Assignment 2

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1 PROBLEM 70 FROM GATE EC

Let X and Y be continuous random variables with the joint probabilitiy distribution function

$f(x,y) = \begin{cases} ae^{-2y}, & 0 < x < y < \infty \\ 0, & \text{otherwise} \end{cases}$

The value of $E(X|Y = 2)$ is

(A) 4   (B) 3   (C) 2   (D) 1

2 SOLUTION

Given two continuous random variables X and Y, whose joint probability distribution function is

$$f(x, y) = \begin{cases} ae^{-2y}, & 0 < x < y < \infty \\ 0, & \text{otherwise} \end{cases} \tag{2.0.1}$$

We are asked to find the value of $E(X|Y = 2)$. Firstly we find the marginal distribution function for $X = x$ given $Y = y$,

$$f_{x,y}(x|y) = \frac{f_{x,y}(x, y)}{f_y(y)} \tag{2.0.2}$$

$$f_y(y) = \int_{-\infty}^{\infty} f_{x,y}(x, y)\, dx \tag{2.0.3}$$

$$= \int_{-\infty}^{0} f_{x,y}(x, y)\, dx + \int_{0}^{y} f_{x,y}(x, y)\, dx$$
$$+ \int_{y}^{\infty} f_{x,y}(x, y)\, dx \tag{2.0.4}$$

$$= 0 + \int_{0}^{y} ae^{-2y}\, dx + 0 \tag{2.0.5}$$

$$f_y(y) = ae^{-2y}y \tag{2.0.6}$$

Therefore,

$$\Pr(X = x|Y = 2) = \frac{f_{x,y}(x, y)}{f_y(2)} \tag{2.0.7}$$

Substituting y=2 in Equation2.0.6

$$= \frac{ae^{-4}}{ae^{-4}2} \tag{2.0.8}$$

$$\Pr(X = x|Y = 2) = \begin{cases} \frac{1}{2}, & 0 < x < y < \infty \\ 0, & \text{otherwise} \end{cases} \tag{2.0.9}$$

Hence, expected value of X is

$$E(X|Y = 2) = \int_{-\infty}^{\infty} x\Pr(X = x|Y = 2)\, dx \tag{2.0.10}$$

$$= \int_{-\infty}^{0} x\Pr(X = x|Y = 2)\, dx$$
$$+ \int_{0}^{2} x\Pr(X = x|Y = 2)\, dx \tag{2.0.11}$$
$$+ \int_{2}^{\infty} x\Pr(X = x|Y = 2)\, dx$$

$$= 0 + \int_{0}^{2} x\frac{e^{4-4}}{2}\, dx + 0 \tag{2.0.12}$$

$$= \int_{0}^{2} x\frac{1}{2}\, dx \tag{2.0.13}$$

$$= \frac{1}{2}\left[\frac{x^2}{2}\right]_{0}^{2} \tag{2.0.14}$$

$$= \frac{1}{2}\frac{4}{2} \tag{2.0.15}$$

$$\boxed{E(X|Y = 2) = 1} \tag{2.0.16}$$