

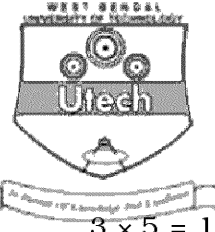




- iv) The sequence  $\{(-1)^n\}$  is
- a) convergent                      b) oscillatory
- c) divergent                      d) none of these.
- v) The point ( 0, 3, 0 ) lies on
- a)  $yz$  plane                      b)  $x$ -axis
- c)  $y$ -axis                      d)  $xz$  plane.
- vi) If  $y=5 \cdot x^{100} + 3$ , then  $y_{100}$  is
- a) 100 !                      b)  $5 \times 100$  !
- c) 0                      d) none of these.
- vii) The centre of the sphere  $x^2 + y^2 + z^2 - 6x - 4y - 6z + 16 = 0$  is
- a) ( 3, 3, 3 )                      b) ( 3, 2, 3 )
- c) ( - 3, 2, - 3 )                      d) ( - 3, - 2, 3 ).
- viii) If  $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ ,  $\vec{b} = \hat{i} - \hat{j}$ , then (  $a, b$  ) .  $a$  is equal to
- a) 1                      b) - 1
- c) 3                      d) - 3.
- ix) If a function  $f ( x, y )$  has critical point at ( 2, 2 ), then  $f_x ( 2, 2 )$  is equal to
- a) 2                      b) 0
- c) any non-zero value                      d) none of these.

$(x, y)$  is equal to

- xv) If  $y = \tan^{-1}\left(\frac{\cos x}{1 + \sin x}\right)$ , then  $y_4$  is equal to



**GROUP – B**  
**( Short Answer Type Questions )**

Answer any *three* of the following.

$$3 \times 5 = 15$$

2. Test the convergence of the following series ( Mention the tests you are using ) :

$$2x + \frac{3x^2}{8} + \frac{4x^3}{27} + \dots \infty, \text{ for } x > 0$$

3. If  $y_n = \frac{d^n}{dx^n}(x^n \log_e x)$ , show that

$$y_n = ny_{n-1} + (n-1)!.$$

4. If  $f(x) = \tan^{-1} \frac{x^2 + y^2}{x - y}$ , prove that by using

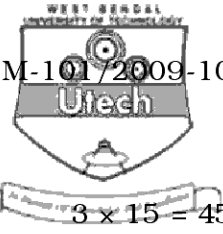
$$\text{Euler's Theorem } x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = \frac{1}{2} \sin 2f.$$

5. Prove that  $\sin 46^\circ \approx \frac{\sqrt{2}}{2} \left( 1 + \frac{\pi}{180} \right)$ . Is the estimate high or low ?

6. Obtain the reduction formula of  $I_n = \int_0^{\frac{\pi}{2}} \sin^m x \, dx$ .

$$\text{Hence evaluate } \int_0^1 \frac{(1-x)^{\frac{5}{2}}}{\sqrt{x}} \, dx.$$

7. In what direction from the point ( 1, 1, -1 ) is the directional derivative of  $f = x^2 - 2y^2 + 4z^2$  a maximum ? What is the magnitude of this directional derivative ?



**GROUP – C**  
**( Long Answer Type Questions )**

Answer any *three* of the following.

3 × 15 = 45

8. a) State Ratio Test for convergence of an infinite series of positive terms. Test the convergence of any one of the following series.

i) 
$$\sum_{n=1}^{\infty} \frac{n! 2^n}{n^n}$$

ii) 
$$\sum_{n=1}^{\infty} \frac{n^2 - 1}{n^2 + 1} x^n$$

- b) If  $y = \sin ( m \sin^{-1} x )$ , then show that

i) 
$$(1 - x^2) y_2 - x y_1 + m_2 y = 0$$

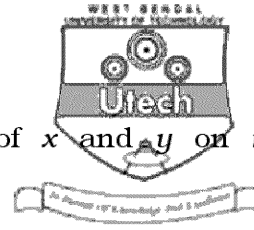
ii) 
$$(1 - x^2) y_{n+2} - (2n + 1) x y_{n+1} + m^2 - n^2 = 0.$$

Also obtain  $y_n(0)$ .

