

Assignment 11.9.5_6Q

EE23BTECH11028 - Kamale Goutham

Question

Find the sum of all two digit numbers which when divided by 4, yields 1 as remainder?

Solution: Input parameters are:

PARAMETER	VALUE	DESCRIPTION
$x(0)$	13	First term
d	4	common difference
$x(n)$	$[13 + 4n]u(n)$	General term of the series

TABLE I
INPUT PARAMETER TABLE

$$x(n) = x(0) + nd \quad (1)$$

$$n = \frac{97 - 13}{4} = 21 \quad (2)$$

$$(3)$$

From ??

$$X(z) = \frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2} \quad (4)$$

$$X(z) = \frac{13 - 9z^{-1}}{(1 - z^{-1})^2}, |z| > 1 \quad (5)$$

$$y(n) = x(n) * u(n) \quad (6)$$

$$Y(z) = X(z)U(z) \quad (7)$$

$$\Rightarrow Y(z) = \frac{13 - 9z^{-1}}{(1 - z^{-1})^3}, |z| > 1 \quad (8)$$

Using contour integration to find the inverse z-transform,

$$y(n) = \frac{1}{2\pi j} \oint_C Y(z)z^{n-1}dz \quad (9)$$

$$y(21) = \frac{1}{2\pi j} \oint_C \frac{(13 - 9z^{-1})z^{20}}{(1 - z^{-1})^3} \quad (10)$$

We can observe that the pole is repeated 3 times and thus $m = 3$,

$$R = \frac{1}{(m-1)!} \lim_{z \rightarrow a} \frac{d^{m-1}}{dz^{m-1}} ((z-a)^m f(z)) \quad (11)$$

$$= \frac{1}{(2)!} \lim_{z \rightarrow 1} \frac{d^2}{dz^2} (13z^{23} - 9z^{22}) \quad (12)$$

$$R = 1210 \quad (13)$$

$$\therefore y(21) = 1210 \quad (14)$$

Therefore, the sum of all two-digit numbers that, when divided by 4, yield a remainder of 1 is 1210.

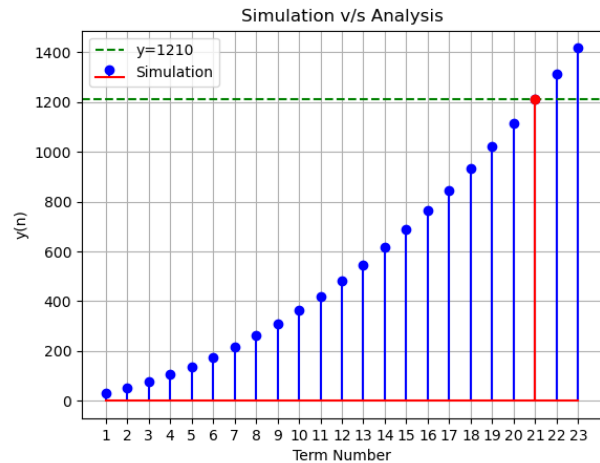


Fig. 1. $y(n) = 13 + 15n + 2n^2$