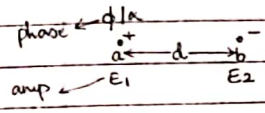
Unit 2

ANTENNA ARRAYS

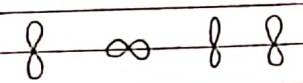
The radiation pattern of antenna depends on

* no. of antenna elements
* distance b/w antenna elements

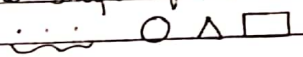


$d = \frac{\lambda}{2}, \frac{\lambda}{4}, \lambda$

* Amplitude of signal fed to each antenna element
* The phase of antenna element.
* Radiation pattern of each antenna



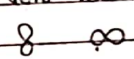
* The shape of the array



Point Source : $l << \lambda$ (dimensionless antenna)

Isotropic

Non-isotropic



* Directivity is 1 (D=1)
* 2D pattern is circle
* 3D pattern is sphere
* Dimensionless antenna

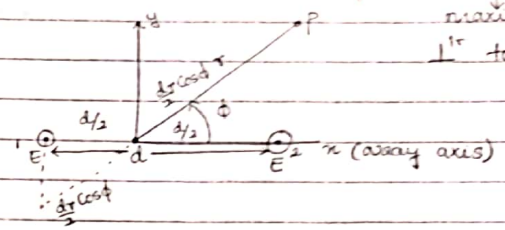
* Dimensionless antenna
* has some direction
* D > 1

Array of two Isotropic Point Sources

* Having equal amplitude & inphase (Broad side array)
* Having equal amplitude & out of phase (End fire array)
* Having equal amplitude & in phase quadrature (extended end fire array)
* Having equal amplitude & any phase (General case)
* Having any amplitude & any phase (Most General case)

1. Having Equal Amplitude & In-phase (Broad Side Array)

maximas are $\perp^r$ to A.A



$d_r = \beta d = \frac{2\pi d}{\lambda}$

$E = E_1 + E_2$

$E_1 = E_0 e^{-j\psi/2}$
$E_2 = E_0 e^{j\psi/2}$ } fields

$\psi = d_r \cos\phi$

$E = E_0 e^{-j\psi/2} + E_0 e^{j\psi/2}$

ψ = path difference

$E = E_0 [e^{j\psi/2} + e^{-j\psi/2}]$

$E = E_0 (2\cos\psi/2)$

$E = 2E_0 \cos\psi/2$

$\frac{E}{2E_0} = E_n = \cos\psi/2 \longrightarrow$ Normalized field

$E_n = \cos\left(\frac{d_r \cos\phi}{2}\right)$

→ Direction of maxima : Angle at which radiation is maximum.

$E_n = 1$

$\cos\left(\frac{d_r \cos\phi}{2}\right) = 1$

$\frac{d_r \cos\phi}{2} = \cos^{-1}(1)$

$d_r \cos\phi = 0$

$\pi \cos\phi = 0$

$\cos\phi = 0$

$\phi = \cos^{-1}(0) = \phi_{max}$

$\phi_{max} = \pm\frac{\pi}{2} \mid \pm 90^\circ$

$d_r = \frac{2\pi d}{\lambda}$

$\because d = \lambda/2$

$d_r = \pi$

$d_r = \beta d$

where $\beta = \frac{2\pi}{\lambda}$

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→ Direction of Null

$$\cos\left(\frac{d_r \cos\phi_0}{2}\right) = 0$$

$$\frac{d_r \cos\phi_0}{2} = \cos^{-1}(0) \qquad \because d_r = \pi$$

$$\frac{d_r \cos\phi_0}{2} = \pm\frac{\pi}{2}$$

$$\frac{\pi}{2}\cos\phi_0 = \pm\frac{\pi}{2}$$

$$\cos\phi_0 = \pm\frac{\pi}{2}\cdot\frac{2}{\pi} = \pm 1$$

$$\phi_0 = \cos^{-1}(\pm 1)$$

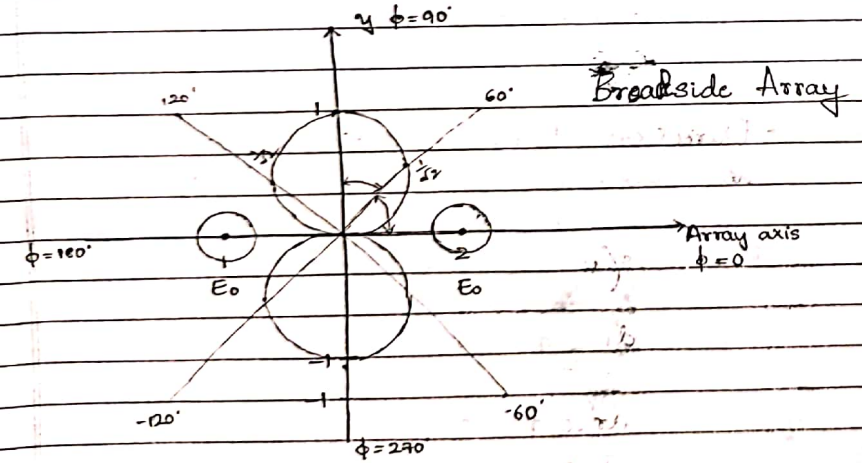
$$\phi_0 = 0^\circ,\ 180^\circ$$

→ Direction of Half Power Point

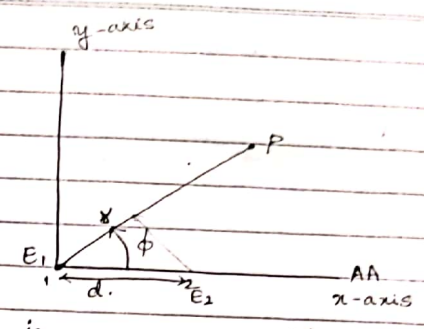
$$\cos\left[\frac{\pi}{2}\cos\phi_{HPP}\right] = \frac{1}{\sqrt{2}}$$

$$\frac{\pi}{2}\cos\phi_{HPP} = \pm\frac{\pi}{3},\ \pm 120^\circ$$

$$\pm 60^\circ,\ \pm 120^\circ$$



$$\frac{4\pi}{\frac{3\pi}{2}+\frac{3\pi}{2}} = \frac{4\pi\times 2}{6\pi}$$



$$E_1 = E_0 e^{j0} \qquad E_2 = E_0 e^{j\psi} \qquad \psi = d_r\cos\phi$$

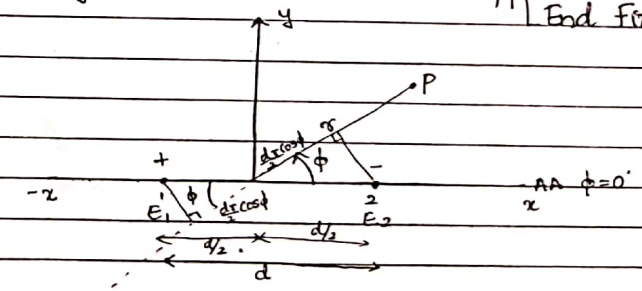
$$E = E_1 + E_2 = E_0 e^{j0} + E_0 e^{j\psi}$$

$$= E_0\left[1 + e^{j\psi}\right] \qquad e^{j\psi} = e^{j\psi/2} + e^{j\psi/2}$$

$$= E_0 e^{j\psi/2}\cos\psi/2$$

$$E = \cos\frac{\psi}{2}\ /\psi/2$$

2. Having Equal Amplitude & Opposite Side Phase [End Fire Array]



$$E = E_1 + E_2 = E_0 e^{-j\psi/2} - E_0 e^{j\psi/2}$$

$$= -E_0\left[\frac{e^{j\psi/2} - e^{-j\psi/2}}{2j}\right] 2j$$

$$= -E_0(2j)\sin\psi/2$$

$$= -2j E_0 \sin\psi/2$$

↳ normalizing (making it 1)

$$\therefore E = \sin\psi/2 = \sin\left[\frac{d_r}{2}\cos\phi\right]$$

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Direction of maxima :

	Maximas	Nulls	HPP
$\cos(x)$	$\pm K\pi$	$\pm(2K+1)\pi/2$	$\pm(2K+1)\pi/4$
$\sin(x)$	$\pm(2K+1)\pi/2$	$\pm K\pi$	$\pm(2K+1)\pi/4$

where $K = 0, 1, 2 \ldots$

Q. Array of 2 Isotropic Point Sources having equal amplitude & out-of-phase & point sources are placed at $d = \lambda/2$. Obtain complete radiation pattern. Also find directivity of the array.

→ $E = \sin \psi/2$ $\qquad d = \lambda/2$

where $\psi = d_r \cos\phi$ $\qquad d_r = \frac{2\pi d}{\lambda} = \frac{2\pi \cdot \lambda}{2\lambda} = \pi$

$$E = \sin\left[\frac{d_r}{2}\cos\phi\right]$$

Maxima :

$$E = 1$$

$$\sin\left(\frac{d_r}{2}\cos\phi\right) = 1$$

$$\frac{d_r}{2}\cos\phi = \sin^{-1}(1)$$

$$\frac{\pi}{2}\cos\phi = \frac{\pi}{2}$$

$$\phi = \cos^{-1}(1)$$

$$\phi = 0^\circ / \pm\pi$$

Direct^n of maxima

$$\frac{\pi}{2}\cos\phi_{max} = \pm(2K+1)\frac{\pi}{2}$$

Direction of nulls

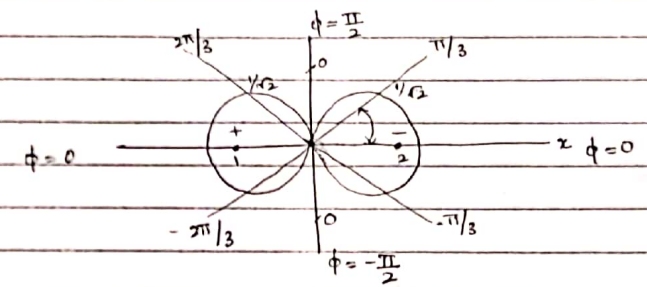
$$\frac{\pi}{2}\cos\phi_n = \pm K\pi$$

$K = 0$, $\phi = \pm\frac{\pi}{2}$

Direct^n of HPP

$$\frac{\pi}{2}\cos\phi_{HPP} = \pm(2K+1)\frac{\pi}{4}.$$

$K = 0$, $\phi_{HPP} = \cos^{-1}\left(\frac{1}{2}\right) = \pm 60^\circ, \pm 120^\circ$



$$D = \frac{4\pi}{\Omega_A}$$

$$\Omega_A = \theta_{HP} \cdot \phi_{HP}$$

$$\theta_{HP} = \phi_{HP}$$

$$D = \frac{4\pi}{\left(\frac{\pi}{3}+\frac{\pi}{3}\right)\cdot\left(\frac{\pi}{3}+\frac{\pi}{3}\right)} = \frac{4\pi}{\left(\frac{2\pi}{3}\right)\left(\frac{2\pi}{3}\right)} = \frac{9}{\pi}$$

$D = 2.86$ (in End fire case)

$D = 11.6$ (in broad side case)

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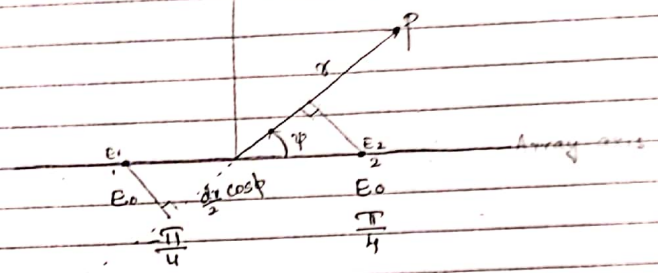
Case 3 Equal Amplitude & In-phase Quadrate



$$E_1 = E_0 e^{-j\psi/2 - \pi/4} \qquad E_2 = E_0 e^{j\psi/2 + \pi/4}$$
$$= E_0 e^{-[j\psi/2 + \pi/4]}$$

$$E = E_1 + E_2$$
$$= E_0 e^{-[j\psi/2+\pi/4]} + E_0 e^{j\psi/2+\pi/4}$$
$$= E_0 \left[e^{j\psi/2+\pi/4} + e^{-j\psi/2+\pi/4} \right] \times 2/2$$
$$= 2E_0 \cos\left(\frac{\psi}{2} + \frac{\pi}{4}\right)$$

By normalizing

$$E = \cos\left(\frac{\psi}{2} + \frac{\pi}{4}\right)$$

$$E = \cos\left(\frac{dr}{2}\cos\phi + \frac{\pi}{4}\right)$$

Q. If $d = \lambda/2$, draw the complete radiation pattern.

→ $d = \frac{2\pi d}{\lambda} = \pi$

$$E = \cos\left[\frac{\pi}{2}\cos\phi + \frac{\pi}{4}\right]$$

Direction of Maxima:

$$\cos\left[\frac{\pi}{2}\cos\phi_{max} + \frac{\pi}{4}\right] = 1$$

$$\frac{\pi}{2}\cos\phi_{max} + \frac{\pi}{4} = \pm K\pi$$

$K = 0$, $\frac{\pi}{2}\cos\phi_{max} = -\frac{\pi}{4}$

$$\phi_{max} = 120^\circ, 240^\circ$$

Direct^n of Nulls:

$$\cos\left[\frac{\pi}{2}\cos\phi + \frac{\pi}{4}\right] = 0$$

$$\frac{\pi}{2}\cos\phi_0 + \frac{\pi}{4} = \pm(2K+1)\frac{\pi}{2}$$

$K = 0$, $\frac{\pi}{2}\cos\phi_0 + \frac{\pi}{4} = \pm\frac{\pi}{2}$

$$\frac{\pi}{2}\cos\phi_0 = \pm\frac{\pi}{2} - \frac{\pi}{4}$$

$+\frac{\pi}{2\times2} - \frac{\pi}{4}$ $\quad$ $-\frac{\pi}{2\times2} - \frac{\pi}{4}$

$\frac{\pi}{4}$ $\quad$ $-\frac{3\pi}{4}$

$$\frac{\pi}{2}\cos\phi_0 = \frac{\pi}{4/2}, \quad \frac{\pi}{2}\cos\phi_0 = \frac{-3\pi}{4/2}$$

$$\phi_0 = \pm 60^\circ \qquad \phi_0 = \pm 120^\circ$$

doesn't exist ∵ maxima is at $\pm 120^\circ$

Direct^n of HPP:

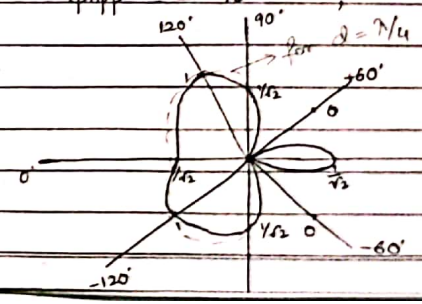
$$\cos\left[\frac{\pi}{2}\cos\phi + \frac{\pi}{4}\right] = \frac{1}{\sqrt{2}}$$

$$\frac{\pi}{2}\cos\phi_{HPP} + \frac{\pi}{4} = \pm(2K+1)\pi/4$$

$K = 0$, $\frac{\pi}{2}\cos\phi_{HPP} = \pm\frac{\pi}{4} - \frac{\pi}{4}$

$$\frac{\pi}{2}\cos\phi_{HPP} = +\frac{\pi}{4} - \frac{\pi}{4}, \quad \frac{\pi}{2}\cos\phi_{HPP} = -\frac{\pi}{4} - \frac{\pi}{4} = -\frac{2\pi}{4} = -\frac{\pi}{2}$$

$$\phi_{HPP} = \pm 90^\circ, \qquad \phi_{HPP} = \pm 180^\circ$$



$$\phi_{HP} = 90^\circ = \theta_{HP}$$

$$D = \frac{4\pi}{\theta_{HP}\,\phi_{HP}} = \frac{16}{\pi}$$

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Case 4: Equal Amplitude & any phase (General case)

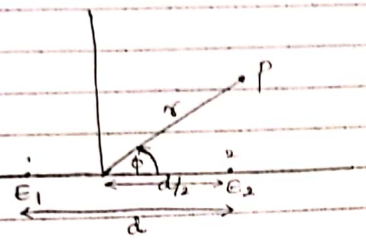
α - phase difference b/w 2 isotropic point sources

$\psi = dr\cos\phi + \alpha$

Inphase : $\alpha = 0$

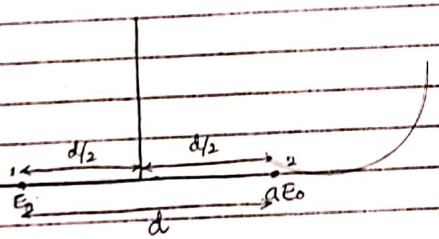
out-of phase : $\alpha = \pi/2$

In-phase quadrature: $\alpha = \pi/4$



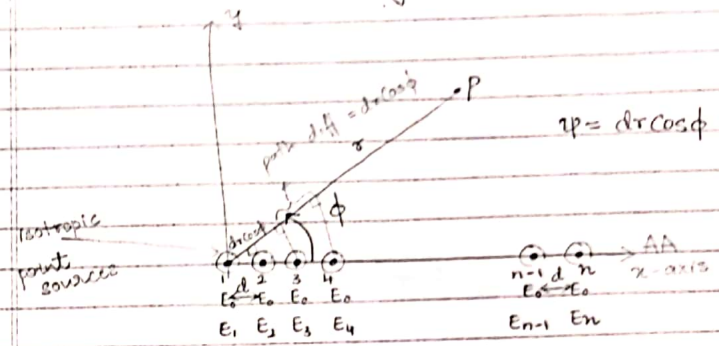
$$E = E_1 + E_2$$
$$= E_0 e^{-j\psi/2} + E_0 e^{j\psi/2}$$
$$= 2E_0 \left[\frac{e^{j\psi/2} + e^{-j\psi/2}}{2}\right]$$
$$E = 2E_0 \cos\left(\frac{\psi}{2}\right) \qquad \psi = dr\cos\phi + \delta$$

Case 5: Any amplitude & any phase (Most General case)



$$E = E_1 + E_2$$
$$= \sqrt{(E_0 + aE_0\cos\psi)^2 + (aE_0\sin\psi)^2}$$
$$|E| = E_0\sqrt{(1 + a\cos\psi)^2 + (a\sin\psi)^2}$$
$$\angle E = \tan\epsilon = \frac{aE_0\sin\psi}{E_0 + aE_0\cos\psi}$$
$$= \frac{a\sin\psi}{1 + a\cos\psi}$$

⇒ Array of 'n' isotropic point sources having equal amplitude & spacing. [Linear Array]



$\psi = dr\cos\phi + \delta$

phase diff

$$E = E_1 + E_2 + E_3 + \cdots + E_{n-1} \quad \text{—(1)}$$
$$= E_0 e^{j0} + E_0 e^{j\psi} + E_0 e^{2j\psi} + \cdots + E_0 e^{(n-1)j\psi}$$
$$E = E_0 + E_0 e^{j\psi} + E_0 e^{2j\psi} + \cdots + E_0 e^{(n-1)j\psi} \quad \text{—(2)}$$

multiply $e^{j\psi}$ on b.s of eqn (2)

$$Ee^{j\psi} = E_0 e^{j\psi} + E_0 e^{2j\psi} + E_0 e^{3j\psi} + \cdots + E_0 e^{nj\psi} \quad \text{—(3)}$$

eqn. (2) − (3)

$$E[1 - e^{j\psi}] = E_0[1 - e^{jn\psi}]$$
$$E = \frac{E_0[1 - e^{jn\psi}]}{[1 - e^{j\psi}]} = \frac{E_0[1 - e^{jn\psi/2} \cdot e^{jn\psi/2}]}{[1 - e^{j\psi/2} \cdot e^{j\psi/2}]}$$
$$= \frac{E_0 e^{jn\psi/2}}{e^{j\psi/2}}\left[\frac{e^{-jn\psi/2} - e^{jn\psi/2}}{e^{-j\psi/2} - e^{j\psi/2}}\right]$$
$$= \frac{-E_0 e^{jn\psi/2}}{e^{j\psi/2}}\left[\frac{e^{jn\psi/2} - e^{-jn\psi/2}}{e^{j\psi/2} - e^{-j\psi/2}}\right] \cdot \frac{2j}{2j}$$
$$E = \frac{E_0 e^{jn\psi/2}}{e^{j\psi/2}}\left[\frac{\sin n\psi/2}{\sin\psi/2}\right]$$

If $E_0 = 1$

$$E = \frac{e^{jn\psi/2}}{e^{j\psi/2}}\left[\frac{\sin n\psi/2}{\sin\psi/2}\right]$$
$$E = e^{j(n-1)\psi/2}\left[\frac{\sin n\psi/2}{\sin\psi/2}\right]$$

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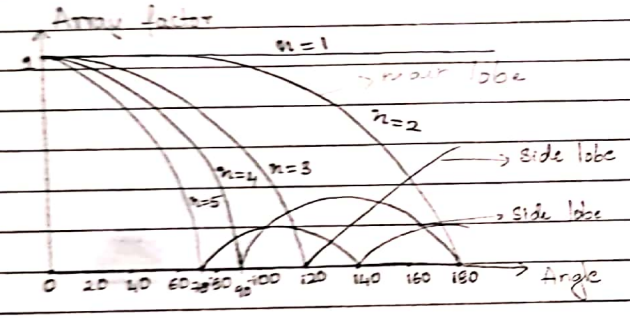
$$E = \frac{\sin n\psi/2}{\sin \psi/2} \angle \xi \quad \text{where } \xi = \frac{\psi(n-1)}{2}$$

Normalized field, $E_n = \frac{E}{E_{max}}$

$$E_{max} = \lim_{\psi \to 0} \left\{ \frac{\sin n\psi/2}{\sin \psi/2} \right\}$$

$$E_{max} = n$$

Array factor, $E_n = \frac{1}{n} \cdot \frac{\sin n\psi/2}{\sin \psi/2}$



If n point sources in the array,

no. of Side lobes $= n-2$

Total no. of lobes (ML + SL) $= n-1$

The Array factor exhibits the following factors:

1. As n increases, main lobe narrows.
2. As $n \uparrow$ ce, there are more side lobes in one (0-180) period of array factor.
3. No. of sidelobes $= n-2$ & no. of complete lobes $= n-1$ & 1 main lobe in each period.
4. The side lobe peaks decreases with increase in n.

Case 1 **Broad Side Array**

(They are in-phase if all the isotropic points are in-phase ($\delta = 0$))

$$\psi = dr\cos\phi + \delta$$

If all sources are in-phase, then $\delta = 0$

$\Rightarrow \psi = dr\cos\phi \longrightarrow$ condition for BSA.

Direction of Maxima

$$\psi = dr\cos\phi$$

$\psi = 0$ (path diff.), then radt^n is max.

$$\therefore dr\cos\phi = 0$$

$$\phi = \cos^{-1}(0)$$

$$\phi = \pm 90^\circ$$

Direction of Nulls

(equate numerator of Array factor to 0)

$$\therefore \sin\frac{n\psi}{2} = 0$$

$$\frac{n\psi}{2} = \sin^{-1}(0) = \pm K\pi$$

$K = 0, 1, 2, \ldots$

$$\Rightarrow \psi = \pm \frac{2K\pi}{n}$$

$$dr\cos\phi_0 = \pm \frac{2K\pi}{n}$$

$$\cos\phi_0 = \pm \frac{2K\pi}{ndr}$$

$ndr \longrightarrow$ dist^n in radians

$$\phi_0 = \cos^{-1}\left(\pm \frac{2K\pi}{ndr}\right)$$

Direction of HPP/ Side lobes

(equate num. of AF to 1)

$$\therefore \sin n\psi/2 = 1$$

$$n\psi/2 = \sin^{-1}(1) = \pm(2K+1)\pi/2$$

$$\psi = \pm \frac{(2K+1)\pi}{n}$$

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$$d_r \cos\phi = \pm \frac{(2(k)+1)\pi}{n}$$

$$\cos\phi = \pm \frac{(2k+1)\pi}{d_r n}$$

$$\phi = \cos^{-1}\left(\pm \frac{(2k+1)\pi}{d_r n}\right)$$

Q. A linear uniform array of isotropic antennas satisfies the following parameters: $n=4$, $\delta=0$, $d=\lambda/2$. Obtain the complete field pattern & find the B.W b/w 1st f/f null, half power B.W & directivity.

