

First let us see the definition of **Euler's Totient Function**.

Euler's totient function denoted as $\phi(n)$, is an arithmetic function that counts the positive integers less than or equal to N that are relatively prime to N .

For this question we need to find the sum of all $\phi(n)$ for all i between 1 to N . We see that the constraints are very high and we have multiple test cases, so we would prefer to pre-process all the values in one go so that we can answer each query in **$O(1)$ time**.

How can we find $\phi(N)$ for a single N ?

Properties of Euler's Totient Function

1. If N is prime then $\phi(N) = N-1$, as all numbers less than N are relatively prime to N .
2. If N is prime and $k > 0$, then $\phi(N^k) = N^k - N^{k-1}$.
3. If x and y are relatively prime, then $\phi(x.y) = \phi(x) \times \phi(y)$.

Now we will use these properties to find $\phi(N)$ when N is not prime.

$N = P_1^{k_1} \cdot P_2^{k_2} \dots P_m^{k_m}$, where P_i are the prime factors of N .

$$\phi(N) = \phi(P_1^{k_1}) \cdot \phi(P_2^{k_2}) \dots \phi(P_m^{k_m})$$

$$\text{Let } X = P_1^{k_1} \cdot P_2^{k_2} \dots P_{m-1}^{k_{m-1}}$$

$$\phi(N) = \phi(X) \cdot \phi(P_m^{k_m})$$

$$\phi(N) = (P_1^{k_1} - P_1^{k_1-1})(P_2^{k_2} - P_2^{k_2-1}) \dots (P_m^{k_m} - P_m^{k_m-1})$$

$$\phi(N) = \phi(X)(P_m^{k_m}(1 - (1/P_m)))$$

Here we see that X will always be less than N thus if we use dynamic programming to solve for $\phi(N)$ we will always have the value of $\phi(X)$. All that remains is to find P_m and $P_m^{k_m}$.

$P_m^{k_m} = N/X$ In order to find P_m we can simply modify the [Sieve of Eratosthenes](#) to store the largest prime that divides any number.

```
for (int i=2; i <= MAX; i++)
    if (prime[i] == 0)
        for (int j = i + i; j <= MAX; j += i)
            prime[j] = i;
```

If **prime[N]** is equal to **0**, then the N is prime or else it represents the largest prime that divides N .

When **prime[N]** is not equal to **0**, we can find X by dividing N with **prime[N]** until N is divisible by **prime[N]**. Now since we have all the requisites to calculate $\phi(N)$ we can do it dynamically by iterating from 1 to N .

Now once we have the values of $\phi(N)$ for all possible N , we can simply pre-process the sum till the i^{th} position so that we can answer each query in **$O(1)$** after preprocessing of **$O(N \log N)$** .