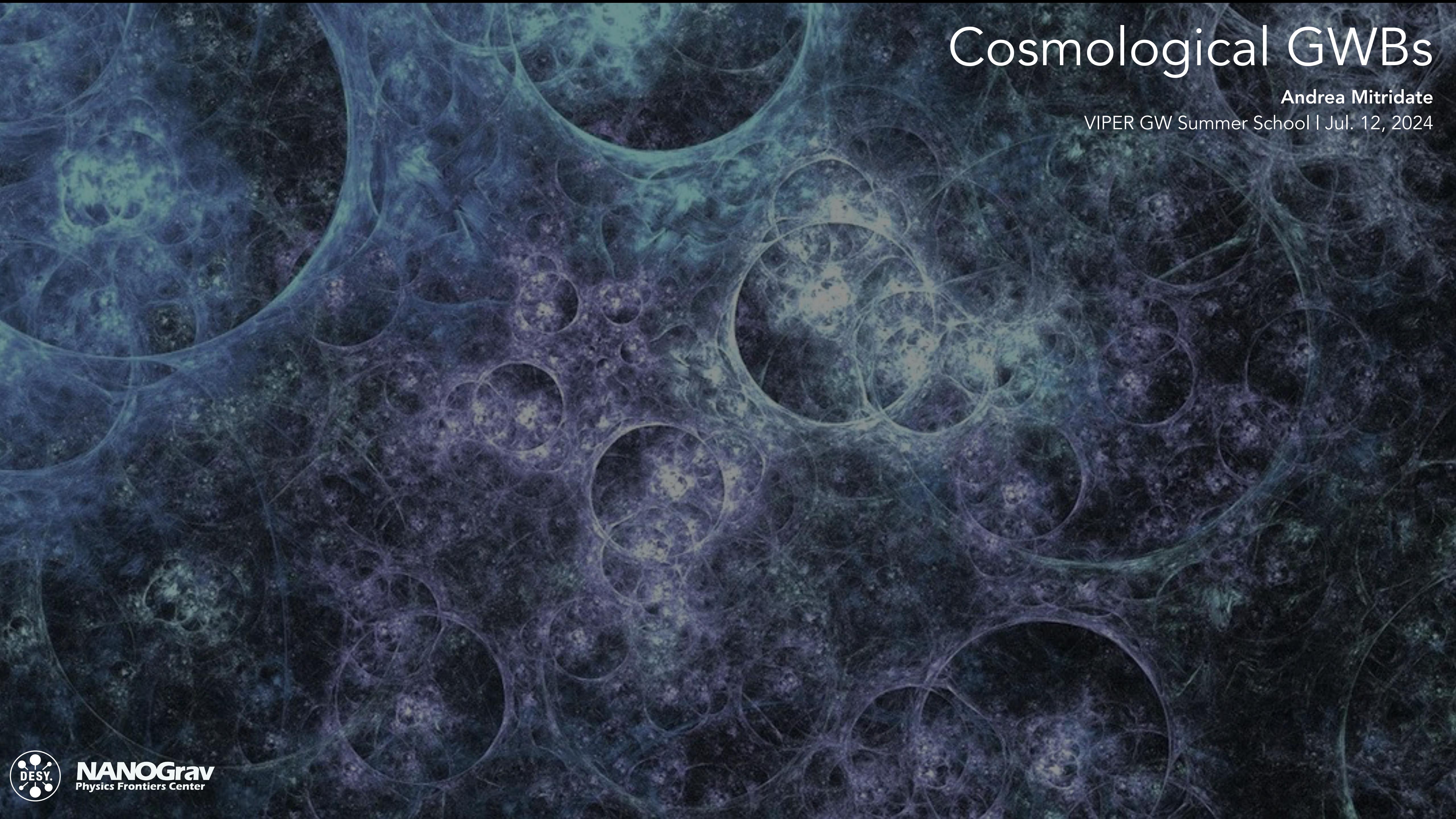


Cosmological GWBs

Andrea Mitridate

VIPER GW Summer School | Jul. 12, 2024



NANOGrav
Physics Frontiers Center

OUTLINE

what is the fuss all about

general properties of cosmological GWBs

two examples

how to search for them with PTArcade

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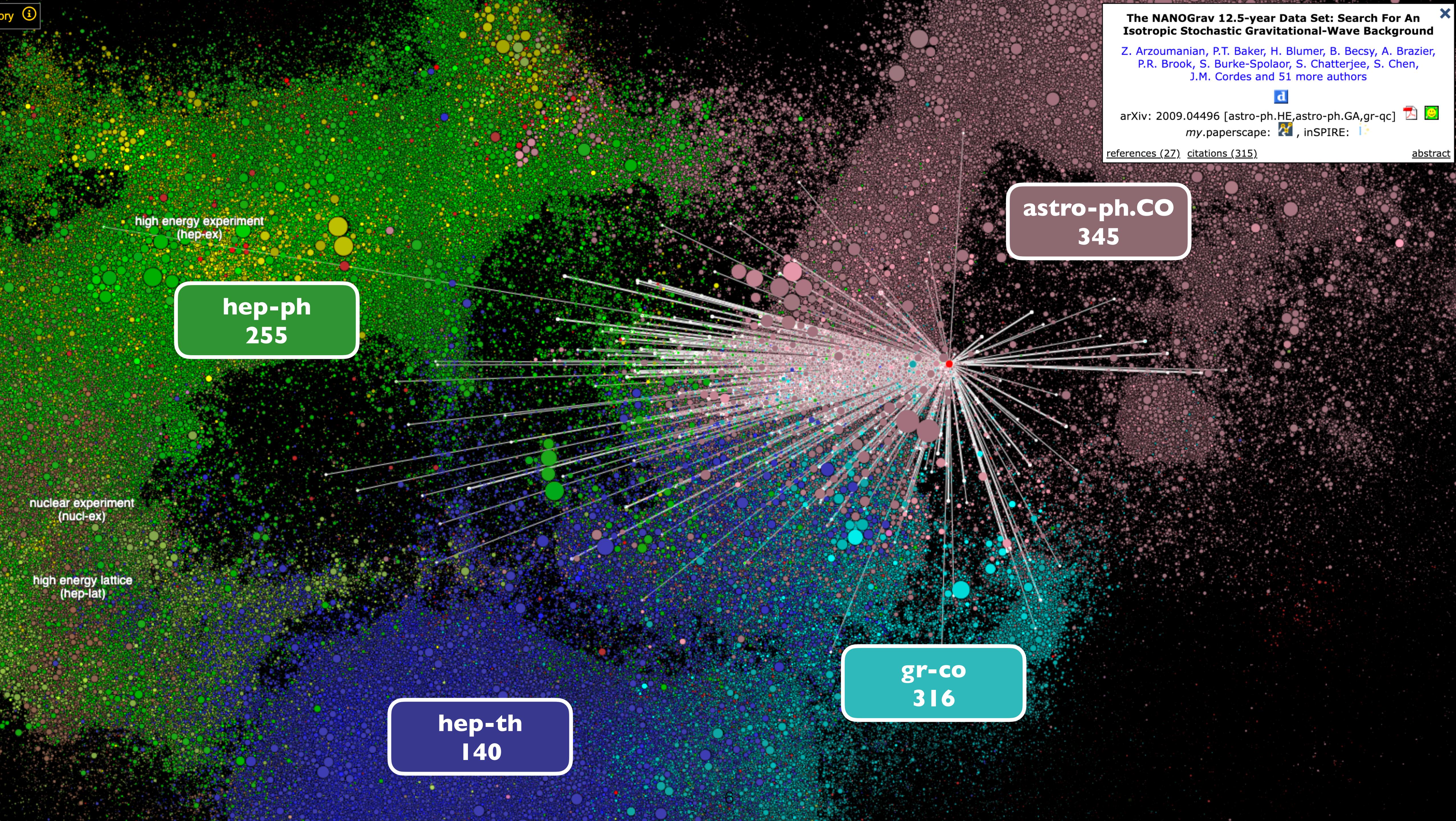
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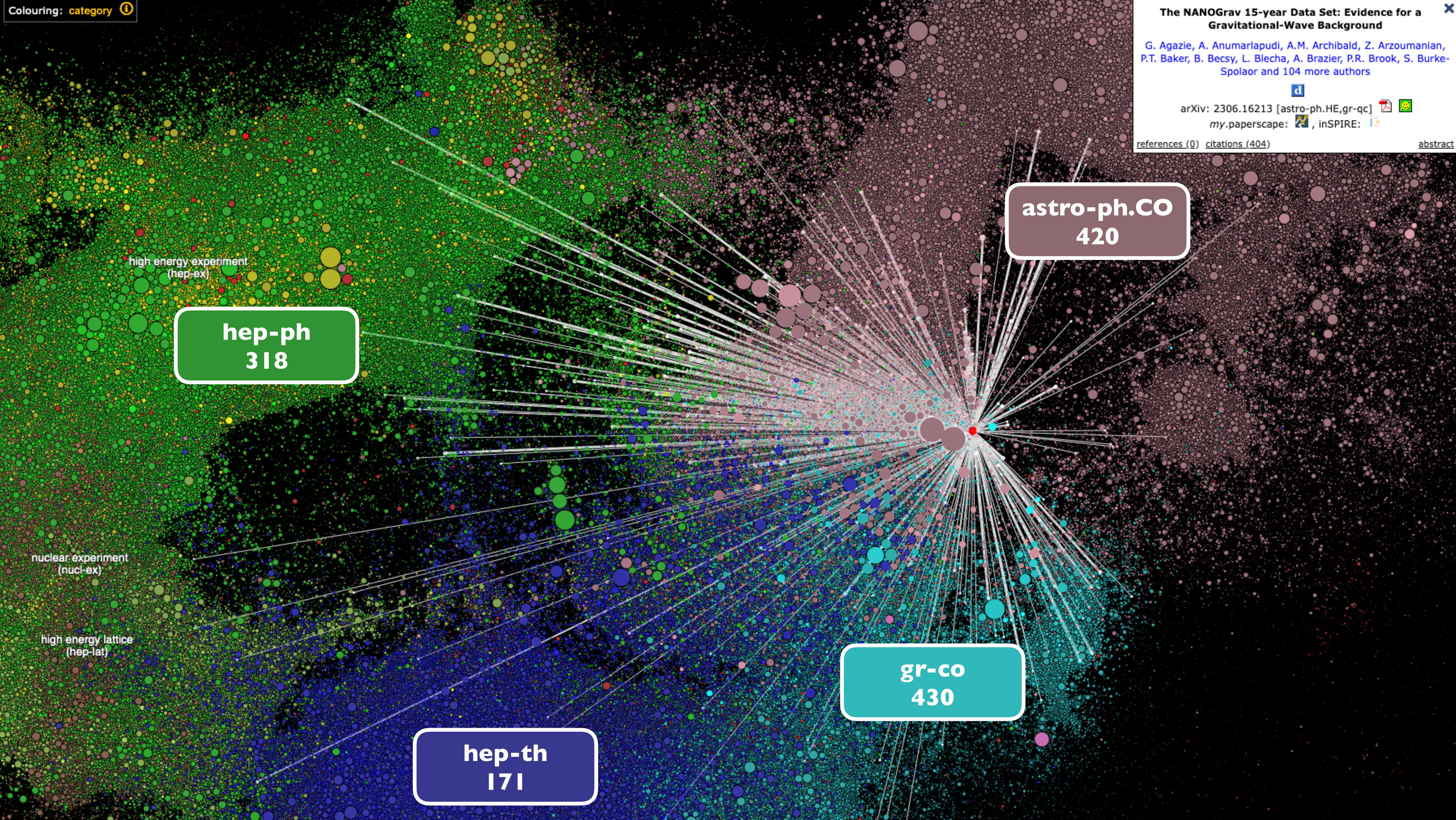
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A NEW WINDOW INTO THE UNIVERSE

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evidence papers

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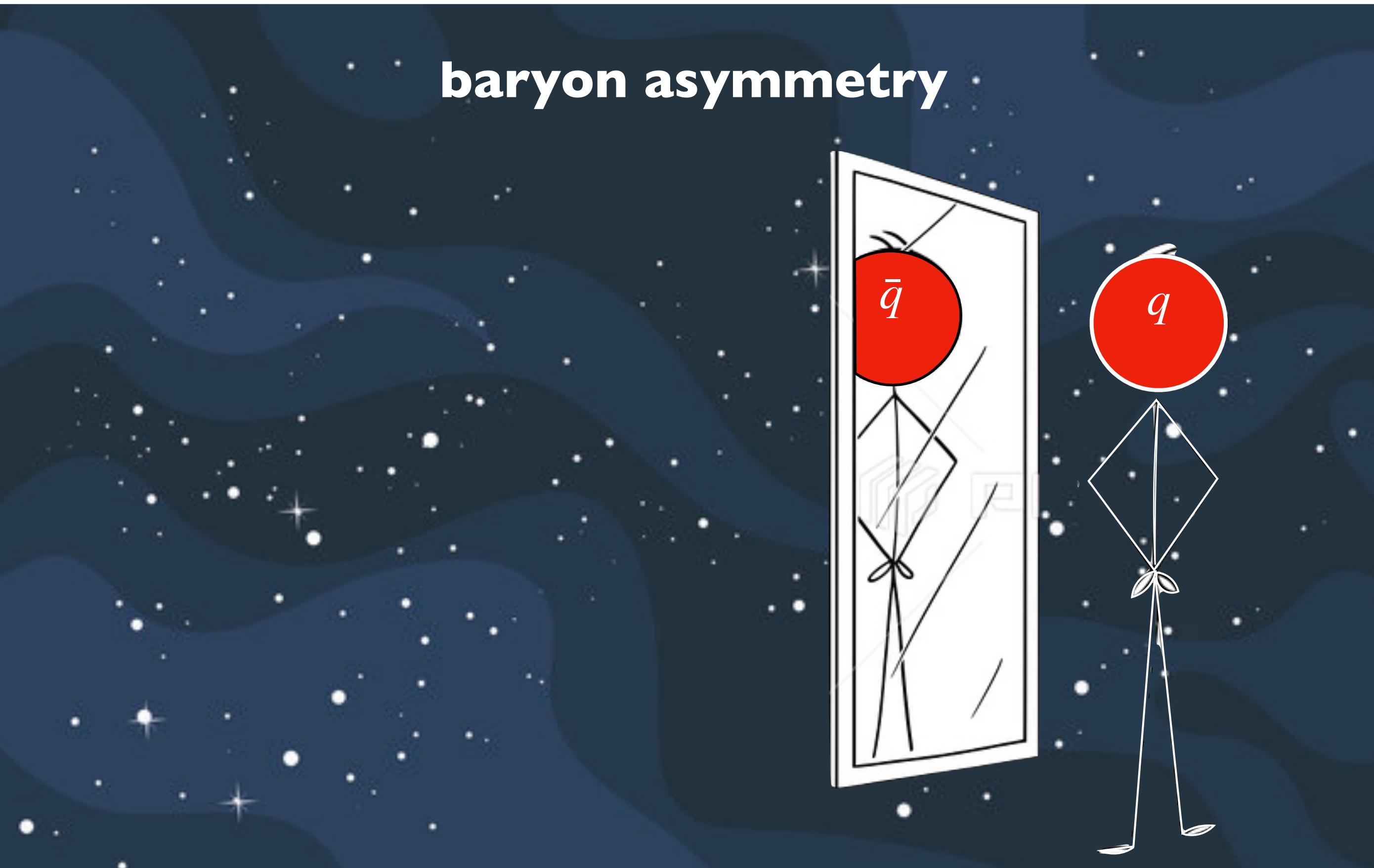
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evidence papers

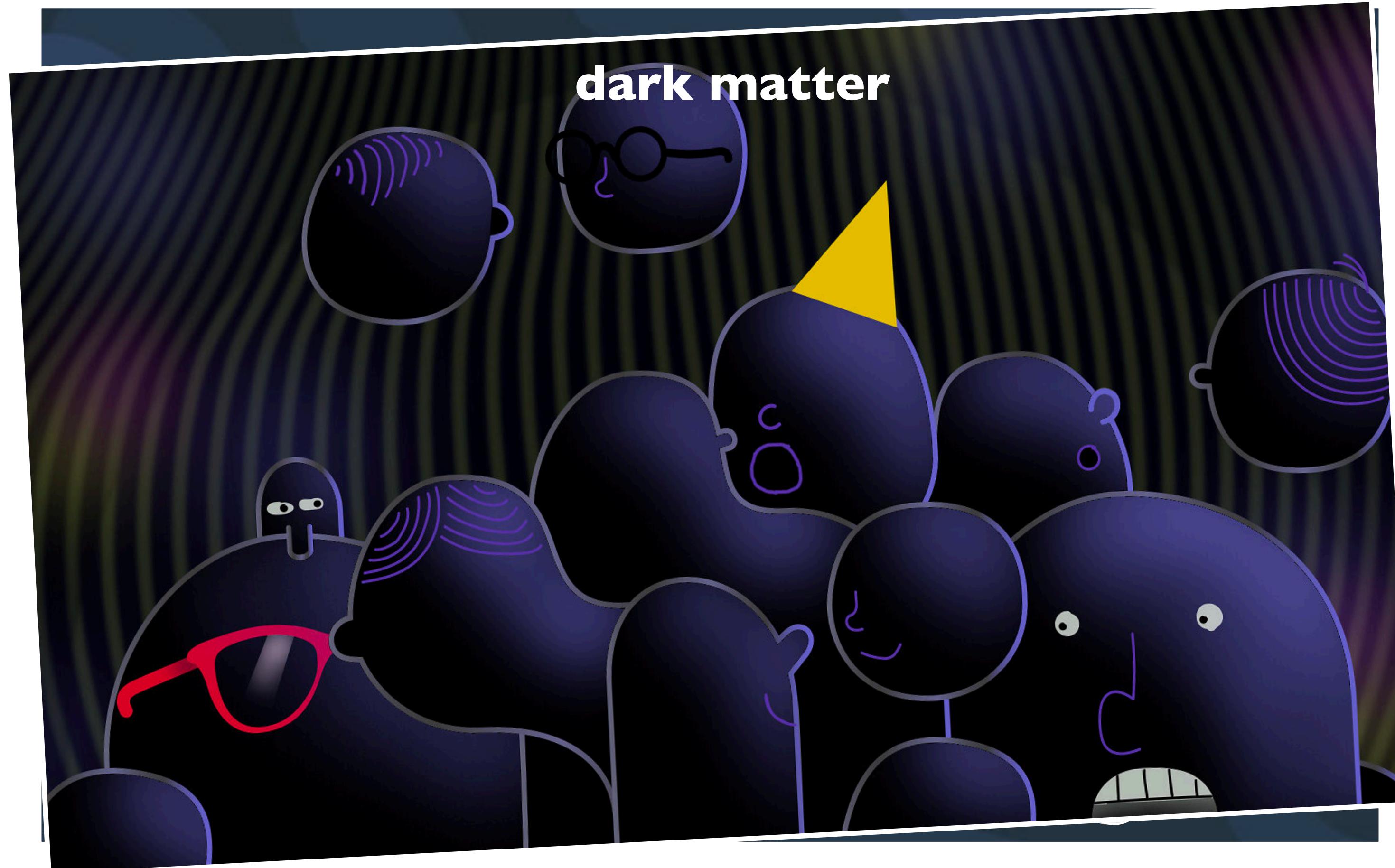
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NG new physics paper

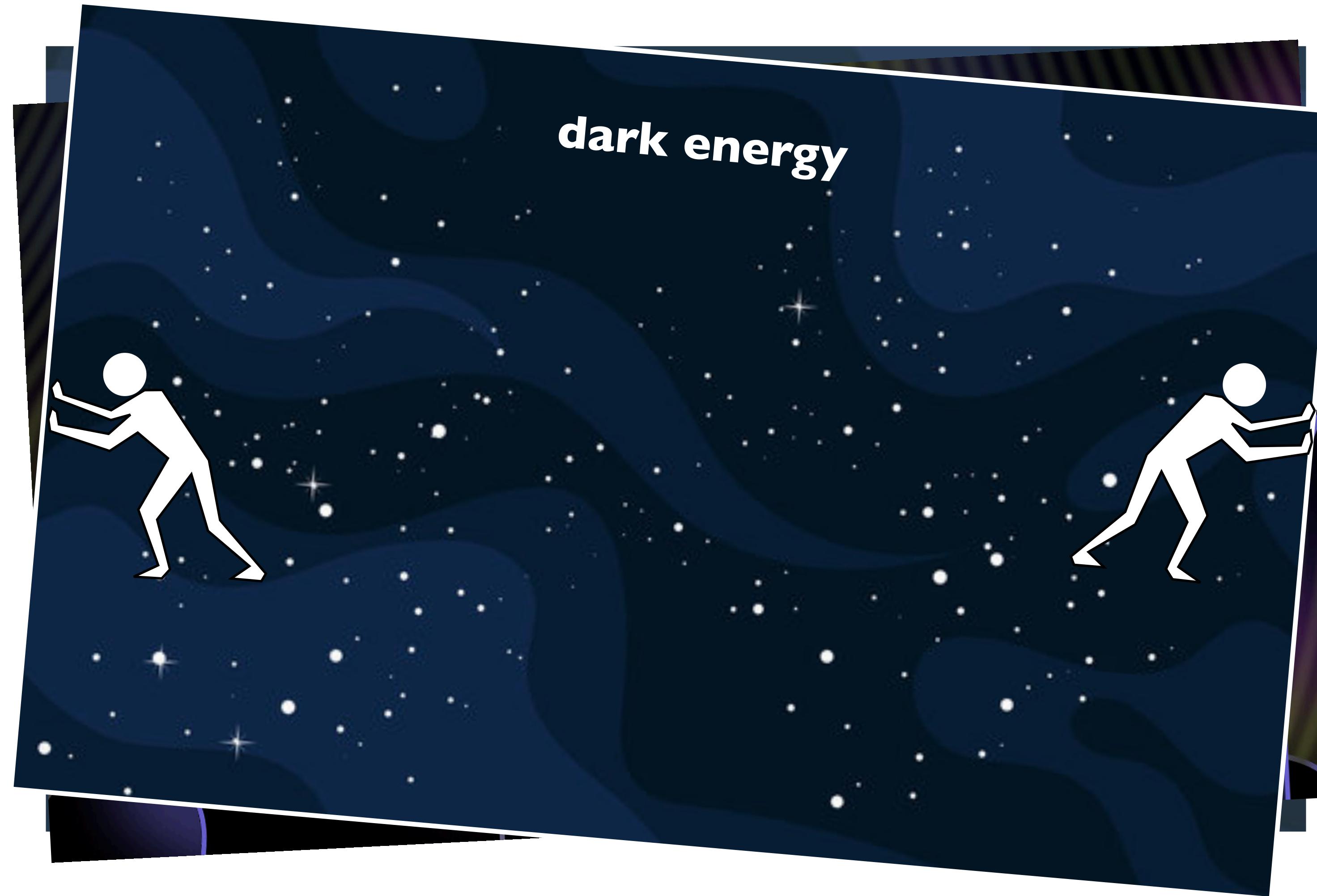
OPEN PROBLEMS IN PARTICLE PHYSICS



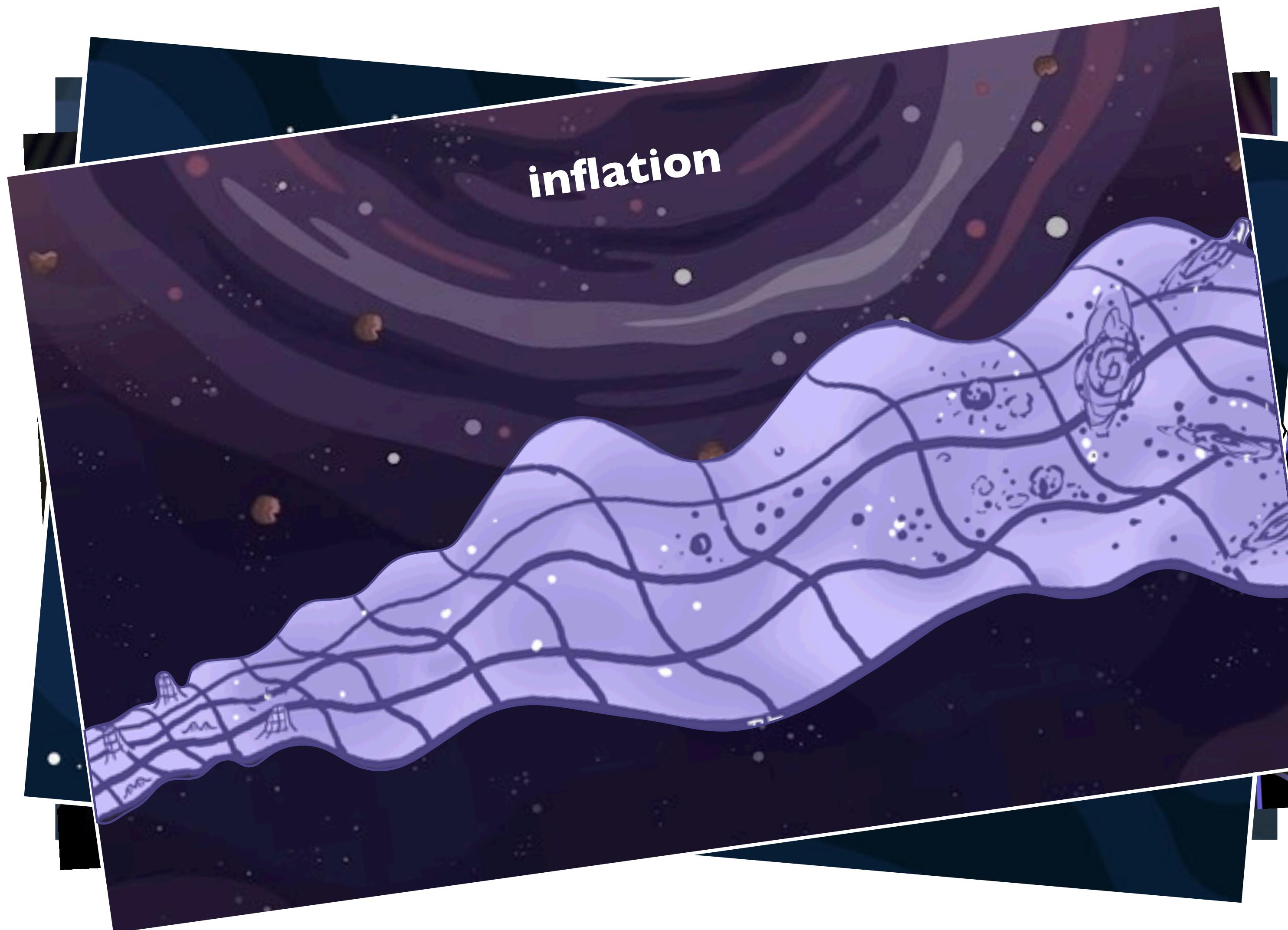
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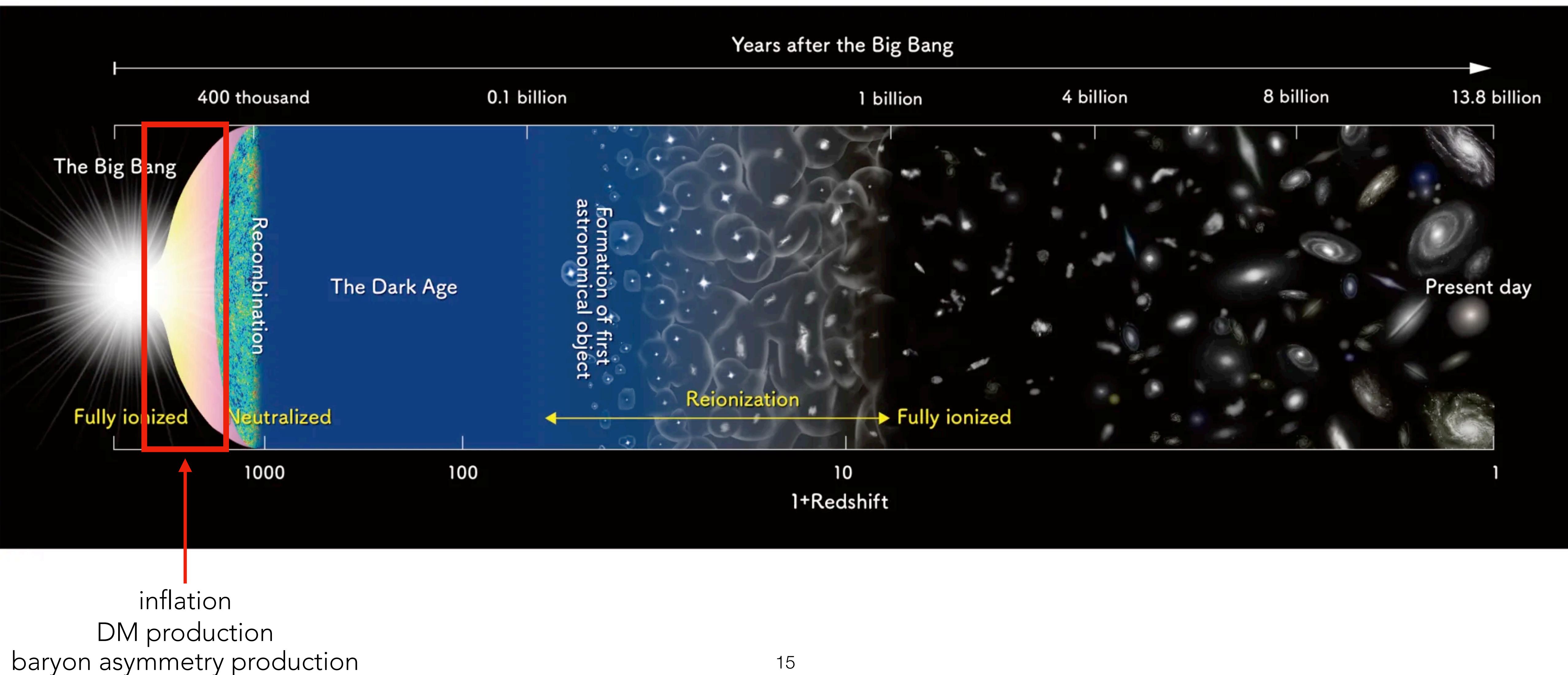
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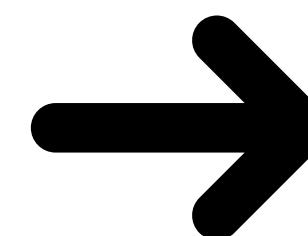


