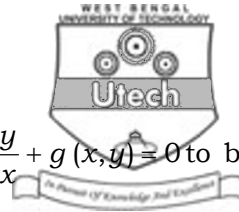


*Invigilator's Signature* : .....



iii) For the differential equation  $f(x,y)\frac{dy}{dx} + g(x,y) = 0$  to be exact if

a)  $\frac{\partial f}{\partial y} = \frac{\partial g}{\partial x}$

b)  $\frac{\partial f}{\partial x} = \frac{\partial g}{\partial y}$

c)  $\frac{\partial^2 f}{\partial x^2} = \frac{\partial^2 g}{\partial y^2}$

d) none of these.

iv) The auxiliary equation of  $\frac{d^2 y}{dx^2} + a^2 y = \sec ax$  ( $a \neq 0$ ) is

a)  $m^2 + a^2 = 0$

b)  $m^2 + 2a^2 = 0$

c)  $m^2 + a = 0$

d) none of these.

v) The general solution of  $y = px + \sqrt{a^2 p^2 + b^2}$ , where  $p = \frac{dy}{dx}$  is

a)  $y = cx + \sqrt{a^2 c^2 + b^2}$

b)  $y = cx - \sqrt{a^2 c^2 + b^2}$

c)  $y = c - x\sqrt{a^2 c^2 + b^2}$

d) none of these.

vi) The maximum number of edges in a simple graph with  $n$  vertices is

a)  $n$

b)  $\frac{n-1}{2}$

c)  $\frac{n(n-1)}{2}$

d) none of these.



- vii) A binary tree has exactly
- two vertices of degree two
  - one vertex of degree 2
  - one vertex of degree one
  - none of these.
- viii) If a graph G has 7 vertices and 9 edges, then the size of the adjacency matrix is
- $7 \times 7$
  - $7 \times 9$
  - $9 \times 9$
  - none of these.
- ix) Dijkstra's algorithm is used to
- find maximum flow in a network
  - scan all vertices of a graph
  - find the shortest path from a specific vertex to another one
  - none of these.
- x) The singularities of the integral  $\int_{-1}^2 \frac{dx}{x(x-1)}$  are
- 0, 1
  - 1, 2
  - 1, 2
  - 0, 2.
- xi) The value of  $\Gamma\left(\frac{1}{2}\right)$  is
- $2\pi$
  - $\sqrt{\pi}$
  - $\frac{\pi}{2}$
  - none of these.



xii) Laplace transform of the function  $\sin at$  is

- a)  $\frac{s}{s^2 + a^2}$                       b)  $\frac{s}{s^2 - a^2}$   
 c)  $\frac{a}{s^2 + a^2}$                       d)  $\frac{a}{s^2 - a^2}$ .

xiii) The value of  $\Gamma(m)\Gamma(1-m)$  is

- a)  $\frac{2\pi}{\sin \pi}$                       b)  $\frac{3\pi}{\sin m\pi}$   
 c)  $\frac{\pi}{\sin m\pi}$                       d) none of these.

xiv) The value of  $\beta\left(\frac{1}{2}, \frac{1}{2}\right)$  is

- a)  $\sqrt{\pi}$                       b)  $\pi$   
 c)  $\frac{\pi}{2}$                       d) none of these.

### GROUP – B

#### ( Short Answer Type Questions )

Answer any *three* of the following                       $3 \times 5 = 15$

2. Solve  $(D^2 - 5D + 6)y = e^x \cos x$  where  $D = \frac{d}{dx}$ .
3. Find the general and singular solution of  $(y - px)(p - 1) = p$   
 where  $p = \frac{dy}{dx}$ .

