Assignment 11.9.5_6Q

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Question

Find the sum of all two digit numbers which when divided by 4,yields 1 as reminder?
Solution: Input parameters are:

PARAMETER	VALUE	DESCRIPTION
$x(0)$	13	First term
d	4	common difference
$x(n)$	$[13+4n]u(n)$	General term of the series
$y(n)$	$[13+15n+2n^2]u(n)$	sum of terms of the series

TABLE I
INPUT PARAMETER TABLE

$$\text{x(n)} = x(0) + n\text{d} \tag{1}$$
$$n = \frac{97-13}{4} = 21 \tag{2}$$
$$X(z) = \sum_{k=-\infty}^{\infty} x(k) \times u(k) \times (z^{-k}) \tag{3}$$
$$\implies nu(n) Z \frac{z^{-1}}{(1-z^{-1})^2}, |z| > 1 \tag{4}$$
$$X(z) = \frac{13-9z^{-1}}{(1-z^{-1})^2}, |z| > 1 \tag{5}$$
$$y(n) = x(n) * u(n) \tag{6}$$
$$Y(z) = X(z)U(z) \tag{7}$$
$$\implies Y(z) = \left(\frac{13-9z^{-1}}{(1-z^{-1})^3}\right), |z| > 1 \tag{8}$$

Using contour integration to find the inverse z-transform,

$$y(n) = \frac{1}{2\pi j} \oint_C Y(z) z^{n-1} dz \tag{9}$$
$$y(21) = \frac{1}{2\pi j} \oint_C \left(\frac{(13-9z^{-1})z^{20}}{(1-z^{-1})^3}\right) \tag{10}$$

We can observe that the pole is repeated 3 times and thus $m = 3$,

$$R = \frac{1}{(m-1)!} \lim_{z\to a} \frac{d^{m-1}}{dz^{m-1}} ((z-a)^m f(z)) \tag{11}$$
$$= \frac{1}{(2)!} \lim_{z\to 1} \frac{d^2}{dz^2} \left(13z^{23} - 9z^{22}\right) \tag{12}$$
$$R = 1210 \tag{13}$$
$$\therefore y(21) = 1210 \tag{14}$$

Therefore, the sum of all two-digit numbers that, when divided by 4, yield a remainder of 1 is 1210.

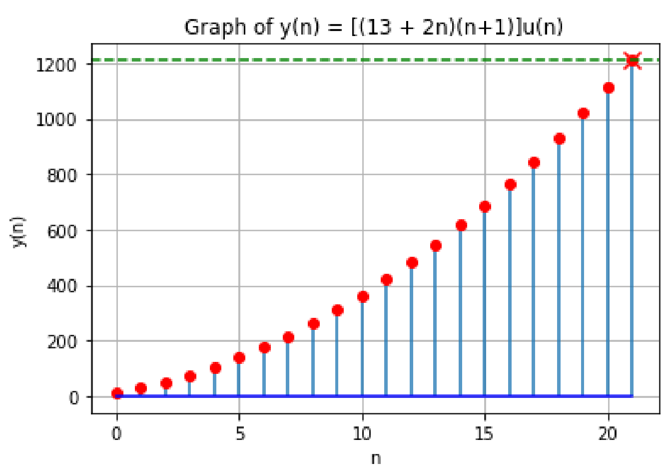

Fig. 1. y(n) = $13 + 15n + 2n^2$