COSMIC ARCHOLOGY



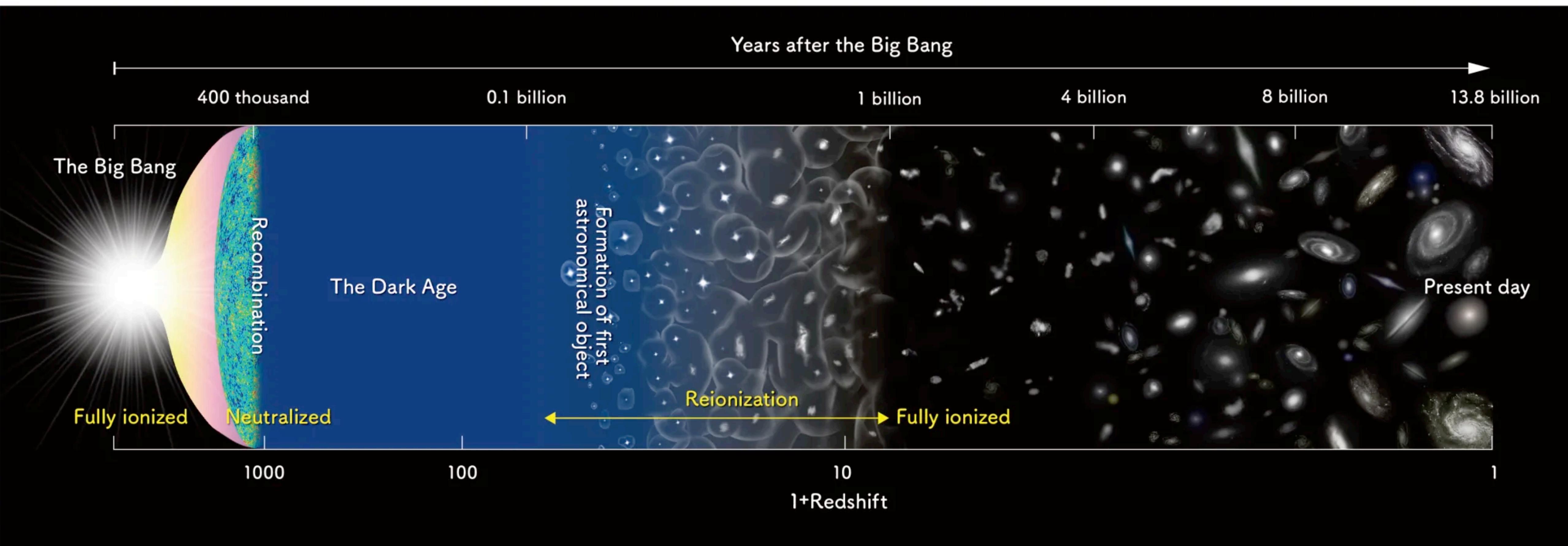
COSMIC ARCHOLOGY

finite speed of light



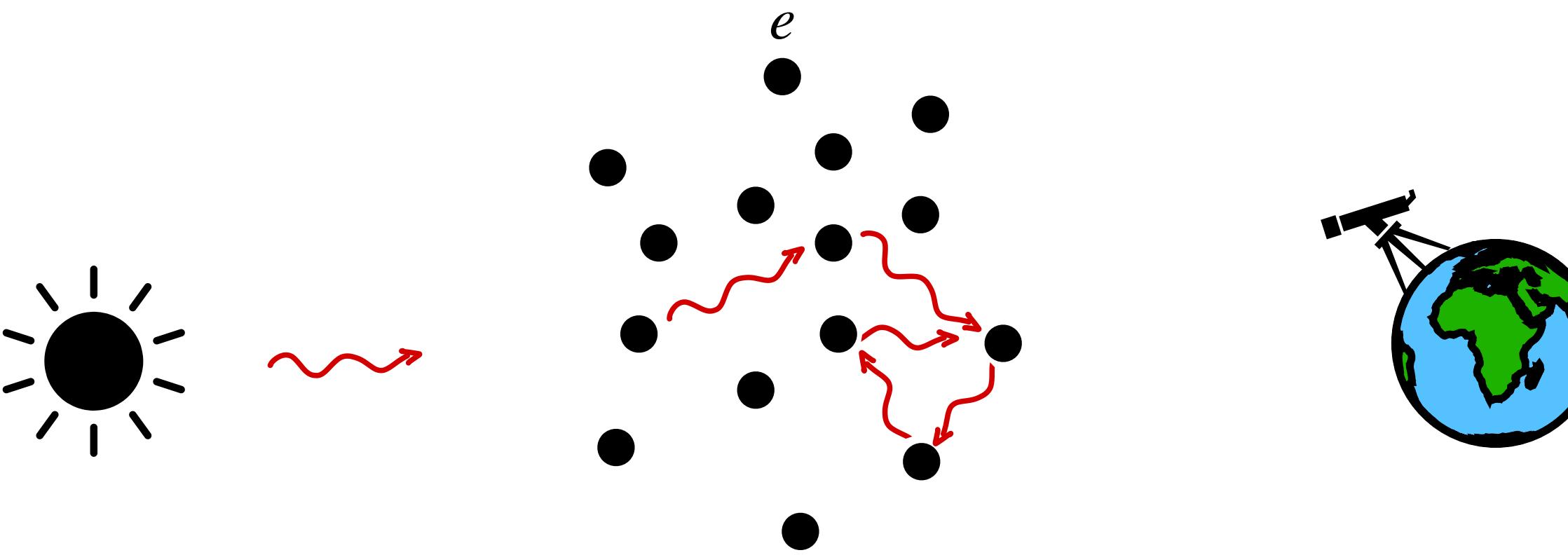
looking far

= looking in the past



THE CMB WALL

photons are not the best messenger in high-density environments

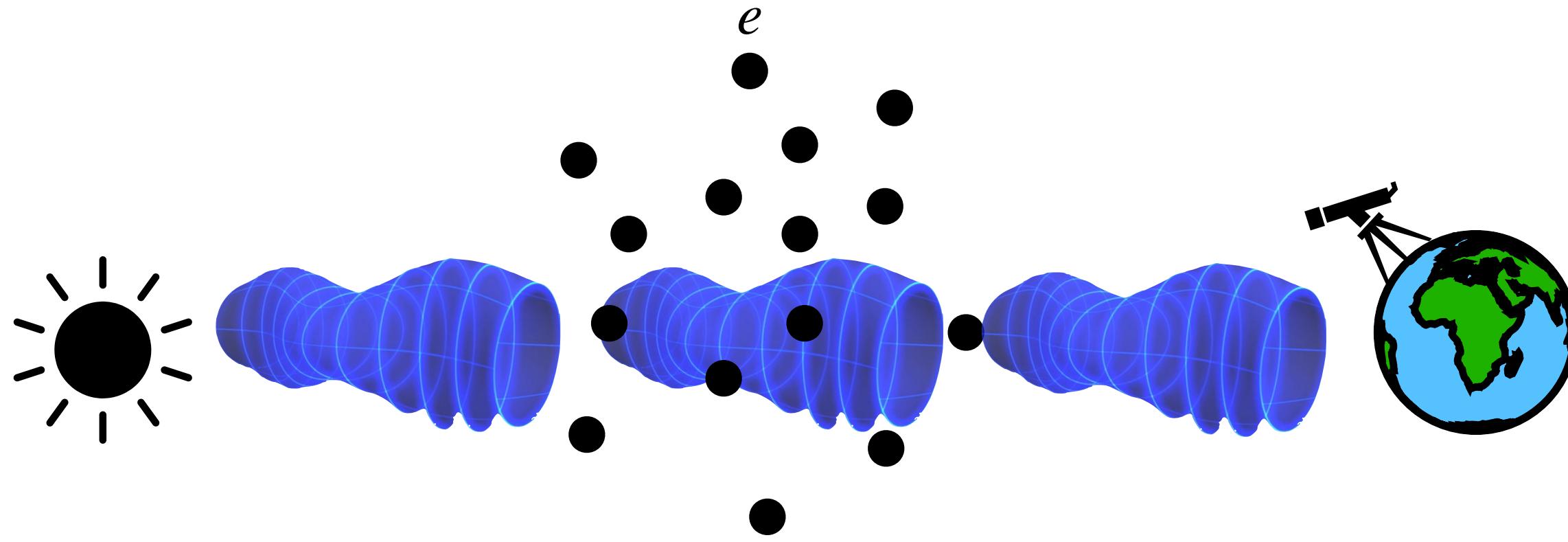


the mean free-path for photons in the primordial plasma is

$$\lambda_\gamma \sim \frac{1}{\sigma_T n_e} \sim \mu\text{m} \left(\frac{\text{MeV}}{T} \right)^3$$

any information encoded in EM radiation before recombination is lost in the electron bath

THE CMB WALL



the GW mean free-path in the primordial plasma is

$$\lambda_{\text{GW}} \sim \left(\frac{G m_e^2}{\alpha} \right)^2 \lambda_\gamma \sim \frac{1}{\sigma_T n_e} \sim 10^{79} \text{m} \left(\frac{\text{MeV}}{T} \right)^3$$

GW do not care about the bath and travel undisturbed trough it

how to describe (stochastic) GWBs

CHARACTERIZING A GWB

$$h_{ij}(t, \vec{x}) = \sum_A \int df \int d\hat{\Omega} \underbrace{\tilde{h}_A(f, \hat{\Omega})}_{\text{"Fourier" components}} e^{-2\pi i f(t - \hat{\Omega} \cdot \vec{x})} \underbrace{e_{ij}^A(\hat{\Omega})}_{\text{polarization tensors}}$$

plane waves

the complex functions $\tilde{h}^A(f, \hat{\Omega})$ are treated as Gaussian random variables

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$$\langle \tilde{h}_A(f, \hat{\Omega}) \rangle = 0$$

$$\langle \tilde{h}_A(f, \hat{\Omega})^* \tilde{h}_A(f', \hat{\Omega}') \rangle = \delta_{AA'} \delta(\hat{\Omega}, \hat{\Omega}') \delta(f - f') H(f) P(f, \hat{\Omega})$$

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plane waves

The diagram illustrates the decomposition of a gravitational wave field $h_{ij}(t, \vec{x})$. It starts with a horizontal line labeled "plane waves" at the top. This line is broken into two segments by a vertical dotted line. The left segment is labeled "Fourier" components and contains a vertical dotted line with a bracket underneath it. The right segment is labeled "polarization tensors" and also contains a vertical dotted line with a bracket underneath it. The middle part of the original horizontal line is labeled $\tilde{h}_A(f, \hat{\Omega})$.

the complex functions $\tilde{h}^A(f, \hat{\Omega})$ are treated as Gaussian random variables

$$\langle \tilde{h}_A(f, \hat{\Omega}) \rangle = 0 \quad \text{GWB power map}$$

$$\langle \tilde{h}_A(f, \hat{\Omega})^* \tilde{h}_A(f', \hat{\Omega}') \rangle = \delta_{AA'} \delta(\hat{\Omega}, \hat{\Omega}') \delta(f - f') H(f) P(f, \hat{\Omega})$$

GWB power spectrum

The diagram shows the relationship between the GWB power map and the GWB power spectrum. On the left, there is a horizontal line with a vertical dotted line in the middle, labeled "GWB power map". On the right, there is a horizontal line with a vertical dotted line in the middle, labeled "GWB power spectrum". An arrow points from the "GWB power map" side towards the "GWB power spectrum" side, indicating a transformation or relationship between them.

CHARACTERIZING A GWB

$$h_{ij}(t, \vec{x}) = \sum_A \int df \int d\hat{\Omega} \underbrace{\tilde{h}_A(f, \hat{\Omega})}_{\text{"Fourier" components}} e^{-2\pi i f(t - \hat{\Omega} \cdot \vec{x})} \underbrace{e_{ij}^A(\hat{\Omega})}_{\text{polarization tensors}}$$

plane waves

the complex functions $\tilde{h}^A(f, \hat{\Omega})$ are treated as Gaussian random variables

$$\langle \tilde{h}_A(f, \hat{\Omega}) \rangle = 0$$

for isotropic background

$$\langle \tilde{h}_A(f, \hat{\Omega})^* \tilde{h}_A(f', \hat{\Omega}') \rangle = \delta_{AA'} \delta(\hat{\Omega}, \hat{\Omega}') \delta(f - f') H(f) P(\hat{\Omega})$$

GWB power spectrum

ONE QUANTITY, MANY NAMES

a Gaussian and isotropic background is fully characterized by its power spectrum

GWB power spectrum

$$\langle \tilde{h}_A^* \tilde{h}_A \rangle = \delta_{AA'} \delta(\hat{\Omega}, \hat{\Omega}') \delta(f - f') H(f)$$

GWB relic density

$$\Omega_{\text{GW}}(f) = \frac{1}{\rho_c} \frac{d\rho_{\text{GW}}}{d \ln f}$$
$$\Omega_{\text{GW}}(f) = \frac{32\pi^3}{3H_0^2} f^3 H(f)$$

characteristic strain

$$\langle h_{ij} h_{ij} \rangle = 2 \int_0^\infty d \log f h_c^2(f)$$
$$h_c^2(f) = 16\pi f H(f)$$

ONE QUANTITY, MANY NAMES

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induced TOAs common process

$$\langle \delta t_a(t_1) \delta t_b(t_2) \rangle = \int df \Gamma_{ab} S(f) e^{(2\pi f(t_1 - t_2))}$$

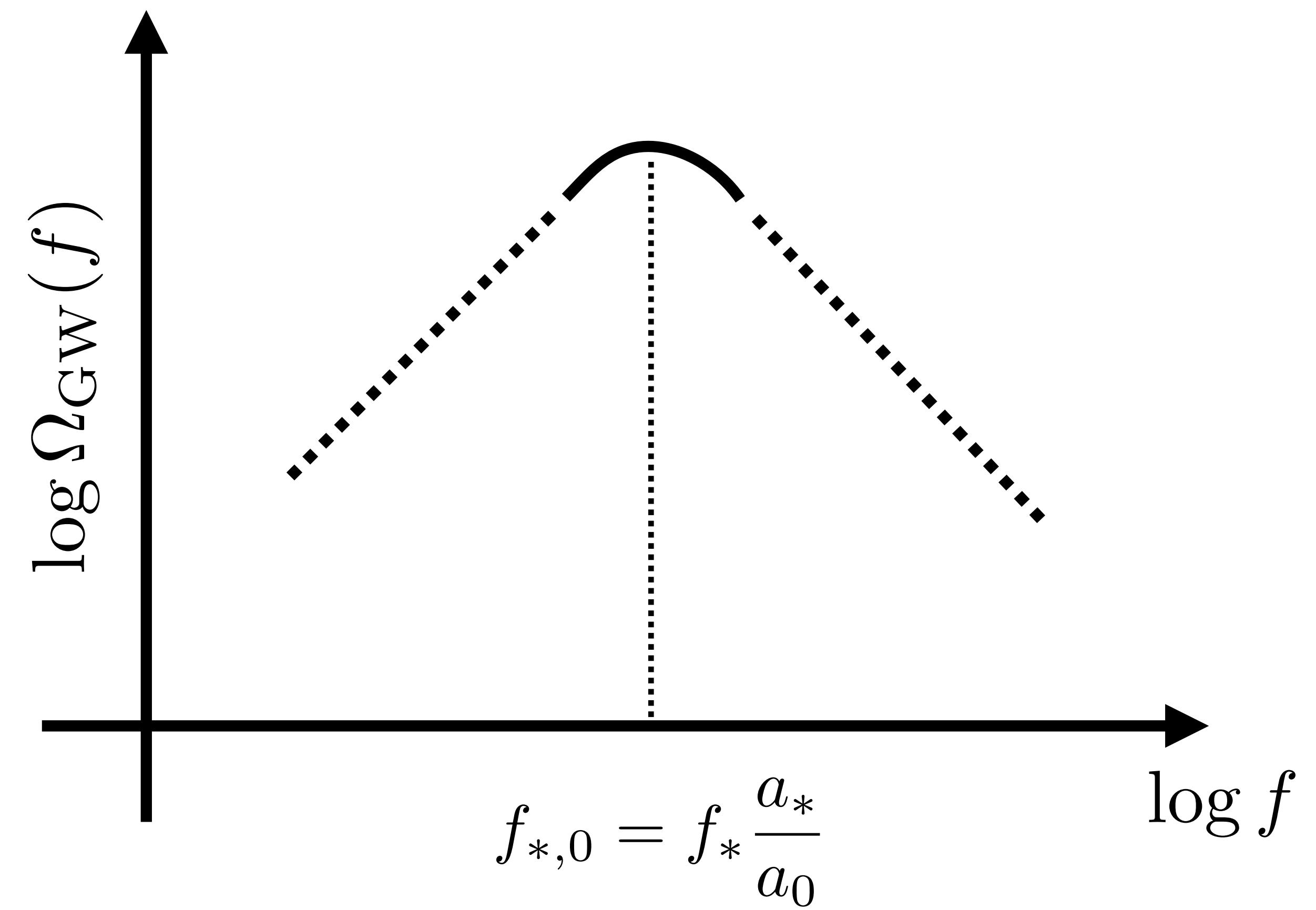
$$S(f) = \frac{H(f)}{(2\pi f)^2}$$

general properties of cosmological GWBs

SPECTRUM: peak frequency

short live sources active at some time $t_* \lesssim H_*^{-1}$

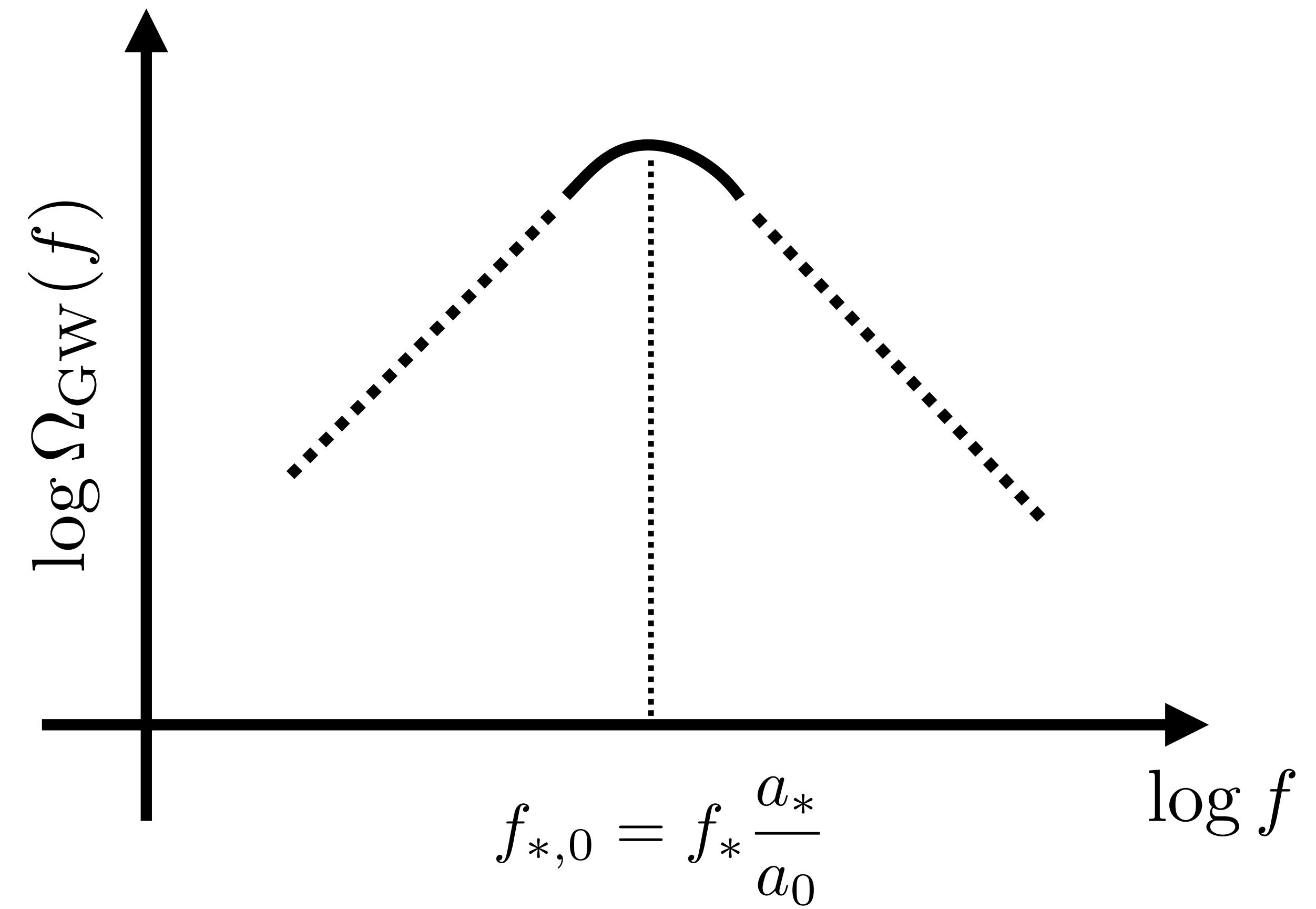
$$f_* = \frac{1}{\text{characteristic length scale}}$$



SPECTRUM: peak frequency

short live sources active at some time $t_* \lesssim H_*^{-1}$

$$f_* = \frac{1}{\text{characteristic length scale}} = \frac{1}{\varepsilon_* H_*^{-1}}$$

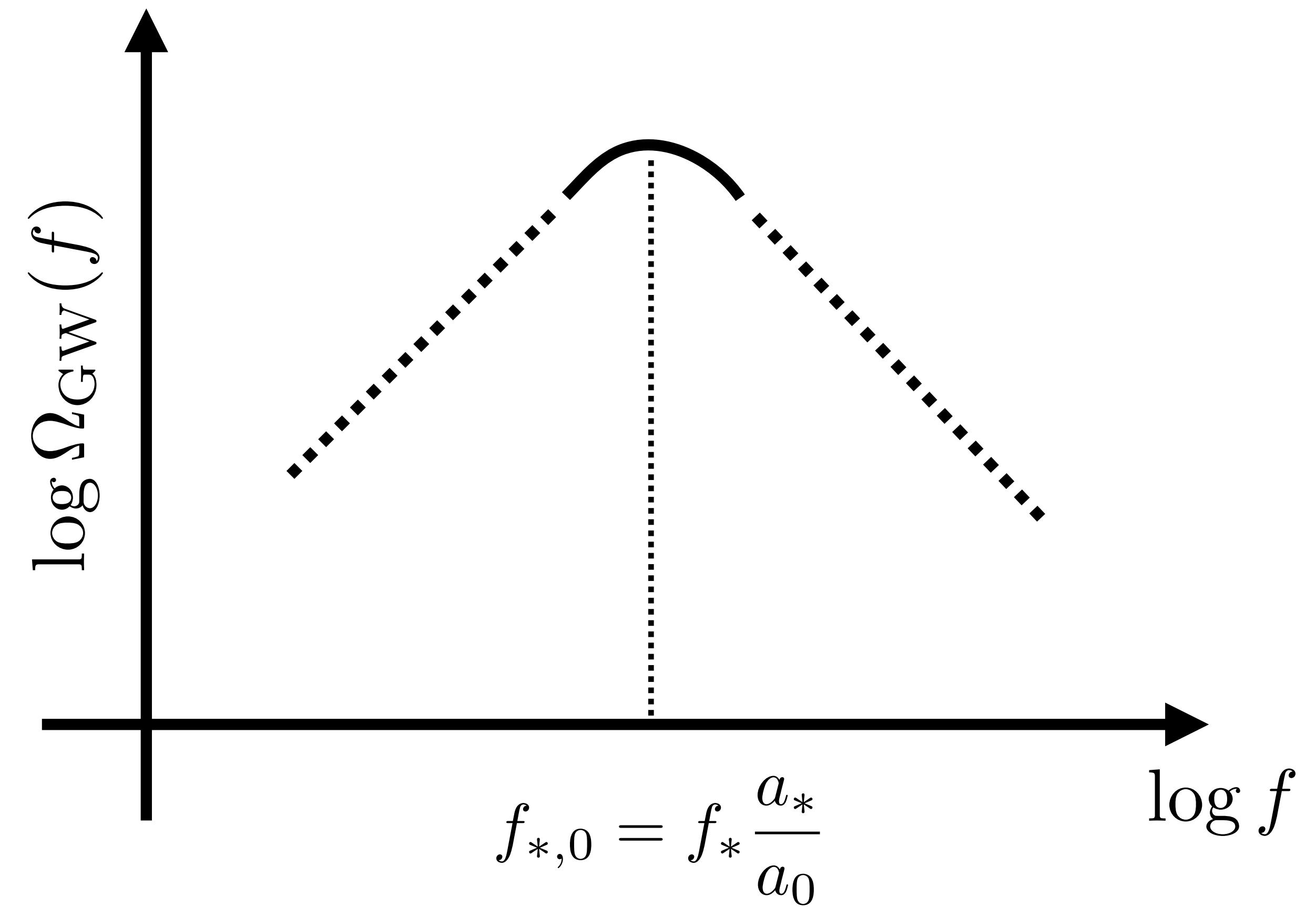


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$$f_* = \frac{1}{\text{characteristic length scale}} = \frac{1}{\varepsilon_* H_*^{-1}}$$

$\varepsilon_* \lesssim 1$
for casual sources



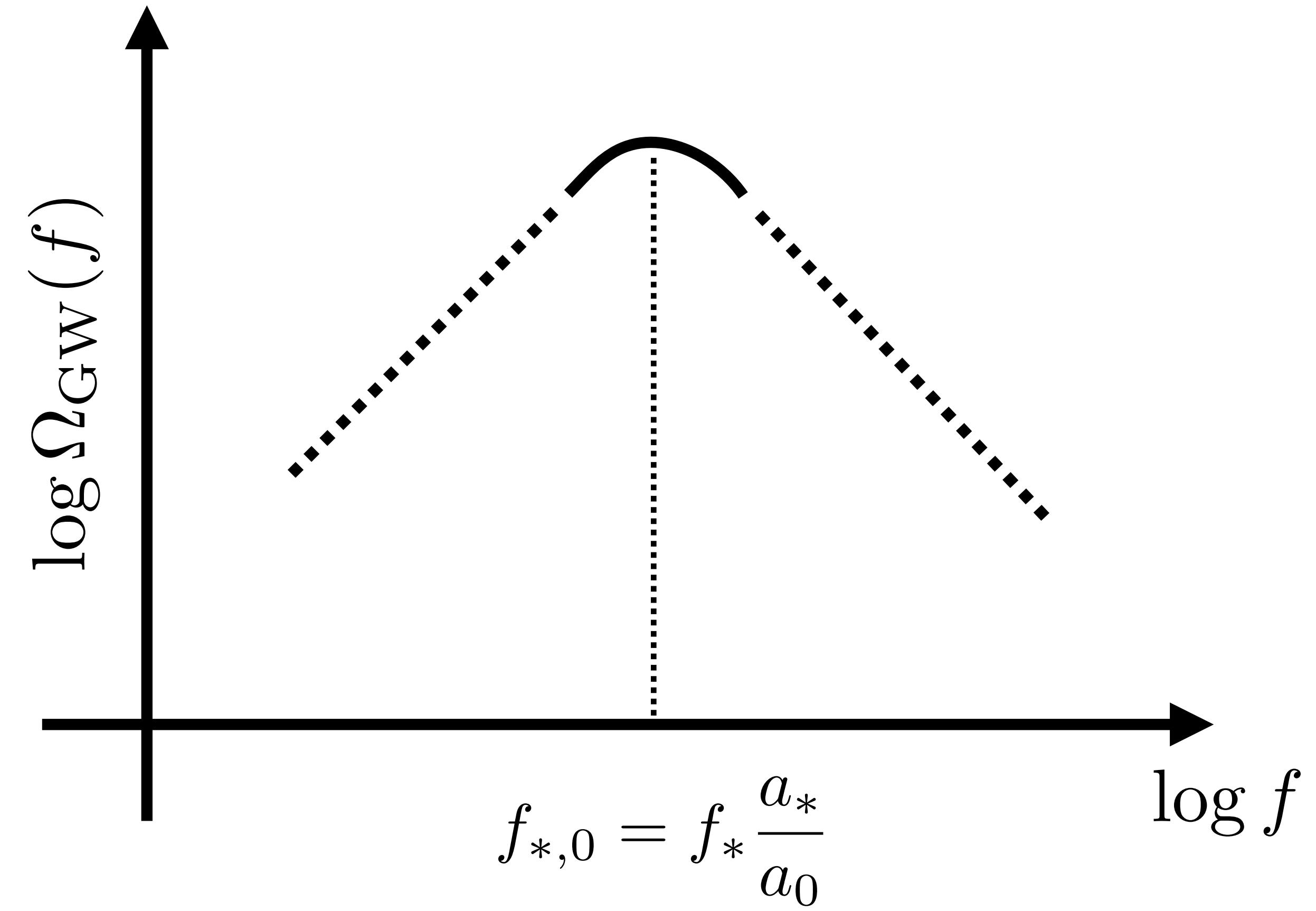
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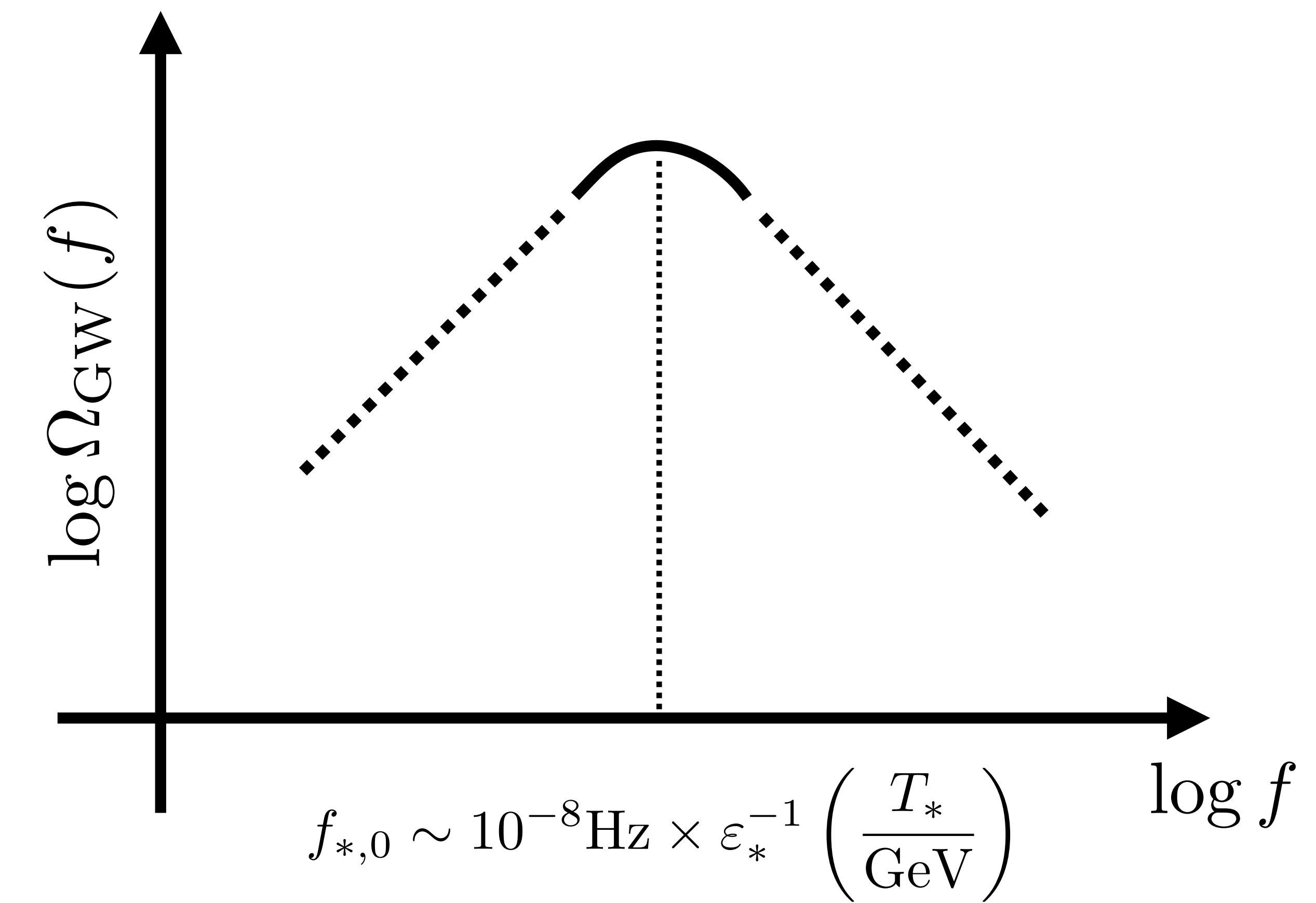
$$f_{*,0} \sim 10^{-8} \text{Hz} \times \varepsilon_*^{-1} \left(\frac{T_*}{\text{GeV}} \right)$$



