Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/B.Tech (EE)/SEM-3/CS-312/2009-10

2009

NUMERICAL METHODS AND PROGRAMMING

Time Allotted : 3 Hours | Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

   i) The value of x that satisfies $f(x) = 0$ is called the
      a) root of an equation $f(x) = 0$
      b) root of a function $f(x)$
      c) zero of equation $f(x) = 0$
      d) none of these.

   ii) Given $A = \begin{pmatrix} 6 & 2 & 3 & 9 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 4 & 5 \\ 0 & 0 & 0 & 6 \end{pmatrix}$, then A is a ..... matrix.
      a) diagonal
      b) identity
      c) lower triangular
      d) upper triangular.

iii) To solve the ordinary differential equation $3\frac{dy}{dx}+5y^2=\sin x, y(0)=5$ by Euler's method, you need to rewrite the equation as

a) $\frac{dy}{dx}=\sin x-5y^2, y(0)=5$

b) $\frac{dy}{dx}=\frac{1}{3}(\sin x-5y^2), y(0)=5$

c) $\frac{dy}{dx}=\frac{1}{3}\left(-\cos x-\frac{5y^2}{3}\right), y(0)=5$

d) $\frac{dy}{dx}=\frac{1}{3}\sin x, y(0)=5.$

iv) $f(x)$ is a polynomial of degree 3 if

a) $\Delta^3 f(x)=0$ b) $\Delta^3 f(x)=$ constant

c) $\Delta f(x)=$ constant d) $F^3 f(x)=$ constant.

v) The equation $AX = B$ has unique solution if

a) Rank (A) ≠ Rank (A, B)

b) Rank (A) < Rank (A, B)

c) Rank (A) = Rank ($A\ B$) = Number of unknowns

d) Rank (A) = Rank ($A\ B$) ≠ Number of unknowns.

vi) An approximate polynomial passes through (n + 1) data points, the degree of the polynomial is

a) n + 1 b) n

c) n or less d) n + 1 or less.

vii) If the interval of differencing is unity and $f(x)=ax^2$ (a is a constant), which one of the following choices is *wrong* ?

a) $\Delta f(x)=a(2x+1)$ b) $\Delta^2 f(x)=2a$

c) $\Delta^3 f(x)=2$ d) $\Delta^4 f(x)=0.$

viii) Output of the following programme code :

```
main ( )
{
    int i;
    for (i=1;1<=2;i++)
    {
        printf ("%d\n",i);
        if (i==1)
        continue;
        printf("ELECTRICAL \n");
    }
    printf("ENGINEERING");
}
```

a) 2 1 ELECTRICAL ENGINEERING

b) 1 2 ELECTRICAL ENGINEERING

c) ELECTRICAL ENGINEERING

d) none of these.

ix) The condition of convergence of Newton-Raphson method when applied to an equation $f(x) = 0$ in an interval is

a) $f'(x) \neq 0$

b) $|f'(x)| < 1$

c) $|f(x)f''(x)| < [f'(x)]^2$

d) $[f''(x)]^2 > |f(x).f'(x)|$.

x) The error in Simpson's one third rule is of

a) $O(h^2)$

b) $O(h^3)$

c) $O(h^4)$

d) $O(h^5)$.

xi) DDL stands for

a) Data Defined Language

b) Data Dictionary Language

c) Data Definition Language

d) Dictionary Defined Lanuage.

xii) For $n = 1$, Lagrange's interpolation formula becomes

a) an equation of a straight line

b) an equation of a parabola

c) an equation of a hyperbola

d) none of these.

xiii) The polynomial function $f(x)$ constructed from the data $f(3) = -1, f(4) = 5, f(5) = 15$ is

a) $2x^2 + 8x + 5$

b) $x^2 - 8x - 5$

c) $x^2 + 8x + 5$

d) $2x^2 - 8x + 5$.

GROUP – B
(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Define absolute error and relative error. Prove that the relative error of a product of several approximate non-zero numbers does not exceed the sum of the relative errors of the numbers.

3. Explain secant method briefly with a diagram and arrive at the secant formula.

4. Solve the equation $x^3 - 2x - 5 = 0$ with $2 \le x \le 3$ using Regula-Falsi (false position) method. Continue up to 3 successive approximations.

5. Construct Lagrange's interpolation polynomial for the function $y = \sin(\pi x)$, choosing the points $X_0 = 0, x_1 = \frac{1}{6}, x_2 = \frac{1}{2}$.

6. Solve the following equations using Gauss-Seidel Method :

$$2x_1 + 2x_2 = 1$$

$$x_1 + 2x_2 + x_3 = 2$$

$$x_2 + x_3 = 4$$

Start with the initial approximated values of $x_1 = 1$, $x_2 = 2$ and $x_3 = 1$ and continue up to 3 successive approximations.

GROUP – C
(Long Answer Type Questions)

Answer any *three* of the following. 3 × 15 = 45

7. a) Explain LU decomposition method for solving linear equation. Calculate L and U for the given set of equations :

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

b) Solve the following equation using LU decomposition method :

$$3x_1 + 2x_2 + x_3 = 10$$

$$2x_1 + 3x_2 + 2x_3 = 14$$

$$x_1 + 2x_2 + 3x_3 = 14$$

3 + 3 + 9

8. a) Explain bisection method (halving method). What is the convergence criterion for bisection method ?

b) Solve $x^2 - 4x - 10 = 0$ using bisection method in the range [– 1, – 2] continued up to 6th approximation.

3 + 3 + 9

9. a) Arrive at Newton's forward interpolation formula.

b) Consider the table below :

x	1	2	3	4	5	6	7	8	9	10
y	2	6	12	20	30	42	56	72	90	110

Evaluate y, when $x = 8{\cdot}4$ using Newton's forward interpolation formula.

c) Define dividend difference expression with two and three arguments. 5 + 6 + 4

10. a) Arrive at the formula for basic Trapezoidal rule.

b) Using basic Trapezoidal rule, arrive at Composite Trapezoidal rule.

c) Calculate the integral $\int_{-1}^{+1} e^x \, dx$ using composite trapezoidal rule for

i) $n = 2$ and

ii) $n = 4$. 5 + 4 + 6

11. a) Using Taylor's series, solve the differential equation $dy/dx = x^2 + y^2$ for $x = 0{\cdot}25$ and $x = 0{\cdot}5$.

b) Arrive at Euler's formula as numerical solution of ordinary differential equation.

c) Using Euler's formula, solve $dy/dx = 2x + 3$, subject to the initial condition $y = 1$ at $x = 1$ for value of x changing from 1 to $1{\cdot}5$ in an interval of $0{\cdot}1$. Calculate y_1, y_2, y_3 and y_4. 4 + 5 + 6