Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/B.TECH (ECE) (Separate Supple)/SEM-7/EC-704B/2011

2011
ADVANCED MATHEMATICS FOR ELECTRONICS ENGINEERING

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives of the following : $10 \times 1 = 10$

i) The solution to the partial differential equation $z = px + qy + p^4 + q^5$ is

a) $z = ax + by + a^4 + b^5$

b) $z = ax + by$

c) $z = ax + by + a^2 + b^2$

d) $z = ax - by$

ii) $\frac{d}{dx}\left(x^{-n} J_n\right)$ is equal to

a) $x^n J_{n-1}$ b) $x^{n-1} J_{n-1}$

c) $x^{n+1} J_{n-1}$ d) $x^{-n} J_{n+1}$

iii) The value of a skew-symmetric determinant of odd order is always

a) 1 b) 0

c) perfect square d) odd.

iv) The value of $\int_{-1}^{1} P_m(x) P_n(x)\, dx,\ m \neq n$ is equal to :

a) 0 b) $\frac{n}{2n+1}$

c) $\frac{n}{2n-1}$ d) $\frac{2}{2n+1}$

v) $L\left(e^{at}\, t^n\right)$ is equal to

a) $\frac{b}{(s-a)^2 - b^2}$ b) $\frac{s-a}{(s-a)^2 + b^2}$

c) $\frac{n!}{(s-a)^{n+1}}$ d) $\frac{s}{(s-a)^2 - b^2}$

vi) The Fourier cosine transform of $\frac{1}{x}$ is :

a) $\frac{1}{\pi}$ b) $\sqrt{\frac{\pi}{2}}$

c) $\sqrt{\frac{2}{\pi}}$ d) $\sqrt{\frac{1}{2}}$.

vii) Inverse transformation $w = \frac{1}{z}$ transforms the straight line ay + bx + 0 into

a) circle

b) straight line through origin

c) straight line

d) none of these.

viii) Let $L\{f(t)\} = \frac{s^2}{4s+1}$, then the Laplace Transform of the function $f(st)$ is

a) $\frac{s^2}{4s+5}$ b) $\frac{s^2}{100s+5}$

c) $\frac{s^2}{25(4s+5)}$ d) $\frac{s^2}{s+1}$.

ix) For Legendre polynomial of degree n, $P_2(x)$ is equal to

a) 1 b) $\frac{1}{2}\left(3x^2-1\right)$

c) $\frac{1}{2}\left(x^2-3\right)$ d) $\frac{1}{2}\left(5x^3-3x\right)$.

x) The value of $\int_c \frac{dz}{z^2(z-3)}$, where $C : |z| = 2$ is

a) 0 b) 1

c) $2\pi i$ d) $-\frac{2\pi i}{9}$.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Find out the eigenvalue and corresponding eigenvectors of the following matrix :

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{pmatrix}_{3\times 3}$$

3. Solve the following partial differential equation :

$$\left(x^2 - yz\right)p + \left(y^2 - zx\right)q = \left(z^2 - xy\right)$$

4. Prove that the function $f(z) = \begin{cases} \dfrac{x^3(1+i) - y^3(1-i)}{x^2+y^2} & \text{if } z \neq 0 \\ = 0 & \text{if } z = 0 \end{cases}$

satisfies C-R equations at the origin but $f'(0)$ does not exist.

5. Using the properties of the determinant show that

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$$

6. Prove that : $4J_n''(x) = J_{n-2}(x) - 2J_n(x) + J_{n+2}(x)$

7. Find : $L^{-1}\left\{\dfrac{2s^2-1}{\left(s^2+1\right)\left(s^2+4\right)}\right\}$

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

8. a) Find the Fourier cosine transform of e^{-x^2}

b) Solve completely the equation $\frac{\partial^2 y}{\partial t^2} = C2 \frac{\partial^2 y}{\partial x^2}$, representing the vibration of a string of length L, fixed at both ends, subject to the condition.

i) $y\,(\,o,\,t\,) = 0$

ii) $y\,(\,l,\,t\,) = 0$

iii) $y\,(\,x,\,o\,) = f\,(\,x\,)$

iv) $\frac{\partial y\,(x,\,o)}{\partial t} = 0\,,\;\; o < x < l$ 7 + 8

9. a) Using residue theorem show that

$$\int_0^{2\pi} \frac{d\theta}{1 + a \sin\theta} = \frac{2\pi}{\sqrt{1-a^2}}\,,\ \left(-1 \langle\, a \,\langle 1\right)$$

b) Using convolution theorem evaluate :

$$L^{-1}\left\{\frac{s}{\left(s^2 + a^2\right)^2}\right\}$$

c) If A and B are Hermitian matrices, then show that (AB – BA) is skew Hermitian. 6 + 5 + 4

10. a) Find the Fourier transform of $f(x)$, given by

$$f(x) = \begin{cases} 1 - x^2, & \text{if } |x| \langle | \\ o, & \text{if } |x| \rangle | \end{cases}$$

Hence show that $\int_o^\infty \left(\frac{x \cos x - \sin x}{x^3} \right) \cos \frac{x}{2} \, dx = -\frac{3\pi}{16}$

b) Solve the partial differential equation.

$$\left(D^2 + DD' - 6D'^2 \right) z = y \cos x$$

where $D \equiv \frac{\partial}{\partial x}$ and $D' \equiv \frac{\partial}{\partial y}$

c) Determine and classify the singular points of

$f(z) = \frac{z}{e^z - 1}$ 6 + 5 + 4

11. a) Show that the transformation $w = \frac{5 - 4z}{4z - 2}$ maps the unit circle $|z| = 1$ into a circle of radius unity and centre $\left(-\frac{1}{2}, o \right)$

b) Given $v(x, y) = x^4 - 6x^2 y^2 + y^4$.

Find $f(z) = u(x, y) + iv(x, y)$ such that $f(z)$ is analytic.

c) Find out the complete integral of the partial differential equation : $\sqrt{p} + \sqrt{q} = 2x$

where, $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$ 5 + 5 + 5

12. a) Expand $f\left(z\right) = \dfrac{z}{(z-1)\,(2-z)}$ in a Laurent series valid for $1 < \left| z \right| < 2$ and $o < \left| z-2 \right| < 1$

b) Prove the Rodrigues formula :

$$P_n\left(x\right) = \frac{1}{2^n \underline{\lfloor n}} - \frac{d^n}{dx^n}\left[\left(x^2-1\right)^n\right]$$

c) Construct Green's function for the boundary value problem

$u^{//} - u = x,\ u\,(\,o\,) = u\,(\,1\,) = 0$ 5 + 5 + 5