SPECTRUM: amplitude

**primordial GWs contribute to the background
expansion as additional radiation**

$$H(a)^2 = H_0^2 \left[\left(\frac{\rho_{\text{GW}}^0}{\rho_c^0} + \Omega_{\text{rad}}^0 \right) \left(\frac{a_0}{a} \right)^4 + \Omega_{\text{mat}}^0 \left(\frac{a_0}{a} \right)^3 + \Omega_{\Lambda}^0 \right]$$



SPECTRUM: amplitude

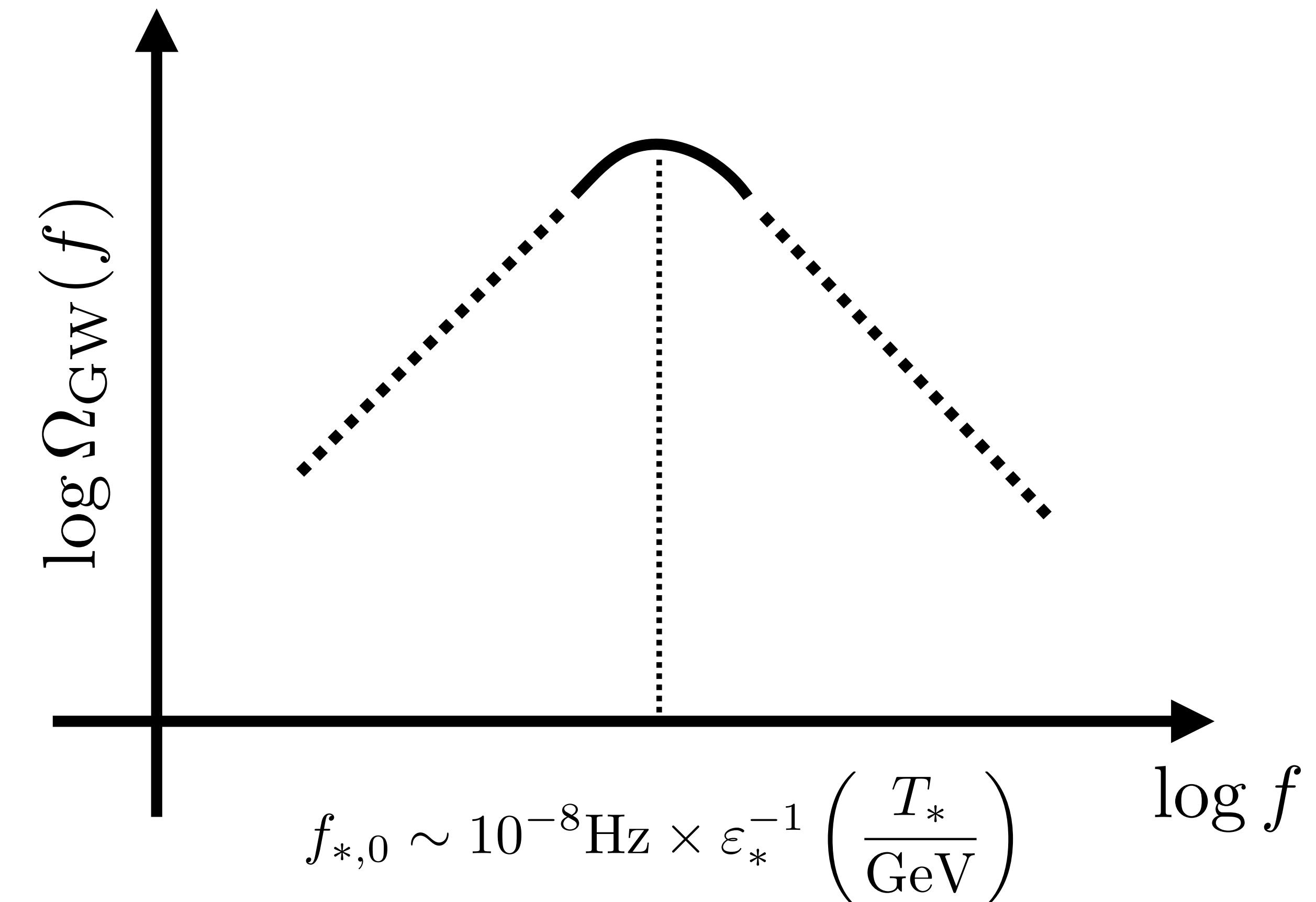
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any observable sensitive to the Universe
expansion constraints the energy in GWs

$$\int \frac{df}{f} h^2 \Omega_{\text{GW}}(f) \lesssim 10^{-6}$$

BBN + CMB



SPECTRUM: amplitude

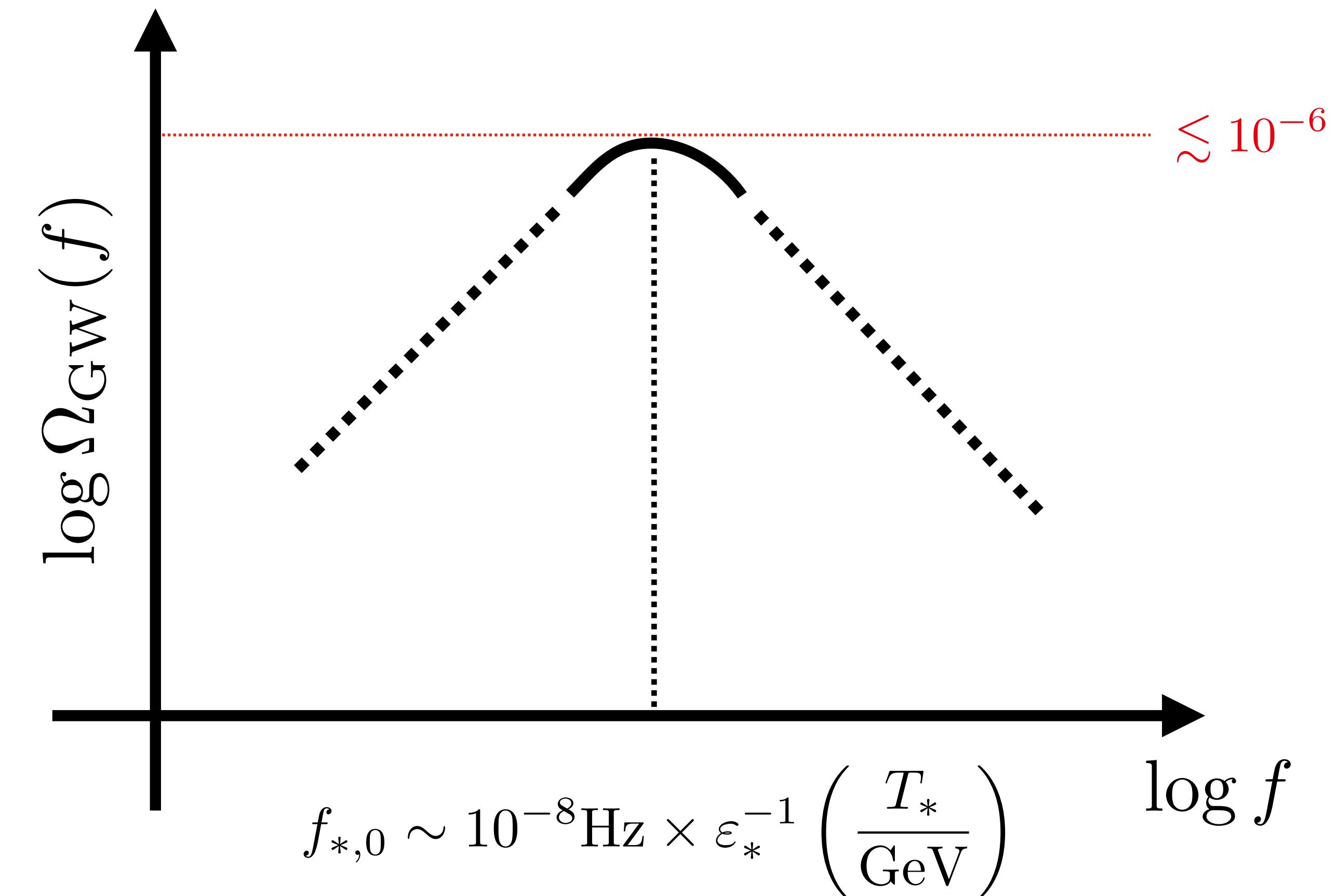
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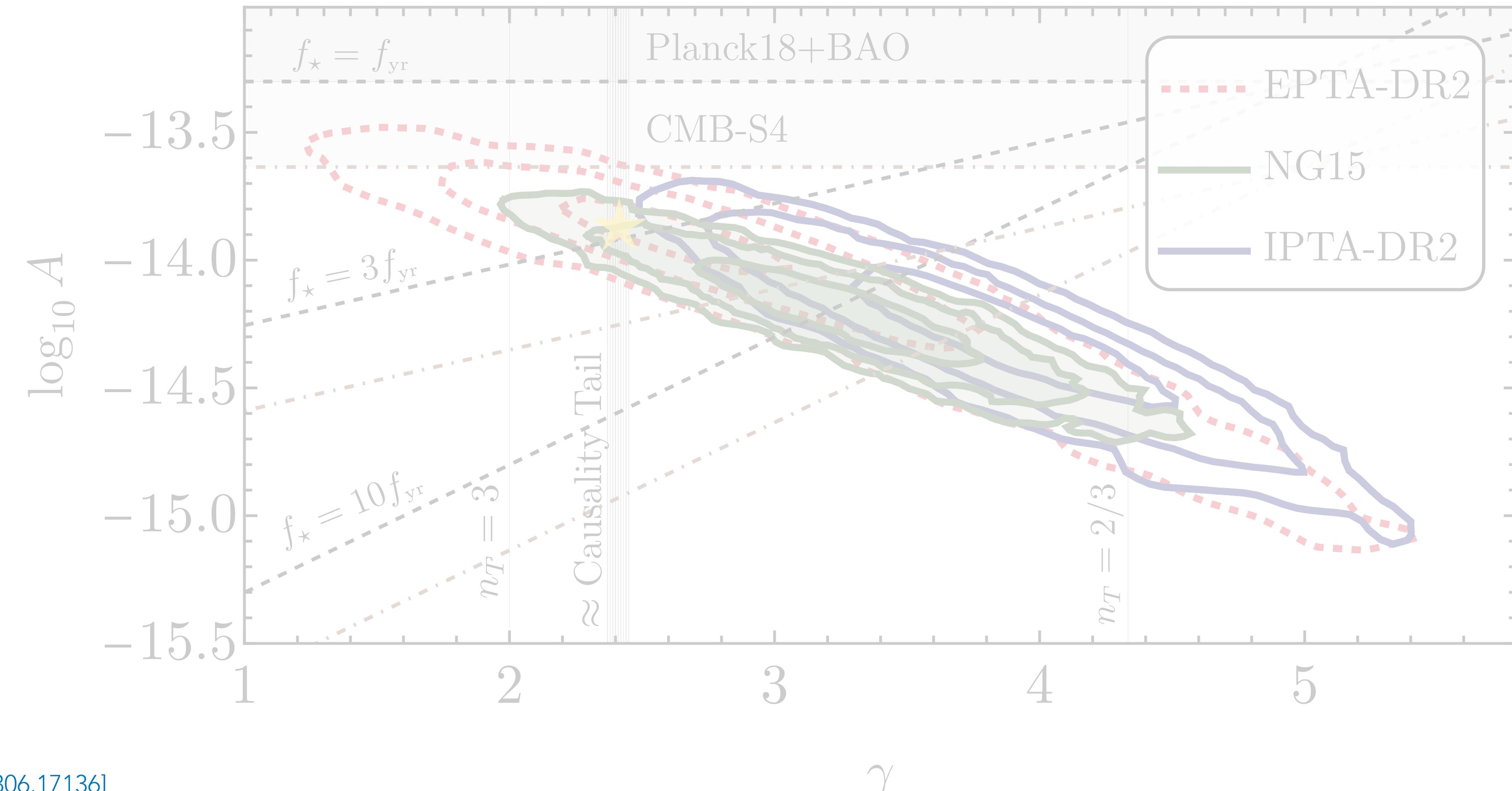
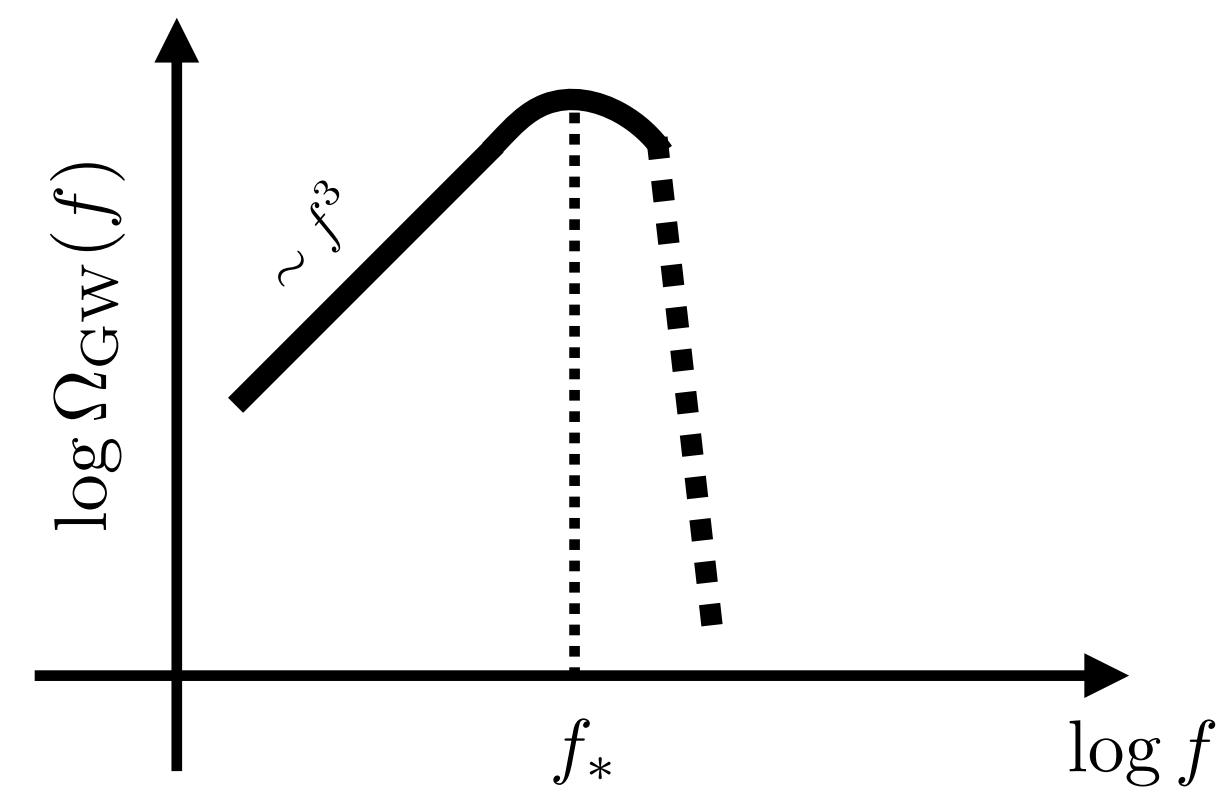
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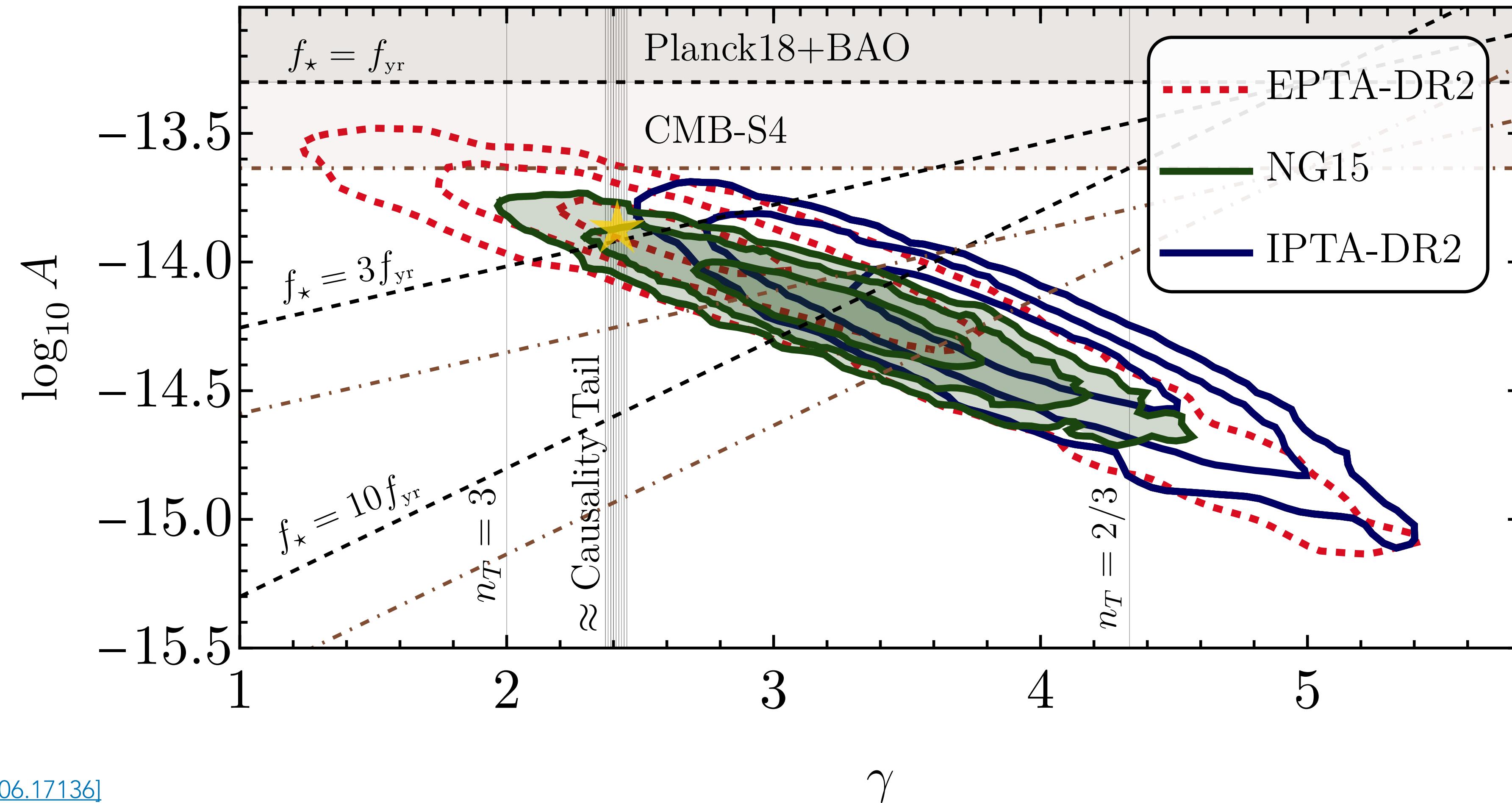
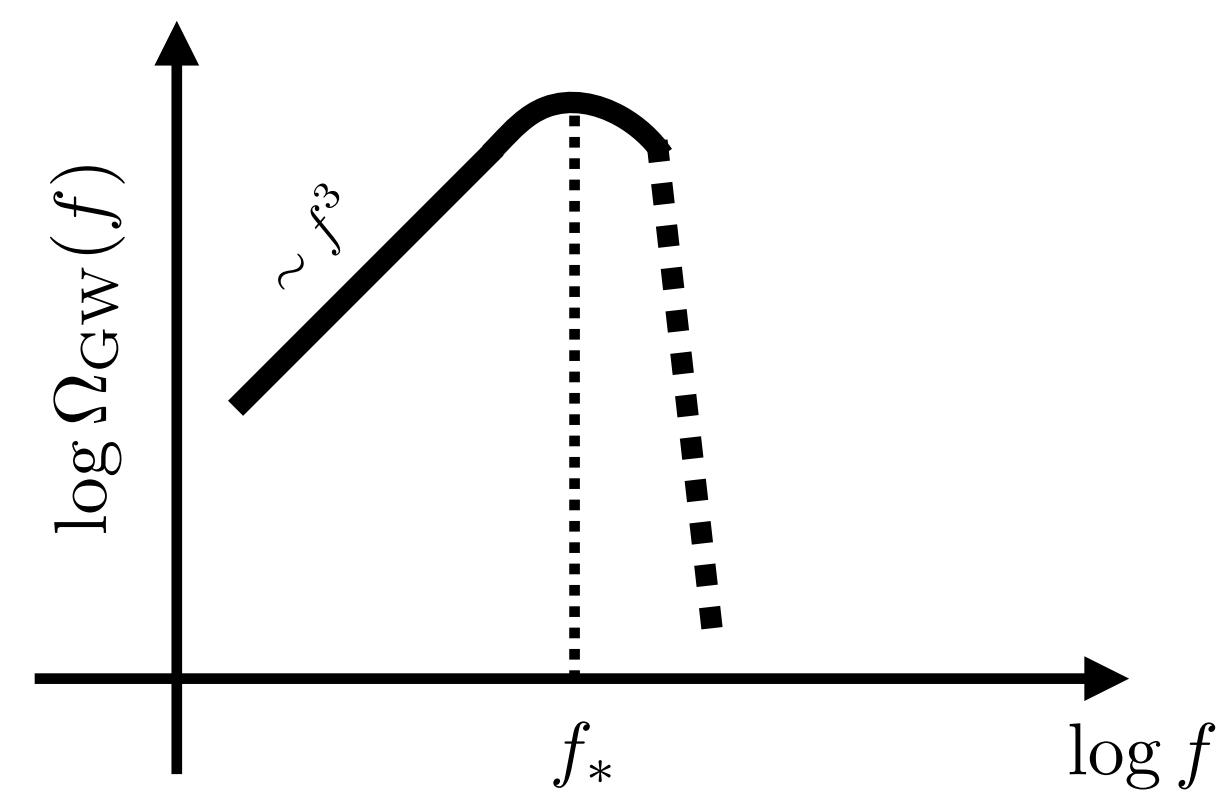
SPECTRUM: amplitude

$$h^2 \Omega_{\text{GW}} = A^2 \left(\frac{f}{\text{yr}^{-1}} \right)^{5-\gamma} \Theta(f_* - f)$$



SPECTRUM: amplitude

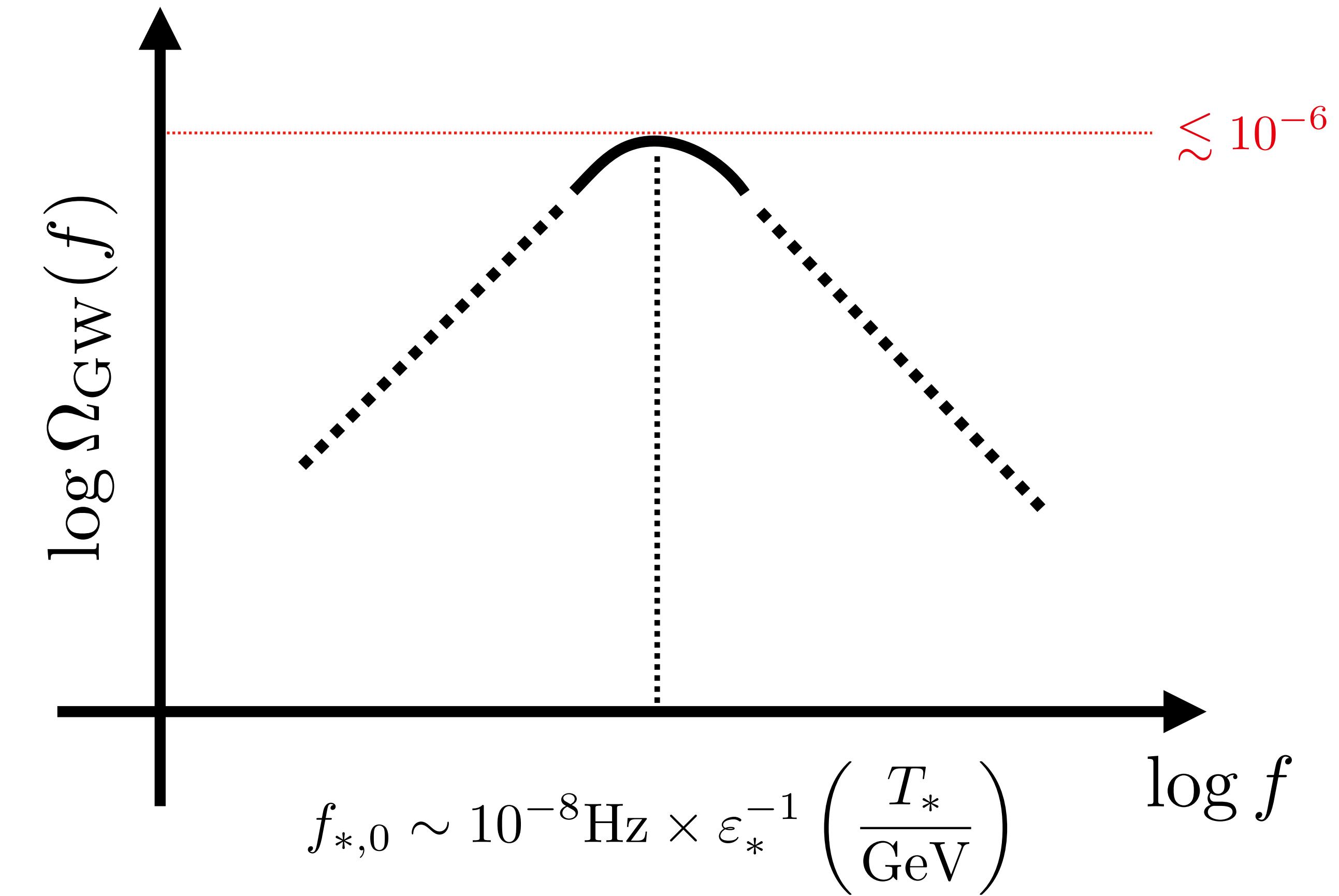
$$h^2 \Omega_{\text{GW}} = A^2 \left(\frac{f}{\text{yr}^{-1}} \right)^{5-\gamma} \Theta(f_* - f)$$



SPECTRUM: slope

anisotropic stress tensor of the (cosmological) source

$$\ddot{h}_{ij}(\mathbf{x}, t) + 3H\dot{h}_{ij}(\mathbf{x}, t) - \frac{\nabla^2}{a^2} h_{ij}(\mathbf{x}, t) = 16\pi G \Pi_{ij}(\mathbf{x}, t)$$



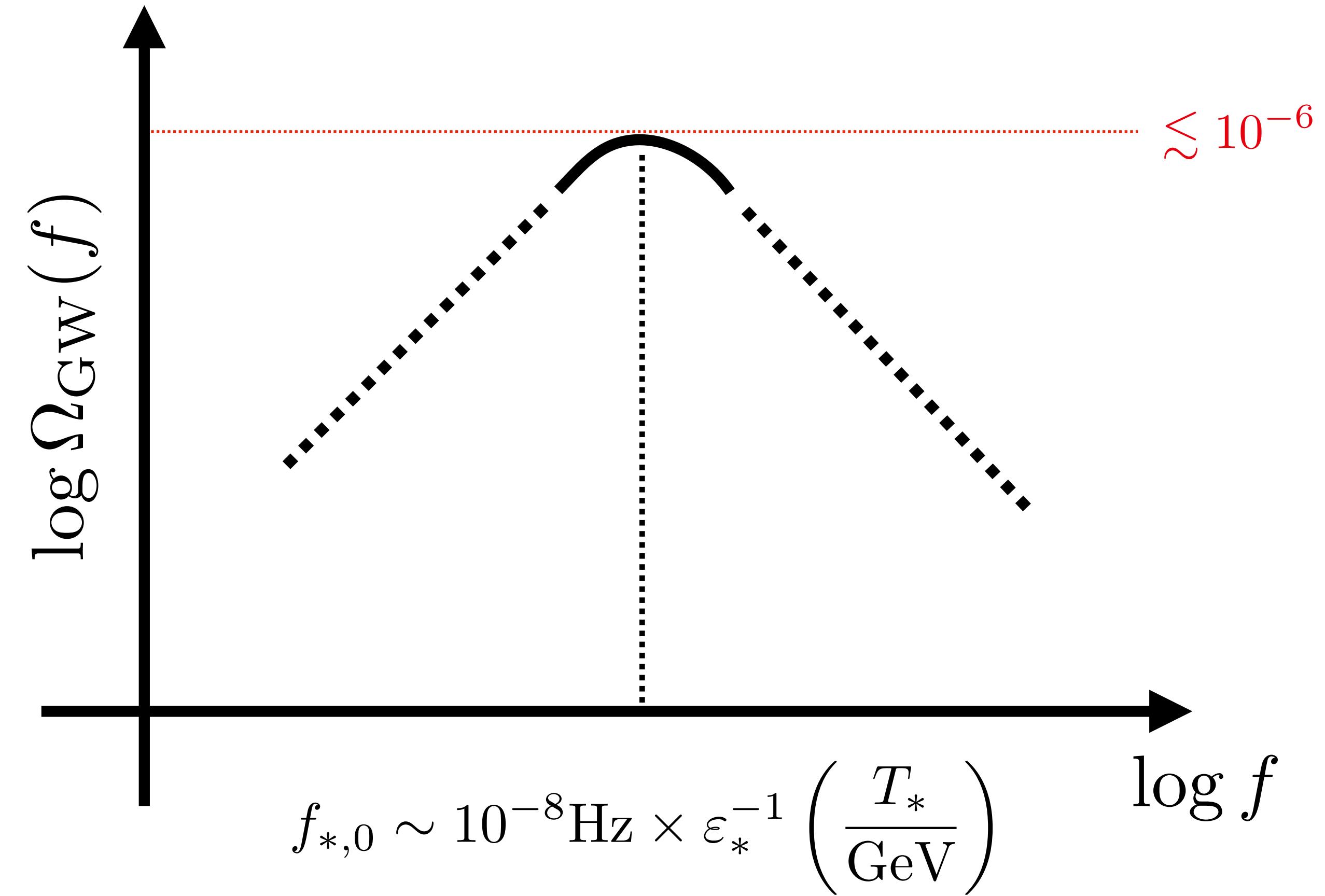
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for homogeneous and isotropic stochastic sources

$$\langle \Pi_{ij}(\mathbf{k}, t) \Pi^{ij}(\mathbf{k}, t') \rangle = (2\pi)^3 \delta(\mathbf{k} - \mathbf{k}') \Pi(\mathbf{k}, t, t')$$



