Maulana Abul Kalam Azad University of Technology, West Bengal

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In Pursuit of Knowledge And Excellence

MAULANA ABUL KALAM AZAD UNIVERSITY OF TECHNOLOGY, WEST BENGAL

Paper Code : BS-M-102

PUID : 01035 (To be mentioned in the main answer script)

MATHEMATICS-IB

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

   i) The value of $\int_0^{\frac{\pi}{2}} \sin^6 x \, dx$ is

      a) $\frac{7\pi}{32}$ b) $\frac{7\pi}{16}$

      c) $\frac{5\pi}{32}$ d) $\frac{5\pi}{16}$.

   ii) The value of $\Gamma(3)$ is

      a) 6 b) 2

      c) 24 d) 1.

[Turn over

iii) The singularity of the integral $\int_{-1}^{2} \frac{dx}{x(x-1)}$ are

a) 1, 2 b) − 1, 2

c) 0, 1 d) 0, 2.

iv) The locus of the centre of curvature is called

a) envelope b) evolute

c) circle of curvature d) involutes.

v) Which of the following functions does not satisfy Rolle's theorem in [− 1, 1] ?

a) x^2 b) $\frac{1}{x^4+2}$

c) $\frac{1}{x}$ d) $\sqrt{x^2+3}$.

vi) The value of $\lim_{x \to 0} \frac{\sin^2 x}{x^2}$ is

a) − 1 b) 1

c) 2 d) does not exist.

vii) All eigenvalues of any nilpotent matrix are

a) 0 b) 1

c) 2 d) none of these.

viii) If $f(x, y) = \dfrac{x^2 + y^2}{\sqrt{x+y}}$ then $xf_x + yf_y =$

a) $\frac{1}{2}$ b) $\frac{1}{2}f$

c) $\frac{3}{2}f$ d) none of these.

ix) The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if

a) $p \geq 1$ b) $p > 1$

c) $p < 1$ d) $p \leq 1$.

x) The value of the determinant $\begin{vmatrix} 100 & 101 & 102 \\ 105 & 106 & 107 \\ 110 & 111 & 112 \end{vmatrix}$ is

a) 2 b) 0

c) 405 d) -1.

xi) If $x = r\cos\theta$, $y = r\sin\theta$, then $\dfrac{\partial(r,\theta)}{\partial(x,y)}$ is

a) r b) 1

c) $\frac{1}{r}$ d) none of these.

xii) The value of t for which

$$\vec{f} = (x+3y)\hat{i} + (y-2x)\hat{j} + (x+tz)\hat{k} \text{ is solenoidal is}$$

a) 2 b) -2

c) 0 d) 1.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Show that $\int_0^\infty \frac{dx}{(1+x^2)^5} = \frac{35\pi}{256}$.

3. The circle $x^2 + y^2 = a^2$ is revolved about the x-axis. Show that the surface area and the volume of the sphere thus generated are respectively $4\pi a^2$ and $\frac{4}{3}\pi a^3$.

4. Evaluate $\lim_{x \to 0} \left(\frac{1}{x^2} - \frac{1}{\sin^2 x} \right)$.

5. Find the maximum value of $x^3 y^2$ subject to the constraint $x + y = 1$, using the method of Lagrange's multiplier.

6. If $f = x^2 y + 2xy + z^2$, then show that *curl grad* $f = 0$.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Expanding the determinant by Laplace's method in terms of minors of 2nd order formed from the first two, prove that

$$\begin{vmatrix} 0 & a & b & c \\ -a & 0 & d & e \\ -b & -d & 0 & f \\ -c & -e & -f & 0 \end{vmatrix} = (af - be + cd)^2 .$$

b) Find the eigenvalues and the eigenvectors corresponding to the smallest eigenvalue of the matrix

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

c) Check the consistency of the given system of equations and solve if possible :

$x + 2y - z = 10; \quad x - y - 2z = -2; \quad 2x + y - 3z = 8.$

8\. a) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 - 3 = z$ at the point $(2, -1, 2)$.

b) Check the convergence of the series $\left(\frac{2^2}{1^2} - \frac{2}{1}\right)^{-1} + \left(\frac{3^2}{2^2} - \frac{3}{2}\right)^{-2} + \left(\frac{4^2}{3^2} - \frac{4}{3}\right)^{-3} + \ldots$.

c) Use Mean-Value theorem to prove the following inequality $\frac{x}{1+x} < \log(1+x) < x$, if $x > 0$. 5 + 5 + 5

9. a) Show that the rectangle of maximum area that can be inscribed in circle is a square.

b) Check whether the matrix $A = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}$ is diagonalizable or not.

c) Using Parseval's identity corresponding to the Half-Range cosine series of the function $f(x) = x$, $0 < x < 2$, find the sum of the series

$$\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \ldots\,.$$ 5 + 5 + 5

10 a) Find the Fourier series of the function

$$f(x) = \begin{cases} \pi + 2x, & -\pi < x < 0 \\ \pi - 2x, & 0 \le x \le \pi \end{cases}$$

and hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots = \frac{\pi^2}{8}$. 8

b) Find the evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. 7

11. a) If $u = xf\left(\frac{y}{x}\right) + g\left(\frac{y}{x}\right)$ then show that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 0\,.$$

b) If $y = \tan^{-1} x$ then prove that

i) $(1+x^2)\, y_1 = 1$

ii) $(1+x^2)\, y_{n+1} + 2nxy_n + n(n-1)y_{n-1} = 0$.

c) Find the directional derivative of $f = xyz$ at (1, 1, 1) in the direction $2\hat{i} - \hat{j} - 2\hat{k}$.