Image Interpolation using Discrete Wavelet Transform

Project report submitted in partial fulfillment of the requirements for the award of the degree of

Master of Sciences in Mathematics

(Specialization in Computer Science)

by

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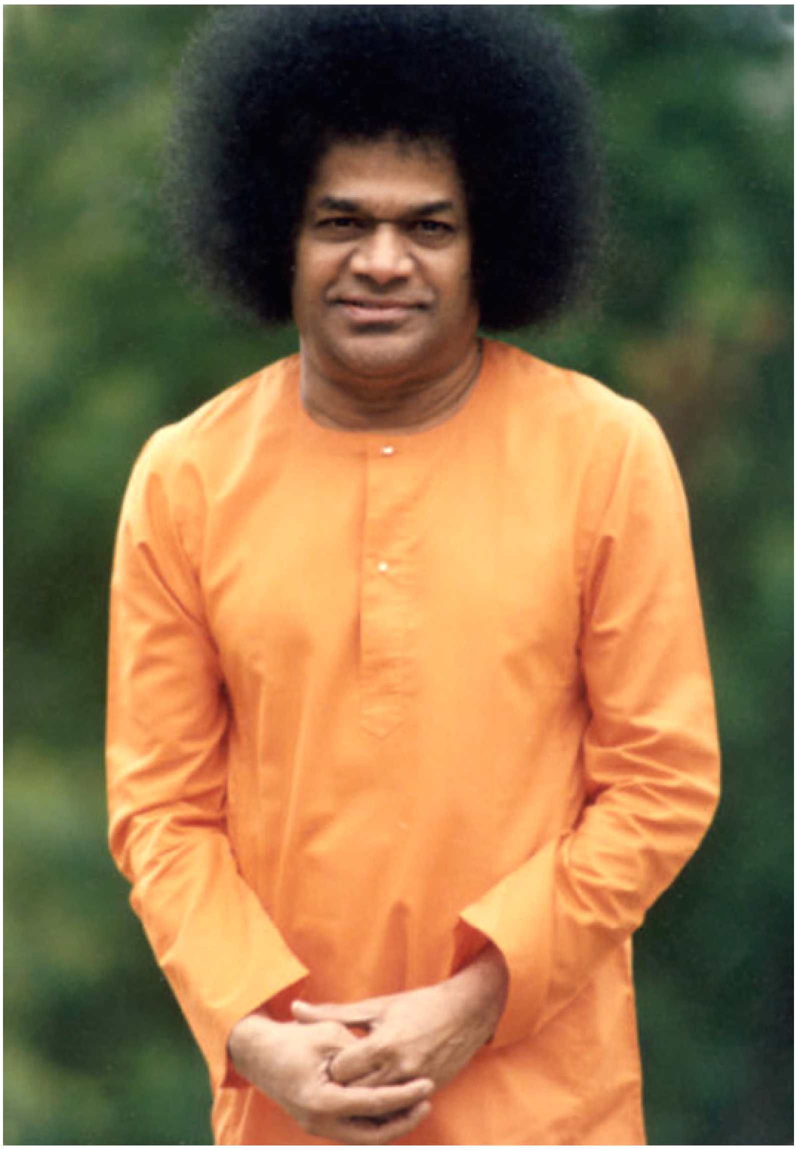

A pen in my hand and my hand in His hands

“Dedicated to Sai Maa”

Sri Sathya Sai Institute of Higher Learning

(Deemed University)

DEPARTMENT OF MATHEMATICS & COMPUTER SCIENCE

Certificate

This is to certify that this project report entitled **Image Enhancement using Discrete Wavelet Transform** being submitted by Sri. **Goutam Kelam** in partial fulfillment of the requirements for the award of the degree **Master of Sciences in Mathematics (specialization in Computer Science)** is a record of bonafide research work carried out by him under our supervision and guidance during the academic year 2016-17 in the Department of Mathematics and Computer Science, Sri Sathya Sai Institute of Higher Learning, Prasanthi Nilayam campus. To the best of our knowledge, the results embodied in this project have not formed the basis of any work submitted to any other University or Institute for the award of any Diploma or Degree.

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DECLARATION

The work embodied in this thesis was carried out by me under the supervision of **Sri Srikanth Khanna**, in the Department of Mathematics and Computer Science, Sri Sathya Sai Institute of Higher Learning, Prasanthi Nilayam. The work is original and the results embodied in this thesis have not been submitted either in part or full to any other University or Institute for the award of any Diploma or Degree.

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ABSTRACT

The things that we perceive through our eyes stays for a long time and gives more information compared to the information we gain by hearing or studying about it. So, in order to enhance what we percieve, we perform various image enhancement techniques among the various image enhancement techniques, we have chosen interpolation method. In this thesis, we will first try to understand the concept of wavelets and their mathematical background. Since the images are discrete signals we will discuss about the wavelets in their discrete forms – discrete wavelet transform and stationary wavelet transform. In the end, we will apply the concepts learnt about DWT and SWT on images to produce an enhanced image by interpolating the subbands produced by the wavelet transforms.

In the first half of the thesis, we have presented a theoretical background about wavelets and also discussed about multi-resolution analysis (MRA). We discussed about discrete and stationary wavelet transform and the mathematical background of a 1D DWT on a signal in brief, then we extended our discussion for the 2D DWT for images. In this part, we also discussed the advantages and disadvantages of SWT and compared the subbands obtained by SWT and DWT.

In the second half, we have implemented and analyzed techniques for image enhancement using DWT-SWT method and satellite images enhancement using DWT based method. The results obtained using both the methods were compared to a tradational method known as wavelet zero padding (WZP) and found that in both the cases the technique used outperform the WZP method.

ABBREVIATIONS

HR	High Resolution
LR	Low Resolution
SR	Super Resolution
MSE	Mean Square Error
SNR	Signal To Noise Ratio
PSNR	Peak Signal to Noise Ratio
2D	2 Dimensional
1D	1 Dimensional
NN	Nearest Neighbour
WZP	Wavelet Zero Padding
DWT	Discrete Wavelet Transform
IDWT	Inverse Discrete Wavelet Transform
SWT	Stationary Wavelet Transform
FT	Fourier Transform
DST	Discrete Sine Transform
DCT	Discrete Cosine Transform
STFT	Short-Time Fourier Transform
DIP	Digital Image Processing
IE	Image Enhancement
MRA	Multi-resolution Analysis

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Chapter 1

Introduction

To write well, express yourself like the common people,
but think like a wise man.
- Aristotle

Images play an important part in our day to day life. Humans perceive most of the information through their vision and images are storehouse of information. The information content of an image is decided by the resolution of the image, for a lower resolution image, the information content is low, for a higher resolution image, the information content is high. Its natural, for us to demand a HR image, but cost and resolution are inversely proportional. There is, and will always be a tradeoff between the cost and resolution. The information present in the LR images can be useful, if we can perform some kind of transformations and extract the information out of the LR images. This can be done by enlarging and enhancing the LR images, the image thus produced will be a HR image.

1.1 Aim

The aim of this dissertation is to study the image enhancement techniques using the wavelet transforms like, WZP, DWT, SWT. In our case, image enhancement is done by enlarging the LR image using wavelet transform based interpolation techniques to acheive an HR image which contains relatively more information.

1.2 Motivation

Image Enhancement (IE) is an invaluable asset for researchers, to exploit a wide variety of fields –

1. IE techniques are great boon for the medical field, as every minute detail is critical for the diagnosis, IE techniques are applied on MRI, ultrasound and various other reports to reduce the noise and improve the image.

2. IE techniques are used in the field of forensic science, to enhance the images obtained in crime scenes, such as fingerprint, videos obtained by CCTV cameras, etc which are generally LR images.

3. IE techniques are used to enhance the satellite images which are useful for military purposes, forest cover predictions, etc.

Recent rise of the drone cameras, have also increased the need of IE. We no longer need to depend upon satellite images for the aerial view, we can develop our own drones with not so sophisticated cameras, and can enhance the LR images taken, and analyze the HR images.

1.3 Chapterization

This thesis is organised as –

- ***Chapter 2:*** This chapter gives the theoretical background of wavelets and their history. It also discusses in brief about the multi-resolution analysis (MRA)

- ***Chapter 3:*** This chapter discusses about the discrete wavelet transform (DWT) and its mathematical background for 1D and 2D discrete signals. This chapter also discusses the stationary wavelet transform (SWT) and its pros and cons.

- ***Chapter 4:*** In this chapter, the DWT-SWT method for IE has been discussed and implemented, the effectiveness of the DWT-SWT method is compared with that of WZP method, by calculating the PSNR values. This chapter also discusses about interpolation and its three standard types namely, nearest-neighbor method, bi-linear method and bi-cubic method.

- ***Chapter 5:*** In this chapter, the DWT method for IE has been discussed, implemented, the effectiveness of the DWT method is compared with that of WZP method, by calculating the PSNR values. This chapter also dicusses the WZP method for IE.
- ***Chapter 6:*** This chapter concludes the thesis with a disussion of the work and lists some future work in this regard.

Chapter 2

Theoretical Background

The more extensive a man's knowledge of what has been done,
the greater will be his power of knowing what to do.
- Benjamin Disraeli

Introduction

In past decades, there has been an abrupt rise in the stature of the wavelets transform in the field of image processing as an alternative to the established **Fourier Transform (FT)** and transforms related to it, such as **Discrete Sine Transform (DST)** and **Discrete Cosine Transform (DCT)**. Fourier's idea was that given any complex function, it can be approximated as a weighted sum of simple functions, which are in turn obtained from basic functions like *sine* and *cosine* functions. Thus, the original signal can be approximated or can be fully represented under certain conditions. In Fourier theory, any signal with a period 2π can be represented as an infinite sum of sines and cosines, making FT highly suitable for infinite and periodic signals. For several years, the FT dominated the field of signal processing. It succeeded in providing the frequency information contained in the analyzed signal, but it failed to give any information about the occurrence time. This shortcoming motivated the scientists to look for new transforms. The first step in this long research journey was to cut the signal of interest in several parts and then to analyze each of these parts separately. The idea at a first glance seemed to be very promising since it allowed the extraction of time information and the localization of different frequency components. This approach is known as the **Short-Time Fourier Transform (STFT)**. The fundamental question, which then arose was, how to cut the signal? The best solution to this dilemma was to find a fully scalable modulated window in which no signal cutting is needed anymore. This goal was achieved successfully by the use of the **wavelet transform** [1].

2.1 Wavelets

Wavelets are mathematical functions that decompose the given signal with respect to different frequency components and then analyze each of the frequency components with a resolution corresponding to their scales.

