

Assignment 2

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Download all python codes from

<https://github.com/GouthamSai22/AI1103/blob/main/Assignment2/Codes>

and latex-tikz codes from

<https://github.com/GouthamSai22/AI1103/blob/main/Assignment2/main.tex>

1 PROBLEM 70 FROM GATE EC

Let X and Y be continuous random variables with the joint probability distribution function

$$f(x,y) = \begin{cases} ae^{-2y}, & 0 < x < y < \infty \\ 0, & \text{otherwise} \end{cases}$$

The value of $E(X|Y = 2)$ is

- 1) 4
- 2) 3
- 3) 2
- 4) 1

2 SOLUTION

Given two continuous random variables X and Y, whose joint probability distribution function is

$$f(x,y) = \begin{cases} ae^{-2y}, & 0 < x < y < \infty \\ 0, & \text{otherwise} \end{cases} \quad (2.0.1)$$

We are asked to find the value of $E(X|Y = 2)$. Firstly we find the marginal distribution function for $X = x$ given $Y = y$,

$$f_{x,y}(x|y) = \frac{f_{x,y}(x,y)}{f_y(y)} \quad (2.0.2)$$

$$f_y(y) = \int_{-\infty}^{\infty} f_{x,y}(x,y) dx \quad (2.0.3)$$

$$= \int_{-\infty}^0 f_{x,y}(x,y) dx + \int_0^y f_{x,y}(x,y) dx \quad (2.0.4)$$

$$+ \int_y^{\infty} f_{x,y}(x,y) dx$$

$$= 0 + \int_0^y ae^{-2y} dx + 0 \quad (2.0.5)$$

$$f_y(y) = ae^{-2y}y \quad (2.0.6)$$

Therefore,

$$\Pr(X = x|Y = 2) = \frac{f_{x,y}(x,y)}{f_y(2)} \quad (2.0.7)$$

Substituting $y=2$ in Equation 2.0.6

$$= \frac{ae^{-4}}{ae^{-4}2} \quad (2.0.8)$$

$$\Pr(X = x|Y = 2) = \begin{cases} \frac{1}{2}, & 0 < x < y < \infty \\ 0, & \text{otherwise} \end{cases} \quad (2.0.9)$$

Hence, expected value of X is

$$E(X|Y = 2) = \int_{-\infty}^{\infty} x \Pr(X = x|Y = 2) dx \quad (2.0.10)$$

$$= \int_{-\infty}^0 x \Pr(X = x|Y = 2) dx$$

$$+ \int_0^2 x \Pr(X = x|Y = 2) dx \quad (2.0.11)$$

$$+ \int_2^{\infty} x \Pr(X = x|Y = 2) dx$$

$$= 0 + \int_0^2 x \frac{e^{4-4}}{2} dx + 0 \quad (2.0.12)$$

$$= \int_0^2 x \frac{1}{2} dx \quad (2.0.13)$$

$$= \frac{1}{2} \left[\frac{x^2}{2} \right]_0^2 \quad (2.0.14)$$

$$= \frac{1}{2} \frac{4}{2} \quad (2.0.15)$$

$$\boxed{E(X|Y = 2) = 1} \quad (2.0.16)$$