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Paper Code : M-101

MATHEMATICS-I

Time Allotted : 3 Hours | *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) $\frac{d^n}{dx^n}(2^{-5x}) =$

a) $(-1)^n 5^n (\log 2)^n 2^{-5x}$ b) 2^{-5x}

c) $(-1)^n 5^n . 2^{-5x}$ d) none of these.

ii) Lagrange's mean value theorem is obtained from Cauchy's mean value theorem for two functions $f(x)$ and $g(x)$ by putting $g(x) =$

a) 1 b) x^2

c) x d) none of these.

iii) The series $\sum \frac{1}{(2x+1)^n}$ is

a) convergent b) divergent

c) oscillatory d) none of these.

iv) The rank of the null matrix is

a) 0 b) 1

c) 2 d) 3.

If the matrix $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & \lambda \end{pmatrix}$ is singular, then the value of λ is

a) 3 b) 5

c) 2 d) 4.

vi) If α, β, γ are the roots of the equation $x^2 - 3x + 2 = 0$ then $\begin{vmatrix} 0 & \alpha & \beta \\ \beta & 0 & 0 \\ 1 & -\alpha & \alpha \end{vmatrix}$ =

a) 3 b) 3/2

c) -6 d) 3.

vii) Which of the following theorems can be applied on $f(x) = |x|$ in the interval $[-1, 1]$?

a) Rolle's theorem

b) Mean Value theorem

c) Cauchy's M. V. theorem

d) none of these.

viii) The sequence $\left\{\frac{1}{3^n}\right\}$ is

a) monotonic increasing

b) oscillatory

c) divergent

d) monotonic decreasing.

ix) If $u = \sin^{-1}\sqrt{\frac{x^{\frac{1}{2}} + y^{\frac{1}{2}}}{x^{\frac{1}{3}} + y^{\frac{1}{3}}}}$ then $\sin u$ is a homogeneous function of degree

a) $\frac{1}{6}$ b) $-\frac{1}{6}$

c) $\frac{1}{12}$ d) $-\frac{1}{12}$.

x) If $u+v=x$, $uv=y$ then the value of $\frac{\partial(u,\ v)}{\partial(x,\ y)}$ =

a) $u-v$ b) $\frac{1}{u-v}$

c) uv d) $\frac{1}{uv}$.

xi) If $u=f\left(\frac{x}{y},\ \frac{y}{z},\ \frac{z}{x}\right)$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}$ is equal to

a) 1 b) −1

c) 3 d) 0.

xii) The value of m for which the vectors $2\hat{i}-\hat{j}-\hat{k}$, $\hat{i}+2\hat{j}-3\hat{k}$, $3\hat{i}+m\hat{j}+5\hat{k}$ are coplanar is

a) 4 b) − 4

c) 5 d) − 5.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Prove that $\begin{vmatrix} (b+c)^2 & c^2 & b^2 \\ c^2 & (c+a)^2 & a^2 \\ b^2 & a^2 & (a+b)^2 \end{vmatrix} = 2(bc+ca+ab)^2$

3. Expand the function $\sin x$ in power of x in infinite series.

4. Test the convergence of the series

$$1+\frac{1}{2^2}+\frac{2^2}{3^3}+\frac{3^3}{4^4}+\frac{4^4}{5^5}+\ldots.$$

5. Verify Cayley-Hamilton theorem for $A=\begin{pmatrix}1 & 2 & 3\\ 3 & -2 & 1\\ 4 & 2 & 1\end{pmatrix}$.

Hence compute A^{-1}.

6. Find $div\vec{F}$ and $curl\vec{F}$ where $\vec{F}=grad(x^3+y^3+z^3-3xyz)$.

GROUP – C
(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) If $y=\sin(m\sin^{-1}x)$, show that

$$(1-x^2)y_{n+2}-(2n+1)xy_{n+1}=(n^2-m^2)y_n=0.$$

b) Using mean value theorem prove that

$$0<\frac{1}{\log(1+x)}-\frac{1}{x}<1 \text{ for all } x>0.$$

c) If $I_n=\int_0^{\pi/2}\sin^{2n+1}\theta\,d\theta$, where n is a positive integer, show that $I_n=\frac{2n}{2n+1}I_{n+1}$. Use this to evaluate $\int_0^{\pi/2}\sin^7\theta d\theta$. $5+5+5$

8. a) Examine the convergence of the series

$$1+\frac{\sqrt{2}-1}{1!}+\frac{(\sqrt{2}-1)^2}{2!}+\frac{(\sqrt{2}-1)^3}{3!}+\ldots..$$

b) Calculate f_x, f_y, $f_x(0,0)$, $f_y(0,0)$ for the function

$$f(x)=\frac{xy}{\sqrt{x^2+y^2}}, \text{ if } x^2+y^2\neq 0$$

$$=0, \quad \text{if } x=y=0$$

c) Find the rank of the matrix $\begin{pmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 1 \end{pmatrix}$.

5 + 5 + 5

9. a) Test the convergence of the series

$$\frac{6}{1.3.5} + \frac{8}{3.5.7} + \frac{10}{5.7.9} + \ldots$$

b) Examine the consistency of the system of the following equations and solve :

$x + 2y - z = 10,\ x - y - 2z = -2,\ 2x + y - 3z = 8$

c) Find all maxima and minima of the function $f(x, y) = x^3 + y^3 - 63(x + y) + 12xy$. 5 + 5 + 5

10. a) If $u = f(y - z, z - x, x - y)$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$

b) If $\vec{p} = t\hat{i} + 2t^2\hat{j} + 3t^3\hat{k}$, evaluate $\left[\dot{\vec{p}}\ \ddot{\vec{p}}\ \dddot{\vec{p}}\right]$.

c) Apply Stokes' theorem to evaluate $\int_c (ydx + zdy + xdz)$ where C is the curve of intersection of $x^2 + y^2 + z^2 = a^2$ and $x + z = a$.

5 + 5 + 5

11. a) If $x = a(u + v)$, $y = b(u - v)$

where $u = r^2 \cos 2\theta$, $v = r^2 \sin 2\theta$, then show that $\frac{\partial(x, y)}{\partial(r, \theta)} = -8abr^3$.

b) Evaluate the following double integral by using suitable transformation :

$\iint_R x^2 y^2 dxdy$ where R is the region of the circle $x^2 + y^2 \leq 1$.

c) Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} dxdydz$. 5 + 5 + 5