Assignment IN_40Q

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Question

The signal $x(t) = (t-1)^2 u(t-1)$,where u(t) is unit-step function,has the Laplace transform X(s).The Value of X(1) is

1) $\frac{1}{e}$
2) $\frac{2}{e}$
3) $2e$
4) e^2

(GATE 2022 IN 40)

Solution:

$$x(t) = (t-1)^2 u(t-1) \tag{1}$$

Taking Laplace-Transform:

$\mathcal{L}\{u(t)\}$

$$u(t) \leftrightarrow \frac{1}{s} \tag{2}$$

$\mathcal{L}\{tu(t)\}$

$$tu(t) \leftrightarrow \frac{1}{s^2} \tag{3}$$

$$\vdots$$

$\mathcal{L}\{t^n u(t)\}$

$$t^n u(t) \leftrightarrow \frac{n!}{s^{n+1}} \tag{4}$$

if X(s) is Laplace transform of x(t) then,

$$x(t - t_0) = e^{-st_0} X(s) \tag{5}$$

using 4 and 5
$\mathcal{L}\{tu(t)\}$

$$(t-1)^2 u(t-1) L \frac{2e^{-s}}{s^3} \tag{6}$$

$$X(s) = \frac{2e^{-s}}{s^3} \tag{7}$$

$$X(1) = \frac{2}{e} \tag{8}$$

$\therefore$ 2 is Correct.