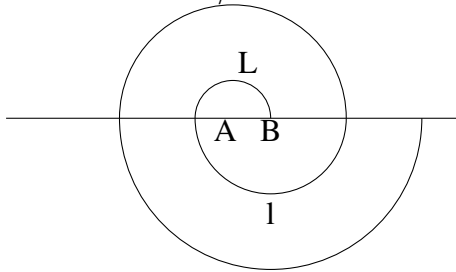


Assignment 10.5.3 _18Q

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Question A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A, of radii 0.5cm, 1.0cm, 1.5cm, 2.0cm, ... as shown in Fig.5.4. what is the total length of such a spiral made up of thirteen consecutive semicircles? (Take $\pi = \frac{22}{7}$)



Solution: Input parameters are:

PARAMETER	VALUE	DESCRIPTION
$x(0)$	$\frac{11}{7}$	First term
d	$\frac{22}{7}$	common difference
$x(n)$	$[\frac{11}{7} + \frac{22}{7}n]u(n)$	General term of the series

TABLE I
INPUT PARAMETER TABLE

From (??) :

$$X(z) = \frac{11 + 11z^{-1}}{7(1 - z^{-1})^2}, |z| > 1 \quad (1)$$

$$y(n) = x(n) * u(n) \quad (2)$$

$$Y(z) = X(z)U(z) \quad (3)$$

$$\Rightarrow Y(z) = \frac{11 + 11z^{-1}}{7(1 - z^{-1})^3}, |z| > 1 \quad (4)$$

Using contour integration to find the inverse z-

transform,

$$y(n) = \frac{1}{2\pi j} \oint_C Y(z)z^{n-1} dz \quad (5)$$

$$y(12) = \frac{1}{2\pi j} \oint_C \frac{11z^{11} + 11z^{10}}{7(1 - z^{-1})^3} \quad (6)$$

We can observe that the pole is repeated 3 times and thus $m = 3$,

$$R = \frac{1}{(m-1)!} \lim_{z \rightarrow a} \frac{d^{m-1}}{dz^{m-1}} ((z-a)^m f(z)) \quad (7)$$

$$= \frac{1}{(2)!} \lim_{z \rightarrow 1} \frac{d^2}{dz^2} \left(\frac{11z^{14} + 11z^{13}}{7} \right) \quad (8)$$

$$R = \frac{1859}{7} \quad (9)$$

$$\therefore y(12) = 265.571428 \quad (10)$$

Therefore, The total length of spiral made up of thirteen consecutive semicircles is 265.571428.

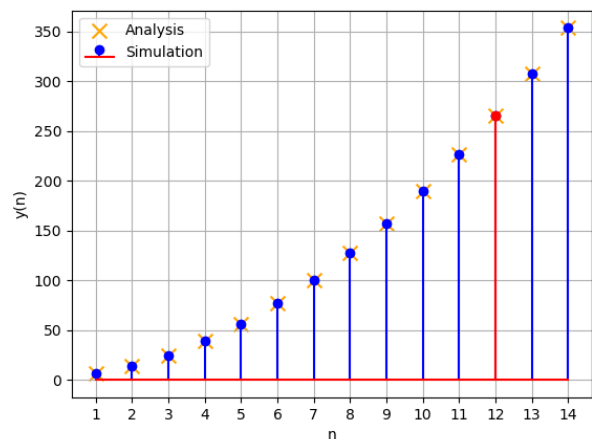


Fig. 1. $y(n) = 11/7n^2$