→ Given: linear array

$\delta=0$ → Broad Side L.A

$$\psi = d_r \cos\phi + \delta$$

$\delta = 0$ for B.A

$$\psi = d_r \cos\phi \qquad d = \lambda/2$$

$$\psi = \pi\cos\phi \qquad d_r = \frac{2\pi d}{\lambda}$$

$$d_r = \pi$$

* maxima

$$\psi = 0$$

$$\pi\cos\phi_m = 0$$

$$\Rightarrow \phi_{max} = \pm 90^\circ$$

$n=4$

$(n-1) = 3$ complete lobes

no. of sidelobes $= n-2 = 2$

* Direct^n of nulls:

WKT $E_n = \frac{1}{n}\frac{\sin n\psi/2}{\sin \psi/2}$

$$\sin n\psi/2 = 0$$

$$n\pi\cos\phi_0 = \pm 2k\pi$$

$$\phi_0 = \cos^{-1}\left[\pm\frac{2k\pi}{n d_r}\right] = \cos^{-1}\left[\pm\frac{2k\pi}{2k\cdot\pi}\right]$$

$k=0$, $n=4$: $\phi_0 = \cos^{-1}\left[\pm\frac{2(0)\pi}{4\pi}\right] = \pm 90^\circ$ not valid

$k=1$: $\phi_0 = \cos^{-1}\left[\pm\frac{1}{2}\right] = \pm 60^\circ / \pm 120^\circ$ → 1st null

$k=2$: $\phi_0 = \cos^{-1}\left[\pm\frac{2}{2}\right] = \pm 0^\circ / \pm 180^\circ$ → 2nd null

$\phi_0 = \pm 90^\circ$ is not valid ∵ it is the direct^n of major lobe.

* Direct^n of side lobes:

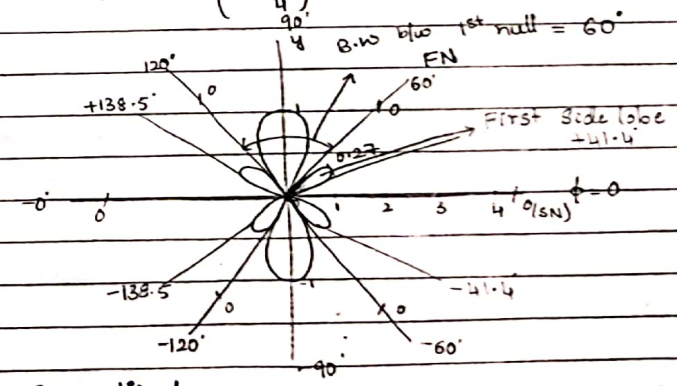
$$\sin n\psi/2 = 1$$

$$\phi = \cos^{-1}\left(\pm\frac{(2k+1)\pi}{n d_r}\right)$$

$d_r = \pi$, $n=4$: $\phi = \cos^{-1}\left(\pm\frac{(2k+1)\pi}{4\cdot\pi}\right) = \cos^{-1}\left(\pm\frac{(2k+1)}{4}\right)$

$k=0$: $\phi = \cos^{-1}\left(\pm\frac{(0+1)}{4}\right) = \pm 75.5^\circ / \pm 104.4^\circ$ not valid

$k=1$: $\phi = \cos^{-1}\left(\pm\frac{3}{4}\right) = \pm 41.4^\circ / \pm 138.5^\circ$



* E → normalised

$$E = \frac{\sin n\psi/2}{n\sin\psi/2} = \frac{1}{4}\frac{\sin\left[\frac{4\pi\cos\phi}{2}\right]}{\sin\left[\frac{\pi\cos\phi}{2}\right]}$$

$n=4$

$\psi = d_r\cos\phi$

$= \pi\cos\phi$

$$E = 0.27$$

$\phi = 41.4$

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$HPBW = \frac{1}{2} FNBW = \frac{1}{2} \cdot 60^\circ = 30^\circ$

* Directivity, $D = \frac{41200}{\theta_{HP} \cdot \phi_{HP}} = \frac{41200}{30^\circ \times 30^\circ}$

$D = 45.77$

Q. Compute the field pattern & find BW b/w first null & HPBW for a linear uniform array of 6 isotropic point sources spaced $\lambda/2$ dist apart. The power is applied in equal amplitude & in-phase. ($\because \delta = 0$)

→ $\delta = 0$ → Broad side Array

$n = 6$

$d = \lambda/2 \Rightarrow dr = \frac{2\pi d}{\lambda} = \pi$

$\therefore \psi = \pi \cos\phi$

$\pi \cos\phi_{max} = 0$

$\phi_{max} = \pm 90^\circ$

$n = 6$

Complete lobes $= n - 1 = 5$

no. of side lobes $= n - 2 = 4$

Direct. of nulls:

$\phi_0 = \cos^{-1}\left[\pm \frac{2k\pi}{ndr}\right] = \cos^{-1}\left[\pm \frac{\cancel{2}k\pi}{3\cancel{6} \cdot \cancel{\pi}}\right]$

$k=0: \phi_0 = \cos^{-1}[\pm 0] = \pm 90^\circ$ → not valid

$k=1: \phi_0 = \cos^{-1}\left[\pm \frac{1}{3}\right] = \pm 70.5^\circ / \pm 109.4^\circ$

$k=2: \phi_0 = \cos^{-1}\left[\pm \frac{2}{3}\right] = \pm 48.1^\circ / \pm 131.8^\circ$

$k = 3: \phi_0 = \cos^{-1}\left[\pm \frac{3}{3}\right] = \pm 0^\circ / \pm 180^\circ$

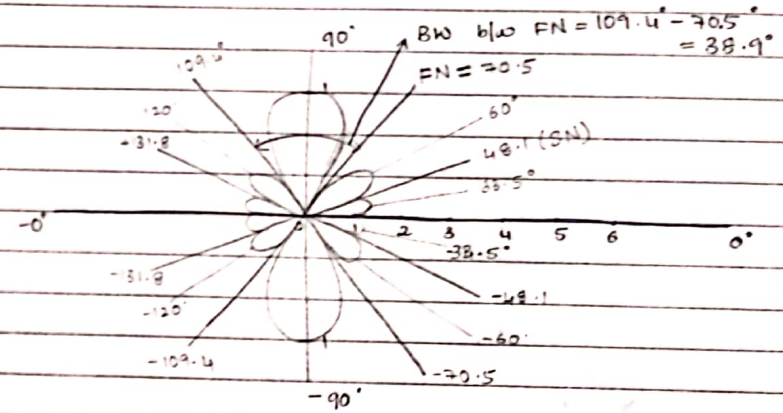
Direct. of Side lobes

$\phi = \cos^{-1}\left[\frac{\pm(2k+1)\pi}{ndr}\right] = \cos^{-1}\left[\frac{\pm(2k+1)}{6}\right]$

$k=0: \phi = \cos^{-1}\left[\pm \frac{1}{6}\right] = \pm 80.4^\circ$ & $\pm 99.5^\circ$ → not valid

$k=1: \phi = \cos^{-1}\left[\pm \frac{\cancel{3}}{2\cancel{6}}\right] = \pm 60^\circ$ & $\pm 120^\circ$

$k=2: \phi = \cos^{-1}\left[\pm \frac{5}{6}\right] = \pm 33.5^\circ$ & $\pm 146.4^\circ$



$HPBW = \frac{1}{2} FNBW = \frac{1}{2}(38.9^\circ) = 19.45^\circ$

Directivity, $D = \frac{41200}{\theta_{HP} \cdot \phi_{HP}} = \frac{41200}{(19.45)(19.45)}$

$D = 108.90$

Case 2 Linear End Fire Array

(maxima are along array axis)

$\phi = \pm 0^\circ$

$\psi = dr \cos\phi + \delta$ → phase diff

In end fire array, $\phi = 0^\circ$

along maxima direct., path diff. $\psi = 0$

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$$d_r \cos(0) + \delta = 0$$
$$d_r + \delta = 0$$
$$\boxed{\delta = -d_r}$$

$$\psi = d_r \cos\phi + \delta$$
$$= d_r \cos\phi - d_r$$
$$\psi = d_r [\cos\phi - 1] \quad \text{— End fire Array condt}^{n}$$

Null Direct^{n}

$$\sin \frac{n\psi}{2} = 0$$
$$\frac{n\psi}{2} = \sin^{-1}(0) = \pm k\pi$$
$$\psi = \frac{\pm 2k\pi}{n}$$
$$d_r [\cos\phi_0 - 1] = \frac{\pm 2k\pi}{n}$$
$$\cos\phi_0 - 1 = \frac{\pm 2k\pi}{n d_r}$$
$$\phi_0 = \cos^{-1}\left[\frac{\pm 2k\pi}{n d_r} + 1\right]$$

Side lobe direct^{n}

$$\sin \frac{n\psi}{2} = 1$$
$$\frac{n\psi}{2} = \sin^{-1}(1) = \pm (2k+1)\frac{\pi}{2}$$
$$\psi = \frac{\pm (2k+1)\pi}{n}$$
$$d_r [\cos\phi - 1] = \frac{\pm (2k+1)\pi}{n}$$
$$\cos\phi - 1 = \frac{\pm (2k+1)\pi}{n d_r}$$
$$\phi = \cos^{-1}\left[\frac{\pm (2k+1)\pi}{n d_r} + 1\right]$$

Q. A linear array consists of 4 Isotropic point sources. The dist^{n} b/w adjacent element is $\lambda/2$. The Power is applied in equal amplitude & phase diff. of d_r. Obtain the complete field pattern & find FNBW, HPBW, D & G if $\eta = 85\%$.

→ Given: $\delta = d_r$ → End fire array.

∴ Maximas are along array axis. $\phi_{max} = \pm 0^\circ$

$n = 4$

⇒ Complete lobes $= n - 1 = 3$

no. of Side lobes $= n - 2 = 2$

Null Direct^{n}

$$d_r = \frac{2\pi d}{\lambda} = \frac{\cancel{2}\pi \cdot \cancel{\lambda}}{\cancel{2} \cdot \cancel{\lambda}} = \pi$$

$$\psi = \pm \frac{2k\pi}{n}$$

$$* \ \phi_0 = \cos^{-1}\left[\frac{\pm 2k\pi}{n d_r} + 1\right] = \cos^{-1}\left[\frac{\pm \cancel{2}k\pi}{\cancel{4}_2 \cdot \pi} + 1\right]$$

$$\phi_0 = \cos^{-1}\left[\pm \frac{k}{2} + 1\right]$$

$k = 0$: $\phi_0 = \cos^{-1}[\pm 1] = 0^\circ$ → invalid

$k = 1$: $\phi_0 = \cos^{-1}\left[\pm \frac{1}{2} + 1\right] = \cos^{-1}\left[\pm \frac{3}{2}\right]$ × $= \pm 60^\circ$

$= \cos^{-1}\left[+\frac{1}{2} + 1\right]$, $\cos^{-1}\left[\frac{-1}{2} + 1\right] = \cos^{-1}\left[\frac{1}{2}\right] = 60^\circ$

$k = 2$: $\phi_0 = \cos^{-1}\left[-\frac{2}{2} + 1\right] = \pm 90^\circ$

$k = 3$: $\phi_0 = \pm 120^\circ$

Side lobe Direct^{n}

$$\sin \frac{n\psi}{2} = 1$$

$$* \ \phi = \cos^{-1}\left[\frac{\pm (2k+1)\pi}{n d_r} + 1\right] = \cos^{-1}\left[\frac{\pm (2k+1)\cancel{\pi}}{4\cancel{\pi}} + 1\right]$$

$k = 0$: $\phi = \cos^{-1}\left[\pm \frac{1}{4} + 1\right] = \pm 41.4^\circ$

$k = 1$: $\phi = \cos^{-1}\left[\pm \frac{3}{4} + 1\right] = \pm 75.52^\circ$

$k = 2$: $\phi = \cos^{-1}\left[\pm \frac{5}{4} + 1\right] = \pm 104.4^\circ$

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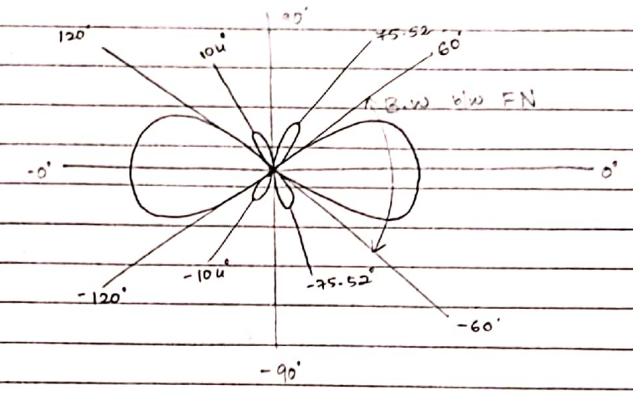
$K = 8 : \quad \phi = \cos^{-1}(-0.75) = \pm 138.59^\circ$



$FNBW = 60^\circ - (-60^\circ) = 120^\circ$

$HPBW = \frac{120^\circ}{2} = 60^\circ$

$D = \frac{41200}{\theta_{HP} \cdot \phi_{HP}} = \frac{41200}{60^\circ \times 60^\circ} = 11.44$

$G = \eta D = 0.85 \times 11.44$

$G = 9.72$ ~~dB~~

$E = \frac{1}{n} \frac{\sin n\psi/2}{\sin \psi/2}$

Q. For linear array, max^m radiation is expected along A.A which consists of 5 isotropic point sources spaced $\lambda/2$ apart. Obtain complete radt^n pattern & find D.

→ Given : max. radt^n along A.A → End fire Array

$\phi_{max} = \pm 0^\circ$

$\delta = dr$

$d = \lambda/2 \Rightarrow dr = \pi$

Side lobes $= n - 2 = 3$

Null direct^n

$\phi_0 = \cos^{-1}\left[\pm \frac{2k\pi}{ndr} + 1\right] = \cos^{-1}\left[\pm \frac{2k}{5} + 1\right]$

$K=0 : \phi_0 = \cos^{-1}[\pm 1] = 0^\circ$ ×

$K=1 : \phi_0 = \cos^{-1}\left[\pm \frac{2}{5} + 1\right] = \cos^{-1}[-0.4+1] = \pm 53.13^\circ$

$K=2 : \phi_0 = \cos^{-1}\left[\pm \frac{4}{5} + 1\right] = \pm 78.46^\circ$

$K=3 : \phi_0 = \cos^{-1}\left[\pm \frac{6}{5} + 1\right] = \pm 101.53^\circ$

$K=4 : \phi_0 = \cos^{-1}\left[\pm \frac{8}{5} + 1\right] = \pm 126.86^\circ$

$K=5 : \phi_0 = \cos^{-1}[\pm 2 + 1] = \pm$ ~~180~~ 180°

Side lobe direct^n

$\phi_1 = \cos^{-1}\left[\pm \frac{(2k+1)\pi}{ndr} + 1\right] = \cos^{-1}\left[\pm \frac{(2k+1)}{5} + 1\right]$

$K=0 : \phi_1 = \cos^{-1}\left(\frac{1}{5} + 1\right) = \pm 86.86^\circ$ ×

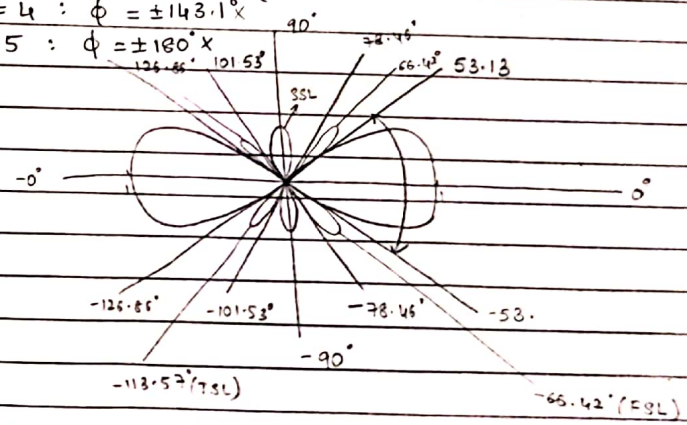
$K=1 : \phi = \cos^{-1}\left(\pm \frac{3}{5} + 1\right) = \pm 66.42^\circ$

$K=2 : \phi = \cos^{-1}(\pm 5/5 + 1) = \pm$ ~~180~~ 90°

$K=3 : \phi = \cos^{-1}\left(\pm \frac{7}{5} + 1\right) = \pm 113.57^\circ$

$K=4 : \phi = \pm 143.1^\circ$ ×

$K=5 : \phi = \pm 180^\circ$ ×



$FNBW = 53.13^\circ - (-53.13^\circ) = 106.26^\circ$

$HPBW = 53.13^\circ = \theta_{HP} = \phi_{HP}$

$D = \frac{41200}{\theta_{HP} \cdot \phi_{HP}} = \frac{41200}{(53.13)(53.13)} = 14.59^\circ$

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Case 3: Extended End Fire Array [Increased Directivity End Fire Array]

$\phi_m = \pm 0^\circ$

BSA	EFA
$\delta = \gamma$	$\delta = -dr$
$\phi_m = \pm 90^\circ$	$\phi_m = \pm 0^\circ$

$11.45 = D_{EFA} < D_{BSA} = 45$

To increase directivity

$$\boxed{\delta = -dr - \frac{\pi}{n}} \Rightarrow D\uparrow$$

$\rightarrow$ condition for Super Directivity

* Hanson Woodyard Array

$$\psi = dr\cos\phi + \delta$$

$$\psi = dr\cos\phi - dr - \frac{\pi}{n}$$

$$\psi = dr\left[\cos\phi - 1\right] - \frac{\pi}{n} \quad \text{—(1)}$$

Total path difference

In linear Array, $E = \dfrac{\sin n\psi/2}{\sin \psi/2}$

* Direction of Nulls

$$\sin n\psi/2 = 0$$

$$n\psi = \pm 2k\pi \quad \text{—(2)}$$

$$n\left[dr(\cos\phi - 1) - \frac{\pi}{n}\right] = \pm 2k\pi$$

$$dr(\cos\phi - 1) - \frac{\pi}{n} = \frac{\pm 2k\pi}{n}$$

$$dr(\cos\phi - 1) = \frac{\pm 2k\pi}{n} + \frac{\pi}{n}$$

$$\cos\phi - 1 = \frac{\pm 2\pi k}{ndr} + \frac{\pi}{ndr}$$

$$\phi_0 = \cos^{-1}\left[\frac{\pm 2k\pi}{ndr} + \frac{\pi}{ndr} + 1\right]$$

* Direction of Side lobes

$$\sin\frac{n\psi}{2} = 1$$

$$\frac{n\psi}{2} = \pm(2k+1)\frac{\pi}{2}$$

$$n\psi = \pm(2k+1)\pi$$

$$n\left[dr(\cos\phi - 1) - \frac{\pi}{n}\right] = \pm(2k+1)\pi$$

$$dr(\cos\phi - 1) - \frac{\pi}{n} = \frac{\pm(2k+1)\pi}{n}$$

$$dr(\cos\phi - 1) = \frac{\pm(2k+1)\pi}{n} + \frac{\pi}{n}$$

$$\cos\phi = \frac{\pm(2k+1)\pi}{drn} + \frac{\pi}{drn} + 1$$

$$\phi = \cos^{-1}\left[\frac{\pm(2k+1)\pi}{ndr} + \frac{\pi}{ndr} + 1\right]$$

Q. If $n = 4$ & $d = \lambda/2$, obtain the complete field pattern & find the directivity.

$\rightarrow$ For Extended End Fire Array

$$\delta = -dr - \frac{\pi}{n}$$

$$\phi_m = \pm 0^\circ$$

$$dr = \frac{2\pi d}{\lambda}$$

$$\delta = -\pi - \frac{\pi}{n}$$

$$dr = \pi$$

$n = 4$

$$\delta = -\pi - \frac{\pi}{4} = \frac{-5\pi}{4}$$

Direct^n of Null:

$$\phi_0 = \cos^{-1}\left[\frac{\pm 2k\pi}{4\cdot\pi} + \frac{\pi}{4\cdot\pi} + 1\right]$$

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$$\phi_0 = \cos^{-1}\left[\pm\frac{k}{2} + \frac{1}{4} + \frac{1\times 1}{4\times 1}\right]$$

$$\phi_0 = \cos^{-1}\left[\pm\frac{k}{2} + \frac{5}{4}\right]$$

$k=0$: $\phi_0 = \cos^{-1}\left[\frac{5}{4}\right]$ – invalid

$k=1$: $\phi_0 = \cos^{-1}\left[\pm\frac{1^2}{2\times 2} + \frac{5}{4}\right] = \cos^{-1}\left[\frac{5}{4} - \frac{1}{2}\right]$

$= \pm 41.41^\circ$

$k=2$: $\phi_0 = \cos^{-1}\left[\pm 1 + \frac{5}{4}\right] = \cos^{-1}\left[\frac{5}{4} - 1\right] = \pm 75.52^\circ$

$k=3$: $\phi_0 = \cos^{-1}\left[\pm\frac{3}{2} + \frac{5}{4}\right] = \cos^{-1}\left[\frac{5}{4} - \frac{3}{2}\right] = \pm 104.47^\circ$

$k=4$: $\phi_0 = \cos^{-1}\left[\frac{5}{4} \pm \frac{4}{2}\right] = \cos^{-1}\left[\frac{5}{4} - 2\right] = \pm 138.6^\circ$

Direction of Sidelobes

$$\phi = \cos^{-1}\left[\pm\frac{(2k+1)\pi}{ndr} + \frac{\pi}{ndr} + 1\right]$$

$$= \cos^{-1}\left[\pm\frac{(2k+1)\cancel{\pi}}{4\cdot\cancel{\pi}} + \frac{\cancel{\pi}}{4\cdot\cancel{\pi}} + 1\right]$$

$$\phi = \cos^{-1}\left[\pm\frac{(2k+1)}{4} + \frac{5}{4}\right]$$

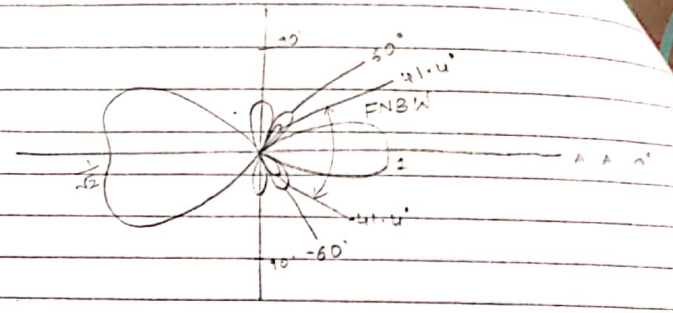
$k=0$: $\phi = \cos^{-1}\left[\pm\frac{1}{4} + \frac{5}{4}\right] = 0$ ×

$k=1$: $\phi = \cos^{-1}\left[\pm\frac{3}{4} + \frac{5}{4}\right] = \cos^{-1}[1.25 - 0.75] = \pm 60^\circ$

$k=2$: $\phi = \cos^{-1}\left[\pm\frac{5}{4} + \frac{5}{4}\right] = \cos^{-1}(0) = \pm 90^\circ$

$k=3$: $\phi = \cos^{-1}\left[\pm\frac{7}{4} + \frac{5}{4}\right] = \cos^{-1}\left[\frac{5}{4} - \frac{7}{4}\right] = \pm 120^\circ$

$k=4$: $\phi = \cos^{-1}\left[\pm\frac{9}{4} + \frac{5}{4}\right] = \cos^{-1}(-1) = \pm 180^\circ$ ×



HPBW $\simeq \frac{1}{2}$ FNBW

$= 41.4^\circ$

$$D = \frac{41200}{\theta_{HP}\times\phi_{HP}} = \frac{41200}{41.4\times 41.4} = \underline{\underline{24.03}}$$

⇒ Long Array

Array length $= nd$

$nd \gg k\lambda \longrightarrow$ Long Array

$\underset{(exact)}{FNBW} \simeq \underset{(approx)}{FNBW}$

→ Approximate FNBW

$\sin\frac{n\psi}{2} = 0$

$\frac{n\psi}{2} = \pm k\pi$

$\psi = \pm\frac{2k\pi}{n}$ —— (1)

$\psi = dr\cos\phi + \delta$ —— (2)

(1) = (2) ⇒ $dr\cos\phi + \delta = \pm\frac{2k\pi}{n}$

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$$d_r \cos\phi = \pm \frac{2K\pi}{n} - \delta$$

$$\phi = \cos^{-1}\left[\pm \frac{2K\pi}{n d_r} - \frac{\delta}{d_r}\right]$$

Case 1: Broad Side Array

$\delta = 0$

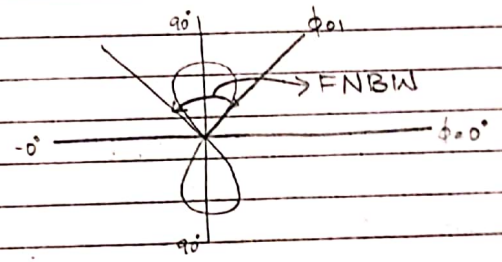
$$\Rightarrow \phi = \cos^{-1}\left[\pm \frac{2K\pi}{n d_r} - 0\right]$$

$$\phi = \cos^{-1}\left(\pm \frac{2K\pi}{n d_r}\right)$$

$K = 0$: $\phi = \cos^{-1}(0) = \pm 90^\circ$ ×

↳ Direct° of maximas

$K = 1$: $\phi_{01} = \cos^{-1}\left(\pm \frac{2 \times 1 \times \pi}{n d_r}\right) = \cos^{-1}\left(\pm \frac{2\pi}{n d_r}\right)$



$$FNBW = 2[90 - \phi_{01}]$$
$$= 2\left[90 - \cos^{-1}\left(\pm \frac{2\pi}{n d_r}\right)\right]$$

$$FNBW = 2\sin^{-1}\left(\pm \frac{2\pi}{n d_r}\right) = 2\sin^{-1}\left(\pm \frac{2\pi}{n \cdot \frac{2\pi d}{\lambda}}\right)$$

$$FNBW = 2\sin^{-1}\left(\pm \frac{\lambda}{nd}\right)_{exact}$$

Long array $nd > K\lambda$

$nd > \lambda$ $(K=1)$

$$\therefore FNBW \simeq \frac{2\lambda}{nd}_{approx} \quad (\because \theta \simeq \text{small} \Rightarrow \sin\theta \simeq \theta)$$

