1

Assignment 11.9.5 6Q

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Question

Find the sum of all two digit numbers which when divided by 4, yields 1 as reminder?

Solution: Input parameters are:

PARAMETER	VALUE	DESCRIPTION
x (0)	13	First term
d	4	common difference
x(n)	[13+4n]u(n)	General term of the series

TABLE I INPUT PARAMETER TABLE

$$\mathbf{x}(\mathbf{n}) = \mathbf{x}(0) + n\mathbf{d} \tag{1}$$

$$n = \frac{97 - 13}{4} = 21\tag{2}$$

(3)

From ??

$$X(z) = \frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2}$$
(4)

$$X(z) = \frac{13 - 9z^{-1}}{(1 - z^{-1})^2}, |z| > 1$$
 (5)

$$y(n) = x(n) * u(n)$$
 (6)

$$Y(z) = X(z)U(z) \tag{7}$$

$$\implies Y(z) = \frac{13 - 9z^{-1}}{(1 - z^{-1})^3}, |z| > 1$$
 (8)

Using contour integration to find the inverse z-transform,

$$y(n) = \frac{1}{2\pi i} \oint_C Y(z) z^{n-1} dz \tag{9}$$

$$y(21) = \frac{1}{2\pi i} \oint_C \frac{(13 - 9z^{-1})z^{20}}{(1 - z^{-1})^3}$$
 (10)

We can observe that the pole is repeated 3 times and thus m = 3,

$$R = \frac{1}{(m-1)!} \lim_{z \to a} \frac{d^{m-1}}{dz^{m-1}} \left((z-a)^m f(z) \right) \tag{11}$$

$$= \frac{1}{(2)!} \lim_{z \to 1} \frac{d^2}{dz^2} \left(13z^{23} - 9z^{22} \right) \tag{12}$$

$$R = 1210$$
 (13)

$$\therefore y(21) = 1210$$
 (14)

Therefore, the sum of all two-digit numbers that, when divided by 4, yield a remainder of 1 is 1210.

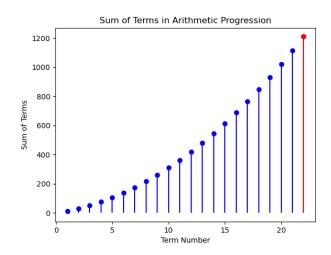


Fig. 1. $y(n) = 13 + 15n + 2n^2$