Assignment 1

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Download all python codes from

https://github.com/GouthamSai22/AI1103/blob/main/Assignment2/Codes

and latex-tikz codes from

https://github.com/GouthamSai22/AI1103/blob/main/Assignment2/main.tex

1 Problem 70 from GATE EC

Let X and Y be continuous random variables with the joint probabilitiy distribution function

f(x,y) = $\begin{cases} ae^{-2y}, & 0 < x < y < \infty \\ 0, & \text{otherwise} \end{cases}$

The value of $E(X|Y = 2)$ is ?

2 Solution

Given two continuous random variables X and Y, whose joint probability distribution function is

$$f(x, y) = \begin{cases} ae^{-2y}, & 0 < x < y < \infty \\ 0, & \text{otherwise} \end{cases} \tag{2.0.1}$$

We are asked to find the value of $E(X|Y = 2)$. Since Y=2, we can say that the probability distribution function of x in this case is

$$\Pr(X = x) = \begin{cases} ae^{-2\times 2}, & 0 < x < 2 \\ 0 & \text{otherwise} \end{cases} \tag{2.0.2}$$

From the properties of the joint probability function,

$$\iint_{-\infty}^{\infty} f(x, y)\, dx\, dy = 1 \tag{2.0.3}$$

From the definition of the joint probability equation, Equation2.0.3 can be written as

$$\int_0^{\infty} \left(\int_0^{y} ae^{-2y}\, dx \right) dy = 1 \tag{2.0.4}$$

$$\int_0^{\infty} \left(ae^{-2y} \int_0^{y} 1\, dx \right) dy = 1 \tag{2.0.5}$$

$$\int_0^{\infty} \left(ae^{-2y} y \right) dy = 1 \tag{2.0.6}$$

$$a \int_0^{\infty} e^{-2y} y\, dy = 1 \tag{2.0.7}$$

$$a \left(-\frac{1}{2} e^{-2y} \left(y + \frac{1}{2} \right) \right)_0^{\infty} = 1 \tag{2.0.8}$$

$$a \left(\frac{1}{4} \right) = 1 \tag{2.0.9}$$

$$\Rightarrow a = 4 \tag{2.0.10}$$

Hence, Equation2.0.2 can be written as

$$\Pr(X = x) = \begin{cases} 4e^{-4}, & 0 < x < 2 \\ 0 & \text{otherwise} \end{cases} \tag{2.0.11}$$

We know, for a continuous random variable,

$$E(X) = \int_{-\infty}^{\infty} x \Pr(X = x)\, dx \tag{2.0.12}$$

$$= \int_{-\infty}^{0} x \Pr(X = x)\, dx + \int_0^{2} x \Pr(X = x)\, dx + \int_2^{\infty} x \Pr(X = x)\, dx \tag{2.0.13}$$

$$= 0 + \int_0^{2} \left(x \times 4e^{-4} \right) dx + 0 \tag{2.0.14}$$

$$= 4e^{-4} \left(\frac{x^2}{2} \right)_0^2 \tag{2.0.15}$$

$$= 4e^{-4} \times 2 \tag{2.0.16}$$

$$\tag{2.0.17}$$

$$\boxed{E(X) = 0.146} \tag{2.0.18}$$