12-带限信道中的无码间干扰传输

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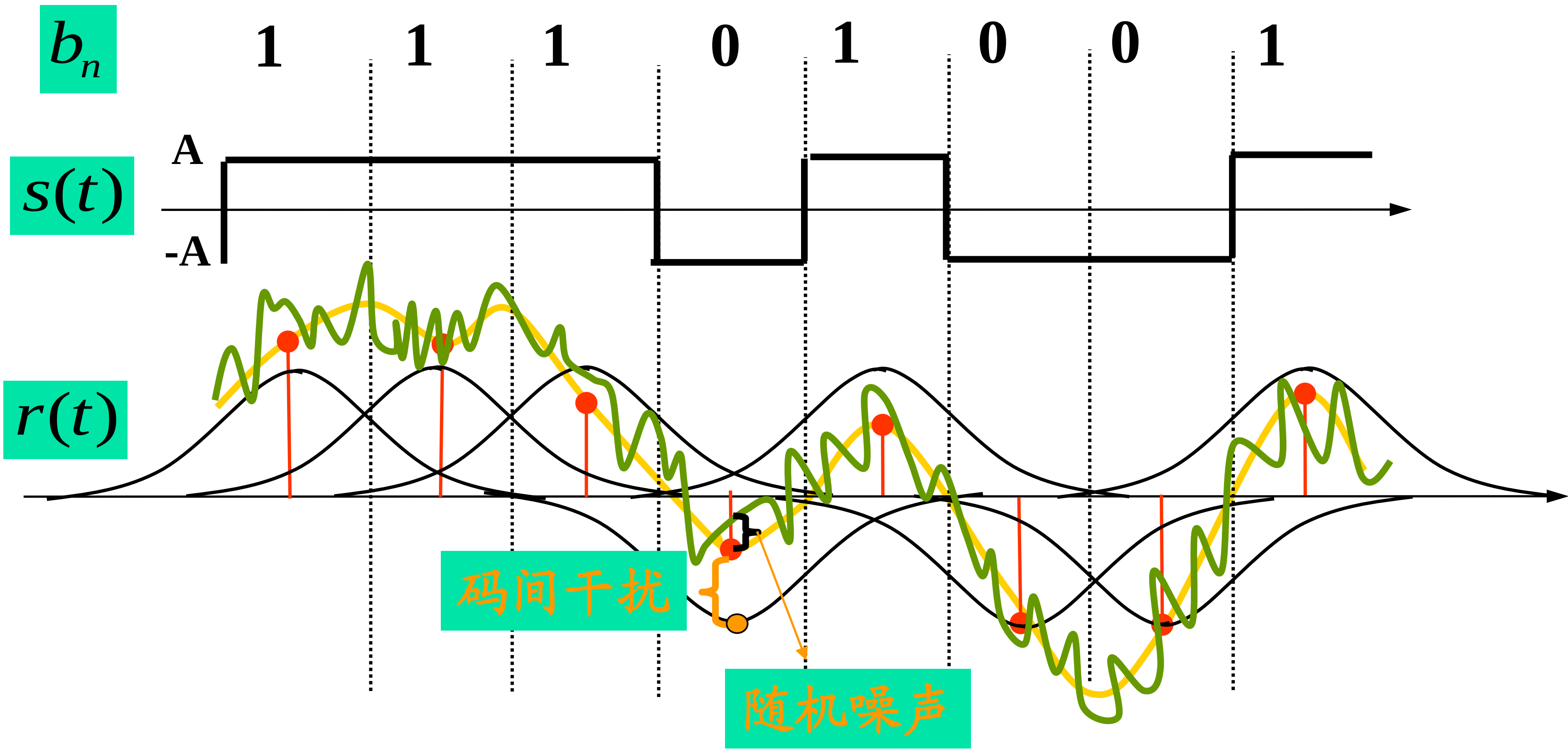
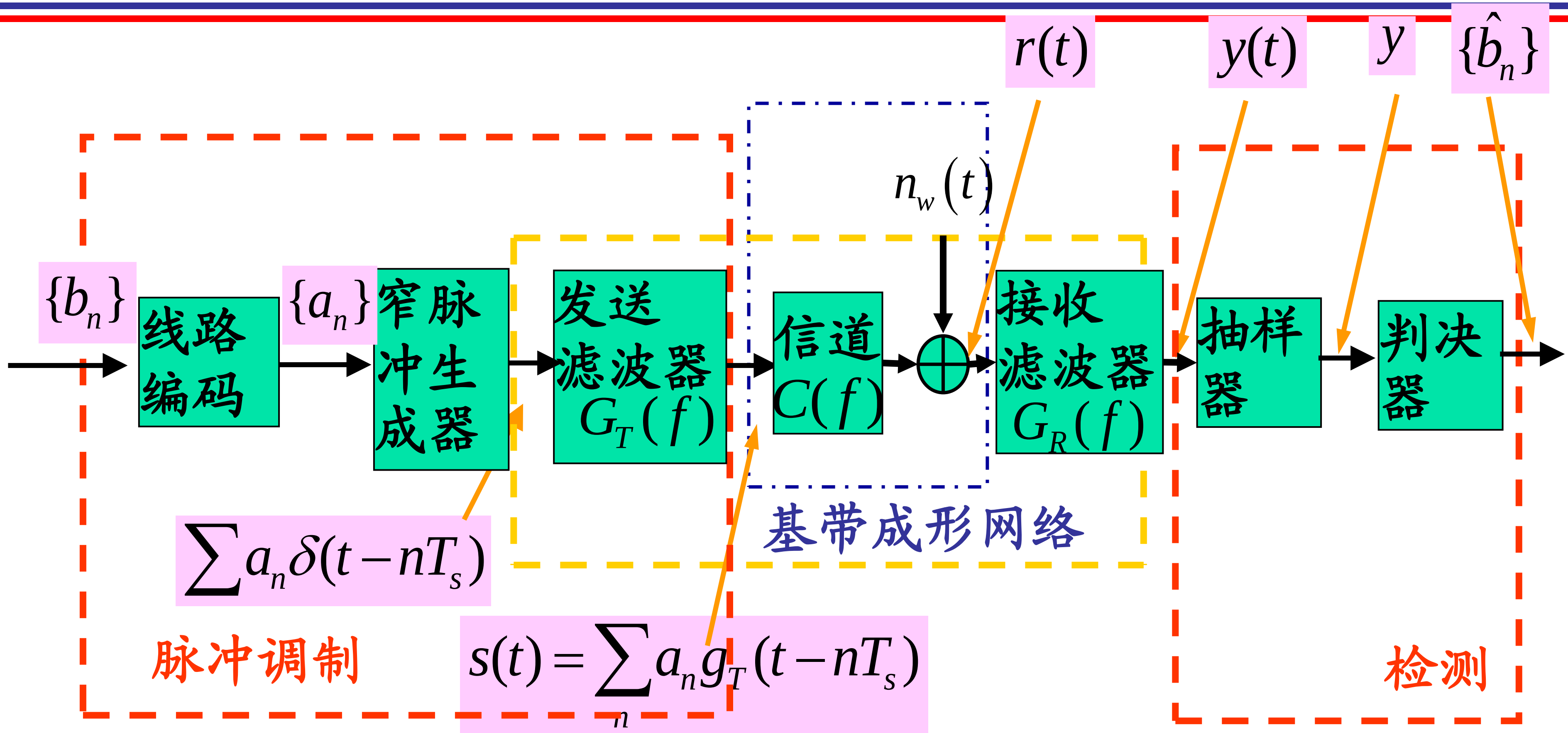
（2）无码间干扰的基带传输特性