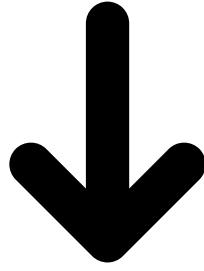
SPECTRUM: slope

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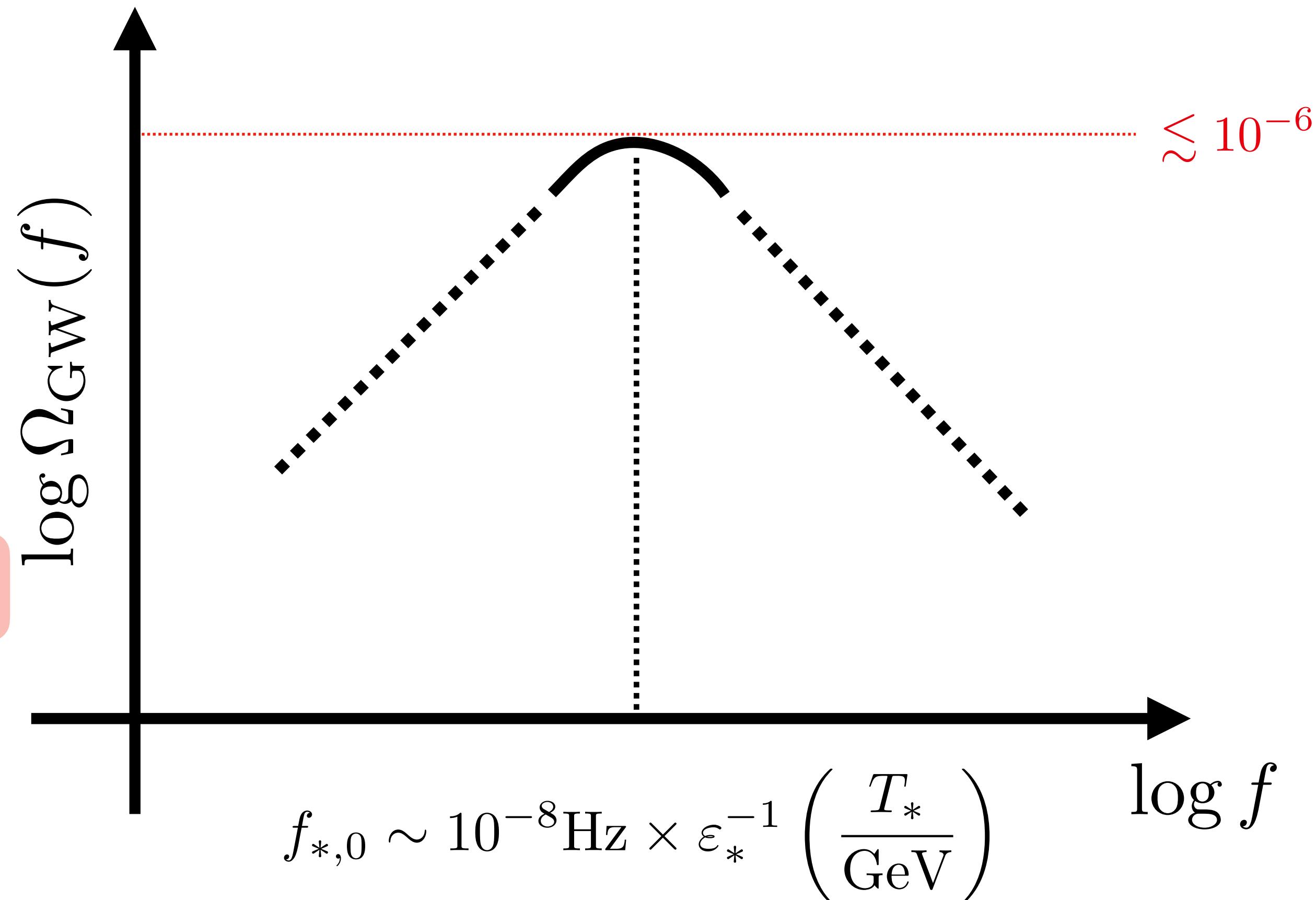
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for homogeneous and isotropic stochastic sources

$$\langle \Pi_{ij}(\mathbf{k}, t) \Pi^{ij}(\mathbf{k}, t') \rangle = (2\pi)^3 \delta(\mathbf{k} - \mathbf{k}') \Pi(\mathbf{k}, t, t')$$



$$\Omega_{\text{GW}}(k) = \frac{4G}{\pi} k^3 \int_{\eta_{\text{in}}}^{\eta_{\text{fin}}} d\eta a^3(\eta) \int_{\eta_{\text{in}}}^{\eta_{\text{fin}}} d\zeta a^3(\zeta) \cos[k(\eta - \zeta)] \Pi(k, \eta, \zeta)$$



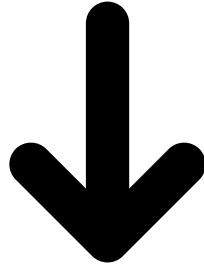
SPECTRUM: slope

anisotropic stress tensor of the (cosmological) source

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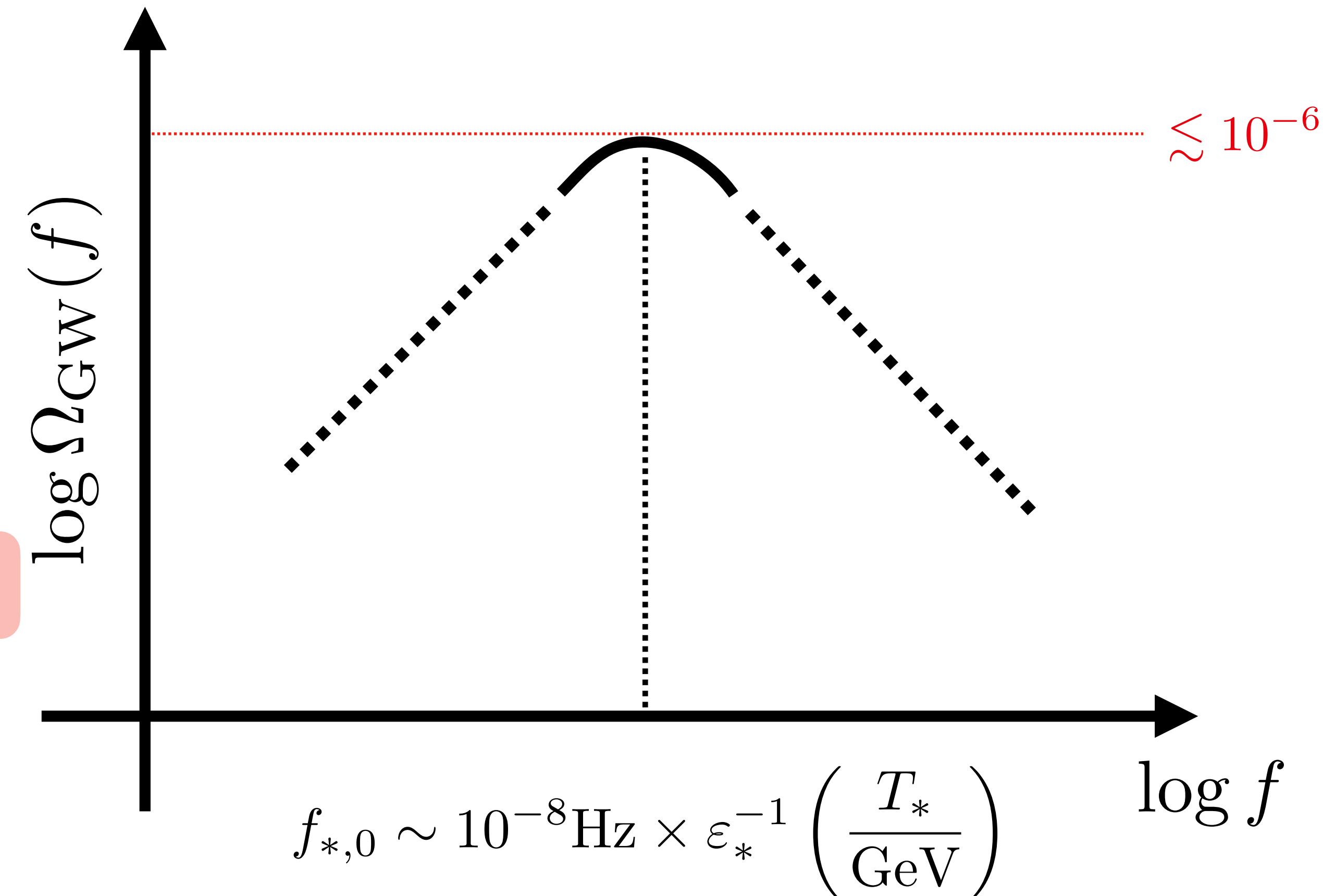
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for casual sources

$$\Pi(k, \eta, \zeta) \xrightarrow[k < 1/H_*]{} \text{const}$$



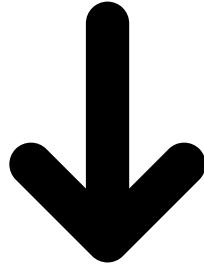
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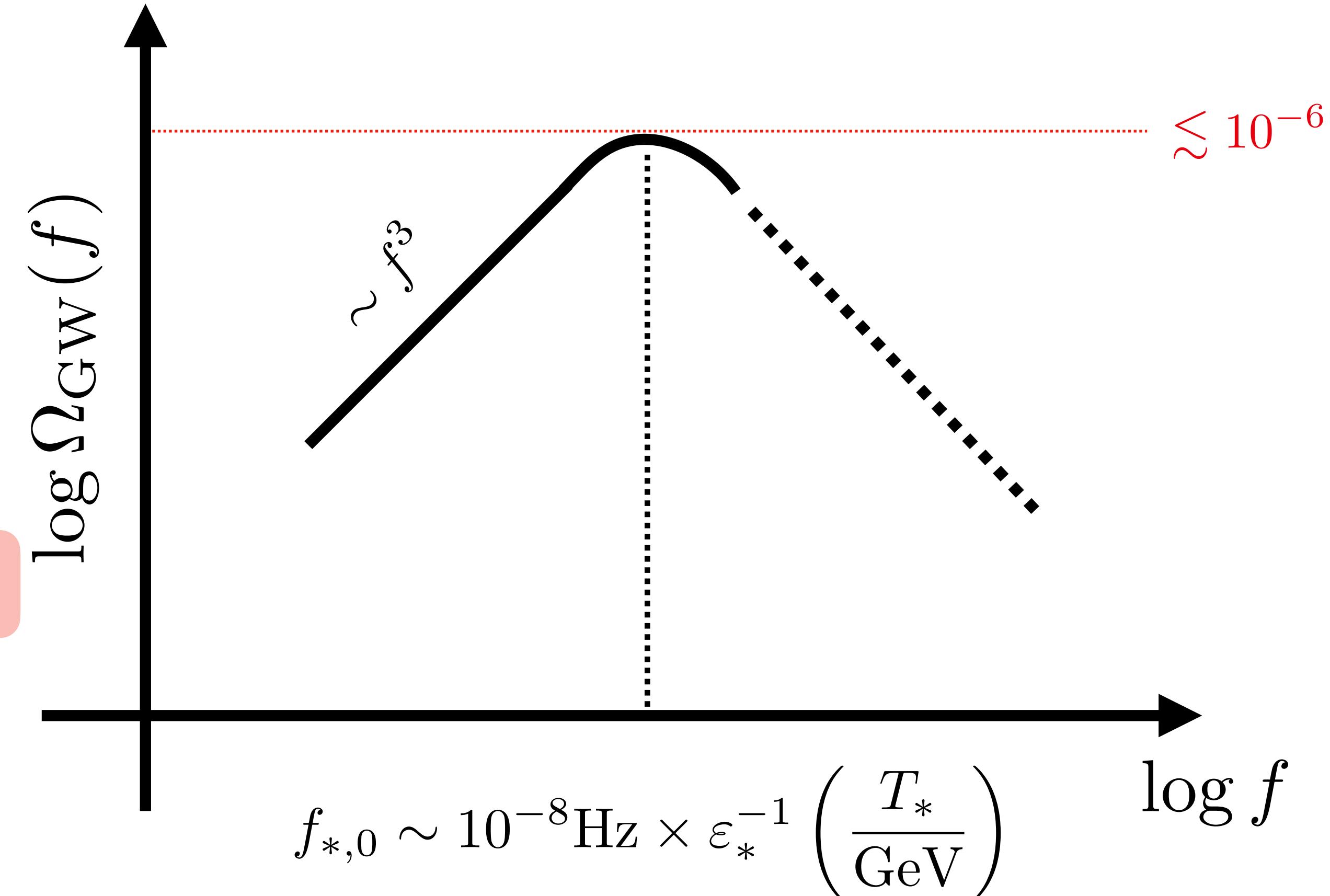


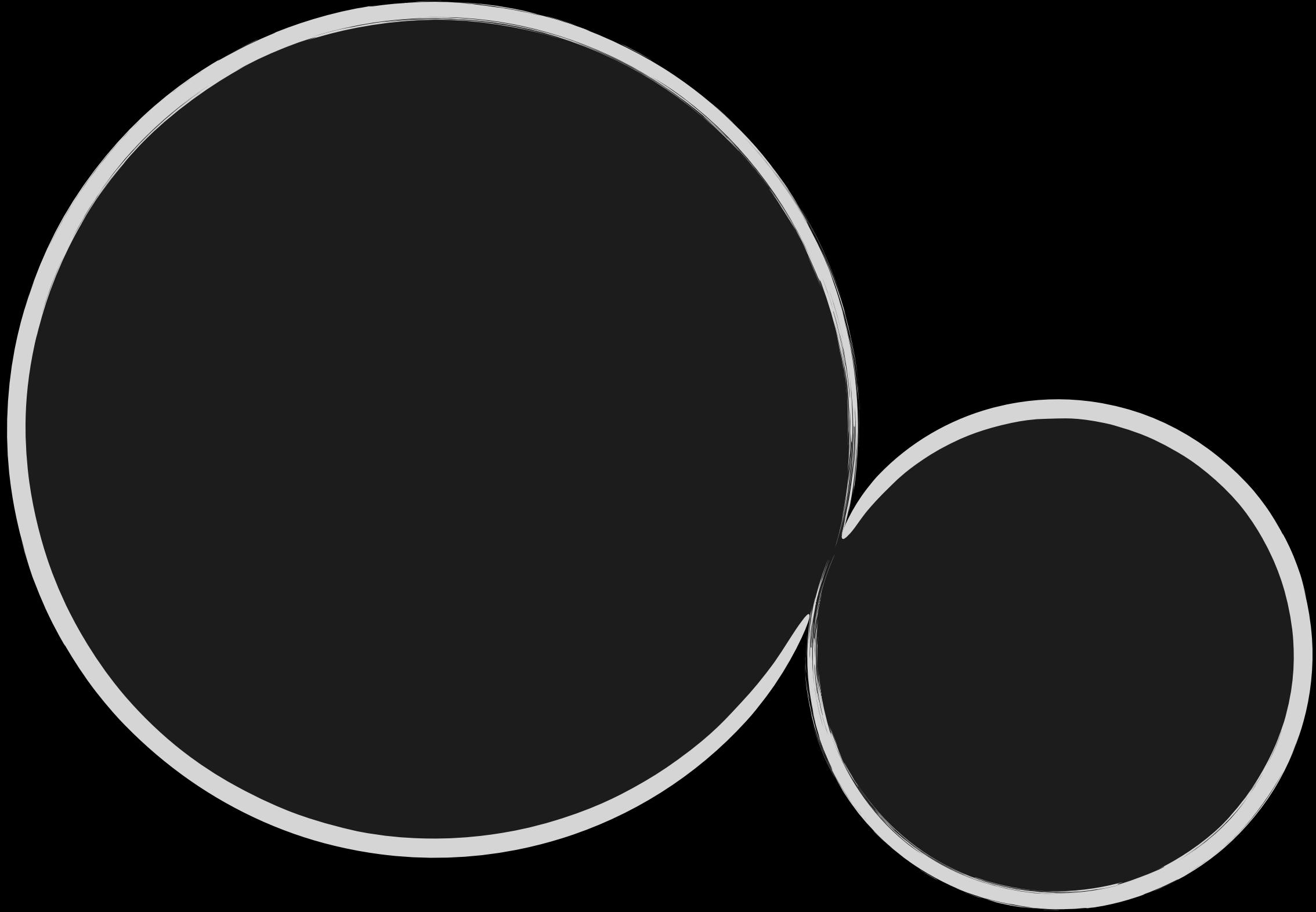
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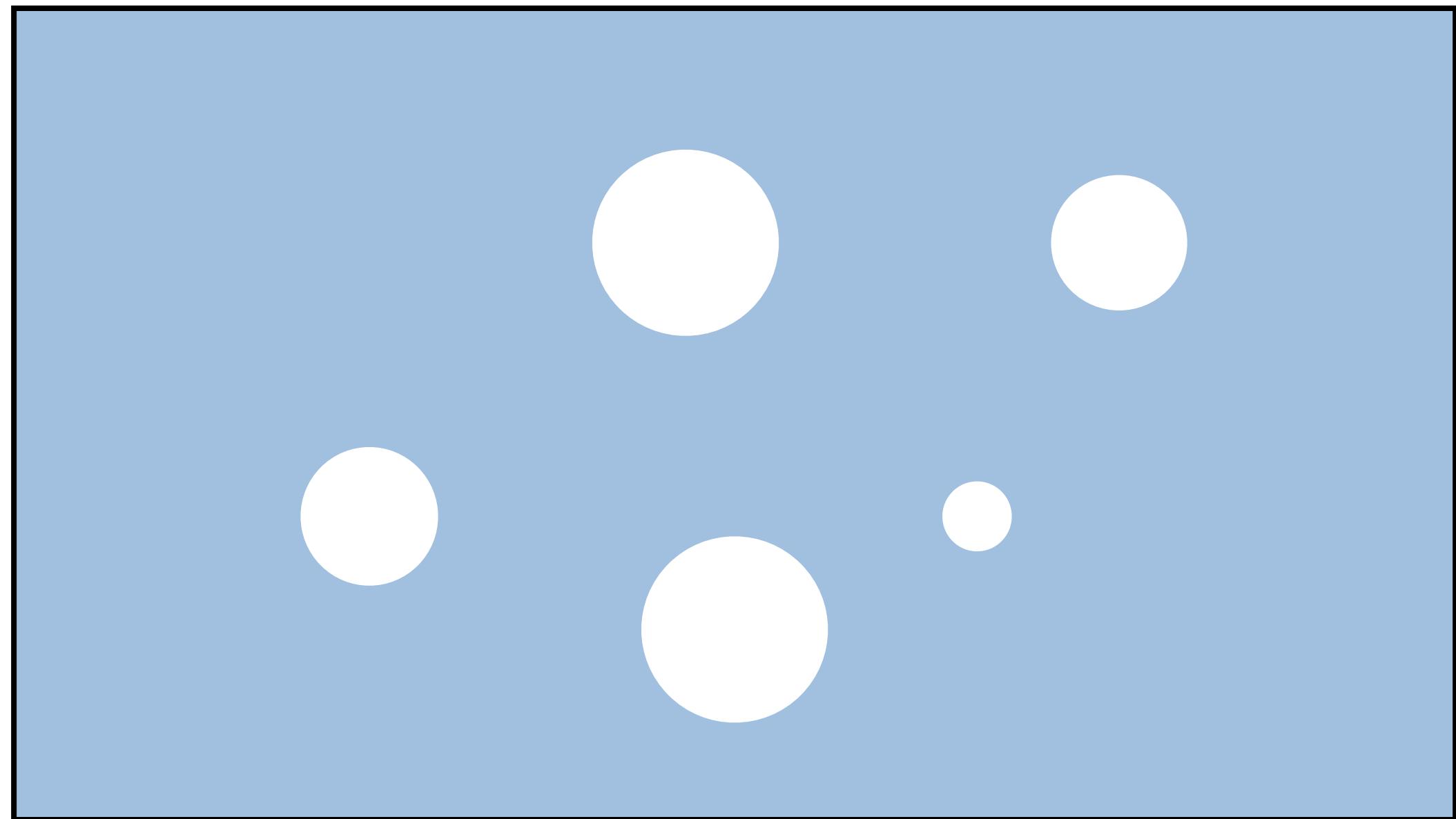
$$\Omega(f)_{\text{GW}} \xrightarrow[k < 1/H_*]{} f^3$$