Suppose $\psi \in L^1 \cap L^2$, be a well localized and regular function. If the function ψ, in the frequency domain, satisfies the property that

$$\int_{\mathbf{R}^+} \frac{|\overline{\psi}(w)|^2}{|w|} dw = \int_{\mathbf{R}^-} \frac{|\overline{\psi}(w)|^2}{|w|} dw < +\infty$$

where $\overline{\psi}$ is the FT of ψ, then the function ψ, is called a **wavelet**. This condition implies that the value of the wavelet integrated over whole of $\mathbf{R}$ is zero.

2.2 History of wavelets

It is assumed that wavelet analysis, has different origins, as per the history of mathematics [2].

Prior to 1930, the mathematical study of wavelets began with Josheph Fourier's theory related to frequencies in 1807. He stated that any periodic function $f(x)$ with period 2π can be represented as the sum

$$a_o + \sum_{n=1}^{\infty} (a_n cos(nx) + b_n sin(nx))$$

where the coefficients are given by

$$a_o = \frac{1}{2\pi} \int_0^{2\pi} f(x) dx$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)cos(nx)dx$$
$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)sin(nx)dx$$

This is known as Fourier series.

In 1909, A.Haar, mentioned about the Haar wavelet for the first time in his thesis. Haar wavelets are defined as

$$\psi(x) = \begin{cases} 1 & x \in (0, \frac{1}{2}) \\ -1 & x \in (\frac{1}{2}, 1) \end{cases}$$

Haar wavelets are compactly supported and are easily computable, but it is not continuously differentiable, limiting their application.

In the 1930's a physicist, Paul Levy, used the Haar basis functions to study the Brownian motion [3] and found Haar basis was better than Fourier basis functions, to investigate the Brownian motion.

In 1980, a physicist and an engineer, Grossman and Morlet defined wavelets explicitly in context of quantum physics. In 1985, Stephane Mallat, discovered the relationship between quadrature mirror filters, pyramid algorithms and orthonormal wavelet bases, which introduced wavelets in digital signal processing. Based on Mallat's results, the first non-trivial, continuously differentiable wavelets were constructed by Y. Meyer, and a set of wavelet orthonormal basis function were created by Ingrid Daubechies.

2.3 Multi-resolution analysis (MRA)

MRA is a concept for constructing the biorthogonal and orthonormal transformation using wavelets [4].

Definition – A set of sub-spaces

$$... \subset V^{(3)} \subset V^{(2)} \subset V^{(1)} \subset V^{(0)} \subset V^{(-1)} \subset V^{(-2)} \subset V^{(-3)} \subset ...$$

on L^2 is called **multi-resolution analysis (MRA)**, if the following conditions are satisfied –

1. $\bigcup_{n \in \mathbf{Z}} V^{(n)} = L^2(\mathbf{R})$
2. $\bigcap_{n \in \mathbf{Z}} V^{(n)} = 0$
3. $x(t) \in V^{(0)} \Leftrightarrow x(2^{-n}t) \in V^{(n)}$
4. $x(t) \in V^{(0)} \Leftrightarrow x(t - n) \in V^{(0)}$
5. *There exist an orthonormal basis* $\{\phi_n\}_{n \in \mathbf{Z}}$, *for* $V^{(0)}$ *such that* $\phi_n(t) = \phi(t - n)$. *This function* $\phi(t)$ *is called* ***scaling function***.

2.4 An Introduction to Wavelets

Amara Graps [2], in this paper, introduced about wavelets very briefly for the beginners. The paper is divided into three parts. The first part of the paper describes the history of wavelets, starting from the early 1800's when Fourier gave his theory on frequencies, and how different wavelets like Haar, Meyers and such wavelets were invented. The second part of the paper compares the wavelet transforms with Fourier transforms, their similarities, their dissimilarities, how wavelet transforms are superior to Fourier transforms. This parts also states the properties and special features like adapted waveforms and wavelet packets, of wavelets. The third part of this paper describes some of the interesting applications in the field of image processing such as image compression, human vision, musical tones and de-noising noisy data.

2.5 Wavelet Analysis for Image Processing

Wavelet transforms have become increasingly important in image compression since wavelets allow both time and frequency analysis simultaneously. Tzu-Heng Henry Lee [5], describes the fundamental concept behind the wavelet transform. The author talks about the Heisenbergs Uncertainty Principle and how it imposes a lower bound on the area of the time frequency window in case of the Windowed Fourier Transform (WFT). This paper also provides an overview of some improved algorithms on the wavelet transform such as pyramidal algorithm, wavelet transform in two dimensions. The latter part of this paper emphasize on lifting scheme which is an improvization technique based on the wavelet transform.

2.6 A Theory of Multiresolution Signal Decomposition - The Wavelet Representation

The multi-resolution decomposition is an effective way to represent a digital image and analyze the information present in it. Analyzing the image at every resolution is absurd as most of the information present is redundant. Stephane Mallat [6], proposed a novel method of decomposing the signal in $L^2(\mathbf{R}^n)$ using the wavelet orthonormal basis, to obtain the difference in the approximation of the given signal at resolutions 2^m and 2^{m+1}.

In $L^2(\mathbf{R})$, the family of functions, defined as

$$(\sqrt{2^m}\psi(2^m x - n))_{(m,n)\in\mathbf{Z^2}}$$

is called as **wavelet orthonormal basis**. This decomposition is defined as **wavelet representation**. This method reduces the effort in analyzing the signal at every resolution level. So, if a signal is processed at a resolution $\mathbf{r}_0$, it is more efficient to analyze only the additional details of the image at a higher resolution $\mathbf{r}_j$. This type of decomposition are more useful in edge detection, signal coding, etc.

2.7 Image Resolution Enhancement by Using Discrete and Stationary Wavelet Decomposition

Over the years, many IE methods have been developed to enhance the satellite images, like interpolation, WZP, etc. Hasan Demirel and Gholamreza Anbarjafari

[7], proposed a method to enhance a LR image by parallely decomposing it using DWT, SWT methods. The corrected subbands are calculated by adding interpolated subbands of DWT with corresponding SWT subbands and IDWT is used to obtain the enhanced image. The paper also demonstrates the superiority of the method wrt the tradational IE techniques such as interpolation, WZP etc.

2.8 Discrete Wavelet Transform-Based Satellite Image Resolution Enhancement

Hasan Demirel and Gholamreza Anbarjafari [8], proposed a method similar to [7], for enhancing the images taken from the satellites. In this method the low resolution images taken by the satellites are decomposed using DWT, the subbands are interpolated and corrected subbands are estimated using the difference of image and the low frequency subbands. IDWT is used to obtain the enhanced image. The SWT used in the [7], produces redundant information of the frequencies in the subband. This method is free from the redundant computation and produces approximately equivalent results produced using DWT-SWT method proposed in [7].

Chapter 3

Discrete Wavelet Transform (DWT)

As a rule, men worry more about what they can't see than about what they can.
- Julius Caesar

3.1 Introduction

In chapter 2, we had defined wavelet ψ and the scaling function ϕ in brief. We can express any continuous function with the wavelet function ψ and scaling functions ϕ, which are by themselves continuous, except the fact that the translation as well as the scaling of ψ and ϕ are discrete.

Digital images are descrete signals. If we want to apply the concept of wavelets on the images, we need to work with the discrete form of the wavelets, namely discrete wavelet transform (DWT).

In the first half of this chapter, we will define DWT and its mathematical background for a 1-D signal and application of DWT on 2-D signals i.e. digital images. In the later part of this chapter, we will briefly discuss the stationary wavelet transform (SWT), its advandtages and disadvantages.

3.2 Discrete Wavelet Transform (DWT)

A discrete wavelet transform for a sequence $x(n)$ with N discrete points, where $0 \leq n \leq (N-1)$ in 1-D , is mathematically defined as

$$DWT = f(t) = W_{\psi}(j_0, k) + W_{\phi}(j, k) \tag{3.1}$$

where

$$W_{\psi}(j_0, k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x(n)\phi_{j_0,k}(n) \tag{3.2}$$

$$W_{\phi}(j, k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x(n)\psi_{j,k}(n) \tag{3.3}$$

and $j \geq j_0$.

The DWT coefficients W_{ϕ} and W_{ψ} can be used to regenerate the discrete sequence $x(n)$ as

$$x(n) = \frac{1}{\sqrt{N}} W_{\phi}(j_0, k)\phi_{j_0,k}(n) + \frac{1}{\sqrt{N}} \sum_{j=j_0}^{\infty} \sum_{k} W_{\psi}(j, k)\psi_{j_0,k}(n) \tag{3.4}$$

Figure 3.1 shows that, given a discrete signal $X[n]$ in the time domain, in order to obtain the DWT, low pass and high pass filters are applied to the signal successively. The DWT decomposes the signal into two categories namely, approximation and details. The detail information determines the resolution aspect of the signal, and the scale at which the DWT is applied is determined by two operations – upsampling and downsampling.

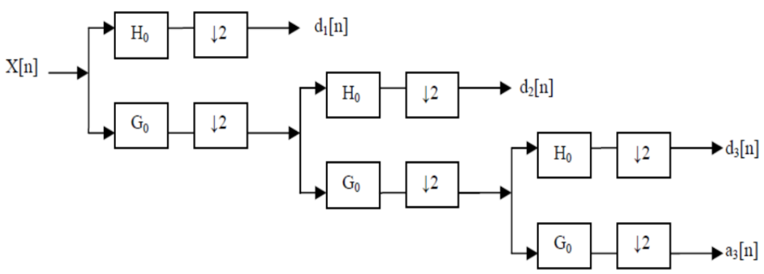


Figure 3.1: Three level wavelet decomposition

In Figure 3.1, the sequence $X[n]$ denotes the time domain signal, where $n \in \mathbf{N}$. G_0 and H_0 represent the low filter and high filter respectively. At every level, the coarser approximations $a_i[n]$ of the signal is produced by the low pass filter G_0 and the detail information $d_i[n]$ is produced by the high pass filter H_0, by downsampling by 2 and i denotes the level of decomposition.

When we downsample by 2, at every level, the filters half the frequency band of the signal, as a result of which there is a reduction of uncertainity in the signals' frequency, which in turn doubles the frequency resolution. As the frequency of the signal is halved, the resolution is also halved, doubling the scaling as a byproduct. The time resolution is also halved, due to downsampling because now using only half of the samples of the signal, we represented the signal.

The summary of the above discussion is that we should use DWT on signals which have either low frequency for more duration, or high frequency for short duration. The reason is that, for a low frequency signal DWT produces better frequency resolution and poor time resolution and for a high frequency signals the time resolution is better whereas the frequency resolution is poor.

