Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/B.TECH(CSE/IT)(OLD)/SEM-4/M-401/2012

2012

MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) Let R be the relation on the set $A = \{ a, b, c \}$ where $R = \{ (a,a), (b,b), (a,b), (b,a) \}$. Then R is

a) an equivalence relation

b) reflexive, symmetric but not transitive

c) reflexive but not symmetric and transitive

d) symmetric, transitive but not reflexive.

ii) A semigroup ($G,*$) will be monoid if

a) associative law holds under (*)

b) commutative law holds under (*)

c) inverse element exists $\forall a \in G$

d) G contains an identity element.

iii) The set of all residue class Z_6 contains

a) 6 elements b) 5 elements

c) 7 elements d) none of these.

iv) Solution of the recurrence relation $a_n = 3a_{n-1}$, $a_0 = 1$ is $a_n =$

a) 3^n b) 3^{n-1}

c) 3^{n+1} d) none of these.

v) In a Boolean Algebra, $(a+b)' + (a+b')$ is

a) b b) a

c) a' d) ab.

vi) In a lattice { 1, 5, 25, 125 }, the complement of 25 is

a) 1 b) 5

c) 25 d) 125.

vii) If a graph has 4 vertices and 7 edges, then the order of Adjacency matrix is

a) 4 × 4 b) 4 × 7

c) 7 × 4 d) 7 × 7.

viii) A complete graph with n vertices has

a) $(n-1)$ edges b) $n(n-1)/2$ edges

c) $2n$ edges d) $n(n+1)/2$ edges.

ix) The degree of any vertex of a null graph is

a) 0 b) 1

c) 2 d) none of these.

x) The hamming distance between 11010 and 10101 is

a) 2 b) 3

c) 4 d) 0.

xi) The order of Dihedral Group D_4 is

a) 4 b) 6

c) 8 d) 64.

xii) If the cyclic group G contains 11 distinct elements, then it has

a) 2 generators b) 7 generators

c) 9 generators d) 10 generators.

xiii) Which of the following sets is closed under multiplication ?

a) $\{ 1, -1, 0, 2 \}$ b) $\{ 1, i \}$

c) $\{1, \omega, \omega^2 \}$ d) $\{1, \omega\}$.

xiv) A tree contains at least

a) one vertex b) two vertices

c) three vertices d) four vertices.

xv) A ring with zero divisors is called an integral domain.

a) True b) False.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Show that centre of a group G, given by

$Z(G) = \{ a \in G : ag = ga \forall g \in G \}$ is a normal subgroup of G.

3. Show that the ring of matrices of the form $\begin{bmatrix} 2\alpha & 0 \\ 0 & 2\beta \end{bmatrix}$, $\alpha, \beta \in Z$ contains divisors of zero. (Z = set of all integers and the operations are matrix addition and multiplication)

4. In a lattice $(L, \wedge, \vee)$ prove that $a \wedge b = a$ if and only if $a \vee b = b,\ a, b \in L$.

5. Express $E = y' + z(x' + y)$ as a full disjunctive normal form.

6. Draw the graph whose incidence matrix is

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

7. Define a planer graph. If G be a connected planner graph with n_v vertices, n_e edges and n_f faces, then show that $n_v - n_e + n_f = 2$.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

8. a) Using generating function, solve the recurrence relation $a_n - 7a_{n-1} + 10a_{n-2} = 0$ for $n > 1$ and $a_0 = 3$, $a_1 = 3$.

   b) Show that the set of all roots of the equation $x^4 = 1$ forms a group under multiplication.

   c) Show that the set of matrices $s = \left\{ \begin{pmatrix} \alpha & 0 \\ \beta & 0 \end{pmatrix} : \alpha, \beta \in \mathbb{R} \right\}$ is a left ideal but not a right ideal of 2 × 2 real matrices.

