MAULANA ABUL KALAM AZAD UNIVERSITY OF TECHNOLOGY, WEST BENGAL

Paper Code : M-101

MATHEMATICS-I

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : 10 × 1 = 10

i) $\dfrac{d^n}{dx^n}(2^{-5x}) =$

a) $(-1)^n 5^n (\log 2)^n 2^{-5x}$ b) 2^{-5x}

c) $(-1)^n 5^n \cdot 2^{-5x}$ d) none of these.

ii) Lagrange's mean value theorem is obtained from Cauchy's mean value theorem for two functions $f(x)$ and $g(x)$ by putting $g(x) =$

a) 1 b) x^2

c) x d) none of these.

iii) The series $\sum \dfrac{1}{(2x+1)^n}$ is

a) convergent b) divergent

c) oscillatory d) none of these.

iv) The rank of the null matrix is

a) 0 b) 1

c) 2 d) 3.

v) If the matrix $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & \lambda \end{pmatrix}$ is singular, then the value of λ is

a) 3 b) 5

c) 2 d) 4.

vi) If α, β, γ are the roots of the equation $x^2 - 3x + 2 = 0$ then $\begin{vmatrix} 0 & \alpha & \beta \\ \beta & 0 & 0 \\ 1 & -\alpha & \alpha \end{vmatrix} =$

a) 3 b) 3/2

c) – 6 d) 3.

vii) Which of the following theorems can be applied on $f(x) = |x|$ in the interval $[-1, 1]$?

a) Rolle's theorem

b) Mean Value theorem

c) Cauchy's M. V. theorem

d) none of these.

viii) The sequence $\left\{\frac{1}{3^n}\right\}$ is

a) monotonic increasing

b) oscillatory

c) divergent

d) monotonic decreasing.

ix) If $u = \sin^{-1}\sqrt{\dfrac{x^{\frac{1}{2}} + y^{\frac{1}{2}}}{x^{\frac{1}{3}} + y^{\frac{1}{3}}}}$ then $\sin u$ is a homogeneous function of degree

a) $\frac{1}{6}$ b) $-\frac{1}{6}$

c) $\frac{1}{12}$ d) $-\frac{1}{12}$.

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x) If $u + v = x$, $uv = y$ then the value of $\frac{\partial(u, v)}{\partial(x, y)} =$

a) $u - v$ b) $\frac{1}{u - v}$

c) uv d) $\frac{1}{uv}$.

xi) If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$ is equal to

a) 1 b) -1

c) 3 d) 0.

xii) The value of m for which the vectors $2\hat{i} - \hat{j} - \hat{k}$ $\hat{i} + 2\hat{j} - 3\hat{k}$, $3\hat{j} + m\hat{i} + 5\hat{k}$ are coplanar is

a) 4 b) -4

c) 5 d) -5.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Prove that $\begin{vmatrix} (b+c)^2 & c^2 & b^2 \\ c^2 & (c+a)^2 & a^2 \\ b^2 & a^2 & (a+b)^2 \end{vmatrix} = 2(bc + ca + ab)^2$

3. Expand the function $\sin x$ in power of x in infinite series.

4. Test the convergence of the series

$$1+\frac{1}{2^2}+\frac{2^2}{3^3}+\frac{3^3}{4^4}+\frac{4^4}{5^5}+\ldots$$

5. Verify Cayley-Hamilton theorem for $A=\begin{pmatrix}1 & 2 & 3\\ 3 & -2 & 1\\ 4 & 2 & 1\end{pmatrix}$.

Hence compute A^{-1}.

6. Find $div\vec{F}$ and $curl\vec{F}$ where $\vec{F}=grad(x^3+y^3+z^3-3xyz)$.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3\times 15=45$

7. a) If $y=\sin(m\sin^{-1}x)$, show that

$$(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2-m^2)y_n=0.$$

b) Using mean value theorem prove that

$$0<\frac{1}{\log(1+x)}-\frac{1}{x}<1 \text{ for all } x>0.$$

c) If $I_n=\int_0^{\pi/2}\sin^{2n+1}\theta\, d\theta$, where n is a positive integer, show that $I_n=\frac{2n}{2n+1}I_{n-1}$. Use this to evaluate $\int_0^{\pi/2}\sin^7\theta d\theta$. 5 + 5 + 5

8. a) Examine the convergence of the series

$$1+\frac{\sqrt{2}-1}{1!}+\frac{(\sqrt{2}-1)^2}{2!}+\frac{(\sqrt{2}-1)^3}{3!}+\ldots$$

b) Calculate f_x, f_y, $f_x(0,0)$, $f_y(0,0)$ for the function

$$f(x)=\frac{xy}{\sqrt{x^2+y^2}}, \text{ if } x^2+y^2\neq 0$$
$$=0, \quad \text{if } x=y=0$$

c) Find the rank of the matrix $\begin{pmatrix}1 & 3 & 4 & 3\\ 3 & 9 & 12 & 3\\ 1 & 3 & 4 & 1\end{pmatrix}$.

5 + 5 + 5

9. a) Test the convergence of the series

$$\frac{6}{1.3.5}+\frac{8}{3.5.7}+\frac{10}{5.7.9}+\ldots$$

b) Examine the consistency of the system of the following equations and solve :

$$x+2y-z=10,\ x-y-2z=-2,\ 2x+y-3z=8$$

c) Find all maxima and minima of the function $f(x,y)=x^3+y^3-63(x+y)+12xy$. 5 + 5 + 5

10. a) If $u=f(y-z, z-x, x-y)$, prove that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$

b) If $\vec{p}=t\hat{i}+2t^2\hat{j}+3t^3\hat{k}$, evaluate $\left[\vec{p}\ \vec{p}\ \vec{p}\right]$.

c) Apply Stokes' theorem to evaluate $\int_c(ydx+zdy+xdz)$ where C is the curve of intersection of $x^2+y^2+z^2=a^2$ and $x+z=a$.

5 + 5 + 5

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11. a) If $x = a(u + v)$, $y = b(u - v)$

where $u = r^2 \cos 2\theta$, $v = r^2 \sin 2\theta$, then show that

$$\frac{\partial(x, y)}{\partial(r, \theta)} = -8abr^3.$$

b) Evaluate the following double integral by using suitable transformation :

$\iint_R x^2 y^2 dxdy$ where R is the region of the circle $x^2 + y^2 \leq 1$.

c) Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} dxdydz$. 5 + 5 + 5

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