



ENGINEERING & MANAGEMENT EXAMINATIONS, DECEMBER - 2008

MATHEMATICS - I

SEMESTER - 1

Time : 3 Hours]

[Full Marks : 70

GROUP - A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any ten of the following :

10 × 1 = 10

i) If $A = \{ 1, 2, 3, 4, 8 \}$, $B = \{ 2, 4, 6, 7 \}$, then $A \Delta B$ isa) $\{ 2, 4 \}$ b) $\{ 1, 2, 3, 4, 6, 7, 8 \}$ c) ϕ d) $\{ 1, 3, 6, 7, 8 \}$

ii) $\lim_{x \rightarrow 0} (1+x)^{1/x}$ is equal to

a) 1

b) e

c) 0

d) ∞

iii) $\frac{d}{dx} (\log_a x)$ is equal toa) $\frac{1}{x}$ b) $\log (1/x)$ c) $(1/x) \log_a e$ d) $x \log e$

iv) If $y = \log x^2$, the value of $\frac{d^2 y}{dx^2}$ isa) $\frac{2}{x^2}$ b) $-\frac{2}{x^2}$ c) $\frac{2}{x}$ d) $2x$



v) The matrix $A = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ is an

- a) orthogonal matrix b) idempotent matrix
c) identity matrix d) none of these.

vi) Derivative of x^4 with respect to x^2 is

- a) $4x^3$ b) $2x^2$
c) $2x$ d) 4.

vii) If the roots of the equation $ax^2 + bx + c = 0$ ($a \neq 0$) are real and unequal, then its discriminant D satisfies

- a) $D > 0$ and D = a perfect square
b) $D = 0$
c) $D > 0$ and $D \neq$ a perfect square
d) $D < 0$.

viii) If $A = \{1, 2, 3\}$, $B = \{2, 3, 6\}$, then $A \cup B$ is

- a) $\{1, 2, 3\}$ b) $\{2, 3\}$
c) $\{1, 2, 3, 6\}$ d) none of these.

ix) If α, β, γ be the roots of $x^3 - 3x^2 + 6x - 2 = 0$, then $\sum \alpha\beta$ is

- a) -3 b) 6
c) 2 d) none of these.

x) If $f(x) = 3 + 2x$; when $x \geq 0$

$$= -3 - 2x; \text{ when } x < 0,$$

then $\lim_{x \rightarrow 0} f(x)$ is

- a) 3 b) -3
c) 0 d) none of these.



xi) If $f(x) = \frac{x}{|x|}$; when $x \neq 0$
 $= 1$; when $x = 0$, then

- a) $f(x)$ is continuous at $x = 0$
- b) $f(x)$ is continuous, but not differentiable at $x = 0$
- c) $f(x)$ is discontinuous at $x = 0$
- d) none of these.

xii) The value of $\int_{-1}^2 |x| dx$ is

- a) 3
- b) 5
- c) 5/2
- d) none of these.

GROUP - B

(Short Answer Type Questions)

Answer any *three* of the following.

3 × 5 = 15

2. If α, β, γ be the roots of the equation, $x^3 + px^2 + qx + r = 0$, then find the value of $\sum \alpha^3$.
3. If $u = \tan^{-1} \left(\frac{x+y}{\sqrt{x} + \sqrt{y}} \right)$, then show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{4} \sin 2u$.
4. Prove that the set of even integers (including zero) forms an additive group.
5. Evaluate $\int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$.
6. If $P = \begin{pmatrix} 9 & 1 \\ 4 & 3 \end{pmatrix}$ and $Q = \begin{pmatrix} 1 & 5 \\ 7 & 12 \end{pmatrix}$, find the matrix R so that $5P + 3Q + 2R$ is a null matrix.

**GROUP - C****(Long Answer Type Questions)**Answer any *three* of the following questions. $3 \times 15 = 45$

7. a) State Rolle's Theorem.

b) Differentiate n times the following equation :

$$(1 + x^2) y_2 + (2x - 1) y_1 = 0.$$

c) If $y = \sin (m \sin^{-1} x)$, show that

$$(1 - x^2) y_{n+2} - (2n + 1) x y_{n+1} + (m^2 - n^2) y_n = 0. \quad 4 + 5 + 6$$

8. a) If p th, q th and r th terms of an A.P. are P , Q and R respectively, show that $p(Q - R) + q(R - P) + r(P - Q) = 0$.b) Show that the centroid of a triangle with vertices (x_1, y_1) , (x_2, y_2) and (x_3, y_3) is $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$.c) Find the equation of a straight line through the point of intersection of lines $2x - 3y + 4 = 0$ and $3x + 4y - 5 = 0$ and that is perpendicular to the line $6x - 7y + 8 = 0$. 5 + 5 + 59. a) Show that $\cos x > 1 - \frac{x^2}{2}$ if $0 < x < \frac{\pi}{2}$.b) If $f(x, y) = \begin{cases} \frac{x^2 y^2}{x^2 + y^2}, & x^2 + y^2 \neq 0 \\ 0, & x^2 + y^2 = 0, \end{cases}$ then show that $f_{xy}(0, 0) = f_{yx}(0, 0)$.c) Evaluate $\int \frac{x^2}{(x^2 + a^2)(x^2 + b^2)} dx$. 4 + 6 + 5

10. a) Reduce the following equation to its canonical form and determine the nature of the conic represented by it :

$$6x^2 - 5xy - 6y^2 + 14x + 5y + 4 = 0.$$

b) Find the equation of the ellipse one of whose foci is $(-1, 1)$, eccentricity is 0.5 and the corresponding directrix is $y = x + 3$. 9 + 6



11. a) Solve the equation by Cardan's method, $2x^3 + 3x^2 + 3x + 1 = 0$.
- b) Let $G = \{ a \in \mathbb{R} / -1 < a < 1 \}$. Define a binary operation \otimes on G by $a \otimes b = \frac{a+b}{1+ab} \quad \forall a, b \in G$. Show that (G, \otimes) is a group.
- c) Find the nature of the roots $x^4 + qx^2 + rx - s = 0$ by Descartes' rule of signs (where q, r, s , being positive). 15
12. a) If by a transformation of one rectangular axis to another with same origin the expression $ax + by$ changes to $a'x' + b'y'$,
prove that $a^2 + b^2 = a'^2 + b'^2$,
- b) Show that $\int_0^{\infty} \frac{dx}{(x+1)(x+2)} = \log 2$.
- c) Use the method of integration to evaluate $\lim_{n \rightarrow \infty} \frac{1^k + 2^k + \dots + n^k}{n^{k+1}} ; k > 0$.

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