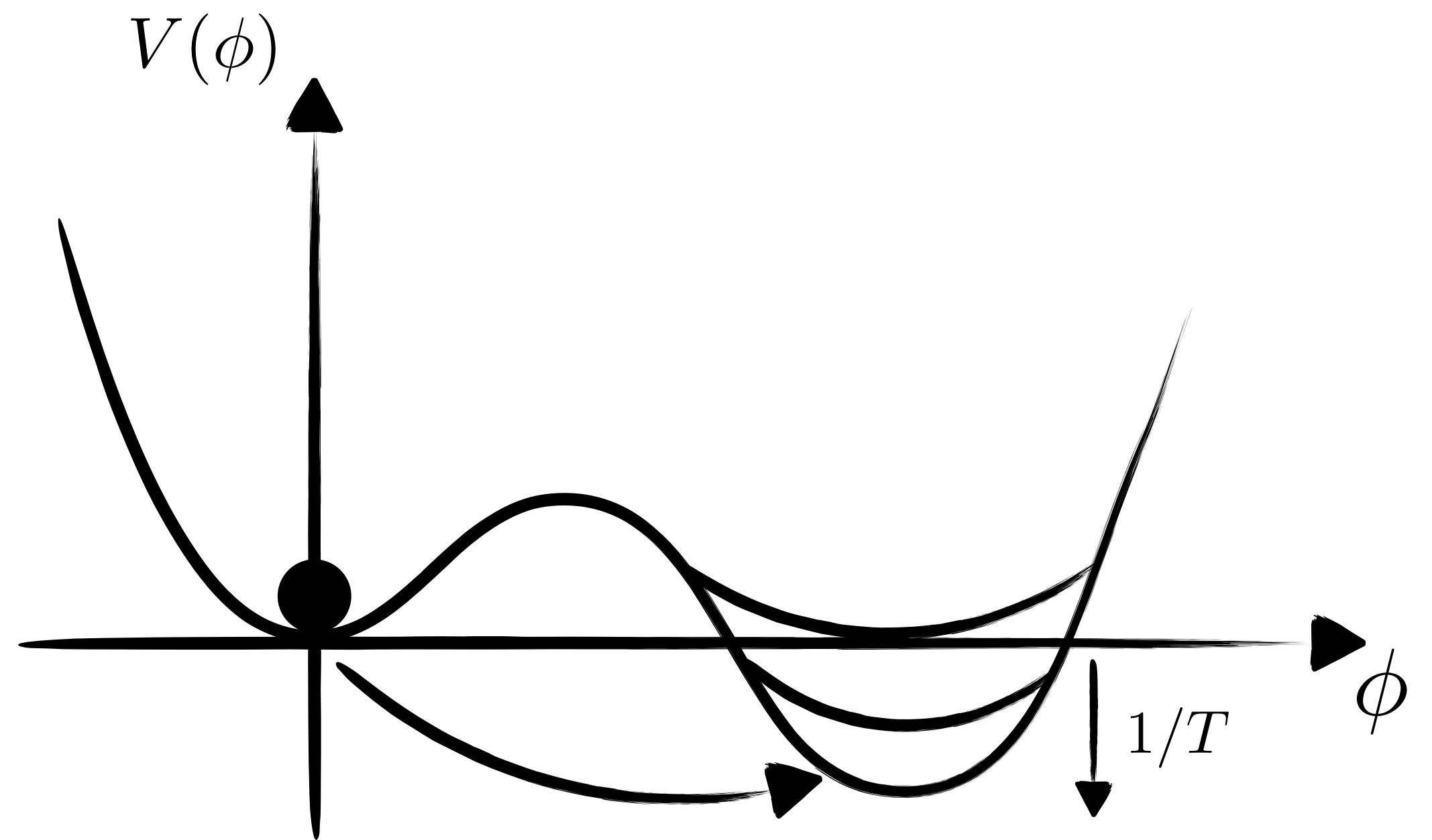


cosmological phase transitions

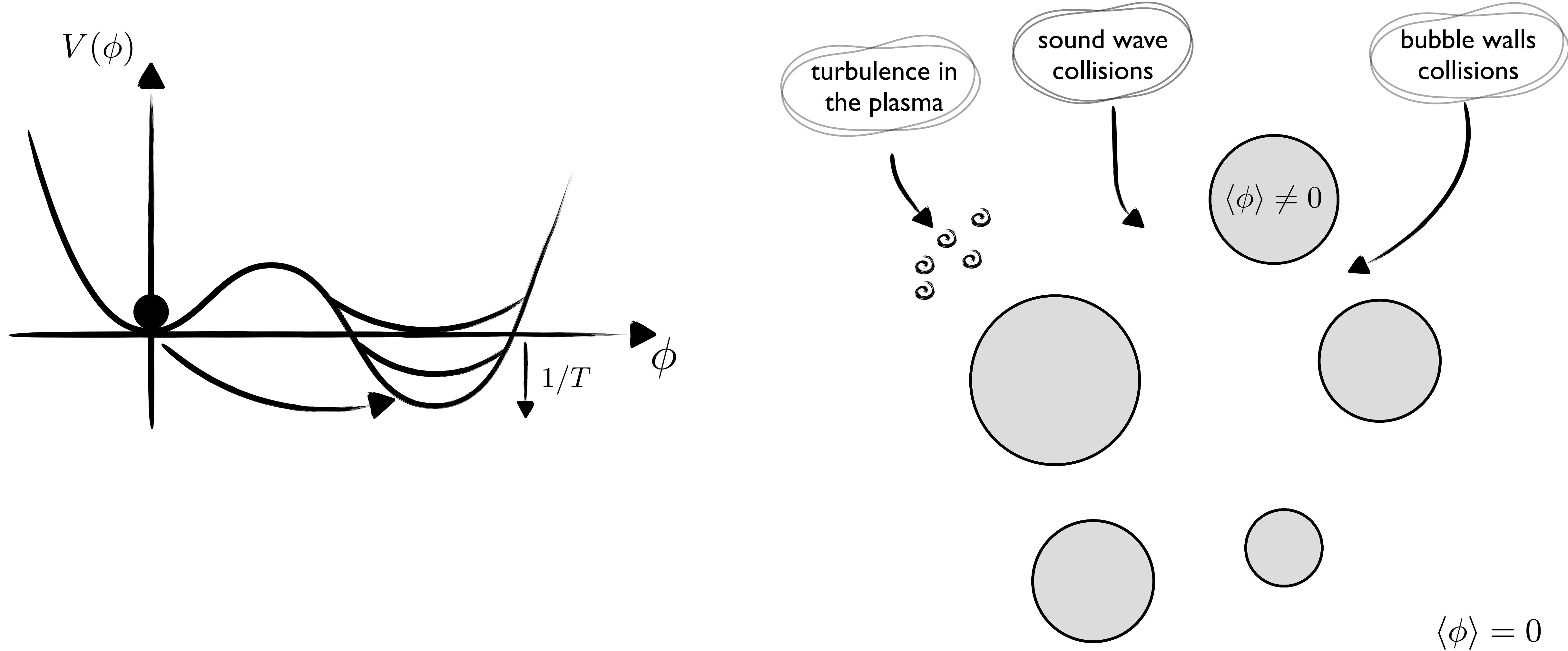
PHASE TRANSITIONS



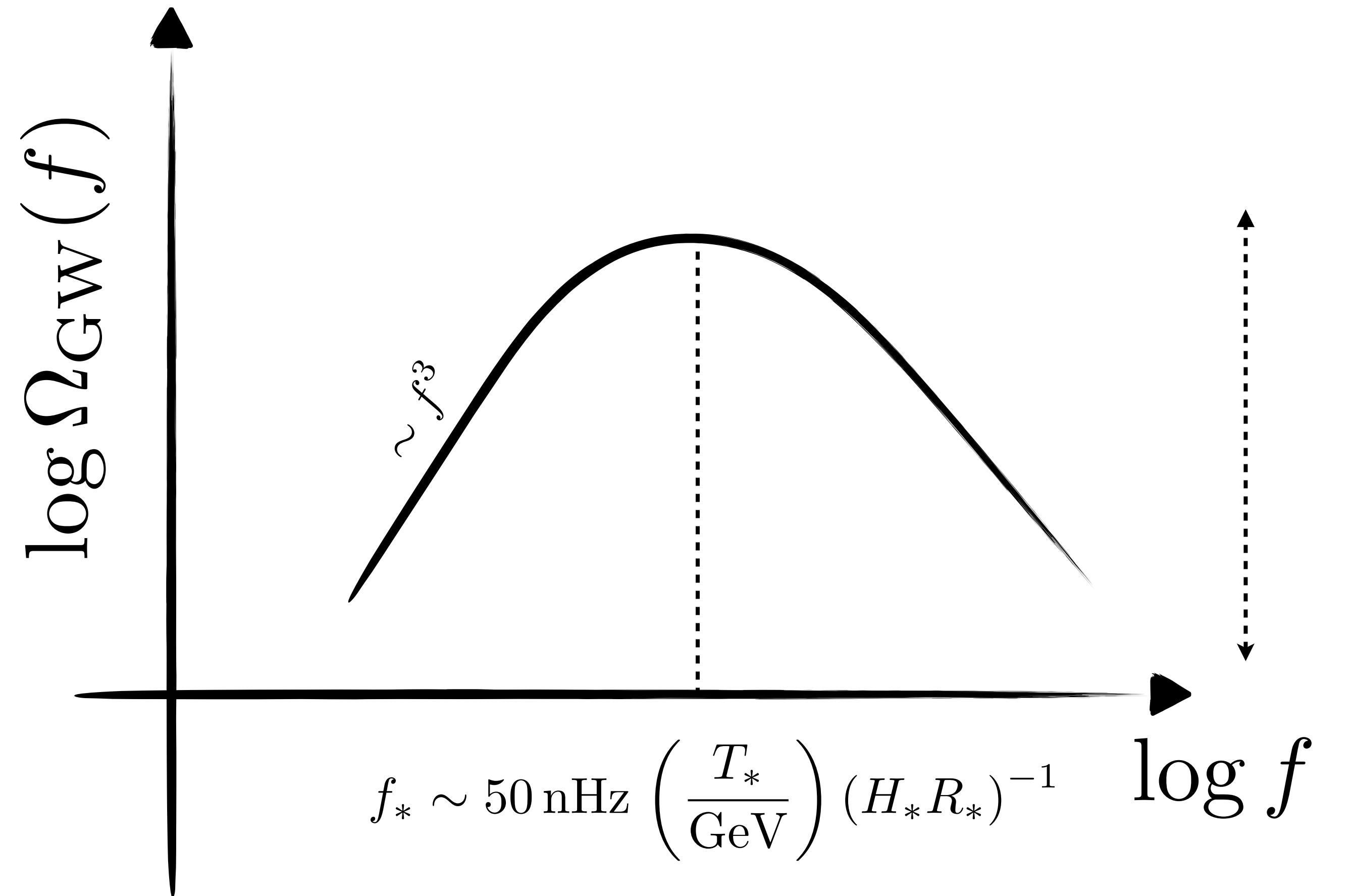
PHASE TRANSITIONS



PHASE TRANSITIONS

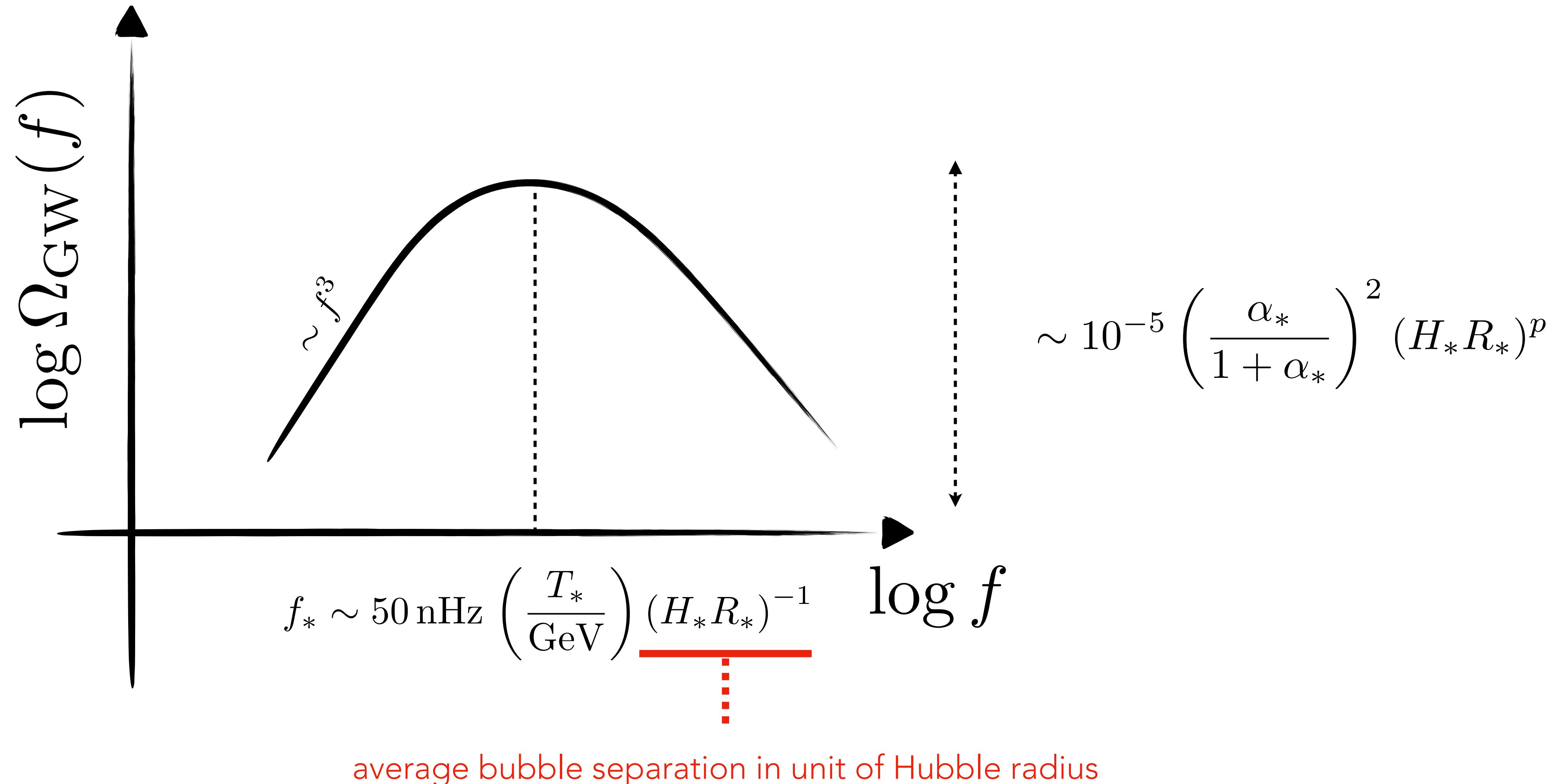


PHASE TRANSITIONS

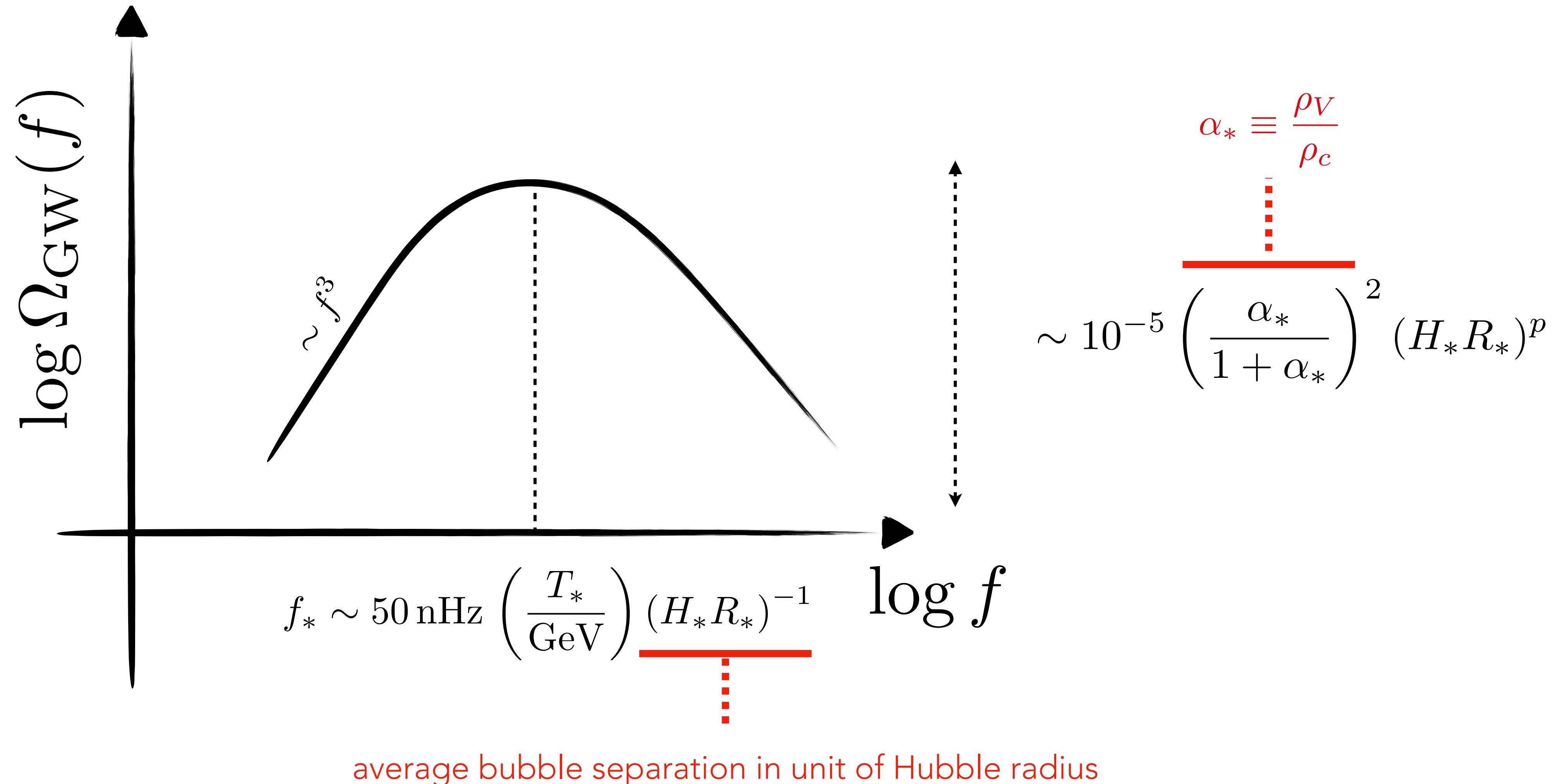


$$\sim 10^{-5} \left(\frac{\alpha_*}{1 + \alpha_*} \right)^2 (H_* R_*)^p$$

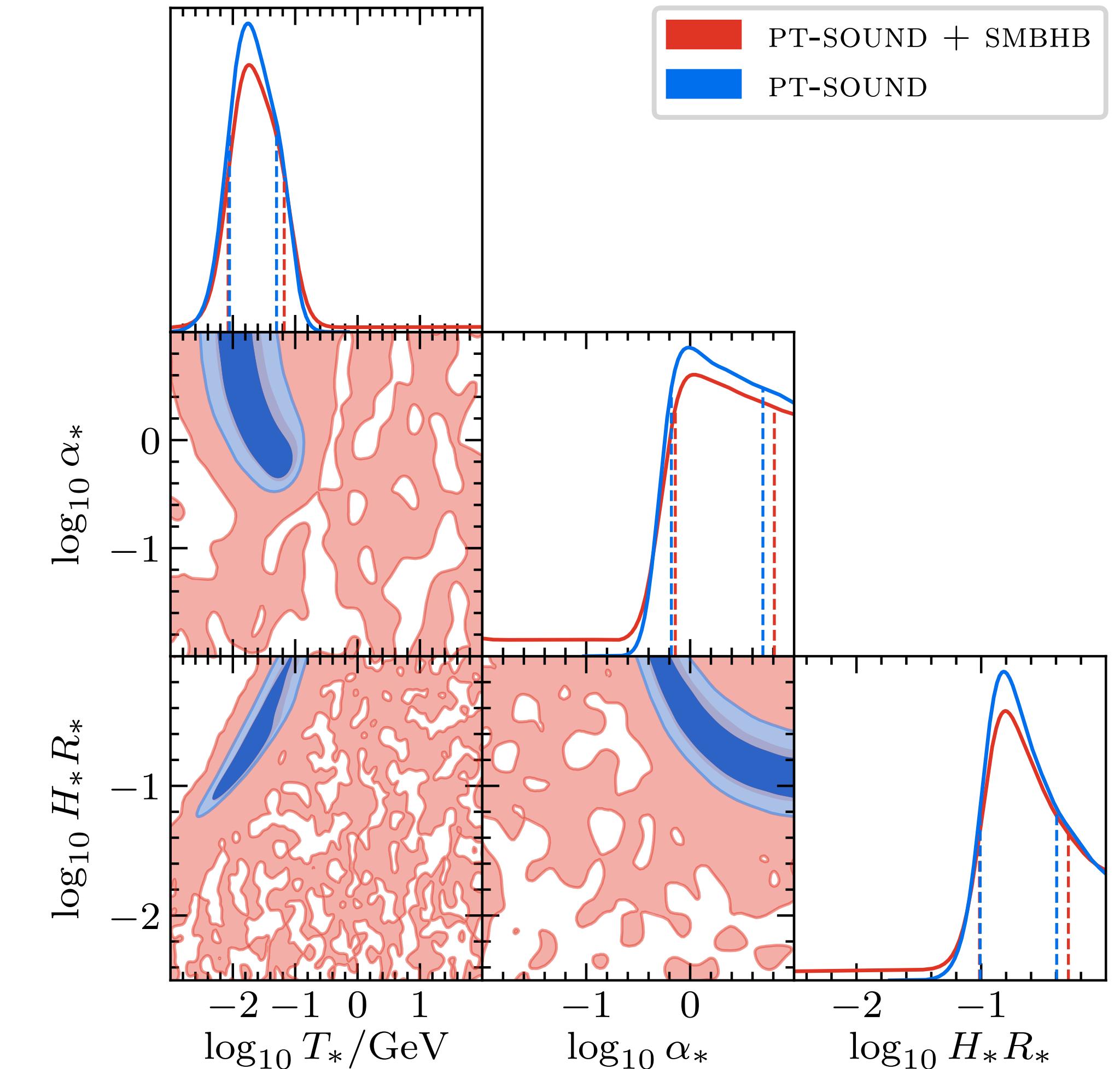
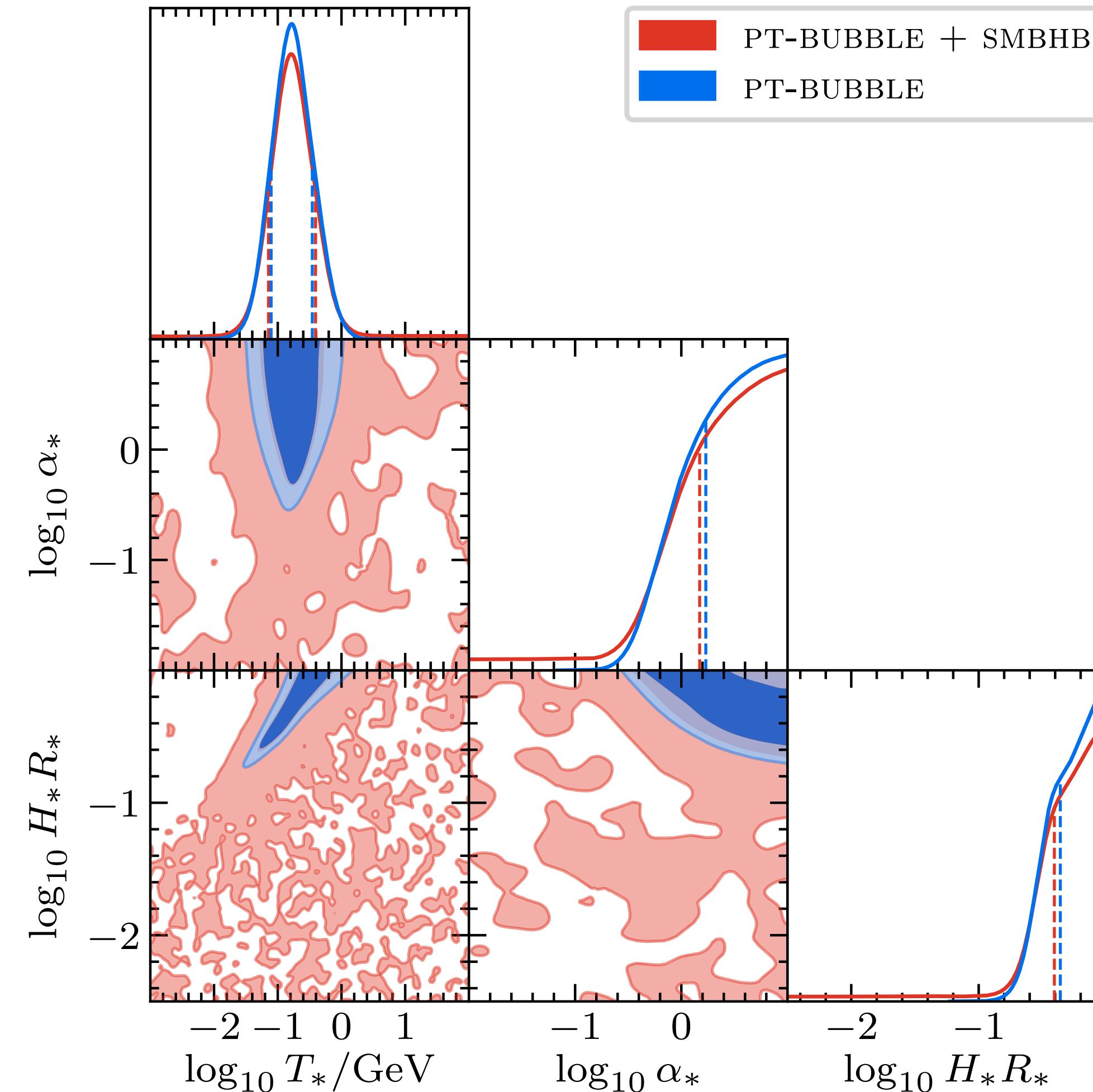
PHASE TRANSITIONS

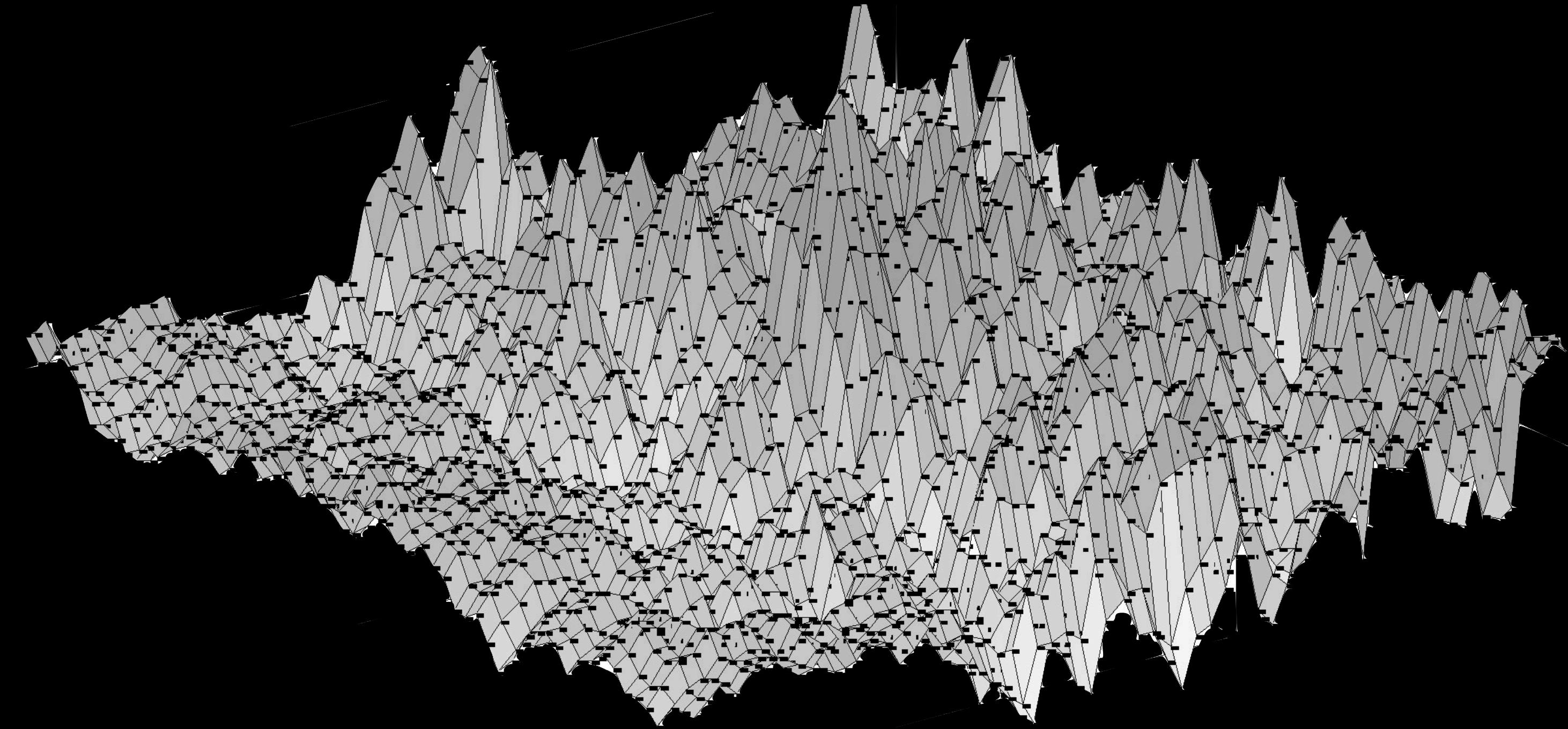


PHASE TRANSITIONS



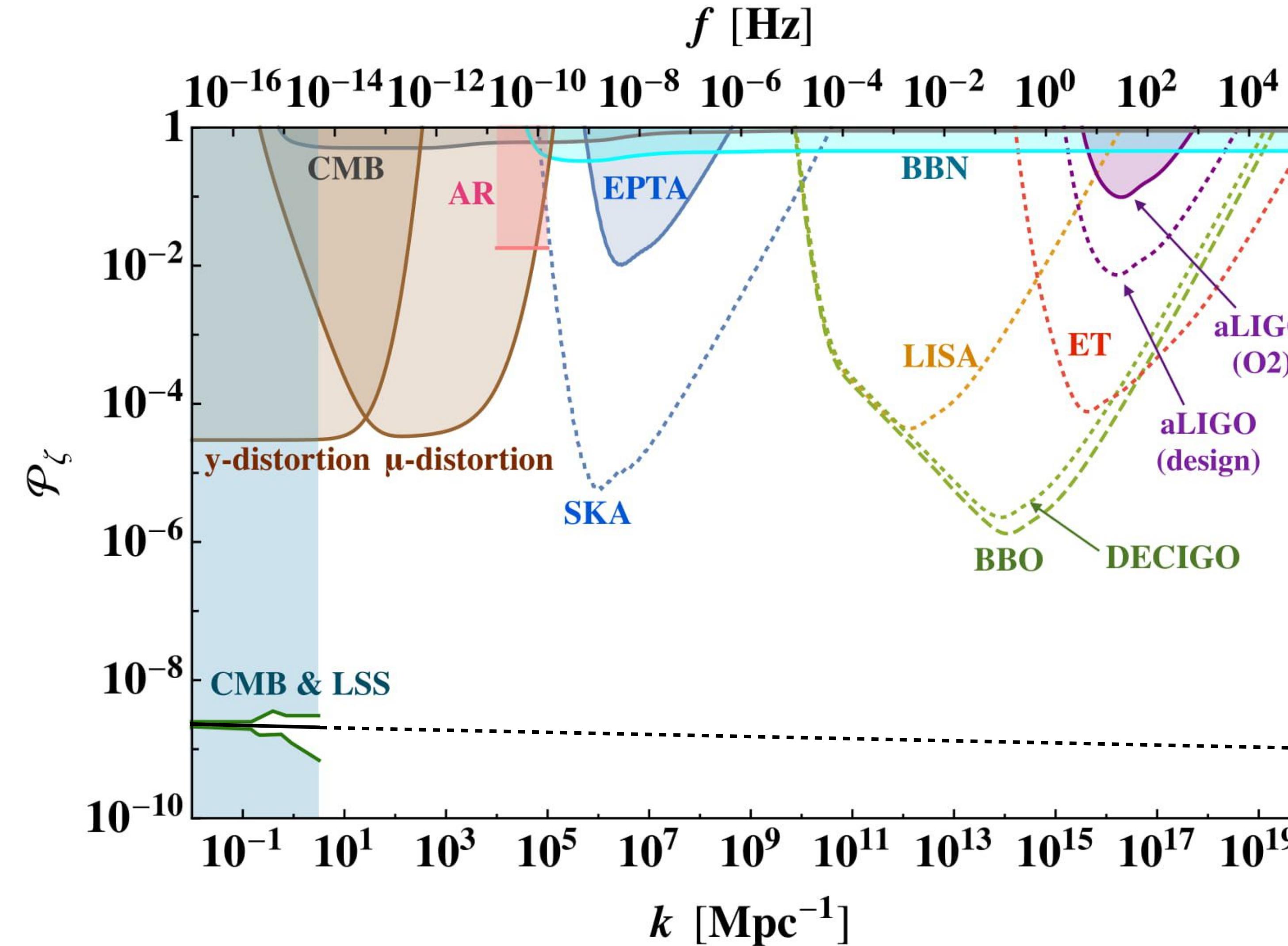
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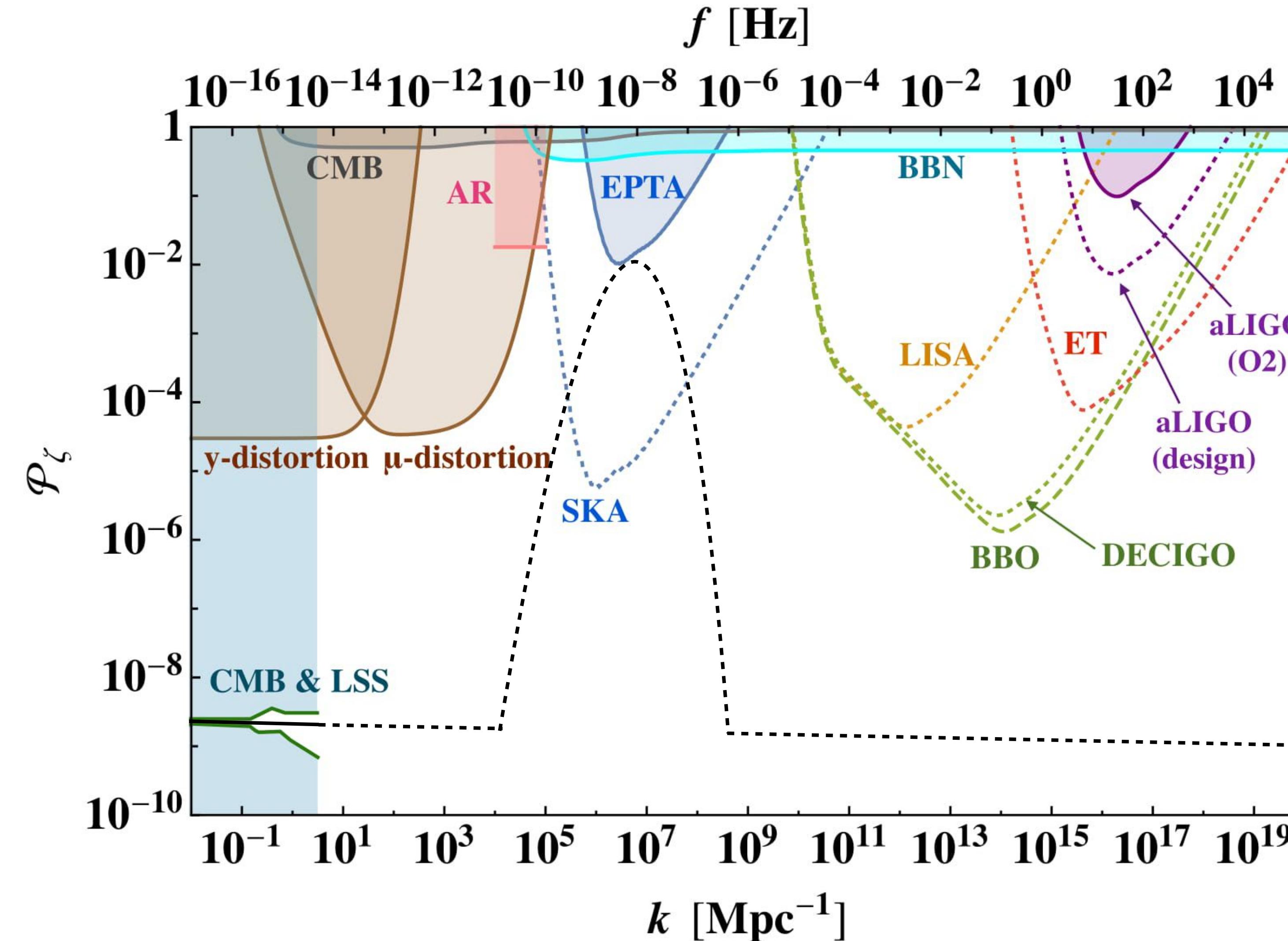


scalar induced GWs

SCALAR INDUCED GW



SCALAR INDUCED GW



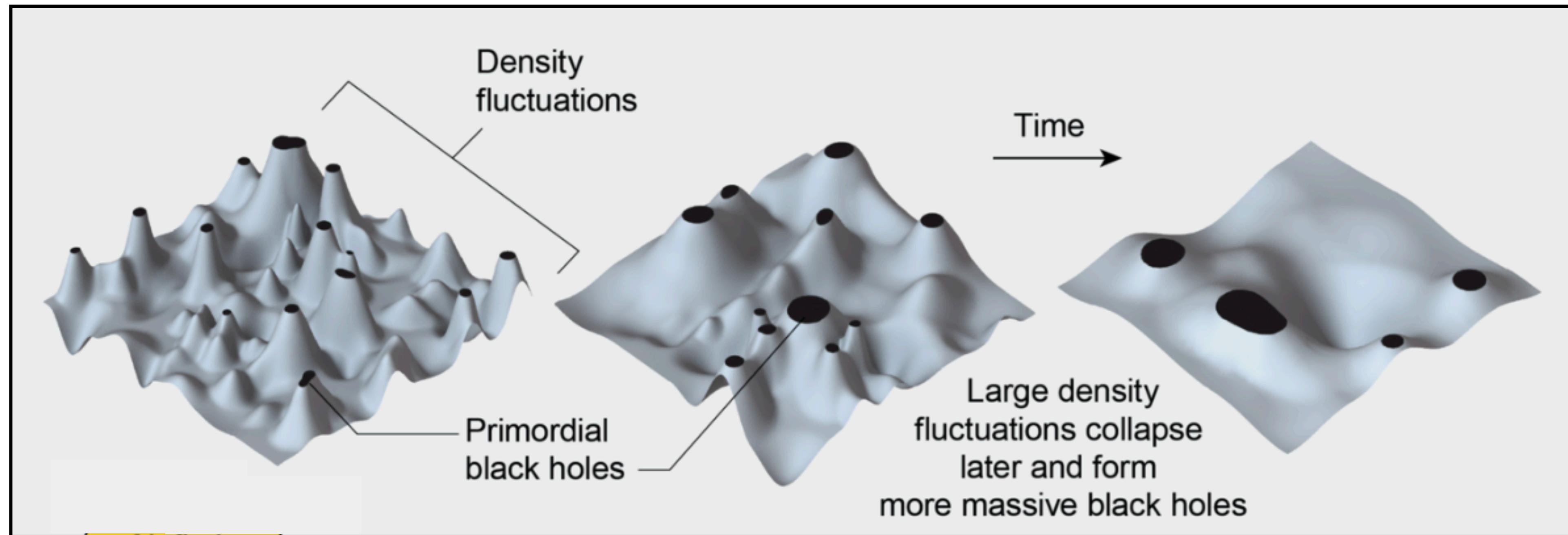
SCALAR INDUCED GW

fraction of patches collapsing at horizon re-entry

$$\beta(M) \sim \frac{1}{\sqrt{2\pi}\sigma(M)} \int_{\delta_c}^{\infty} d\delta \exp\left(-\frac{\delta^2}{2\sigma(M)^2}\right)$$

