Formulation for vehicle routing problem with simultaneous delivery and pickups

Goutam Agarwal

Parameters:

N : Total number of customers

DC/CC : Node 0 representing the distribution center/central depot

V : Total number of available vehicles $c_{i,j}$: Travel distance (routing cost) between node i and j

$t_{i,j}$: Travelling time between node i and j

a_0 : Earliest start time of any vehicle from the distribution center

a_i : Earliest start time of service at customer i b_0 : Latest arrival time of any vehicle to the central depot b_i : Latest start time of service at customer i

C_k : Capacity of vehicle k (heterogeneous vehicles assumed) s_i : Service time of customer i p_i : Pickup quantity of customer i

d_i : Delivery quantity of customer i

fixed_k : Dispatching cost of vehicle k (fixed/overhead cost) a : Constant representing the trade-off between dispatching cost and routing cost **Decision Variables:**

$x_{i,j}$: Assigned the value 1 when the arc between customer i and j is selected as part of the routing pl

$\text{del}_{i,k}$: Assigned the value 1 when customer i is assigned to vehicle k, 0 otherwise

del_k : Assigned the value 1 when vehicle k is used in the solution, 0 otherwise

$\text{load}_{0,k}$: Load of vehicle k when it starts from the distribution center (DC)

$\text{start}_{0,k}$: Starting time of vehicle k when it starts from the DC

$\text{load}_{i,k}$: Load on vehicle after completing the service at customer i

dist_i : Total distance traveled up to customer i

$start_i$: Starting time of the service at customer i

Objective Function:

$$Z = \alpha \sum_{i=1}^{N} \sum_{j=1}^{N} (x_{ij} * c_{ij}) + \left((1-\alpha) \sum_{k=1}^{V} (\mathrm{del}_k * \right) \quad !$$

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Subject to the following constraints:

$$\sum_{k=1}^{V} \mathrm{del}_{i,k} = 1 \quad , \quad \text{for } i = 1,2,\ldots,N \qquad (2)$$

$$\mathrm{del}_{i,k} \le \mathrm{del}_k, \quad \text{for } i = 1,2,\ldots,N \text{ and } k = 1,2,\ldots,V \qquad (3)$$

$$\mathrm{del}_{i,k} - \mathrm{del}_{j,k} \le (1 - x_{ij}), \quad \text{for } i,j = 1,2,\ldots,N,\ i \neq j, \text{ and } k = 1,2,\ldots,V \qquad (4)$$

$$\sum_{j=1}^{N} x_{i,j} = 1, \quad \text{for } i = 1,2,\ldots,N \qquad (5)$$

$$\sum_{i=1}^{N} x_{i,j} = 1, \quad \text{for } j = 1,2,\ldots,N \qquad (6)$$

$$x_{i,i} = 0, \quad \text{for } i = 1,2,\ldots,N \qquad (7)$$

$$x_{i,j} + x_{j,i} \le 1, \quad \text{for } i = 1,2,\ldots,N-1,\ j = i+1, i+2,\ldots,N \qquad (8)$$

$$\mathrm{dist}_j \ge \mathrm{dist}_i + c_{i,j} - M(1 - x_{i,j}), \quad \text{for } i,j = 1,2,\ldots,N,\ i \neq j \qquad (9)$$

$$\mathrm{st}_i \ge \mathrm{start}_{0,k} + t_{0,i}, \quad \text{for } i = 1,2,\ldots,N \text{ and } k = 1,2,\ldots,V \qquad (10)$$

$$\mathrm{st}_j \ge \mathrm{st}_i + s_i + t_{i,j} - M(1 - x_{i,j}) \quad \text{for } i,j = 1,2,\ldots,N,\ i \neq j \qquad (11)$$

$$\mathrm{load}_{0,k} = \sum_{i=1}^{N} \mathrm{del}_{i,k} \cdot d_i, \quad \text{for } k = 1,2,\ldots,V \qquad (12)$$

$$\mathrm{ld}_i \ge \mathrm{load}_{0,k} + d_i + p_i \quad \text{for } i = 1,2,\ldots,N \text{ and } k = 1,2,\ldots,V \qquad (13)$$

$$\mathrm{ld}_j \ge \mathrm{ld}_i + d_i + p_i - M(1 - x_{i,j}) \quad \text{for } i,j = 1,2,\ldots,N,\ i \neq j \qquad (14)$$

Constraints (15)–(19) represent the bounds for the respective decision variables:

$$a_0 \le \text{start}_{0,k} \le b_0, \quad \text{for } k = 1,2,\ldots,V \tag{15}$$

$$a_i \le \text{st}_i \le b_i, \quad \text{for } i = 1,2,\ldots,N \tag{16}$$

$$0 \le \text{load}_{0,k} \le C_k, \quad \text{for } k = 1,2,\ldots,V \tag{17}$$

$$0 \le \text{ld}_i \le \max_{1 \le k \le V} C_k, \quad \text{for } i = 1,2,\ldots,N \tag{18}$$

$$0 \le \text{dist}_i \le M_1, \quad \text{for } i = 1,2,\ldots,N \tag{19}$$

Constraints

1. Constraint (2): Each customer should be allocated to only a single vehicle.
2. Constraint (3): A customer can be allocated to a vehicle only when thecorresponding vehicle is used in the final solution.
3. Constraints (4): Customers belonging to the same route are allocated tothe same vehicle.
4. Constraints (5) and (6): Each customer need to be visited only once.
5. Constraints (7)–(8): Restrictions to avoid subtours in the route sequence
6. Constraints (9): If the arc between two nodes is selected/available as partof the travel plan, then the travel distance should be equal to the distance between them.
7. Constraints (10) and(11): If the arc between customers *i* and *j* is selected in the routing plan, then the service time at customer *j* should be later than the service time at customer *i*.
8. Constraint (12): The load of a vehicle at any point in the route should beless than the capacity of the vehicle.
9. Constraint (13): Limits the initial load of the vehicle when it starts fromthe DC.
10. Constraints (14) : Represent the cumulative load of a vehicle after visitinga particular customer.