Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/B.TECH (CE/OLD)/SEM-4/CE-401/2013

2013

MATHEMATICS – II

Time Allotted : 3 Hours | Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) In Simpson's one-third formula for the integration $\int_a^b f(x)\,dx$, the degree of precision is

a) 0 b) 1

c) 2 d) 3.

ii) The n^{th} order forward difference of the n^{th} degree polynomial is

a) $n!$ b) $(n+1)!$

c) 0 d) none of these.

iii) The rate of convergence of the Newton-Raphson method for solving $f(x) = 0$ is

a) quadratic b) biquadratic

c) cubic d) linear.

iv) The inherent error for Simpson's $\frac{1}{3}$ rule of integration is as (the notations have their usual meanings)

a) $-\frac{nh^5}{180} f^{//}(x_0)$

b) $-\frac{nh^5}{140} f^{//}(x_0)$

c) $-\frac{nh^3}{12} f^{//}(x_0)$

d) none of these.

v) If a number be rounded to m-decimal places, then for the absolute error (E_A),

a) $E_A > \frac{1}{2} \times 10^{-m}$

b) $E_A \leq \frac{1}{2} \times 10^{-m}$

c) $E_A = 10^{-m}$

d) $E_A \geq \frac{1}{2} \times 10^{-m}$.

vi) In fixed point interation method [$x = \phi(x)$] for the equation $\pi x = \sin(x)$, the appropriate choice of $\phi(x)$ such that sequence $x_0, x_1, x_2, \ldots, x_n$ converges to the root is

a) $\frac{\sin(x)}{\pi}$ b) $\frac{\cos(x)}{\pi}$

c) $\cos(x)$ d) none of these.

vii) The condition of convergence of approximations for a real foot $\phi(x) = x$ by method of iteration is

a) $max\,[\,|\phi'(a)|, |\phi'(b)|\,] = 0$ in (a, b)

b) $max\,[\,|\phi'(a)|, |\phi'(b)|\,] < 1$ in (a, b)

c) $min\,[\,|\phi'(a)|, |\phi'(b)|\,] = 0$ in (a, b)

d) $min\,[\,|\phi'(a)|, |\phi'(b)|\,] < 1$ in (a, b).

viii) Runge-Kutta method is used to

a) interpolate

b) solve a transcendental equation numerically

c) integrate a definite integral numerically

d) solve a first order ordinary differential equation numerically.

ix) Which of the following numbers has greatest precision ?

4·3201, 4·32, 4·320106, 4·3.

a) 4·3201 b) 4·32

c) 4·320106 d) 4·3.

x) An approximate polynomial passes through $(n + 1)$ data points is of degree

a) $n + 1$ b) n

c) n or less d) $n + 1$ or less.

xi) If E_a is the absolute error in a quantity whose true and approximate values are given by x_t and x_a, then the relative error is given by

a) $\left| \frac{E_a}{x_a} \right|$ b) $\left| \frac{E_a}{x_t} \right|$

c) $\left| \frac{E_a}{x_t - x_a} \right|$ d) $| E_a |$.

xii) $(\Delta - \nabla)\, x^2$ is equal to (the notations have their usual meanings)

a) h^2 b) $-2h^2$

c) $2h^2$ d) none of these.

xiii) Rounding off the number 0·0063945 correct up to 4 significant figures is

a) 0·0064 b) 0·0063

c) 0·006395 d) 0·006394.

xiv) To find out the numerically largest eigenvalue of a matrix which of the following methods is used ?

a) Newton-Raphson method

b) Trapezoidal rule

c) Runge-Kutta method of second order

d) Power method.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. 3 × 5 = 15

2. Assuming that the following values of y belong to a polynomial of degree 4, compute the next three values :

x	0	1	2	3	4	5	6	7
y	1	– 1	1	– 1	1	—	—	—

3. If $\Delta x = 0{\cdot}005$ and $\Delta y = 0{\cdot}001$ be the absolute errors in $x = 2{\cdot}11$ and $y = 4{\cdot}15$, find the relative error in computation of $x + y$.

4. Find out the integration value, correct up to 5 decimal places of the integration $\int_0^1 \frac{1}{1 + x^2}\, dx$, taking number of sub-intervals $n = 10$ by Simpson's $\frac{1}{3}$ rule.

5. Use inverse interpolation formula to find out x when $f(x) = 0{\cdot}645$ correct up to one decimal place from the following table :

x :	0·3	0·5	0·6
$f(x)$:	0·6179	0·6915	0·7257

6. Solve the following equation by Gauss elimination method :

$$-2x + 2y + z + 2t = 7$$

$$-x + 2y = -2$$

$$-3x + y + 2z + t = -3$$

$$x + 2t = 0.$$

7. Find the moment generating function of the Binomial distribution and from it determine its mean and variance.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

8. a) Derive Trapezoidal integration formula from Newton-Cotes formula.

   b) Evaluate $\int_0^1 \frac{1}{1+x^2}\,dx$, by Trapezoidal rule taking number of sub-intervals $n = 10$, correct to 4 decimal places.

   c) Write down the inherrent error formula for trapezoidal rule for numerical integration. $7 + 7 + 1$

9. a) Derive Newton's Backward difference interpolation formula.

   b) Find a real root of the equation $x^3 - x - 11 = 0$ correct up to 4 decimal places using Newton-Raphson in the interval $(2, 4)$ and estimate the error in this method.

$5 + (7 + 3)$

10. a) Derive the Newton-Raphson formula for computing the real root of an equation $f(x) = 0$.

b) Explain the geometrical significance of Newton-Raphson method.

c) Show that Newton-Raphson method has a quadratic convergence. 5 + 5 + 5

11. a) Use Euler's method to find an approximate value of y corresponding to $x = 1$, given that, $\frac{dy}{dx} = x + y$ and

$y = 1$, when $x = 0$.

b) With usual notations, prove that,

$$\Delta^n \left(\frac{1}{x}\right) = (-1)^n \, \frac{n! \, h^n}{x(x+h)\ldots(x+nh)}$$ 5 + 10

12. a) Given that the mode of the following frequency distribution of 70 observations is 58·75. Find the missing frequencies f_1 and f_2.

Class	*Frequency*
52 — 55	15
55 — 58	f_1
58 — 61	*25*
61 — 64	f_2

b) Find one real root of $10^x + \sin(x) + 2x = 0$, by the bisection method in the interval $(-1, 0)$, correct up to 3 significant figures. 5 + 10

13. Use the method of least squares to fit the curve

$$f(x) = c_0 x + \left(\frac{c_1}{\sqrt{x}} \right)$$ for the following data :

x :	0·2	0·3	0·5	1	2
$f(x)$:	16	14	11	6	3

Find also the least square errors (correct up to 3 decimal places).