ENGINEERING & MANAGEMENT EXAMINATIONS, DECEMBER – 2007

MATHEMATICS

SEMESTER – 1

Time : 3 Hours] [Full Marks : 70

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) $\displaystyle\lim_{x \to \frac{\pi}{2}} (1 - \sin x) \tan x =$

a) 1 b) 0

c) $\frac{1}{2}$ d) e.

ii) $\frac{dy}{dx}$ of $y = \sin^{-1} x + \sin^{-1} \sqrt{(1 - x^2)}$ is

a) 0 b) $\frac{1}{2}$

c) $\frac{x}{\sqrt{1 - x^2}}$ d) none of these.

iii) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{2n + 1}$ is

a) convergent

b) divergent

c) neither convergent nor divergent

d) none of these.

iv) If $u + v = x$, $uv = y$ then $\frac{\partial(x, y)}{\partial(u, v)} =$

a) $u - v$ b) uv

c) $u + v$ d) $\frac{u}{v}$.

v) A normal vector to the plane $x + 2y + 3z - 1 = 0$ is

a) $2\hat{i} + \hat{j} + 3\hat{k}$
b) $\hat{i} + 2\hat{j} + 3\hat{k}$
c) $\hat{i} - 2\hat{j} - 3\hat{k}$
d) $\hat{i} + 2\hat{j} - 3\hat{k}$.

vi) The value of x which makes the vector $x\hat{i} + 2\hat{j} + 8\hat{k}$ and $2\hat{i} + 3\hat{j} - \hat{k}$ perpendicular is

a) 0
b) -1
c) 1
d) 2.

vii) If $x = r\cos\theta$, $y = r\sin\theta$, then $\dfrac{\partial(x, y)}{\partial(r, \theta)}$ is

a) $-\dfrac{1}{r}$
b) $-r$
c) $+r$
d) $\dfrac{1}{r}$.

viii) The n^{th} derivative of $(ax + b)^{10}$, when $n > 10$ is

a) a^{10}
b) $\lfloor 10 \, a^{10}$
c) 0
d) $\lfloor 10$.

ix) The reduction formula of $I_n = \int_0^{\pi/2} \cos^n x \, dx$ is

a) $I_n = \left(\dfrac{n-1}{n}\right) I_{n-1}$
b) $I_n = \left(\dfrac{n}{n-1}\right) I_n$
c) $I_n = \left(\dfrac{n-1}{n}\right) I_{n-2}$
d) none of these.

x) The equation of the straight line passing through (1, 1, 1) and (2, 2, 2) is $(x-1) = (y-1) = (z-2)$

a) True
b) False.

xi) If $f(x, y) = x^3 + 3xy^2 + y^3 + x^2$, then $x\dfrac{df}{\partial x} + y\dfrac{df}{\partial y} = 3f$

a) True
b) False.

xii) $\int_a^b \left[\int_c^d f(x, y)\, dx \right] dy = \int_c^d \left[\int_a^b f(x, y)\, dy \right] dx$ is always true.

a) True b) False.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following : $3 \times 5 = 15$

2. For what values of x the following series is convergent ?

$$\frac{x}{1.3} + \frac{x^2}{3.5} + \frac{x^3}{5.7} + \ldots\ldots\ldots\ldots .$$

3. Find the moment of inertia of a thin uniform rectangular lamina of adjacent sides of lengths $2a$ and $2b$ and of mass M about an axis of symmetry through its centre.

4. Evaluate $\int_C (3xy\, dx - y^2\, dy)$ where C is the arc of the parabola $y = 2x^2$ from $(0, 0)$ to $(1, 2)$.

5. If $y_n = \frac{d^n}{dx^n} \{ x^n \log_e x \}$, show that $y_n = ny_{n-1} + \underline{|n-1}$

6. If $y = a \cos(\log x) + b \sin(\log x)$, prove that

$$x^2 y_{n+2} + (2n+1) xy_{n+1} + (n^2+1) y_n = 0.$$

7. If $f(x) = \sin^{-1} x$, $0 < a < b < 1$, use mean value theorem to prove that

$$\frac{(b-a)}{\sqrt{1-a^2}} < \sin^{-1} b - \sin^{-1} a < \frac{(b-a)}{\sqrt{1-b^2}}.$$

GROUP – C

(Long Answer Type Questions)

Answer any *three* questions. $3 \times 15 = 45$

8. a) Examine continuity and differentiability of $f(x)$ at $x = 0$ where $f(x) = x \cos \frac{1}{x}$, $x \neq 0$ and $f(0) = 0$.

b) If $u = xf\left(\frac{y}{x}\right) + g\left(\frac{y}{x}\right)$ then show that $xu_x + yu_y = xf\left(\frac{y}{x}\right)$ and

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 0.$$

c) Find the extrema of the following function :

$f(x, y) = x^3 + 3xy^2 - 3y^2 - 3x^2 + 4.$ 5 + 5 + 5

9. a) State d' Alembert's ratio test for convergence of an infinite series. Examine the convergence and divergence of the series $1 + \frac{x}{2} + \frac{x^2}{5} + \frac{x^3}{10} + \ldots \infty$.

b) Test the convergence of the following series (any *one*) :

i) $\sum_{n=1}^{\infty} \sqrt[3]{n^3 + 1} - n,$

ii) $\left(\frac{1}{3}\right)^2 + \left(\frac{1.2}{3.5}\right)^2 + \left(\frac{1.2.3}{3.5.7}\right)^2 + \ldots\ldots\infty$.

c) State Leibnitz's test for convergence of an alternating series. Prove that the series $x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots\ldots\infty$ is absolutely convergent when $|x| < 1$ and conditionally convergent when $|x| = 1$. 5 + 5 + 5

10. a) Show that the equation of the plane through the points (1, 0, – 1), (3, 2, 2) and parallel to the straight line $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ is $3y - 2z = 2$.

b) Determine the value of k so that the straight lines $\frac{x-1}{2} = \frac{y}{1} = \frac{z-5}{2}$ and $\frac{x-2}{1} = \frac{y-8}{k} = \frac{z-11}{4}$ may intersect.

c) Find the vlolume of the solid bounded by the paraboloid of revolution $x^2 + y^2 = az$, the xy plane and $x^2 + y^2 = 2ax$. 5 + 5 + 5

11. a) Change the order of integration and hence evaluate

$$\int_0^1 \int_{e^x}^{e} \frac{dx\,dy}{y^2 \log y}.$$

b) Find the centroid of the area of the right loop of the Lemniscate $r^2 = a^2 \cos 2\theta$.

c) In what direction from the point (1, 1, − 1) is the directional derivative of $f = x^2 - 2y^2 + 4z^2$ a maximum ? What is the magnitude of this directional derivative ? 5 + 5 + 5

12. a) Verify Gauss divergence theorem for the vector field $\bar{F} = y\hat{i} + x\hat{j} + z^2\hat{k}$ over the cylindrical region bounded by $x^2 + y^2 = 9$, $z = 0$, $z = 2$.

b) Verify Stokes' theorem for $\vec{A} = (y - z + 2)\vec{i} + (yz + 4)\vec{j} - xz\vec{k}$ over the surface of the cube $x = y = z = 0$ and $x = y = z = 2$ above xy plane. 7 + 8

END