Case 2: End Fire Array

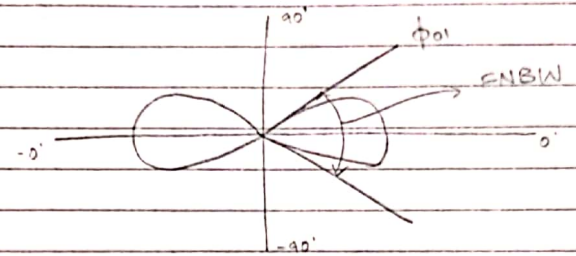
$\delta = -d_r$

$$\Rightarrow \phi = \cos^{-1}\left[\pm \frac{2K\pi}{n d_r} - \frac{(-d_r)}{d_r}\right]$$

$$\phi = \cos^{-1}\left[\pm \frac{2K\pi}{n d_r} + 1\right]$$

$K = 0$: $\phi = \cos^{-1}(+1) = 0^\circ$ ×

$K = 1$: $\phi_{01} = \cos^{-1}\left(\pm \frac{2\pi}{n d_r} + 1\right)$



$$FNBW = 2\phi_{01} = 2\cos^{-1}\left(\pm \frac{2\pi}{n d_r} + 1\right)$$

$$= 2\cos^{-1}\left(\pm \frac{2\pi}{n \cdot \frac{2\pi d}{\lambda}} + 1\right)$$

$$FNBW \simeq 2\cos^{-1}\left(\pm \frac{\lambda}{nd} + 1\right)$$

Exact method: $FNBW = 4\sin^{-1}\left[\sqrt{\frac{\lambda}{2nd}}\right]$

Approx. method: $FNBW = 2\sqrt{\frac{2\lambda}{nd}}$

Case 3: Extended End Fire Array

$$\delta = -d_r - \frac{\pi}{n}$$

$$\psi = d_r \cos\phi + \delta$$
$$\psi = d_r[\cos\phi - 1] - \frac{\pi}{n}$$

$$\sin\frac{n\psi}{2} = 0$$

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$$\frac{n\psi}{2} = \pm K\pi$$

$$\frac{n}{2}\left[d_r(\cos\phi - 1) - \frac{\pi}{n}\right] = \pm K\pi$$

$$d_r(\cos\phi - 1) = \pm\frac{2K\pi}{n} + \frac{\pi}{n}$$

$$\cos\phi = \pm\frac{2K\pi}{nd_r} + \frac{\pi}{nd_r} + 1$$

$$\phi = \cos^{-1}\left[\pm\frac{2K\pi}{nd_r} + \frac{\pi}{nd_r} + 1\right]$$

$K = 0$: $\phi = \cos^{-1}\left[\frac{\pi}{nd_r} + 1\right]$ $\qquad \cos\theta = 1 - 2\sin^2\frac{\theta}{2}$

$K = 1$: $\phi = \cos^{-1}\left[\pm\frac{2\pi}{nd_r} + \frac{\pi}{nd_r} + 1\right]$

FNBW

$$\cancel{1} - 2\sin^2\frac{\phi}{2} = \pm\frac{2K\pi}{nd_r} + \frac{\pi}{nd_r} + \cancel{1}$$

$$2\sin^2\frac{\phi}{2} = \mp\frac{2K\pi}{nd_r} - \frac{\pi}{nd_r}$$

$2\sin^2\frac{\phi}{2} = \mp\frac{\pi}{nd_r}(2K-1)$

$\sin^2\frac{\phi}{2} = \mp\frac{\pi}{2nd_r}(2K-1)$

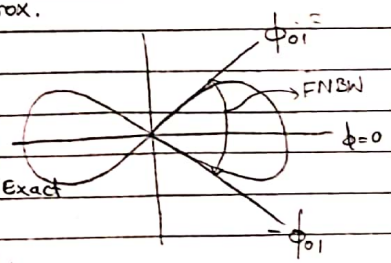
$\frac{\phi}{2} = \sin^{-1}\sqrt{\mp\frac{\pi}{(2nd_r)}(2K-1)}$

$$\phi = 2\sin^{-1}\left[\pm\sqrt{\frac{\pi}{2nd_r}(2K-1)}\right]$$

$K = 1$: $\phi_{01} = 2\sin^{-1}\left[\mp\sqrt{\frac{\pi}{2nd_r}}\right]$ exact

$nd_r \gg \lambda$

$\therefore \phi_{01} = 2\sqrt{\frac{\pi}{2nd_r}}$ approx.



$FNBW = 2\phi_{01}$

$\Rightarrow FNBW = 4\sin^{-1}\left[\mp\sqrt{\frac{\pi}{2nd_r}}\right]$ Exact

$FNBW \simeq 4\sqrt{\frac{\pi}{2nd_r}}$ approx.

$$\simeq 4\sqrt{\frac{\pi}{\ldots}} = 4\sqrt{\frac{\lambda}{\ldots}} = 2\sqrt{\frac{\lambda}{\ldots}}$$

$D_{Ext.\,EFA} > D_{EFA}$

Calculate the Directivity

Q. Show that $D_{E^2FA} > D_{EFA}$

$\rightarrow$ WKT, $D = \frac{4\pi}{\theta_{HP} \times \phi_{HP}}$

for Ext. EFA

$$FNBW = 2\sqrt{\frac{\lambda}{nd}}$$

WKT $\quad HPBW \simeq \frac{1}{2}[FNBW] = \frac{1}{\cancel{2}} \cdot \cancel{2}\sqrt{\frac{\lambda}{nd}}$

$$\theta_{HP} = \left(\frac{\lambda}{nd}\right)^{1/2} = \phi_{HP}$$

$$\therefore D_{E^2FA} = \frac{41200}{\left(\frac{\lambda}{nd}\right)^{1/2} \times \left(\frac{\lambda}{nd}\right)^{1/2}} = \frac{41200}{\left(\frac{\lambda}{nd}\right)^{1/4}}$$

Assuming that in θ-plane, the pattern is Omni-directional,

$$\theta = 2\pi$$

$$\phi_{HP} = \frac{FNBW}{2} = \frac{\cancel{2}\left(\sqrt{\frac{2\lambda}{nd}}\right)}{\cancel{2}} = \sqrt{\frac{2\lambda}{nd}}$$

$$D_{EFA} = \frac{4\pi^2}{2\pi \times \sqrt{\frac{2\lambda}{nd}}} = 2\sqrt{\frac{nd}{2\lambda}} = \sqrt{\frac{4nd}{2\lambda}}$$

$$= \sqrt{\frac{2nd}{\lambda}}$$

array length expressed in wavelength $\leftarrow L\lambda = \frac{nd}{\lambda}$

$$\therefore D_{EFA} = \sqrt{2L\lambda}$$

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for E^2FA,

$$\phi_{HP} = \frac{FNBW}{2} = \sqrt{\frac{\lambda}{nd}}$$

$$\theta_{HP} = 2\pi$$

$$D_{E^2FA} = \frac{4\pi^2}{2\pi \times \sqrt{\frac{\lambda}{nd}}} = 2\sqrt{\frac{nd}{\lambda}} = \sqrt{\frac{4nd}{\lambda}}$$

$$L\lambda = \frac{nd}{\lambda}$$

$$\therefore D_{E^2FA} = \sqrt{4L\lambda}$$

$$\therefore D_{E^2FA} > D_{EFA}$$

for BSA:

$$\theta_{HP} = 2\pi$$

$$\phi_{HP} = \frac{FNBW}{2} = \frac{\frac{2\lambda}{nd}}{2} = \frac{\lambda}{2nd} = \frac{\lambda}{nd}$$

$$D_{BSA} = \frac{4\pi}{2\pi \times \frac{\lambda}{nd}} = \frac{2nd}{\lambda}$$

$$D_{BSA} = 2L\lambda$$

$$D_{BSA} > D_{E^2FA} > D_{EFA}$$

Q. Find the Directivity of a Broad Side Uniform Array with 10 isotropic elements with a spacing of $\frac{\lambda}{4}$.

→ (for $n > 6$, using approx method)

Given: $n = 10$, $d = \frac{\lambda}{4}$

for BSA, $FNBW \simeq \frac{2\lambda}{nd}$

$$D = 2L\lambda = \frac{2nd}{\lambda} = \frac{2 \times 10 \times \lambda/4}{\lambda}$$

$$D = 5$$

Q. An uniform L.A consists of 16 isotropic point sources with a spacing of $\frac{\lambda}{4}$ with a phase difference of $-90°$ (δ). Calculate HPBW, D, beam solid angle (Ω_A), Ae.

→ [If $\delta = -d\tau$ → EFA

$\delta = 0$ → BSA

$\delta = -d\tau - \frac{\pi}{n}$ → E^2FA]

Given: $\delta = -90° = -\frac{\pi}{2}$

$$d = \frac{\lambda}{4} \Rightarrow d\tau = \frac{2\pi d}{\lambda} = \frac{2\pi(\lambda/4)}{\lambda} = \frac{\pi}{2}$$

$\delta = -d\tau \Rightarrow$ End Fire Array

for EFA, $FNBW = 2\sqrt{\frac{2\lambda}{nd}}$

$$HPBW = \frac{1}{2} FNBW = \sqrt{\frac{2\lambda}{16 \times \lambda/4}} = \sqrt{\frac{1}{2}}$$

$$\therefore HPBW = 0.7071 \text{ rad} = \phi_{HP}$$

$$D = \frac{4\pi}{\theta_{HP} \times \phi_{HP}} = \frac{4\pi}{2\pi \times 0.7071} = 2.83$$

$$\Omega_A = \theta_{HP} \times \phi_{HP} = 2\pi \times 0.7071 = 4.44$$

$$A_e = \frac{\lambda^2}{\Omega_A} = \frac{\lambda^2}{4.44} = 0.225\lambda^2$$

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⇒ Scanning Array

$\psi = d_r \cos\phi_1 + \delta$, $\psi = 0$, $\delta = ?$

max. radt^n is along some arbitrary direct^n

for $\phi_1 = 60^\circ$

maxima is not along $\pm 0^\circ / \pm 90^\circ$

Q. An array of 4 isotropic point sources placed along a straight line, the dist^n b/w adjacent elements is $\lambda/2$. The peak is to be obtained in a direct^n 60° from the A.A. What should be the phase diff. b/w adjacent elements. Obtain complete field pattern & find FNBW, HPBW

→ Given: $n = 4$, $d = \lambda/2$

maxima along 60° → Scanning array

$\phi_1 = 60^\circ$, $\psi = 0$

$\psi = d_r \cos\phi_1 + \delta$ $\qquad d_r = \frac{2\pi d}{\lambda} = \pi$

$0 = d_r \cos(60^\circ) + \delta$

$\delta = -\pi \cos(60^\circ)$

$\delta = -\frac{\pi}{2}$

$\therefore \psi = \pi \cos\phi - \frac{\pi}{2}$

Direct^n of Nulls:

$\sin n\psi/2 = 0$

$n\psi/2 = \pm k\pi$

$\psi = \pm \frac{2k\pi}{n}$

$\pi \cos\phi - \frac{\pi}{2} = \pm \frac{k\pi}{2}$

$\pi \cos\phi_0 = \pm \frac{k\pi}{2} + \frac{\pi}{2}$

$\cos\phi_0 = \pm \frac{k}{2} + \frac{1}{2}$

$\phi_0 = \cos^{-1}\left(\pm \frac{k}{2} + \frac{1}{2}\right)$

$k = 0$: $\phi_0 = \cos^{-1}\left[\frac{1}{2}\right] = 60^\circ$ X maxima direct^n

$k = 1$: $\phi_0 = \cos^{-1}\left(\pm \frac{1+1}{2}\right) = \pm 90^\circ, \pm 0^\circ$

$k = 2$: $\phi_0 = \cos^{-1}\left(\pm \frac{2+1}{2}\right) = \pm 120^\circ$

$k = 3$: $\phi_0 = \cos^{-1}\left(\pm \frac{3+1}{2}\right) = \pm 180^\circ$

Direct^n of Sidelobes:

$\sin n\psi/2 = 1$

$\frac{n\psi}{2} = \pm(2k+1)\frac{\pi}{2}$

$\psi = \pm \frac{(2k+1)\pi}{n}$

$\pi \cos\phi - \frac{\pi}{2} = \pm \frac{(2k+1)\pi}{n}$

$\pi \cos\phi = \pm \frac{(2k+1)\pi}{n} + \frac{\pi}{2}$

$\phi = \cos^{-1}\left[\pm \frac{(2k+1)}{4} + \frac{1}{2}\right]$

$k = 0$: $\phi = \cos^{-1}\left[\pm \frac{1}{4} + \frac{1}{2}\right] = \pm 41.4^\circ, \pm 75.52^\circ$ not valid FSL should exist after FN

$k = 1$: $\phi = \cos^{-1}\left[\pm \frac{3}{4} + \frac{1}{2}\right] = \pm 104.47^\circ$

$k = 2$: $\phi = \cos^{-1}\left[\pm \frac{5}{4} + \frac{1}{2}\right] = \pm 138.59^\circ$

$k = 3$: $\phi = \cos^{-1}\left[\pm \frac{7}{4} + \frac{1}{2}\right]$ X

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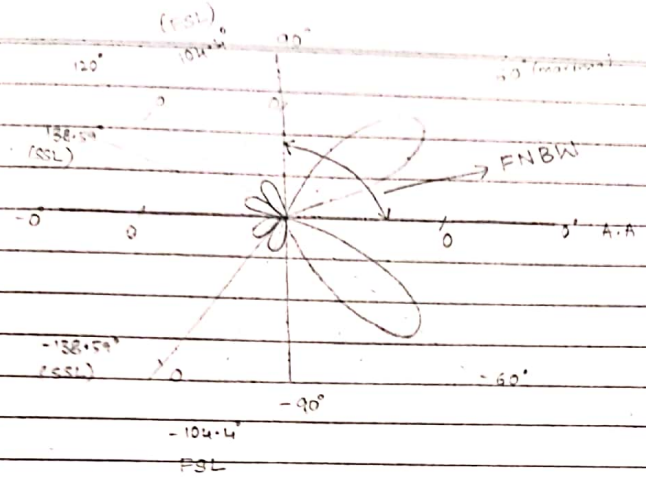

$FNBW = 90^\circ$

$\therefore HPBW = \frac{1}{2} FNBW = 45^\circ$

$$D = \frac{41200}{\theta_{HP} \times \phi_{HP}} = \frac{41200}{45 \times 45} = \underline{\underline{20.34}}$$

Q. An array consists of 4 isotropic point sources & distⁿ b/w adjacent sources is $\lambda/3$ & the phase diff. is $\pi/3$. Obtain complete field pattern.

→ Given : $d = \lambda/3$, $n = 4$, $\delta = \pi/3$

$$d_r = \frac{2\pi d}{\lambda} = \frac{2\pi \lambda/3}{\lambda} = \frac{2\pi}{3}$$

$$\psi = d_r \cos\phi + \delta$$

$$\psi = \frac{2\pi}{3}\cos\phi + \frac{\pi}{3}$$

$$\psi = \frac{\pi}{3}[2\cos\phi + 1]$$

Peaks :

$$\psi = 0$$

$$\frac{\pi}{3}(2\cos\phi + 1) = 0$$

$$2\cos\phi_{max} = -1$$

$$\cos\phi_{max} = \frac{-1}{2}$$

$$\phi_{max} = \pm 120^\circ$$

Direction of Nulls :

$$\sin n\psi/2 = 0$$

$$\frac{n\psi}{2} = \pm K\pi$$

$$\psi = \pm \frac{2K\pi}{4/2}$$

$$\frac{\pi}{3}[2\cos\phi_0 + 1] = \pm \frac{K\pi}{2}$$

$$2\cos\phi_0 = \pm \frac{3K}{2} - 1$$

$$\cos\phi_0 = \pm \frac{3K}{4} - \frac{1}{2}$$

$$\phi_0 = \cos^{-1}\left[\pm \frac{3K}{4} - \frac{1}{2}\right]$$

$K=0$: $\phi_0 = \cos^{-1}[-1/2] = \pm 120^\circ$ X

$K=1$: $\phi_0 = \cos^{-1}\left[\pm \frac{3}{4} - \frac{1}{2}\right] = \pm 75.52^\circ$,

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Q. An uniform array is req. to produce an end fire beam when it is operated at 10 GHz. It contains 50 radiators spaced at 0.5λ. Find phase shift & array length.

→ $\delta = -dr$

$\therefore \delta = -\pi$

$dr = \frac{2\pi d}{\lambda} = \frac{2\pi(0.5)\lambda}{\lambda}$

$= \pi$

$nd \rightarrow$ Array length

Given : $n = 50$

$d = 0.5\lambda$

$\therefore \lambda = \frac{c}{f} = \frac{3\times10^8}{10\times10^9} = 0.03\,m$

$\therefore$ Array length, $nd = 50\times0.03\times0.5$

$= 0.75\,m$

⇒ Pattern Multiplication

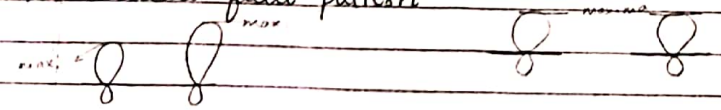
↳ determine field pattern of non-isotropic point sources

radiation is not uniform

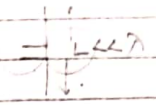
eg: short dipole ($L << \lambda$)

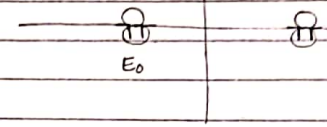
Non-isotropic pt Sources

Similar	Identical
* unequal peaks / different maximum radiation	* equal peaks
* identical field pattern	* identical field pattern

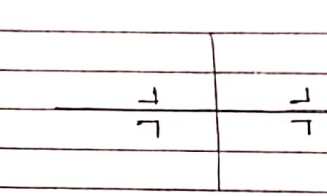


* 2-D pattern (Short dipole) is such that its maxima is ⊥ to length





$E_0 = E_0' \sin\phi$

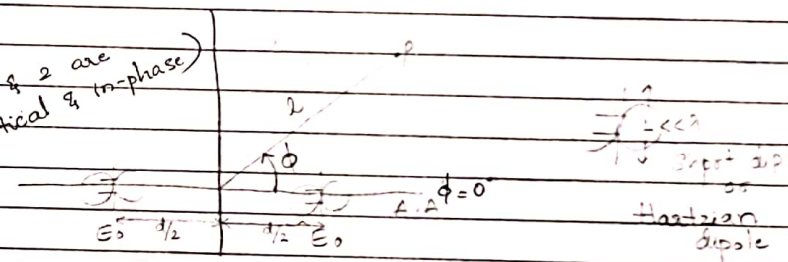


$E_0 = E_0' \cos\phi$

multiply field pattern with Array pattern.

⇒ Non-Isotropic Point Sources & Principle of Pattern Multiplication

(dipole 1 & 2 are identical & in-phase)



⊥ to A.A & equi-distant from reference pt.

Individual pattern of the dipole

$E_0 = E_0' \cos\phi$ —(1)

Array of 2 isotropic pt. sources having equal amplitude & any phase, $E = 2E_0 \cos\psi/2$

where $\psi = dr\cos\phi + \delta$

$\delta = 0$,

then $\psi = dr\cos\phi$

$$E = 2E_0 \cos\left(\frac{dr}{2}\cos\phi\right) \quad \text{——— (2)}$$

$$= 2E_0' \cos\phi \cdot \cos\left(\frac{dr}{2}\cos\phi\right)$$

Array of 2 identical point sources is

$$E_n = \underbrace{\cos\phi}_{\text{individual path}} \cdot \underbrace{\cos\left[\frac{dr}{2}\cos\phi\right]}_{\text{Array pattern}}$$

If dipoles ① & ② are not identical (they are similar)

Array pattern

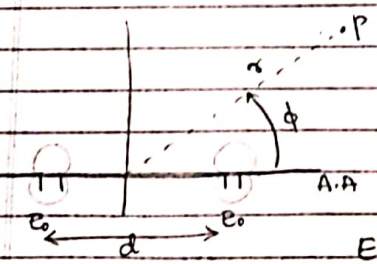
$$E = E_0\sqrt{(1+a\cos\psi)^2 + a^2\sin^2\psi}, \quad 0<a<1$$

$$= E_0'\cos\phi\sqrt{(1+a\cos\psi)^2 + a^2\sin^2\psi}$$

$\psi = dr\cos\phi + \delta$

$$E_n = \cos\phi\sqrt{(1+a\cos\psi)^2 + a^2\sin^2\psi)} \qquad \delta = 0$$

If dipoles are arranged as shown



$E_0 = E_0'\sin\psi$

$\psi = dr\cos\phi + \delta$

If both dipoles are identical

$$E_n = \sin\phi \cdot \cos\left(\frac{dr}{2}\cos\phi\right)$$

If both the dipoles are similar

$$E_n = \sin\phi\sqrt{(1+a\cos\psi)^2 + a^2\sin^2\psi}$$

→ Pattern Multiplication

$$E = \underbrace{f(\theta,\phi)}_{\text{Field pattern}}\, \underbrace{F(\theta,\phi)\angle f_p(\theta,\phi) + F_p(\theta,\phi)}_{\text{phase pattern}}$$

where

$f(\theta,\phi)$ = Field pattern of individual

$f_p(\theta,\phi)$ = phase "

$F(\theta,\phi)$ = Field pattern of array of isotropic sources

$F_p(\theta,\phi)$ = Phase "

* The total field pattern of an array of non-isotropic but similar sources is the product of the individual source pattern & pattern of an array of isotropic point sources each located at the phase centre of the individual source & having same relative amplitude & phase. while the total phase pattern is the sum of the phase pattern of individual source & the phase pattern of isotropic pt. sources.

Q. Assume 2 identical pt. sources separated by a dist. d, each source having field pattern $E_0 = E_0'\sin\phi$. Let $d = \lambda/2$ & $\delta = 0$, then obtain the complete field pattern of the array.

