Assignment 1

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Download all python codes from

https://github.com/GouthamSai22/AI1103/blob/main/Assignment2/Codes

and latex-tikz codes from

https://github.com/GouthamSai22/AI1103/blob/main/Assignment2/main.tex

1 Problem 70 from GATE EC

Let X and Y be continuous random variables with the joint probabilitiy distribution function
f(x,y) = $\begin{cases} ae^{-2y}, & 0 < x < y < \infty \\ 0, & \text{otherwise} \end{cases}$
The value of $E(X|Y = 2)$ is
(A) 4 (B) 3 (C) 2 (D) 1

2 Solution

Given two continuous random variables X and Y, whose joint probability distribution function is

$$f(x, y) = \begin{cases} ae^{-2y}, & 0 < x < y < \infty \\ 0, & \text{otherwise} \end{cases} \tag{2.0.1}$$

We are asked to find the value of $E(X|Y = 2)$. Firstly we find the marginal distribution function for $X = x$ given $Y = y$,

$$f_{x,y}(x|y) = \frac{f_{x,y}(x, y)}{f_y(y)} \tag{2.0.2}$$

$$f_y(y) = \int_{-\infty}^{\infty} f_{x,y}(x, y)\, dx \tag{2.0.3}$$

$$= \int_{-\infty}^{0} f_{x,y}(x, y)\, dx + \int_{0}^{y} f_{x,y}(x, y)\, dx + \int_{y}^{\infty} f_{x,y}(x, y)\, dx \tag{2.0.4}$$

$$= 0 + \int_{0}^{y} ae^{-2y}\, dx + 0 \tag{2.0.5}$$

$$f_y(y) = ae^{-2y}y \tag{2.0.6}$$

Therefore,

$$\Pr(X = x|Y = 2) = \frac{f_{x,y}(x, y)}{f_y(2)} \tag{2.0.7}$$

Substituting y=2 in Equation2.0.6

$$= \frac{ae^{-4}}{ae^{-4}2} \tag{2.0.8}$$

$$\Pr(X = x|Y = 2) = \begin{cases} \frac{1}{2}, & 0 < x < y < \infty \\ 0, & \text{otherwise} \end{cases} \tag{2.0.9}$$

Hence, expected value of X is

$$E(X|Y = 2) = \int_{-\infty}^{\infty} x \Pr(X = x|Y = 2)\, dx \tag{2.0.10}$$

$$= \int_{-\infty}^{0} x \Pr(X = x|Y = 2)\, dx + \int_{0}^{2} x \Pr(X = x|Y = 2)\, dx + \int_{2}^{\infty} x \Pr(X = x|Y = 2)\, dx \tag{2.0.11}$$

$$= 0 + \int_{0}^{2} x \frac{e^{4-4}}{2}\, dx + 0 \tag{2.0.12}$$

$$= \int_{0}^{2} x \frac{1}{2}\, dx \tag{2.0.13}$$

$$= \frac{1}{2}[\frac{x^2}{2}]_0^2 \tag{2.0.14}$$

$$= \frac{1}{2}\frac{4}{2} \tag{2.0.15}$$

$$\boxed{E(X|Y = 2) = 1} \tag{2.0.16}$$