

Name :

Roll No. :

Invigilator's Signature :

**CS/B.TECH(BME(N)/ECE(N)/EE(N)/EEE(N)/EIE(N)/
ICE(N)/PWE(N))/SEM-3/M(CS)-301/2011-12
2011**

NUMERICAL METHODS

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :

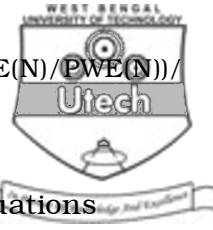
$$10 \times 1 = 10$$

i) Which of the following is not a computational error ?

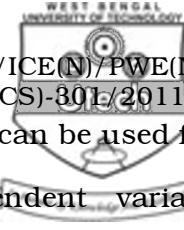
- | | |
|---------------------|--------------------|
| a) Truncation error | b) Round-off error |
| c) Inherent error | d) None of these. |

ii) Newton-Raphson method fails when

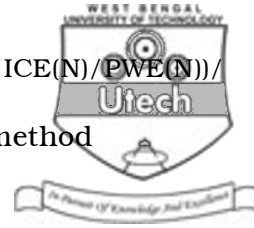
- | | |
|-----------------|-------------------|
| a) $f'(x) = 1$ | b) $f'(x) = 0$ |
| c) $f'(x) = -1$ | d) $f''(x) = 0$. |



- iii) Finite difference method is used to solve
- a system of linear simultaneous equations
 - a system of non-linear simultaneous equations
 - partial differential equations
 - non-linear equations.
- iv) Regula-falsi method has a convergence rate of the order of
- 2
 - 1.62
 - 1
 - none of these.
- v) Gauss-Seidel method for solution of a system of linear simultaneous equations converges if
- $|a_{ii}| \geq \sum_{\substack{j=1 \\ j \neq i}}^n |a_{ij}|$
 - $|a_{ii}| > \sum_{\substack{j=1 \\ j \neq i}}^n |a_{ij}|$
 - $|a_{ii}| / |a_{nn}| = 1$
 - none of these.
- vi) Modified Euler's method has a truncation error of the order of
- h
 - h^2
 - h^4
 - h^3 .



- vii) Divided difference interpolation formula can be used for
- the tabular values with independent variable unequally spaced
 - inverse interpolation
 - both (a) and (b)
 - none of these.
- viii) Truncation error in Simpson's $\frac{1}{3}$ rd rule is given by
- $\frac{b-a}{180} h^4 f^{iv}(\xi)$, $a \leq \xi \leq b$
 - $\frac{b-a}{90} h^5 f^{iv}(\xi)$, $a \leq \xi \leq b$
 - $\frac{b-a}{6} h^4 f'''(\xi)$, $a \leq \xi \leq b$
 - $\frac{b-a}{90} h^4 f^{iv}(\xi)$, $a \leq \xi \leq b$.
- ix) Which of the following relations is true ?
- $E = 1 - \Delta$, $\Delta - \square = \Delta \square$
 - $E = 1 - \Delta$, $\Delta + \square = \Delta \square$
 - $E = 1 + \Delta$, $\Delta + \square = \Delta \square$
 - $E = 1 + \Delta$, $\Delta - \square = \Delta \square$.
- x) Trapezoidal method can be used to integrate numerically a function represented in tabular form
- with odd number of intervals only
 - with even number of intervals only
 - both (a) and (b)
 - none of these.



xi) Condition of convergence for Euler's method

a) $|1 + hf'(x_i, y_i)| < 1$

b) $|1 + hf'(x_i, y_i)| \leq 1$

c) $|1 + hf'(x_i, y_i)| > 1$

d) $|1 + hf'(x_i, y_i)| \geq 1.$

xii) Milne's corrector formula is

a) $y_{n+1} = y_n + \frac{h}{3} (y'_{n-1} + 4y'_n + 4y'_{n+1})$

b) $y_{n+1} = y_{n-1} + \frac{h}{3} (y'_{n-1} + 4y'_n + 4y'_{n+1})$

c) $y_{n+1} = y_n + \frac{4h}{3} (y'_{n-1} + 4y'_n + 4y'_{n+1})$

d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following.

3 × 5 = 15

2. Given the function $y = \frac{1}{x}$, show that the divided difference of n^{th} order

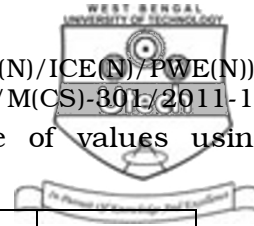
$$y[x_0, x_1, x_2, \dots, x_n] = (-1)^n / (x_0 x_1 x_2 \dots x_n)$$

3. Solve the following system of linear equations by Gauss-Seidel iterative method :

$$9x + 2y + 3z = -7$$

$$x - 6y + 2z = -2$$

$$x + y + 3z = 5.$$



4. Fit a polynomial to the following table of values using Lagrange interpolation formula :

x :	0	1	3	4
y :	- 12	0	6	12

Find the value of y when

- a) $x = 2$
b) $x = 3.5$.
5. Find the value of $\frac{1}{23}$ using Newton-Raphson method. Result is required to be corrected up to 4 decimal places.
6. Solve the following equation using bisection method :

$$3x + \sin x - e^x = 0$$

Take $x_0 = 1$ and $x_1 = 0$.

Result is required to be corrected up to 2 decimal places.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

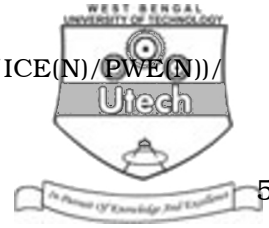
7. a) Derive the order of convergence for Newton-Raphson method. 5
b) Solve the following initial value problem using Euler's method :

$$\frac{dy}{dx} = x^2 + y \text{ with } y(0) = 1.$$

Compute the first 5 steps of the solution with $h = 0.1$.

Compare the results (% relative error) with those obtained from the exact solution

$$y = 3e^x - x^2 - 2x - 2. \quad 10$$



8. a) Prove by the method of induction :

$$\Delta^m y_r = \square^m y_{r+m}.$$

5

- b) Use Newton's formula to find the area of a circle of diameter 98 cm.

5

D (cm) :	80	85	90	95	100
A (cm²)	5026	5674	6362	7088	7854

- c) Derive Lagrange interpolation formula. 5
9. a) Derive the expression for total truncation error associated with Simpson's $\frac{1}{3}$ rd method. 8
- b) Evaluate the following integral using trapezoidal method :

$$I = \int_0^2 \left(1 / (x^2 + 4) \right) dx.$$

Take $h = 0.125$. Hence obtain the value of π . 7

10. a) Solve the following system of equations using LU factorization method. 8

$$3x - y + 2z = 12$$

$$x + 2y + 3z = 11$$

$$2x - 2y - z = 2.$$

- b) Find the inverse of the following matrix : 7

$$A = \begin{bmatrix} 8 & -4 & 0 \\ -4 & 8 & -4 \\ 0 & -4 & 8 \end{bmatrix}.$$



11. a) Define Δ , \square and E . 5
- b) Derive Newton's Backward difference interpolation formula. 5
- c) Derive 4th order Runge-Kutta formula for solution of initial value problem of ordinary differential equation. 5
-