	Utech
Name :	
Roll No.:	An Agranus (y' Exercising 2 and Experient
Invigilator's Signature :	

MATHEMATICS

Time Allotted: 3 Hours Full Marks: 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any ten of the following : $10\times 1=10$

- i) The generators of the cyclic group (Z, +) are
 - a) 1, -1

b) 0, 1

c) 0, -1

- d) 2, -2.
- ii) The mapping $f : R \varnothing R$ given by $f(x) = |x|, x \square R$ is
 - a) Injective
- b) Surjective
- c) Bijective
- d) None of these.
- iii) Let S be a finite set of n distinct elements. The number of bijective mapping from S to S is
 - a) n^2

b) *n*!

c) 2^n

d) None of these.

4101 [Turn over



- If three Boolean variables x, y and z are defined on Boolean Algebra B, then which one of the following is a fundamental product?
 - a) $xy^{\prime}z$

- b) хy
- c) xy(x+y)
- d) none of these.
- If G is binary tree on n vertices, the G has edges v)
 - a) n(n-1)
- b) n-1

c) n

- d) $\frac{n(n-1)}{2}$.
- Solution of the recurrence relation $S_n = 2S_{n-1}$ with vi) $S_0 = 1 \text{ is } S_n =$
 - a)

b) 2^{n-1}

 2^{n+1} c)

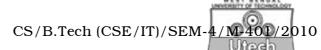
- d) none of these.
- vii) A complete graph is
 - regular a)
- connected simple b)

c) circuit

- d) planar graph.
- viii) On the set $A = \{1, 2, 3\}$, the relation $R = \{ (2, 1), (1, 2), (3, 3) \}$. Then R is
 - symmetric a)
- b) reflexive
- transitive
- d) not a relation at all.
- In the additive group $Z_{\,6}\,$ the order of the element [4] is ix)
 - a) 0

b) 2

c) 3 d) 6.



x)	Let G be a group and $a \ \square$ G. If o (a)	= 17, then
	o (a ⁸) is	A American Constitution

a) 17

b) 16

c) 8

- d) 5.
- xi) If S and T are two subgroup of a group G, then which of the following is a subgroup?
 - a) $S \cup T$

- b) $S \cap T$
- c) S Td

- G-S.
- xii) The dual of a planar graph is dual. It is
 - a) Trueb)

False.

- xiii) A binary tree should have at least
 - a) one vertex
- b) two vertices
- c) three vertices
- d) four vertices.
- xiv) A connected graph is Eulerian iff it has no vertex of odd degree. It is
 - a) Trueb)

False.

- xv) The number of idempotent element in Z is
 - a) 0

b) 1

c) 2

d) none of these.

GROUP - B

(Short Answer Type Questions)

Answer any *three* of the following.

 $3 \times 5 = 15$

- 2. Let $G = \{ (a, b) : a \neq 0, b \mid R \}$ and * be a binary compostion defined on G by (a, b) * (c, d) = (ac, bc + d). Show that (G, *) is a non Abelian group.
- 3. Show that for any two subgroups H and K of a group $G H \cap K$ is also a subgroup of G.



- Let G be a group, if a, b \square G such that $a^4 = e$, the 4. element of G and $ab = ba^2$, prove that a = e.
- Prove that every cyclic group is an Abelian group. 5.
- Show that the mapping $F:(Z, \bullet) \varnothing (R, \bullet)$ defined by 6. $f(x) = x^2 \ \forall x \mid Z$ is a monomorphism but not isomorphism.
- If in a ring R with unity, $(xy)^2 = x^2 y^2 \forall x, y \mid R$, then 7. show that R is a commutative.

GROUP - C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

Examine whether the following two graphs are 8. isomorphic.

Dia.

5

Draw the dual of the graph. b)

Dia.

5



c) Determine the adjacency matrix of the following di-graph :

Dia.

5

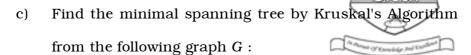
9. a) Construct a simple logic circuit which would satisfy the truth table.

х	у	f
1	1	1
0	1	1
1	0	0
0	0	1

5

b) Prove that a graph G has a spanning tree if and only if G is connected.5

4101 5 [Turn over



Dia.

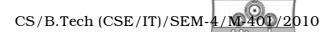
5

- 10. a) Consider the lattice $L = \{1, 2, 3, 4, 6, 12\}$, the divisors of 12 ordered by divisibility. Find the lower and upper bound of L. Is L a complemented lattice?
 - b) For any Boolean Algebra, show that.

$$(xy' + xz') + x' = (x' + y + z)(x' + y + z')(x' + y' + z').$$

5

c) Using generating function solve the recurrence relation, $a_n - 7a_{n-1} + 10a_{n-2} = 0, \text{ for } n > 1 \text{ and } a_0 = 3,$ $a_1 = 3.$



- 11. a) Prove that the number of vertices in a binary tree is always odd.
 - b) Find the truth table of the Boolean function

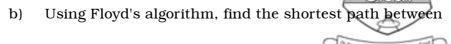
$$f = z'xy + xy' + y. 5$$

- c) Prove that a complete graph with n vertices consist of $\frac{n(n-1)}{2}$ number of edges. 5
- 12. a) Prove that the identity elements and the inverse of an element in a group is unique. 5
 - b) Prove that in a group (G, *), (a * b) $^{-1}$ = b^{-1} * a^{-1} .
 - c) Prove that the set of matrices

$$H = \left\{ \begin{pmatrix} x & 0 \\ 0 & x \end{pmatrix} : x \, \Box \, R, \, x \neq 0 \right\} \text{ forms a normal}$$
 subgroup of GL (2 , R), the group of all real non-singular $2 \infty 2$ matrices under multiplication.

13. a) Using Ford-Fulkerson's algorithm, find the maximum flow in the following network :

Dia.



- i) w_2 and w_6
- ii) w_1 and w_6 .

Dia.

7