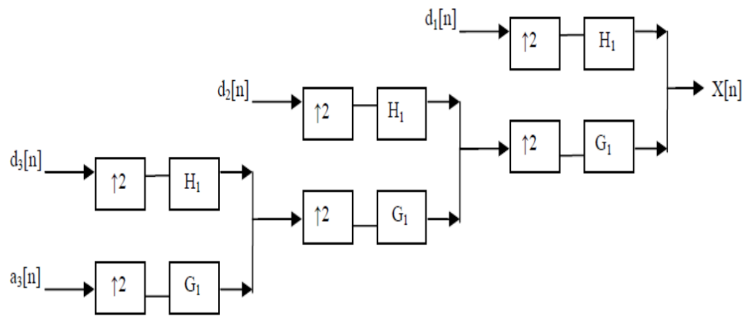


Figure 3.2: Three level wavelet reconstruction

Figure 3.2, shows the three level block diagram for the reconstruction of the signal using the wavelet decomposition coefficients. In reconstruction method, at every level, we upsample the detail as well as the approximation coefficients by a factor of 2. The coefficents are then filtered using the synthesis filters G_1 and H_1, then the coefficients are added to produce the original signal.

3.3 Discrete Wavelet Transformation in Two Dimensions

In previous section 3.2, we saw DWT in 1D. Images are two dimensional discrete signals, to apply DWT on images we need to use a 2D DWT which is separable. In 2D DWT, the wavelet coefficents become a function of two variables.

The wavelet and scaling function for a 2D DWT is denoted as $\psi(x, y)$ and $\phi(x, y)$. We define the translation basis function and the scaling basis functions as –

$$\psi_{j,m,n}(x, y) = 2^{\frac{j}{2}}\psi^{i}(2^{j}x - m, 2^{j}y - n), i = \{H, V, D\} \tag{3.5}$$

$$\phi_{j0,m,n}(x, y) = 2^{\frac{j}{2}}\phi(2^{j}x - m, 2^{j}y - n) \tag{3.6}$$

Thus we see that, in a 2D DWT we have three wavelet functions $\psi^{H}(x, y)$,$\psi^{V}(x, y)$, $\psi^{D}(x, y)$, of two variables, unlike a single wavelet function of one variable in case of 1D DWT, and only one scaling function but of two variables.

The scaling and the wavelet functions are separable due to the translation basis function and the scaling basis function, and we know that if the function is separable, it can be written as $f(x, y) = g(x)h(y)$. Similaly, the scaling and the wavelet functions can be written as –

$$\phi(x, y) = \phi(x)\phi(y) \tag{3.7}$$

$$\psi^{H}(x, y) = \psi(x)\phi(y) \tag{3.8}$$

$$\psi^{V}(x, y) = \phi(x)\psi(y) \tag{3.9}$$

$$\psi^{D}(x, y) = \psi(x)\psi(y) \tag{3.10}$$

Let us consider an image of size (MxN), then the horizontal, vertical and diagonal directions are denoted as *H,V,D*, $W_\phi(j_0, m, n)$ denotes the approximation coefficient at j_0^{th} scale. The horizontal, vertical and diagonal detail coefficients are represented as $W_\psi^H(j, m, n)$, $W_\psi^V(j, m, n)$, $W_\psi^D(j, m, n)$, where j is the scale and $j \geq j_0$[5].

The wavelet coefficients are defined as –

$$W_\phi(j_0, m, n) = \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y)\phi_{j_0,m,n}(x, y) \tag{3.11}$$

$$W_\psi^H(j, m, n) = \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y)\psi_{j,m,n}^H(x, y) \tag{3.12}$$

$$W_\psi^V(j, m, n) = \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y)\psi_{j,m,n}^V(x, y) \tag{3.13}$$

$$W_\psi^D(j, m, n) = \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y)\psi_{j,m,n}^D(x, y) \tag{3.14}$$

The inverse discrete wavelet transform (IDWT) to reconstruct the image from the wavelet coefficients is defined as –

$$f(x, y) = \frac{1}{\sqrt{MN}} \sum_m \sum_n K_\phi + \frac{1}{\sqrt{MN}} \sum_{i=H,V,D} \sum_{j=j_0}^{\infty} \sum_m \sum_n K_\psi \tag{3.15}$$

where, $K_\phi = W_\phi(j_0, m, n)\phi_{j_0,m,n}(x, y)$

$K_\psi = W_\psi^i(j, m, n)\psi_{j,m,n}(x, y)$

It is interesting to note that a 2D DWT is similar to applying 1D DWT serially. When we apply 2D DWT on the original image, high pass and low pass filters are applied on the columns of the original image first and then the high pass and low pass filters are applied on the filtered image. This happens for the 1st level of decomposition, if we want to decompose the image at 2^{nd} level then the approximate coefficients become the input image for the 2^{nd} iteration of DWT. The rows and the columns are reduced by a multiple of 2 when the filters are applied at every level.

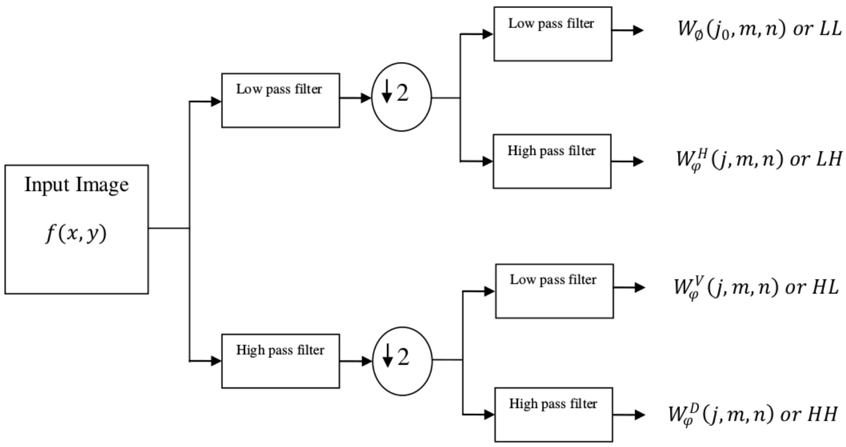


Figure 3.3: One level 2D DWT

Figure 3.3 shows that when we apply 2D DWT on the input image, we will obtain four subimages namely approximate, horizontal, vertical and diagonal. The approximation coefficients are obtained when the low-pass filter is applied on both rows and columns. The approximate image is similar to the input image, but with reduced resolution. When the low-pass filter is applied on the rows and the high-pass filter is applied on the input image, we obtain the horizontal coefficients, which captures the details about the horizontal edges. Similarly, when high-pass filter is applied on the rows and low-pass filter is applied on the columns of the input image, we obtain the vertical coefficients, which captures the details about vertical edges. Finally when high-pass filter is applied on both rows and columns of the input image, we obtain the diagonal coefficients, which captures the details about diagonal edges.

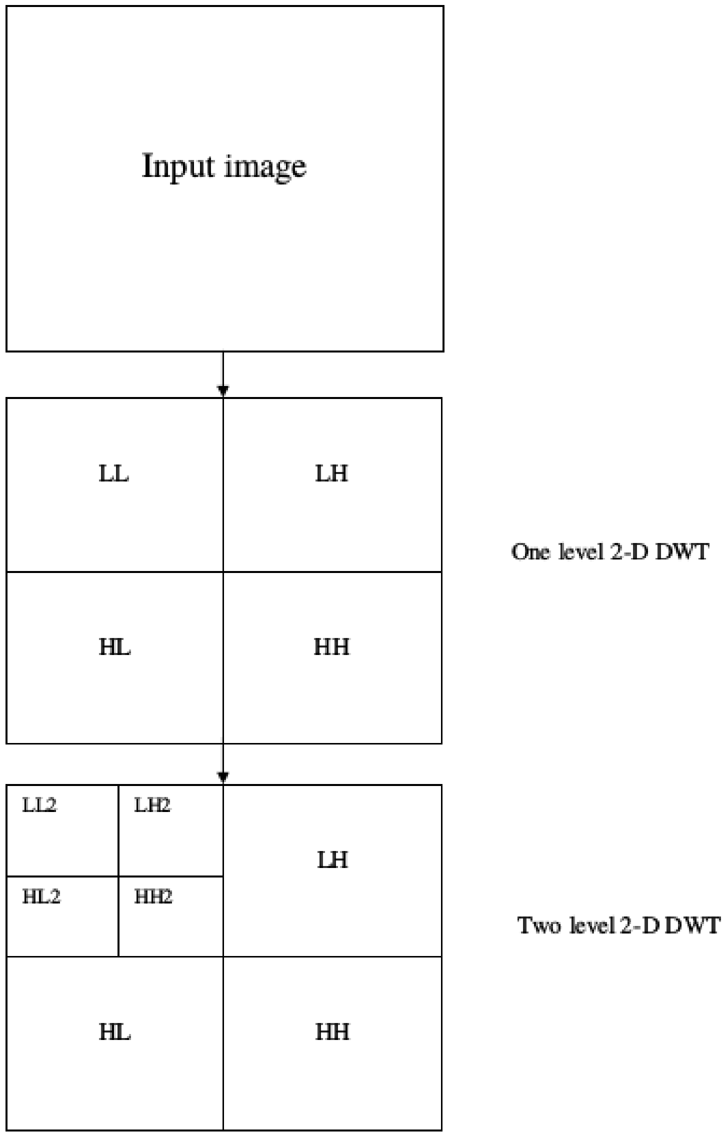


Figure 3.4: Block diagram of subbands generated by 2D DWT

In Figure 3.4, number of subbands generated for one level 2D DWT is four, with one approximation coefficient (LL) and three detail coefficients (LH, HL, HH). In case of two level 2D DWT, the approximation coefficient (LL) acts as an input image so the number of subbands generated will be seven, and three detailed coefficients (LH, HL and HH) in first level, at the second level we obtain one approximation coefficient (LL2) and three detail coefficients (LH2, HL2 and

HH2).

3.4 Stationary Wavelet Transform (SWT)

The SWT is a wavelet transform algorithm proposed by Holschneider *et.al.* [9]. DWT is time invariant, however it is not translation invariant. To overcome this shortcoming of the DWT, the SWT algorithm was designed.

In SWT algorithm, the filtering coefficients in the j^{th} resolution level are up-sampled by a factor of $2^{(j-1)}$ and the up samplers as well as the down samplers of the DWT method were removed to obtain the translation invariance of the wavelet decomposition [10].

