Name : ……………………………………………………………

Roll No. : ……………………………………………………………

Invigilator's Signature : ……………………………………….

CS / B.TECH (EE (N), EIE, EEE, PWE, BME, ICE, ECE) / SEM-3 / M-302 / 2010-11

2010-11

MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :

10 × 1 = 10

i) If $F[f(x)] = F(s)$ represents the Fourier transform of the function $f(x)$, then $F[f(ax)]$ ('a' being a constant) equals

a) $F(s/a)$ b) $a\,F(s)$

c) $(1/|a|)F(s/a)$ d) $(1/a^2)F(as)$.

ii) A function $f(x)$, $a < x < b$, can be expanded in a Fourier series

a) only if it is continuous everywhere

b) even if it is discontinuous at a finite number of points in (a, b)

c) even if it is unbounded in (a, b)

d) only if it is both continuous & bounded in (a, b).

iii) Three unbiased coins are tossed simultaneously. This is repeated four times. Then the probability of getting at least one head each time is

a) $(1/8)^4$ b) $(2/8)^4$

c) $(7/8)^4$ d) $(3/8)^4$.

iv) For a Poisson distribution $P(X)$ is $P(1) = P(2)$, then $P(0)$ is

a) $1/e$ b) $1/e^2$

c) $1/e^3$ d) none of these.

v) A graph has 10 vertices and 15 edges. Its circuit rank is

a) 25 b) 12

c) 6 d) 5.

vi) A binary tree has 11 vertices. The minimum and maximum height of the tree is

a) (4, 5)
b) (3, 5)
c) (3, 10)
d) (4, 10).

vii) If $f(x)$ is an odd function then $\mathcal{F}(f(x))$ is given by

a) $F(s) = 2F_s(s)$
b) $F(s) = 2iF_s(s)$
c) $F(s) = 0 \cdot 5iF_s(s)$
d) $2F(s) = iF_s(s)$,

where $\mathcal{F}$ denotes Fourier Transform.

viii) The order of pole $z = 0$ of the function $\frac{\cos z}{z^3}$ is

a) 2
b) 1
c) 3
d) 4.

ix) If X is normally distributed with zero mean and unit variable, then the expectation of X^2, is

a) 1
b) 0
c) 8
d) 2.

x) The maximum and minimum values for correlation coefficient are

a) 1, 0
b) 2, 1
c) 0, –1
d) 1, –1.

xi) If a simple graph has 15 edges then sum of the degrees of all the vertices is

a) 25 b) 24

c) 50 d) 30.

xii) A closed walk in which no vertex (except is terminal vertices) appear more than once is called

a) path b) Eulerian circuit

c) circuit d) trail.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following $3 \times 5 = 15$

2. If $f(z) = \dfrac{xy^2(x+iy)}{x^2+y^4}, z \neq 0$ & $f(0) = 0$, then prove that $\dfrac{f(z) - f(0)}{z} \to 0$ as $z \to 0$ along any radius vector but not as $z \to 0$ in any manner.

3. If f is analytic function then show that $\nabla^2 \, | f(z) |^2 = 4 \dfrac{\partial (u,v)}{\partial (x,y)}$ where $f(z) = u + iv$ and $z = x + iy$.

4. Expand the following function in a Fourier series in $[-\pi, \pi]$

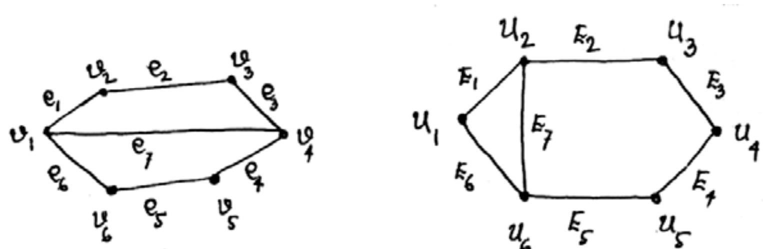
$$f(x) = \begin{cases} -\frac{1}{2}(\pi + x) & \text{when } -\pi \leq x < 0 \\ \frac{1}{2}(\pi - x) & \text{when } 0 \leq x < \pi \end{cases}$$

5. Show that $f(x)$ given by

$$f(x) = \begin{cases} x & \text{for } 0 < x < 1 \\ k - x & \text{for } 1 < x < 2 \\ 0 & \text{elsewhere} \end{cases}$$

is a probability density function for a suitable value of k. Calculate the probability that the random variable lies between 1/2 and 3/2.