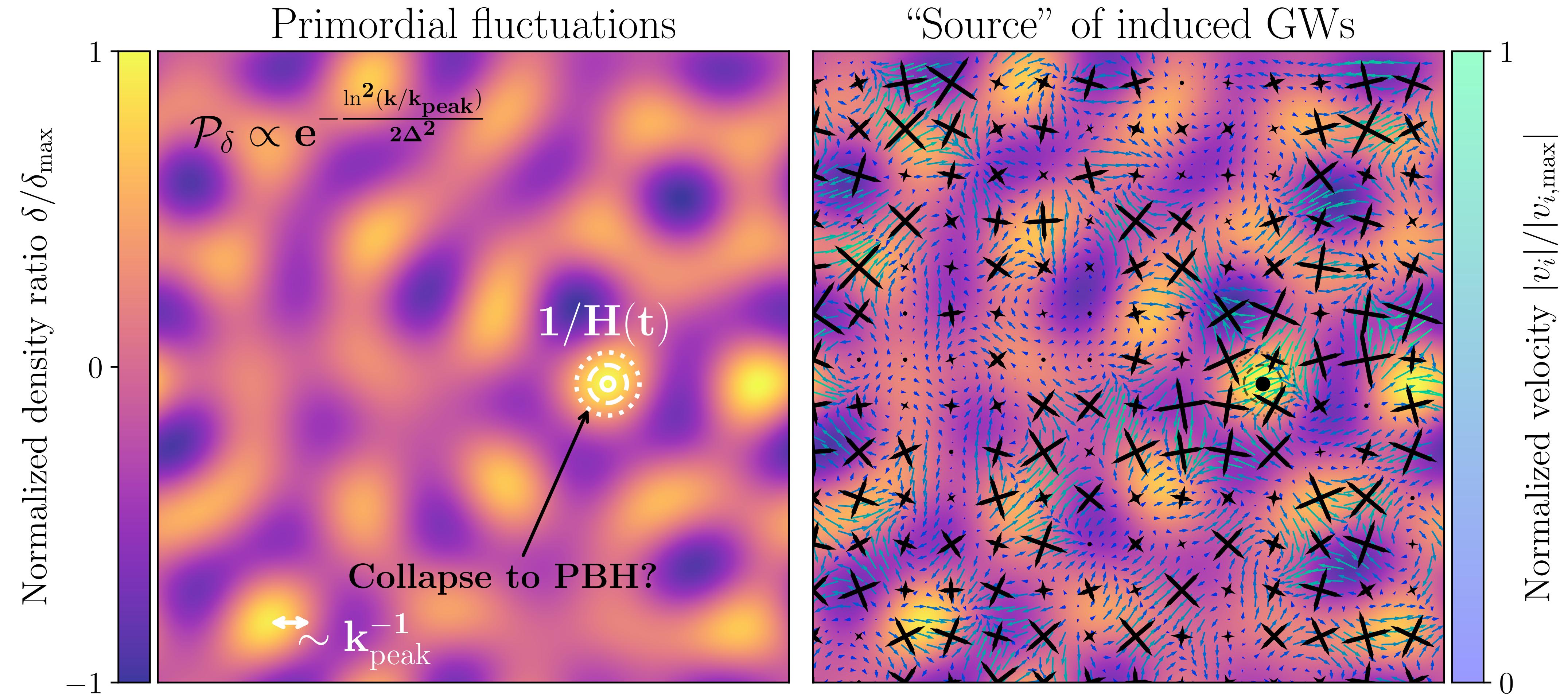
variance of density perturbations

$$\sigma(M)^2 \sim \int \frac{dk}{k} P_\delta(k) W^2(kR(M))$$



SCALAR INDUCED GW

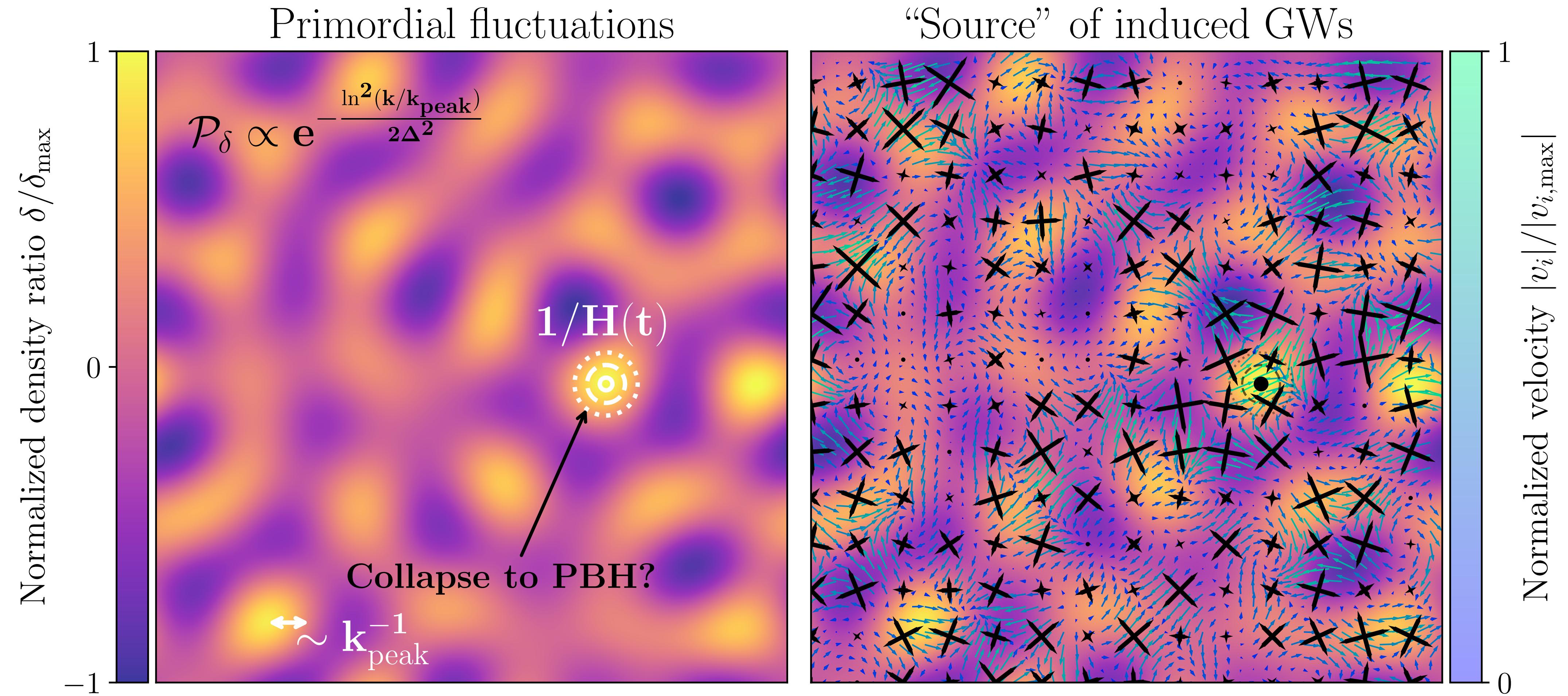
$$h_{ij}'' + 2Hh_{ij}' - \Delta h_{ij} = [4\partial_i\Phi\partial_j\Phi + 2a^2(\rho + P)v_iv_j]^{TT}$$



SCALAR INDUCED GW

$$h_{ij}'' + 2Hh_{ij}' - \Delta h_{ij} = [4\partial_i\Phi\partial_j\Phi + 2a^2(\rho + P)v_iv_j]^{\text{TT}}$$

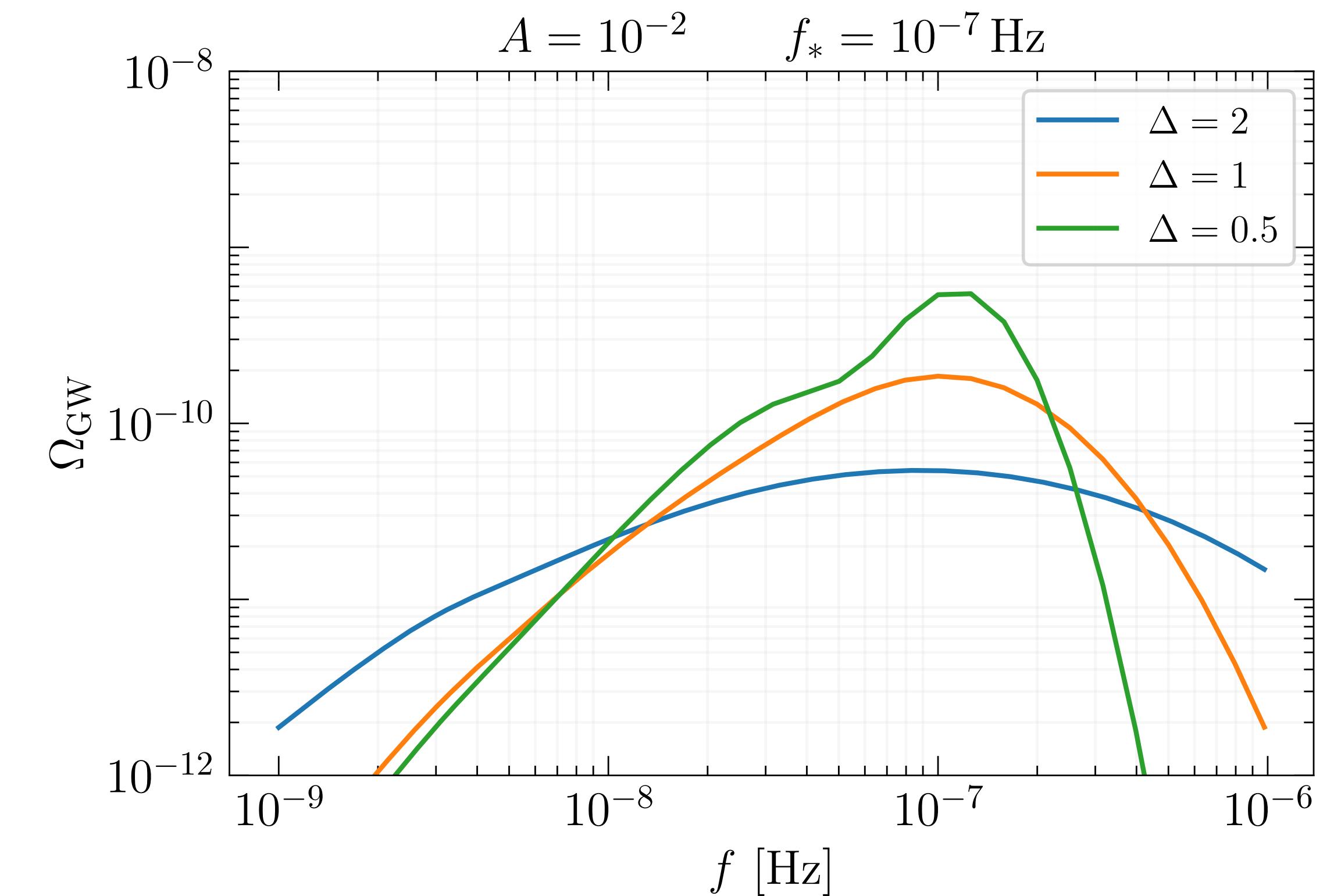
density fluctuations act as a source for GWs



SCALAR INDUCED GW

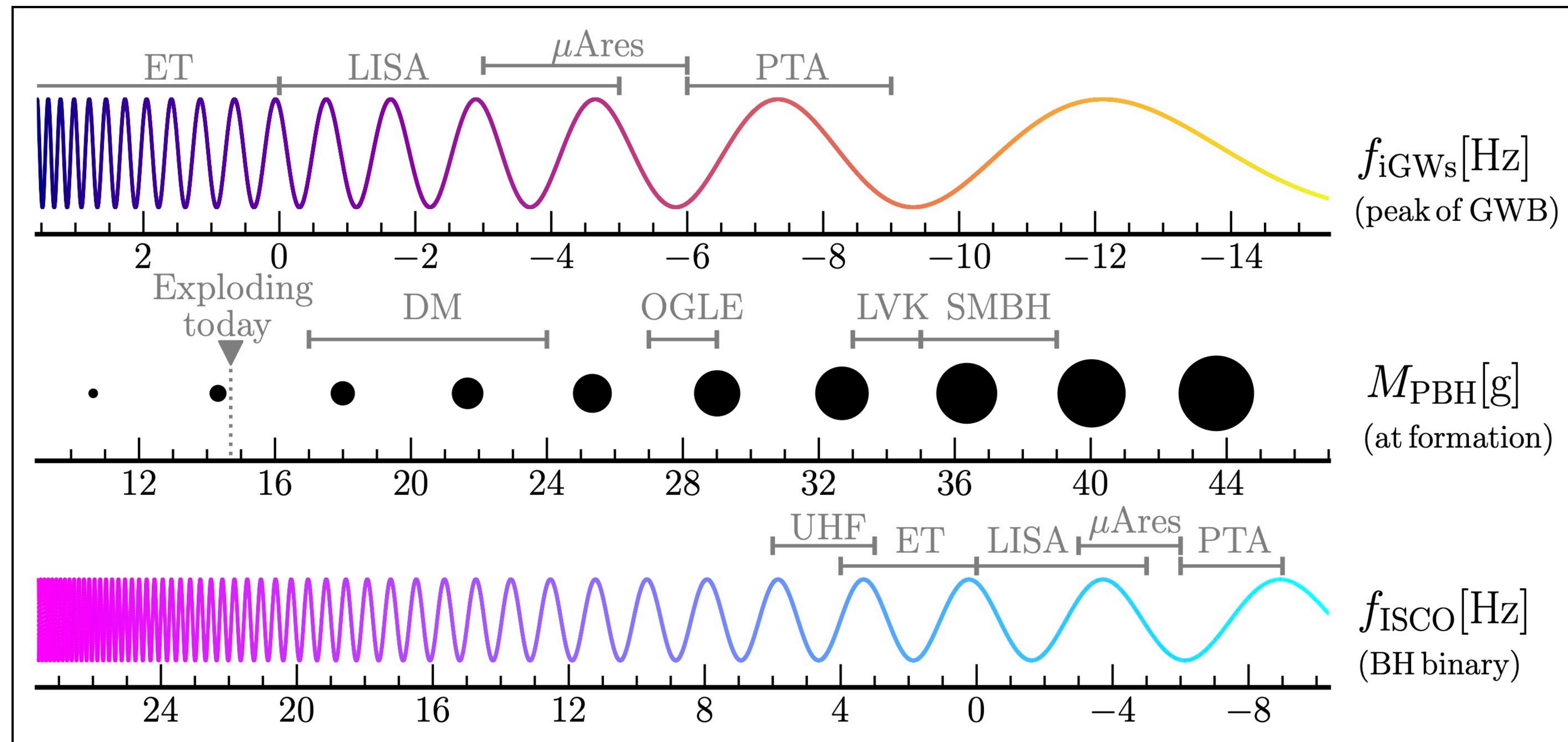
$$\Omega_{\text{GW}}(f) \simeq \Omega_r \int_0^\infty dv \int_{|1-v|}^{1+v} du \mathcal{K}(u, v) \mathcal{P}_{\mathcal{R}}(uk) \mathcal{P}_{\mathcal{R}}(vk)$$

$$P_{\mathcal{R}}(k) = \frac{A}{2\pi\Delta} \exp \left[-\frac{1}{2} \left(\frac{\ln k - \ln k_*}{2\Delta^2} \right)^2 \right]$$

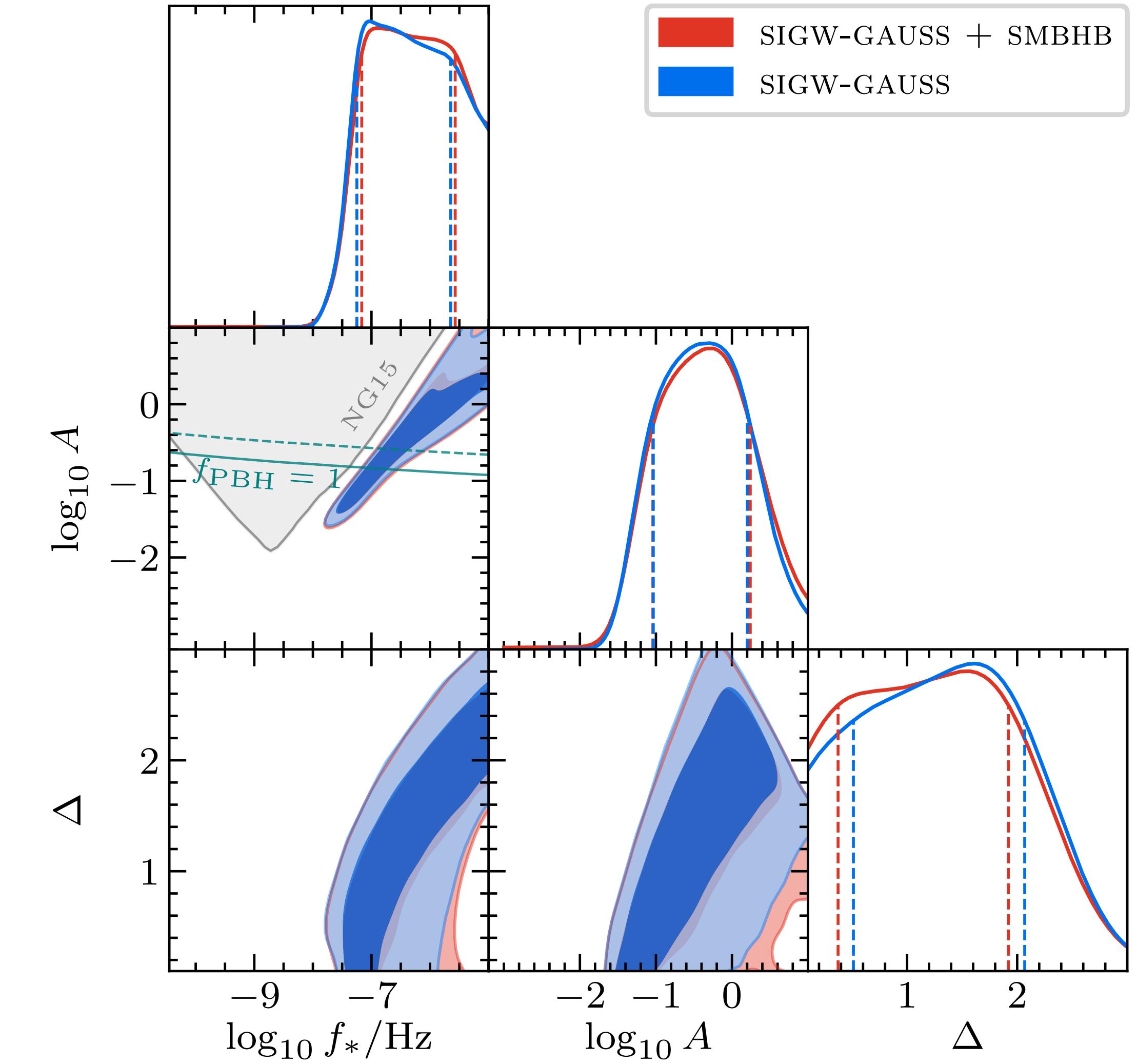
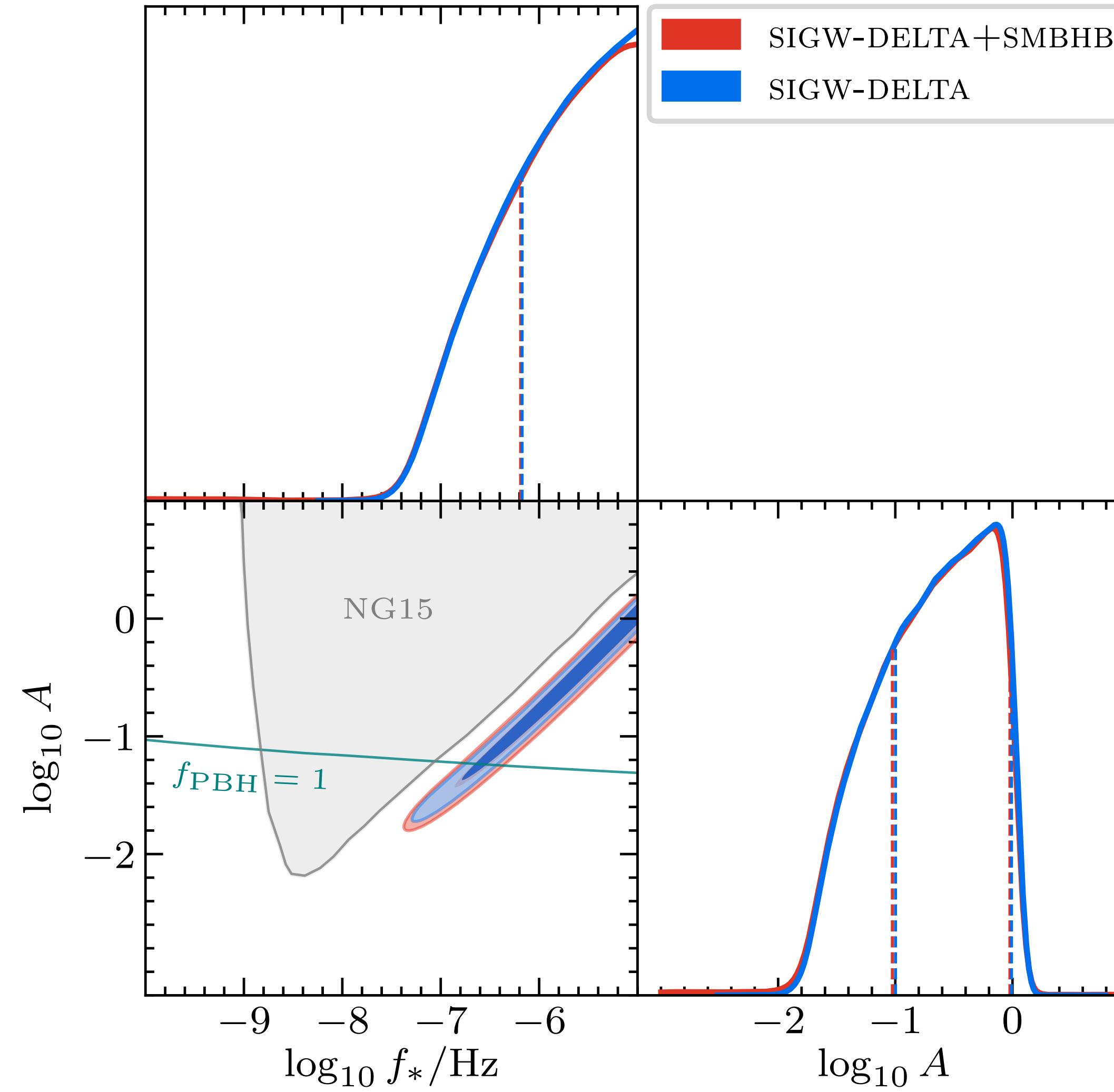


SCALAR INDUCED GW

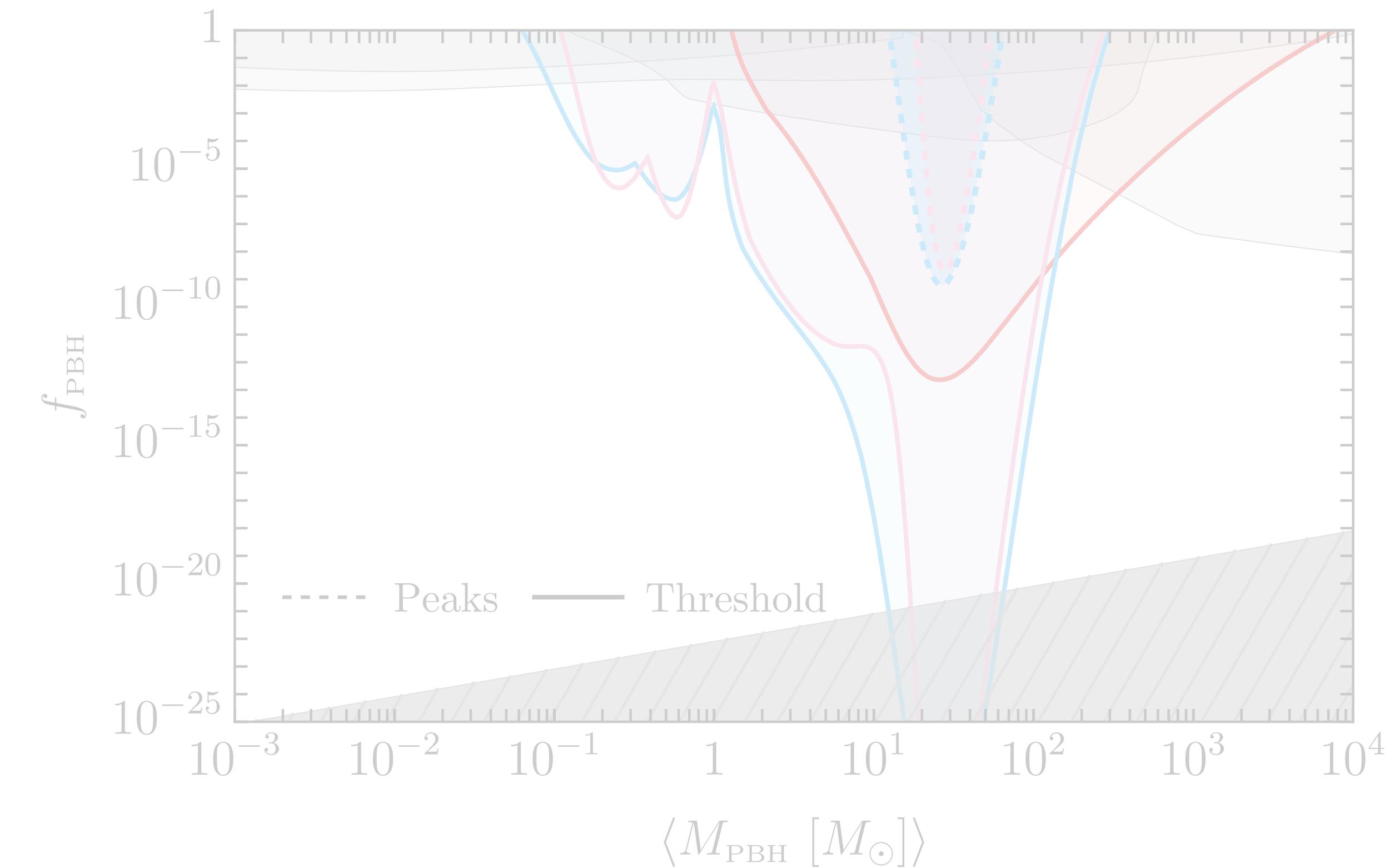
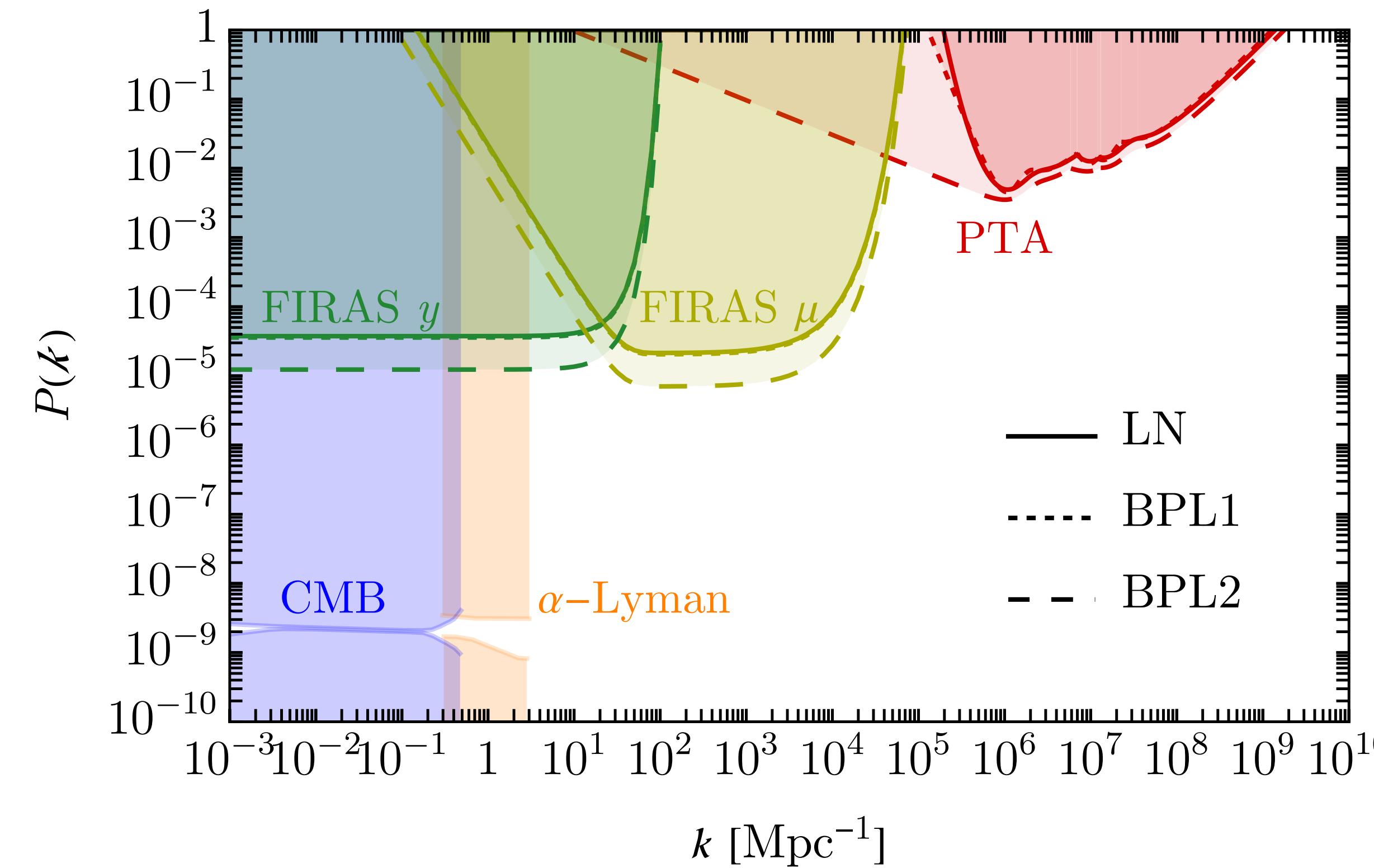
scalar induced GW-PBH connection