In every resolution level, the number of samples are equal for both the input and the output produced by the SWT. Whereas the output of the DWT contains half the number of input samples. Thus, the output of the SWT contains a large amount of redundant data, because there will be redundancy of N in the wavelet coefficients for a N level decomposition using SWT.

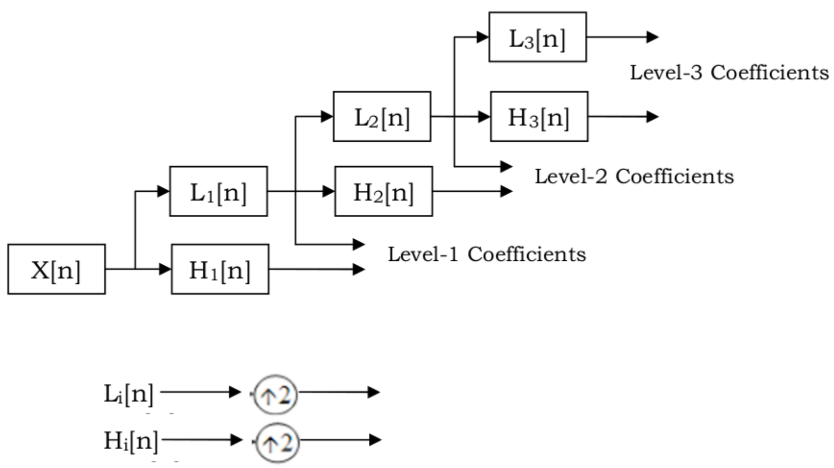


Figure 3.5: 3 level SWT filter bank

Figure **??**, shows the block diagram of the 3 level SWT filter bank. This diagram clearly shows the rendundancy in the SWT algorithm, because the filters obtained in the previous level are upsampled to obtain new filters in the next level.

Advantages of SWT

1. Unlike DWT, SWT is translation invariant. i.e. the wavelet coefficients obtained using SWT remains unchanged even though the input signal is shifted.

2. Since DWT halves the number of samples present in the output, so DWT is efficient only on images of size 2^n where $n \in \mathbf{N}$, but we can apply SWT on images of arbitrary size also.

3. As SWT is linear, translation invariant and contains redundant information, it provides better approximation as compared to the DWT.

4. Due to its redundant nature, SWT performs better in edge detection, image fusion etc, compared to DWT, where there is a loss of information as the number of output samples are less.

Disadvantages of SWT

1. Time taken to decompose an image is high in SWT as compared to DWT.

2. Since SWT contains same number of wavelet coefficients at every level, SWT requires larger space for storage and also has high computational complexity compared to DWT.

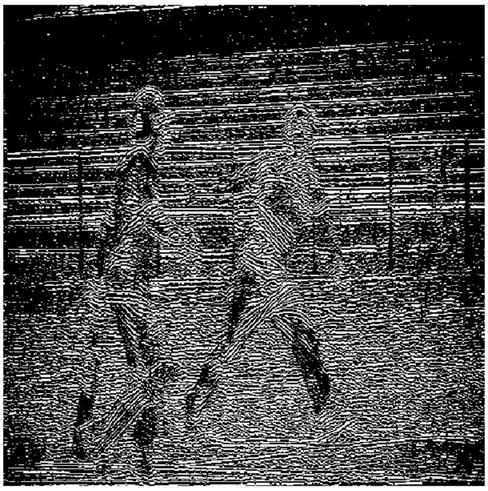
3. Due to the high computational complexity and high time complexity, SWT is ineffective for real time applications.

3.5 Comparision of SWT and DWT

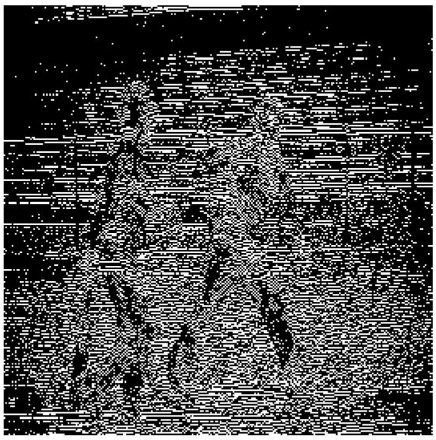
(a) SWT (512x512)

(b) DWT (256x256)

Figure 3.6: Approximation coefficients for image of size (512x512)

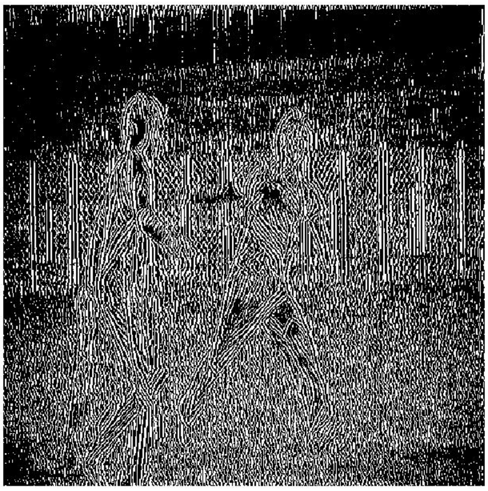


(a) SWT (512x512)

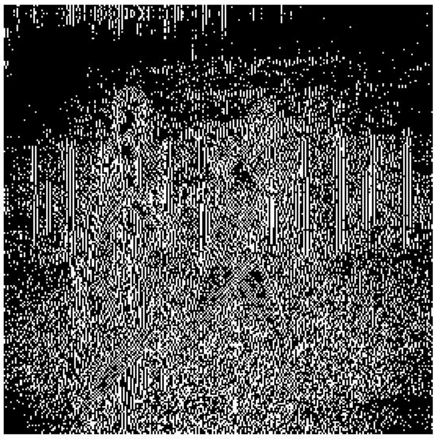


(b) DWT (256x256)

Figure 3.7: Horizontal coefficients for image of size (512x512)



(a) SWT (512x512)



(b) DWT (256x256)

Figure 3.8: Vertical coefficients for image of size (512x512)

(a) SWT (512x512)

(b) DWT (256x256)

Figure 3.9: Diagonal coefficients for image of size (512x512)

3.6 Discussion

The images in figures 3.6, 3.7, 3.8 and 3.9 are scaled by 0.3 of their original size. The input image taken was of size (512x512) and DWT as well as SWT is applied on the input image. We find that in figure 3.6, the approximation coefficient produced by SWT is of larger resolution. In figure 3.7, the horizontal details in case of SWT is much finer than that of DWT. The bowler's hand movement is captured better in SWT where as its difficult to distinguish the bowler's arms in horizontal coefficient produced by DWT. In figure 3.8, the vertical coefficient in SWT is more clearer than that of DWT. In figure3.9, the outline of both the batsman as well as bowler is clearly captured by the diagonal coefficients of SWT than that of DWT.

We clearly see that SWT outperforms DWT in the feild of decomposition, this is all due to the redundant nature of SWT. However, DWT is much faster to compute the approximation and details coefficients, so is much better than SWT for real time scenarios. So, both DWT and SWT have their pros and cons. In next chapter, we will see how the merits of both SWT and DWT can be combined to produce an enhanced image.

Chapter 4

Image Resolution Enhancement Using Discrete and Stationary Wavelet Decomposition

By three methods we may learn wisdom;
First, by reflection, which is noblest;
Second, by imitation, which is easiest;
and third by experience, which is the bitterest.
- Confucius

4.1 Introduction

Humans depends heavily on their vision to picturize and understand the world around them. Humans can process a large amount of visual data very quickly. Images have become one of our most important medium to convey this visual information. An image can be assumed as a two dimensional function $f(x, y)$ where x, y are spatial co-ordinates.The amplitude of f at (x, y) is called the gray level or the intensity of the image at that point. Suppose x, y and the intensity value of f are all finite and descrete quantities, then the image is called, a **digital image** [5].

Like how images play an important role in our life, resolution has the same importance in an Image. **Resolution** refers to the sharpness of the details of an image, smoothness of its curved lines and a faithful reproduction of image. Resolution is a primal aspect for an image, it decides the clarity of an image. If an image has a high resolution then they are clearer and it is easier to extract more detailed information from them. Every image need not have high resolution, the information extracted from a low resolution image may not contain a lot of information. However, they still contain some amount of useful data to be extracted. As the saying goes "In God we believe, but In Data we trust". Each and every little detail is important. In order to extract more information from a low resolution image we need to increase its resolution, this is called **Resolution Enhancement**. The output of a resolution enhancement process is a super resolved image. **Super resolution** is a process in which multiple low resolution images are combined to produce a high resolution image.

Image enhancement technique is a part of Digital Image Processing (DIP). The image enhancement techniques improve the quality of the image as perceived by a human. Image enhancement techniques are being used in various image and video processing, such as content based image retrieval, machine vision, feature extraction, etc. In order to enhance the resolution of an image we need to perform image transformation or increase the number of pixels of the image using interpolation.

Image resolution enhancement finds its application in many fields. Few of which are listed below

- Satellite imaging.
- Video standard conversion.
- Medical image enhancement.
- Video enhancement and restoration.
- Astronomical imaging.

This chapter is divided into the following sections —

Section 2. Brief discussion about interpolation.

Section 3. Methodology of SWT-DWT technique for image enhancement.

Section 4. Algorithm.

Section 5. Experimental results.

Section 6. Conclusion.

4.2 Interpolation

Interpolation is a method which determines the value of a given function at a point which lies in between the sample points. An interpolation function can be imagined as a special type of approximation function, with a property that the functions must coincide with the data sample that was sampled at the interpolation nodes. In simple terms, suppose f is a function that is sampled, then the function g is called an interpolation function iff $g(x_k) = f(x_k)$, where x_k is the interpolated node. Suppose that the data is equally spaced then we can write the interpolation function as

$$g(x) = \sum_k c_k u\left(\frac{x - x_k}{h}\right)$$

where g is the interpolation function, c_k are parameters dependent on sample data, u is the interpolation kernel, x_k are the interpolated nodes and h denotes the increment while sampling. The values are chosen in such a way that the fundamental property of interpolation i.e $g(x_k) = f(x_k)$, is always satisfied for any arbitrary x_k.

Figure 4.1, shows that if we convolute a discrete signal $y(n)$ with n points with an interpolating function $h(t)$, we obtain the interpolated signal $y(t)$. This is the interpolation step. In figure 1.(b), if the interpolated signal is again convoluted with a sampling function with n' discrete points, then we obtain a resampled signal containing n' points.

