



Name : .....

Roll No. : .....

Invigilator's Signature : .....

**CS/B.Tech(N)/SEM-1/M-101/2012-13**

**2012**

**MATHEMATICS-I**

Time Allotted : 3 Hours

Full Marks : 70

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words  
as far as practicable.*

**GROUP - A**

**( Multiple Choice Type Questions )**

1. Choose the correct alternatives for any *ten* of the following :

$$10 \times 1 = 10$$

i) The sequence  $\left\{ (-1)^n \frac{1}{n} \right\}$  is

- a) Convergent
- b) Oscillatory
- c) Divergent
- d) none of these.

ii) The matrix  $\begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$  is

- a) Symmetric
- b) Skew-symmetric
- c) Singular
- d) Orthogonal.

$\vec{f} = (x + 3y) \hat{i} + (y - 2z) \hat{j} + (x + tz) \hat{k}$  is solenoidal is

- iv) The series  $\sum \frac{1}{n^p}$  is convergent if

- v) The two eigenvalues of the matrix

$$A = \begin{bmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix} \text{ are } 2 \text{ and } -2. \text{ The third}$$

eigenvalue is

- vi) If Rolles theorem is applicable to  $f(x) = x(x^2 - 1)$  in  $[0, 1]$ , then  $c =$

- a) 1                      b) 0  
c)  $-\frac{1}{\sqrt{3}}$                 d)  $\frac{1}{\sqrt{3}}$ .

a) 0                      b) 2

c)  $\frac{1}{2}$                      d) none of these.

a)  $5^n \cos(5x + 3)$

b)  $5^n \sin\left(\frac{n\pi}{2} + 5x + 3\right)$

c)  $5^n \cos\left(\frac{n\pi}{2} + 5x + 3\right)$

d) none of these.

( 0,1 ) to ( 1, 0 ) is

a) 0                      b)  $\frac{3}{2}$   
c)  $\frac{1}{2}$                      d)  $\frac{2}{3}$ .

x) The value of  $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 \theta \, d\theta$  is

a) 0

b)  $\frac{6.4.2}{7.5.3.1}$

c)  $\frac{6!}{7!}$

d) none of these.

- ## GROUP – B

Answer any *three* of the following.

$$3 \times 5 = 15$$

- $$\vec{F} = \text{grad} (x^3 + y^3 + z^3 - 3xyz).$$



**GROUP - C**  
**( Long Answer Type Questions )**

Answer any *three* of the following.

3 × 15 = 45

7. a) If  $u = x^2 - 2y$ ,  $v = x + y + z$ ,  $w = x - 2y + 3z$ ,  
find  $\frac{\partial (u, v, w)}{\partial (x, y, z)}$ .

b) Prove that 
$$\begin{vmatrix} 1 & \alpha & \alpha^2 - \beta\gamma \\ 1 & \beta & \beta^2 - \gamma\alpha \\ 1 & \gamma & \gamma^2 - \alpha\beta \end{vmatrix} = 0.$$

- c) If  $v = f(x^2 + 2yz, y^2 + 2zx)$ , prove that

$$(y^2 - zx) \frac{\partial v}{\partial x} + (x^2 - yz) \frac{\partial v}{\partial y} + (z^2 - xy) \frac{\partial v}{\partial z} = 0.$$

5 + 5 + 5

8. a) If  $\theta = t^n e^{-\frac{r^2}{4t}}$ , find what value of  $n$  will make  
 $\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial \theta}{\partial r} \right) = \frac{\partial \theta}{\partial t}$ .

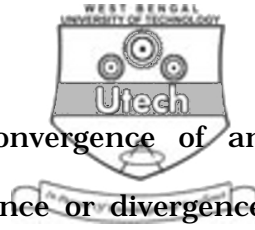
- b) Using mean value theorem prove that

$$0 < \frac{1}{x} \log \left( \frac{e^x - 1}{x} \right) < x.$$

- c) If  $I_n = \int_0^{\frac{\pi}{2}} x^n \sin x \, dx$  ( $n > 1$ ), then show that

$$I_n + n(n-1)I_{n-2} = n \left( \frac{\pi}{2} \right)^{n-1}.$$

5 + 5 + 5



9. a) State D'Alembert's ratio test for convergence of an infinite series. Examine the convergence or divergence of the series  $\left(\frac{1}{3}\right)^2 + \left(\frac{1.2}{3.5}\right)^2 + \left(\frac{1.2.3}{3.5.7}\right)^2 + \dots$

b) If  $y = e^{\tan^{-1}x}$ , then show that  $(1 + x^2) y_{n+2} + (2nx + 2x - 1) y_{n+1} + n(n+1) y_n = 0$ .

c) Find the extreme value of the function

$$f(x, y) = x^3 + y^3 - 3x - 12y + 20. \quad 5 + 5 + 5$$

10. a) If  $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ , then verify that A satisfies its

own characteristic equation. Hence find  $A^{-1}$  and  $A^9$ .

b) If  $u = \tan^{-1} \left( \frac{x^3 + y^3}{x - y} \right)$ , then show that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = (1 - 4 \sin^2 u) \sin 2u.$$

c) Given the system of equation :

$$x_1 + 4x_2 + 2x_3 = 1, \quad 2x_1 + 7x_2 + 5x_3 = k,$$

$$4x_1 + mx_2 + 10x_3 = 2k + 1. \text{ Find for what values of}$$

$k$  and  $m$ , the system has (i) an unique solution,

(ii) no solution (iii) many solution.



11. a) Show that  $\vec{\nabla} r^n = nr^{n-2} \vec{r}$ ,

where  $\vec{r} = \vec{i}x + \vec{j}y + \vec{k}z$ .

b) Evaluate  $\int \int \sqrt{4x^2 - y^2} \, dx dy$  over the triangle formed by the straight lines  $y = 0$ ,  $x = 1$  and  $y = x$ .

c) Verify Stokes theorem for

$\vec{F} = (2x - y) \hat{i} - yz^2 \hat{j} - y^2 z \hat{k}$ , where  $S$  is the upper half surface of the sphere  $x^2 + y^2 + z^2 = 1$  and  $C$  is its boundary.

5 + 5 + 5

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