Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS / B.TECH(CE-OLD) / SEM-3 / CE-301 / 2012-13

2012

MATHEMATICS

Time Allotted : 3 Hours *Full Marks :* 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) Find out the A.M. of the following data set :

5, 25, 36, 74, 45, 60, 52.

a) 51 b) 42·43

c) 46·75 d) none of these.

ii) Standard deviation

a) varies between 0 to 1 b) is a positive quantity

c) is a negative quantity d) none of these.

iii) Probability of an event

a) can be any number b) lies between 0 and 1

c) can be negative d) none of these.

iv) Normal curve is

a) bell shaped b) positively sloped

c) negatively sloped d) none of these.

v) The function $x.f(x) . \cos(x)$ in the interval $[-\pi, \pi]$, where $f(x)$ is an even function, is

a) even b) odd

c) neither even nor odd d) both even and odd.

vi) If A and B are independent events, $P(B) = 0{\cdot}14$ and $P(A \mid B) = 0{\cdot}24$, then the value of $P(A)$ is

a) 0·14 b) 0·0336

c) 0·38 d) 0·24.

vii) If X is normally distributed with zero mean and unit variance, then the expectation of X^2 is

a) 0 b) x^2

c) 1 d) none of these.

viii) If $F(s)$ is the Fourier transform of $f(t)$, then the Fourier transform of $f(t) . \cos(\omega t)$ is

a) $F(s-\omega) + F(s+\omega)$

b) $\frac{1}{2}[F(s-\omega) + F(s+\omega)]$

c) $F(s-\omega) - F(s+\omega)$

d) none of these.

ix) D'Alembert's solution of the wave equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ is

a) $f(x+ct) + f(x-ct)$

b) $\frac{1}{2}[f(x+ct) + f(x-ct)]$

c) $f(x+ct) - f(x-ct)$

d) $\frac{1}{2}[f(x+ct) - f(x-ct)]$,

x) The solution of the partial differential equation (notations have their usual meanings)

$z = px + qy + f(p, q)$ is

a) $z = ax + by + f(a, b)$

b) $z = a + b + f(a, b)$

c) $z = f(a, b)$

d) none of these

where a and b are real constants.

xi) The period of the function

$f(x) = \sin(x) + \frac{1}{2}\sin(2x) + \frac{1}{3}\sin(3x) + \frac{1}{4}\sin(4x)$

is

a) π b) $\frac{2\pi}{3}$

c) 2π d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Eliminate the arbitrary functions f and g to find the partial differential equation for

$z = f(x - at) + g(x + at)$, a being a constant.

3. Expand the function $f(x) = x \cdot \sin(x)$ as a Fourier series in $[-\pi, \pi]$.

4. If the mean of a binomial distribution is 3 and the variance is $\frac{3}{2}$, find the probability of obtaining at most 3 successes.

5. Using the method of separation of variables, solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ where $u(x, 0) = 6e^{-3x}$.

6. If X has Binomial distribution with parameter n and p, then find out its mean and variance.

7. The probability that a student passes a Physics test is $\frac{2}{3}$ and the probability that he passes both a Physics test and an English test is $\frac{14}{45}$. The probability that he passes at least one test is $\frac{4}{5}$. What is the probability that he passes the English test ?

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following questions.

$3 \times 15 = 45$

8. a) Find the Fourier series for the function $f(x) = x + x^2$ in the interval $-\pi < x < \pi$. Hence show that

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots = \frac{\pi^2}{12}.$$

b) Show that $Var(aX + b) = a^2 \, Var(X)$, where a and b are real constants. 10 + 5

9. A tight string of length 1 has its ends $x = 0$ and $x = 1$ fixed. The midpoint is taken to a small height h and released from rest at time $t = 0$. Find the displacement function $y(x, t)$.

10. Obtain the solution of the one dimensional heat equation assuming that the ends $x = 0$ and $x = 1$ of the bar are kept at the temperature zero and the initial temperature be $f(x) = c$, c is a constant, $c > 0$.

11. a) From the Fourier series expansion of $f(x) = |x|$ in $-\pi \le x \le \pi$, prove that

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)^4} = \frac{\pi^4}{96}.$$

b) Find the Fourier cosine transform of $f(x) = \frac{1}{1+x^2}$, $0 < x < +\infty$. 10 + 5