Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/B.Tech/(EE-NEW)/SEM-6/EE-605A/2013

2013

DIGITAL SIGNAL PROCESSING

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : 10 × 1 = 10

i) The sampling frequency of the following analog signal

$x(t) = 3 \sin(150 \pi t) + 5 \cos(200 \pi t)$ should be

a) greater than or equal to 150 Hz

b) greater than or equal to 200 Hz

c) greater than or equal to 600 Hz

d) less than or equal to 200 Hz.

ii) The magnitude response $\left| H\left(e^{j\omega} \right) \right|$ of digital filter is a

a) periodic and odd function of ω

b) periodic and even function of ω

iii) A discrete-time system is said to be causal if and only if its impulse response $h[n]$ is

a) zero for $n < 0$ b) zero for $n > 0$

c) zero at $n = 0$ d) zero at $n \neq 0$.

iv) Fourier transform of $x[n]$ is $x(\omega)$; then the Fourier transform of $n\,x[n]$ is

a) $-j\,\frac{d\times(\omega)}{d\omega}$ b) $\frac{d\times(\omega)}{d\omega}$

c) $j\,\frac{d\times(\omega)}{d\omega}$ d) $\omega\,.\,\frac{d\times(\omega)}{d\omega}$.

v) A signal is an energy signal if

a) $E < \infty,\ P = 0$ b) $E < \infty,\ P = \infty$

c) $P < \infty,\ E = \infty$ d) $E = \infty,\ P = 0$.

vi) The overall impulse response of a cascaded connection of two systems with impulse responses $h_1[n]$ and $h_2[n]$ is

a) $h_1[n] + h_2[n]$

b) $h_1[n]\,.\,h_2[n]$

c) $h_1[n] \ast h_2[n]$

d) $h_1[n] - h_2[n]$.

vii) If $x[n]$ and $h[n]$ are two finite length sequences with length 6 and 5 respectively then their convolution has length

a) 8 b) 9

c) 10 d) 11.

viii) The value of the twiddle factor W_{16}^{2} is given by

a) $0 + j$ b) $0{\cdot}707 - j\,0{\cdot}707$

c) $0 - j$ d) $-0{\cdot}707 + j\,0{\cdot}707$.

ix) If DFT $\{ x[n] \} = X(K)$, then for circularly shifted sequence DFT $\{ x(n-2)_N \}$ is

a) $X(K)\, e^{-j\,4\pi K/N}$ b) $X(K)\, e^{-j\,\pi K/N}$

c) $X(K)\, e^{j\,4\pi K/N}$ d) $X(K)\, e^{j\,\pi K/N}$.

x) The first three points of a 4-point DFT of a real valued sequence are $\{ 6, -2 + j\,2, -2 \}$. The remaining point in the DFT is

a) $2 - j\,2$ b) $2 + j\,2$

c) $6 - 2j$ d) $-2 - j\,2$.

xi) An increase in the length of the rectangular window function

a) increases the main lobe width and computational burden

b) increases the main lobe width and decreases computational burden

c) decreases the main lobe width and computational burden

d) decreases the main lobe width and increase computational burden.

xii) If $x[n] \leftrightarrow X(z)$ then,

a) $x[-n] \leftrightarrow X(-z)$

b) $x[-n] \leftrightarrow z\,X(z)$

c) $x[-n] \leftrightarrow X(z)/z$

d) $x[-n] \leftrightarrow X(1/z)$.

xiii) The best approximation of a derivative function dx/dt of a continuous system, while discretising is made as

a) $(x_n - x_{n-1})/T$

b) $\frac{x_n + x_{n-1}}{T}$

c) $(x_{n+1} - x_n)/T$

d) $\frac{x_{n+1} - x_{n-1}}{2T}$.

GROUP – B
(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. The step response of an LTI system is

   $s[n] = \left(\frac{1}{z}\right)^{n-2} u[n-2]$. Find the impulse response of the system.

3. Consider the following two sequences to show how their ROCs are different although their z-transform are same.

   $x_1[n] = a^n u[n]$.

