

Name :

Roll No. :

Invigilator's Signature :

CS/B.Tech (OLD)/SEM-1/M-101/2011-12

2011

MATHEMATICS

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) The series $\sum x^n = 1 + x + x^2 + \dots \infty$ is convergent if

- a) $x \geq 1$ b) $x \leq -1$
c) $-1 < x < 1$ d) none of these.

ii) A positive term series cannot be oscillatory.

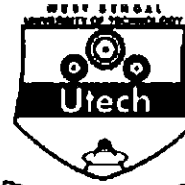
- a) True
b) False.

iii) The limit $\lim_{x \rightarrow 0} |x|/x$ does not exist.

- a) True
b) False.

As a person, I am a member of the following organizations:

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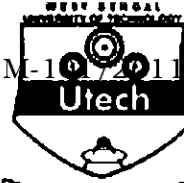
GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following.

$3 \times 5 = 15$

2. If $y = \tan^{-1}x$, then prove that
$$(1+x^2)y_{n+1} + 2nx y_n + n(n-1)y_{n-1} = 0$$
. Find also the value of $(y_n)_0$.
3. Test the convergence of the series $1 + \frac{1}{2^2} + \frac{2^2}{3^3} + \frac{3^3}{4^4} + \frac{4^4}{5^5} + \dots$.
4. Obtain the reduction formula for $\int_0^{\pi/2} \cos^n x \, dx$ and hence evaluate $\int_0^{\pi/2} \cos^7 x \, dx$.
5. Find the directional derivative of $p = 4e^{2x-y+z}$ at the point $(1, 1, -1)$ in a direction towards the point $(-3, 5, 6)$.
6. Find the points of intersection of the line $x + y + z + 1 = 0 = 14x + 9y - 7z - 1$ with XY and YZ planes and hence put down the symmetrical form of its equations.
7. Find a point in the plane $x + 2y + 3z = 13$ nearest to the point $(1, 1, 1)$ using the method of Lagrange's multipliers.



GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

8. a) Examine if the function

$$f(x) = x \tan^{-1}\left(\frac{1}{x}\right), \quad x \neq 0$$

$$= 0, \quad x = 0$$

is derivable at $x = 0$.

- b) State Leibnitz's theorem. If $y = e^{m \cos^{-1} x}$, show that

$$(1 - x^2) y_{n+2} - (2n + 1) x y_{n+1} - (n^2 + m^2) y_n = 0.$$

- c) If $y^{1/m} + y^{-1/m} = 2x$, prove that

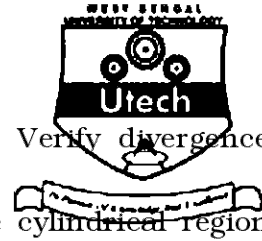
$$(x^2 - 1) y_{n+2} + (2n + 1) x y_{n+1} + (n^2 - m^2) y_n = 0.$$

9. a) State Rolle's theorem. Verify Rolle's theorem for the function $f(x) = \sin x \cos x$ in $[0, \pi/2]$.

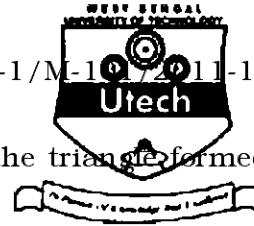
- b) Prove that

$$(\text{if } 0 < a < b < 1), \quad (b - a)/(1 + b^2) < \tan^{-1} b - \tan^{-1} a < (b - a)/(1 + a^2)$$

- c) Expand $\sin x$ in power of x in infinite series stating the condition under which the expansion is valid.



10. a) State divergence theorem of Gauss. Verify divergence theorem for $\vec{F} = y\vec{i} + x\vec{j} + z\vec{k}$ over the cylindrical region bounded by $x^2 + y^2 = 9$, $z = 0$, $z = 2$.
- b) State Green's theorem in plane. Verify Green's theorem in plane for $\int_C (3x^2 - 8y^2) dx + (4y - 6xy) dy$, where C is the boundary of the region defined $y = x^{1/2}$, $y = x^2$.
- c) Find the work done in moving a particle in the force field $\vec{F} = 3x^2\vec{i} + (2xz - y)\vec{j} + z\vec{k}$ along the curve defined by $x^2 = 4y$, $3x^3 = 4z$ from $x = 0$ to $x = 2$.
11. a) Show that $\vec{\nabla} r^n = nr^{n-2}\vec{r}$ where $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$.
- b) Prove that $\text{curl} [\varphi \text{grad } \varphi] = 0$.
- c) Find the co-ordinate of the foot of the perpendicular from $(1, 2, 3)$ on the line $(x - 2)/1 = (y - 1)/2 = z/3$. Find also the length of the perpendicular and its equation.



12. a) Evaluate $\iint (4x^2 - y^2)^{1/2} dx dy$ over the triangle formed by the straight lines $y = 0$, $x = 1$ and $y = x$.

b) Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} dz dy dx$.

c) If $f(x, y) = (x^2 + y^2)^{1/3}$, use Euler's theorem to find the value of $x (\partial f / \partial x) + y (\partial f / \partial y)$ and hence prove that, $x^2 (\partial^2 f / \partial x^2) + 2xy (\partial^2 f / \partial x \partial y) + y^2 (\partial^2 f / \partial y^2) + (2/9) f = 0$.