- c) Expand  $a^x$  in an infinite series by using Maclaurin's Theorem, mentioning the conditions required.

9. a) State Lagrange's mean value theorem. Prove that

$$\frac{x}{1+x} < \log(1+x) < x, x > 0$$



- b) If  $u$  be an homogeneous function of  $x$  and  $y$  on  $n$  dimensions, then prove that

$$x^2 \frac{\partial^2 u}{\partial u^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = n(n-1)u.$$

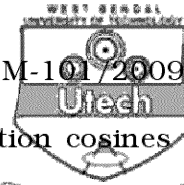
- c) Find the extrema of the function  $f(xy) = xy + a^3 \left( \frac{1}{x} + \frac{1}{y} \right)$ .

10. a) Evaluate :  $\iint_R xy(x+y) dx dy$ , where  $R$  is the region enclosed by the curves  $y = x^2$  and  $y = x$ .

- b) Obtain the reduction formula for  $\int_0^{\frac{\pi}{4}} \tan^n x dx$ . Hence  $\int_0^{\frac{\pi}{4}} \tan^7 x dx$ .

- c) The smaller of the two arcs into which the parabola  $y^2 = 8ax$  divides the circle  $x^2 + y^2 = 9a^2$  is rotate  $x$ -axis. Show that the volume of the solid generated is  $\frac{28}{3} \pi a^3$ .

11. a) Find the equation of the plane through the intersection of the planes  $x + 2y + 3z = 4$  and  $2x + y + z = 5$  is perpendicular to the plane  $5x + 3y + 6z + 8 = 0$ .



- b) Show that the pair of lines whose direction cosines are given by  $\frac{1}{l} + \frac{1}{m} + \frac{1}{n} = 0$  and  $2l - m + 2n = 0$  are

perpendicular.

- c) Show that the straight lines  $\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5}$  and  $\frac{x-8}{7} = \frac{y-4}{1} = \frac{z-5}{3}$  are coplanar. Find the point of

intersection of the two lines and also find the equation of the plane on which they lie.

12. a) Show that normal to the surface

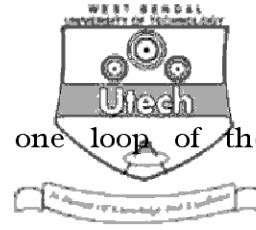
$$\Phi(x, y, z): 2xz^2 - 3xy - 4x = 7 \text{ at } (1, -1, 2) \text{ is } 7\hat{i} - 3\hat{j} + 8\hat{k}.$$

- b) Show that  $\vec{V} = (x + 2y + 4z)\hat{i} + (2x - 3y - z)\hat{j} + (4x + 2z - y)\hat{k}$  is irrotational. Obtain a scalar function  $\Phi(x, y, z)$ , such that  $\vec{V} = \vec{\nabla}\Phi$ .

- c) Verify Green's theorem for  $\oint_C \vec{F} \cdot d\vec{r}$ , where  $\vec{F} = x^2 y\hat{i} + x^2 \hat{j}$

and  $C$  is the boundary described in counter-clockwise direction of the triangle with vertices  $(0, 0)$ ,  $(1, 0)$ ,  $(1, 1)$ .

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13. a) Find the centroid of the area of one loop of the lemniscate  $r^2 = a^2 \cos 2\theta$ .

b) Evaluate  $\iiint z^2 dx dy dz$  over the region defined by  $z \geq 0, x^2 + y^2 + z^2 \leq a^2$ , using suitable transformation.

c) Is the following series absolutely convergent ? (any one)

i) 
$$\sum u_n = \sum (-1)^{n-1} \left( \frac{n^n}{n^{n+1}} - \frac{n+1}{n} \right)^{-n}$$

ii) 
$$\frac{1}{2} - \frac{2}{5} + \frac{3}{10} - \frac{4}{17} + \frac{5}{26} - \dots \infty.$$

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