4. Evaluate  $L^{-1}\left(\frac{s^2}{(s^2 + a^2)(s^2 + b^2)}\right)$ .



5. Prove that a tree with  $n$  vertices has  $(n - 1)$  edges.

6. Find the value of the improper integral  $\int_0^{\infty} \frac{dx}{(1+x)\sqrt{x}}$ .

### GROUP – C

#### ( Long Answer Type Questions )

Answer any *three* of the following.  $3 \times 15 = 45$

7. a) Apply the variation of parameters to solve

$$\frac{d^2 y}{dx^2} + 4y = \sin 2x.$$

b) Solve :  $(x^2 D^2 - xD + 4)y = x \sin(\log x)$  where  $D = \frac{d}{dx}$ .

c) Show that  $\int_{-\infty}^{\infty} x e^{-x^2} dx = 0$ .  $5 + 5 + 5$

8. a) State convolution theorem. Using convolution theorem

$$\text{prove that } L^{-1}\left(\frac{s}{(s^2 + a^2)^2}\right) = \frac{t \sin at}{2a}.$$

b) Solve the following differential equation using Laplace

$$\text{transform } (D^2 + 6D + 9)y = 1 :$$

$$y(0) = 0, y'(0) = 1 \left[ D = \frac{d}{dx} \right].$$

c) Evaluate  $\int_0^{\infty} e^{-3t} \sin t \cos t dt$  using Laplace transform.

$6 + 5 + 4$



9. a) Prove that  $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ .

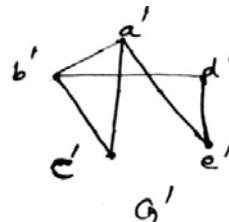
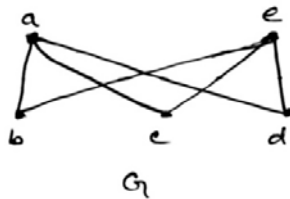
b) Evaluate  $\int_0^1 x^2 (1-x^2)^{\frac{7}{2}} dx$ .

c) Prove that the number of odd degree vertices in a graph is always even. 5 + 5 + 5

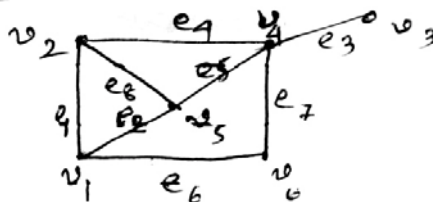
10. a) Draw the graph whose incidence matrix is

$$\begin{matrix} & e_1 & e_2 & e_3 & e_4 & e_5 \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \end{pmatrix} \end{matrix}$$

b) Define isomorphism. Examine whether the following two graphs are isomorphic or not.



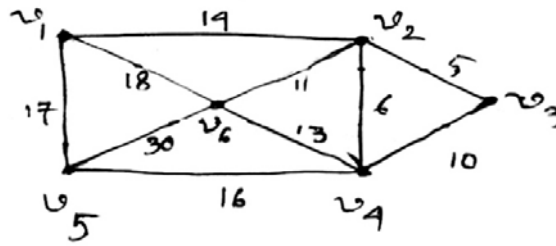
c) Determine the adjacency matrix of the given graph :



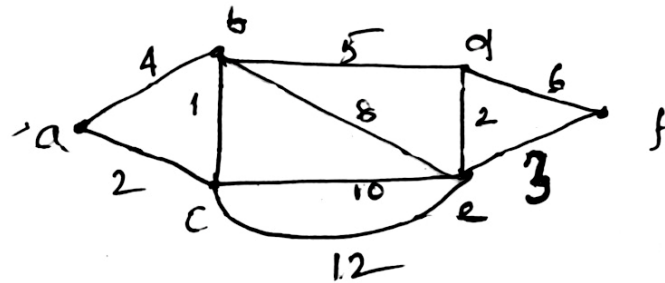
5 + 5 + 5



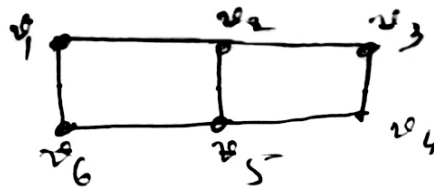
11. a) Apply Prim's algorithm to find the shortest spanning tree of the following graph and find the corresponding minimum weight.



- b) Find the shortest path from the vertex  $a$  to  $f$  in the following graph using Dijkstra's algorithm.



- c) Construct the spanning tree of the following graph by BFS algorithm.



$$6 + 5 + 4$$

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