



Time Allotted : 3 Hours

Full Marks :70

The Figures in the margin indicate full marks.

Candidate are required to give their answers in their own words as far as practicable

Group-A (Very Short Answer Type Question)

1. Answer any ten of the following :

[1 x 10 = 10]

(I) Solve for x and y using CRAMER'S RULE

$$3x + 2y = 16$$

$$7x + y = 19$$

(II) What are the necessary condition for the maclaurin's expansion to be true for function f(x) is

(III) If $f(x) = 2|x| + |x-2|$ then find the value of $f'(1)$

(IV) If $y = 10^{5x}$ then find the value of y_n

(V) Calculate the value of

$$\int_0^{\infty} e^{-x} dx$$

(VI) What are the coefficients of the equation obtained during the elimination called

(VII) For which value of x will $(x-1)(3-x)$ have its maximum

(VIII) If A is matrix of order 3x5 then rank of A^T is at most

(IX) Find the Wronskian of $1, e^x, e^{2x}$

(X) If p1 be an eigen value of the matrix A then what is the eigen value of the matrix $A+1$

(XI) If $A^2 - A + I = 0$, then $A^{-1} =$

(XII) If the eigen values of the matrix

$$\begin{pmatrix} 2 & 3 \\ x & y \end{pmatrix}$$

Group-B (Short Answer Type Question)

Answer any three of the following

[5 x 3 = 15]

2. Prove that set of all second order real square matrices is a vector space with respect to addition and multiplication of a matrix by a real number [5]

3. Prove that the Eigen values of a real symmetric matrix are all real [5]

4. Find the value of [5]

$$\int_0^{\pi/2} \sin^5 \theta \cos^4 \theta d\theta$$

5. Show that the maximum value of $x+1/x$ is less than its minimum value [5]

6. State Cayley Hamilton theorem and verify for matrix [5]

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 10 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Group-C (Long Answer Type Question)

Answer any three of the following

[15 x 3 = 45]

7. (a) Verify Rolle's theorem for the function [5]

$$f(x) = e^{-x} \sin x \text{ on } [0, \pi].$$

(b) Using MVT prove that

[5]

$$0 < \frac{1}{x} \log \frac{e^x - 1}{x} < 1.$$

(c) Using MVT prove that

[5]

$$\sin 46^\circ \sim \frac{1}{2} \sqrt{2} \left(1 + \frac{\pi}{180} \right).$$

8. (a) Examine the consistency of the following system of equations and solve $2x-2y-4z=8, 2x+3y+2z=8, -x+y-z=7/2$

[5]

(b) Find the rank of

[5]

$$\begin{pmatrix} 0 & 0 & 5 & -3 \\ 2 & 4 & 3 & 5 \\ -1 & -2 & 6 & -7 \end{pmatrix}.$$

(c) If A is skew symmetric matrix then prove that $(I-A)(I+A)^{-1}$ is orthogonal matrix

[5]

9. (a) Using definition of beta function prove that

[5]

$$\int_0^{\pi/2} \cos^4 x dx = 3\pi/16$$

(b) Evaluate

[5]

$$\int_0^{\infty} e^{-x^2} dx$$

(c) show that

[5]

$$\int_0^1 x^{3/2} (1-x)^{3/2} dx = 3\pi/128$$

10. (a) Using Lagrange's MVT find the approximate value of

[5]

$$\sqrt[6]{65}$$

(b) Using MVT prove that

[5]

$$x < \sin^{-1} x < \frac{x}{\sqrt{1-x^2}}, \text{ if } 0 < x < 1.$$

(c) Expand the function $f(x)=\sin x$ in powers of x with Lagrange's form of remainder

[5]

11. (a) Find the value of

[5]

$$\lim_{x \rightarrow 0} x^{2 \sin x}$$

(b) Find the value of

[5]

$$\lim_{x \rightarrow 0} \cot x^{\sin x}$$

(c) Find the value of

[5]

$$\lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^{\frac{1}{x^2}}$$