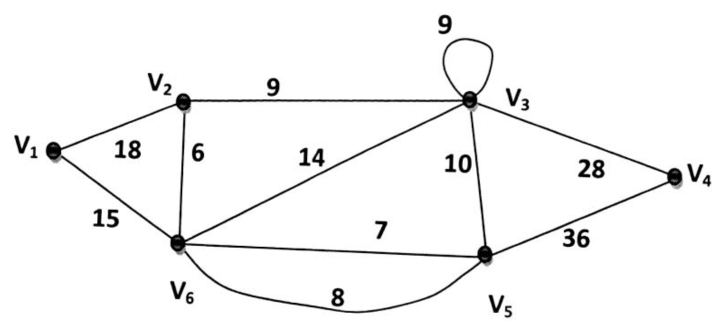
   5 + 5 + 5

9. a) Show that for any two subgroups H and K of a group G, $H \cap K$ is also a subgroup of G.

   b) Show that the mapping $f : (\mathbb{Z}_6, +) \rightarrow (\mathbb{Z}_6, +)$ defined by $f(x) = 5x,\ x \in \mathbb{Z}_6$ is a group homomorphism. Find the ker f.

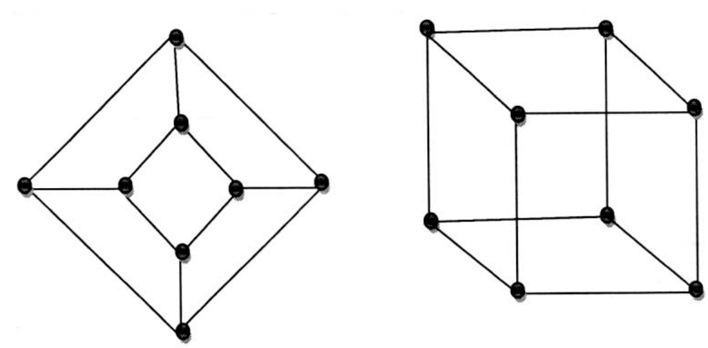
c) Prove that in a lattice $(L, \leq)$, for any $a, b, c \in L$, if $a \leq b \leq c \Rightarrow a \vee b = b \wedge c$ and $(a \wedge b) \vee (b \wedge c) = b$ $= (a \vee b) \wedge (a \vee c)$. 5 + 5 + 5

10. a) Applying Dijkstra's algorithm, find the shortest path from v_1 to v_4 in the following graph :



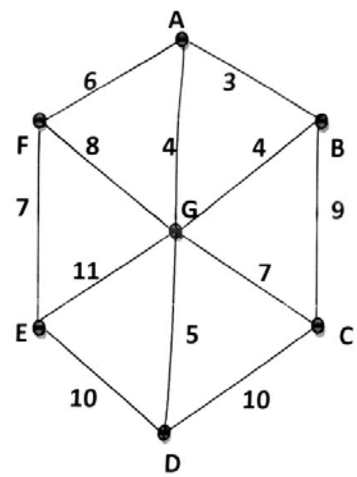
b) Prove that every cut set in a connected graph contains at least one branch of every spanning tree of graph.

c) Define Isomorphic graph. Examine whether the following two graphs are isomorphic or not.



6 + 4 + 5

11. a) Apply Prim's algorithm to find a minimal spanning tree for the following weighted graph :



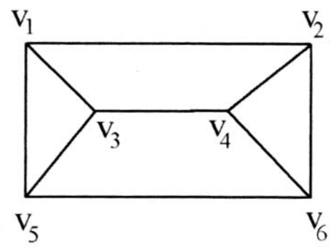
b) Show that if every element of a group is its own inverse, then it is an Abelian group.

c) Show that $f : R \to R, f(x) = 2x + 3$ is a bijective mapping. 6 + 4 + 5

12. a) Show that a ring R satisfies cancellation law if and only if R is without zero divisor. 5

b) A relation ρ is defined on Z by "$a \rho b$ if and only if $a^2 - b^2$ is divisible by 5" for $a, b \in Z$. Prove that ρ is an equivalence relation on Z. Show that there are three distinct equivalence classes. 3 + 2

c) Draw the dual of the following graph :



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