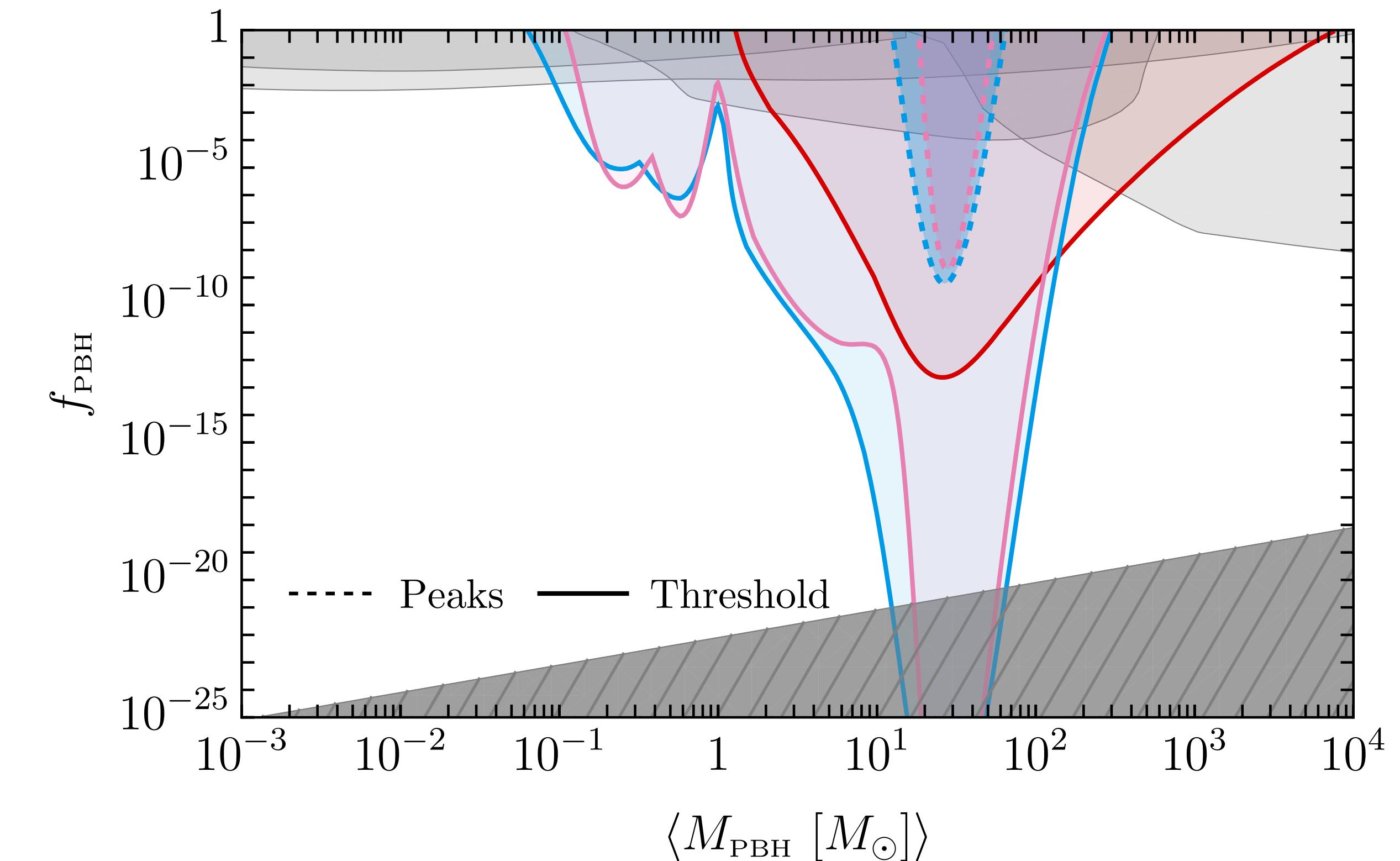
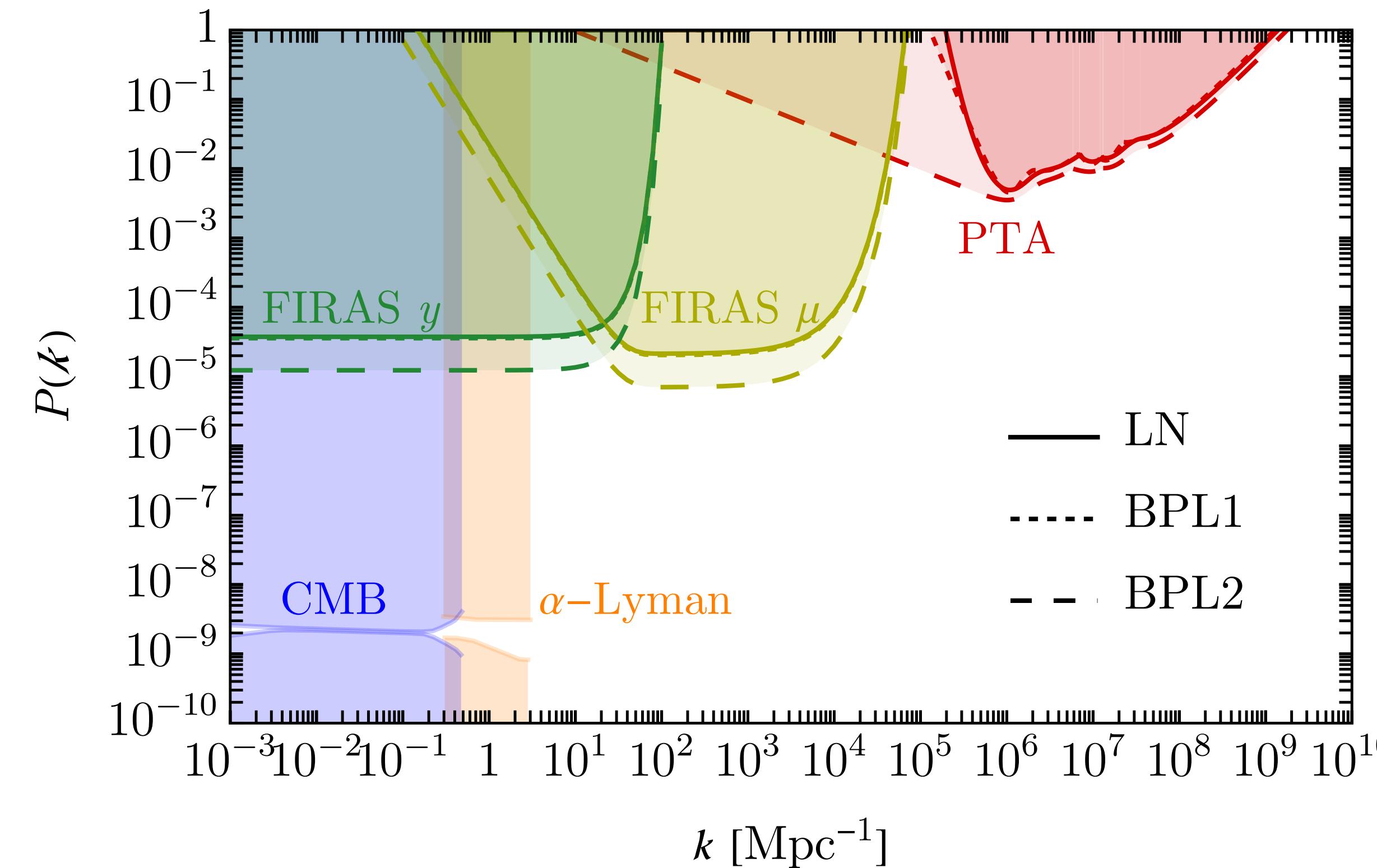
SCALAR INDUCED GW



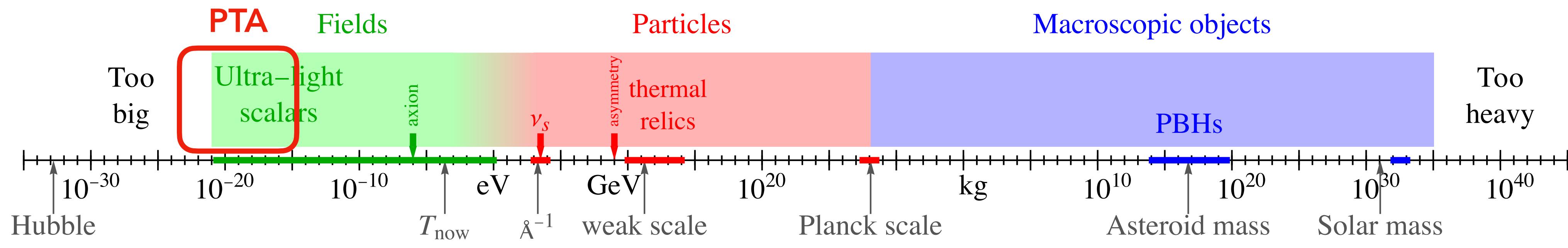
SCALAR INDUCED GW



SCALAR INDUCED GW



not only GWs



ULDM feature **large occupation numbers**

$$N_{\text{occ}} \sim \frac{\rho_{\text{DM}}}{m} \lambda_{\text{dB}}^3 \sim 10^{93} \left(\frac{10^{-20} \text{eV}}{m} \right)$$



within the Galactic halo, we can treat ULDM as a **classical field**

$$\phi(\vec{x}, t) = A(\vec{x}) \cos (mt + \alpha(\vec{x}))$$

ULDM: GRAVITATIONAL SIGNALS

ULDM density and pressure fluctuations

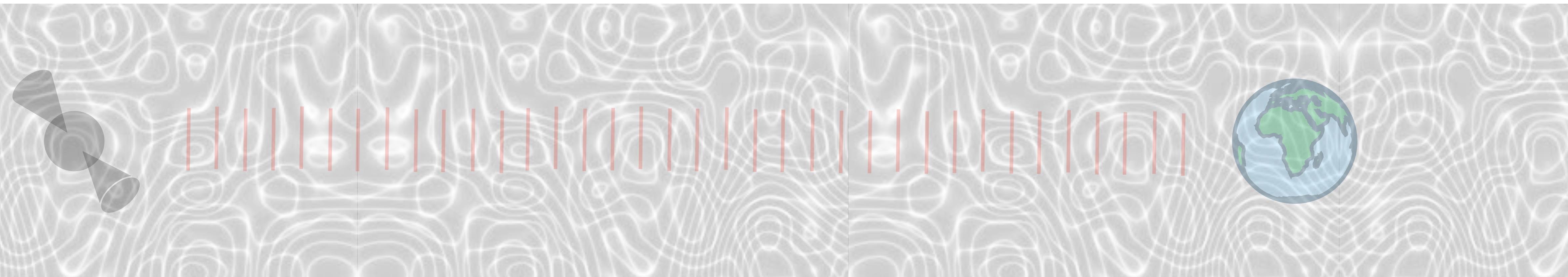
$$\begin{aligned} p_\phi &\approx -\rho_0 \cos(2mt + \alpha) \\ \rho_\phi &\approx \rho_0 [1 - v^2 \cos(2mt + \alpha)] \end{aligned}$$

source gravitational potentials fluctuations

$$\begin{aligned} \nabla^2 \Psi &= 4\pi G \rho_\phi \\ 6\ddot{\Psi} + 2\nabla^2(\Phi - \Psi) &= 24\pi G p_\phi \end{aligned}$$

hence metric fluctuations

$$ds^2 = [1 + 2\Phi(t, \vec{x})] dt^2 - [1 - 2\Psi(t, \vec{x})] d\vec{x}^2$$



metric fluctuations induce a deterministic timing delay

$$h(t) = A \left[\hat{\phi}_E^2 \sin(2m_\phi t + \gamma_E) - \hat{\phi}_P^2 \sin(2m_\phi t + \gamma_P) \right]$$

ULDM: GRAVITATIONAL SIGNALS

ULDM density and pressure fluctuations

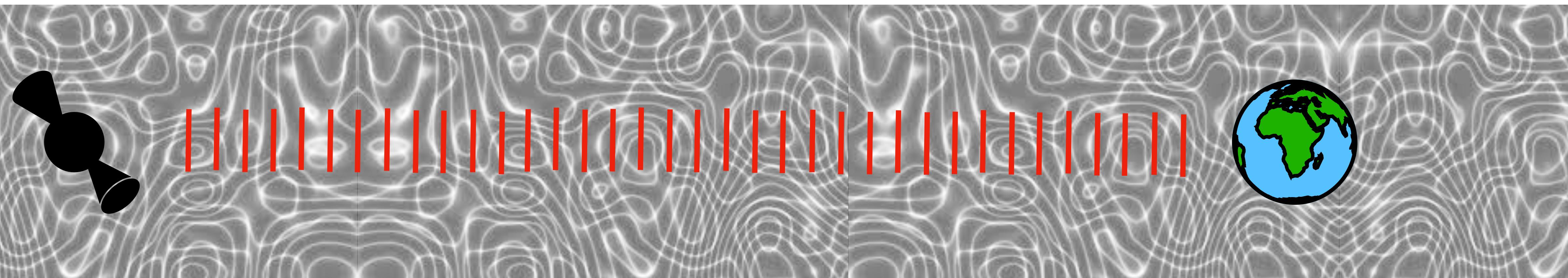
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metric fluctuations induce a monochromatic and deterministic timing delay

$$h(t) = A \left[\hat{\phi}_E^2 \sin(2m_\phi t + \gamma_E) - \hat{\phi}_P^2 \sin(2m_\phi t + \gamma_P) \right]$$

ULDM: GRAVITATIONAL SIGNALS

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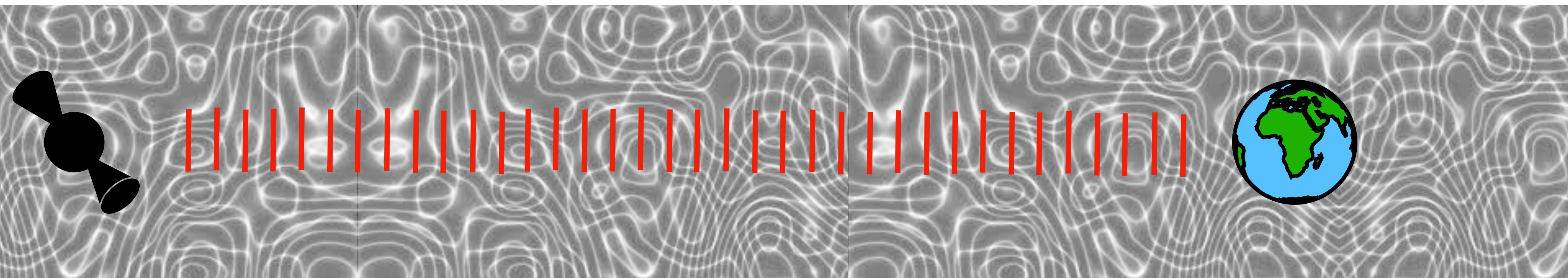
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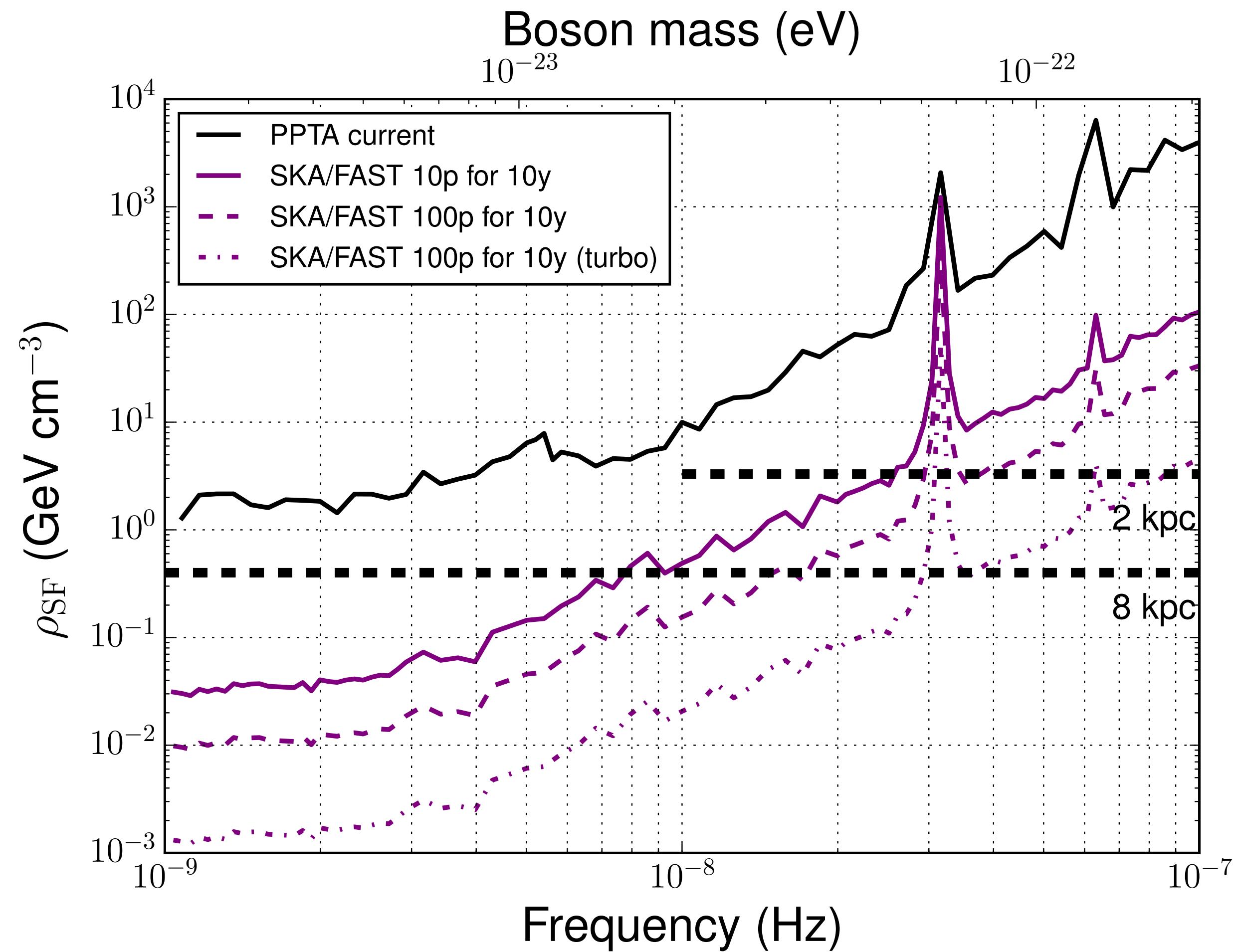
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$$h(t) = A [\hat{\phi}_E^2 \sin(2m_\phi t + \gamma_E) - \hat{\phi}_P^2 \sin(2m_\phi t + \gamma_P)]$$

$$A \sim \frac{G\rho_\phi}{m_\phi^3}$$

ULDM: GRAVITATIONAL SIGNALS

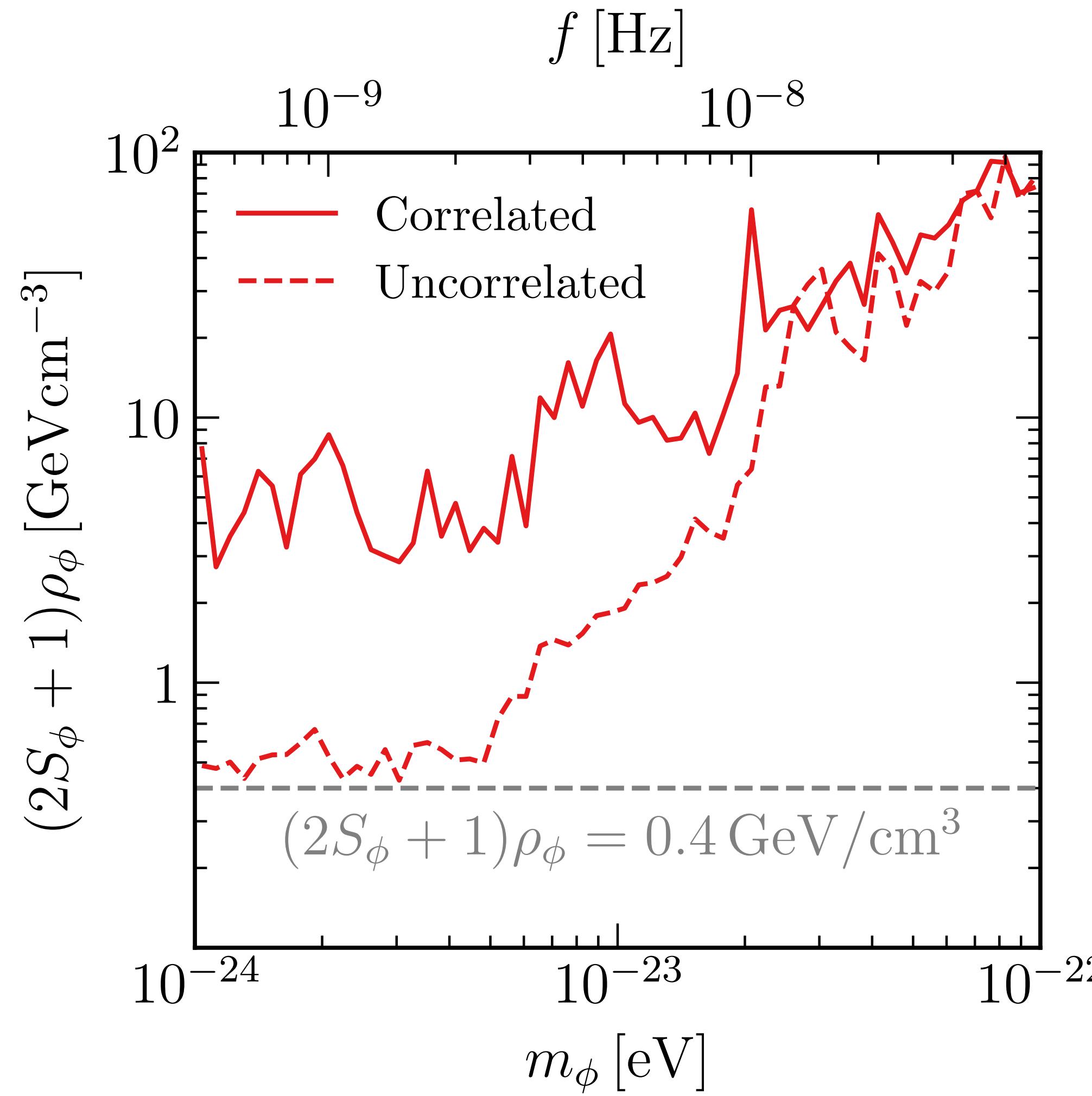
PPTA (2018)
Porayko et al. [1810.03227]



ULDM: GRAVITATIONAL SIGNALS

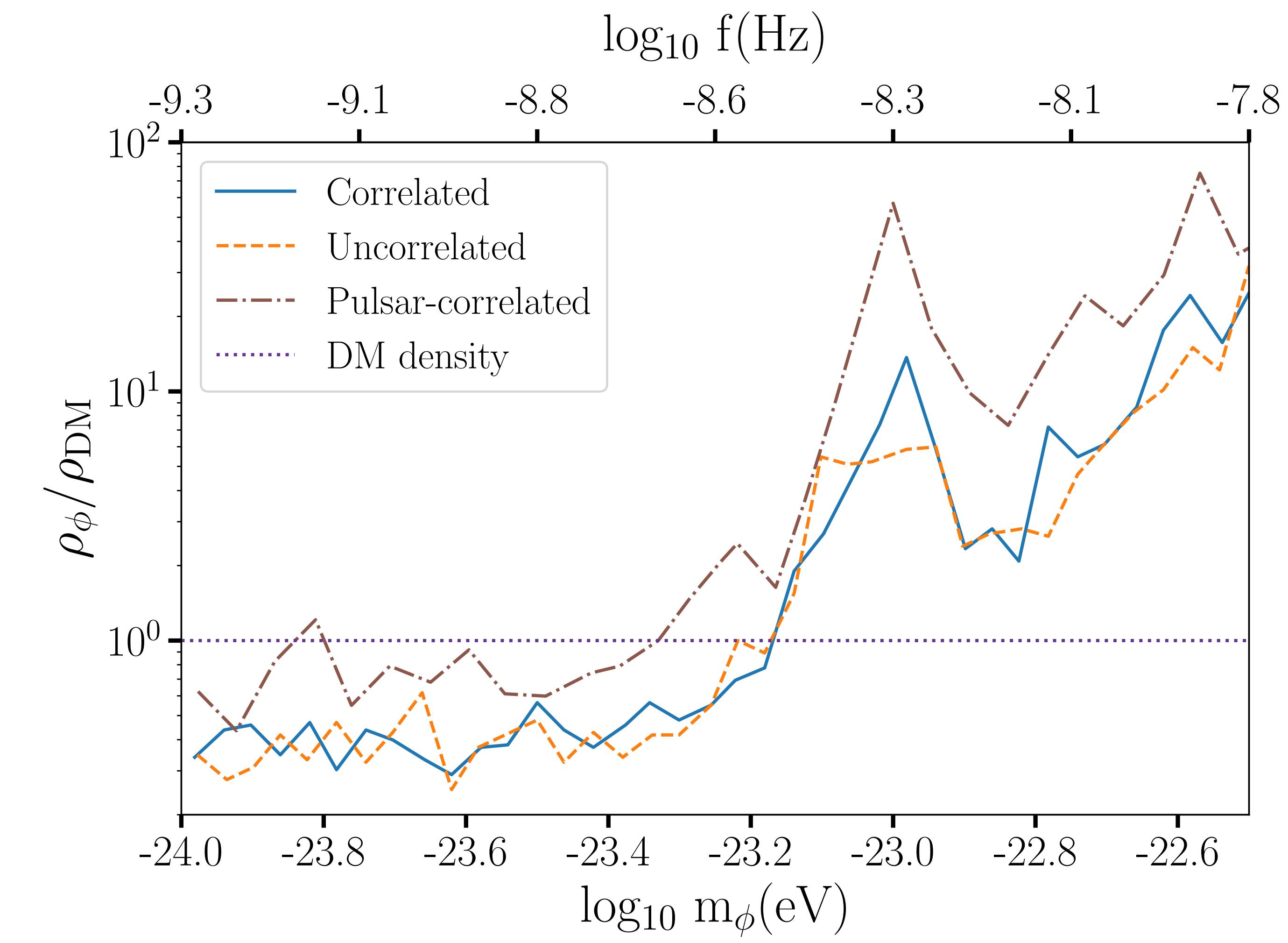
NANOGrav 15-year

Afzal et al. [2306.16219]

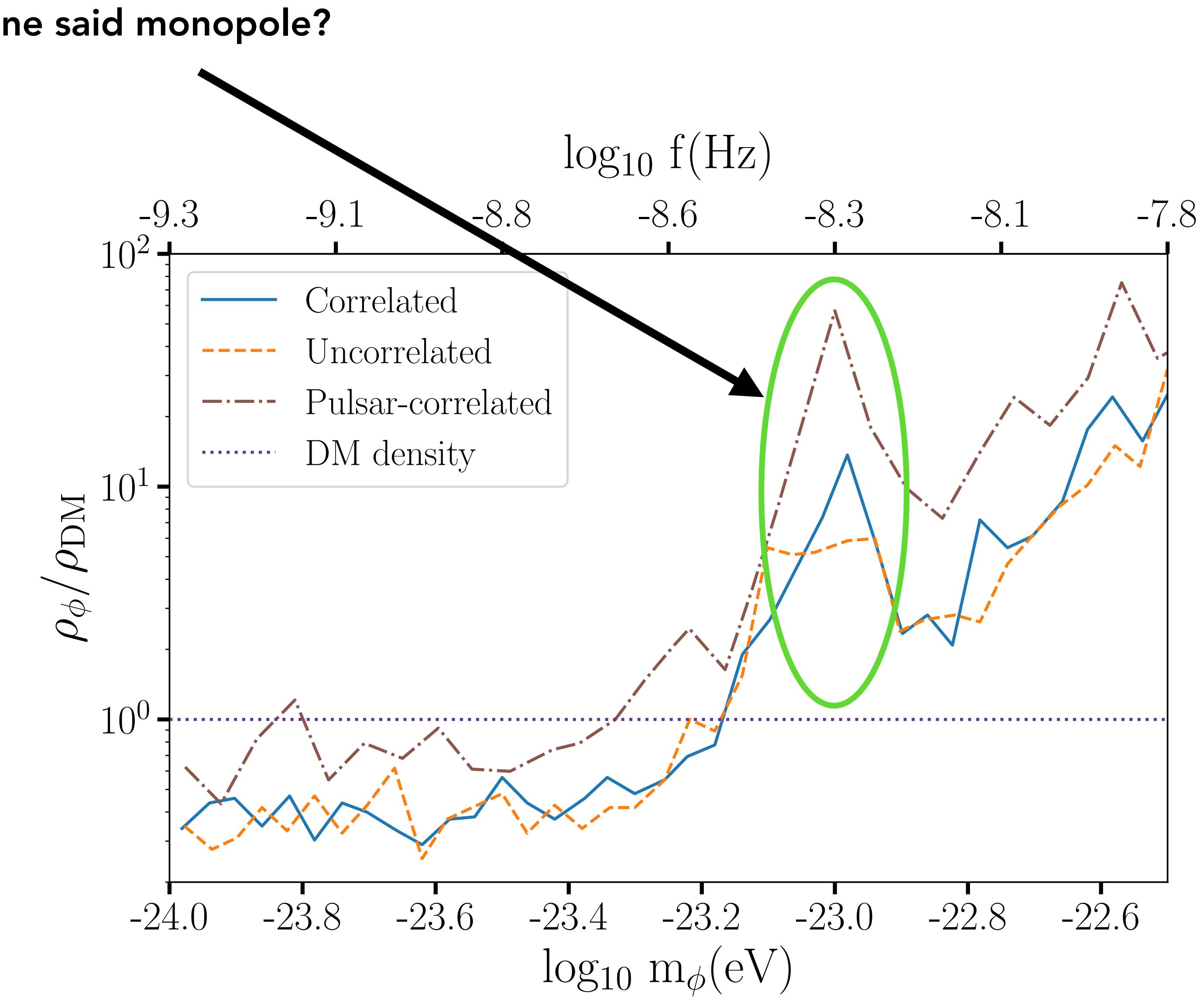
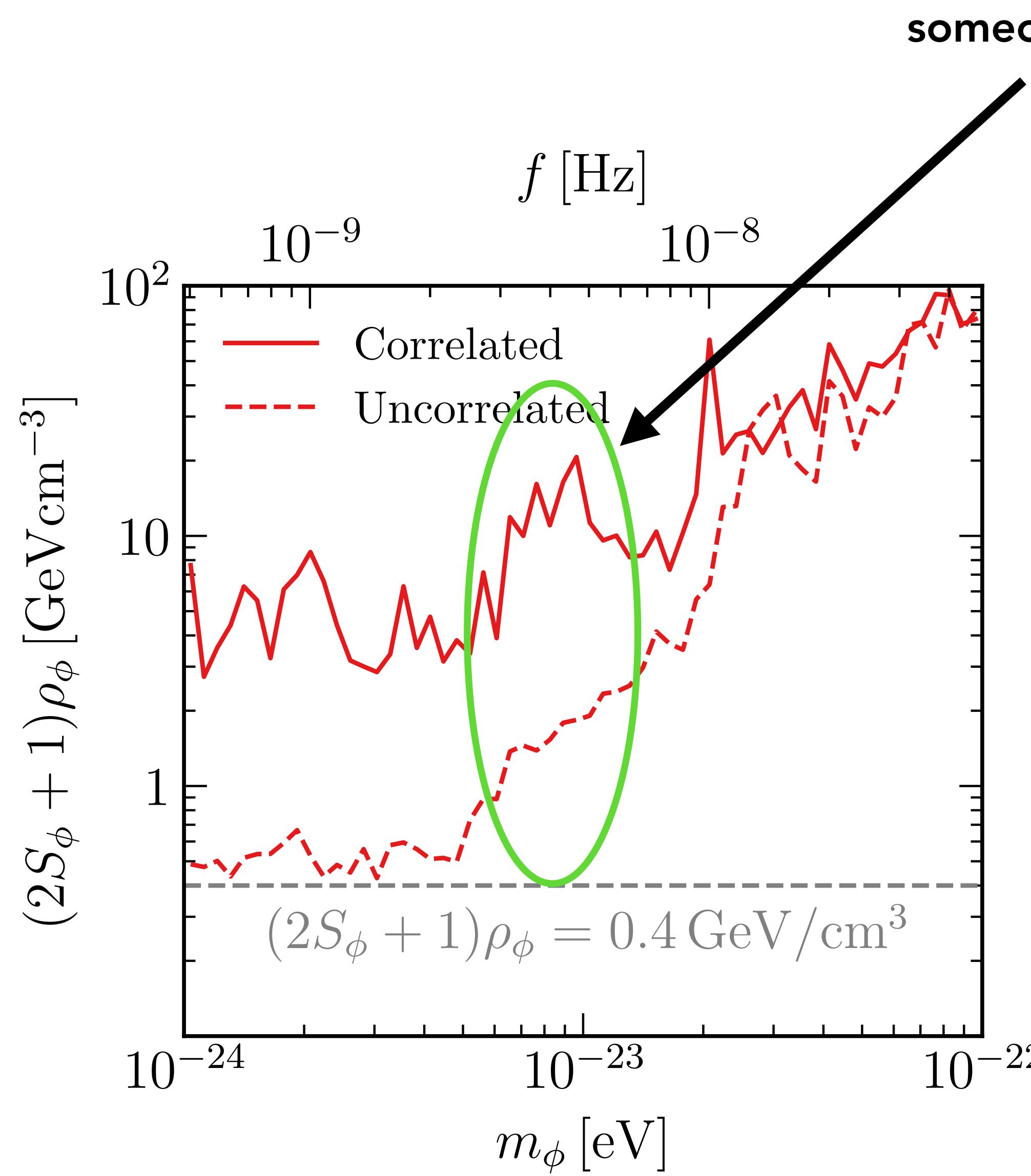