→ Given: Individual field pattern, $E_0 = E_0'\sin\phi$

$d = \lambda/2$, $\delta = 0$

$$E = 2E_0'\sin\phi \cdot \cos\psi/2$$

$\psi = dr\cos\phi + \delta$ $\qquad dr = \frac{2\pi d}{\lambda} = \pi$

$\delta = 0$

$$\psi = \pi\cos\phi$$

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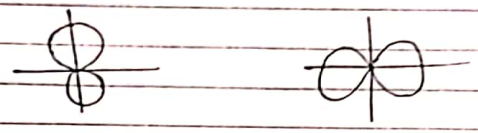
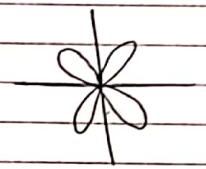
$$E_n = \sin\phi \cdot \cos\left(\frac{\pi}{2}\cos\phi\right)$$

if $E = E_0' \sin\phi$ & out of phase



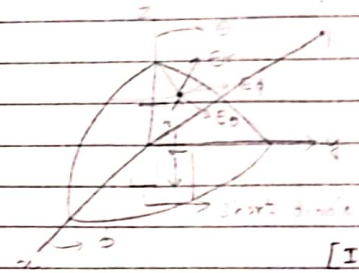
Total pattern :



Unit 3

Fields

Short dipole



$$I = \frac{dQ}{dt} = I_0 e^{j\omega t}$$

r is large

$[I]$ - retarded current

$$[I] = I_0 e^{j\omega(t - r/c)}$$

Vector potential - A

Scalar potential - V

Using Maxwell's eqn., $E_r \quad E_\theta \quad E_\phi$

$H_r \quad H_\theta \quad H_\phi$

* Near Field Components of Short Dipole

The expressions, $E_r = \frac{[I]\, l \cos\theta}{2\pi\varepsilon_0}\left[\frac{1}{r^2 c} + \frac{1}{j\omega r^3}\right]$ -(1)

$$E_\theta = \frac{[I]\, l \sin\theta}{4\pi\varepsilon_0}\left[\frac{j\omega}{r c^2} + \frac{1}{r^2 c} + \frac{1}{j\omega r^3}\right] \text{-(2)}$$

$E_\phi = 0 \rightarrow$ for short dipole

$H_r = H_\theta = 0$

$$H_\phi = \frac{[I]\, l \sin\theta}{4\pi}\left[\frac{j\omega}{r c} + \frac{1}{r^2}\right]$$

* Far Field Components of Short Dipole

If r is large, r^2 & r^3 are very large

Then $\frac{1}{r} > \frac{1}{r^2} > \frac{1}{r^3}$ are very small & hence neglected

$\therefore \quad E_r = E_\phi = 0 = H_r = H_\theta$

$$E_\theta = \frac{[I]\, l \sin\theta}{4\pi\varepsilon_0}\left[\frac{j\omega}{r c^2}\right]$$

$$H_\phi = \frac{[I]\, l \sin\theta}{4\pi}\left[\frac{j\omega}{r c}\right]$$

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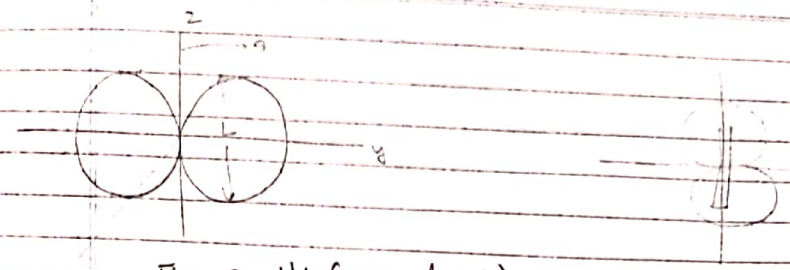


E_θ & H_ϕ (near field)

Near field pattern of E_r comp.

$$* \quad Z_0 = \frac{E}{H} = \frac{E_\theta}{H_\phi} = \frac{1}{\varepsilon_0 E} \quad -(1) \qquad \varepsilon = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \quad -(2)$$

$$E_\theta = \frac{j\omega I_0 e^{j\omega(t - r/c)} \cdot L \cdot \sin\theta}{4\pi \varepsilon_0 c^2}$$

$$H_\phi = \frac{j\omega I_0 e^{j\omega(t - r/c)} \cdot L \cdot \sin\theta}{4\pi r c}$$

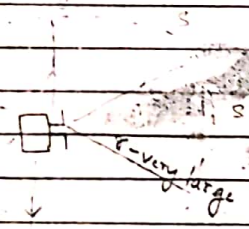
$$Z_0 = \frac{E_\theta}{H_\phi} = \sqrt{\frac{\mu_0 \varepsilon_0}{\varepsilon_0}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \sqrt{\frac{4\pi \times 10^{-7}}{8.85 \times 10^{-12}}}$$

$$= 120\pi$$

(or)

$$377\,\Omega$$

⇒ Radiation Resistance of Short Dipole (R_r)



$$S = S_r$$

$$S_\theta = S_\phi = 0$$

$$P_r = S + S + \cdots$$

total power radiated

$$P_r = \iint_{\phi\,\theta} S_r \cdot dS$$

Avg. Poynting vector, $S = \frac{1}{2} Re(E \times H)$

Under far field condition, $S_\theta = S_r$

$$S_r = \frac{1}{2} Re(E_\theta H_\phi)$$

$$\frac{E_\theta}{H_\phi} = Z_0 \rightarrow E_\theta = Z_0 H_\phi = \sqrt{\frac{\mu_0}{\varepsilon_0}} H_\phi$$

$$\therefore S_r = \frac{1}{2} |H_\phi|^2 \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

Total power radiated, $P_r = \iint S_r \, dS$

$$= \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \frac{1}{2} |H_\phi|^2 \sqrt{\frac{\mu_0}{\varepsilon_0}} \, dS$$

$$= \frac{1}{2} \times 120\pi \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} |H_\phi|^2 r^2 \sin\theta \, d\theta \, d\phi$$

$$= 60\pi \times 2\pi \int_{\theta=0}^{\pi} \left(\frac{\omega I_0 L \sin\theta}{4\pi r c}\right)^2 r^2 \sin\theta \, d\theta$$

$$= \frac{\cancel{120}^{30} \pi^{\cancel{2}} \omega^2 I_0^2 L^2}{\cancel{16}^{2}\,4\pi^{\cancel{2}} c^2} \int_{\theta=0}^{\pi} \sin^3\theta \, d\theta$$

$$= \frac{\cancel{15}^{5}\, \omega^2 I_0^2 L^2}{\cancel{2}_{1}\, c^2} \times \frac{\cancel{4}^{2}}{3}$$

$$\beta = \frac{\omega}{c} = \frac{2\pi \cancel{c}}{\lambda}$$

$$P_r = 10 \left(\frac{2\pi \cancel{c}}{\lambda}\right)^2 I_0^2 L^2$$

$$= \frac{10 \times 4\pi^2 \times I_0^2 L^2}{\lambda^2}$$

$$\boxed{P_r = \frac{40\pi^2 I_0^2 L^2}{\lambda^2}}$$

$$P_r = I_{rms}^2 R_r = \left(\frac{I_0}{\sqrt{2}}\right)^2 R_r$$

$$\boxed{R_r = \frac{80\pi^2 L^2}{\lambda^2}}$$

Q. Find the length of an elementary dipole having a radt. rest. of 5Ω & freq. of 5 MHz.

→ Given : $R_r = 5\Omega$, $f = 5$ MHz

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$$f = \frac{c}{\lambda} \Rightarrow \lambda = \frac{c}{f} = \frac{3\times10^8}{5\times10^6} = 60\ m$$

W.K.T, $R_r = \frac{80\pi^2 L^2}{\lambda^2}$

$$\Rightarrow L^2 = \frac{R_r \times \lambda^2}{80\pi^2}$$

$$= \frac{5\times60^2}{80\times\pi^2}$$

$$L = 4.77\ m$$

$L << \lambda \rightarrow$ Short dipole

Q. For a short dipole $\lambda/15$ long, find radiation rest^n efficiency, effective aperture if loss rest^n is 1Ω.

→ Given: $L = \lambda/15$, $R_l = 1\Omega$

$R_r = ?$

$\eta = ?$

$A_e = ?$

$$R_r = \frac{80\pi^2 L^2}{\lambda^2} = \frac{80\pi^2(\lambda^2/15^2)}{\lambda^2}$$

$$R_r = 3.51\ \Omega$$

$$\boxed{\eta\% = \frac{R_r}{R_r + R_l} \times 100}$$

$$= \frac{3.51}{3.51+1} \times 100$$

$$\eta = 77.8\ \%$$

$$\boxed{A_e = \frac{v^2}{4 S R_r}} = \frac{(EL)^2}{4\cdot\left(\frac{E^2}{Z_0}\right)R_r} = \frac{E^2\times(\lambda/15)^2}{4\times\frac{E^2}{120\pi}\times 3.51}$$

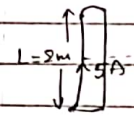
$$A_e = 0.119\ \lambda^2$$

Q. A 2m long vertical wire carries a current of 5A at 1MHz. Find the strength of radiated field at 30 kms with a direct^n of right angle $(\theta = 90°)$ to the axis of the wire assuming the wire is situated in free space.

→ $L = 2m$, $I_0 = 5A$, $f = 1MHz$, $r = 30\ km$

$E = ?$

(Determine whether it is a short dipole or vertical $(\lambda/2)$ antenna)



$$\lambda = \frac{c}{f} = \frac{3\times10^8}{1\times10^6} = 300\ m$$

$L = 2m << \lambda = 300\ m$

$\Rightarrow$ short dipole

$R > \frac{2L^2}{\lambda} \rightarrow$ far field

$$\frac{2L^2}{\lambda} = \frac{2\times2^2}{300} = 0.0266 < r = 30\ km$$

∴ Far field

$$E_\theta = \frac{j\omega I_0 e^{j\omega(t - r/c)}}{4\pi\varepsilon_0 c^2 r}\cdot L\sin\theta$$

$$= \frac{j(2\pi f)(I_0 e^{j2\pi f(t-r/c)})}{4\pi\varepsilon_0 c^2 r}\cdot L\sin\theta \quad \sim I_0$$

$$= \frac{2\pi\times1\times10^6\times[I_0 = 5A]\times2\times\sin(90°)}{4\pi\times8.85\times10^{-12}\times(3\times10^8)^2\times30\times10^3}$$

$$E_\theta = 2.092\times10^{-4}$$

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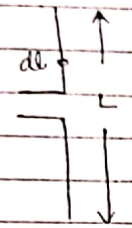
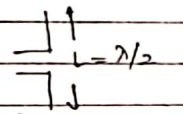
⇒ The Thin Linear Antenna

Length is comparable with wavelength



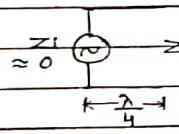
$L \approx \lambda$ & $d = \frac{\lambda}{100}$

dl – short dipole.



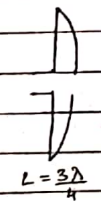
λ/2 antenna

derived from o.c quarter t/p wave transm. line & it is an impedance inverter.



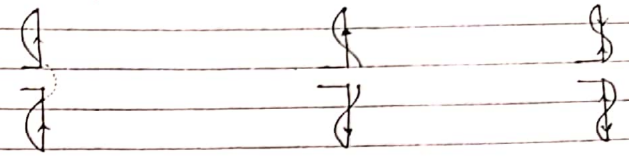
$Z_0 = \infty$ → o.c λ/4 antenna

if $L = 3\lambda/4$, Current distribution is



if $L = \lambda$ → Full wave antenna (2 HW antennas)

If $L = \frac{5}{4}\lambda$, $L = 3/2\,\lambda$, $L = 2\lambda$



→ Thin Linear Antenna Expression

$$[I] = I_0 \sin\left[\frac{2\pi}{\lambda}\left(\frac{l}{2} + x\right)\right] e^{j\omega(t - r/c)}$$

Under far field condition

$$H_\phi = \frac{j[I]}{2\pi r}\left[\frac{\cos\left(\frac{\beta l}{2}\cos\theta\right) - \cos\left(\frac{\beta l}{2}\right)}{\sin\theta}\right]$$

$$E_\theta = Z_0 H_\phi = \frac{j60[I]}{r}\left[\frac{\cos\left(\frac{\beta l}{2}\cos\theta\right) - \cos\left(\frac{\beta l}{2}\right)}{\sin\theta}\right]$$

$(\because Z_0 = 120\pi)$

$\beta = \frac{2\pi}{\lambda}$

for λ/2 antenna, WKT

$$H_\phi = \frac{j[I]}{2\pi r}\left[\frac{\cos\left(\frac{\beta \lambda/2}{2}\cos\theta\right) - \cos\left(\frac{\beta \cdot \lambda/2}{2}\right)}{\sin\theta}\right]$$

$$= \frac{j[I]}{2\pi r}\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right) - \cos\left(\frac{\pi}{2}\right)}{\sin\theta}\right]$$

$$\therefore H_\phi = \frac{j[I]}{2\pi r}\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]$$

$$E_\theta = \frac{j60[I]}{r}\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]$$

Q. Find the field pattern for λ/2 antenna.

→ $E_\theta = \frac{j60[I]}{r}\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]$

Direct" of maxima :

$E_\theta = 1$

$j60[I]\left[\cos\left(\frac{\pi}{2}\cos\theta\right)\right] = 1$

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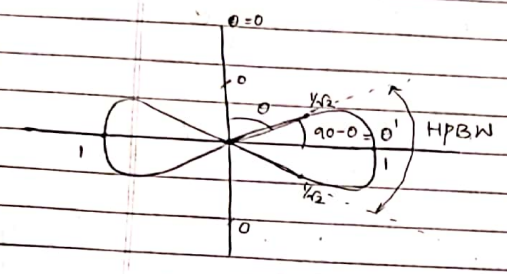
$$\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} = \frac{r}{j60[I]}$$

$$\cos\left(\frac{\pi}{2}\cos\theta\right) = \frac{r\sin\theta}{j60[I]}$$

$$\frac{\pi}{2}\cos\theta = \cos^{-1}\left(\frac{r\sin\theta}{j60[I]}\right)$$

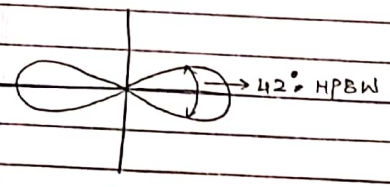


$\theta = 51^\circ$

$\theta' = 90^\circ - 51^\circ = 39^\circ$

$HPBW = 2\theta' = 78^\circ$

$H\phi$ - plane

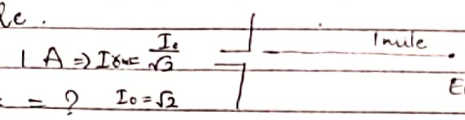


Q. A Half wave dipole in a free space is radiating with a current of 1A rms at the antenna terminals. Find angle θ for max. field strength & determine the field strength & power density at a point 1 mile from the antenna & the corresponding angle.

→ $I_{rms} = 1A \Rightarrow I_{0rms} = \frac{I_0}{\sqrt{2}}$

$\theta_{max} = ?$ $I_0 = \sqrt{2}$

$L = \lambda/2$



$$E_\theta = \frac{j60[I]}{r}\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]$$

WKT $E_\theta = 0$ at $\theta = 0^\circ$

maximas along $\theta = 90^\circ / \pi/2 \Rightarrow E_\theta = max.$

$\theta = \pi/2$

$$|E_\theta| = \frac{60\times\sqrt{2}A}{1\text{ mile}}\left[\frac{\cos\left(\frac{\pi}{2}\cos\frac{\pi}{2}\right)}{\sin\frac{\pi}{2}}\right]$$

$$= \frac{60\times\sqrt{2}}{1\text{ mile}}(1)$$

$$|E_\theta| = \frac{60\times1.414}{1.6\text{ km}} = \frac{84.84}{1.6\times10^3}$$

$$|E_\theta| = 53.03\text{ mV/m}$$

⇌ Radiation Resistance of λ/2 antenna

$$H\phi = \frac{j[I]}{2\pi r}\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]$$

$$|H\phi| = \frac{I_0}{2\pi r}\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right] \quad \text{—— (1)}$$

$$S_r = \frac{1}{2}|E_\theta||H\phi| \quad \text{—— (2)}$$

$$Z_0 = \frac{E_\theta}{H\phi} \quad \text{—— (3)}$$

$$E_\theta = Z_0 H\phi \quad \text{—— (4)}$$

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$$\Rightarrow S_r = \frac{1}{2} Z_0 |H_\phi|^2 \qquad \{Z_0 = 120\pi\}$$

$$S_r = 60\pi |H_\phi|^2 \quad \text{—— (5)}$$

Total power radiated,

$$P_r = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} S_r \cdot ds$$

$$= \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} 60\pi |H_\phi|^2 \cdot ds$$

$$= 60\pi \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \left(\frac{I_0}{2\pi r}\right)^2 \left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right]^2 \cdot r^2 \sin\theta \, d\theta \, d\phi$$

$$= \frac{\cancel{60}^{15}\cancel{\pi} I_0^2}{\cancel{4}\cancel{\pi^2}} \cdot 2\pi \int_{\theta=0}^{\pi} \left[\frac{\cos^2(\pi/2 \cos\theta)}{\sin^{\cancel{2}}\theta} \cdot \cancel{\sin\theta} \, d\theta\right]$$

$$P_r = 30 I_0^2 \int_{\theta=0}^{\pi} \frac{\cos^2(\pi/2 \cos\theta)}{\sin\theta} \, d\theta$$

Let $\cos\theta = u$

$-\sin\theta \, d\theta = du$

$\Rightarrow d\theta = \frac{-du}{\sin\theta}$

$\theta = 0 \Rightarrow u = 1$

$\theta = \pi \Rightarrow u = -1$

$$\therefore P_r = 30 I_0^2 \int_{u=1}^{-1} \frac{\cos^2(\frac{\pi}{2}u)}{\sin\theta} \cdot \left(\frac{-du}{\sin\theta}\right)$$

$$= +30 I_0^2 \int_{u=-1}^{1} \frac{\cos^2(\frac{\pi}{2}u)}{\sin^2\theta} \cdot du$$

$\because \sin^2\theta = 1 - \cos^2\theta$

$= 1 - u^2$

$$P_r = 30 I_0^2 \int_{u=-1}^{1} \frac{\cos^2(\frac{\pi}{2}u)}{1-u^2} \, du$$

$$\left\{\frac{1}{1-u^2} = \frac{1}{2}\left(\frac{1}{(1+u)} + \frac{1}{(1-u)}\right)\right\}$$

$$P_r = 30 I_0^2 \int_{u=-1}^{1} \cos^2\left(\frac{\pi}{2}u\right) \cdot \frac{1}{2}\left[\frac{1}{(1+u)} + \frac{1}{(1-u)}\right] du$$

$$= 15 I_0^2 \int_{u=-1}^{1} \cos^2\left(\frac{\pi}{2}u\right)\left[\frac{1}{(1+u)} + \frac{1}{(1-u)}\right] du$$

Let $1+u = \frac{v}{\pi}$, $\quad 1-u = \frac{v'}{\pi}$

$du = \frac{dv}{\pi}$ $\quad du = \frac{-dv'}{\pi}$

$u = -1 \Rightarrow v = 0$ $\quad u = -1 \Rightarrow v' = 2\pi$

$u = 1 \Rightarrow v = 2\pi$ $\quad u = 1 \Rightarrow v' = 0$

$$P_r = 15 I_0^2 \left[\int_{v=0}^{2\pi} \frac{\cos^2\left(\frac{\pi}{2}\left(\frac{v}{\pi} - 1\right)\right)}{\frac{v}{\pi}} \frac{dv}{\pi} + \int_{v'=2\pi}^{0} \frac{\cos^2\left(\frac{\pi}{2}\left(\frac{v'}{\pi} + 1\right)\right)}{\frac{v'}{\pi}} \left(-\frac{dv'}{\pi}\right)\right]$$

$$= 15 I_0^2 \left[\int_{v=0}^{2\pi} \frac{\cos^2\left(\frac{v}{2} - \frac{\pi}{2}\right)}{v} \cdot dv + \int_{v'=0}^{2\pi} \frac{\cos^2\left(\frac{v'}{2} + \frac{\pi}{2}\right)}{v'} \cdot dv'\right]$$

$$\frac{v - \pi}{\pi} = \frac{\pi - v'}{\pi}$$

$$P_r = \frac{30 I_0^2}{2} \cdot 2 \int_0^{2\pi} \frac{\cos^2\left(\frac{v-\pi}{2}\right)}{v} \cdot dv$$

$$= \frac{60}{2} \cdot \left(\frac{I_0}{\sqrt{2}}\right)^2 \int_0^{2\pi} \frac{\cos^2\left(\frac{v-\pi}{2}\right)}{v} \cdot dv$$

$$= \frac{60}{2} I_{rms}^2 \int_0^{2\pi} \frac{1 + \cos(v - \pi)}{v} \, dv$$

$$= 30 I_{rms}^2 \int_0^{2\pi} \frac{1 - \cos v}{v} \, dv$$

$$= 30 I_{rms}^2 \int_0^{2\pi} \frac{1 - \cos v}{v} \, dv$$

$$= 30 I_{rms}^2 \cdot (2.44)$$

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$$P_r = 73\, I_{rms}^2$$

$$P_r = I_{rms}^2\, R_r$$

$$\cancel{I_{rms}^2}\, R_r = 73\, \cancel{I_{rms}^2}$$

$$\therefore R_r = 73\ \Omega$$

Q. Determine the magnitude of E & H of a $\lambda/2$ antenna operated at a freq. of 300MHz at a dist^n of 100m in the broad side plane for maximum radiation. Input current to the antenna is 100 with an angle 0 i.e., $100\angle 0^\circ$ mA. How much avg. power is radiated by antenna?

→ $$|E_\theta| = \frac{60\, j[I]}{r}\left[\frac{\cos(\pi/2 \cos\theta)}{\sin\theta}\right]$$

$\theta = 90^\circ$, $r = 100\,m$, $I_0 = 100\,mA$

$$|E_\theta| = \frac{60 \times 100\,m}{100}\left[\frac{\cos(\pi/2\, \cancelto{1}{\cos(\pi/2)})}{\sin \pi/2}\right]$$

$$E_\theta = 60 \times 10^{-3}$$

$$E_\theta = 60\ mV/m$$

$$Z_0 = \frac{E_\theta}{H_\phi}$$

$$H_\phi = \frac{60\,m}{120\pi}$$

$$H_\phi = 0.159\,mA/m$$

$$H_\phi = 159.1\ \mu A/m$$

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{100}{\sqrt{2}}$$

$$P_{av} = R_r (I_{rms})^2$$

WKT, $R_r = 73\,\Omega$

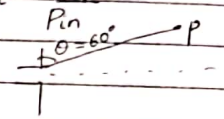
$$P_{av} = 73\left(\frac{100m}{\sqrt{2}}\right)^2 = 0.365\ W$$

Q. A half wave dipole is radiating into free space. The system is defined so that the origin is at the center of the dipole & z-axis is aligned with the dipole. I/p power to the dipole is 100W. Assuming an overall efficiency of 50%, find the power density at $r = 500m$, $\theta = 60^\circ$ & $\phi = 0$.

→ $$P_r = \eta P_{in} \qquad \because \eta = \frac{P_r}{P_{in}}$$

$$= 0.5 \times 100$$

$$P_r = 50\ W$$



WKT $$P_r = (I_{rms})^2 \cdot R_r$$

$$50 = \left(\frac{I_0}{\sqrt{2}}\right)^2 \cdot 75$$

$$I_0 = \sqrt{\frac{50 \times 2}{75}}$$

$$I_0 = 1.17\ A$$

$$|H_\phi| = \frac{I_0}{2\pi r}\left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right]$$

$$= \frac{1.17}{2\pi \times 500}\left[\frac{\cos(\pi/2\, \cos(60^\circ))}{\sin(60^\circ)}\right]$$

$$= 429.89\ \mu A/m$$

$$= 0.3038\ mA/V$$

$$S_r = \frac{1}{2}(120\pi)\,|H_\phi|^2$$

$$S_r = 17.89\ \mu V/m^2$$

Q. Find radiation rest^n & power radiated of length 0.1m carrying a current of $I = 10\cos 2\pi \times 10^9 t$

→ $I = I_0 \cos\omega t = I_0 \cos(2\pi f)t$

$\therefore I_0 = 10\,A$

$f = 10^9\ Hz$, $L = 0.1\,m$

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$$\frac{2L^2}{\lambda} = \frac{2L^2}{\frac{c}{f}} = \frac{2\times(0.1)^2\times10^9}{3\times10^8}$$

$$= 0.066 << \lambda$$

$$\lambda = \frac{c}{f} = 0.3$$

$$R_r = \frac{80\pi^2 L^2}{\lambda^2} = \frac{80\pi^2(0.1)^2}{0.3^2}$$

$$R_r = 87.73\ \Omega$$

$$P_r = I_{rms}^2 R_r$$

$$= \left(\frac{I_0}{\sqrt{2}}\right)^2 R_r$$

$$= \left(\frac{10}{\sqrt{2}}\right)^2 (87.73)$$

$$P_r = 4.386\ KW$$

Q. Find the length of the wire carrying a current of 6A peak at 1.5 MHz. The field strength measured at a distⁿ 20 Km in a direction at right angle to the axis of the wire is 0.5 mV/m. The wire is being situated in free space.

→ $r = 20$ km

$E_0 = 0.5$ mV/m , $\theta = 90°$

$I_0 = 6$ A

$f = 1.5$ MHz $\Rightarrow \lambda = \frac{c}{f} = \frac{3\times10^8}{1.5\times10^6} = 200$

$L = ?$

Assuming it is short dipole

$$|E_0| = \frac{60\pi I_0 L \sin\theta}{r\lambda}$$

$$0.5\times10^{-3} = \frac{60\pi\times6\times L\times\sin 90}{20\times10^3\times200}$$

$$\therefore L = 1.768\ m$$

Q. An isotropic radiator has a field strength given by $E = \frac{10I}{r}$ V/m where I is terminal current, r is distⁿ in m. Find radiation restⁿ.

→ $$P_r = \int_0^{2\pi}\int_0^{\pi} S_r \cdot dS$$

$$S_r = \frac{1}{2}\frac{E^2}{Z_0} = \left(\frac{10I}{r}\right)^2 \cdot \frac{1}{240\pi}$$

$$P_r = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \frac{1}{240\pi}\frac{100I^2}{r^2} r^2 \sin\theta\, d\theta\, d\phi$$

$$= \frac{1}{240\pi}\cdot 100I^2 \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \sin\theta\, d\theta\, d\phi$$

$$= \frac{2\pi\cdot100I^2}{240\pi}\int_{\theta=0}^{\pi}\sin\theta\, d\theta$$

$$= \frac{5}{6}I^2\left[-\cos\theta\right]_0^{\pi}$$

$$= -\frac{5}{6}I^2\left[-1-1\right]$$

$$= \frac{10}{6}I^2$$

$$P_r = 1.66\, I^2$$

$$P_r = I_{rms}^2 R_r$$

$$1.66\, I^2 = \frac{I^2}{2} R_r$$

$$\therefore R_r = 3.32\ \Omega$$

XQ. A $\lambda/2$ dipole in free space is radiating with a current of 1 A RMS at the ant

Q. A magnetic field strength of 5 μA/m is req. at a point at $\theta = \pi/2$ & 2 km away from an antenna in free space. Neglecting

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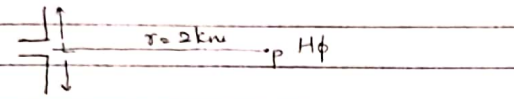
ohmic losses, how much power at antenna terminals if it is

a) Hertzian dipole of length $\lambda/25$

b) $\lambda/2$ dipole.

