# Assignment CS_15Q

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QUESTION

The Lucas sequence $L_n$ is defined by the recurrence relation:

$$L_n = L_{n-1} + L_{n-2}, \text{ for } n \geq 3$$

with $L_1=1$ and $L_2=3$
Which one of the option given is TRUE?

1) $L_n = \left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n$

2) $L_n = \left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{3}\right)^n$

3) $L_n = \left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{3}\right)^n$

4) $L_n = \left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n$

(GATE 2023 CS 15)

**Solution:**

Initial condition $L_1=1$ and $L_2=3$

$$L_n = L_{n-1} + L_{n-2} \qquad (1)$$

Assume $L_{n+1} = x(n)$

$$x(n) = [x(n-1) + x(n-2) - 3]u(n-2) + u(n) + 2u(n-1) \qquad (2)$$

$$X(z) = [z^{-1}X(z) + z^{-2}X(z) - 3]\frac{2z^{-1}}{1-z^{-1}} + \frac{1}{1-z^{-1}} + 2\frac{z^{-2}}{1-z^{-1}} \qquad (3)$$

$$X(z)[1 - z^{-1} - 2z^{-2} - 2z^{-3}] = 1 - 6z^{-1} + 2z^{-2} \qquad (4)$$