



Name :

Roll No. :

Invigilator's Signature :

CS/B.TECH(CE-OLD)/SEM-3/CE-301/2012-13

2012

MATHEMATICS

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

GROUP - A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :

$$10 \times 1 = 10$$

i) Find out the A.M. of the following data set :

5, 25, 36, 74, 45, 60, 52.

- | | |
|----------|-------------------|
| a) 51 | b) 42.43 |
| c) 46.75 | d) none of these. |

ii) Standard deviation

- | | |
|---------------------------|---------------------------|
| a) varies between 0 to 1 | b) is a positive quantity |
| c) is a negative quantity | d) none of these. |

iii) Probability of an event

- | | |
|----------------------|-------------------------|
| a) can be any number | b) lies between 0 and 1 |
| c) can be negative | d) none of these. |

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[Turn over



- iv) Normal curve is
- bell shaped
 - positively sloped
 - negatively sloped
 - none of these.
- v) The function $x.f(x) \cdot \cos(x)$ in the interval $[-\pi, \pi]$, where $f(x)$ is an even function, is
- even
 - odd
 - neither even nor odd
 - both even and odd.
- vi) If A and B are independent events, $P(B) = 0.14$ and $P(A|B) = 0.24$, then the value of $P(A)$ is
- 0.14
 - 0.0336
 - 0.38
 - 0.24.
- vii) If X is normally distributed with zero mean and unit variance, then the expectation of X^2 is
- 0
 - x^2
 - 1
 - none of these.
- viii) If $F(s)$ is the Fourier transform of $f(t)$, then the Fourier transform of $f(t) \cdot \cos(\omega t)$ is
- $F(s - \omega) + F(s + \omega)$
 - $\frac{1}{2} [F(s - \omega) + F(s + \omega)]$
 - $F(s - \omega) - F(s + \omega)$
 - none of these.
- ix) D'Alembert's solution of the wave equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ is
- $f(x + ct) + f(x - ct)$
 - $\frac{1}{2} [f(x + ct) + f(x - ct)]$
 - $f(x + ct) - f(x - ct)$
 - $\frac{1}{2} [f(x + ct) - f(x - ct)],$



- x) The solution of the partial differential equation (notations have their usual meanings)

$$z = px + qy + f(p, q) \text{ is}$$

- a) $z = ax + by + f(a, b)$
- b) $z = a + b + f(a, b)$
- c) $z = f(a, b)$
- d) none of these

where a and b are real constants.

- xi) The period of the function

$$f(x) = \sin(x) + \frac{1}{2} \sin(2x) + \frac{1}{3} \sin(3x) + \frac{1}{4} \sin(4x)$$

is

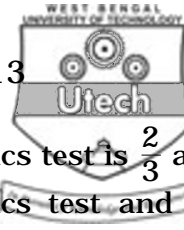
- a) π
- b) $\frac{2\pi}{3}$
- c) 2π
- d) none of these.

GROUP - B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Eliminate the arbitrary functions f and g to find the partial differential equation for
 $z = f(x - at) + g(x + at)$, a being a constant.
3. Expand the function $f(x) = x \cdot \sin(x)$ as a Fourier series in $[-\pi, \pi]$.
4. If the mean of a binomial distribution is 3 and the variance is $\frac{3}{2}$, find the probability of obtaining at most 3 successes.
5. Using the method of separation of variables, solve $\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u$
 where $u(x, 0) = 6e^{-3x}$.
6. If X has Binomial distribution with parameter n and p , then find out its mean and variance.



7. The probability that a student passes a Physics test is $\frac{2}{3}$ and the probability that he passes both a Physics test and an English test is $\frac{14}{45}$. The probability that he passes at least one test is $\frac{4}{5}$. What is the probability that he passes the English test ?

GROUP - C

(Long Answer Type Questions)

Answer any *three* of the following questions.

$$3 \times 15 = 45$$

8. a) Find the Fourier series for the function $f(x) = x + x^2$ in the interval $-\pi < x < \pi$. Hence show that
- $$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots = \frac{\pi^2}{12}.$$
- b) Show that $\text{Var}(aX + b) = a^2 \text{Var}(X)$, where a and b are real constants. 10 + 5
9. A tight string of length 1 has its ends $x = 0$ and $x = 1$ fixed. The midpoint is taken to a small height h and released from rest at time $t = 0$. Find the displacement function $y(x, t)$.
10. Obtain the solution of the one dimensional heat equation assuming that the ends $x = 0$ and $x = 1$ of the bar are kept at the temperature zero and the initial temperature be $f(x) = c$, c is a constant, $c > 0$.
11. a) From the Fourier series expansion of $f(x) = |x|$ in $-\pi \leq x \leq \pi$, prove that

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)^4} = \frac{\pi^4}{96}.$$

- b) Find the Fourier cosine transform of $f(x) = \frac{1}{1+x^2}$, $0 < x < +\infty$. 10 + 5