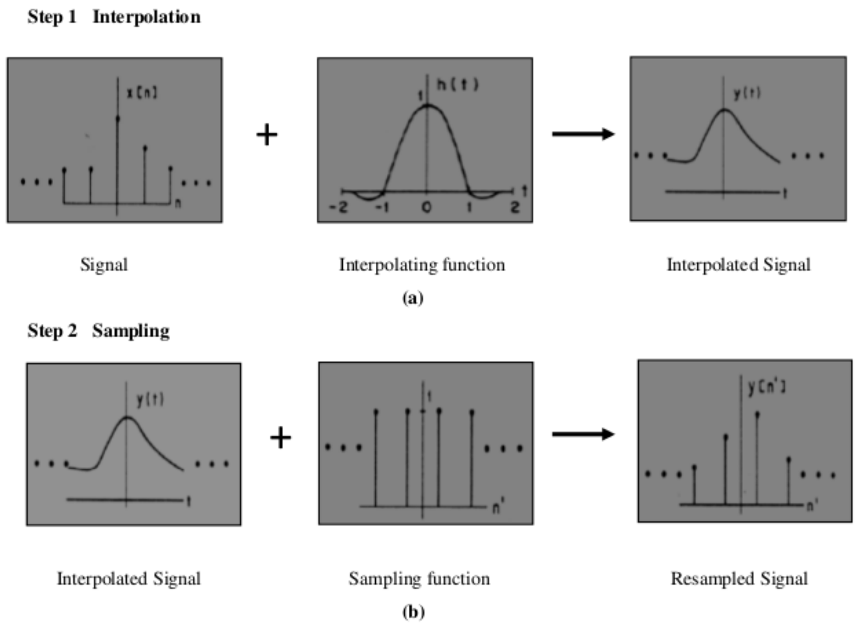


Figure 4.1: Resampling Process: (a) Interpolation, (b) Sampling

4.2.1 Image Interpolation

Image interpolation is a way to produce a larger image from a smaller image. Interpolation is a method to compute the missing pixel value in an image by considering the neighboring pixels. The operations in image processing can be broadly divided into two categories – spatial domain techniques and frequency domain techniques. The spatial domain techniques operate directly on the pixel values whereas the frequency domain techniques operate on the frequency information of an image or a video. Suppose we have a low resolution matrix of size 3x3, then a matrix of 6x6 is generated by adding new pixels through interpolation, as shown in figure 4.2.

4.2.2 Interpolation Methods

Over the years, many interpolation methods has been developed for enhancing the image resolution, but the nearest neighbor, Bi-linear and Bi-cubic interpolation methods are frequently used.

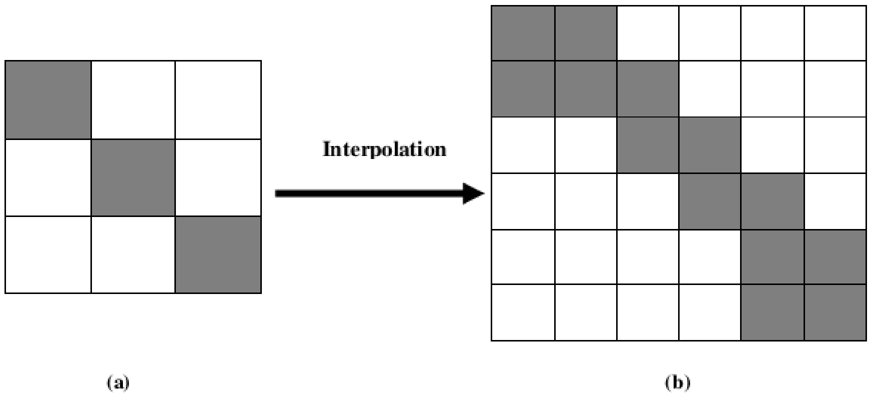


Figure 4.2: Resolution enhancement of (a) 3x3 image to (b) 6x6 image, using interpolation

4.2.2.1 Nearest Neighbor Interpolation

Nearest Neighbor interpolation is one of the simplest interpolation from computational point of view. In this method the interpolated pixel is assigned a value of the nearest sample point in the input image. This technique is also known as pixel replication or point shift algorithm. Figure 4.3 shows the effect of nearest neighbor interpolation, the missing pixels takes the value of the nearest pixel.

In figure 4.4, the original image is of size (256x256) and the interpolated image obtained is of size(512x512). The interpolation technique used is nearest neighbor. The images are displayed at a scale of 0.3 of the actual size of the images. The interpolated image is bigger in size compared to the input image, we also see that the interpolated image has lot of block appearence.

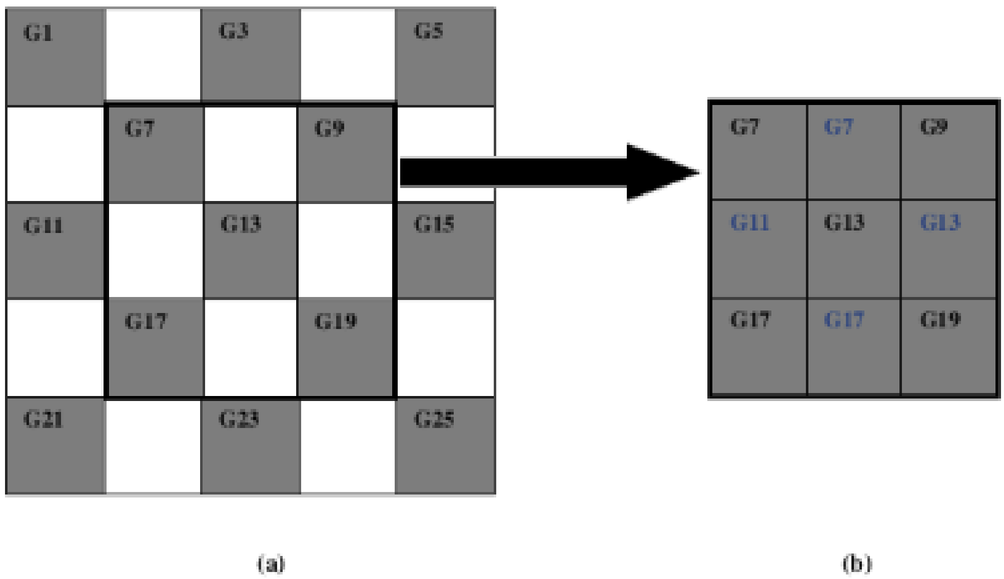
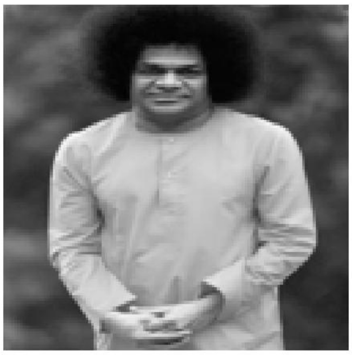
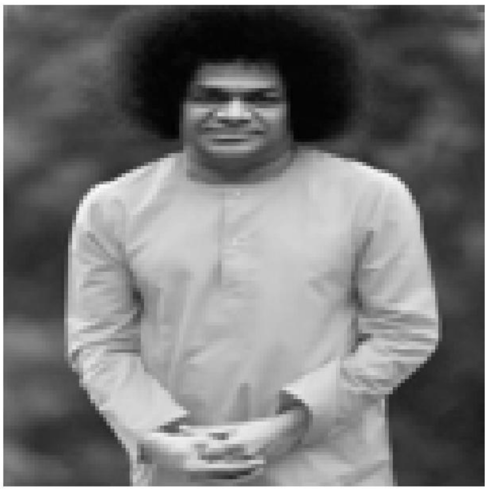


Figure 4.3: Nearest Neighbor Interpolation (a) Before (b) After, interpolation



(a) Original image (256x256)



(b) Interpolated image (512x512)

Figure 4.4: Nearest neighbor interpolation

4.2.2.2 Bi-linear Interpolation

In bi-linear interpolation the new pixel which is generated is the weighted average of the nearest 4 pixel values in the original image. Figure 4.5 shows the basic concept of the bi-linear interpolation.

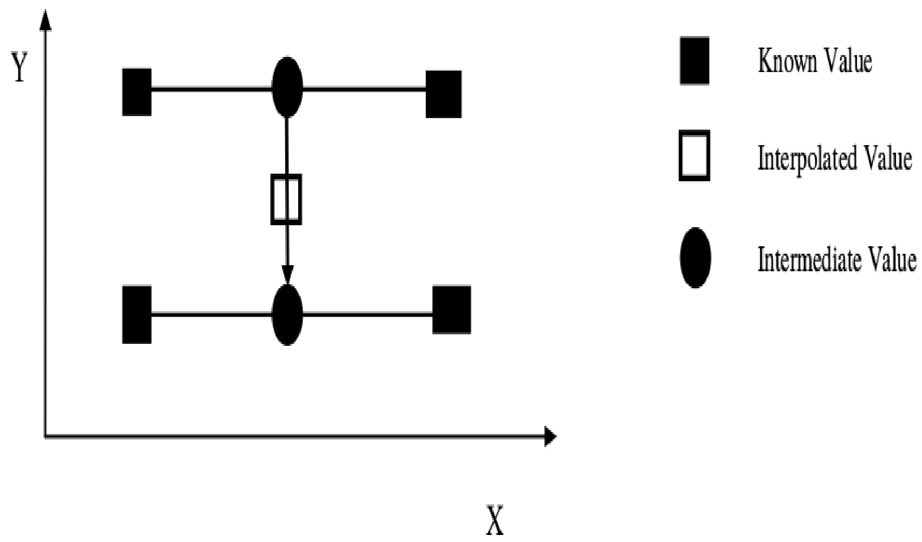
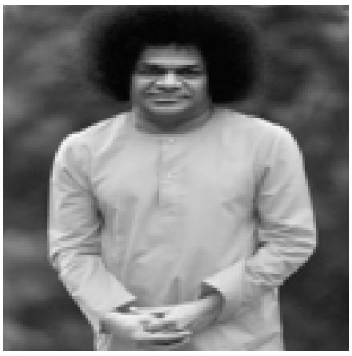


Figure 4.5: Bi-Linear Interpolation

In figure 4.6, the original image is of size (256x256) and the interpolated image obtained is of size(512x512). The interpolation technique used is bi-linear. The images are displayed at a scale of 0.3 of the actual size of the images. Image generated by bi-linear interpolation have much smoother appearance than the image generated by nearest neighbor in figure 4.4, however, we see that in figure 4.6, the image is blurred and there is a loss in image resolution.



