Tutorial on Z Algorithm

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Z algorithm is a linear time string matching algorithm which runs in $O(n)$ complexity. It is used to find all occurrence of a pattern P in a string S, which is common string searching problem.

Algorithm

Given a string S of length n, the Z Algorithm produces an array Z where $Z[i]$ is the length of the longest substring starting from $S[i]$ which is also a prefix of S, i.e. the maximum k such that $S[j] = S[i + j]$ for all $0 \le j < k$. Note that $Z[i] = 0$ means that $S[0] \neq S[i]$. For easier terminology, let's refer to substrings which are also a prefix as prefix-substrings.

The algorithm relies on a single, crucial invariant. Iterate over the letters in the string (index i from 1 to $n - 1$) and maintain an interval $[L,\ R]$ which is the interval with maximum R such that $1 \le L \le i \le R$ and $S[L \ldots R]$ is a prefix-substring (if no such interval exists, just let $L = R = \ -1$). For $i = 1$, simply compute L

and R by comparing $S[0...]$ to $S[1...]$. $Z[1]$ is obtained during this process.

Now, suppose the correct interval $[L, \ R]$ for $i - 1$ and all of the Z values up to $i - 1$. Compute $Z[i]$ and the new $[L, \ R]$ by the following steps:

- If $i > R$, then there does not exist a prefix-substring of S that starts before i and ends at or after i. If such a substring existed, $[L, \ R]$ would have been the interval for that substring rather than its current value. Thus "reset" and compute a new $[L, \ R]$ by comparing $S[0...]$ to $S[i. . .]$ and get $Z[i]$ at the same time ($Z[i] = R - L + 1$).

- Otherwise, $i \leq R$, so the current $[L, \ R]$ extends at least to i. Let $k = i - L$. It is known that $Z[i] \geq min(Z[k], \ R - i + 1)$ because $S[i. . .]$ matches $S[k. . .]$ for at least $R - i + 1$ characters (they are in the $[L, \ R]$ interval which is known to be a prefix-substring).

- If $Z[k] < R - i + 1$, then there is no longer prefix-substring starting at $S[i]$ (or else $Z[k]$ would be larger), meaning $Z[i] = Z[k]$ and $[L, \ R]$ stays the same. The latter is true because $[L, \ R]$ only changes if there is a prefix-substring starting at $S[i]$ that extends beyond R, which is not the case here.

- If $Z[k] \geq R - i + 1$, then it is possible for $S[i...]$ to match $S[0...]$ for more than $R - i + 1$ characters (i.e. past position R). Thus, there's a need to update $[L,\ R]$ by setting $L = i$ and matching from $S[R + 1]$ forward to obtain the new R. Again, $Z[i]$ is obtained during this process.

The process computes all of the Z values in a single pass over the string, so we're done. Correctness is inherent in the algorithm and is pretty intuitively clear.

Time Complexity

The algorithm runs in $O(n)$ time. Characters are never compared at positions less than R, and every time a match is found, R is increased by one, so there are at most n comparisons.

Implementation

Note that the optimization $L = R = i$ is used when $S[0] \neq S[i]$ (it doesn't affect the algorithm since at the next iteration $i > R$ regardless).

```
int L = 0, R = 0;
for (int i = 1; i < n; i++)
{
    if (i > R)
    {
```

```
            L = R = i;
            while (R < n && s[R-L] == s[R])
            {
                R++;
            }
            z[i] = R-L;
            R--;
        }
        else
        {
            int k = i-L;
            if (z[k] < R-i+1)
            {
                z[i] = z[k];
            }
            else
            {
                L = i;
                while (R < n && s[R-L] == s[R])
                {
                    R++;
                }
                z[i] = R-L;
                R--;
            }
        }
}
```

Applications

One application of the Z Algorithm is for the standard string matching problem of finding matches for a pattern T of length m in a string S of length n. This can be done in $O(n + m)$ time by using the Z

Algorithm on the string $T\ \Phi\ S$ (that is, concatenating T, Φ, and S) where Φ is a character that matches nothing. The indices i with $Z[i] = m$ correspond to matches of T in S.

TEST YOUR UNDERSTANDING

Count Substring Occurrences

Given 2 Strings C, S. Find the number of occurrences of C in S. Length of is N, length of C is M.

Input:

Given 2 Strings C, S.

Output:

Print the number of occurrences of C in S.

Constraints:

$1 \leq M \leq N \leq 10^5$.

SAMPLE INPUT

```
abc
abcabdabc
```

SAMPLE OUTPUT

```
2
```

Enter your code or Upload your code as file.

All changes saved

Visual Basic (mono vbnc 4.0.1)

```
Loading...
```

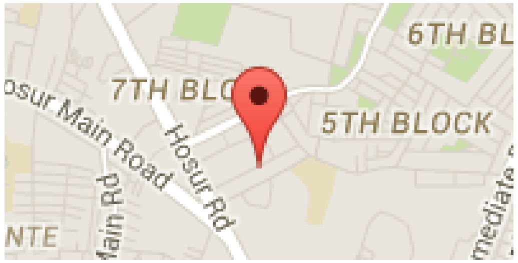
1:1

COMPILE & RUN

SUBMIT

Press ctrl-space for autocomplete suggestions.

POWERED BY codetable



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