   $x_2[n] = -a^n u[-n-1]$.

4. For the following system determine whether they are linear, causal and time-invariant.

   a) $y[n] = e^{x[n]}$

   b) $y[n] = x[n] + 3x[n+1]$. $2 \times 2\frac{1}{2}$

5. Determine the periods of the sequences

   a) $x_1[n] = 2\cos(0{\cdot}15\,\pi n) + 5\sin(0{\cdot}25\,\pi n + \pi/3)$

   b) $x_2[n] = e^{-j\,0{\cdot}2\,\pi n} + e^{0{\cdot}7\,\pi n}$. $2 \times 2\frac{1}{2}$

6. Obtain a Cascade realisation of the following system :

   $y[n] = 0{\cdot}8\,y[n-1] + 0{\cdot}12\,y[n-2]$

   $= x[n] + x[n-1] - 2x[n-2]$.

7. Given $x[n] = (1 + n/5)$, $-5 \le n \le -1$

$= 1$ $0 \le n \le 5$

$= 0$ otherwise.

a) Sketch the function $x_1[n] = n\,x[n]$

b) Show the even and odd part of $x_1[n]$. 3 + 1 + 1

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. 3 × 15 = 45

8. a) What do you understand by BIBO stable system ? Show that an LTI sysgtem is said to be BIBO stable if its impulse response $h[n]$ is absolutely summable

$$i.e., \quad \sum_{n=-\infty}^{\infty} |h[n]| < \infty$$

b) Determine the impulse response of the system given by

$y[n] = 0{\cdot}6\,y[n-1] - 0{\cdot}8\,y[n-2] + x[n]$.

c) Use *Z*-transform to perform convolution of the two sequences

$x[n] = \delta[n] - \delta[n-2]$

$y[n] = 2\delta[n] - 2\delta[n-1] + \delta[n-3]$ 5 + 5 + 5

9. a) Show the outputs of a basic butterfly graph of a radix-2 DIF - FFT for inputs x and y.

b) Find the (outputs) 8 pt DFT of the following sequence using Radix-2 DIF - FFT algorithm after padding with necessary zero.

$$x[n] = \sum_{k=0}^{4} (k-2)\,\delta[n-k].$$

c) Show how the time complexity of finding the DFT of 256 point data sequence improves by using Radix-2 FFT algorithm instead of using direct computation.

10. a) How is the overlap save method useful in the convolution of sequences ?

b) Show that time domain convolution is equivalent to Z-domain multiplication of sequences

c) $x[n] = \{2, -1, 3, 4, 1\}$

$h[n] = \{1, 2, -1\}$.

Find $x[n] \ast h[n]$ using Z-transform.

11. a) Design a Butterworth digital filter using a bilinear transformation for the following spencification :

$$0{\cdot}9 \leq \left| H\left(e^{j\omega}\right) \right| \leq 1 \qquad 0 \leq \omega \leq \frac{\pi}{4}$$

$$\left| H\left(e^{j\omega}\right) \right| \leq 0{\cdot}2 \qquad \frac{3\pi}{4} \leq \omega \leq \pi$$

Sampling time $T = 1$ sec.

b) Discuss how frequency warping effect is taken into account in the design of an IIR filter using bilinear transformation. 10 + 5

12. a) Design a linear phase FIR filter, approximating the ideal frequency response

$$Hd\left(e^{j\omega}\right) = e^{-j\propto\omega} \quad \text{for } |\omega| \leq \frac{\pi}{6}$$

$$= 0 \qquad \frac{\pi}{6} \leq |\omega| \leq \pi$$

Determine the filter co-efficient for $N = 13$, with a Bartlett window.

b) Show a realisation of the filter. 12 + 3

13. Write short notes on any *three* of the following : 3 × 5

a) Impulse in variant method for design of IIR filter

b) Gibbs phenomenon and its effect

c) Aliasing and sampling rate in signal processing

d) Quantisation and its effect on digital filters.