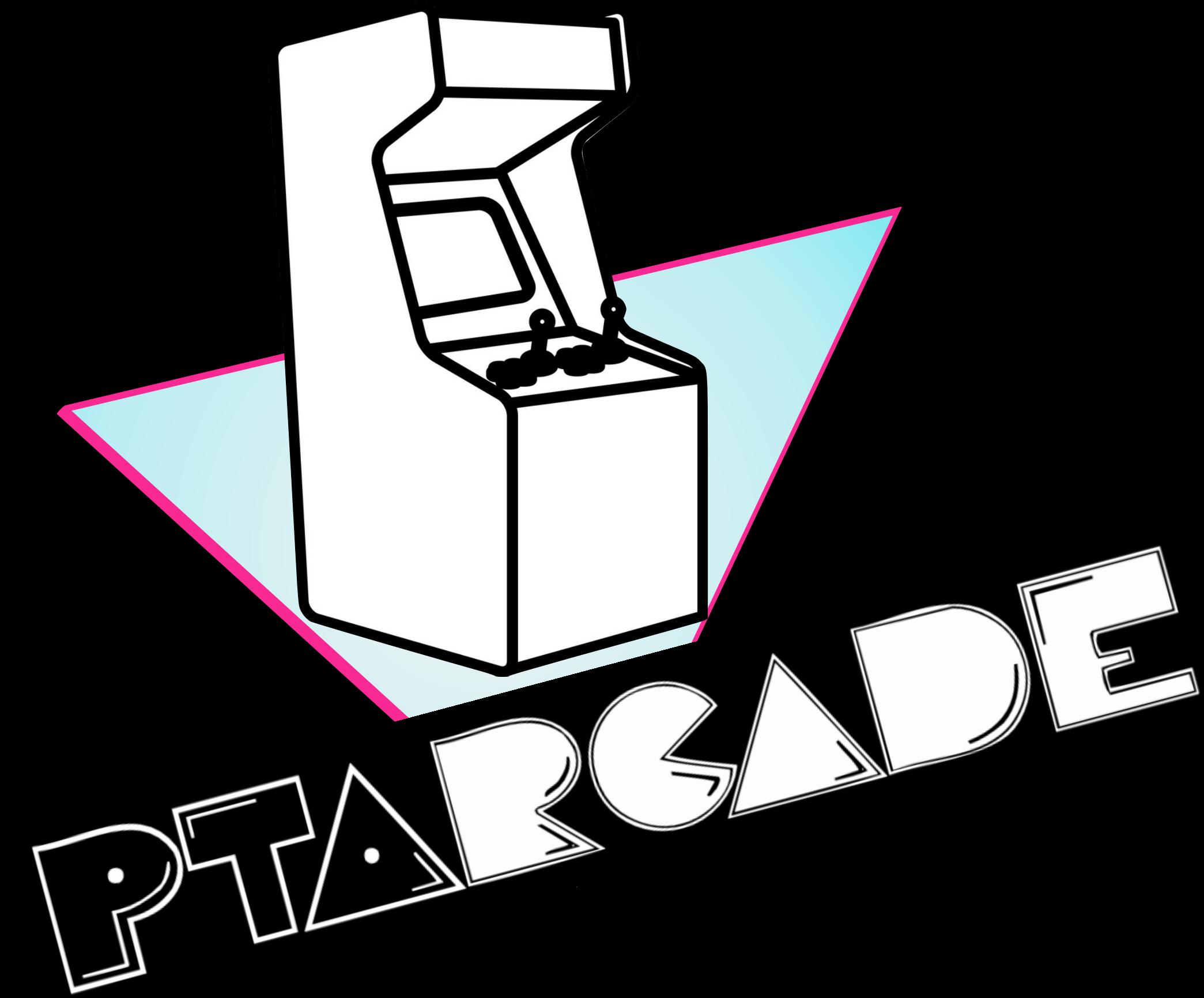
EPTA DR2

Smarra et al. [2306.16228]



ULDM: GRAVITATIONAL SIGNALS





have a model you want to test against PTA data?
say hello to PT Arcade



you have a PTA signal

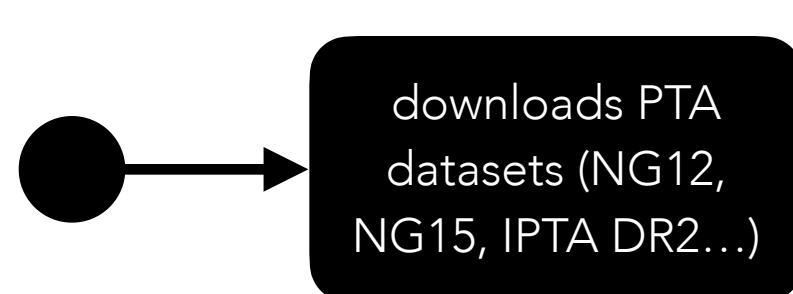
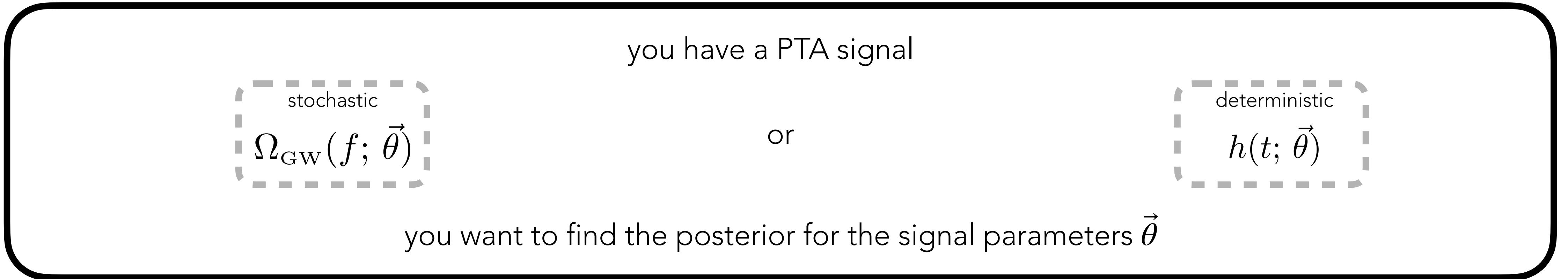
A dashed gray square frame containing the text "stochastic" at the top and the mathematical expression $\Omega_{\text{GW}}(f; \vec{\theta})$ below it.

or

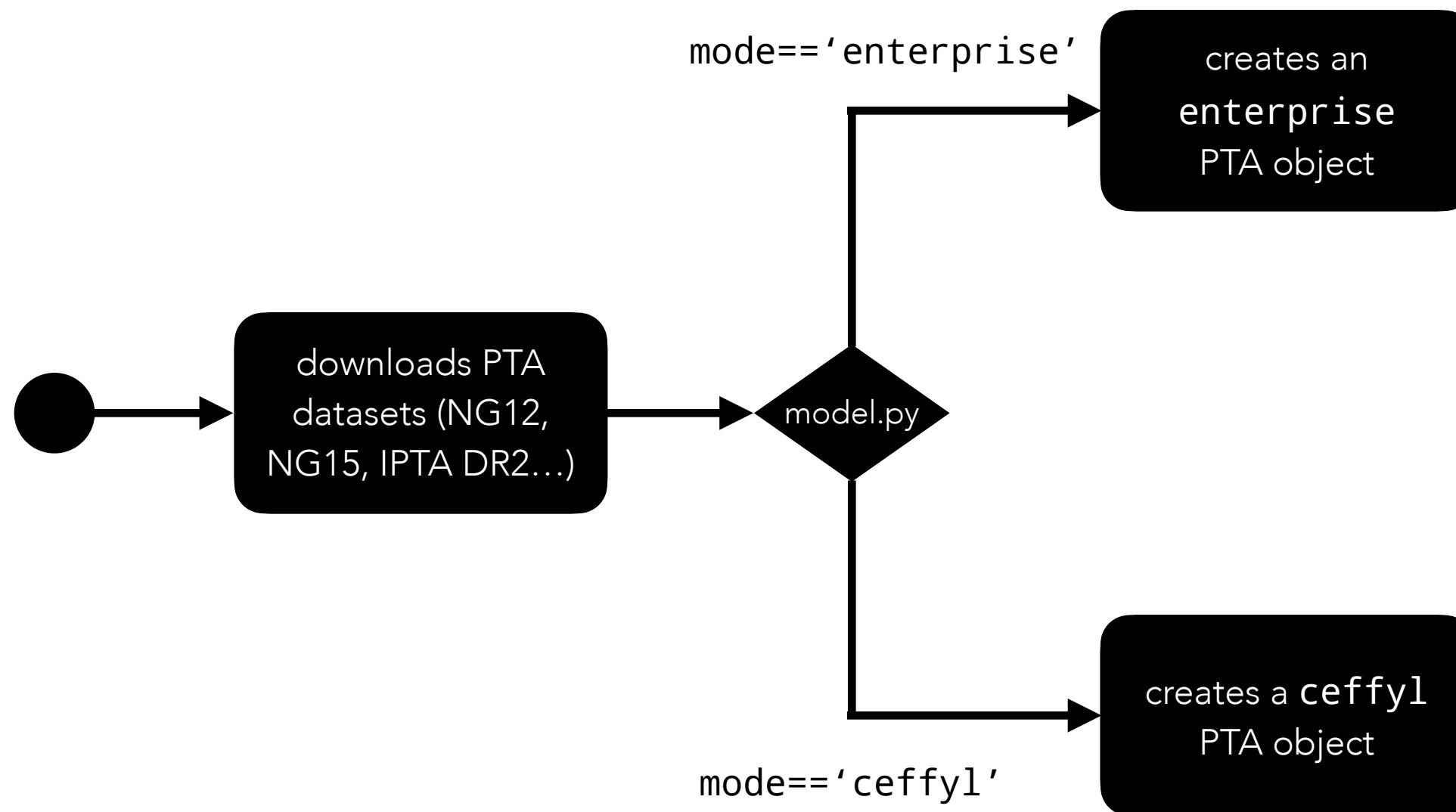
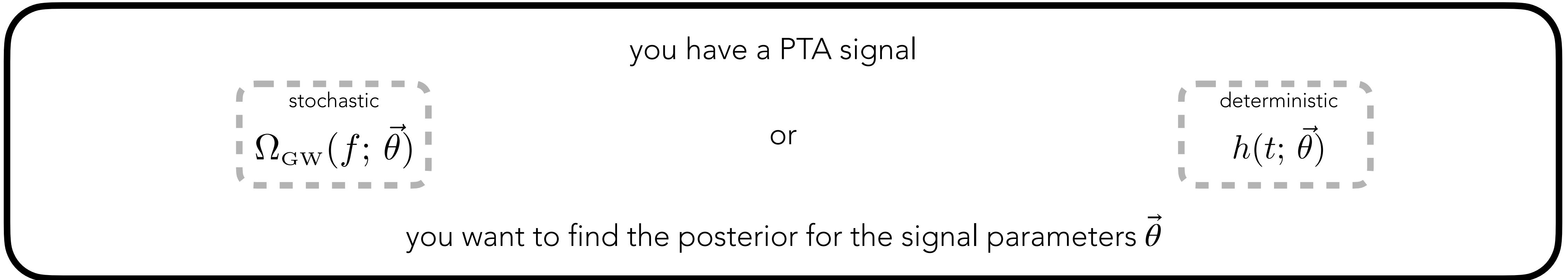
A dashed gray square frame containing the text "deterministic" at the top and the mathematical expression $h(t; \vec{\theta})$ below it.

you want to find the posterior for the signal parameters $\vec{\theta}$

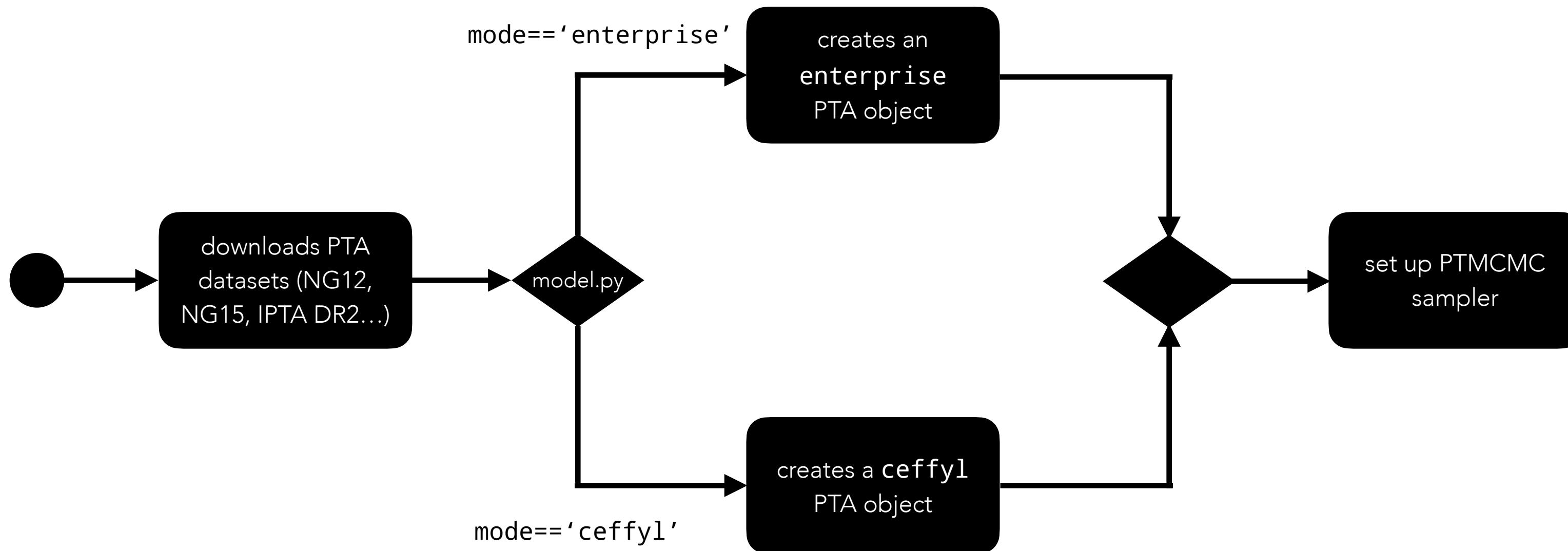
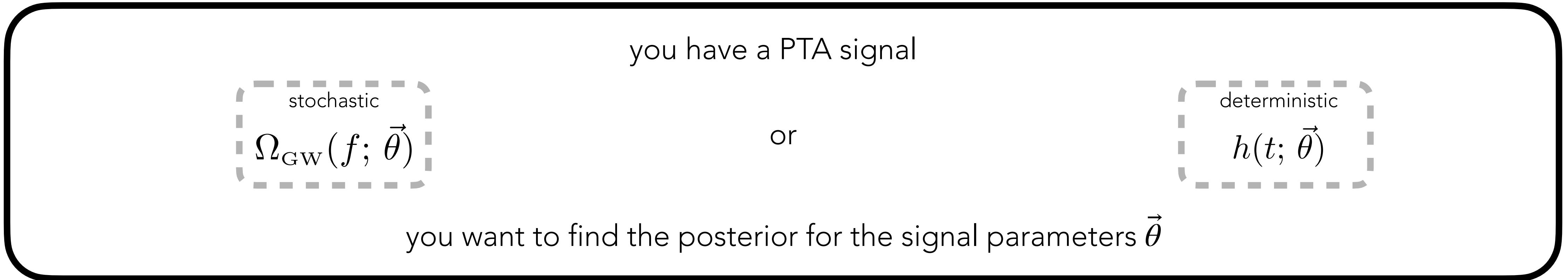
PTArcade



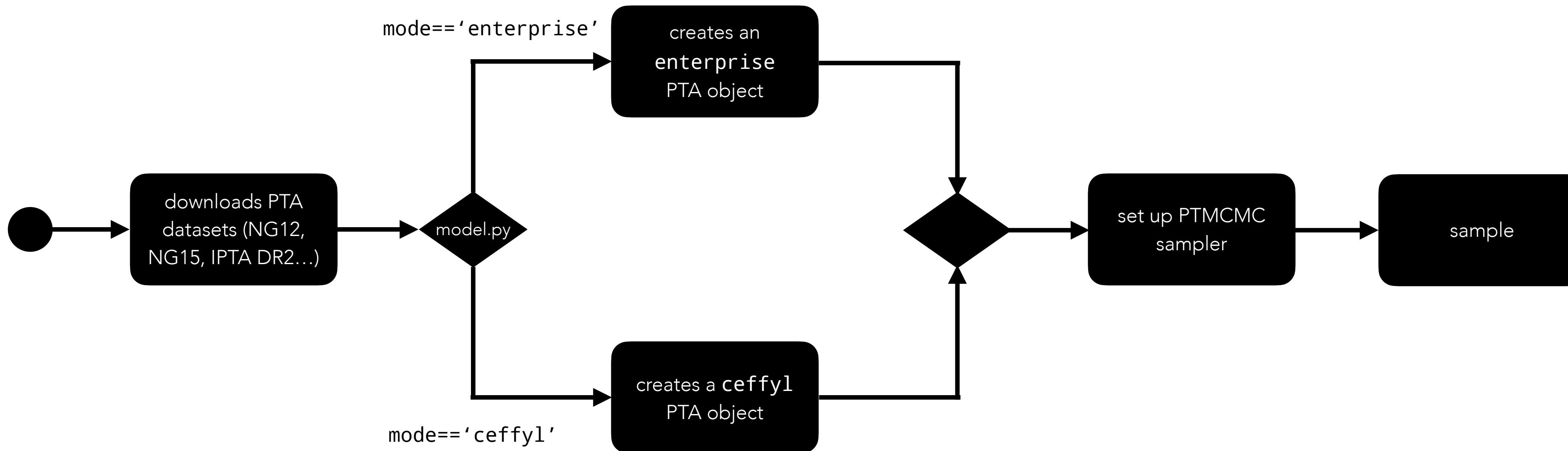
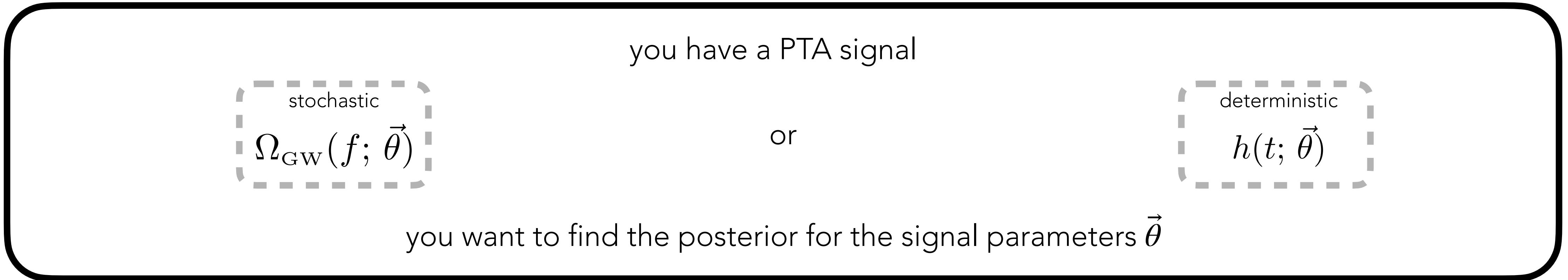
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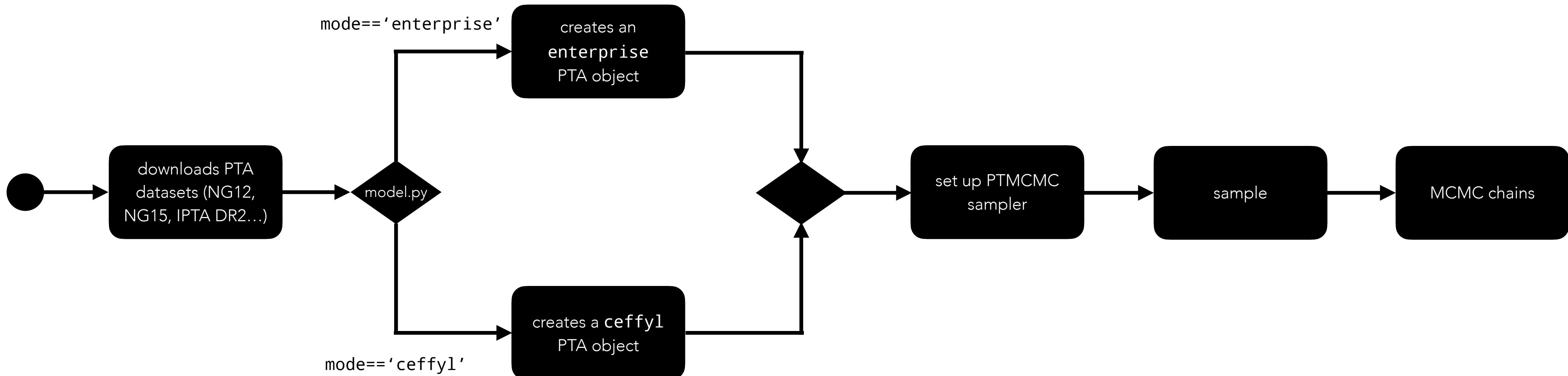
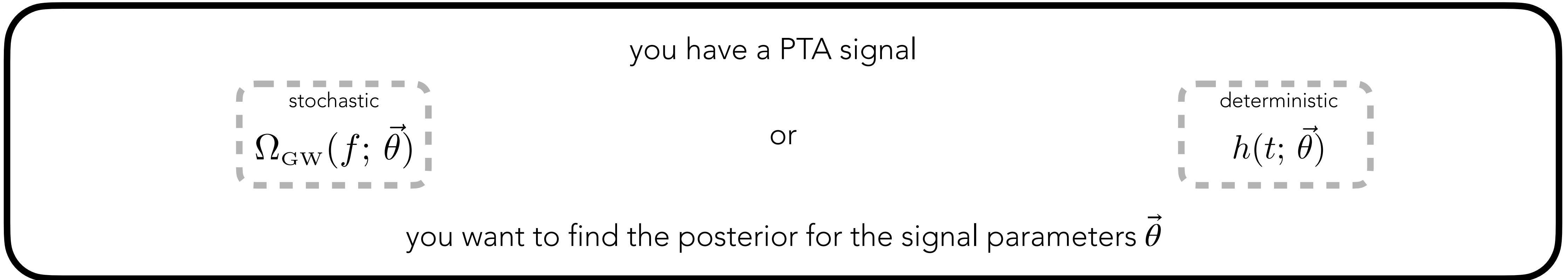
PTArcade



PTArcade



PTArcade



STEP I: installation

on linux and iOS (with intel chips)



```
conda install -c conda-forge ptarcade
```

on iOS (with apple silicon chips)



```
conda create -n ptarcade --platform osx-64 -c conda-forge python=3.10 ptarcade
```

STEP II: model file

$$h^2\Omega_{GW}(f) = \frac{A_*}{f/f_* + f_*/f}$$

REQUIRED CONTENT

- parameter priors
- signal function
 - power spectrum (for stochastic signals)
 - time series (for deterministic ones)

OPTIONAL CONTENT

- modified ORF
- model name
- if add SMBHB signal on top

model.py

```
● ● ●  
from ptarcade.models_utils import prior  
  
parameters = {  
    'log_A_star' : prior("Uniform", -14, -6),  
    'log_f_star' : prior("Uniform", -10, -6)  
}  
  
def S(x):  
    return 1 / (1/x + x)  
  
def spectrum(f, log_A_star, log_f_star):  
    A_star = 10**log_A_star  
    f_star = 10**log_f_star  
  
    return A_star * S(f/f_star)
```

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STEP II: model file

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def spectrum(f, log_A_star, log_f_star):  
    A_star = 10**log_A_star  
    f_star = 10**log_f_star  
  
    return A_star * S(f/f_star)  
  
def orf(f, pos1, pos2, log_A_star, log_f_star):  
    cos_ab = np.dot(pos1, pos2)  
  
    return cos_ab**2  
  
name = 'choose_your_model_name'  
smbhb = True
```

STEP II: model file

$$h^2\Omega_{GW}(f) = \frac{A_*}{f/f_* + f_*/f}$$

REQUIRED CONTENT

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def orf(f, pos1, pos2, log_A_star, log_f_star):  
    cos_ab = np.dot(pos1, pos2)  
  
    return cos_ab**2  
  
name = 'choose_your_model_name'  
smbhb = True
```

STEP II: model file

$$h^2\Omega_{GW}(f) = \frac{A_*}{f/f_* + f_*/f}$$

REQUIRED CONTENT

- parameter priors
- signal function
 - power spectrum (for stochastic signals)
 - time series (for deterministic ones)

OPTIONAL CONTENT

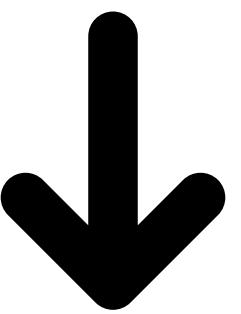
- modified ORF
- model name
- **if add SMBHB signal on top**

model.py

```
● ● ●  
from ptarcade.models_utils import prior  
  
parameters = {  
    'log_A_star' : prior("Uniform", -14, -6),  
    'log_f_star' : prior("Uniform", -10, -6)  
}  
  
def S(x):  
    return 1 / (1/x + x)  
  
def spectrum(f, log_A_star, log_f_star):  
    A_star = 10**log_A_star  
    f_star = 10**log_f_star  
  
    return A_star * S(f/f_star)  
  
def orf(f, pos1, pos2, log_A_star, log_f_star):  
    cos_ab = np.dot(pos1, pos2)  
  
    return cos_ab**2  
  
name = 'choose_your_model_name'  
smbhb = True
```

STEP III: run

```
● ● ●  
ptarcade -m model.py
```

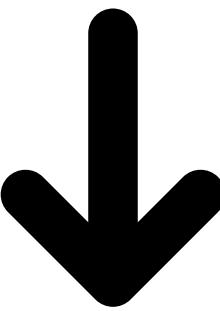


output structure

```
out_dir/  
└── model_name/  
    └── chain_0/  
        ├── chain_1.txt  
        ├── pars.txt  
        ├── priors.txt  
        └── ...
```

STEP III: run

```
● ● ●  
ptarcade -m model.py
```

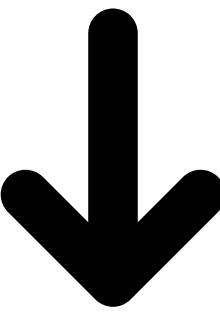


output structure

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        ├── chain_1.txt  
        ├── pars.txt  
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        └── ...
```



usual enterprise/ceffyl output

OPTIONAL CONFIGURATIONS

TUNABLE PARAMETERS

- enterprise or ceffyl mode
- PTA data set
- include or not cross correlations
- # of frequency components for GWB
- # of frequency components for iRN
- resume old chains
- output directory
- # MCMC samples
- model comparison option
- SMBHB parameter priors

config.py

```
mode = 'ceffyl'  
pta_data = 'NG15'  
  
corr = False  
gwb_components = 14  
  
red_components = 30  
  
resume = False  
out_dir = './chains/'  
N_samples = int(2e6)  
  
mod_sel = False  
bhb_th_prior = True  
A_bhb_logmin = None  
A_bhb_logmax = None  
gamma_bhb = None
```

ceffyl mode can only be used for stochastic signals

OPTIONAL CONFIGURATIONS

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```

options are: 'NG12', 'NG15', 'IPTA2'

coming soon: 'EPTA2_full', 'EPTA2_new'

OPTIONAL CONFIGURATIONS

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- SMBHB parameter priors

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```

$$\Omega_{\text{GW}} \propto A_{\text{BHB}}^2 \left(\frac{f}{\text{year}^{-1}} \right)^{5-\gamma_{\text{BHB}}} \text{year}^{-2}$$

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- **SMBHB parameter priors**

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```

$$\Omega_{\text{GW}} \propto A_{\text{BHB}}^2 \left(\frac{f}{\text{year}^{-1}} \right)^{5-\gamma_{\text{BHB}}} \text{year}^{-2}$$

CONCLUSIONS

strong evidence for a GWB

if cosmological origin it will provide the first glimpse into the early Universe

independently from the origin of the GWB, PTA can be used to set complementary constraints on new physics models

have a model that you want to test? PTArcade comes to rescue