



**MAULANA ABUL KALAM AZAD UNIVERSITY OF  
TECHNOLOGY, WEST BENGAL**

Paper Code : M-201

**MATHEMATICS-II**

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own  
words as far as practicable.

**GROUP - A**

**( Multiple Choice Type Questions )**

1. Choose the correct alternatives for any ten of the  
following : 10 × 1 = 10

i) The differential equation

$M(x, y) dx + N(x, y) dy = 0$  is exact if

- a)  $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$  ~~b)  $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$~~   
c)  $\frac{\partial^2 M}{\partial y^2} = \frac{\partial^2 N}{\partial x^2}$  d)  $\frac{\partial^2 M}{\partial x^2} = \frac{\partial^2 N}{\partial y^2}$

ii) The integrating factor of the differential equation  
 $\frac{dy}{dx} + 5y = \cos 2x$  is (6)

- ~~a)  $e^{5x}$~~  b)  $e^{-5x}$   
c)  $e^{2x}$  d)  $e^{-2x}$

iii) The general solution of  $p = \log(px - y)$ , where  
 $p = \frac{dy}{dx}$ , is

- a)  $y = cx - c$  ~~b)  $y = cx - e^c$~~   
c)  $y = c^2 x - e^{-c}$  d) none of these.

iv) The value of  $\frac{\Gamma(\frac{5}{2})}{\Gamma(\frac{3}{2})}$  is equal to

- a)  $5/2$  b)  $5/4$   
c)  $3/4$  ~~d)  $3/2$~~

v) The value of the integral  $\int_0^\infty e^{-x^2} dx$  is

- a) 1 b)  $\sqrt{\frac{\pi}{2}}$   
c)  $\frac{\pi}{3}$  d)  $\sqrt{\pi}$

vi) The value of  $B(\frac{5}{2}, 2)$  is equal to

- a)  $3/35$  b)  $4/35$   
~~c)  $6/35$~~  d) none of these.

vii)  $L\{t^4\}$  is equal to

- a)  $\frac{4}{s^5}$                       b)  $\frac{5}{s^5}$   
 c)  $\frac{24}{s^4}$                       d)  $\frac{24}{s^5}$

viii)  $L^{-1}\left\{\frac{1}{(s+1)^5}\right\}$  is equal to

- a)  $\frac{t^5 e^{-t}}{5!}$                       b)  $\frac{t^4 e^{-t}}{4!}$   
 c)  $\frac{t^4 e^{-t}}{4!}$                       d)  $\frac{t^5 e^{-t}}{5!}$

ix) A simple graph with 6 vertices and 3 components has at least ..... edges.

- a) 2                      b) 3  
 c) 4                      d) 9

x) Adjacency matrix of a graph is

- a) singular                      ~~b) symmetric~~  
 c) skew-symmetric                      d) none of these.

xi) The number of edges in a tree with  $n$  vertices is

- a)  $n$                       ~~b)  $n-1$~~   
 c)  $n+1$                       d) 0.

xii) Tree is a connected graph without any

- a) odd vertex                      b) even vertex  
 c) pendant vertex                      ~~d) circuit.~~

### GROUP - B

#### ( Short Answer Type Questions )

Answer any *three* of the following.  $3 \times 5 = 15$

- Solve :  $\frac{d^2 y}{dx^2} - 9y = e^{3x} \cos x$ .
- Use Laplace transformation to solve  $\frac{d^2 y}{dt^2} + 9y = 1$ , where  $y(0) = 1$  and  $y\left(\frac{\pi}{2}\right) = -1$ .
- Examine the convergence of the improper integral  $\int_0^1 \frac{dx}{\sqrt{x(1-x)}}$ .
- Prove that in a simple graph with  $n$  number of vertices and  $m$  number of components can have maximum  $\frac{1}{2}(n-m)(n-m+1)$  number of edges.
- ~~Prove that the number of vertices in a binary tree is always odd.~~

