Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/B.TECH (NEW)(CSE/IT)/SEM-4/M-401/2012

2012
MATHEMATICS - III

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct answers for any *ten* of the following : $10 \times 1 = 10$

i) If $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$ and $P(A \cup B) = \frac{1}{2}$, then $P(B|A)$ is

a) $\frac{3}{4}$ b) $\frac{4}{3}$

c) $\frac{1}{4}$ d) $\frac{1}{3}$.

ii) The variance of a random variable x is

a) $\{E(x)\}^2$ b) $E(x^2)$

c) $E(x^2) - \{E(x)\}^2$ d) $E(x^2) - E(x)$.

iii) A statistic t is said to be an unbiased estimator of a population parameter θ when

a) $E(t) = \theta$ b) $E(t^2) = \theta$

c) $E(t^2) = \{E(\theta)\}^2$ d) $\{E(t)\}^2 = E(\theta^2)$.

iv) The maximum likelihood estimate is a solution of the equation

a) $\frac{\partial \angle(\theta)}{\partial \theta} = 0$ b) $\frac{\partial \angle(\theta)}{\partial \theta} = \text{constant}$

c) $\frac{\partial \angle(\theta)}{\partial \theta} = \theta$ d) none of these.

v) If H_1 ($\mu > 60$) is an alternative hypothesis, then the null hypothesis is

a) H_0 ($\mu < 60$) b) H_0 ($\mu \geq 60$)

c) H_0 ($\mu \leq 60$) d) none of these.

vi) A random variable x has the following p.d.f :

$$f(x) = \begin{cases} k, & -2 < x < 2 \\ o, & \text{otherwise} \end{cases}$$

then the value of k is

a) $\frac{1}{12}$ b) $\frac{1}{2}$

c) $\frac{1}{4}$ d) $\frac{1}{8}$.

vii) A complete graph is called Kuratowski's first graph if it has

a) 5 vertices b) 4 vertices

c) 6 vertices d) 7 vertices.

viii) If G is a non-planar graph then the possible number of vertices of G is

a) 2 b) 3

c) 4 d) 6.

ix) The chromatic number of a graph containing an odd circuit is

a) 3

b) 2

c) greater than or equal to 3

d) greater than or equal to 2.

x) The generators of the cyclic group $(Z, +)$ are

a) $1, -1$ b) $0, 1$

c) $0, -1$ d) $2, -2$.

xi) The inverse of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}$ is

a) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}$ b) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix}$

c) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix}$ d) none of these.

xii) If R is a ring without zero divisors, then $x.\ y = 0$ implies

a) $x = 0$ or $y = 0$ b) $x = 0$ and $y = 0$

c) $x = 0, y \neq 0$ d) $x \neq 0, y = 0$.

xiii) Which of the following sets in closed under multiplication ?

a) $\{ 1, -1, 0, 2 \}$ b) $\{ 1, i \}$

c) $\{ 1, w, w^2 \}$ d) $\{ w, 1 \}$.

xiv) A group G is commutative iff

a) $ab = ba$ b) $(ab)^{-1} = b^{-1} a^{-1}$

c) $(ab)^{-1} = a^{-1} b^{-1}$ d) $(ab)^2 = ab$.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. If $P(A \cap B) = P(A) P(B)$, then prove that

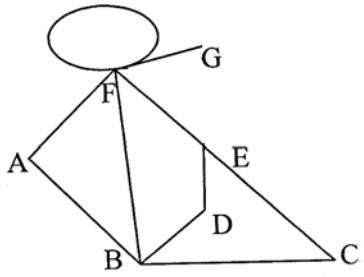
$P(A^c \cap B^c) = P(A^c) P(B^c)$

3. Find the mean and variance of Poisson distribution with parameter λ.

4. If G the a group such that $(ab)^2 = a^2 b^2$ for all $a, b \in G$; show that the group G is Abelian.

5. A normal population has a mean 0.1 and standard deviation 2·1. Find the probability that the mean of a sample of size 900 will be negative. Given that $P(|z| = 1{\cdot}43) = 0{\cdot}847$.

6. Draw the dual of the following graph :



GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following questions.

$3 \times 15 = 45$

7. a) A regular graph G determines 8 regions, degree of each vertex being 3. Find the number of vertices of G.

   b) Prove that the chromatic polynomial of a tree with n vertices is $x(x-1)^{n-1}$, whose x is the no. of colours.

   c) Prove that every finite integral domain is a field.

8. a) Prove that a subgroup H of a group G is said to be normal if $aH = Ha$ for all $a \in G$.

   b) A box contains 5 red balls and 10 white balls. Two balls are drawn at random from the box without replacement. What is the probability that

      i) the second ball is white,

      ii) the first ball drawn is red, given the second ball drawn is white ?

   c) Define a cyclic group. Prove that every cyclic group is abelian.

9. a) Show that every planar graph is 6 colourable.

   b) If G be a connected graph with n vertices , e edges and r faces, prove that $n-e+r=2$.

   c) If T is an unbiased estimator of θ, show that $\sqrt{T}$ is biased estimate of $\sqrt{\theta}$.

10. a) State and prove Baye's theorem.

   b) A random sample with observations 65, 71, 64, 71, 70, 69, 64, 63, 67, 68 is drawn from a normal population with variance 7·056. Test the hypothese's that the population mean is 69 at 1% level of significance. [Given that $P(0 < z < 2{\cdot}58) = 0{\cdot}495$].

   c) If a population has normal distribution with parameter μ and σ, then prove that the statistic $\frac{1}{n}\sum_{i=1}^{n}(x_i-\mu)^2$ is maximum likelihood estimate of σ^2 where μ is known.

11. a) Show that the group (z_9, +) is a homomorphic image of the group (z, +).

   b) In a bolt factory, machines A, B, C manufacture respectively 25%, 35%, 40%. Of the total of their output 5%, 4%, 2% are defective bolts. A bolt is drawn at random from the product and is found to be defective. What are the probability that it was manufactured by machines A, B and C ?

   c) The lifetime of a certain brand of an electric bulb may be considered as a random variable with mean 1200h and s.d. 250h. Find the probability, using Central Limit theorem, that the average lifetime of 60 bulbs exceeds 1250h.

12. a) Prove that the set of all even integers form a commutative ring.

b) Prove that the intersection of two subrings is a subring.

c) Prove that the sample mean $\bar{x}$ is an unbiased estimator of the population mean.