6. Define isomorphism of two graphs. Show whether the following graphs are isomorphic or not :



$G_1(V_1, E_1)$ $\qquad$ $G_2(V_2, E_2)$

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Consider Heavyside unit function

$$h(1-|t|) = 0, |t| > 1$$
$$= 1, |t| \leq 1$$

Prove that $F^{-1}(\sin s / s) = h(1-|x|)$ where F^{-1} is the inverse Fourier transform i.e., $F^{-1}(F(s)) = f(t)$.

b) Using Parseval's identity of Fourier transform prove that

$$\int_0^{\infty}(1-\cos x/x)^2 dx = \pi/2$$

c) Using Fourier transform solve the heat equation

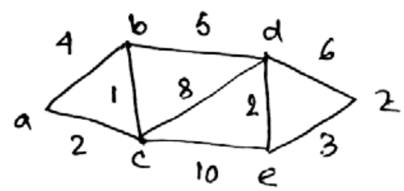
$\delta^2 u/\delta x^2 = (1/c^2)(\delta u/\delta x), -\infty < x < \infty, t > 0$

with boundary condition $u(x,t) \to 0,\ \delta u(x,t)/\delta x \to 0$

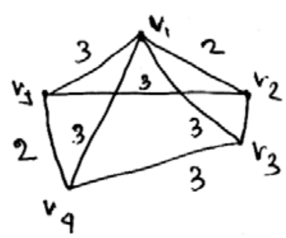
as $|x| \to \infty$ & initial condition $u(x,0) = e^{-x^2/4c^2}, -\infty < x < \infty$

3 + 4 + 8

8. a) Using Dijkstra's algorithm find the length of the shortest path of the following graph :



b) Find by Prim's Algorithm a minimum spanning tree from the following graph :



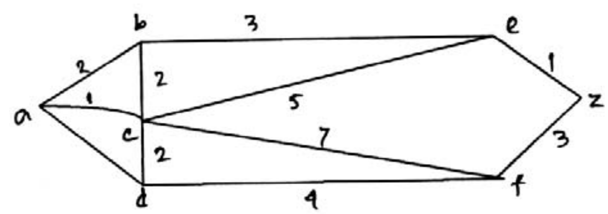
8 + 7

9. a) Solve the differential equation :

$k\,\partial^2 u/\partial x^2 = \partial u/\partial t,\ -\infty < x < \infty,\ t > 0$

with $u(x,t) = 0$ as $x \to \pm\infty$, $\partial u/\partial t = 0$ as $x \to \pm\infty$ and $u(x,0) = f(x),\ -\infty < x < \infty$.

b) Apply Dijkstra's algorithm to determine a shortest path between a to z in the following graph.



10. a) The probability density function of a random variable X is $f(x) = K(x-1)(2-x)$, for $1 \le x \le 2$.

$= 0$, otherwise.

Determine –

(i) the value of the constant k and

(ii) $P\left(\frac{5}{4} \le X \le \frac{3}{2}\right)$.

b) In a normal distribution, 31% of the items are under 45 and 8% are above 64. Find the mean and standard deviation. [Given that $P(0 < Z < 1\cdot405) = 0\cdot42$ and $P(-0\cdot496 < Z < 0) = 0\cdot19$]

c) If the equations of two Regression lines obtained in a correlation analysis are $3x + 12y - 19 = 0$ and $9x + 3y = 46$. Determine which one is Regression equation of y on x and which one is the regression equation of x on y. Find the means of x on y and correlation coefficient between x and y. 4 + 5 + 6

11. a) If $f(x) = \begin{cases} 0 & -\pi \le x \le 0 \\ \sin x & 0 \le x \le \pi \end{cases}$, prove that

$$f(x) = \frac{1}{\pi} + \frac{1}{2}\sin x - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\cos 2nx}{4n^2 - 1}$$

Hence show that

$$\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \ldots\ldots = \frac{1}{2}.$$

b) Evaluate $\int_C \frac{4 - 3z}{(z-1)z(z-3)} dz$, where C is the circle $|z| = \frac{5}{2}$.

c) Show that $u(x, y) = x^3 - 3xy^2$ is harmonic in C and find a function $v(x, y)$ such that $f(z) = u + iv$ is analytic.

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