

Assignment 11.9.5 _6Q

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QUESTION

Find the sum of all two digit numbers which when divided by 4, yields 1 as remainder?

Solution: Input parameters are:

PARAMETER	VALUE	DESCRIPTION
$x(0)$	13	First term
d	4	common difference
$x(n)$	$[13 + 4n]u(n)$	General term of the series

TABLE I
INPUT PARAMETER TABLE

$$x(n) = x(0) + nd \quad (1)$$

$$n = \frac{x(n) - x(0)}{d} \quad (2)$$

$$n = \frac{97 - 13}{4} = 21 \quad (3)$$

$$X(z) = \sum_{k=-\infty}^{\infty} x(k) \times u(k) \times (z^{-k}) \quad (4)$$

$$\Rightarrow nu(n) \xleftrightarrow{Z} \frac{z^{-1}}{(1 - z^{-1})^2} \quad (5)$$

$$R.O.C \quad |z| > 1 \quad (6)$$

$$X(z) = \frac{13 - 9z^{-1}}{(1 - z^{-1})^2}, |z| > 1 \quad (7)$$

$$y(n) = x(n) * u(n) \quad (8)$$

$$\Rightarrow Y(z) = X(z)U(z) \quad (9)$$

$$Y(z) = \left(\frac{13 - 9z^{-1}}{(1 - z^{-1})^3} \right), |z| > 1 \quad (10)$$

Using contour integration to find the inverse z-transform,

$$y(21) = \frac{1}{2\pi j} \oint_C Y(z) z^{20} dz \quad (11)$$

$$= \frac{1}{2\pi j} \oint_C \left(\frac{(13 - 9z^{-1})z^{20}}{(1 - z^{-1})^3} \right) \quad (12)$$

We can observe that the pole is repeated 3 times and thus $m = 3$,

$$R = \frac{1}{(m-1)!} \lim_{z \rightarrow a} \frac{d^{m-1}}{dz^{m-1}} ((z-a)^m f(z)) \quad (13)$$

$$= \frac{1}{(2)!} \lim_{z \rightarrow 1} \frac{d^2}{dz^2} \left((z-1)^3 \frac{(13 - 9z^{-1})z^{23}}{(z-1)^3} \right) \quad (14)$$

$$= \frac{1}{(2)!} \lim_{z \rightarrow 1} \frac{d^2}{dz^2} (13z^{23} - 9z^{22}) \quad (15)$$

$$= \frac{1}{(2)!} (13 \times 23 \times 22 - 9 \times 22 \times 21) \quad (16)$$

$$= 1210 \quad (17)$$

Therefore, the sum of all two-digit numbers that, when divided by 4, yield a remainder of 1 is 1210.

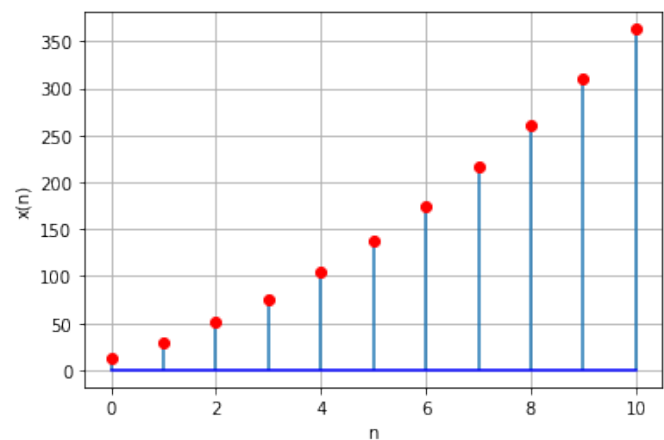


Fig. 1. $y(n) = 13 + 15n + 2n^2$