$ and $q = (a,b)$ is $|c-a| + |d-b|$). 5
- b) Given a set S of n points in the plane, already sorted by x-coordinates, give the outline of an efficient algorithm to find all points in S which are not dominated by any other point. Indicate the running time of your algorithm. 5



- c) Given a set S of n points in the plane, give the outline of an efficient algorithm to decide if the convex hull $CH(S)$ of S is a triangle. (You are given the point set and not the convex hull). Indicate the running time of your algorithm. 5
8. a) We using the fact that the centre of gravity of a triangle is at its centroid, design and analyze an algorithm to compute the centre of gravity of a metallic plate in the shape of a simple polygon, assuming the thickness of the metal is uniform. Assume the polygon has n vertices. 5
- b) We are given a set G of k simple polygons P_1, P_2, \dots, P_k with a total of n vertices. Give the outline of an $O(n \log k)$ algorithm to compute the convex hull of the vertices of the polygons in G . 5
- c) i) Give an algorithm to determine whether a point is inside a polygon or not. 3
- ii) What improvements can you suggest if it is known in advance that the polygon is convex? 2
9. a) Describe briefly the Bentley-Ottman algorithm for finding segment intersections. 5
- b) Describe briefly the segment tree data structure. 5
- c) Describe briefly the Graham's Scan algorithm for finding the convex hulls of a set of points in the plane. 5



10. a) Give an $O(n)$ algorithm to test if a given point q is in the convex hull of a set S of n points in the plane. The point q is not in S . Only the points in set S are given (in no particular order), not its convex hull. Obviously you cannot compute the hull and then test if q is in it. 5
- b) We are given two sets of points S and A with n points each. We wish to find for each point in A , the second nearest neighbour amongst points in S . A Voronoi diagram for S $VOR(S)$ is given and also an associated planar point location data structure. Design and analyze an algorithm to show how the task can be achieved on $O(n \log n)$ time. 5
- c) We wish to preprocess a set S of horizontal line segments in the plane such that given a query vertical line segment q , the segments in S intersecting q can be reported efficiently. Give the outline of a data structure for this problem that uses an interval tree. What is the space complexity of the data structure ? 5
11. a) Write a short note on Voronoi diagrams. 5
- b) Describe an $O(n \log n)$ sweep line algorithm for finding the closest pair of points for a set of points in the plane. 5
- c) Describe briefly the dynamic programming algorithm for minimum weighted triangulation in the plane. 5

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