(a) Original image (256x256)

(b) Interpolated image (512x512)

Figure 4.6: Bi-linear interpolation

4.2.2.3 Bi-cubic Interpolation

Bi-cubic interpolation is more or less similar to bi-linear interpolation, the difference is instead of using only 4 known pixel values to determine the unknown pixel value like in bi-linear case, bi-cubic uses 4x4 area surrounding the unknown pixel value to be determined. i.e. it uses weighted average of 16 known pixel values to determine the unknown pixel. The image produced by bi-cubic interpolation in Figure 4.7 shows the basic concept of bi-cubic interpolation.

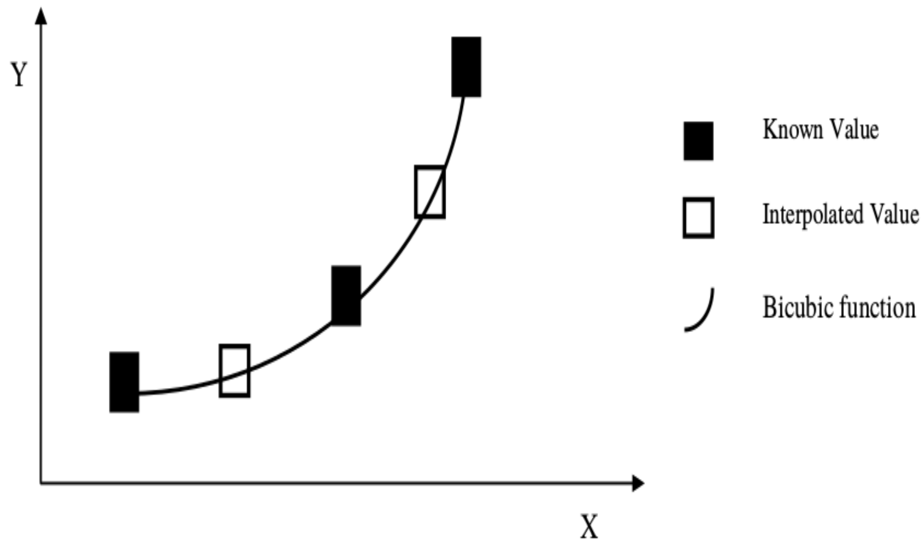
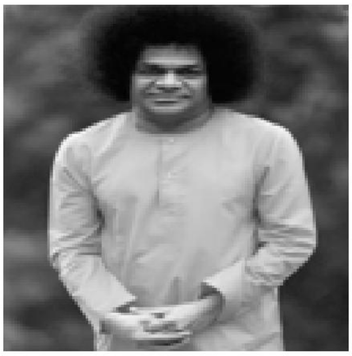


Figure 4.7: Bi-cubic Interpolation

In figure 4.8, the original image is of size (256x256) and the interpolated image obtained is of size(512x512). The interpolation technique used is bi-linear. The images are displayed at a scale of 0.3 of the actual size of the images. The bicubic interpolated image in figure 4.8 is sharper than that produced by the bi-cubic in Figure 4.6, and also there are no block apperance in the image as in case of nearest neighbor as in case of Figure 4.4. Thus we conclude that the bi-cubic interpolation produces the best missing pixel value and is more sophisticated.



(a) Original image (256x256)

(b) Interpolated image (512x512)

Figure 4.8: Bi-cubic interpolation

Even though the interpolation method enhances the resolution of an image, but they alone are of no use since they cannot produce high frequency parts of an image. The interpolation techniques cannot predict better information of the edges since, the pixels at the edges have lesser pixels surrounding it than the pixels at the centre.

After stating the limitation of image resolution enhancement using interpolation in spatial domain, we will study the effect of image resolution enhancement in frequency domain. In the frequency domain, as a mathematical tool, wavelets are used to extract information from the images. Wavelet transforms can be classified as continuous or discrete wavelet and multiresolution based transforms.

4.3 Overview of the DWT-SWT Method for Image Enhancement

The method proposed by Hasan Demirel and Gholamreza Anbarjafari [7] uses DWT, SWT and bi-cubic interpolation techniques to enhance a low resolution input image to produce a sharper high resolution image. DWT is applied to decompose the low resolution image into 4 subbands. One of the four subbands produced is low frequency subband whereas rest all are high frequency subbands. The high frequency subbands are interpolated using bi-cubic interpolation technique. Meanwhile, SWT is also applied to decompose the low resolution image into different subbands. The subbands of SWT and the interpolated subbands of DWT are added to estimate high frequency subbands. Parallely, the low resolution input image is also interpolated, then the estimated high frequency subbands and the interpolated input image are combined using inverse DWT (IDWT), to produce a high resolution output image. The enhanced image produced by SWT-DWT technique is compared by the high resolution image produced by wavelet zero padding (WZP) method.

4.3.1 Algorithm

Image enhancement algorithms use degraded high resolution image as input low resolution image. The input image is produced after downsampling the image obtained from low pass filtering the high resolution image.

Figure 4.9 gives an overall view of the DWT-SWT method

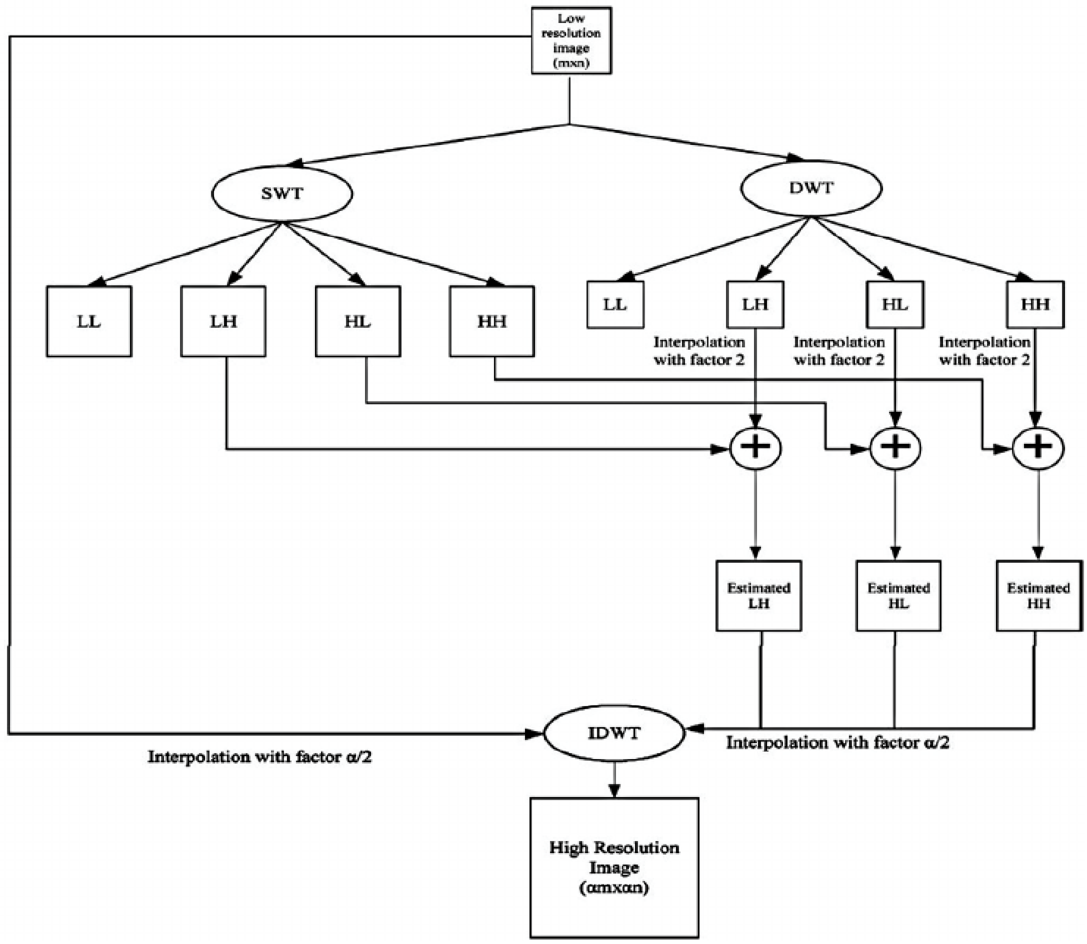


Figure 4.9: Block Diagram of DWT-SWT method

Steps to enhance low resolution image to high resolution image using the DWT-SWT method –

1. Take a high resolution image and convert it into a low resolution image of dimension 256 x 256.

2. DWT is applied to the low resolution image, the output of this transform are four frequency sub-bands i.e. low frequency component (LL), low-high frequency component (LH), high-low frequency component (HL) and high frequency component (HH). The dimension of all the four sub-bands obtained is half of the dimension of the input image.

3. SWT is applied to the low resolution image, to get four sub-bands LL, LH, HL and HH, but these four sub-bands will have dimensions as that of the input image.

4. Interpolate the high frequency sub-bands LH, HL and HH obtained by DWT decomposition of the input image by a factor of 2 using bi-cubic interpolation.

5. The interpolated sub-bands of the DWT decomposition are added to their corresponding SWT sub-bands, to get the estimated LH, HL and HH frequency component.

6. Interpolate the estimated sub-bands as well as the low resolution input image using bi-cubic interpolation with a factor $\frac{1}{2}$. The interpolated input image acts as estimated LL subband.

7. Apply inverse discrete transform on the four estimated sub-bands, the result obtained is a high resolution image.

4.3.2 Discussion

When we interpolate a low resolution image to produce a super resolved image, we lose the high frequency components i.e the edge information about the image, this is due to the smoothening effect of the bi-cubic interpolation. The edge informations are highly important in a super resolved image, to protect the high frequency components of the image we will apply DWT on the image. The DWT decomposes the low resolution input image into 4 image subbands namely LL, LH, HL and HH. The size of the subbands thus produced is half of that of the input image due to downsampling by DWT. Thus there is loss of information in each of the subbands, to compensate for the loss of information we decompose the low resolution input image using SWT. SWT also produces the 4 subbands, however there is no downsampling in case of SWT as in that of DWT. Each of the high frequency components have same size as that of the input image. The high frequency subbands i.e LH, HL, HH, produced by DWT are interpolated using bi-cubic interpolation by a factor 2, to approximate each of the high frequency subbands such that they have the same size as that of the subbands produced by the SWT.

