Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/B.Tech(ECE)/SEM-7/EC-704B/2011-12

2011

ADVANCED ENGINEERING MATHEMATICS FOR ELECTRONICS ENGINEERING

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : 10 × 1 = 10

i) Cauchy-Riemann equations are

a) $u_x = -v_y$, $u_y = v_x$

b) $u_x = v_y$, $u_y = -v_x$

c) $u_x = v_y$, $u_y = v_x$

d) $u_x = -v_y$, $u_y = -v_x$.

ii) If α, β are the roots of the equation $x^2 - 3x + 2 = 0$

then $\begin{vmatrix} 0 & \alpha & \beta \\ \beta & 0 & 0 \\ 1 & -\alpha & \alpha \end{vmatrix}$ is

a) 6 b) $\frac{3}{2}$

c) -6 d) 3.

iii) The residue of $f(z) = \dfrac{1}{(z+1)^2 (z-2)}$ *at* $z = -1$ is

a) $\frac{1}{3}$ b) $\frac{1}{9}$

c) $-\frac{1}{3}$ d) $-\frac{1}{9}$.

iv) If $L[f(t)] = F(s)$ then $L[f(2t)]$ will be

a) $\frac{1}{2} F\left(\frac{s}{2}\right)$ b) $2F\left(\frac{s}{2}\right)$

c) $\frac{1}{2} F(2s)$ d) $2F(2s)$.

v) $L\left\{e^{-2t} \cos t\right\}$ is equal to

a) $\dfrac{p}{p^2 + 4p + 5}$ b) $\dfrac{p+2}{p^2 + 4p + 5}$

c) $\dfrac{p+3}{p^2 + 4p + 5}$ d) $\dfrac{p+1}{p^2 + 4p + 5}$.

vi) The differential equation $p^2 + q^3 - pq + zxy = 0$ is

a) degree = 1, order = 1, linear

b) degree = 2, order = 2, linear

c) degree = 3, order = 1, non-linear

d) degree = 4, order = 2, non-linear.

vii) If the Fourier transform of a function

$$f(x) \text{ is } \bar{F}(s) = F\{f(x)\} = \int_{-\infty}^{\infty} e^{isx} f(x)\, dx,$$

then the inversion formula for Fourier transform is

a) $\frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-isx} \bar{F}(s)\, ds$

b) $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-isx} \bar{F}(s)\, ds$

c) $\frac{1}{2\pi} \int_{-\infty}^{\infty} e^{isx} \bar{F}(s)\, ds$

d) $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{isx} \bar{F}(s)\, ds$.

viii) For Legendre polynomial of degree n, $P_2(x)$ is equal to

a) 1

b) $\frac{1}{2}(3x^2 - 1)$

c) $\frac{1}{2}(x^2 - 3)$

d) $\frac{1}{2}(5x^3 - 3x)$.

ix) The equation $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - 1)\, y = 0$ is

a) Bessel's equation of order zero

b) Bessel's equation of order one

c) Legendre's equation of order zero

d) Legendre's equation of order one.

x) The solution of the differential equation

$$\left(\frac{y^2 z}{x} \right) p + xzq = y^2 \text{ is}$$

a) $\phi \left(x^2 - z^2, x^3 - y^3 \right) = 0$

b) $\phi \left(x^2 - z^2, x^2 - y^2 \right) = 0$

c) $\phi \left(x^2 - z^2, x - y \right) = 0$

d) $\phi \left(x^2 - z^2, 1 \right) = 0$.

xi) Let $f(z) = \sin \frac{1}{z}$. Then $Z = 0$ is

a) a pole

b) removable singularity

c) essential singularity

d) none of these.

xii) The bilinear transformation that maps the points $z_1 = 0, z_2 = -i, z_3 = -1$ into $w_1 = i, w_2 = 1, w_3 = 0$ respectively is

a) $w = \left(\frac{z+1}{z-1} \right)$

b) $w = i \left(\frac{z-1}{z+1} \right)$

c) $w = -i \left(\frac{z-1}{z+1} \right)$

d) $w = -i \left(\frac{z+1}{z-1} \right)$.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Show that $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cdot \cos x$.

3. Solve $4 \frac{\partial^2 z}{\partial x^2} - 4 \frac{\partial^2 z}{\partial x \partial} + \frac{\partial^2 z}{\partial y^2} = 16 \log(x + 2y)$.

4. Determine the row rank and the column rank of the marix A and verify that the row rank of the matrix A equals to column rank of the matrix A, where

$$A = \begin{pmatrix} 2 & 1 & 4 & 3 \\ 3 & 2 & 6 & 9 \\ 1 & 1 & 2 & 6 \end{pmatrix}$$

5. If $f(z)$ is analytic, prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) |f(z)|^2 = 4 |f'(z)|^2 .$$

6. Evaluate $L^{-1}\left\{ \frac{1}{s^3 (s+1)^3} \right\}$ using convolution theorem.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Prove that if A & B are orthogonal matrices of the same order, then AB is also orthogonal. 3

b) Find the rank of the matrix $\begin{pmatrix} 0 & 0 & 2 & 2 & 0 \\ 1 & 3 & 2 & 4 & 1 \\ 2 & 6 & 2 & 6 & 2 \\ 3 & 9 & 1 & 10 & 6 \end{pmatrix}$ 6

c) Find the Fourier sine integral for $f(x) = e^{-\beta x}$, hence show that $\frac{\pi}{2} \cdot e^{-\beta x} = \int_0^{\infty} \frac{\lambda \sin \lambda x}{\beta^2 + \lambda^2}\, d\lambda$. 6

8. a) Show that the line $y = \frac{x}{3}$ is mapped onto the circle under the bilinear transformation $w = \frac{iz + 2}{4z + i}$. Find the centre and radius of the image circle. 5

b) Applying residue theorem evaluate

$$\int_C \frac{3z^2 + z - 1}{(z^2 - 1)(z - 3)}\, dz$$ where c is the circle $|z| = 2$. 5

c) Prove that $\int_0^{\alpha} \frac{\cos x}{1 + x^2}\, dx = \frac{\pi}{2e}$. 5

9. a) Find the Fourier cosine transform of the function $f(x) = \dfrac{1}{1+x^2}$. 5

b) Applying binomial theorem $(x^2 - 1)^n$, differentiating n times term by term and comparing with Legendre coefficient, prove the Rodrigues Formula

$$P_n(x) = \frac{1}{2^n . n!} \frac{d^n}{dx^n} \left[(x^2 - 1)^n \right] .$$ 5

c) Find the Laurent's series expansion of $f(z) = z^2 e^{\frac{1}{z}}$. 5

10. a) Show that generating function for Bessel Function $J_n(x)$ is $e^{x/2\left(t - \frac{1}{t}\right)}$. 10

b) Prove that $J_{3/2} = \sqrt{\dfrac{2}{\pi x}} \left(\dfrac{\sin x}{x} - \cos x \right)$. 5

11. a) Solve by Fourier transform

$$K \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} , \; 0 < x < \bullet, \; t > 0$$

with $u(0, t) = 0, \; t > 0$

$$u(x, 0) = f(x), \; 0 < x < \bullet$$

and u & $\dfrac{\partial u}{\partial x}$ $\varnothing$ 0 as x $\varnothing$ $\bullet$. 9

b) Using contour integration evaluate

$$\int_0^\pi \frac{1 + 2\cos\theta}{5 + 4\cos\theta} \, d\theta.$$ 6