### GROUP - C

#### ( Long Answer Type Questions )

Answer any *three* of the following.  $3 \times 15 = 45$

- a) Find the Laplace Transform of the following function :

$$f(t) = \begin{cases} t, & 0 < t < c \\ 2c - t, & c < t < 2c \end{cases} \text{ having period } 2c. \quad 8$$

( Turn over

b) Show that

$$\int_a^b (x-a)^{m-1} (b-x)^{n-1} dx = (b-a)^{m+n-1} \beta(m, n).$$

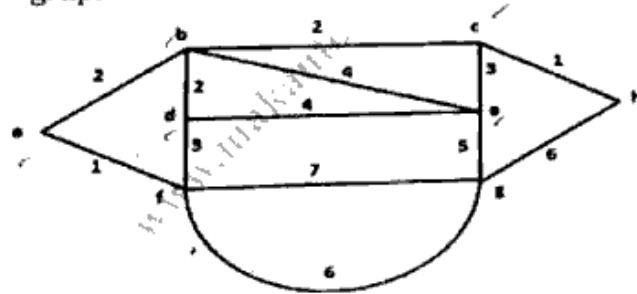
Using this result evaluate the value of the integral

$$\int_3^5 (x-3)^3 (5-x)^4 dx. \quad 5+2$$

8. a) Solve by the method of variation of parameters

$$\frac{d^2 y}{dx^2} + a^2 y = \sec ax.$$

b) Apply Dijkstra's algorithm to find shortest path between the vertices 'a' and 'h' in the following graph :



8+7

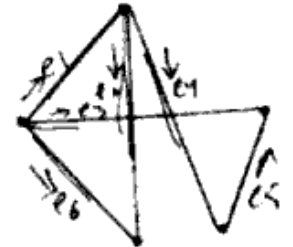
9. a) Draw the graph whose incidence matrix is :

$$\begin{matrix} & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{matrix} & \begin{bmatrix} -1 & -1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \end{matrix}$$

b) Show that :

$$\int_0^{\infty} e^{-x^4} x^2 dx \times \int_0^{\infty} e^{-x^4} dx = \frac{\pi}{8\sqrt{2}}.$$

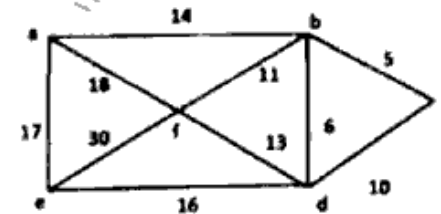
c) Show whether the following two graphs are isomorphic or not :



5+5+5

10. a) Show that the minimum number of edges in a connected simple graph with  $n$  vertices is  $n-1$ .

b) Apply Prim's algorithm to find the shortest spanning tree of the following graph and find the corresponding minimum weight.



c) Apply convolution theorem of Laplace transformation to evaluate  $L^{-1} \left\{ \frac{p}{(p^2 + a^2)^2} \right\}$ .

5+5+5

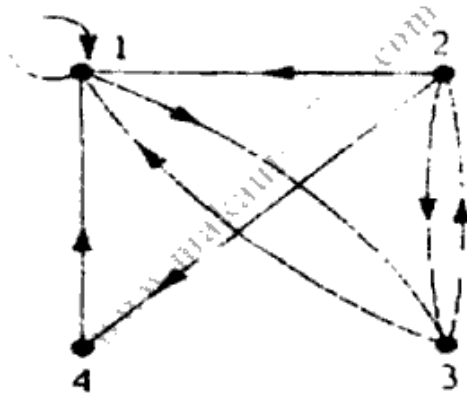
11. a) Find the general and singular solution of  
 $y = px + \sqrt{a^2 p^2 + b^2}$ ,  $p = \frac{dy}{dx}$ .

b) Solve the following simultaneous equations :

$$\frac{dx}{dt} + 3x + y = e^t$$

$$\frac{dy}{dt} - x + y = e^{2t}$$

c) Find the adjacency and incidence matrix of the following graph :



5 + 5 + 5

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