Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/B.Tech (NEW)/SEM-1/M-101/2011-12

2011

MATHEMATICS - I

Time Allotted : 3 Hours Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :

10 × 1 = 10

i) The least upper bound of the sequence $\{\frac{n}{n+1}\}$ is

a) 0 b) $\frac{1}{2}$

c) 1 d) 2.

ii) The value of $\begin{vmatrix} 2000 & 2001 & 2002 \\ 2003 & 2004 & 2005 \\ 2006 & 2007 & 2008 \end{vmatrix}$ is

a) 2000 b) 0

c) 45 d) none of these.

iii) If $\lambda^3 - 6\lambda^2 + 9\lambda - 4$ is the characteristic equation of a square matrix A, then A^{-1} is equal to

a) $A^2 - 6A + 9I$ b) $\frac{1}{4}A^2 - \frac{3}{2}A + \frac{9}{4}I$

c) $A^2 - 6A + 9$ d) $\frac{1}{4}A^2 - \frac{3}{2}A + \frac{9}{4}$

iv) If x = r cos θ, y=r sin θ, then $\frac{\partial(r,\theta)}{\partial(x,y)}$ is

a) r b) 1

c) $\frac{1}{r}$ d) none of these.

v) $f(x,y) = \frac{\sqrt{y}+\sqrt{x}}{y+x}$ is a homogeneous function of degree

a) $\frac{1}{2}$ b) $-\frac{1}{2}$

c) 1 d) 2.

vi) If $\vec{\alpha}\cdot(\vec{\beta}\times\vec{\gamma}) = 0$, then the vectors $\vec{\alpha}$, β, $\vec{\gamma}$ are

a) coplanar b) independent

c) collinear d) none of these.

vii) The nth derivative of $(ax+b)^{10}$ is (where n>10)

a) a^{10} b) $\underline{|10}\; a^{10}$

c) 0 d) $\underline{|10}$.

viii) If for any *two* vectors $\vec{a}$ and $\vec{b}$, $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$, then $\vec{a}$ and $\vec{b}$ are

a) parallel b) collinear

c) perpendicular d) none of these.

ix) If $A^{-1} = \frac{1}{7}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, then A=

a) $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ b) $\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$

c) $\frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$ d) $\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$.

x) The reduction formula of $I_n = \int^{\frac{\pi}{2}} \cos^n x dx$ is

a) $I_n = \left(\frac{n-1}{n}\right) I_{n-1}$ b) $I_n = \left(\frac{n}{n-1}\right) I_n$

c) $I_n = \left(\frac{n-1}{n}\right) I_{n-2}$ d) none of these.

xi) The series $\sum_{n=1}^{\infty} \frac{n^2}{2n^2+1}$ is

a) convergent b) divergent

c) oscillatory d) none of these.

xii) Lagrange's Mean Value Theorem is obtained from Cauchy's Mean Theorem for *two* functions f(x) and g(x) by putting g(x)=

a) 1 b) x^2

c) x d) $\frac{1}{x}$.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Prove that every square matrix can be expressed as the sum of a symmetric matrix and a skew symmetric matrix :

3. By Laplace's method, prove that

$$\begin{bmatrix} a & b & c & d \\ -b & a & d & -c \\ -c & -d & a & b \\ -d & c & -b & a \end{bmatrix} = (a^2+b^2+c^2+d^2)^2$$

(consider minors of order 2).

4. If $2x = y^{\frac{1}{m}} + y^{\frac{-1}{m}}$, then prove that

$$\left(x^2-1\right)y_{n+2} + (2n+1)xy_{n+1} + (n^2-m^2)y_n = 0$$

5. If $u = xf\left(\frac{y}{x}\right) + g\left(\frac{y}{x}\right)$, then show that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 xy}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 0$$

6. Show that the bounded by a simple closed curve C is given by $\frac{1}{2}\oint_C (xdy - ydx)$

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. i) If $f(x,y) = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right)$, verify $f_{xy} = f_{yx}$.

ii) State the Rolle's theorem and examine if you can apply the same for $f(x) = \tan x$ in $[0, \pi]$

iii) Find the value of λ and μ for which

$x + y + z = 3$

$2x - y + 3z = 4$

$5x - y + \lambda z = \mu$ have

(a) unique solution

(b) many solution

(c) no solution.

8. i) Find the maxima and minima of the function

$$f(x,y) = x^3 + y^3 - 63(x+y) + 12xy.$$

Find also the saddle points.

ii) State Leibnitz's test for alternating series and apply it to examine the convergence of $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots \alpha$.

iii) Applying Lagrange's Mean Value Theorem prove that

$$\frac{x}{1+x} \le \log(1+x) < x, \text{ for all } x>0.$$

9. i) If $y = e^{m\sin^{-1}x}$, show that

$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$ Hence find y_n when x = 0.

ii) Prove that [a+b b+c c+a] = 2[a b c], where a, b, c are three vectors.

iii) Find the directional derivative of $f = xyz$ at (1,1,1) in the direction $2\vec{i} - \vec{j} - 2\vec{k}$.

10. i) Prove that $\begin{vmatrix} b^2+c^2 & a^2 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix} = 4a^2b^2c^2$

ii) State Divergence Theorem of Gauss. Verify divergence theorem for $\vec{F} = y\vec{i} + x\vec{j} + z\vec{k}$ over the cylindrical region bounded by $x^2 + y^2 = 9$, $z = 0$, $z = 2$.

iii) Test the series for convergence

$$\frac{1^p}{2^q} + \frac{2^p}{3^q} + \frac{3^p}{4^q} + \ldots\ldots$$

11. i) Obtain a reduction formula for $\int_0^{\frac{1}{2}} \sin^n x dx$. Hence obtain

$$\int_0^{\frac{\pi}{2}} \sin^9 x dx$$

ii) Given two vectors $\vec{\alpha} = 3\vec{i} - \vec{j}$, $\vec{\beta} = 2\vec{i} + \vec{j} - 3\vec{k}$. Express $\vec{\beta}$ in the form $\vec{\beta_1} + \vec{\beta_2}$, where $\vec{\beta_1}$ is parallel to $\vec{\alpha}$ and $\vec{\beta_2}$ is perpendicular to $\vec{\alpha}$.

iii) Show that $\vec{A} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$ is irrotational. Find the scalar function ϕ, such that $\vec{A} = \vec{\nabla}\phi$.