The interpolated high frequency subbands of DWT and the high frequency subbands of SWT are combined together to produce estimated high frequency components. since, as discussed the LL subband will contain fewer information about the input image, so instead of interpolating the LL using bi-cubic with a factor of 2, we can use the low resolution image itself as the LL subband equivalent. So all the subbands are of the size of the input image. However, we need the super resolved image to have the same size as that of input image. If we combine all these subbands using IDWT then the image produced will be double the size of the input image.We avoid this by interpolating all the three estimated high frequency and the interpolated input image by a factor of $\frac{\alpha}{2}$ and then combine all the subband using IDWT to produce a high resolution image of size (αMxαN), given the input size of the image is (MxN).

4.3.3 Experimental Results

We have taken three high resolution images and resized them using imresize to 256x256 and then used the obtained low resolution images as input for the DWT and SWT transform. We obtain the satisfactory results. We have also computed the PSNR value for each of the super resolution images produced by wavelet zero padding (WZP) and the proposed DWT-SWT method with respect to the original low resolution input image. Figures 4.10, 4.11 and 4.12 shows the results of HR image obtained by WZP and DWT-SWT method.

PSNR is one of the most important parameters to measure the quality of the super resolved image. PSNR stands for ***peak signal-to-noise ratio***, it is a ratio between maximum possible power of a signal and the power of the perturbed signal. Mathematically, it is defined as

$$PSNR = 10\log_{10}\left(\frac{peakval^2}{MSE}\right)$$

where peakval is 256, and MSE is called the mean square error. The MSE is defined as

$$MSE = \frac{1}{MN}\sum_{m=1}^{M-1}\sum_{n=1}^{N-1}\left(I(m,n) - J(m,n)\right)^2$$

where I is the original image and J is the super resolved image, both of size (MxN).

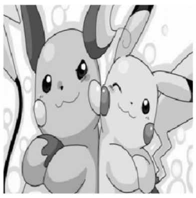
We observe that the PSNR value obtained for the proposed DWT-SWT transform is greater than the PSNR value for the WZP, indicating the proposed DWT-SWT algorithm produces better high resolution image than the WZP.

(a) LR image (256x256)

(b) SR image using WZP (512x512), (PSNR = 20.4134 dB)

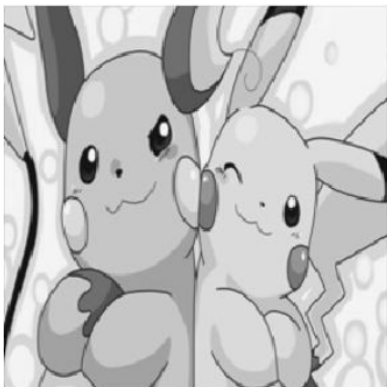
(c) SR image using DWT-SWT (512x512), (PSNR = 30.4561 dB)

Figure 4.10: Results of WZP and DWT-SWT on a noisy LR image of a dog



(a) LR image (256x256)

(b) SR image using WZP (512x512), (PSNR = 15.9982 dB)



(c) SR image using DWT-SWT (512x512), (PSNR = 26.0923 dB)

Figure 4.11: Results of WZP and DWT-SWT on a LR image of Pikachu

(a) LR image (256x256)

(b) SR image using WZP (512x512), (PSNR = 16.3554 dB)

(c) SR image using DWT-SWT (512x512), (PSNR = 28.4221 dB)

Figure 4.12: Results of WZP and DWT-SWT on a LR image of a person

4.3.4 Conclusion

In this chapter, we have seen some of the ways to perform image resolution enhancement. We found that bi-cubic interpolation is better than nearest neighbor and bi-linear interpolation. The interpolated images loses the high frequency components of the image due to smoothening caused by the interpolation techniques. We also found that the super resolved image produced by the DWT-SWT method is better than that produced by WZP, the reason for this is that in the proposed method the edge information i.e. the high frequency components of the image were preserved due to DWT and SWT transformations. In chapter 3, it is discussed that SWT is not suitable for real time applications due to its high time complexity. In the next chapter, we will see how in a real time application, DWT alone can be used to produce an enhanced image. We know SWT is better than DWT in image enhancement, but by using the LR image as the LL subband in IDWT the DWT gives equivalent result as that of SWT.

Chapter 5

Discrete Wavelet Transform-Based Satellite Image Resolution Enhancement

Research is to see what everbody else has seen,
and to think what nobody else has thought
- Albert Szent-Gyorgyi

5.1 Introduction

Satellites are manmade objects, which are placed in the orbit of celestial bodies, such as the Earth, moon or other planets, to collect information or to facilitate communications. Satellites affect our life in numerous ways like, they keep us safer, help in our day to day communication and is a medium for broadcasting the entertainment.

Satellite images are images of a part or whole of the earth taken using artificial satellites. Recently, a lot of research is based on satellite images, the reason is that, satellite images are being used in many applications such as astronomy, military, weather forcasting, anaysis of land cover, geosciences, meteorology, forestry, oceanography and so on. Since, these images are taken from space they suffer from resolution problems caused due to atmospheric conditions, scattering, etc. Thus though the satellite images contain the complete information they are not visibly clear. However most of the applications need clear and high resolution images but building such a satellite is going to be costly. So there is always a tradeoff between the resolution of an image and the cost to achieve it. Instead of building a satellite which takes higher resolution images, we use image enhancing techniques to produce a SR image from the satellite's LR images. One more reason why we prefer to enhance an LR image is that, suppose we got a HR image at the broadcast station and we want to send it to the users, a lot of bandwidth is required to send the image. Instead we can send a LR image and then convert the LR image to a SR image at receiver's end which is economically cheaper and faster.

The SR image produced after enhancement of the LR image taken by satellite, are used for further processing of the image, such as segmentation, analysis, recognition and detection. Figure 5.1 shows an image of a stadium and its surroundings taken from a satellite.

Figure 5.1: Satellite image

This chapter is divided into the following sections —
Section 2. Overview of the chapter
Section 3. Discussion on WZP
Section 4. The wavelet based method for image enhancement.
Section 5. Algorithm.
Section 6. Experimental results.
Section 7. Conclusion.

5.2 Overview of the Chapter

In this chapter, we will learn a technique for enhancing the resolution of a satellite image. In this technique, we will apply interpolated DWT to the input low resolution (LR) image. Inverse discrete wavelet transform (IDWT) is then applied to combine the interpolated high frequency subbands, generated by the DWT decomposition of the LR image and the original LR image to produce an enhanced image.

We have used the difference between input LR image and the interpolated LL subband, and used this difference to estimate the high frequency subbands. This process helps in acheiving a sharper image.

DWT based technique is compared with WZP, which is one of the standard interpolation techniques. The output produced by the proposed technique was found to be a sharper image of the input LR image. We also demonstrate that the DWT based technique outperforms the WZP.

5.3 Wavelet Zero Padding (WZP)

WZP is one of the most simplest image enhancement technique. In this method, it is assumed that the signal is zero outside the given support. Zero padding when used for spectral analysis with wavelet transforms, helps in increasing the accuracy of the amplitudes of the signal, and it does not help in increasing the frequency resolution [11]. Zero padding adds a string of samples with zero values at the end of a given time-domain sequence. When we perform zero padding in a time domain it is equivalent to perform interpolation in frequency domain. We know that, wavelet transforms are defined for signal with infinite length, so when we want to extend a signal of finite length, one of the most simple way is to pad the signals with zeros and extend it before transformation.

In image resolution enhancement, given an LR image, we produce an enhanced image by applying IDWT on the interpolated subbands LL, LH, HL, HH. In WZP, the LR image is used as LL subband and the coefficients of all the three high frequency subbands are replaced by zeros.

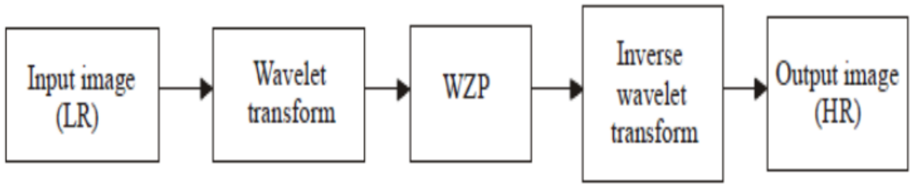


Figure 5.2: Block diagram for WZP

Figure 5.2 shows a block diagram of the steps involved in producing a HR image from an LR image using WZP method.

Drawbacks –

i WZP artificially creates a lot of border discontinuities.

ii Shifting caused by WZP may alter the array positions relative to the concerned frequencies.

Some of the results obtained using WZP on the input LR image are shown below. The images in figures 5.3 and 5.4 are scaled by 0.2 of their original size. –

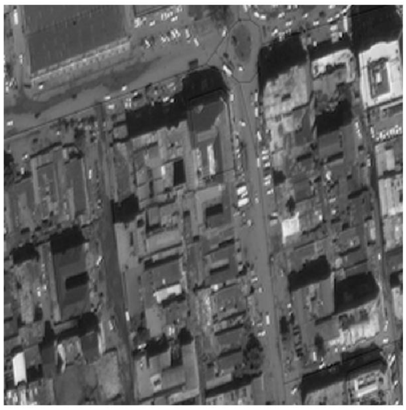
(a) Input LR image (256x256)

(b) SR image (512x512)

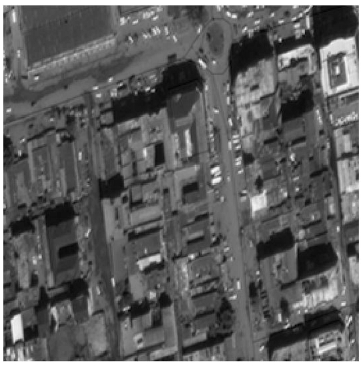
(c) SR image (256x256),(PSNR = 21.994 dB)

Figure 5.3: Result of SR images produced by WZP

(a) Input LR image (256x256)



(b) SR image (512x512)



(c) SR image (256x256),(PSNR = 29.465 dB)

Figure 5.4: Result of SR images produced by WZP

5.4 DWT based image enhancement technique

The proposed method uses DWT and bi-cubic interpolation techniques to enhance an LR input image to produce a sharper HR image. DWT is applied to decompose the low resolution image into 4 subbands. One of the four subbands produced is a low frequency subband whereas rest all are high frequency subbands. The subbands are interpolated using bi-cubic interpolation technique. The difference between the interpolated LL subband and input LR image is calculated. The image difference and the interpolated subbands of DWT are added to get estimated high frequency subbands. Parallely, the LR image is also interpolated, and the estimated high frequency subbands and the interpolated LR image are combined using IDWT, to produce a HR output image. The DWT based technique is compared with the high resolution image produced by wavelet zero padding (WZP) method.

5.4.1 Algorithm

Image enhancement algorithms use degraded high resolution image as input low resolution image. The input image is produced after downsampling the image obtained from low pass filtering the high resolution image. Figure 5.5 gives an overall view of the DWT based algorithm.