（1）数字PAM基带传输及码间干扰

- 带限信道对基带传输信号的影响



码间干扰分析模型



$$g(t)=g_T(t)*c(t)\xleftrightarrow{F.T.}G(f)=G_T(f)C(f)$$

$$h(t)=g(t)*g_R(t)\xleftrightarrow{F.T.}H(f)=G_T(f)C(f)G_R(f)$$

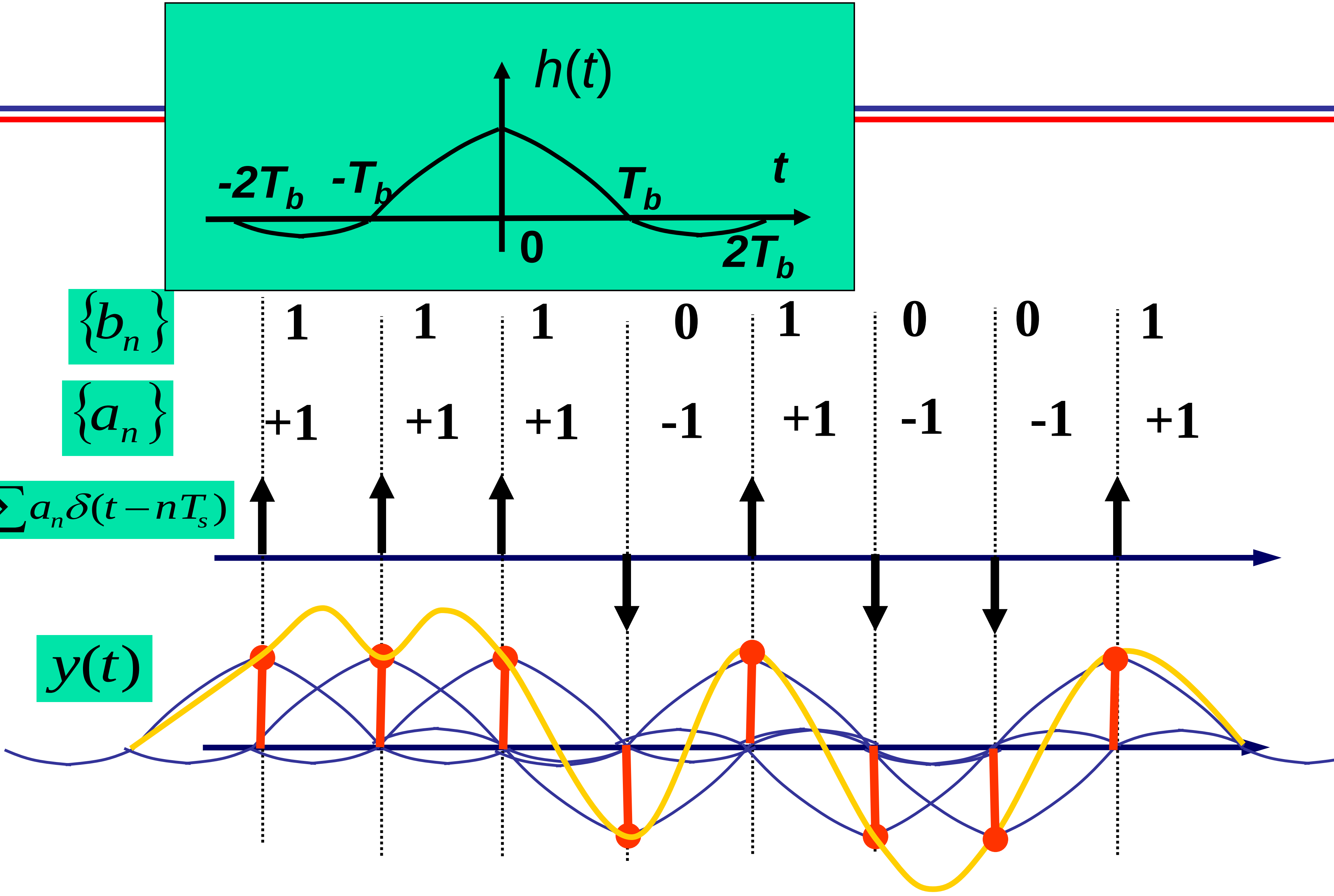
码间干扰分析模型

$$r(t)=\left[\sum_{n=-\infty}^{\infty}a_n\delta(t-nT_s)\right]*g(t)+n_w(t)=\sum_{n=-\infty}^{\infty}a_n g(t-nT_s)+n_w(t)$$

$$y(t)=\left[\sum_{n=-\infty}^{\infty}a_n\delta(t-nT_s)\right]*h(t)+n_R(t)=\sum_{n=-\infty}^{\infty}a_n h(t-nT_s)+n_R(t)$$

$$n_R(t)=n_w(t)*g_R(t)$$

$$\begin{aligned}y(mT_s+t_0)&=\sum_{n=-\infty}^{\infty}a_n h(mT_s+t_0-nT_s)+n_R(mT_s+t_0)\\&=\underbrace{a_m h(t_0)}_{\text{有用信号}}+\underbrace{\sum_{n=-\infty,n\neq m}^{\infty}a_n h\left[(m-n)T_s+t_0\right]}_{\text{码间干扰(ISI)}}+\underbrace{n_R(mT_s+t_0)}_{\text{随机噪声}}\end{aligned}$$



h(t)
t
-2T_b
-T_b
T_b
0
2T_b
{b_n}
1 1 1 0 1 0 0 1
{a_n}
+1 +1 +1 -1 +1 -1 -1 +1
∑a_nδ(t−nT_s)
y(t)

