IE 501 : Optimization Models

Improving profits for Supermarkets through shelf Optimization

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Content

Motivation Behind the Project

Efficient shelf space allocation is vital for maximizing supermarket profitability. With limited shelf space and increasing customer demands, optimizing product placement ensures:

- Maximizing Profits: Optimize shelf space for high-demand, high-margin products.
- Customer Satisfaction: Ensure product availability and an enhanced shopping experience.
- Cost Efficiency: Minimize restocking and inventory costs.
- Complex Challenges: Address issues such as product fragility, and dependencies.
- Scalable Solutions: Use data-driven optimization for real-world impact.

Problem Statement

Product Shelf Space Allocation and Optimization

The Problem focuses on allocating products to shelves in a retail store to maximize revenue while considering various parameters and constraints. These include shelf capacity, height, weight, visibility, and accessibility, as well as product-specific parameters such as revenue, space usage, fragility, turnover, and facings. The goal is to determine the optimal number of facings for each product on each shelf, while ensuring that shelf and product constraints are met.

Methodology

Approach Overview

- Formulated as a Mixed-Integer Linear Programming (MILP) model.

- Objective: Maximize profitability while adhering to real-world constraints.

1 Data Simluation

Simulated product details (demand, size, fragility, profit margins).

2 Model Formulation

Defined decision variables, constraints, and the objective function.

3 Optimization Technique

Solved using Pyomo/GLPK for efficient allocation.

4 Final output

Found optimized allocation of products to each shelves

Formulation

1. Parameters

- R_i: Revenue per unit of product i.
- L_i: Loss per unit of product i if unsold.
- S_i: Space occupied by one facing of product i.
- H_i: Height of product i.
- W_i: Weight of product i.
- F_i: Fragility of product i.
- mf_i: Minimum facings required for product i on any shelf (if assigned).

Formulation

1. Parameters

- MF_i: Maximum facings allowed for product i on any shelf (if assigned).
- $F_{\text{req,min},i}$: Minimum total facings required for product i across all shelves.
- $F_{\text{req,max},i}$: Maximum total facings allowed for product i across all shelves.
- C_j: Capacity of shelf j (in terms of space).
- SH_j: Height limit of shelf j.
- SW_j: Weight limit of shelf j.
- $F_{\text{threshold}}$: Fragility threshold for product placement on shelves.

Formulation

2. Decision Variables

$x_{ij} \in \{0, 1\}$: Binary variable indicating whether product i is assigned to shelf j.

$f_{ij} \in \mathbb{Z}_{\geq 0}$: Number of facings of product i on shelf j.

$z_{ij} \in \mathbb{R}_{\geq 0}$: Auxiliary variable for linearization of $x_{ij} \cdot f_{ij}$.

Formulation

3. Constraints

First constraint is for shelf space capacity and the second one is for fragility constraint

$$\sum_{i \in P} S_i \cdot f_{ij} \leq C_j, \quad \forall j \in S$$

$$x_{ij} = 0, \quad \forall i \in P,\, j \in S \text{ where } F_i > F_{\text{threshold}}$$

Formulation

3. Constraints

The first constraint ensures the lower bound on the number of facings for a product on a single shelf, while the second constraint establishes its upper bound.

$$f_{ij} \geq mf_i \cdot x_{ij}, \quad \forall i \in P,\, j \in S$$

$$f_{ij} \leq MF_i \cdot x_{ij}, \quad \forall i \in P,\, j \in S$$

Formulation

3. Constraints

The first constraint ensures the lower bound on the number of facings for a product across all shelves, while the second constraint establishes its upper bound.

$$\sum_{j \in S} f_{ij} \geq F_{\text{req,min},i}, \quad \forall i \in P$$

$$\sum_{j \in S} f_{ij} \leq F_{\text{req,max},i}, \quad \forall i \in P$$

Formulation

3. Constraints

The first one is the Weight Constraint while the second one is the Height constraint.

$$\sum_{i \in P} W_i \cdot f_{ij} \leq SW_j, \quad \forall j \in S$$

$$H_i \cdot x_{ij} \leq SH_j, \quad \forall i \in P, j \in S$$

Formulation

3. Constraints

These are the linearization constraints which we need to ensure that the objective function remains linear and our model does not become non linear in nature.

$$z_{ij} \leq f_{ij}, \quad \forall i \in P,\, j \in S$$

$$z_{ij} \leq x_{ij} \cdot MF_i, \quad \forall i \in P,\, j \in S$$

$$z_{ij} \geq f_{ij} - (1 - x_{ij}) \cdot MF_i, \quad \forall i \in P,\, j \in S$$

$$z_{ij} \geq 0, \quad \forall i \in P,\, j \in S$$

Formulation

3. Constraints

First one is product- shelf assignment constraint while the second one is diversity constraint

$$\sum_{j \in S} x_{ij} \geq 1 \quad \forall i \in P$$

$$\sum_{i \in P} x_{ij} \geq 2 \quad \forall j \in S$$

Formulation

4. Objective Function

$$\text{Maximize} \sum_{i \in P} \sum_{j \in S} \left(R_i \cdot z_{ij} - L_i \cdot z_{ij} \right)$$

Here Z_ij represents product of x_ij and f_ij, To maintain a linear formulation, the product was not directly included in the function.

Final Output

After running our code, the output provides an optimized allocation, detailing which products and the exact quantities of each should be placed on each shelf.

Shelf	Products Assigned
S1	P10(10.0), P22(11.0), P40(3.0), P45(2.0), P48(15.0)
S2	P4(1.0), P5(4.0), P11(1.0), P21(1.0), P23(1.0), P34(11.0), P35(9.0), P38(7.0), P39(9.0), P40(1.0), P47(2.0)
S3	P7(5.0), P8(13.0), P11(1.0), P26(1.0), P28(1.0), P35(3.0), P36(7.0), P37(10.0), P39(1.0), P40(6.0), P41(10.0)
S4	P2(2.0), P5(1.0), P6(4.0), P10(2.0), P12(1.0), P19(2.0), P24(6.0), P25(1.0), P26(6.0), P29(6.0), P31(4.0), P39(1.0), P42(8.0), P43(1.0), P45(1.0), P49(4.0), P50(6.0)
S5	P7(3.0), P11(1.0), P17(9.0), P19(2.0), P23(10.0), P25(6.0), P26(1.0), P27(11.0), P28(9.0), P31(1.0), P33(1.0), P49(3.0)
S6	P4(9.0), P6(1.0), P8(1.0), P13(10.0), P15(4.0), P18(1.0), P21(1.0), P43(6.0), P45(5.0), P47(10.0)
S7	P1(10.0), P3(2.0), P11(1.0), P15(9.0), P18(12.0), P31(6.0), P41(1.0), P43(2.0), P47(3.0)
S8	P9(8.0), P11(1.0), P12(6.0), P14(13.0), P16(10.0), P25(2.0), P43(1.0), P46(10.0)
S9	P2(3.0), P4(3.0), P7(1.0), P13(4.0), P19(10.0), P32(15.0), P36(1.0), P44(15.0), P45(1.0)
S10	P3(10.0), P11(1.0), P20(5.0), P21(6.0), P26(1.0), P29(3.0), P30(10.0), P33(7.0), P36(7.0), P49(1.0)

Thanks