

3) Mathematically derive the average runtime complexity of the non-random pivot version of quicksort.

The time complexity of recurrence relation in non-random pivot:

$$T(n) = T(k) + T(n-k-1) + O(n)$$

where k = no. of elements less than pivot.

On average, pivot divides array equally into two halves

$$k \approx \frac{n}{2}$$

Substitute k in $T(n)$

$$T(n) \approx T\left(\frac{n}{2}\right) + T\left(n - \frac{n}{2} - 1\right) + O(n)$$

$$\approx 2T\left(\frac{n}{2}\right) + O(n)$$

Using Master's Theorem

$$\underline{T(n) = O(n \log n)}$$