CS-503

Discrete Mathematics

me Allotted: 3 Hours Full Marks: 70

The questions are of equal value
The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

GROUP A
(Multiple Choice Type Questions)

Answer any *ten* questions. $10 \times 1 = 10$

(i) The numeric sequence of the generating function $\frac{z^5+1}{z+1}$ is

(A) 1, −1, 1, −1 (B) 1 − 1 + 1 − 1 + 1 ...
(C) 1, 1, 1, 1 (D) none of these

(ii) A tree has 21 vertices, then $\chi(G)$ is

(A) 20 (B) 10
(C) 40 (D) none of these

ii) The chromatic polynomial of Peterson graph is

(A) 3 (B) 4
(C) 5 (D) none of these

) The number of ways that a tree with 5 vertices can be coloured by 4 colours is

(A) 324 (B) 350
(C) 20 (D) none of these

[Turn over]

http://www.makaut.com

(v) The proposition $p \wedge (q \wedge \sim q)$ is a

(A) contradiction (B) tautology
(C) both (A) and (B) (D) none of these

(vi) $A \wedge B$ is equivalent to

(A) $\sim A \to \sim B$ (B) $\sim A \to B$
(C) $\sim B \to A$ (D) $\sim(A \to \sim B)$

(vii) $(S, \leq)$ is a poset if and only if

(A) "≤" is reflexive, antisymmetric and transitive
(B) "≤" is reflexive, symmetric and transitive
(C) "≤" is reflexive and transitive
(D) "≤" is antisymmetric and transitive

(viii) Every non-empty subset of $\mathbb{N}$ contains a

(A) maximal element (B) minimal element
(C) least element (D) greatest element

(ix) If k is a positive integer, gcd(ka, kb) =

(A) $k \cdot$ gcd(ka, b) (B) $k \cdot$ gcd(a, b)
(C) $k \cdot$ gcd(a, kb) (D) none of these

(x) If gcd(a, b) = d, then $\frac{a}{d}$, $\frac{b}{d}$ are

(A) relatively prime (B) prime
(C) composite (D) both (A) and (B)

(xi) In the Lattice {1, 5, 25, 125} w.r.t to the order relation "divisibility" the complement of 1 is

(A) 1 (B) 5
(C) 25 (D) 125

(xii) The solution of the recurrence relation $a_n = 2a_{n-1} + 1$ with $a_0 = 0$ is

(A) $1 - 2^n$ (B) $2^n - 2$
(C) $2^{n-1} - 1$ (D) $2^n - 1$

http://www.makaut.com

http://www.makaut.com

GROUP B
(Short Answer Type Questions)

Answer any *three* questions. $3 \times 5 = 15$

2. Solve the difference equation $y_{n+4} - y_n = 2^n$.

3. Consider $\mathbb{N}$, the set of all natural numbers and a relation "$\leq$" defined on $\mathbb{N}$ by $a \leq b$ holds if and only if a|b, $\forall a, b \in \mathbb{N}$. Then show that $(\mathbb{N}, \leq)$ is a poset.

4. In an area the custom is that girls give their choice to whom they will marry. One such choice is given below. Give your decision for matching and decide whether each girl will be married within their choice.

$g_1 \to (b_1, b_3, b_5), g_2 \to (b_1, b_2, b_4), g_3 \to (b_3, b_4, b_5), g_4 \to (b_2, b_3, b_5), g_5 \to (b_2, b_3, b_4)$

5. Find whether the argument $p \to \neg q, r \to q, r \models \neg p$ is valid or not.

6. Prove that the product of any m consecutive integers is divisible by m.

GROUP C
(Long Answer Type Questions)

Answer any *three* questions. $3 \times 15 = 45$

7. (a) If $S = \{1,2,3\}$ then draw the Hasse diagram of the poset $(\wp(S), \subseteq)$, where $\wp(S)$ is the power set of S. 5

(b) Using the laws of propositional logic show that $\models (p \wedge q) \to (p \vee q)$. 5

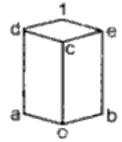
(c) Using generating function solve the difference equation $y_{n+2} - 5y_{n+1} + 6y_n = 0$ when $y_0 = 1, y_1 = 0$. 5

8. (a) Show that every bipartite graph is 2 chromatic. 5

(b) Find integers u and v satisfying gcd(272, 119) = $272u + 119v$. 5

http://www.makaut.com

(c) Show that the poset given in the following Hasse diagram is a lattice. Is it distributive and complemented? 5



(a) In an under graduate Science college offering a course in which every student will have to take English and one of the following groups (Botany, Zoology, Microbiology), (Botany, Chemistry and Zoology), (Physics, Zoology and Botany), (Maths, Physics, Chemistry), (Maths, Physics, Statistics), (Maths, Statistics, Computer), (Computer, Chemistry, Microbiology). 5

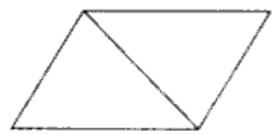
Find the period allocation in a day such that students can attend their class without any hindrance.

(b) Show that number of primes is infinite. 5

(c) If $\gcd(a, b) = 1$, show that $\gcd(a + b, a^2 - ab + b^2) = 1$ or 3. 5

).(a) State principle of inclusion and exclusion and use it to find the total number of integers between 1 and 1000 which are neither perfect squares nor perfect cubes. 1+4

(b) State decomposition theorem for obtaining chromatic polynomial and find the chromatic polynomial of the following graph using it. 1+4



(c) Show that $(p \to q) \wedge (q \to r) \to (p \to r)$ is a tautology. 5

(a) Cq is a cycle (i.e. a circular chain) with the nine vertices $a, b, c, d, e, f, g, h, i$. How many distinct maximal matchings of size 4 in Cq contain the edge ab? 5

b) Prove that $(19)^{20} \equiv 1 \pmod{181}$. 5

c) Find the closed form of the generating function for numeric function $f_r = \frac{r}{2}(r+1), r > 0$. 5