（2）无码间干扰的基带传输特性

- 抽样判决基带信号波形取决于基带传输系统冲激响应h(t)

$$h(t) \xleftrightarrow{F.T.} H(f) = G_T(f)C(f)G_R(f)$$

- 什么样的$h(t)$或者$H(f)$能够形成最小码间干扰？

$$h(nT_s) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

$$h(nT_s) = \int_{-\infty}^{\infty} H(f) e^{j2\pi f n T_s} df$$

奈奎斯特第一准则

定理：为使$h(t)$满足

$$h(nT_s)=\begin{cases}1 & n=0\\ 0 & n\neq 0\end{cases}$$

其充分必要条件是$h(t)$的傅立叶变换$H(f)$必须满足

$$\sum_{m=-\infty}^{\infty} H(f+\frac{m}{T_s})=T_s$$

$$Z(f)=\sum_{m=-\infty}^{\infty} H(f+m\cdot r_s)=T_s$$

奈奎斯特第一准则证明(1/3)

$$h(t)=\int_{-\infty}^{\infty}H(f)e^{j2\pi ft}df$$

$$h(nT_s)=\int_{-\infty}^{\infty}H(f)e^{j2\pi nT_s t}df=\sum_i\int_{(2m-1)/2T_s}^{(2m+1)/2T_s}H(f)e^{j2\pi fnT_s}df$$

令$f'=f-\frac{m}{T_s},df'=df,f=f'+\frac{m}{T_s}$

$$h(nT_s)=\sum_m\int_{-1/2T_s}^{1/2T_s}H\left(f'+\frac{m}{T_s}\right)e^{j2\pi f'nT_s}e^{j2\pi mn}df'$$

$$=\sum_m\int_{-1/2T_s}^{1/2T_s}H\left(f'+\frac{m}{T_s}\right)e^{j2\pi f'nT_s}df'$$

$$=\int_{-1/2T_s}^{1/2T_s}\sum_m H\left(f+\frac{m}{T_s}\right)e^{j2\pi fnT_s}df$$

奈奎斯特第一准则证明(2/3)

对于周期为$f_0=1/T_s$的周期信号

$$G(f)=\sum_n g_n e^{j\frac{2\pi nf}{f_0}}$$

其中

$$g_n=\frac{1}{f_0}\int_{-f_0/2}^{f_0/2} G(f)e^{\frac{j2\pi nf}{f_0}}df=T_s\int_{-1/2T_s}^{1/2T_s} G(f)e^{j2\pi nfT_s}df$$

$$h_n=h(nT_s)=T_s\int_{-1/2T_s}^{1/2T_s}\boxed{\left\{\frac{1}{T_s}\sum_m H\left(f+\frac{m}{T_s}\right)\right\}}e^{j2\pi fnT_s}df$$

⇓

h_n *is the Fourier coefficient of* $G(f)=\frac{1}{T_s}\sum_m H\left(f+\frac{m}{T_s}\right)$

奈奎斯特第一准则证明(3/3)

$$\underbrace{\frac{1}{T_s}\sum_m H\left(f+\frac{m}{T_s}\right)}_{\text{Periodic function}} = \underbrace{\sum_n \overbrace{h_n}^{\text{Fourier Coefficients}} e^{-j2\pi f n T_s}}_{\text{Fourier Series}}$$

$$h(n)=\begin{cases}1 & n=0\\ 0 & n\neq 0\end{cases} \Longrightarrow \frac{1}{T_s}\sum_m H\left(f+\frac{m}{T_s}\right)=1$$

