Assignment CS_15Q

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Question

The Lucas sequence $L_n$is defined by the recurrence relation:

$$L_n = L_{n-1} + L_{n-2}, for n \geq 3$$

with L_1=1 and L_2=3
Which one of the option given is TRUE?

1) $L_n = \left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n$
2) $L_n = \left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{3}\right)^n$
3) $L_n = \left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{3}\right)^n$
4) $L_n = \left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n$

(GATE 2023 CS 15)

Solution:
Initial condition L_1=1 and L_2=3

$$L_n = L_{n-1} + L_{n-2} \tag{1}$$

Assume $L_n = r^n$

$$r^n = r^{n-1} + r^{n-2} \tag{2}$$
$$r^2 = r^1 + 1 \tag{3}$$
$$r_1, r_2 = \frac{1+\sqrt{5}}{2}, \frac{1-\sqrt{5}}{2} \tag{4}$$

now we have, L_n=Ar_1^n+Br_2^n

$$L_1 = Ar_1^1 + Br_2^1 = 1 \tag{5}$$
$$L_2 = Ar_1^2 + Br_2^2 = 3 \tag{6}$$
$$A = 1, B = 1 \tag{7}$$
$$L_n = \left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n \tag{8}$$

$\therefore$ option 1 is correct