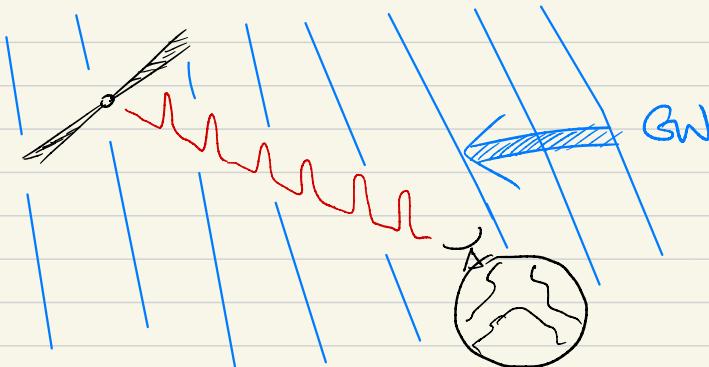


Timing Response Due to GWs

- Timing response to GWs



GW spacetime line element ($G=c=1$)

$$ds^2 = -dt^2 + [g_{ab} + h_{ij}^{TT}(t, \vec{x})] dx^i dx^j$$

\Rightarrow a^i, b^j = spatial components
 a, b = different pulsars.

Near photon path ... $ds^2 = 0$

↳ along x-axis toward observer (@ origin)

$$0 = -dt^2 + (1 + h_x^{TT}(t, \vec{x})) dx^2$$

$$dx^2 = (1 + h_{xx}^{TT}(t, \vec{x}))^T dt^2$$

$$\therefore dx \approx - \left(1 - \frac{1}{2} h_{xx}^{TT}(t, \vec{x})\right) dt$$

moving towards origin

⇒ Consider Earth-pulsar distance, L.

$$\int_{x_p}^{x_E} dx = -L = - \int_{t_{em}}^{t_{obs}} dt' \left(1 - \frac{1}{2} h_{xx}^{TT}(t', \vec{x})\right)$$

$$L = (t_{obs} - t_{em}) - \frac{1}{2} \int_{t_{em}}^{t_{obs}} h_{xx}^{TT}(t', \vec{x}) dt'$$

• Assumption : $h_{xx}^{TT}(t, \vec{x}) \ll 1$

$$\therefore t_{obs} \approx t_{em} + L$$

photon path \approx along unperturbed trajectory

$$\vec{x}(t) = (t_{obs} - t) \hat{p}$$

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\Rightarrow generalize to arbitrary photon origin direction

$$h_{\alpha\alpha}^{TT} \longleftrightarrow p^i p^j h_{ij}^{TT}$$

\neq

Thus, $t_{obs} = t_m + L + \frac{p^i p^j}{2} \int_{t_m}^{t_m+L} dt' h_{ij}^{TT}[t', (t_m+L-t')\hat{p}]$

PULSE #1

- Consider a 2nd pulse emitted one rotational period after the first.

$$\Rightarrow t'_{an} = t_m + P$$



$$t'_{obs} - t_{obs} = P + \Delta P$$

extra from GWS.

- Replace $t_m \rightarrow t_m + P$ and insert into t_{obs}

$$\Rightarrow t'_{obs} = t_m + P + L + \frac{p^i p^j}{2} \int_{t_m+P}^{t_m+P+L} dt' h_{ij}^{TT}[t', (t_m+P+L-t')\hat{p}]$$

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$$t'' = t' - p \Rightarrow \text{redefine variables}$$

$$t'_{abs} = t_{em} + p + L + \frac{p^i p^j}{2} \int_{t_{em}+p}^{t_{em}+p+L} dt' h_{ij}^{TT} [t', (t_{em}+p+L-t')\hat{p}]$$

$$= t_{em} + p + L + \frac{p^i p^j}{2} \int_{t_{em}}^{t_{em}+L} dt'' \cdot h_{ij}^{TT} [t'' + p, (t_{em} + L - t'')\hat{p}]$$

Thus $t'_{abs} - t''_{abs} = p + \frac{p^i p^j}{2} \int_{t_{em}}^{t_{em}+L} dt' \left\{ h_{ij}^{TT}(t'+p, \vec{x}_o(t)) - h_{ij}^{TT}(t, \vec{x}_o(t)) \right\}$

$\vec{x}_o(t) = (t_{em} + L - t)\hat{p}$

$$\Rightarrow P \sim 10^{-3} \text{ s}$$

$$P_{GW} \sim 1/10^9 \text{ Hz} \sim 10^9 \text{ s}$$

$$S_0 \approx h_{ij}^{TT}(t+p, \vec{x}_o(t)) \approx h_{ij}^{TT}(t, \vec{x}_o(t))$$

Taylor expanding

$$+ \left. \frac{\partial h_{ij}^{TT}(t, \vec{x})}{\partial t} \right|_{\vec{x}=\vec{x}_o(t)} \times p$$

$$\Rightarrow \frac{\Delta P}{P} = \frac{p^i p^j}{2} \int_{t_{em}}^{t_{em}+L} dt' \left[\frac{1}{8t'} \cdot h_{ij}^{TT}(t, \vec{x}) \right]$$

$\vec{x} = \vec{x}_o(t)$

\Rightarrow Consider monochromatic GW propagating in direction $\hat{\Omega}$.

$$h_{ij}^{TT}(t, \vec{x}) = A_{ij}(\hat{\Omega}) \cos[\omega_{gw}(t - \hat{\Omega} \cdot \vec{x})]$$

$$\begin{aligned} \therefore \frac{\Delta P}{P} &= \frac{1}{2} \frac{p^i p^j A_{ij}}{(1 + \hat{\Omega} \cdot \hat{p})} \times \left\{ \begin{array}{l} \cos(\omega_{gw} t_{abs}) \\ -\cos(\omega_{gw} t_{em} - \omega_{gw}(t_{abs} - t_{em}) \hat{\Omega} \cdot \hat{p}) \end{array} \right\} \\ &= \frac{1}{2} \frac{p^i p^j A_{ij}}{1 + \hat{\Omega} \cdot \hat{p}} \left\{ \cos(\omega_{gw} t_{abs}) - \cos(\omega_{gw}(t_{em} - L \hat{\Omega} \cdot \hat{p})) \right\} \end{aligned}$$

Finally \Rightarrow GW-induced redshift to the arrival rate of radio pulses ... $z(t) = \frac{\Delta P}{P}$.

$$\begin{aligned} \therefore z(t, \hat{\Omega}) &= \frac{r_0 - r(t)}{r_0} \underbrace{\text{EARTH TERM}}_{\downarrow} \underbrace{\text{PULSAR TERM}}_{= \frac{1}{2} \frac{p^i p^j}{(1 + \hat{\Omega} \cdot \hat{p})} [h_{ij}(t, \vec{x}_{\text{earth}}) - h_{ij}(t - L, \vec{x}_{\text{pulsar}})]} \end{aligned}$$

$$z(t, \hat{\Omega}) = \frac{1}{2} \frac{p^i p^j}{(1 + \hat{\Omega} \cdot \hat{p})} \cdot \Delta h_{ij}$$