CS/B.Tech (CE)/SEM-4/CE-401/2011

2011
MATHEMATICS – II

Time Allotted : 3 Hours   *Full Marks :* 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :   10 × 1 = 10
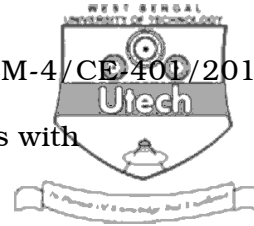
    i)   Newton's forward formula is used for interpolating the value of *y* near the

         a)   end of a set   b)   beginning of a set

         c)   both (a) and (b)   d)   none of these.

    ii) As soon as a new value of a variable is found by iteration it is used immediately in the following equation, this method is called

         a)   Gauss-Jacobi method b)   Gauss-Seidal method

         c)   Relaxation method   d)   None of these.

iii) The order of convergence in Newton-Raphson method is

    a)   2   b)   3

    c)   0   d)   none of these.

iv) When rounded off correct to 5 significant digits 41·79992 becomes

    a)   41·800   b)   41·7990

    c)   42·799   d)   none of these.

v) In case of bisection method the convergence is

    a)   linear   b)   quadratic

    c)   very slow   d)   none of these.

vi) The degree of precision of Simpson's $\frac{1}{3}$ rd rule is

    a)   1   b)   2

    c)   3   d)   4.

vii) If the number 436·268184 is rounded off to 436·2682 (correct to 7 significant figure) the relative error in rounding off is

    a) $0 \cdot 000016$ b) $3 \cdot 667469 \times 10^{-8}$

    c) $3 \cdot 667469 \times 10^{-6}$ d)   none of these.

viii) Lagrange's interpolation formula deals with

    a)   Equispaced argument

    b)   Unequispaced argument

    c)   both (a) and (b)

    d)   none of these.

ix)   For trapezoidal rule of numerical integration the number of sub-intervals should be

    a)   even   b)   even or odd

    c)   odd   d)   multiples of three.

x)   Newton-Raphson method fails if

    a) $f'(x) = 0$ b) $f'(x) > 0$

    c) $f'(x) < 0$ d)   none of these.

xi)   The method of iteration formula for the function $\phi(x)$ must satisfy

    a) $\left|\phi'(x)\right| < 1$ b) $\left|\phi'(x)\right| > 1$

    c) $\left|\phi'(x)\right| = 1$ d) $\left|\phi'(x)\right| = 2.$

xii) Regula-Falsi method is used to

   a) solve the differential equation of boundary value problem

   b) solve transcendental equation numerically

   c) to find the real roots of an equation $f(x) = 0$

   d) none of these.

xiii) Runge-Kutta method has a truncation error, which is of order

   a) h^2 b) h^3

   c) h^4 d) none of these.

xiv) An example of iterative method is

   a) Gauss-elimination   b) Gauss-Jordan

   c) Gauss-Seidel method   d) None of these.

xv) If $f(x) = 2x^3 - 3x^2 + 4x + 5$ lets $\Delta^3 f(x)$ [where $h = 1$] is

   a) 8   b) 200

   c) 12   d) 1000.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Evaluate the missing term from the following table :

x	0	1	2	3	4	5
$f(x)$	0	—	8	15	—	35

3. Prove that :

   i) $E = e^{hD}$ where $D = \dfrac{\mathrm{d}}{\mathrm{d}x}$ and E is shift operator.

   ii) Find the absolute, relative and percentage errors if $\dfrac{1}{3}$ is

   approximated by 0·333. $2 + 3$

4. Compute the root of $10^x + \sin x + 2x = 0$ by method of bisection correct up to three significant figures.

5. Find the positive root of the equation $x^2 + 2x - 2 = 0$ correct up to two significant figures by Newton-Raphson method.

6. Obtain a function whose first differences is $2x^3 - 3x^2 + 3x - 10$.

7. Using Taylor's series method, find y (0·1) and y (0·2) where $\dfrac{dy}{dx} = 2y + 3e^x$ with y (0) = 0.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

8. a) Construct the forward difference table for a function $f(x)$ given by the following table :

x	2·0	2·2	2·4	2·6	2·8	3·0	3·2
y = f(x)	0·135335	0·110803	0·090718	0·074274	0·060810	0·049787	0·040762

b) Discuss the advantages and disadvantages of Lagrange's interpolation.

c) Construct Lagrange's interpolation formula by using the following table :

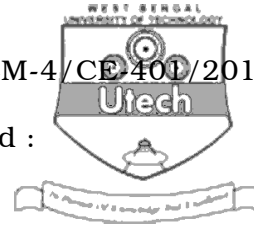| x	40	45	50	55
y = f(x)	15·22	13·99	12·62	11·13

$5 + 4 + 6$

9. a) Solve by Jacobi's method :

$x - y + 4z = 9$

$8x - 3y + 2z = 20$

$4x + 11y - z = 33.$

b) Solve by Gauss-Seidel iterative method :

$3x + 9y - 2z = 11$

$4x + 2y + 13z = 24$

$4x - 2y + z = -8.$

c) Calculate mean, mode and median of the following data relating to weights of 120 articles :

Weight (gm)	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
No of articles	14	17	22	26	23	18

$4 + 5 + 6$

10. a) Using Euler's method find the solution of the differential equation $\dfrac{dy}{dx} = x^2 - y$, $y(0) = 1$ for $x = 0·3$ taking $h = 0·1$. Compare the result with exact solution.
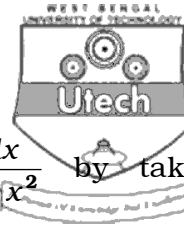
b) Use Runge-Kutta fourth order method to solve the following equation $\dfrac{dy}{dx} = x + y$, $y(0) = 1$ at $x = 0·1$.

c) Fit the parabola $y = a + bx + cx^2$ to the following data and also the goodness of fit, by least square method :

x :	1	2	3	4	5	6	7	8	9
y :	2	6	7	8	10	11	11	10	9

$5 + 5 + 5$

11. a) Find the numerical value of $\int_{0}^{6} \dfrac{dx}{1+x^2}$ by taking $h = 1$ using Simpson's rule.

   b) Use trapezoidal rule to evaluate $\int_{0}^{1} x^3 dx$ considering 5 sub-intervals.

   c) Prove that for small value of h $\Delta^{x+1} f(x_0) \approx h^{x+1} f^{x+1}(x_0)$ where the symbols have their usual meaning.   5 + 5 + 5

12. a) Find first derivative of $f(x)$ at $x = 15$ if

x	15	20	25	30	35	40
$f(x)$	3·37	7·0	13·62	24·00	38·87	59·00

   b) Find the numerically largest eigenvalue of the matrix $A = \begin{bmatrix} 3 & -5 \\ -2 & 4 \end{bmatrix}$ by power method.

   c) Derive Newton's forward difference interpolation formula.   5 + 5 + 5