Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/B.TECH (NEW)/SEM-1/M-101/2013-14

2013

MATHEMATICS – I

Time Allotted : 3 Hours — Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :

$10 \times 1 = 10$

i) The value of the determinant

$$\begin{vmatrix} 100 & 101 & 102 \\ 105 & 106 & 107 \\ 110 & 111 & 112 \end{vmatrix} \text{ is}$$

a) 0 b) 10

c) 100 d) 1000

ii) The equation $x + y + z = 0$ has

a) infinite number of solutions

b) no solution

c) unique solution

d) two solutions

iii) The value of $\int_{1}^{0} \int_{0}^{1} (x + y)\, dx\, dy =$

a) 2   b) 3

c) 1   d) 0

iv) $f(x, y) = \dfrac{\sqrt{y} + \sqrt{x}}{y + x}$ is a homogeneous function degree

a) $\dfrac{1}{2}$ b) $\dfrac{-1}{2}$

c) 1   d) 2

v) In the MVT $f(h) = f(0) + hf'(\theta h)$, $0 < \theta < 1$, $f(x) = \dfrac{1}{1 + x}$ and $h = 3$, then the value of θ is

a) 1   b) $\dfrac{1}{3}$

c) $\dfrac{1}{\sqrt{2}}$ d) none of these.

vi) If $y = e^{ax + b}$ then $(y_5)\, 0 =$

a) ae^{a} b) $a^5 e^b$

c) $a^b e^{ax}$ d) none of these

vii) The series $\sum \dfrac{1}{(2n+1)^{[illegible]}}$ is

a) convergent   b) divergent

c) oscillatory   d) none of these

viii) $\int_{0}^{\frac{\pi}{2}} \cos^6 x\, dx$ is equal to

a) $\dfrac{7\pi}{12}$ b) $\dfrac{5\pi}{32}$

c) $\dfrac{\pi}{32}$ d) $\dfrac{3\pi}{16}$

ix) If $[\vec{a}\ \vec{b}\ \vec{c}] = 0$ then the vectors $\vec{a}, \vec{b}, \vec{c}$ are

a) colinear   b) coplanar

c) orthogonal   d) none of these.

x) If $u(x, y) = \tan^{-1}\left(\dfrac{y}{x}\right)$, then the value of $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y}$ is

a) 0   b) $2u(x, y)$

c) $u(x, y)$ d) none of these

xii) The centre of the sphere given by the equation

[illegible] is

a) $\left(-\frac{b}{a}, -\frac{c}{a}, -\frac{d}{a} \right)$

b) $(-b, -c, -d)$

c) $\left(-\frac{b}{2a}, -\frac{c}{2a}, \frac{d}{2a} \right)$

d) $\left(\frac{b}{2a}, \frac{c}{2a}, \frac{d}{2a} \right)$

GROUP - B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 1$

2. Prove that every square matrix can be expressed as the sum of a symmetric matrix and a skew symmetric matrix.

3. Show that

$$\vec{f} = \left(6xy + z^2 \right) \hat{i} + \left(3x^2 - z \right) \hat{j} + \left(3xz^2 - y \right) \hat{k}$$

is irrotational. Hence find a scalar function ϕ such that

$$\vec{f} = \vec{\nabla} \phi .$$

4. Using Mean Value Theorem prove that

$$x < \sin^{-1} x < \frac{x}{\sqrt{1 - x^2}}, \quad 0 < x < 1$$

5. Show that the area bounded by a simple closed curve C i given by $\frac{1}{2} \oint_C (x\,dy - y\,dx)$

Prove that the function

[illegible]

has neither maxima for minima at the origin.

GROUP - C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

a) If $f = | \vec{r} |$ where $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$,

prove that $\vec{\nabla} \left(\frac{1}{r} \right) = -\frac{\vec{r}}{r^3}$

b) Prove that

$$\begin{vmatrix} b^2 + c^2 & a^2 & a^2 \\ b^2 & c^2 + a^2 & b^2 \\ c^2 & c^2 & a^2 + b^2 \end{vmatrix} = 4a^2 b^2 c^2$$

c) If $y = \cos \left(m \sin^{-1} x \right)$ then prove that

$$\left(1 - x^2 \right) y_{n+2} - (2n + 1)\, x y_{n+1} + \left(m^2 - n^2 \right) y_n = 0.$$

$5 + 5 + 5$

a) If the vector function $\vec{F}$ and $\vec{G}$ are irrotational, prove that $\vec{F} \times \vec{G}$ is solenoidal.

b) If $f(x, y) = x^2 \tan^{-1} \left(\frac{y}{x} \right) - y^2 \tan^{-1} \left(\frac{x}{y} \right)$, verify that $f_{xy} = f_{yx}$.

c) Find the maxima and minima of the function $x^3 + y^3 - 3x + 12y + 20$. Also find the saddle point.

$5 + 5 + 5$

9\. a) Evaluate $\begin{vmatrix} 0 & a & b & c \\ -a & 0 & d & e \\ -b & -d & 0 & f \\ -c & -e & -f & 0 \end{vmatrix}$ by Laplace expansion method.

b) Verify Green's theorem for

$$\oint_C \left[\left(3x - 8y^2 \right) dx + \left(4y - 6x \right) dy \right]$$ where C region bounded by $x = 0$, $y = 0$ and $x + y = 1$.

c) For what values of λ and μ the system of equations

$x + y + z = 6$

$x + 2y + 3z = 10$, has (i) Unique solution, (ii) solution, (iii) Infinite solutions. $x + 2y + \lambda z = \mu$

5 + 5

10\. a) If $u_n = \int_0^{\frac{\pi}{4}} \tan^n \theta \, d\theta$, then prove that

$$n \left(u_{n+1} + u_{n-1} \right) = 1.$$

b) Prove that

(if $0 < a < b$), $\frac{(b - a)}{\left(1 + b^2\right)} < \tan^{-1} b - \tan^{-1} a < \frac{(b - a)}{\left(1 + a^2\right)}$

Hence show that $\frac{\pi}{4} + \frac{3}{25} < \tan^{-1} \frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$.

c) Test the convergence of the series

$$\frac{6}{1.3.5} + \frac{8}{3.5.7} + \frac{10}{5.7.9} + \ldots$$ 5 + 5

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11\. a) State Leibnitz's theorem for convergence of a series. Hence test the convergence of the following series :

$$1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{5^2} - \frac{1}{6^2} + \ldots\ldots$$

b) If $z = f(x, y)$ where $x = e^u \cos v$ and $y = e^u \sin v$, show that $y \frac{\partial z}{\partial u} + x \frac{\partial z}{\partial v} = e^{2u} \frac{\partial z}{\partial y}$.

c) Evaluate

$$\int_a^0 \int_0^x \int_0^{x+y} e^{x+y+z} \, dx \, dy \, dz.$$ 5 + 5 + 5

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