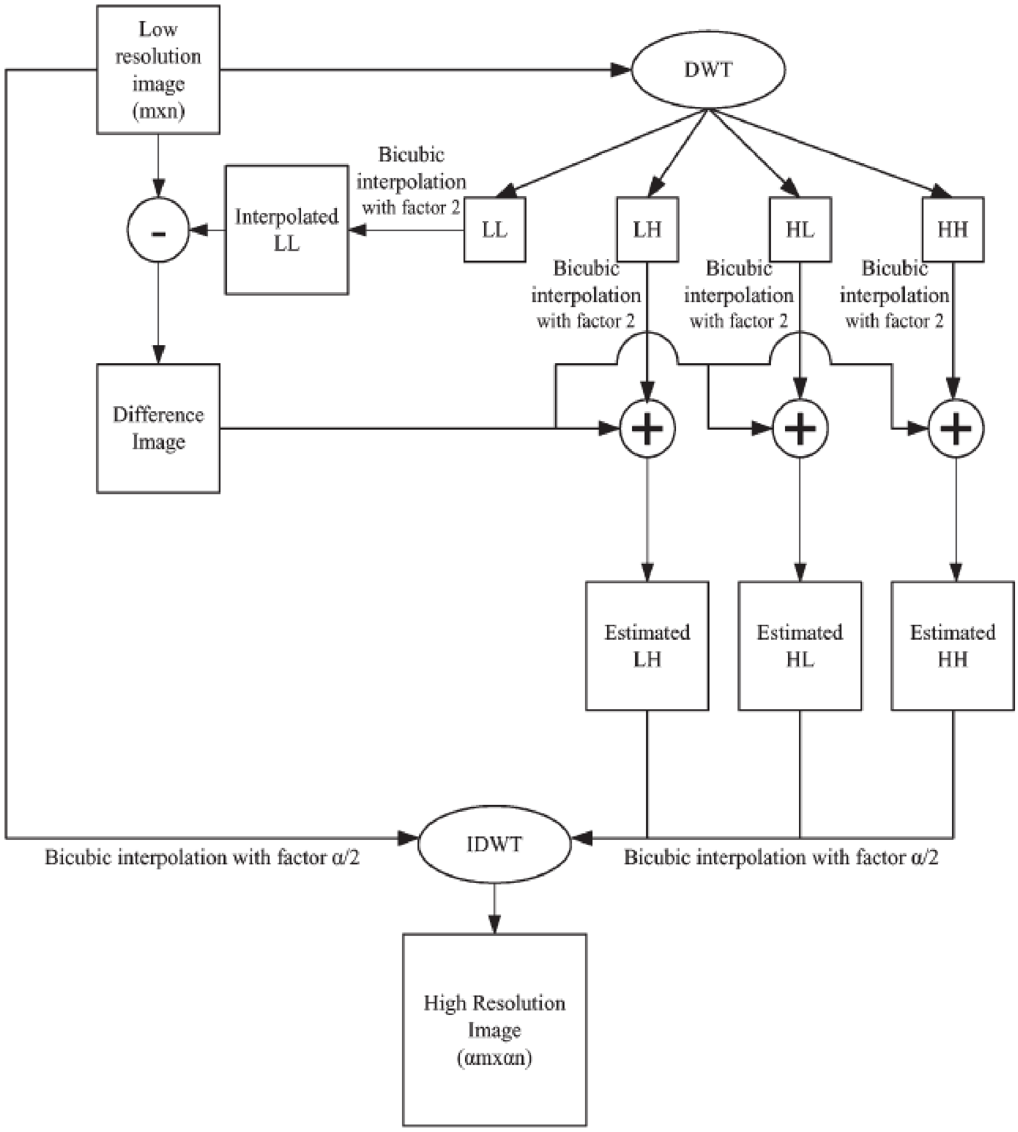


Figure 5.5: Block Diagram of the DWT based Algorithm

Steps to enhance low resolution image to high resolution image using the proposed method –

1. Take a high resolution image and convert it into a low resolution image of dimension 256 x 256.

2. DWT is applied to the low resolution image. The output of this transform are four frequency sub-bands i.e. low frequency component (LL), low-high frequency component (LH), high-low frequency component (HL) and high frequency component (HH). The dimension of all the four sub-bands obtained is half of the dimension of the input image.

3. Interpolate all the sub-bands LL, LH, HL and HH obtained by DWT decomposition of the input image by a factor of 2 using bi-cubic interpolation.

4. Compute the difference between the input LR image and the interpolated LL image.

5. Compute the estimated high frequency subband by adding the difference computed in previous step, to the interpolated high frequency subbands.

6. Interpolate the estimated sub-bands as well as the low resolution input image using bi-cubic interpolation with a factor $\frac{\alpha}{2}$. The interpolated input image acts as estimated LL.

7. Apply inverse discrete wavelet transform on the four estimated sub-bands. The result obtained is a high resolution image.

5.4.2 Discussion

The DWT is applied to the LR image to preseve the edge information while enhancing the image. DWT decomposes the low resolution input image into 4 image subbands namely LL, LH, HL and HH. The size of the subbands thus produced is half of that of the input image due to downsampling by DWT. Thus there is loss of information in each of the subbands, to compensate for the loss of information, in the previous chapter we have used SWT, which decomposes the image into 4 subbands and where all the subbands are of the size of the LR image, since there is no downsampling. Hence all the edge information is intact and is used to produce enhanced high frequency subbands after interpolating the high frequency subbands produced by DWT.

However as we discussed, there is a small problem using SWT, that there is no downsampling in case of SWT decomposition. There is a lot of data redundancy in each of the subbands. Since most of the high frequency values produced using SWT are actually present in the high frequency subbands, it is computationally expensive to decompose the LR image again using SWT. So, in the DWT based method, we compute the difference of the LR image and the inerpolated LL, the result of this contains approximately all the missing information of the edges. This difference of the image is then used to estimate high frequency sunbband by combining each of the interpolated high frequency subbands produced by DWT decomposition.

Since, the LL subband will contain lesser information about the input image, so instead of interpolating the LL using bi-cubic with a factor of 2, we can use the low resolution image itself as the LL subband equivalent. So all the subbands are of the size of the input image. However, we need the super resolved image to have the same size as that of input image. If we combine all the subbands using IDWT then the image produced will be double the size of the input image. We avoid this by interpolating all the three estimated high frequency bands and the interpolated input image by a factor of $\frac{\alpha}{2}$ and then combine all the subband using IDWT to produce a high resolution image of size (αMxαN), given the input size of the image is (MxN).

Theoretically speaking, when we perform the DWT decomposition, our subbands must be of half the size of the input image, however this is not the case most of the time, so we need to resize each of the subbands according to our needs to get the desired outputs. If ignored, computational errors occur, such as mismatch in matrix dimensions, etc. and sometimes, this introduces unwanted artifacts in the HR image.

5.4.3 Experimental Results

We have taken two high resolution satellite images and resized them using imresize to 256x256 and then used the obtained low resolution image as input for the DWT based IE method. We have also computed the PSNR value for each of the super resolution images produced by wavelet zero padding (WZP) and the proposed DWT based method with respect to the original low resolution input image.Figures 5.3 and 5.4 shows the result of HR image obtained by WZP method. Figures 5.6 and 5.7 shows the result of HR image obtained by DWT method .

PSNR is one of the most important parameters to measure the quality of the super resolved image. PSNR stands for ***peak signal-to-noise ratio*** [**?**]. It is a ratio between maximum possible power of a signal and the power of the perturbed signal. Mathematically, it is defined as

$$PSNR = 10\log_{10}\left(\frac{peakval^2}{MSE}\right)$$

where peakval is 256, and MSE is called the mean square error. The MSE is defined as

$$MSE = \frac{1}{MN}\sum_{m=1}^{M-1}\sum_{n=1}^{N-1}\left(I(m,n) - J(m,n)\right)^2$$

where I is the original image and J is the super resolved image, both of size (MxN).

5.4.4 Conclusion

In this chapter, we have seen how to produce an enhanced image using WZP and the DWT based method. We found that the SWT produces a lot of redundancy in each of the subbands. In the proposed method, the compensation for the loss of edge information is done using the difference of LR image and interpolated LL subband, as it contains most of the missing information. We also found that the super resolved image produced by the DWT based method is better than that produced by WZP. The reason for this is that in the DWT based method the edge information i.e. the high frequency components of the image were preserved due to DWT transformations.

5.4.5 Results–

(a) Input LR image (256x256)

(b) HR image using DWT based method (512x512)

Figure 5.6: Result of SR of satellite image using DWT method (PSNR = 24.4134 dB)

(a) Input LR image (256x256)

(b) HR image using DWT based method (512x512)

Figure 5.7: Result of SR of satellite image using DWT method (PSNR = 33.9982 dB)

Chapter 6

Summary and Future Work

Now this is not the end. It is not even the beginning of the end.
But it is, perhaps, the end of the beginning
- Winston Churchill

6.1 Summary

In this thesis, we presented a brief literature review on wavelets and their history. We discussed how wavelet analysis can be used in image processing [12] and the representation of wavelets at higher resolution [6].

The images produced after interpolation are enhanced, compared to the input image, however the edges are not sharp and the reason is the smoothening effect caused by the interpolation methods. On our search for getting a better enhancement technique, we moved from the spatial domain to the frequency domain. In the frequency domain, wavelets are used for extracting information from the images. So we used wavelet transformation to preserve the edge information of the images. We studied about discrete wavelet transform and stationary wavelet transform and checked for ourselves that SWT is better in edge preservation than DWT.

We found that even though SWT is better in edge preservation than DWT, its time complexity is more than DWT which computes the approximation and details quickly. We then wanted to combine the merits of both the wavelet transform and produce an enhanced image. In this regard, we implemented and analyzed two works [8] and [7] of Hasan Demirel and Gholamreza Anbarjafari. The results of enhanced images obtained by DWT-SWT method as proposed in [7] and DWT based method as proposed in [8] are compared with the wavelet zero padding methods. The images produced by DWT-SWT and DWT based method are better enhanced than the images produced by WZP. PSNR metric is used to measure the enhancement of the image with respect to the original image.

6.2 Future Work

We have compared both DWT-SWT method and DWT based method with WZP, however we have not compared the DWT-SWT method with DWT based method. Hasan Demirel and Gholamreza Anbarjafari [8], claims that the DWT based method outperform the DWT-SWT method . So we would like to include this in our future work. We have not tried enhancing LR images with noise, and we would like to work on enhancing the noisy LR images in future.

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