ENGINEERING & MANAGEMENT EXAMINATIONS, DECEMBER – 2006

MATHEMATICS

SEMESTER – 1

Time : 3 Hours] [Full Marks : 70

GROUP – A

(Objective Questions)

1. Answer any *ten* of the following : $10 \times 1 = 10$

A. Choose the correct alternatives :

i) The sequence $\{(-1)^n\}$ is

a) convergent b) oscillatory

c) divergent d) none of these.

ii) If $\vec{\alpha} = 3i - 2j + k$, $\vec{\beta} = 2\vec{i} - \vec{k}$, then $(\vec{\alpha} \times \vec{\beta}) \cdot \vec{\alpha}$ is equal to

a) $i + j + k$ b) $i + k$

c) 0 d) 2.

iii) If $f(x) = \dfrac{\sin x}{x}$ $(x \neq 0)$, then $\lim\limits_{x \to 0} f(x)$ is equal to

a) 0 b) 1

c) $\dfrac{1}{2}$ d) -1.

iv) The series $\sum \dfrac{1}{n^p}$ is convergent if

a) $p \geq 1$ b) $p > 1$

c) $p < 1$ d) $p \leq 1$.

B. Fill in the blanks :

v) If $x = r\cos\theta$, $y = r\sin\theta$, then $\dfrac{\partial(x, y)}{\partial(r, \theta)}$ is equal to ....

vi) If A, B and C are angles made by a line with positive directions of co-ordinate axes, then $\cos^2 A + \cos^2 B + \cos^2 C$ is equal to ....

C. Answer the question very briefly :

vii) Give an example of a sequence which is bounded but not convergent.

D. Choose the correct alternatives :

viii) The moment of inertia of a thin uniform rod of mass M and length $2a$ about an axis perpendicular to the rod at its centre is

a) $\dfrac{Ma^2}{3}$ b) $\dfrac{Ma^2}{2}$

c) Ma^2 d) $\dfrac{Ma^2}{4}$.

ix) If $\phi = 3x^2y - y^3 z^2$, then Grad ϕ at (1, – 2, – 1) is

a) $-16i + 12j - 9k$ b) $-9i - 12j + 16k$

c) $-12i - 9j - 16k$ d) $12i + 16j - 9k$.

E. Fill in the blank :

x) Degree of homogeneity of $ax^2 + 2hxy + by^2$ is equal to ....

F. Answer the following questions very briefly :

xi) Evaluate $\int_0^{\pi/2} \frac{\cos x}{\sin x + \cos x}\, dx$.

xii) A, B, C and D are points of (α, 3, – 1), (3, 5, – 3), (1, 2, 3) and (3, 5, 7) respectively. If AB is perpendicular to CD then find the value of α.

Group - B
(Short Answer Questions)

Answer any *three* questions. $3 \times 5 = 15$

2. If a, b, c are three vectors, show that

$[\, a \times b,\ b \times c,\ c \times a \,] = [\, a, b, c \,]^2$. Symbols have their usual meanings.

3. Find the length of the perimeter of Asteroid $x = a\cos^3\theta$, $y = a\sin^3\theta$.

4. State Rolle's theorem and examine if you can apply the same for $f(x) = \tan x$ in $[0, \pi]$.

5. Show that
$$f(x, y) = \frac{2xy}{x^2 + y^2} \quad \text{for } (x, y) \neq (0, 0)$$
$$= 0 \quad \text{for } (x, y) = (0, 0).$$
is not continuous at (0, 0).

6. If $y = \left(x^2 - 1\right)^n$, show that $\left(x^2 - 1\right) y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0$.

7. Find the extrema of $f(x, y) = x^3 + y^3 - 63(x + y) + 12xy$.

Group - C
(Long Answer Questions)

Answer any *three* questions. $3 \times 15 = 45$

8. a) Examine continuity and differentiability of $f(x)$ at $x = 0$, when $f(x) = x\sin\frac{1}{x}$; $(x \neq 0)$ and $f(0) = 0$.

b) If $u = \tan^{-1}\frac{x^2 + y^2}{x - y}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$.

c) Test for convergence of $\sum_{n=1}^{\infty} \frac{n^2 - 1}{n^2 + 1} x^n$; $x > 0$. $3 \times 5 = 15$

9. a) Expand $\log_e (1 + x)$ in ascending power of x stating the condition of convergence.

b) Evaluate $\iint \sqrt{\frac{1 - x^2 - y^2}{1 + x^2 + y^2}}\, dx\, dy$ over the positive quadrant of the circle $x^2 + y^2 = 1$.

c) State Leibnitz theorem and apply it to examine the covergence of

$1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots\ldots$. $3 \times 5 = 15$

10. a) Obtain the length of loop $5y^2 = (x - 1)(x - 2)^2$.

b) Using divergence theorem evaluate $\iint_S \vec{u} \cdot \vec{n}\, ds$

where $\vec{u} = xi + yj + zk$ and S is the sphere $x^2 + y^2 + z^2 = 9$ and $\vec{n}$ is outward normal to S.

c) A variable plane is at a constant distance p from origin and meets co-ordinate axes in A, B and C. The planes are drawn through A, B and C and parallel to co-ordinate axes. Show that locus of their point of intersection shall be

$\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{p^2}$. $3 \times 5 = 15$

11. a) If $u = \frac{x + y}{1 - xy}$ and $v = \tan^{-1}x + \tan^{-1}y$, find $\frac{\partial(u, v)}{\partial(x, y)}$.

b) If vector functions $\vec{F}$ and $\vec{G}$ are irrotational, show that $\vec{F} \times \vec{G}$ is solenoidal.

c) Verify Stoke's theorem for $\vec{F} = (2x - y)\,i - yz^2 j - y^2 zk$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. $3 \times 5 = 15$

12. a) Obtain a reduction formula for $\int_0^{\pi/2} \sin^n x\, dx$ and evaluate $\int_0^{\pi/2} \sin^5 x\, dx$.

b) If $z = f(x, y)$ where $x = e^u \cos v$, $y = e^u \sin v$, show that

$y \frac{\partial z}{\partial u} + x \frac{\partial z}{\partial v} = e^{2u} \frac{\partial z}{\partial y}$.

c) Obtain the equation of the plane through straight line $3x - 4y + 5z - 10 = 0$,

$2x + 2y - 3z - 4 = 0$ and parallel to the line $x = 2y = 3z$. $3 \times 5 = 15$