2003

DISCRETE MATHEMATICAL STRUCTURE

Time Allotted: 3 hours *Full Marks: 70*

The questions are of equal value.
The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

<u>Answer any seven Questions</u>

a) Confirm or disprove the equality 2

$$(A-B)\cup(A\cap B)\cap(B-A)\cup(A\cup B)'=\phi$$

for any two finite Sets A and B.

b) For any three Sets A, B, C, prove that 3

$$A-(B\cup C)=(A-B)\cap(A-C)$$

c) Define Cartesian product of two sets. If $B\subseteq A$, then prove that $B\times B=(B\times A)\cap(A\times B)$ 2+3

a) Let $f: X\rightarrow Y$ and $g: Y\rightarrow Z$. Let f and g be one-one mapping. Prove that $g_0 f: X\rightarrow Z$ is one-one mapping. 3

b) Let $f: N\rightarrow N$ is a mapping, where $f(n)=n-(-1)^n\ \forall n\in N$. Is this mapping a one-one mapping? 3

c) On the Set R of all real numbers by setting aρb iff $(a-b)$ is an integer. Show that ρ is an equivalence relation on R.. 4

a) Define Poset. Considering the poset 2+3

$P = (\{1, 2, 3, 4, 6, 9, 12, 18, 36\}/)$, find the *l.u.b* and *g.l.b* of the subset $A=\{4,6,9\}$.

b) Draw the Hasse diagram for the divisibility relation on the Set $A = \{2, 3, 6, 12, 24, 36\}$ and find the maximal and minimal elements. 3+2

a) Discuss tautology and contradiction with examples. 3

b) Prove that $[(p\rightarrow q)\wedge(r\rightarrow s)\wedge(p\vee r)]\rightarrow(q\vee s)$ is a tautology. 3

c) Obtain the truth table of 4

$$[\sim p\vee(q\wedge r)]\vee[(p\vee\sim q)\wedge s]\rightarrow(p\wedge q)$$

5. a) Using Mathematical Induction, prove that for all integers $n \geq 4, \quad 3^n \geq n^3$ 5

b) Prove by Mathematical Induction that for all positive integer n, $\frac{n^7}{7}+\frac{n^5}{5}+\frac{2n^3}{3}-\frac{n}{105}$ is an integer. 5

6. a) Solve the following recurrence relation using generating functions: 5

$$a_n + a_{n-1} - 16a_{n-2} + 20a_{n-3} = 0$$

for $n \geq 3$ and $a_0 = 0$, $a_1 = 1$, $a_2 = -1$

b) Solve the recurrence relation 5

$$a_n + 5a_{n-1} + 6a_{n-2} = 3n^2 - 3n + 1$$

with the initial conditions $a_0 = 0$, $a_1 = 1$.

7. a) Eleven papers are set for the engineering examination of which two are in Mathematics. In how many ways can the papers be arranged when the two Mathematics papers do not come together? 5

b) Find the number of parallelograms formed by the intersection of two sets of m and n parallel straight lines. 5

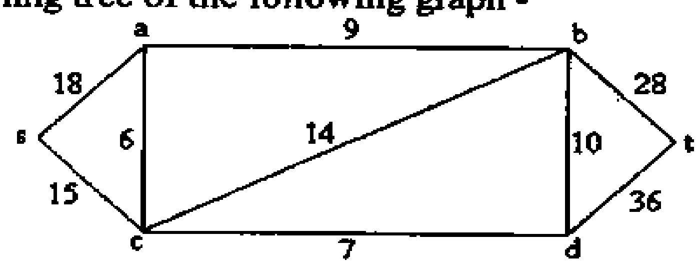
8. a) Define degree of a vertex in a graph. When a graph is called regular? 2+

b) Prove that the number of vertices of odd degree in a graph is always even. 4

c) Suppose G is a non-directed graph with 12 edges. If G has 6 vertices each of degree 3 and the rest have degree less than 3, find the minimum number of vertices G can have. 3

9. a) Define binary tree. Prove that the number of vertices in a binary tree is always odd. 1+

b) Using Dijkstra's algorithm, find the minimum weight spanning tree of the following graph - 6



0. a) Define 'grammar'. Using the grammar G given as 2+3

$$G = (\{S, A, B\}, \{a, b\}, P, S)$$

Where $P = \{(S \to AB), (S \to bA), (A \to a), (A \to aS),$

$(A \to bAA), (B \to b), (B \to bS), (B \to aBB)\}$

construct the derivation tree for the string $aaabbb$.

b) Let M be the finite state machine with state table appearing below: 2+3

	f			g		
S	a	b	c	a	b	c
S_0	S_0	S_1	S_2	0	1	0
S_1	S_1	S_1	S_0	I	1	1
S_2	S_2	S_1	S_0	1	0	0

i) Find the input set A, the state set S, the output set O, and initial state of M.

ii) Draw the state diagram of M.

1. a) Define Moore machine and Mealy machine. 4

b) Construct a DFA from the NDFA 6

$A' = (\{S_0, S_1, S_2\}, \{a, b\}, \delta^1, S_0, F^1)$ where $F = (S_1)$ and the transition function is given by the Table below:

	Inputs	
δ^1	a	b
S_0	(S_1, S_2)	ϕ
S_1	ϕ	(S_2)
S_2	ϕ	(S_2)

2. a) Define Convex fuzzy Set. 2

b) If the two fuzzy Sets F_1 and F_2 are defined by: 2+2

$F_1 = \{(4, 0.2), (6, 0.2), (8, 0.4), (10, 0.5)\}$ and

$F_2 = \{(0, 0.3), (2, 0.5), (4, 0.7), (5, 0.9), (8, 0.7)\}$

Find (i) $F_1 \cap F_2$ and (ii) $F_1 \cup F_2$

c) Write short notes on: 2+2

i) Context free grammar,

ii) Parse tree

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