→



Given, $H\phi = 5\,\mu A/m$

a) $P_r = \iint_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} S_r \cdot ds$

$S = \frac{1}{2} E_\theta H_\phi$

$= \frac{1}{2} Z_0 H_\phi^2 =$

$\frac{E_\theta}{H_\phi} = Z_0$

H

$|H\phi| = \frac{I_0 L \sin\theta}{2\lambda r}$

$L = \frac{\lambda}{25}$

$5\times10^{-6} = \frac{I_0 \lambda/25 \sin 90}{2\lambda \times 2\times10^3}$

$I_0 = 0.5\ A$

for S.D, $R_r = \frac{80\pi^2 L^2}{\lambda^2} = \frac{80\pi^2 \times (\frac{\lambda}{25})^2}{\lambda^2}$

$R_r = 1.26\ \Omega$

$P_r = I_{rms}^2 R_r$

$P_r = \left(\frac{0.5}{\sqrt{2}}\right)^2 \times 1.26$

$P_r = 0.1575\ W$

$P_r = 157.5\ mW$

b) for $\lambda/2 = L$

$|H\phi| = \frac{I_0}{2\pi r}\left[\frac{\cos(\pi/2 \cos\theta)}{\sin\theta}\right]$

$H\phi = 5\times10^{-6} = \frac{I_0}{2\pi \times 2\times10^3}$

$I_0 = 0.06283\ A$

$R_r = \frac{80\pi^2 L^2}{\lambda^2} = \frac{80\pi^2 (\lambda/2)^2}{\lambda^2}$

for $\lambda/2$ antenna, $R_r = 73\ \Omega$

$P_r = I_{rms}^2 R_r = \left(\frac{0.06283}{\sqrt{2}}\right)^2 \times 73$

$P_r = 144.08\ mW$

Q. An electric field strength $10\,\mu V/m$ is to be measured at observation point $\theta = \pi/2$, 500 kms from a half wave dipole antenna operating in air at 50 MHz.

a) What is the length of an antenna?

b) Calculate current that must be fed to the antenna.

c) Find avg. power density radiated by antenna.

→ a) $\lambda = \frac{c}{f} = \frac{3\times10^8}{50\times10^6} = 6$

$L = \frac{\lambda}{2} = 3\ m$

b) $|E_\theta| = \frac{60[I]}{r}\left[\frac{\cos(\pi/2 \cos\theta)}{\sin\theta}\right]$

$10\times10^{-6} = \frac{60 I_0}{r}\left[\frac{\cos(\pi/2 \cos(90^\circ))}{\sin(90^\circ)}\right]$

$I_0 = \frac{10\times10^{-6} \times 500\times10^3}{60}$

$I_0 = 83.33\ mA$

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c) $P_r = (I_{rms})^2 R_r$

for $\lambda/2$ antenna, $R_r = 73\ \Omega$

$$\therefore P_r = \left(\frac{83.33\times10^{-3}}{\sqrt{2}}\right)^2 \times 73$$

$$P_r = 0.253\ W$$

Q. Calculate the dimensions of half wave dipole antenna operating at 100 MHz in free space. What is the radtn resst. Give the total power radiated if it is fed with current amplitude of 25 mA.

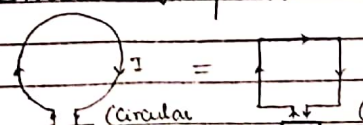
Q An antenna located on surface of flat earth transmits an avg. power of 200 KW. Assuming that all the power is radiated uniformly over the surface of hemisphere with the antenna at the center. Calculate

a) The time average poynting vector at 50 km.
b) The max. electric field at the location.

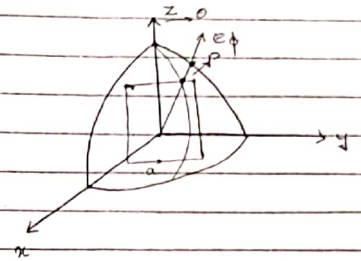
Q. A $\lambda/2$ wave dipole radiator is elevated 100 ft. above the ground. A receiving dipole 3m distn. is elevated at 30 ft. Determine the space wave field strength & surface wave field strength at the receiving antenna when the transmitting antenna carries a current of 5A

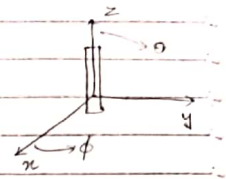
Loop Antenna → produces E_ϕ & H_θ components (because of orientation of dipoles)

1. Small Loop Antenna $a << \lambda$

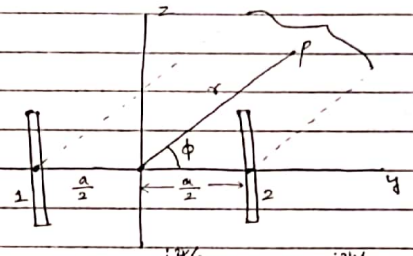


$A_{(circular\ loop)} = A_{(square\ loop)}$





dipole produces E_θ & H_ϕ components



$$E_\phi = -E_{\phi_0} e^{j\psi/2} + E_{\phi_0} e^{-j\psi/2}$$

$\psi = ar\sin\theta$

$\psi = dr\cos\phi$

$\cos\phi = \sin\theta$

$\Rightarrow \psi = ar\sin\theta$

$\psi = \frac{2\pi a}{\lambda}\sin\theta$

E_{ϕ_0} → E_θ component of short dipole

$$E_\phi = E_{\phi_0}\left[e^{-j\psi/2} - e^{j\psi/2}\right]$$

$$E_\phi = -2j E_{\phi_0} \sin\psi/2$$

normalizing

$$E_\phi = -2j E_{\phi_0} \sin\left(\frac{ar}{2}\sin\theta\right)$$

$$E_{\phi_0} = \frac{j\cdot 60\pi [I] a}{r\lambda}$$

$$\Rightarrow E_\phi = -j E_{\phi_0} \sin\theta \cdot ar$$

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$$E_\phi = \frac{-60\pi j^2 [I] a \sin\theta \, a_r}{r\lambda}$$

$$= \frac{-60\pi j^2 [I] a \left(\frac{2\pi a}{\lambda}\right) \sin\theta}{r\lambda} \qquad a = a_r, \quad a_r = \frac{2\pi a}{\lambda}$$

$$= \frac{60\pi [I] a^2 (2\pi) \sin\theta}{r\lambda^2}$$

$$= \frac{120\pi^2 [I] a^2 \sin\theta}{r\lambda^2}$$

area of square

$$\boxed{E_\phi = \frac{120\pi^2 [I] \sin\theta}{r} \cdot \frac{A}{\lambda^2}} \qquad A = a^2$$

$$\frac{E_\phi}{H_\theta} = Z_0$$

$$\Rightarrow H_\theta = \frac{E_\phi}{Z_0} = \frac{E_\phi}{120\pi}$$

$$\therefore H_\theta = \frac{\frac{120\pi^2 [I] \sin\theta}{r} \cdot \frac{A}{\lambda^2}}{120\pi}$$

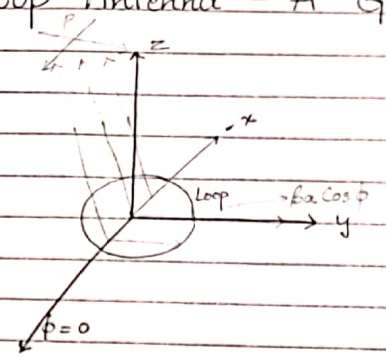
$$H_\theta = \frac{\pi [I] \sin\theta}{r} \cdot \frac{A}{\lambda^2}$$

→ Components of far field of Short dipole & Small Loop

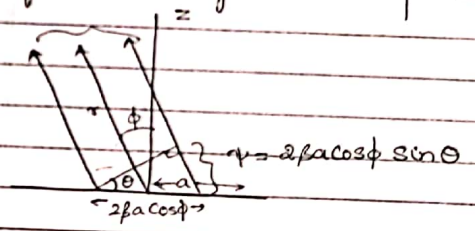
Field	Short dipole	Small loop
Electric	$E_\theta = \frac{j60\pi [I] \sin\theta}{r} \frac{L}{\lambda}$	$E_\phi = \frac{120\pi^2 [I] \sin\theta}{r} \frac{A}{\lambda^2}$
Magnetic	$H_\phi = \frac{j[I] \sin\theta}{2r} \times \frac{L}{\lambda}$	$H_\theta = \frac{\pi [I] \sin\theta}{r} \cdot \frac{A}{\lambda^2}$

The presence of the operator j in the dipole expressions & its absence in the loop expressions indicates that the fields of the electric dipole & of the loop are in time phase quadrature, the current I being in the same phase in both dipole & loop antennas.

⇒ Loop Antenna – A General Case



* In general case, size of the loop ~~antenna~~ is not restricted to a small value compared to wave length
* Assuming that uniform & in-phase current flows through the loop antenna



$$dA_\phi = \frac{\mu dM}{4\pi r} \Rightarrow A_\phi = \int \frac{\mu dM}{4\pi r} \quad \text{—— (1)}$$

dist. from loop to dest. pt.

dM is current movement due to one pair of diagonally infinitesimal dipole of length $a d\phi$

$$[I] a \, d\phi \cos\phi$$

where $[I] = I_0 e^{j\omega(t - r/c)}$

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$$dM = 2j[I]a\, d\phi \cos\phi \sin\psi/2$$

where $\psi = 2\beta a \cos\phi \sin\theta$

$$E_\phi = \frac{60\pi\beta[I]a}{r} J_1(\beta a \sin\theta)$$

$$H_\theta = \frac{\pi\beta[I]a}{2r} J_1(\beta a \sin\theta)$$

→ Far field patterns of circular loop Antenna with uniform current

$$\left.\begin{aligned} E_\phi &= \frac{60\pi\beta[I]a}{r} J_1(\beta a \sin\theta) \\ H_\theta &= \frac{\pi\beta[I]a}{2r} J_1(\beta a \sin\theta) \end{aligned}\right\} \text{General case}$$

for a loop of given size,

$$J_1(c\lambda \sin\theta)$$

For small loop antenna

$$E_\phi = \frac{60\pi\beta a[I]\beta a \sin\theta}{2r}$$

→ Radiation Resist^n of loop Antenna

Q. Calculate the radt^n resist^n of small loop antenna of radius $\lambda/25$ cm & having 8 turns. Assume medium to be free space

→ $$R_r = 31200\left(\frac{nA}{\lambda^2}\right)^2$$

$n = 8$

$a = \frac{\lambda}{25} = r$

$A = \pi a^2 = \frac{\pi\lambda^2}{25^2}$

$$\therefore R_r = 31200\left(\frac{8\times\frac{\pi\lambda^2}{25^2}}{\lambda^2}\right)^2 = \underline{\underline{50.45\ \Omega}}$$

Q. The radius of a circular loop antenna is 0.02λ. How many turns of an antenna will give radt^n resist^n of 35Ω

→ $R_r = 35\Omega$

$a = 0.02\lambda$

$A = \pi a^2 = \pi(0.02\lambda)^2 = 1.25\times10^{-3}\lambda^2$

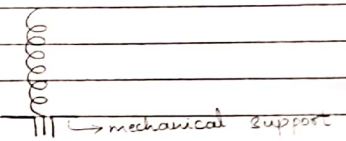
$$R_r = 31200\left(\frac{nA}{\lambda^2}\right)^2$$

$$\sqrt{\frac{35}{31200}} = \frac{n\times 0.4\times10^{-3}\pi\lambda^2}{\lambda^2}$$

$n = 26.65 \approx 27$ turns

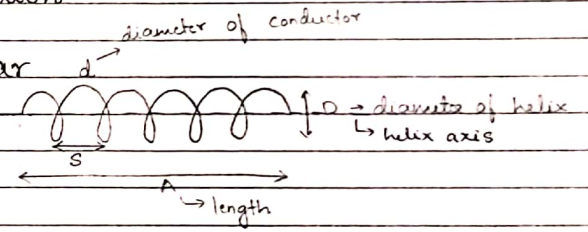
⇒ Helical Antenna < mono-filar / multi (array of helical antennas)

produces circularly polarized waveform



→ Construction

Monopilar

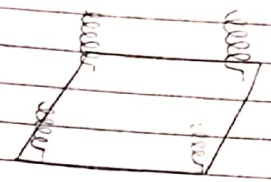


L – length of 1 turn

A – ~~Kelgl~~ Axial length

S – Spacing b/w 2 turns

Multifilar



→ Different modes of helical antennas

1. Normal Mode – $C < \lambda$, $\alpha << \lambda$

circumference of helix → pitch angle

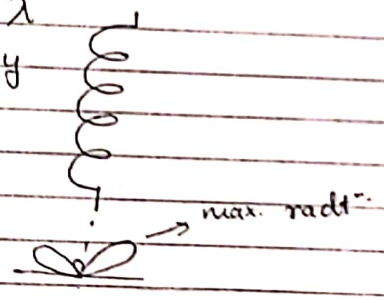
* produces max. radtⁿ ⊥ⁿ to helical axis

2. Axial Mode – $C \simeq \lambda$ (comparable)

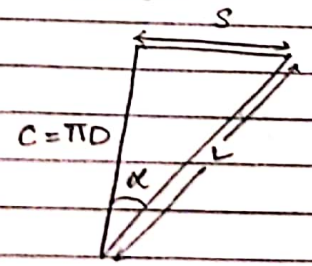
* produces circularly polarized waves

3. Conical Mode – $C > \lambda$

max. radtⁿ in arbitrary directⁿ



pitch angle



pitch angle, $\alpha = \tan^{-1}\left[\frac{S}{\pi D}\right]$

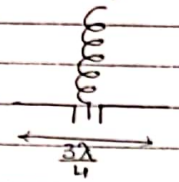
* if $\alpha = 0$, helical antenna becomes loop antenna

* if $\alpha = 90°$, helical antenna becomes antenna.

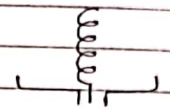
* Axial mode is widely used since the bw width is more & the directivity can be increased by increasing the no. of turns.
* It also produces circularly polarized wave.

→ Different Ground Planes of Monofilar helical antenna (avoid ~~backward radtⁿ~~)

1. Flat gnd plane



2. Cup-Shaped gnd plane



3. Deep Conical

Reduces backward radtⁿ & sidelobes.

→ Design Considerations of Axial Mode H.A.

1. Half Power Beam width $= \frac{52}{C\lambda\sqrt{nS\lambda}}$ (degrees)

$= \frac{52}{C}\sqrt{\frac{\lambda^3}{nS}}$

$0.8 < C\lambda < 1.15$

$12° < \alpha < 14°$

$n > 3$

no. of turns ∝ D

2. BWFN $= \frac{115}{C\lambda\sqrt{nS\lambda}} = \frac{115}{C}\sqrt{\frac{\lambda^2}{nS}}$

3. $D = 15C^2\lambda nS\lambda = \frac{15nSC^2}{\lambda^3}$

4. Axial ratio $= \frac{2n+1}{2n}$

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→ Applications of Helical Antenna.

* HA can be used as feeding antenna for the parabolic reflector.
* HAs are widely used in Satellite Commⁿ.
* Used as ground station antennas for TV transmⁿ & telephone applⁿ.
* The mutual impedance b/w HAs is very small. Hence, they can be used as an array.

Q. A helical antenna has 100 turns, 0.1 m diameter & 70 mm turn spacing. The operating freqⁿ is 1 GHz. Find the Directivity & Polarization state.

→ $D = \frac{15 n S C^2}{\lambda^3}$

$n = 100$

$D = 0.1\,m \Rightarrow C = \pi D = 0.1\pi$

$S = 70\,mm = 70 \times 10^{-3}\,m$

$f = 1\,GHz \Rightarrow \lambda = \frac{c}{f} = \frac{3\times10^8}{1\times10^9} = 0.3$

Circumference, $C = 0.1\,m$

i) $\therefore D = \frac{15 \times 100 \times 70\times10^{-3} \times (0.1\pi)^2}{0.3^3}$

? $D = 383.8$

ii) $D = 0.1\,m$

$\pi D \approx \lambda$

$0.314\,m \approx 0.3\,m$ → Circularly polarized

Q. A 16 turn helical antenna has a circumference of λ & turn spacing is $\lambda/4$. Find HPBW, Axial ratio & the power pattern.

→ $HPBW = \frac{52}{C}\sqrt{\frac{\lambda^3}{nS}}$

$n = 16,\ C = \lambda,\ S = \lambda/4$

$\therefore HPBW = \frac{52}{\lambda}\sqrt{\frac{\lambda^{32}}{16 \cdot \lambda/4}}$

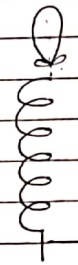
$= \frac{52}{\lambda} \cdot \frac{\lambda}{2}$

$HPBW = 26\ dB$

Axial ratio $= \frac{2n+1}{2n} = \frac{2(16)+1}{2(16)} = 1.0313$

power pattern

$\because C = \lambda$ → Axial mode



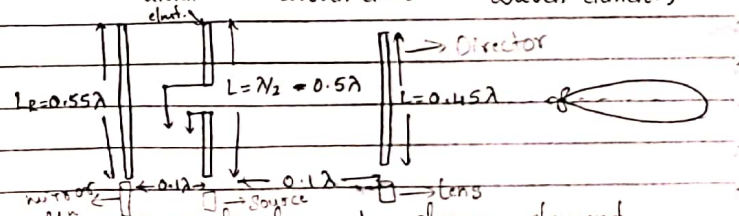
⇒ Yagi-Uda Antenna [Beam antenna]

↳ produces high beam radⁿ pattern

Driven element – dipole (or) folded dipole.

Parasitic element – Reflector, Director.

(length > driven elmt) (length is less than driven element)



beam radⁿ → perpendicular to driven element.

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* Collinear arrays
* Used in TV reception, direct" finder

* Length of ~~Director~~ ~~Reflector~~ Driven element $= \frac{475}{f(MHz)}$ (feet)

" Director " $= \frac{455}{f(MHz)}$ (ft)

" Reflector " $= \frac{500}{f(MHz)}$ (ft

* Dist" b/w the elements is 0.1λ

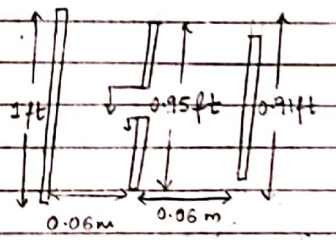
Q Design 3-element Yagi-Uda antenna for freq" of 500 MHz

→ Length of Driven element $= \frac{475}{f} = \frac{475}{500} = 0.95$ ft

" Director " $= \frac{455}{f} = \frac{455}{500} = 0.91$ ft

" Reflector " $= \frac{500}{f} = \frac{500}{500} = 1$ ft

Spacing $= 0.1\lambda = 0.1 \times \frac{3\times10^8}{500\times10^6} = 0.06$ m



Unit 4

Antenna Types

1. Horn Antenna (Aperture type antenna)
   - derived from waveguides
     1. Rectangular w/g
     2. Circular w/g

* Horn antenna is an Aperture type antenna because of its large mouth area

Design Equations of Patch antenna

$$W = \frac{c}{2f_r}\sqrt{\frac{2}{\varepsilon_r+1}}, \quad \varepsilon_{reff} = \frac{\varepsilon_r+1}{2} + \frac{\varepsilon_r-1}{2}\left(1+\frac{12h}{W}\right)^{-1/2}$$

resonant freq"

$$\Delta L = 0.412h\frac{(\varepsilon_{reff}+0.3)}{(\varepsilon_{reff}-0.258)}\frac{\left(\frac{W}{h}+0.264\right)}{\left(\frac{W}{h}+0.8\right)}$$

$$L = \frac{\lambda}{2} - 2\Delta L$$

Q. Design a rectangular microstrip antenna for the following specifications. The substrate used is (RT/Duroid 6010.2) substrate, $\varepsilon_r = 10.2$, substrate thickness is 0.127 cm (h), $f_r = 2$ GHz.

→ $$W = \frac{c}{2f_r}\sqrt{\frac{2}{\varepsilon_r+1}} = \frac{3\times10^8}{2\times2\times10^9}\sqrt{\frac{2}{10.2+1}}$$

$W = 0.03169$ m

$W = 3.169$ cm

$$\varepsilon_{reff} = \frac{\varepsilon_r+1}{2} = \frac{10.2+1}{2} = 5.6$$

$$\varepsilon_{reff} = \frac{\varepsilon_r+1}{2} + \frac{\varepsilon_r-1}{2}\left(1+\frac{12h}{W}\right)^{-1/2} = 5.6+4.6\left(1+\frac{12(0.127)}{3.169}\right)^{-1/2}$$

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$$\Delta L = 0.412h \frac{(\varepsilon_{reff}+0.3)}{(\varepsilon_{reff}-0.258)} \frac{\left(\frac{w}{h}+0.264\right)}{\left(\frac{w}{h}+0.8\right)}$$

$$= 0.412(0.127)\frac{(9.38+0.3)}{(9.38-0.258)}\left(\frac{24.95+0.264}{24.95+0.8}\right)$$

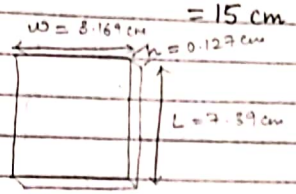
$$\Delta L = 0.0544 \text{ cm}$$

$$L = \frac{\lambda}{2} - 2\Delta L$$

$$L = \frac{15}{2} - 2(0.0544)$$

$$\lambda = \frac{c}{f} = \frac{3\times10^{10}}{2\times10^{9}} = 15 \text{ cm}$$

$$L = 7.39 \text{ cm}$$



⇒ Long Periodic Antenna $\alpha = 30°$, $\tau = 0.7$
(freq. independent / Broad band antenna)

Q Design a 54 - 216 MHz Log Periodic Dipole Antenna for a desired gain of 6.5 dB assuming 1 dB loss, $\tau = 0.822$, $\sigma = 0.149$

→ $$\alpha = 2\tan^{-1}\left[\frac{1-\tau}{4\sigma}\right]$$

$$\alpha = 2\tan^{-1}\left[\frac{1-0.822}{4(0.149)}\right]$$

$$\alpha = 33.25°$$

$f_{min} = 54$ MHz

$$\lambda = \frac{c}{f} = \frac{3\times10^{8}}{54\times10^{6}} = 5.55 \text{ m}$$

$$L_{max} = L_1 = 0.5\lambda$$

$$= 0.5\times5.55$$

$$L_1 = 2.78 \text{ m}$$

length of the longest dipole

length of the shortest dipole

$f_{max} = 216$ MHz

$$\lambda = \frac{c}{f} = \frac{3\times10^{8}}{216\times10^{6}} = 1.38 \text{ m}$$

$$L_{min} = 0.5\times\lambda = 0.5\times1.38$$

$$L_{shortest\ dipole} = 0.694 \text{ m}$$

The lengths of other elements are calculated by scaling the longest dipole (2.78 m) until the smallest dipole (0.694 m) is obtained.

$$\tau = \frac{L_{n+1}}{L_n} = \frac{R_{n+1}}{R_n} = \frac{S_{n+1}}{S_n}$$

$$0.822 = \frac{L_2}{L_1}$$

$L_2 = 0.822\times2.78 = 2.285$ m

$L_3 = 0.822\times2.285 = 1.878$ m

$L_4 = 0.822\times1.878 = 1.543$ m

$L_5 = 0.822\times1.543 = 1.268$ m

$L_6 = 0.822\times1.268 = 1.042$ m

$L_7 = 0.822\times1.268 = 0.857$ m

$L_8 = 0.822\times0.857 = 0.704$ m

$L_9 = 0.822\times0.857 = 0.579$ m

∴ The array consists of 9 dipoles

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Q. Consider a log periodic dipole array with a design factor (τ) of 0.7. 10 dipoles are used in the structure, the smallest dipole having dim. $(\frac{L}{2}) \leftarrow 0.3$ m. Determine

a) Resonant freq^{n} of the smallest dipole.

b) " largest dipole.

c) length of the largest dipole.

d) Passband of the array ($B.W = f_H - f_L$)

$\rightarrow$ $\tau = 0.7$

a) $L_{shortest\ dipole} = 0.3\ m = L_{min}$

$$L_1 = \frac{\lambda_1}{2} = \frac{c}{2f_1}$$

$$L_{min} = 0.5\lambda$$

$$L = (2 \times 0.3) = \frac{0.5 \times c}{f_{max}}$$

$$f_{max} = \frac{0.5 \times 3 \times 10^8}{0.3 \times 2} = 0.5\ GHz = 500\ MHz$$

$$f_{max} = 250\ MHz$$

b) $\cancel{\tau = \frac{L_{n+1}}{L_n}}$

$\cancel{0.7 = \frac{L_{n+1}}{L_{min}}}$

$\cancel{L_{n+1} = 0.7 \times 0.3 = 0.21}$

$\tau = 0.7$

Resonant freq^{n} of largest dipole $= f_1(\tau)^{n-1}$

$$= 250(0.7)^9$$

$$f_{min} = 10.08\ MHz$$

c) $L_1 = L_{max} = 0.5\lambda = \frac{0.5 \times 3 \times 10^8}{10.08 \times 10^6}$

$$L_1 = 14.88\ m$$

d) Passband $= 250\ M - 10.08\ M$

$$= 239.92\ MHz$$

Q. Design resonant microstrip antenna for WLAN appl^{n} (the freq^{n} used is 2.5 GHz – 2.58 GHz). Choose substrate $\varepsilon_r = 2.32$ & $h = 0.176$ cm & $\tan\delta = 0.001$.

