Assignment 11.9.5_6Q

EE23BTECH11028 - Kamale Goutham

Question

Find the sum of all two digit numbers which when divided by 4,yields 1 as reminder?
Solution:

$$\text{x(n)} = x(0) + n \times \text{d} \tag{1}$$

$$\text{n} = \frac{\text{x(n)} - \text{x(0)}}{\text{d}} \tag{2}$$

$$n = \frac{97 - 13}{4} = 21 \tag{3}$$

Input parameters are:

PARAMETER	VALUE	DESCRIPTION
$x(0)$	13	First term
d	4	common difference
$x(n)$	$[13 + 4n]u(n)$	General term of the series

TABLE I
Input Parameter TABLE

$$x(z) = \sum_{k=-\infty}^{\infty} x(k) \times u(k) \times (z^{-k}) \tag{4}$$

R.O.C$\rightarrow$ mod $z \geq 1$:

$$x(z) = 13 \times (z)(z-1)^{-1} + 4 \times (z)(z-1)^{-2} \tag{5}$$

$$x(z) = \frac{13 - 9z^{-1}}{(1 - z^{-1})^2}, |z| > 1 \tag{6}$$

$$y(n) = x(n) * u(n) \tag{7}$$

$$\implies Y(Z) = X(Z)U(Z) \tag{8}$$

$$Y(Z) = \left(\frac{13 - 9z^{-1}}{(1 - z^{-1})^2}\right)\left(\frac{1}{1 - z^{-1}}\right) \tag{9}$$

$$Y(Z) = \left(\frac{13 - 9z^{-1}}{(1 - z^{-1})^3}\right), |z| > 1 \tag{10}$$

Using contour integration to find the inverse z-transform,

$$y(21) = \frac{1}{2\pi j} \oint_C Y(z) z^{20} dz \tag{11}$$

$$= \frac{1}{2\pi j} \oint_C \left(\frac{(13 - 9z^{-1}) z^{20}}{(1 - z^{-1})^3}\right) \tag{12}$$

We can observe that the pole is repeated 3 times and thus $m = 3$,

$$R = \frac{1}{(m-1)!} \lim_{z \to a} \frac{d^{m-1}}{dz^{m-1}} \left((z-a)^m f(z)\right) \tag{13}$$

$$= \frac{1}{(2)!} \lim_{z \to 1} \frac{d^2}{dz^2} \left((z-1)^3 \frac{(13 - 9z^{-1}) z^{23}}{(z-1)^3}\right) \tag{14}$$

$$= \frac{1}{(2)!} \lim_{z \to 1} \frac{d^2}{dz^2} \left(13z^{23} - 9z^{22}\right) \tag{15}$$

$$= \frac{1}{(2)!} (13 \times 23 \times 22 - 9 \times 22 \times 21) \tag{16}$$

$$= 1210 \tag{17}$$

Therefore, the sum of all two-digit numbers that, when divided by 4, yield a remainder of 1 is 1210.