*Name* : .....

*Roll No.* : .....

*Invigilator's Signature* : .....

**CS/B.Tech (CSE/IT)/SEM-4/M-401/2010**

# 2010

# MATHEMATICS

*Time Allotted* : 3 Hours *Full Marks* : 70

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

## GROUP – A

## ( Multiple Choice Type Questions )

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) The generators of the cyclic group $( Z, + )$ are

a) $1, -1$ b) $0, 1$

c) $0, -1$ d) $2, -2$.

ii) The mapping $f : R \varnothing R$ given by $f ( x ) = | x |, x \square R$ is

a) Injective b) Surjective

c) Bijective d) None of these.

iii) Let $S$ be a finite set of $n$ distinct elements. The number of bijective mapping from $S$ to $S$ is

a) $n^2$ b) $n!$

c) $2^n$ d) None of these.

iv) If three Boolean variables $x$, $y$ and $z$ are defined on Boolean Algebra $B$, then which one of the following is a fundamental product ?

a) $xy'z$ b) $xy$

c) $xy\,(x+y)$ d) none of these.

v) If $G$ is binary tree on $n$ vertices, the $G$ has edges

a) $n\,(n-1)$ b) $n-1$

c) $n$ d) $\frac{n\,(n-1)}{2}$ .

vi) Solution of the recurrence relation $S_n = 2S_{n-1}$ with $S_0 = 1$ is $S_n =$

a) $2^n$ b) $2^{n-1}$

c) $2^{n+1}$ d) none of these.

vii) A complete graph is

a) regular b) connected simple

c) circuit d) planar graph.

viii) On the set $A = \{1, 2, 3\}$, the relation $R = \{(2, 1), (1, 2), (3, 3)\}$. Then $R$ is

a) symmetric b) reflexive

c) transitive d) not a relation at all.

ix) In the additive group $Z_6$ the order of the element [ 4 ] is

a) 0 b) 2

c) 3 d) 6.

x) Let $G$ be a group and $a \in G$. If $o(a) = 17$, then $o(a^8)$ is

a) 17 b) 16

c) 8 d) 5.

xi) If $S$ and $T$ are two subgroup of a group $G$, then which of the following is a subgroup ?

a) $S \cup T$ b) $S \cap T$

c) $S - T$ d) $G - S$.

xii) The dual of a planar graph is dual. It is

a) True b) False.

xiii) A binary tree should have at least

a) one vertex b) two vertices

c) three vertices d) four vertices.

xiv) A connected graph is Eulerian iff it has no vertex of odd degree. It is

a) True b) False.

xv) The number of idempotent element in $Z$ is

a) 0 b) 1

c) 2 d) none of these.

## GROUP – B

### ( Short Answer Type Questions )

Answer any *three* of the following. $3 \times 5 = 15$

2. Let $G = \{ (a, b) : a \neq 0, b \in R \}$ and * be a binary compostion defined on $G$ by $(a, b) * (c, d) = (ac, bc + d)$. Show that $(G, *)$ is a non Abelian group.

3. Show that for any two subgroups $H$ and $K$ of a group $G$ $H \cap K$ is also a subgroup of $G$.

4. Let $G$ be a group, if $a, b \in G$ such that $a^4 = e$, the identity element of $G$ and $ab = ba^2$, prove that $a = e$.

5. Prove that every cyclic group is an Abelian group.

6. Show that the mapping $F : (Z, \bullet) \to (R, \bullet)$ defined by $f(x) = x^2 \ \forall x \in Z$ is a monomorphism but not isomorphism.

7. If in a ring $R$ with unity, $(xy)^2 = x^2 y^2 \ \forall x, y \in R$, then show that $R$ is a commutative.

## GROUP – C
### ( Long Answer Type Questions )

Answer any *three* of the following. $3 \times 15 = 45$

8. a) Examine whether the following two graphs are isomorphic.

Dia.

5

b) Draw the dual of the graph.

Dia.

5

c) Determine the adjacency matrix of the following di-graph :



Dia.

5

9. a) Construct a simple logic circuit which would satisfy the truth table.

| $x$ | $y$ | $f$ |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 0 | 0 | 1 |

5

b) Prove that a graph $G$ has a spanning tree if and only if $G$ is connected. 5

c) Find the minimal spanning tree by Kruskal's Algorithm from the following graph $G$ :

Dia.

5

10. a) Consider the lattice $L = \{ 1, 2, 3, 4, 6, 12 \}$, the divisors of 12 ordered by divisibility. Find the lower and upper bound of $L$. Is $L$ a complemented lattice ? 5

b) For any Boolean Algebra, show that.

$( xy' + xz' ) + x' = ( x' + y + z ) ( x' + y + z' ) ( x' + y' + z' )$ .

5

c) Using generating function solve the recurrence relation, $a_n - 7a_{n-1} + 10a_{n-2} = 0$, for $n > 1$ and $a_0 = 3$, $a_1 = 3$. 5

11. a) Prove that the number of vertices in a binary tree is always odd. 5

b) Find the truth table of the Boolean function

$$f = z'xy + xy' + y.$$ 5

c) Prove that a complete graph with $n$ vertices consist of $\frac{n(n-1)}{2}$ number of edges. 5

12. a) Prove that the identity elements and the inverse of an element in a group is unique. 5

b) Prove that in a group $(G, *)$, $(a * b)^{-1} = b^{-1} * a^{-1}$. 5

c) Prove that the set of matrices $H = \left\{ \begin{pmatrix} x & 0 \\ 0 & x \end{pmatrix} : x \in R, x \neq 0 \right\}$ forms a normal subgroup of $GL(2, R)$, the group of all real non-singular $2 \times 2$ matrices under multiplication. 5

13. a) Using Ford-Fulkerson's algorithm, find the maximum flow in the following network :

Dia.

8

b) Using Floyd's algorithm, find the shortest path between

i) $w_2$ and $w_6$

ii) $w_1$ and $w_6$ .

Dia.

7