Name : ....
Roll No. : ....
Invigilator's Signature : ....

CS/B.Tech(ECE,EE,EIE,EEE,PWE,BME,ICE)/SEM-3/M-302/2009-10

2009

MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) The probability that a leap-year selected at random will contain 53 sundays is

a) $\frac{3}{7}$ b) $\frac{2}{7}$

c) $\frac{5}{7}$ d) $\frac{4}{9}$.

ii) If a coin is tossed 6 times in succession, the probability of getting at least one head is

a) $\frac{63}{64}$ b) $\frac{3}{64}$

c) $\frac{7}{63}$ d) None of these.

iii) The probability that the 4 children of a family have different birthdays is

a) 0·9836 b) 0·4735

c) 0·9 d) 0·757.

iv) A tree has n vertices. The number of its edges is

a) $n + 1$ b) $n - 1$

c) $2n$ d) none of these.

v) The value of m such that $3y - 5x^2 + my^2$ is a harmonic function is

a) 5 b) -5

c) 0 d) 3.

vi) Let X and Y be two random variables such that $Y = a + bx$ where a and b are constants. Then, Var (y) is

a) b^2 Var (X) b) Var (X)

c) a^2 Var (X) d) (b/a) Var (X).

vii) The value of $\int_C \frac{dz}{z+3}$ where C is a circle $|z| = 1$ is

a) 0 b) 1

c) 2 d) -1.

viii) If $f(z) = \frac{1}{z^4 - 2z^3}$, then $z = 0$ is a pole of order

a) 3 b) 2

c) 1 d) 4.

ix) The order of the pole $z = 0$ of the function $\frac{\sin z}{z^3}$ is

a) 1 b) 2

c) 3 d) 4.

x) The period of the function $f(x) = \sin 2\pi x$ is

a) $\frac{1}{2}$ b) 1

c) 0 d) $\frac{1}{3}$.

xi) If $f(x) = x \sin x, \; -\pi \le x \le \pi$, be presented in Fourier series as $\frac{a_o}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$,

then the value of a_o will be

a) 2 b) 0

c) 4 d) 1.

xii) If two variables x and y are uncorrelated, then r_{xy} is

a) 1 b) 2

c) 3 d) 0.

xiii) If $x = 4y + 5$ be a regression line of x on y then bxy is

a) $\frac{1}{4}$ b) 4

c) 0 d) 1.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Show that $f(x)$ given by

$$f(x) = x ; \quad 0 < x < 1$$

$$= k - x ; \quad 1 < x < 2$$

$$= 0 ; \quad elsewhere,$$

is a probability density function for a suitable value of k. Calculate the probability that the random variable lies between $\frac{1}{2}$ and $\frac{3}{2}$.

3. Find the Fourier sine transform of $\frac{e^{-ax}}{x}$.

4. Evaluate $\int_c \frac{3z^2 - 2}{z - 1}\, dz$, where c is the circle $|z| = \frac{1}{2}$.

5. An urn contains 3 white and 5 black balls. One ball is drawn and its colour is unnoted, kept aside and then another ball is drawn. What is the probability that it is (i) black (ii) white ?

6. Find the mean and standard deviation of a bionomial distribution.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) If A and B are mutually independent events, prove that A^C and B^C are also mutually independent events.

b) There are three identical urns containing white and black balls. The first urn contains 3 white and 4 black balls, the 2nd urn contains 4 white and 5 black balls and the 3rd urn contains 2 white and 3 black balls. An urn is chosen at random and a ball is drawn from it. If the drawn ball is white, what is the probability that the 2nd urn chosen ?

c) A random variable X has the following p.d.f.

$$f(x) = cx^2 \quad 0 \le x \le 1$$
$$= 0, \; otherwise.$$

Find (i) c (ii) $P\left(0 \le X \le \frac{1}{2}\right)$. 5 + 5 + 5

8. a) Find the Fourier series expansion of the periodic function of period 2π,

$f(x) = x^2$, $-\pi \le x \le \pi$. Hence deduce

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots\dots\dots + \dots\dots = \frac{\pi^2}{12}.$$

b) The following marks have been obtained by students in Mathematics and Statistics (out of 100) :

Maths	45	55	56	58	60	65	68	70	75	80	85
Stats	56	50	48	60	62	64	65	70	74	82	90

Compute the co-efficient of correlation for the above data. Find also the equations of the lines of regrssion.

7 + 8

9. a) Solve

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}, \; x > 0, \; t > 0,$$

if $u(0, t) = 0$, $u(x, 0) = e^{-x}$, $x > 0$, $u(x, t)$ is *unbounded.*

b) If $f(z)$ is a regular function of z, then prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) | f(z) |^2 = 4 \, | f'(z) |^2 .$$ 8 + 7

10. a) Apply *Dijkstra's algorithm* to determine a shorterst path between s to z in the following graph :

Dia.

b) Define isomorphism of two graphs. Examine whether the following graph G and G' are isomorphic. Give reasons.

Dia.

8 + 7

11. a) Use residue theorem to evaluate $\int_C \frac{3z^2 + z - 1}{(z^2 - 1)(z - 3)}\, dz$ around the circle $|z| = 2$.

b) Expand the function $f(z) = \frac{1}{(z^2 + 1)(z + 2)}$ in the region $|z| < 1$.

c) Show that the function $f(z) = \begin{cases} \frac{3xy^2}{x^2 + y^2} & \text{for } z \neq 0 \\ 0 & \text{for } z = 0 \end{cases}$ is continuous at $z = 0$. 5 + 7 + 3

12. a) Show that a simple graph with n vertices and k-components can have at most $\frac{(n - k)(n - k + 1)}{2}$ edges.

b) Find the incidence matrix of the following graph.

Dia.

c) Find the Fourier sine transform of the functon

$$f(x) = \begin{cases} 1 & \text{for } 0 < x \leq \pi \\ 0 & \text{for } x > \pi \end{cases}$$

and hence evaluate the integral

$$\int_0^\infty \frac{1 - \cos p\pi}{p} \sin pxdp.$$ 5 + 5 + 5