$\rightarrow$ $$W = \frac{c}{2f_r}\sqrt{\frac{2}{\varepsilon_r+1}} = \frac{3 \times 10^8}{2 \times 2.5 \times 10^9}\sqrt{\frac{2}{2.32+1}}$$

$$= 0.0465\ m$$

$$W = 4.65\ cm$$

$$\varepsilon_{reff} = \frac{\varepsilon_r+1}{2} + \frac{\varepsilon_r-1}{2}\left(1 + \frac{12h}{W}\right)^{-1/2}$$

$$= 1.66 + 0.66\left(1 + \frac{12(0.176)}{4.65}\right)^{-1/2}$$

$$\varepsilon_{reff} = 2.207$$

$$L_{eff} = \frac{c}{2f_0\sqrt{\varepsilon_{reff}}} = \frac{3 \times 10^8}{2 \times 2.5 \times 10^9\sqrt{2.207}}$$

$$L_{eff} = 4.04\ cm$$

$$L = L_{eff} - 2\Delta L$$

$\tan\delta = 0.001$

$\delta = 0.05729$

$$\Delta L = 0.0914$$

$$L = 4.04 - 2(0.094)$$

$$L = 3.857\ cm$$

Reflector Type Antennas (Aperture type)

Reflectors are used to change the radt^{n} from particular antennas.

Types of Reflectors

1. Flat Sheet
2. Corner reflector
3. Paraboloid

- Flat Sheet

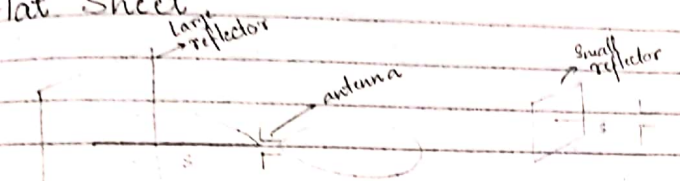


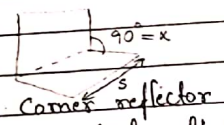
fig.(a) fig.(b)

* Reflector is a perfect conductor. By changing the shape of the reflector, the radt^n pattern can be changed.
* The dist^n b/w the antenna & reflector decides radt^n pattern
* Radt^n towards the reflector gets reflected towards the antenna & makes uni-direct^n radt^n pattern

* fig.(a) - is insensitive to freq^n changes & small reflector is very sensitive to freq^n changes, but the gain of both are almost same
* The reflector can be made to a thin wire but it is sensitive to the freq^n. So size & shape of the reflector decides radt^n pattern

for Flat Sheet $\alpha = 180°$

2. $\alpha = 180°, 90°, 60°$

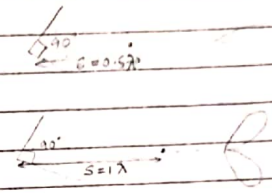


* If $\alpha = 90°$, such type are called flat sheet reflectors
* If $\alpha < 180°$ — Corner reflector

$\alpha = 90°$ - square reflector

If $\alpha < 90°$, gain ↑es as α ↓es but practical limitations in implementation.

* Corner reflector can be used if aperture dimension is less than 2λ.
* If aperture dimension $> 2\lambda$ - parabolic/hyperbolic reflectors are used.
* Shape of the pattern depends on the dist^n S
If $S = 0.5$ - unidirectional radt^n pattern (no side lobes)



bidirect^n pattern

$S = 1.5\lambda$ → unidirect^n pattern with S.L

* By changing the spacing S, the radt^n pattern can be changed.
* Gain is max. for $\alpha = 60°$ - practical limit^n
" min " $\alpha = 180°$ (flat sheet)
* To avoid the wind rest^n grids are instead of solid metallic sheets.

Appl^n of Corner Reflectors
* Used in point-to-point comm^n
* Used in TV receivers.

3. Parabolic reflectors
* convert spherical wavefront to plane wavefront. This process is called Collimation. Its reverse process is called focusing.
* The advantages of plane wavefront -
* phases of all the fields are same.

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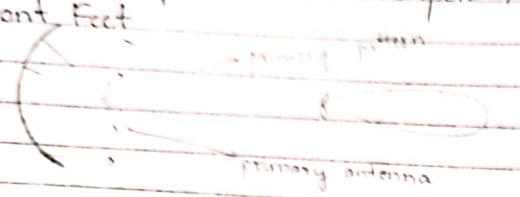
Parabolic Reflector with Dipole Antenna.

(i) Front Feed



Disadvantages of Parabolic reflector

* If the primary antenna is kept at focus, the reflected signals are blocked in that region ∵ the primary antenna acts as an obstacle, thus reduces the aperture size.
* Standing waves are generated due to direct signals & reflected signals. Thus, there maybe impedance mismatch & interference

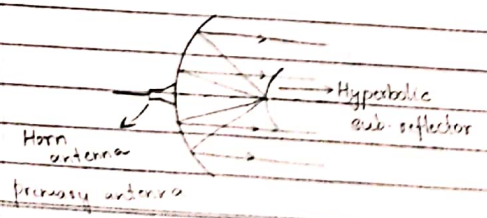
(ii) Offset Feed

* No standing waves are formed, ∴ no impedance mismatch.

Disadvt^n

* Only a portion of the parabola is used & aperture size reduces.

(iii) Cassegrate feed



* Primary antenna is kept at the vertex, sub-reflector is kept at focus.
* Horn antenna radiates in the horizontal direct^n & sub-reflector reflects it.
* To reduce the sub-reflector size, the focal length should be reduced but it may not illuminate the parabolic reflector throughout its length. Thus it has to be kept at longer dist^n.

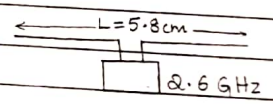
Advt^n

* Standing waves can be reduced, ∴ impedance mismatch is reduced.

Appl^n

* Used for Microwave freq^n
* Point-to-Point Commu^n, TV rcr. station.

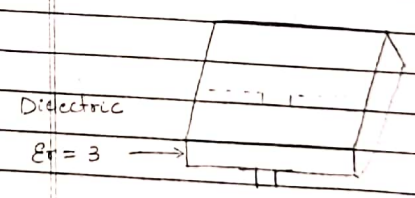
⇒ Embedded Antenna



$f = 2.6\,GHz$

$\lambda = 11.56\,cm$

$L = \frac{\lambda}{2} = 5.7\,cm$



$L = \frac{\lambda}{2\sqrt{\varepsilon_r}}$

$\varepsilon_r \uparrow$, physical size $\downarrow$

⇒ Parabolic Antenna

→ For circular aperture antenna

$BWFN = \frac{140\lambda}{d}$

$HPBW = \frac{70\lambda}{d}$

= 239.92 MHz

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Directivity over isotropic antenna

$$D = 9.87 d_\lambda^2$$

diameter expressed in wavelength

Gain over $\lambda/2$ dipole

$$G = 6 d_\lambda^2$$

→ Rectangular aperture parabalic antenna

$$BWFN = \frac{115\lambda}{d}$$

d → length of aperture

$$HPBW = \frac{58\lambda}{d}$$

Directivity over isotropic antenna

$$D = 12.6 L_\lambda^2$$

Gain over $\lambda/2$ antenna

$$G = 7.7 L_\lambda^2$$

Q. Calculate the BWFN, HPBW & Gain of 2m diameter paraboloidal reflector operating at 6GHz → {Circular aperture antenna}

→ $d = 2m$, $f = 6\,GHz$

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{6\times10^9} = 0.05$$

$$BWFN = \frac{140\lambda}{d} = \frac{140\times0.05}{2} = 3.5^\circ$$

$$HPBW = \frac{70\lambda}{d} = \frac{70\times0.05}{2} = 1.75^\circ$$

$$G = 6d_\lambda^2 = 6(40)^2 = 9600 = 39.82\,dB$$

$$d_\lambda = \frac{d}{\lambda} = \frac{2}{0.05} = 40$$

$$D = 9.87 d_\lambda^2 = 15792 = 41.98\,dB$$

Q. If the length of the rectangular aperture of a paraboloidal reflector is 10λ, Calculate BWFN, HPBW, D & G.

→ $L = 10\lambda$

$$BWFN = \frac{115\lambda}{L} = \frac{115\lambda}{10\lambda} = 11.5^\circ$$

$$HPBW = \frac{58\lambda}{L} = \frac{58\lambda}{10\lambda} = 5.8^\circ$$

$$D = 12.6 L_\lambda^2 = 12.6(10)^2 = 1260$$

$$L_\lambda = \frac{L}{\lambda} = \frac{10\lambda}{\lambda}$$

$$G = 7.7 L_\lambda^2 = 770 = 28.86\,dB$$

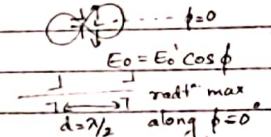
⇒ Pattern Multiplication

Q. 2 identical point sources separated by a distance d each source having the field pattern given by $E_0 = E_0' \cos\phi$, if the dist^n b/w the sources is $\lambda/2$ & the phase angle is 0, find the total field pattern & also determine BWFN & Directivity

→ $E_0 = E_0' \cos\phi$, $d = \lambda/2$, $\delta = 0$

$$\psi = d_r \cos\phi + \delta$$

$$d_r = \frac{2\pi d}{\lambda} = \frac{2\pi\times\lambda/2}{\lambda} = \pi$$



$$\psi = \pi\cos\phi +$$

Array pattern

$$E = E_0 \cos\psi/2 = E_0 \cos(\pi/2 \cos\phi)$$

Total pattern

$$E = \cos\phi \cdot \{\cos(\pi/2 \cos\phi)\}$$

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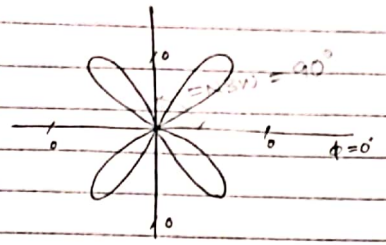


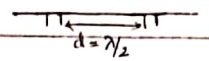
$FNBW = 90^\circ$

$HPBW = \frac{FNBW}{2} = 45^\circ$

$D = \frac{41200}{\theta_{Hp} \times \phi_{Hp}} = \frac{41200}{45 \times 45}$

$D = 20.37$

(ii) if dipoles are placed parallel to A.A ?

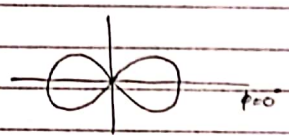


(iii) if out of phase ?

Array pattern

$E = \sin \psi/2$

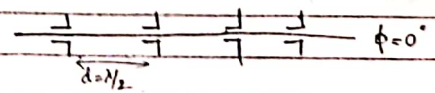
$= \sin(\pi/2 \cos\phi)$



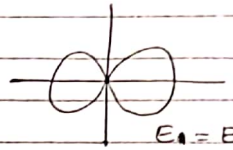
Total pattern

$E = \cos\phi \{\sin(\pi/2 \cos\phi)\}$

Q. 4 vertical radiators ($\perp^r$ to A.A) spaced $\lambda/2$ dist. apart are placed along a straight line. The power is fed with equal magnitude & in-phase. Obtain the field pattern & find the BWFN & directivity.



→ Primary Pattern



$E_1 = E_0' \cos\phi$

equal mag. & in-phase → BSA

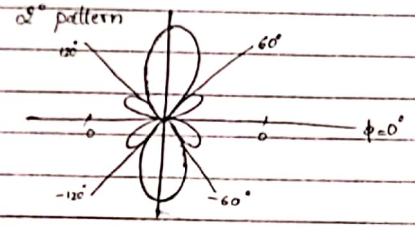
$E_2 = E_0 \times \frac{\sin n\psi/2}{\sin \psi/2}$ $\qquad \psi = dr\cos\phi$

$= \frac{E_0 \sin 4(\frac{\pi}{2}\cos\phi)}{\sin(\frac{\pi}{2}\cos\phi)}$

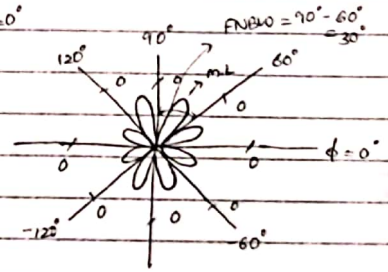
$E_2 = \frac{E_0 \sin(2\pi\cos\phi)}{\sin(\frac{\pi}{2}\cos\phi)}$

$E = E_1 \times E_2$ → pattern multipl^n.

$= \cos\phi \cdot \frac{\sin(2\pi\cos\phi)}{\sin(\frac{\pi}{2}\cos\phi)}$



Total pattern = 1° × 2°



$FNBW = 30^\circ$

$HPBW = 15^\circ$

$D = \frac{41200}{15 \times 15} = 183.11$

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UNIT - 2

Antenna Arrays

Arrays: No. of cascaded antennas.

1D Array

2D Array - spherical

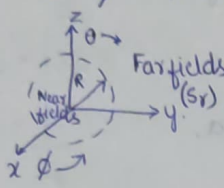
3D Array

Point sources:

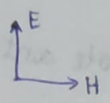
source - antenna.

The Point source is a antenna whose dimension is a point. (Dimensionless antenna).

S_r, S_θ, S_ϕ - Power density components.



$R = \frac{2L^2}{\lambda}$



When the point of observation is far away from antenna then the radiation pattern is independent of distance. (so the signal strength is good).

Short dipole (Hertzian antenna); $l = \frac{\lambda}{4}$.

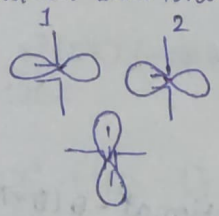
It is a point source.

Point sources are of two types:

i) similar point sources.

ii) Identical

Consider two short dipole antennas



shapes of the radiation pattern is same but ~~str~~ peaks are different - similar.

Shapes of radiation pattern and also peaks are same - Identical point source.

Consider array of point sources

Resultant pattern depends on

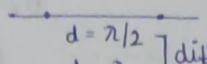
i) Number of point sources in the array.

As no. of point source increase, Directivity increases.

ii) Shape of the array

iii) The individual pattern of each antenna.

iv) The distance between point sources.



$d = \lambda/2$, $d = \lambda$] different effect

v) Power magnitude of each antenna

vi) Phase difference with respect to power field.

vii) Array Dimension.

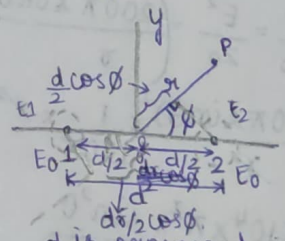
Array of Two point sources.

Isotropic - Uniform radiation in all directions. (dimension is very small).

Array of Two isotropic point sources

Case 1: Array of Two isotropic point sources are having equal amplitude and inphase (rotating (θ, ϕ) in same direction)

Phase difference, $\delta = 0$.



x = Array axis

$\phi = 0°$

d is expressed in radians.

$d_r = \beta d = \frac{2\pi d}{\lambda}$

d_r - distance between p.s in rad.

P - observation point.

Since Point P is near to P.S 2 so the radiation pattern is advanced and that of P.S 1 is delayed.

$E = E_1 + E_2$

$E = E_0 e^{-\frac{j\psi}{2}} + E_0 e^{+\frac{j\psi}{2}}$

$\psi = d_r \cos\phi + \delta \rightarrow$ Path difference (ψ)

$\psi = d_r \cos\phi \quad (\delta = 0)$

$E = E_0 \left(e^{-\frac{j\psi}{2}} + e^{+\frac{j\psi}{2}} \right)$

$E = \frac{2E_0}{2} \left(\frac{e^{\frac{j\psi}{2}} + e^{-\frac{j\psi}{2}}}{2} \right)$

$E = 2E_0 \cos \psi/2$

$E = 2E_0 \cos\left[\frac{d_r}{2}\cos\phi\right]$ $\quad \psi = d_r \cos\phi \rightarrow$ Angle measured from array axis

Angle

$$E_n = \frac{E}{E_{max}} = \frac{2E_0 \cos\left[\frac{d_r}{2}\cos\phi\right]}{2E_0} = \cos\left[\frac{d_r}{2}\cos\phi\right]$$

$E = E_{max}$, $\cos\frac{\psi}{2} = 1$.

	$\sin(x)$	$\cos(x)$
Maximum	$\pm(2K+1)\frac{\pi}{2}$	$\pm K\pi$
Nulls	$\pm(K\pi)$	$\pm(2K+1)\frac{\pi}{2}$
HP (1/2 dy)	$\pm(2K+1)\frac{\pi}{4}$	$\pm(2K+1)\frac{\pi}{4}$

The complete radiation pattern of two array isotropic point sources having equal amplitude & inphase. If the distance between isotropic point source is $\lambda/2$.

$$E_n = \cos\left(\frac{d_r}{2}\cos\phi\right) \quad d = \frac{\lambda}{2} \quad d_r = \frac{2\pi d}{\lambda} = \frac{2\pi\lambda/2}{\lambda} = \pi$$

$$E_n = \cos\left(\frac{\pi}{2}\cos\phi\right)$$

Direction of maximum

$$\cos\left(\frac{\pi}{2}\cos\phi_m\right) = 1.$$

$$\frac{\pi}{2}\cos\phi_m = 0$$

$$\cos\phi_m = 0$$

$$\boxed{\phi_m = \pm\frac{\pi}{2}}$$

Direction of nulls.

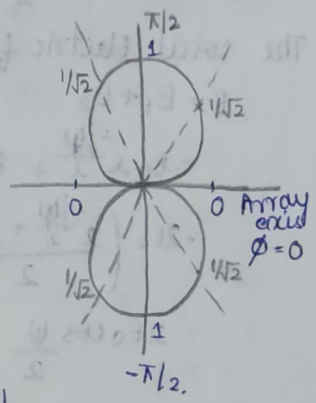
$$\cos\left(\frac{\pi}{2}\cos\phi_0\right) = 0.$$

$$\frac{\pi}{2}\cos\phi_0 = \frac{\pi}{2}$$

$$\cos\phi_0 = \frac{2}{\pi}\times\frac{\pi}{2}$$

$$\phi_0 = \cancel{\cos^{-1}\left(\frac{2}{\pi}\right)}$$

$$\pm\pi$$



Half Power Angle.

$$\Omega_A = \theta_{HP} \times \phi_{HP}.$$

$$= \frac{\pm\pi}{3} \times \frac{\pm\pi}{3}$$

$$= \frac{\pm\pi^2}{9}$$

$$\cos\left(\frac{\pi}{2}\cos\phi_{HP}\right) = \frac{1}{\sqrt{2}}$$

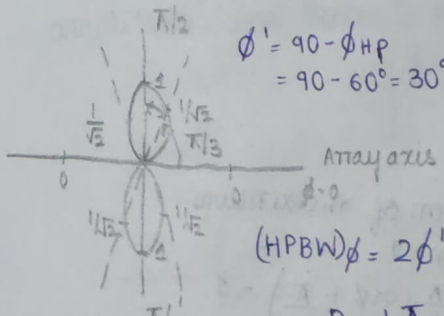
$$\frac{\pi}{2}\cos\phi_{HP} = \frac{\pi}{4}$$

$$\cos\phi_{HP} = \frac{\pi}{4}\times\frac{2}{\pi}$$

$$\phi_{HP} = \cos^{-1}\left(\frac{1}{2}\right)$$

$$\boxed{\phi_{HP} = \pm\frac{\pi}{3}} \quad \boxed{\pm\frac{2\pi}{3}}$$

$$\theta_{HP} = \phi_{HP}$$



$$\phi' = 90 - \phi_{HP}$$
$$= 90 - 60^\circ = 30^\circ = \frac{\pi}{6}$$

$$(HPBW)_\phi = 2\phi' = 2\times\frac{\pi}{6} = \frac{\pi}{3} = 60^\circ$$

$$D = \frac{4\pi}{\Omega_A} = \frac{4\pi 200}{60^\circ_A} = \frac{41200}{60\times 60} = 11.444 \simeq 11.45$$

symmetrical in both the plane

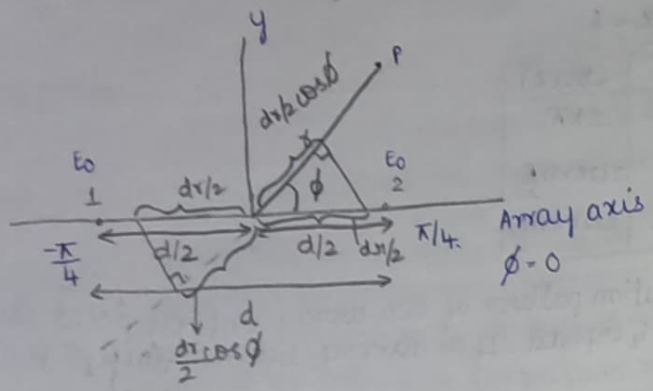
So Directivity is increased due to series of antennas.

Since the radiation is maximum ⊥lar to array axis. This is called Broad side array ϕ_m is ⊥lar to array axis.

Case 2: Equal amplitude & Inphase quadrature

Inphase: Phase difference is zero.

Inphase Quadrature: phase difference is $\pi/2$.



$$\psi = dr\cos\phi + \delta$$

$$\delta = \frac{\pi}{2}$$

$$\psi = dr\cos\phi + \frac{\pi}{2}$$

The total Electric field due to isotropic sources at point P is

$$E = E_1 + E_2$$

$$= E_0 e^{\frac{-j\psi}{2}} + E_0 e^{\frac{+j\psi}{2}}$$

$$= 2E_0 \left(\frac{e^{\frac{-j\psi}{2}} + e^{+j\psi/2}}{2} \right)$$

$$= 2E_0 \cos\frac{\psi}{2}$$

$$E = 2E_0 \cos\left(\frac{dr\cos\phi}{2} + \frac{\pi}{4}\right)$$

$$E_n = \frac{2E_0 \cos\left(\frac{dr\cos\phi}{2} + \pi/4\right)}{2E_0}$$

$$E_n = \cos\left(\frac{dr}{2}\cos\phi + \frac{\pi}{4}\right)$$

if d

If $d = \lambda/2$. Find HPBW

$$dr = \frac{2\pi d}{\lambda} \qquad d = \lambda/2$$

$$= \frac{2\pi \times \lambda}{\lambda \times 2} = \pi$$

$$E_n = \cos\left(\frac{\pi}{2}\cos\phi + \frac{\pi}{4}\right)$$

Direction of maximum

$$E_n = 1.$$

$$\cos\left(\frac{\pi}{2}\cos\phi + \frac{\pi}{4}\right) = 1.$$

$$\frac{\pi}{2}\cos\phi + \frac{\pi}{4} = \pm k\pi$$

$$\frac{\pi}{2}\cos\phi = -\frac{\pi}{4} \pm k\pi \qquad \frac{\pi}{2}\cos\phi = \pm k\pi - \frac{\pi}{4}$$

$$\cancel{\cos\phi = -\frac{\pi}{4} \times \frac{2}{\pi}} \qquad \cos\phi = \frac{\pm 4k - 1}{4} \times \frac{2}{\pi}$$

$$\cancel{\cos\phi = -\frac{1}{m}\ \frac{1}{2}}$$

$\phi_m = \cos^{-1}\left(\frac{-1}{2}\right)$ $\quad K=0,$

$\phi_m = \pm 120^\circ$ $\quad \phi = \cos^{-1}\left(\frac{\pm(4K-1)}{2}\right)$

$\phi_m = \pm \frac{2\pi}{3}$

Direction of Null.

$E_n = 0$

$\cos\left(\frac{\pi}{2}\cos\phi + \frac{\pi}{4}\right) = 0$

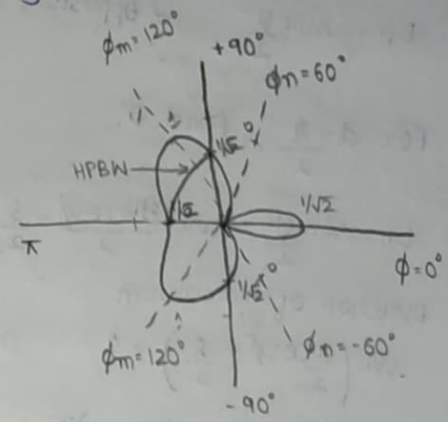
$\frac{\pi}{2}\cos\phi + \frac{\pi}{4} = \frac{\pi}{2} \pm (2K+1)\frac{\pi}{2}$ $\quad K=0,$

$\phi = \pm\frac{\pi}{3}$

$\frac{\pi}{2}\cos\phi = \frac{\pi}{4}$

$\cos\phi = \frac{1}{2}$

$\boxed{\phi_n = \pm\frac{\pi}{3}}$



Direction of Half Power angle.

