Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/B.TECH(NEW)(APM/CSE/IT/AUE/CHE/BT/ME/PE/CE/CT/LT/TT/FT/SEM-4/M(CS)-401/2012

2012

NUMERICAL METHODS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) If $\frac{5}{3}$ is approximated to 1·6667, then absolute error is

a) 0·000033 b) 0·000043

c) 0·000034 d) none of these.

ii) If $f(x) = \frac{1}{x^2}$ then the divided difference $f\ (a, b)$ is

a) $\frac{(a + b)}{(ab)^2}$ b) $\frac{(a - b)}{(ab)^2}$

c) $\frac{1}{a^2} - \frac{1}{b^2}$ d) $\frac{1}{a^2 - b^2}$.

iii) The value of $\frac{\Delta^2}{E}(x^3)$ is

a) x b) $6x$

c) $3x$ d) x^2.

iv) If $\frac{dy}{dx} = x + y$ and $y(1) = 0$, then $y(1.-1)$ according to Euler's method is, $h = 0 \cdot 1$ (say),

a) 0·1 b) 0·3

c) 0·5 d) 0·9.

v) If $y_0 = 2$, $y_1 = 4$, $y_2 = 8$, $y_4 = 32$ then y_3 is equal to

a) 5 b) 15

c) 6 d) 16·5.

vi) The order of h in the error expression of Trapezoidal rule is

a) 1 b) 2

c) 3 d) 4.

vii) Regula-Falsi method is

a) conditionally convergent

b) linearly convergent

c) divergent

d) none of these.

viii) Pivoting is very much essential because

a) determinant of the coefficient matrix should be grater than zero

b) pivot element should not have very large value compared to the elements of the matrix

c) it reduces the possibility of division by zero

d) change of convergence is higher.

ix) Which of the following is true ?

a) $\Delta^n x^n = (n+1)!$ b) $\Delta^n x^n = n!$

c) $\Delta^n x^n = 0$ d) $\Delta^n x^n = n$.

x) An $n \times n$ matrix A is said to be diagonally dominant if

a) $\left| a_{ii} \right| \geq \sum_{\substack{j=1 \\ i \neq j}}^{n} \left| a_{ij} \right|$ b) $\left| a_{ii} \right| \leq \sum_{\substack{j=1 \\ i \neq j}}^{n} \left| a_{ij} \right|$

c) $\left| a_{ii} \right| > \sum_{\substack{j=1 \\ i \neq j}}^{n} \left| a_{ij} \right|$ d) $\left| a_{ii} \right| < \sum_{\substack{j=1 \\ i \neq j}}^{n} \left| a_{ij} \right|$.

xi) The condition of convergence of Newton-Raphson's method is

a) $|f(x).f'(x)| < \{f''(x)\}^2$

b) $|f(x).f''(x)| < \{f'(x)\}^2$

c) $|f(x).f'(x)| > \{f''(x)\}^2$

d) $|f(x).f''(x)| > \{f'(x)\}^2$.

xii) For $\frac{dy}{dx} = xy$ and $y(0) = 2$, the value of k_2 according to Runge-Kutta method of 2nd order is (h = 0·2)

a) 0·1 b) 0·01

c) 0·4 d) 0·04.

GROUP – B
(Short Answer Type Questions)

Answer any *three* of the following. 3 × 5 = 15

2. Given $u_0 + u_6 = 3$, $u_1 + u_5 = 5$, $u_2 + u_4 = 7$. Find u_3, where v_x is a function of *x*.

3. Using the following table find $\frac{dy}{dx}$ at x = 0 & 1·5.

x :	0	1	2	3
y :	1	2	11	34

4. Solve the following system of equations using Gaussian elimination method :

$$x + y + z = 9$$
$$2x - 3y + 4z = 13$$
$$3x + 4y + 5z = 40$$

5. Find the value of $(19)^{\frac{1}{3}}$ correct to four decimal points by Newton-Raphson method.

6. Find the cubic polynomial by Lagrange's interpolation formula which takes the following value :

x :	0	4	5	8
$f(x)$:	1	2	1	10

GROUP – C
(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Find a root of the equation $x^4 - x - 10 = 0$ that lies between 1 & 2 using Newton-Raphson method correct to 3 places of decimal.

   b) Solve the system of equations

$$x + y + 54z = 110$$
$$27x + 6y - z = 85$$
$$6x + 15y + 2z = 72$$

   by Gauss-Seidel method. 7 + 8

8. a) Solve the following system of equations by LU-factorization method :

$$3x + 2y + 7z = 4$$
$$2x + 3y + z = 5$$
$$3x + 4y + z = 7$$

b) Using Runge-Kutta method of order 4, final y (0·2) given that $\frac{dy}{dx} = 3e^x + 2y$, y (0) = 0,taking h = 0·1.

7 + 8

9. a) Find the root of the equation $3x - \cos x - 1 = 0$ by Regula-falsi method, correct to three decimal places.

b) Evaluate $\int_0^{\frac{\pi}{2}} \sqrt{\cos x}\, dx$ by using (i) Trapezoidal and (ii) Simpson's $\frac{1}{3}$rd rule, where h = 15°. 7 + 8

10. a) Compute $y = (1 \cdot 4)$ by Milne's predictor & corrector's method from $\frac{dy}{dx} = \frac{1}{2}(x + y)$ where $y(1) = 3 \cdot 595$, $y(1 \cdot 1) = 3 \cdot 833$, $y(1 \cdot 2) = 4 \cdot 088$, $y(1 \cdot 3) = 4 \cdot 362$.

b) Derive Newton's divided difference formula.

c) Given that $\frac{dy}{dx} = \log_{10}(x + y)$ with the initial condition that $y = 1$ when x = 0. Find y for $x = 0{\cdot}2$ and $x = 0{\cdot}5$ using Euler's modified formula. 5 + 5 + 5

11. a) If $y = f(x)$ is a polynomial degree 5 with $y_0 = f(0) = 0$, $y_1 = f(1) = 3$, $y_2 = f(2) = 14$, $y_3 = f(3) = 45$, $y_4 = f(4) = 84$, $y_5 = f(5) = 170$, $y_6 = f(6) = 258$. It is found that there is one error in the value of y_3. Find the correct value of y_3.

b) Why implicit method is preferred over explicit method though it requires more computations ?

c) Show that the rate of convergence in Newton-Raphson method is quadratic. 8 + 3 + 4

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