$\cos\left(\frac{\pi}{2}\cos\phi + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

$\frac{\pi}{2}\cos\phi + \frac{\pi}{4} = \pm(2K+1)\frac{\pi}{4}$

$K=0$

$\frac{\pi}{2}\cos\phi + \frac{\pi}{4} = \pm\frac{\pi}{4}$ $\quad \frac{\pi}{2}\cos\phi + \frac{\pi}{4} = -\frac{\pi}{4}$

$\frac{\pi}{2}\cos\phi = 0$ $\quad \frac{\pi}{2}\cos\phi = -\frac{\pi}{2}$

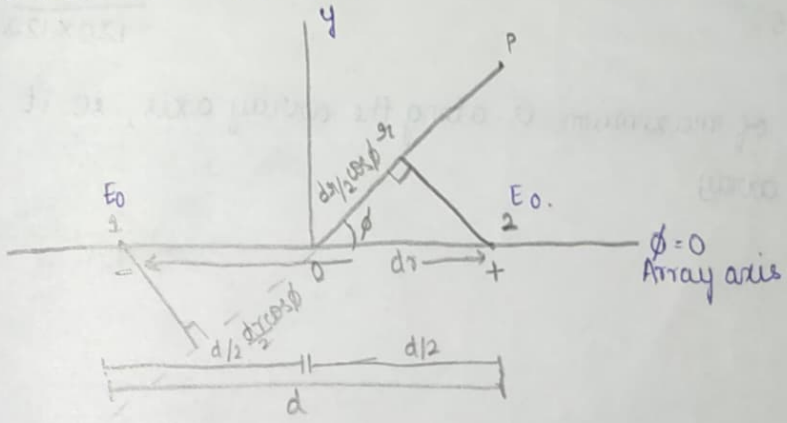
$\cos\phi = 0$ $\quad \cos\phi = -1$

$\boxed{\phi_{HP} = \pm\pi/2}$ $\quad \boxed{\phi_{HP} = \pm\pi}$

HPBW = 90°

$D = \frac{4\pi}{\Omega_A} = \frac{4\pi}{\pi/2 \times \frac{\pi}{2}} = \frac{8\times 2}{\pi} = \frac{16}{\pi} = 5.093$

Case 3: Array of two isotropic point sources having Equal amplitude and Opposite phase.



$\frac{\pi}{2}\cos\phi = \pm K\pi - \frac{\pi}{4}$

$\cos\phi = \frac{\pm(4K-1)\pi \times 2}{4 \times 2 \times \pi}$

$\psi = dr\cos\phi + \delta$

The total Electric field

$E = E_1 + E_2$

$E_0 e^{j\psi/2} - E_0 e^{-j\psi/2}$.

$= 2j\, E_0 \dfrac{(e^{j\psi/2} - e^{-j\psi/2})}{2j}$

$E = 2j\, E_0 \sin\psi/2$

$E_n = \sin\dfrac{\psi}{2}$ opposite phase $= \pi = \delta$

For $d = \dfrac{\lambda}{2}$, Find D.

$E_n = \sin\dfrac{\psi}{2} = \sin\left(\dfrac{dr\cos\phi}{2} + \dfrac{\delta}{2}\right)$ $\quad dr = \dfrac{2\pi d}{\lambda} = \dfrac{2\pi \times \lambda}{2\lambda} = \pi$

Direction of maximum

$\sin\left(\dfrac{\pi}{2}\cos\phi + \dfrac{\delta}{2}\right) = 1$ put $\delta = 0$

$\dfrac{\pi}{2}\cos\phi + \dfrac{\delta}{2} = \pm(2K+1)\dfrac{\pi}{2}$

$K = 0$

$\dfrac{\pi}{2}\cos\phi + \dfrac{\cancel{\delta}}{2} = \pm\dfrac{\pi}{2}$

$\dfrac{\pi}{2}\cos\phi + \dfrac{\pi}{2} = \dfrac{\pi}{2}$

$\dfrac{\pi}{2}\cos\phi = 0$

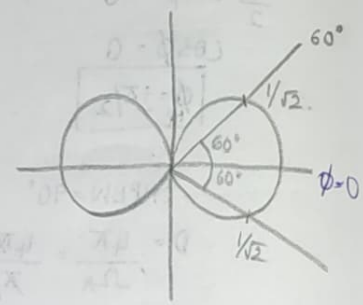
$\cos\phi = 0$

$\phi = \pm\dfrac{\pi}{2}$

$\dfrac{\pi}{2}\cos\phi = -\dfrac{\pi}{2} - \dfrac{\pi}{2}$

$\dfrac{\pi}{2}\cos\phi = -\pi$

$\cos\phi = -\dfrac{\cancel{\pi}2}{\cancel{\pi}}$



$\phi_m = \pm 0$

$\phi_0 = \pm 90°$

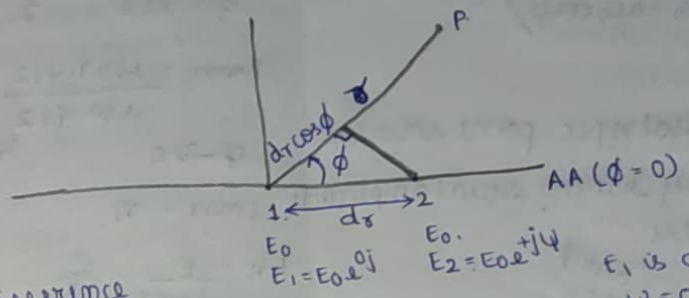
$\phi_{HP} = \pm 60°$

HPBW $= 60° + 60° = 120°$

$D = \dfrac{41200}{120 \times 120} = 2.86$.

Direction of maximum is along the array axis, so it is Endfile array

Case 4: Array of two isotropic point sources having Equal amplitude and any phase.



E_1 is at reference.
$\psi = 0$

The total path difference
$\psi = d_r\cos\phi + \delta$

out of phase $-\frac{\pi}{2} = \delta$
Inphase $\delta = 0$
Inphase quadrature, $\delta = \frac{\pi}{4}$
δ - Phase difference.

$E = E_1 + E_2$

$\cancel{E = E_0 e^{j\psi/2} + E_0 e^{-j\psi/2}}$

$E = E_0 e^{0j} + E_0 e^{j\psi}$

$= E_0(1 + e^{j\psi})$

$= \cancel{E_0(1 + e^{j}}$

$= 2E_0\left(e^{j\psi/2}\left(\frac{e^{-j\frac{\psi}{2}} + e^{+j\psi/2}}{2}\right)\right)$

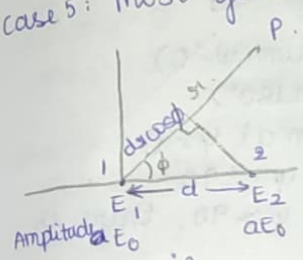
$= 2E_0 e^{j\psi/2}\cos\frac{\psi}{2}$

$E = 2E_0\cos\psi$

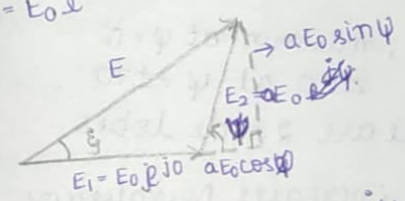
$E_n = \cos\psi$

Case 5: Most general case having any amplitude and any case.

$0 < a < 1$.



$E_1 = E_0 e^{j0}$



ξ - Total Phase difference.

$E_1 = E_0 e^{j0} = E_0$, $E_2 = aE_0 e^{j\psi}$

$E = \sqrt{(E_0 + aE_0\cos\psi)^2 + (aE_0\sin\psi)^2}$

$= \sqrt{E_0^2 + a^2E_0^2\cos^2\psi + 2aE_0^2\cos\psi + a^2E_0^2\sin^2\psi}$

$= E_0\sqrt{1 + a^2(\cos^2\psi + \sin^2\psi) + 2a\cos\psi} = E_0\sqrt{1 + a^2 + 2a\cos\psi}$

$|E| = E_0$

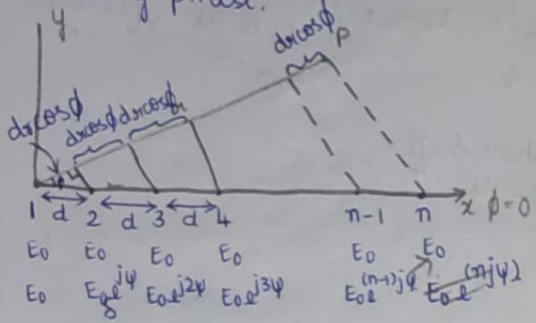
$$\xi = \tan^{-1}\left(\frac{a E_0 \sin\psi}{E_0 + a E_0 \cos\psi}\right)$$

[6M]

Array of n isotropic point sources

or

Linear array having equal amplitude and any phase.



r is the distance b/w point P & origin (reference point).

$$E = E_1 + E_2 + \dots + E_{n-1} + E_n$$

$$E = E_0\left(1 + e^{j\psi} + e^{j2\psi} + \dots + e^{j(n-1)\psi}\right) \quad \rightarrow 1.$$

both multiply $e^{j\psi}$ on both sides.

$$E e^{j\psi} = E_0\left(e^{j\psi} + e^{j2\psi} + e^{j3\psi} + \dots + e^{jn\psi}\right) \quad \rightarrow 2.$$

Equation 1. − Equation 2.

$$E(1 - e^{j\psi}) = E_0\left(1 + e^{j\psi} + e^{j2\psi} + \dots e^{j(n-1)\psi} - e^{j\psi} - e^{j2\psi} \dots e^{jn\psi}\right)$$

$$E(1 - e^{j\psi}) = E_0(1 - e^{jn\psi})$$

$$E = E_0\left[\frac{1 - e^{jn\psi}}{1 - e^{j\psi}}\right]$$

$$E = E_0\left[\frac{1 - e^{\frac{jn\psi}{2}} \cdot e^{-\frac{jn\psi}{2}}}{1 - e^{j\psi/2} \cdot e^{-j\psi/2}}\right]$$

$$E = \frac{E_0 e^{\frac{jn\psi}{2}}}{e^{j\psi/2}}\left[\frac{e^{\frac{jn\psi}{2}} - e^{-jn\psi/2}}{e^{j\psi/2} - e^{-j\psi/2}}\right]$$

$$E = E_0 e^{\frac{j(n-1)\psi}{2}}\left[\frac{\sin n\psi/2}{\sin\psi/2}\right]$$

$$E_n = \frac{E}{E_{max}}$$

$$|E| = \frac{\sin n\psi/2}{\sin\psi/2}$$

$$\xi = E_0 e^{j(n-1)\frac{\psi}{2}}$$

$$E_{max} = \frac{\sin n\psi/2}{\sin\psi/2} \quad \psi \to 0$$

$\psi \to 0$

$$E_{max} = n$$

$$E_n = \frac{E}{E_{max}} = \frac{E}{n}$$

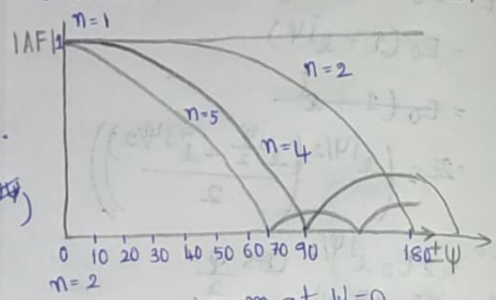
Array Factor (AF).

$$\boxed{E_n = \left[\frac{\sin\frac{n\psi}{2}}{\sin\psi/2}\right] \times \frac{1}{n}}$$

n − number of isotropic point sources.

This is also called Array Factor

Universal Field Pattern Chart.



|AF| is maximum at $\psi = 0$

|AF| is 0 at $\psi = 180°$

If n = 1, isotropic.

If n = 2, maximum ($\psi = 0$).

0 ($\psi = \pm 180°$)

n = 4, maximum at $\psi = 0$

zero at $\psi = \pm 90$

It increases $\psi > 90$, there is a sidelobe.

n = 5, maximum at $\psi = 0$

zero at $\psi = \pm 70$

There are 2 side lobes.

As n increases, Number of side lobes increases, Main lobe narrows. main lobe beam width decreases.

1. As n increases main lobe narrows.
2. As n increases there are more side lobes in one period of array factor.
3. Number of side lobes are $(n-2)$.
Number of complete lobe is $(n-1)$ and one main lobe in each period.
4. The side lobe peak (amplitude) decreases as n increases.

$$\psi = dr\cos\phi + \delta$$
$$dr = \frac{2\pi d}{\lambda}$$

Case 1: Broadside Array : n - number of isotropic point sources in array.

condition: $\delta = 0$ - Inphase.

$$\psi = dr\cos\phi + \delta.$$
$$\psi = dr\cos\phi$$

Direction of Maximum.

$$dr\cos\phi = 0.$$
$$dr = \frac{2\pi d}{\lambda} = \pi$$

Direction of Maximum	Direction of Null	Direction of side lobes
$\psi = 0$ $\psi = dr\cos\phi$ $dr\cos\phi = 0$ $\phi_{max} = \pm 90^\circ$ or $\frac{\pm\pi}{2}$	$\lvert E\rvert = 0.$ $\sin\frac{n\psi}{2} = 0$ $\frac{n\psi}{2} = \pm K\pi$ $\psi = \frac{\pm 2K\pi}{n}$ $dr\cos\phi = \frac{\pm 2K\pi}{n}$ $\phi_0 = \frac{\pm 2K\pi}{n\,dr}$	$\sin\frac{n\psi}{2} = 1.$ $\frac{n\psi}{2} = \pm\frac{(2K+1)\pi}{2}$ $\psi = \frac{\pm(2K+1)\pi}{n}$

Maximum is perpendicular to Array axis.

Null is a point at which electric field intensity is 0.

$$\phi_0 = \frac{\pm 2K\pi}{n\,dr} = \frac{\pm 2K\pi}{n \times \frac{2\pi d}{\lambda}} = \frac{\pm K\lambda}{n\,d}$$

Direction of Maximum:

$$\psi = 0$$
$$dr\cos\phi = 0$$
$$\phi_{max} = \pm 90^\circ \text{ or } \frac{\pi}{2}.$$

Direction of Minimum:

$$\lvert E\rvert = 0$$
$$\sin\frac{n\psi}{2} = 0$$
$$\frac{n\psi}{2} = \pm K\pi$$
$$\psi = \frac{\pm 2K\pi}{n}$$
$$dr\cos\phi = \frac{\pm 2K\pi}{n}$$
$$\phi_0 = \frac{\pm 2K\pi}{n\,dr}$$
$$\boxed{\phi_0 = \pm\cos^{-1}\left[\frac{2K\pi}{n\,dr}\right]}$$

Direction of side lobes.

$$\sin\frac{n\psi}{2} = 1$$
$$\frac{n\psi}{2} = \pm\frac{(2K+1)\pi}{2}.$$
$$\psi = \pm\frac{(2K+1)\pi}{n}$$
$$dr\cos\phi = \pm\frac{(2K+1)\pi}{n}$$
$$\cos\phi = \pm\frac{(2K+1)\pi}{n\,dr}$$
$$\phi = \pm\cos^{-1}\left(\frac{\pm(2K+1)\pi}{n\,dr}\right)$$

consider array of point sources $n=4$, placed at $\lambda/2$ distance and $\delta=0$. Obtain complete radiation pattern and also find HPBW and Directivity.

Direction of maximum:

$$\phi_m = \pm\frac{\pi}{2} \text{ or } 90^\circ$$

Direction of nulls:

$$\phi_0 = \pm\cos^{-1}\left[\frac{\pm(2K\pi)}{n\,d_r}\right]$$

$$d_r = \frac{2\pi d}{\lambda} = \frac{\cancel{2}\pi \times \frac{\cancel{\lambda}}{\cancel{2}}}{\cancel{\lambda}} = \pi$$

$$\phi_0 = \pm\cos^{-1}\left[\frac{\pm(2K\cancel{\pi})}{\cancel{4}_{2}\times\cancel{\pi}}\right]$$

$$\phi_0 = \pm\cos^{-1}\left[\frac{\pm K}{2}\right]$$

$K = 0, 1, 2, 3 \ldots$

$K=0$ $\phi_0 = \pm\frac{\pi}{2}$ Not valid because maximum is at $\pi/2$.

$K=1$, $\phi_{01} = \pm\frac{\pi}{3}, \frac{2\pi}{3}$ First Null $(\pm 60, \pm 120)$

$K=2$, $\phi_{02} = \cancel{\pm K\pi}\ 0$

$K=3$, $\phi_{03} \rightarrow$ Not possible (>1).

Number of side lobes are always $(n-2) = 2$

Number of complete lobes (Main + side) are always $(n-1) = 3$

side lobes

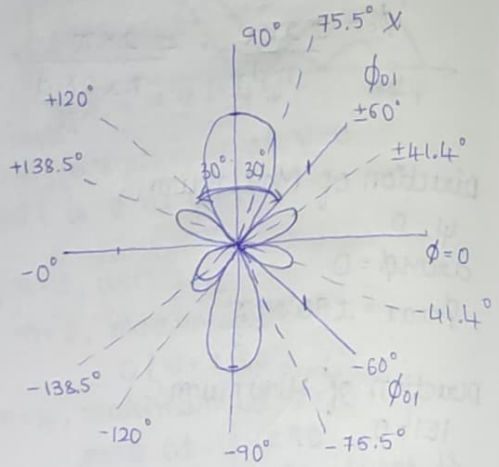
$$\phi = \pm\cos^{-1}\left(\frac{\pm(2K+1)\cancel{\pi}}{n\cancel{\pi}}\right)$$

$$\phi = \pm\cos^{-1}\left(\frac{\pm(2K+1)}{4}\right)$$ either in degree or radians

$K=0$, $\phi_{00} = \pm 75.52^\circ, \pm 104.4^\circ$ Not valid

$K=1$, $\phi_{01} = \pm 41.40^\circ, \pm 138.59^\circ$ valid

ϕ_{00} is not valid because it is before first Null



Total number of lobes = 3.

Major lobe – First Null – First sidelobe – second null – second sidelobe.

First sidelobe exists between first null and second null.

Half Power Beam Width

$$HPBW \simeq \frac{FNBW}{2}$$

$BWFN = 60^\circ$

$$HPBW = \frac{60^\circ}{2} = 30^\circ = \phi_{HP}$$

Assuming pattern is symmetrical $\theta_{HP} = \phi_{HP}$.

$$D = \frac{4\pi 200}{\cancel{\phi_{HP}}\ \theta_{HP}} = \cancel{1373.33}.$$

$D = \frac{41200}{30 \times 30} = 45.78$.

Compute the field pattern and determine beamwidth between first Null and HPBW of a linear uniform array of 6 isotropic point sources spaced $\lambda/2$ distance apart. The Power is applied with equal amplitude and in phase.

$n = 6, \; d = \frac{\lambda}{2}$

$d_x = \frac{2\pi d}{\lambda} = \frac{\cancel{2}\pi \times \frac{\lambda}{\cancel{2}}}{\lambda} = \pi$

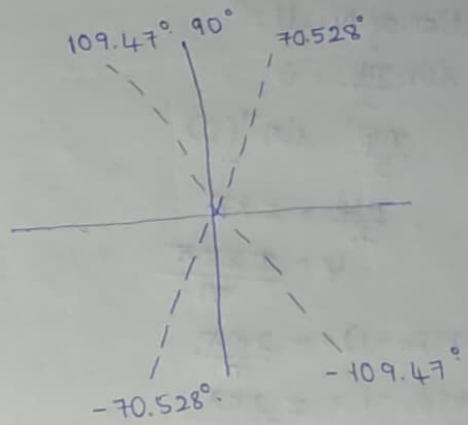
Direction of maximum.

$\phi_m = \pm \frac{\pi}{2}$ or $90°$

Direction of Null.

$\phi_0 = \pm \cos^{-1}\left[\frac{\pm(2K\cancel{\pi})}{n \times \cancel{\pi}}\right]$.

$\phi_0 = \pm \cos^{-1}\left[\frac{\pm 2K}{6}\right] = \pm \cos^{-1}\left[\frac{\pm K}{3}\right]$



$K = 0, 1, 2, 3$

$K=0 \; \phi_0 = \pm \pi/2 \rightarrow$ Not Valid.

$K=1, \phi_{01} = 70.528°, 109.47°$.

$K=2, \phi_{02} = 48.189°, 131.81°$.

$K=3, \phi_{03} = \pm 0, \pm 180°$

Number of side lobes $= (n-2) = 4$.

Number of complete lobes $= (n-1) = 5$

Side lobes

$\phi = \pm \cos^{-1}\left(\frac{\pm(2K+1)\cancel{\pi}}{6\cancel{\pi}}\right)$

$K=0, \; \phi_{01} = \pm 80.405°, 99.59°$ Not Valid (First side lobe after first null).

$K=1, \; \phi_{02} = 60°, 120°$

$K=2, \; \phi_{03} = 33.557°, 146.442°$.

~~$K=3, \phi_{03} =$~~

First Null $= 70.528°$

$90° - 70.528 = 20.528°$.

$FNBW = 2 \times 20.528 = 40°$.

$\phi_{HP} = HPBW = 20°$ $\quad \theta_{HP} = \phi_{HP}$.

$D = \frac{41200}{20° \times 20°} = \frac{41\cancel{200}}{4\cancel{00}} = 103$.

End fire array.

Condition of end fire array: $\delta = -dr$.

$\psi = dr\cos\phi + \delta$

$\psi = dr\cos\phi - dr$

$\psi = dr[\cos\phi - 1] \rightarrow 1.$

Direction of Maximum (ϕ_m).

$\psi = 0$

$dr[\cos\phi - 1] = 0$

$[\cos\phi - 1] = 0$

$\cos\phi_m = 1$

$\phi_m = \pm\cos^{-1}(1)$

$\phi_m = \pm 0^\circ$

Direction of Null:

$\sin\frac{n\psi}{2} = 0$

$\frac{n\psi}{2} = \sin^{-1}(0)$

$\frac{n\psi}{2} = \pm K\pi$

$\psi = \frac{\pm 2K\pi}{n}$

$dr(\cos\phi_0 - 1) = \pm\frac{2K\pi}{n}$

$\cos\phi_0 - 1 = \pm\frac{2K\pi}{ndr}$

$\cos\phi_0 = \pm\frac{2K\pi}{ndr} + 1$

$\phi_0 = \pm\cos^{-1}\left(\pm\frac{2K\pi}{ndr} + 1\right)$

Direction of side lobe:

$\sin\frac{n\psi}{2} = 1$

$\frac{n\psi}{2} = \pm\frac{(2K+1)\pi}{2}$

$\psi = \frac{\pm(2K+1)\pi}{n}$

$dr(\cos\phi_0 - 1) = \frac{\pm(2K+1)\pi}{n}$

$\cos\phi_0 - 1 = \frac{\pm(2K+1)\pi}{ndr}$

$\cos\phi_0 = \frac{\pm(2K+1)\pi}{ndr} + 1$

$$\boxed{\phi_0 = \pm\cos^{-1}\left(\frac{\pm(2K+1)\pi}{ndr} + 1\right)}$$

A linear array of 4 isotropic point sources the distance b/w adjacent element is $\lambda/2$. The Power is applied with eq. Amplitude and Phase difference of dr. Obtain the field pattern and find directivity.

$n = 4,\ d = \lambda/2,\ \delta = -dr$

$dr = \frac{2\pi d}{\lambda} = \frac{2\pi \times \lambda/2}{\lambda} = \pi$

Maximum:

$\phi_m = \pm 0^\circ$

Null.

$\phi_0 = \pm\cos^{-1}\left(\frac{\pm 2K\pi}{2\pi} + 1\right)$

$\phi_0 = \pm\cos^{-1}\left(\pm\frac{K}{2} + 1\right)$

$K = 0,\ \phi_0 = 0, \pm 180^\circ$ – Not valid

$K = 1,$ → not possible.

$K = 2,\ \phi_{01} = \pm 60^\circ, \pm 90^\circ, \pm 120^\circ$

$K = 2$ $K = 3$

Sidelobes.

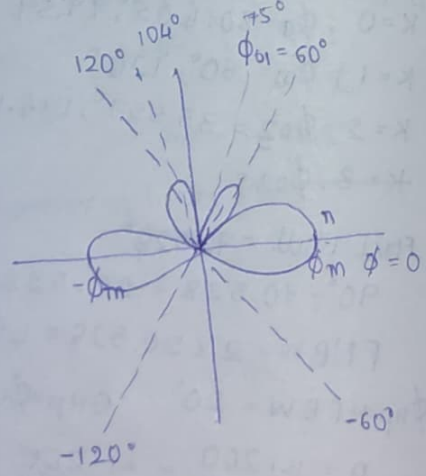
$\phi = \pm\cos^{-1}\left(\frac{\pm(2K+1)\pi}{4\pi} + 1\right)$

$K = 0,\ \phi_1 = \pm\cos^{-1}\left(\frac{\pm 1}{4} + 1\right)$

$\phi_1 = \pm 41.4096^\circ$ → Not valid

$K = 1,\ \phi_2 = \pm 75.5224^\circ$

$K = 2,\ \phi_3 = \pm 104.4775^\circ$



Peak value of major lobe $= n$
$= 4$

ϕ_{01} = Beamwidth.

FNBW $= 2 \times \phi_{01}$
$= 2 \times 60^\circ = 120^\circ$

$HPBW = \frac{FNBW}{2} = \frac{120^\circ}{2} = 60^\circ$

$D = \frac{41200}{60 \times 60} = \frac{41200}{3600} = 11.44$

2. $n = 5$, $d = \frac{\lambda}{2}$ $\delta = -dr$ obtain complete radiation pattern.

$dr = \frac{2\pi d}{\lambda} = \frac{2\pi \times \frac{\lambda}{2}}{\lambda} = \pi$.

Maximum:

$\phi_m = \pm 0^\circ$.

Null.

$\phi_0 = \pm \cos^{-1}\left(\pm \frac{2K\cancel{\pi}}{5\cancel{\pi}} + 1\right)$

$\phi_0 = \pm \cos^{-1}\left(\pm \frac{2K}{5} + 1\right)$

$K = 0$, $\phi_0 = \pm 0^\circ, \pm 180^\circ$

$K = 1$, $\phi_{01} = \pm 53.130^\circ$.

$K = 2$, $\phi_{02} = \pm 78.46^\circ$ $K = 3$, $\phi_{03} = \pm 101.536^\circ$.

~~$K = 3$, $\phi_{03} = \pm$~~ $K = 4$ $\phi_{04} = \pm 126.8^\circ$

Sidelobes

$\phi = \pm \cos^{-1}\left(\pm \frac{(2K+1)\cancel{\pi}}{5\cancel{\pi}} + 1\right)$

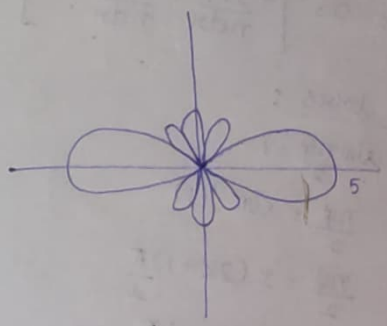
$K = 0$, $\phi = \pm 36.86^\circ$

$K = 1$, $\phi = \pm 66.42^\circ$

$K = 2$, $\phi = \pm 90^\circ$

$K = 3$, $\phi = \pm 103^\circ.578^\circ$.

$K = 4$, $\phi = \cancel{101.43}^\circ$ 143.130.



Linear array:

$E = \frac{\sin n\psi/2}{\sin \psi/2}$

$\psi = dr\cos\phi + \delta$.

$AF = \frac{1}{n} \frac{\sin n\psi/2}{\sin \psi/2}$.

1. Broad side array: $\delta = 0$

$\psi = dr\cos\phi$

$\phi_M = \perp^{lar}$ to array axis

It produces large directivity.

ΩA = small.

Ordinary Endfile array:

2. Endfile array: $\delta = -dr$

$\psi = dr(\cos\phi - 1)$.

ϕ_M = max. radiation of array axis

ΩA is very large, directivity is large.

Q- Endfile array > Extended endfile array.

3. Hansen-Woodyard array or Increase directivity End file array or extended enfile array.

$\delta = -dr - \frac{\pi}{n}$

$\psi = dr\cos\phi + \delta$.

$\psi = dr\cos\phi - dr - \frac{\pi}{n}$.

$\psi = dr[\cos\phi - 1] - \frac{\pi}{n}$

ψ = Total Power difference.

ϕ_M - along 0° (or) array axis

Null : $\sin\frac{n\psi}{2} = 0$.

$\frac{n\psi}{2} = \pm K\pi$

$\psi = \frac{\pm 2K\pi}{n}$.

$dr[\cos\phi - 1] - \frac{\pi}{n} = \pm\frac{2K\pi}{n}$.

$dr[\cos\phi - 1] = \pm\frac{2K\pi}{n} + \frac{\pi}{n}$.

$\cos\phi_0 = \frac{\pm 2K\pi}{ndr} + \frac{\pi}{ndr} + 1$

$\phi_0 = \cos^{-1}\left[\frac{\pm 2K\pi}{ndr} + \frac{\pi}{ndr} + 1\right]$ $K = 0, 1, 2, \ldots$

Side lobes :

$\sin\frac{n\psi}{2} = 1$

$\frac{n\psi}{2} = \sin^{-1}(1)$

$\frac{n\psi}{2} = \pm\frac{(2K+1)\pi}{2}$

$\psi = \frac{\pm(2K+1)\pi}{n}$

$dr[\cos\phi - 1] = \frac{\pm(2K+1)\pi}{n} + \frac{\pi}{n}$

$\cos\phi = \left[\pm\left[\frac{(2K+1)\pi}{ndr} + \frac{\pi}{ndr}\right] + 1\right]$.

$\phi_0 = \cos^{-1}\left[\frac{\pm(2K+1)\pi}{ndr} + \frac{\pi}{ndr} + 1\right]$

$K = 0, 1, 2, \ldots$

4. Scanning array or array with ϕ_{max} in arbitary direction.

$\psi = dr\cos\phi + \delta$.

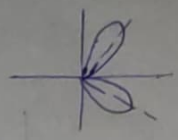
ϕ_M = direction at which maximum direction exist using this what must be the δ.

$\psi = dr\cos\phi_M + \delta$.

max direction, $\psi = 0$

$\delta = -dr\cos\phi_m \quad \phi_m = 60°$

$\delta = -\frac{dr}{2}$



An array of 4 isotropic antennas placed along straight line distance between adjacent element is $\lambda/2$. the peak is to be obtained in a direction 60° from the array axis what should be the phase difference between adjacent element and also obtained complete radiation pattern and find i) FNBW ii) Directivity.

$n = 4$

$dr = \frac{2\pi d}{\lambda} \quad d = \frac{\lambda}{2}$

$dr = \pi$

$\delta = -\frac{dr}{2} = -\frac{\pi}{2}$

Lobes $= n-1 = 4-1 = 3$

Null: $\sin\frac{n\psi}{2} = 0$

$\frac{n\psi}{2} = \pm K\pi$

$\psi = \pm\frac{2K\pi}{n}$

where, $\psi = dr\cos\phi_m - \frac{\pi}{2}$

$dr\cos\phi - \frac{\pi}{2} = \pm\frac{2K\pi}{n}$

$dr\cos\phi = \pm\frac{2K\pi}{n} + \frac{\pi}{2}$

$\cos\phi = \pm\frac{2K\pi}{ndr} + \frac{\pi}{2dr}$

$\phi = \cos^{-1}\left[\pm\frac{2K\pi}{ndr} + \frac{\pi}{2dr}\right]$

$dr = \pi$

$\phi_0 = \cos^{-1}\left[\pm\frac{2K\cancel{\pi}}{n\cancel{\pi}} + \frac{\cancel{\pi}}{2\cancel{\pi}}\right]$

$\phi_0 = \cos^{-1}\left[\pm\frac{2K}{4} + \frac{1}{2}\right]$

$\phi_0 = \cos^{-1}\left[\pm\frac{K}{2} + \frac{1}{2}\right]$

$K = 0, \ \phi = 60°$

$K = 1, \ \phi = \pm 90° \& 0°$

$K = 2, \ \phi = \pm 120°$

$K = 3, \ \phi = \pm 180°$

Side lobes:

$\sin\frac{n\psi}{2} = 1 \rightarrow$ do

$\phi = \pm\cos^{-1}\left[\frac{1}{2} \pm \left(\frac{2k+1}{2}\right)\right]\frac{\pi}{2}$

$k=0$, $\phi = \pm 41.5^\circ$ & $\pm 15.5^\circ$

$k=1$, $\phi = \pm 104.5^\circ$

$k=2$, $\phi = \pm 138.3^\circ$

For a given array $n=4$, $d = \lambda/3$ and $\delta = \frac{\pi}{3}$. Find 1st Null Beam width.

$n=4$, $\alpha = \frac{2\pi d}{\lambda} = \frac{2\pi \times \lambda/3}{\lambda} = \frac{2\pi}{3}$

$\psi = \alpha\cos\phi_m + \frac{\pi}{3}$

$\psi = \frac{2\pi}{3}\cos\phi_m + \frac{\pi}{3}$

$\sin\frac{n\psi}{2} = 0$

$\frac{n\psi}{2} = \pm k\pi$

$\psi = \pm\frac{2k\pi}{n}$

$\frac{2\pi}{3}\cos\phi_m = \pm\frac{2k\pi}{n} - \frac{\pi}{3}$

$\cos\phi_m = \pm\left(\frac{2k\pi}{4} - \frac{\pi}{3}\right)\times\frac{3}{2\pi}$

$\cos\phi_m = \pm\left[\frac{3k}{4} - \frac{1}{2}\right]$

$\phi_m = \cos^{-1}\left(\pm\frac{3k}{4} - \frac{1}{2}\right)$

$k=0$, $\phi_0 = 120^\circ, 60^\circ$

$k=1$, $\phi_1 = 75.5^\circ$ –

$k=2$, $\phi_2 = 0$ –

$k=3$, $\phi_3 =$ – –

$\sin\frac{n\psi}{2} = 1$

$\frac{n\psi}{2} = \pm(2k+1)\frac{\pi}{2}$

$\psi = \pm\frac{(2k+1)\pi}{n}$

$\frac{2\pi}{3}\cos\phi + \frac{\pi}{3} = \pm\frac{(2k+1)\pi}{n}$

$\frac{2\pi}{3}\cos\phi = \pm\frac{(2k+1)\pi}{n} - \frac{\pi}{3}$

$$\cos\phi = \pm\left(\frac{(2k+1)\pi}{n} - \frac{\pi}{3}\right) \times \frac{3}{2\pi}$$

$$\cos\phi = \pm\left(\frac{3(2k+1)}{2\times 4} - \frac{1}{2}\right)$$

$$\phi = \cos^{-1}\left[\pm\left[\frac{3(2k+1)}{8} - \frac{1}{2}\right)\right.$$

$k=0$, $\phi_0 = 97.18°, 151.0°$

$k=1$, $\phi_1 = 51.31°$ —

$\phi_2 =$ — —

$\phi_3 =$ — —

Long Array:

~~If~~ array length = nd

If nd is very large, then such array is called long array.

n - number of isotropic point sources

d - ~~number~~ distance between the isotropic point sources.

i) Broad side array.

$\delta = 0$

$\psi = dr\cos\phi + \delta$

$\psi = dr\cos\phi \rightarrow 1.$

More Beamwidth, less Directivity.

To find first Null angle.

$\sin\frac{n\psi}{2} = 0$

$\frac{n\psi}{2} = \pm k\pi$

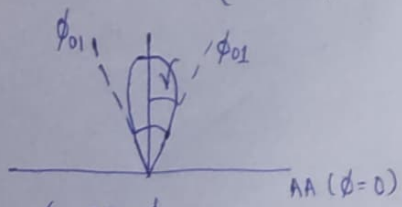
$\psi = \pm\frac{2k\pi}{n} \rightarrow 2.$

$dr\cos\phi = \frac{\pm 2k\pi}{n}$

$\cos\phi = \frac{\pm 2k\pi}{ndr}$

$$\boxed{\phi_0 = \pm\cos^{-1}\left(\frac{\pm 2k\pi}{ndr}\right)}$$

$k = 1 \quad \phi_{01} = \pm\cos^{-1}\left(\frac{\pm 2\pi}{ndr}\right)$



$\gamma = 90 - \phi_{01}$

$90° - \left[\pm\cos^{-1}\left(\frac{\pm 2\pi}{ndr}\right)\right]$

$\gamma = \pm\sin^{-1}\left(\frac{\pm 2\pi}{ndr}\right)$

WKT

$dr = \frac{2\pi d}{\lambda}$

$\gamma = \pm\sin^{-1}\left(\frac{\pm 2\pi}{n \times \frac{2\pi d}{\lambda}}\right)$

$$\boxed{\gamma = \pm\sin^{-1}\left(\frac{\lambda}{nd}\right)}$$

BWFN =

$2\gamma = \pm 2\sin^{-1}\left[\frac{\lambda}{nd}\right] \rightarrow$ Exact expression

to find BWFN of Broad side array

if nd is very large, ~~BWFN~~ $\frac{\lambda}{nd}$ is very small

$BWFN = 2\gamma \times \frac{\lambda}{nd}$ | $\sin\theta \simeq \theta$, θ very small

$BWFN \simeq \left[\frac{2\lambda}{nd}\right]$

$D = \frac{4\pi}{\Omega_A} = \frac{4\pi}{\lambda} \times nd = \frac{4\pi nd}{\lambda}$

$\Omega_A = \phi_{HP} \times \theta_{HP}$

$\phi_{HP} \simeq \frac{\lambda}{nd}$ Assume $\theta_{HP} = \phi_{HP}$.

$D = \frac{4\pi}{\frac{\lambda}{nd} \times \frac{\lambda}{nd}}$

$A_e = \frac{\lambda^2 D}{4\pi}$

Determine FNBW for ordinary endfile array & extended endfile array and also show that extended endfile array is greated than ordinary endfile array.

For endfile array,

$\psi = dr\cos\phi + \delta$

$\delta = -dr$

$\psi = dr(\cos\phi - 1)$

To find First null,

$\sin\frac{n\psi}{2} = 0$

$\frac{n\psi}{2} = \pm k\pi$

$\psi = \pm\frac{2k\pi}{n}$

$dr(\cos\phi - 1) = \pm\frac{2k\pi}{n}$

$\cos\phi - 1 = \pm\frac{2k\pi}{ndr}$

$\cos\phi = \pm\frac{2k\pi}{ndr} + 1$

$\cancel{\left(\phi_0 = \cos^{-1}\left(\pm\frac{2k\pi}{ndr} + 1\right)\right)}$

$$\psi = 90^\circ - \left[\pm\cos^{-1}\left(\frac{\pm 2K\pi}{ndr} + 1\right)\right]$$

$$\psi = \pm\sin^{-1}\left[\pm\left(\frac{2K\pi}{ndr} + 1\right)\right]$$

$$dr = \frac{2\pi d}{\lambda}$$

$$\psi = \pm\sin^{-1}\left[\pm\left(\frac{2K\pi}{n \times \frac{2\pi d}{\lambda}} + 1\right)\right]$$

$K = 1$

$$\psi = \pm\sin^{-1}\left[\pm\left(\frac{\lambda}{nd} + 1\right)\right]$$

$$BWFN = 2\psi = \pm 2\sin^{-1}\left[\pm\left(\frac{\lambda}{nd} + 1\right)\right]$$

If nd is very large.

$$BWFN = 2\left(\frac{\lambda}{nd} + 1\right)$$

$$D = \frac{4\pi}{\phi_{HP} \times \theta_{HP}}$$

$$\phi_{HP} = \left(\frac{\lambda}{nd} + 1\right) \qquad \phi_{HP} = \theta_{HP}$$

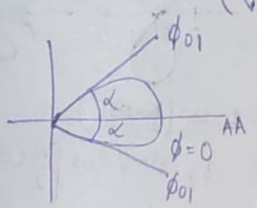
$$\cancel{1} - 2\sin^2\frac{\phi}{2} = \pm\frac{2K\pi}{ndr} + \cancel{1}$$

$$\frac{\sin\phi}{2} = \pm\sqrt{\frac{K\pi}{ndr}}$$

$$= \pm\sqrt{\frac{\pi}{n \times \frac{2\pi d}{\lambda}}}$$

$$\sin\left(\frac{\phi}{2}\right) = \pm\sqrt{\frac{\lambda}{2nd}}$$

$$\phi_{01} = \pm 2\sin^{-1}\left(\pm\sqrt{\frac{\lambda}{2nd}}\right)$$



$$BWFN = 2\alpha$$

$$= \pm 4\sin^{-1}\left(\pm\sqrt{\frac{\lambda}{2nd}}\right)$$

Exact,

If $nd >> 1$

$$BWFN \simeq 4\sqrt{\frac{\lambda}{2nd}}$$

$$BWFN \simeq 2\sqrt{\frac{2\lambda}{nd}}$$

Extended End fire array.

$$\psi = dr\left[\left[\cos\phi - 1\right] - \frac{\pi}{n}\right]$$

$$\delta = -dr - \frac{\pi}{n}$$

$$\psi = dr\cos\phi - dr - \frac{\pi}{n}$$

$$\boxed{\psi = dr(\cos\phi - 1) - \frac{\pi}{n}}$$

To find first Null angle

$$\sin\frac{n\psi}{2} = 0$$

$$\frac{n\psi}{2} = \pm K\pi$$

$$\psi = \frac{\pm 2K\pi}{n}$$

$$dr(\cos\phi - 1) - \frac{\pi}{n} = \frac{\pm 2K\pi}{n}$$

$$dr(\cos\phi - 1) = \frac{\pm 2K\pi}{n} + \frac{\pi}{n}$$

$$\cos\phi - 1 = \frac{\pm 2K\pi}{ndr} + \frac{\pi}{ndr}$$

$$\cos\phi = \frac{\pm 2K\pi}{ndr} + \frac{\pi}{ndr} + 1$$

$$\cancel{1} - 2\sin^2\frac{\phi}{2} = \frac{\pm 2K\pi}{ndr} + \frac{\pi}{ndr} + \cancel{1}$$

$$\sin^2\frac{\phi}{2} = \frac{\pm K\pi}{ndr} \mp \frac{\pi}{2ndr}$$

$$\sin\frac{\phi}{2} = \pm\sqrt{\frac{K\pi}{ndr} \mp \frac{\pi}{2ndr}}$$

$$\frac{\phi}{2} = \sin^{-1}\left(\pm\sqrt{\frac{K\pi}{ndr} \mp \frac{\pi}{2ndr}}\right)$$

$$\phi = \pm 2\sin^{-1}\left(\pm\sqrt{\frac{K\pi}{ndr} \mp \frac{\pi}{2ndr}}\right)$$

$$dr = \frac{2\pi d}{\lambda} \qquad K = 1$$

$$\phi = \pm 2\sin^{-1}\left(\pm\sqrt{\frac{K\pi}{n \times \frac{2\pi d}{\lambda}} \mp \frac{\pi}{2n \times \frac{2\pi d}{\lambda}}}\right)$$

$$\phi = \pm 2\sin^{-1}\left(\pm\sqrt{\frac{\lambda}{2nd} \mp \frac{\lambda}{4nd}}\right)$$

$$\phi_{01} = \pm 2\sin^{-1}\left(\pm\sqrt{\frac{\lambda}{4nd}}\right)$$

$$\phi_{01} = \pm 2\sin^{-1}\left(\sqrt{\frac{\lambda}{4nd}}\right)$$

$K = 1$

$$-2\sin^2\frac{\phi_0}{2} = \frac{\pi}{ndr} - \frac{2\pi}{ndr}$$

$$\sin^2\frac{\phi_0}{2} = \frac{-\pi}{2ndr}$$

$$\phi_0 = \sin^{-1}\sqrt{\frac{\cancel{2}\pi}{\cancel{2}ndr}}$$

$$\phi_0 = \sin^{-1}\sqrt{\frac{\pi}{ndr}}$$

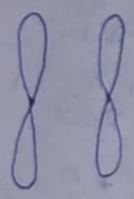
Non Isotropic point Sources:

It is the point source that produces non uniform radiation.

Identical point sources.

Similar point sources

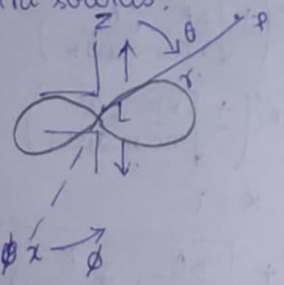
Identical point source produce radiation pattern such that both are similar & amplitudes are equal. (The variation of electric field intensity over θ, ϕ is ~~differ~~ same).



Similar point sources:

Radiation pattern is same but amplitudes are not equal.

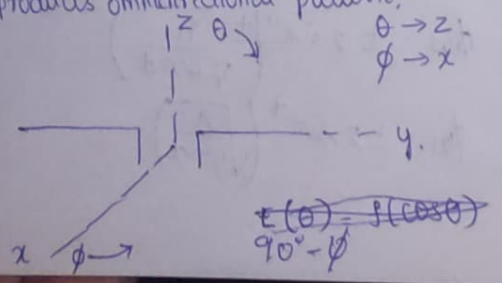
Array of two Non-isotropic but Identical point sources.



The dipole antenna produces Electric field at far field as a function of $\sin\theta$.

$E(\theta) = f(\sin\theta)$

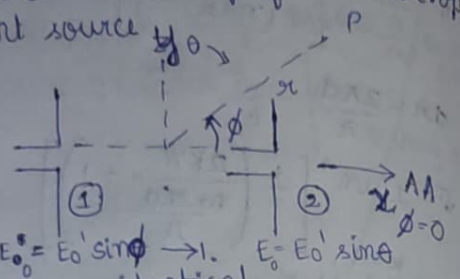
It produces omnidirectional pattern.



Dipole along z-axis, $E(\theta) = f(\sin\theta)$

Dipole along y-axis, ~~$E(\theta) = f(\cos\theta)$~~

Now consider array of two isotropic point source



$E_0' = E_0'\sin\phi \rightarrow 1.$ $\quad E_0 = E_0'\sin\theta$

1 & 2 are identical

Equal amplitude and any phase.

$$E = 2E_0\cos\psi/2$$

$$\psi = dr\cos\phi + \delta \qquad \delta = 0$$

$$E = 2E_0\cos\left[\frac{dr\cos\phi}{2}\right] \rightarrow 2.$$

$$\cancel{E = E_0[dr\cos\phi]}$$

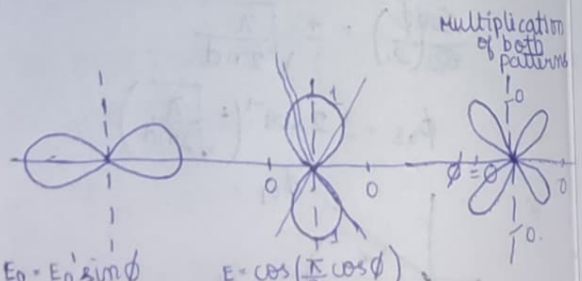
$$E = 2E_0'\sin\phi\cos\left[\frac{dr}{2}\cos\phi\right]$$

$$\cancel{E = \frac{dr}{2}E_0'\sin 2\phi}$$

$$\cancel{E = \frac{E_0'}{2}}$$

$$E = 2E_0'\sin\phi\cos\left[\frac{dr}{2}\cos\phi\right]$$

$$\boxed{E_n = \sin\phi\cos\left[\frac{dr}{2}\cos\phi\right]}$$



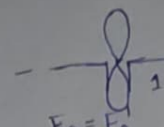
$E_0 = E_0'\sin\phi$ $\qquad E = \cos\left(\frac{\pi}{2}\cos\phi\right)$

$d = \lambda/2$ $\qquad E_n = \sin\phi\cos\left[\frac{\pi}{2}\cos\phi\right]$

$$dr = \frac{2\pi \times \cancel{\lambda}}{\cancel{\lambda}\,2}$$

$$dr = \pi$$

short dipoles p



$E_0 = E_0$

$E_0 = E_0'\cos\phi$

$E = 2E_0\cos$

$E = 2E_0'$ co

$\delta = 0,\ d = \lambda$

$\psi = \frac{\pi}{2}$ co

$E = 2E_0'$ c

$E_n = \cos\phi$

Prima Patt

Array of

$E = E_0$

Pattern N

$E = f(\theta$

$f(\theta, \phi)$ – sour

$f_p(\theta, \phi)$

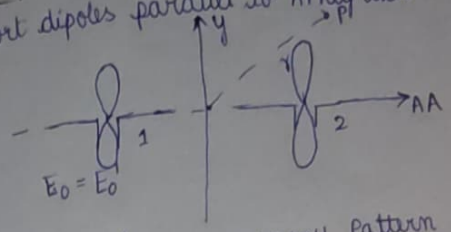
$F(\theta, \phi$

$F_p(\theta,$

$E = f(\theta, \phi)$

field

short dipoles parallel to Array axis



$E_0 = E_0' \cos\phi$ — Primary Pattern

$E = 2E_0 \cos \psi/2$ — Array Pattern

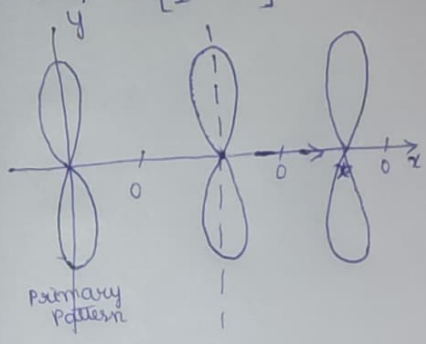
$E = 2E_0' \cos\phi \cos \psi/2$

$\delta = 0,\ d = \lambda/2,\ \psi = dr\cos\phi$

$\psi = \frac{\pi}{2} \cos\phi$

$E = 2E_0' \cos\phi \cos\left[\frac{\pi}{2} \cos\phi\right]$

$E_n = \cos\phi \cos\left[\frac{\pi}{2} \cos\phi\right]$



Array of similar point sources:

$E = E_0 \sqrt{1 + a^2 + 2a\cos\phi}$

Pattern Multiplication:

$E = f(\theta,\phi) F(\theta,\phi) \angle f_p(\theta,\phi) + F_p(\theta,\phi)$

$f(\theta,\phi) \rightarrow$ field pattern of individual sources.

$f_p(\theta,\phi)$ - Phase patterns of individual sources.

$F(\theta,\phi)$ = Field pattern of array of isotropic point sources and

$F_p(\theta,\phi)$ = Phase pattern of array of isotropic point sources.

$E = \underbrace{f(\theta,\phi) F(\theta,\phi)}_{\text{field pattern}} \underbrace{\angle f_p(\theta,\phi) + F_p(\theta,\phi)}_{\text{phase pattern}}$