$$Z(f)=\sum_m H\left(f+\frac{m}{T_s}\right)=T_s$$

Nyquist first criterion

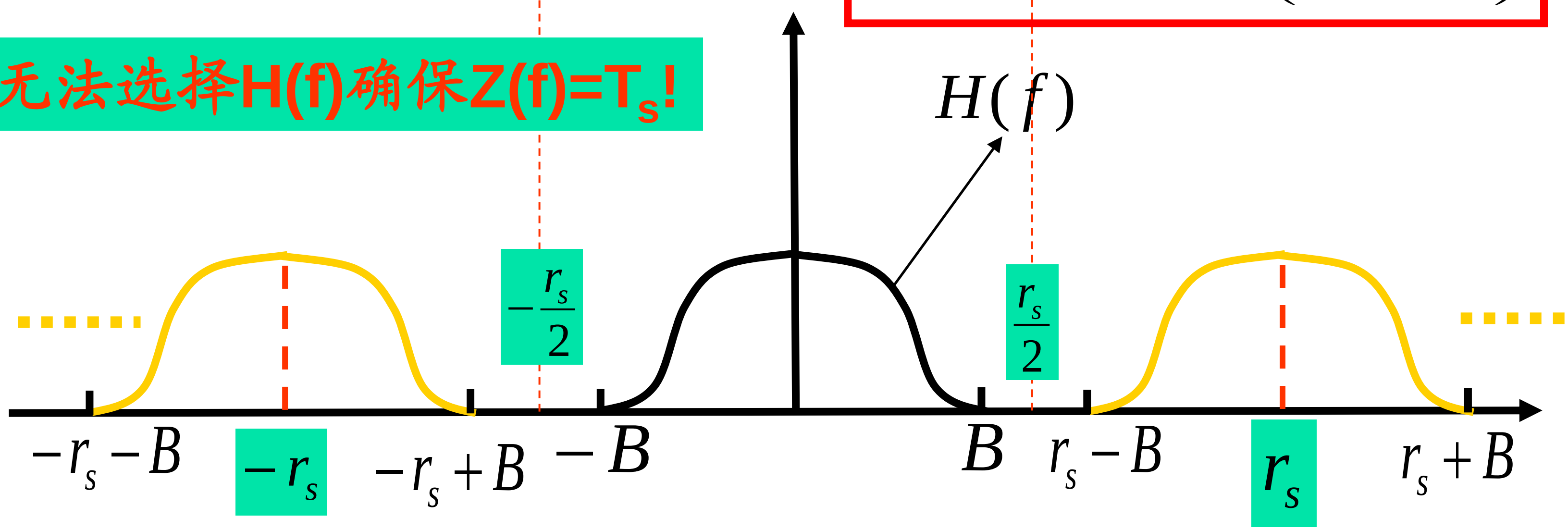
奈奎斯特第一准则讨论(1/3)

假定基带传输系统带宽为B，符号传输速率为r_s，符号间隔为 $T_s=1/r_s$,显然当f>B时，H(f)=0。分三种情况讨论如下:

- 情况1：$r_s>2B$

$$Z(f)=\sum_m H\left(f+\frac{m}{T_s}\right)$$

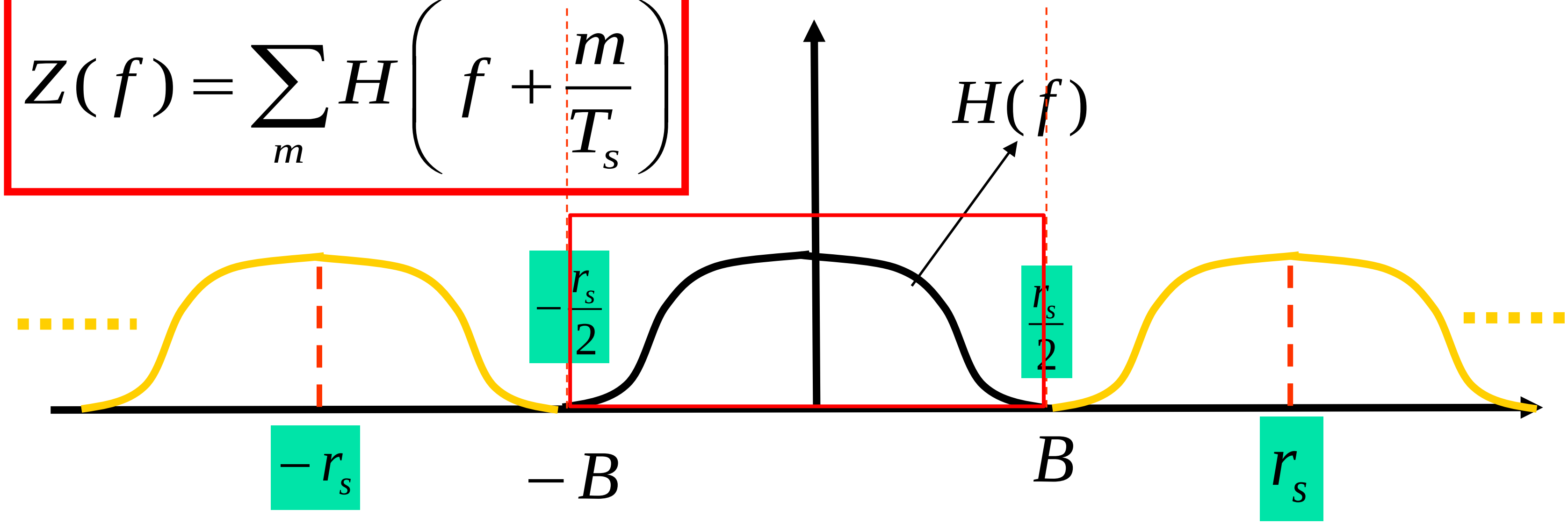
无法选择H(f)确保$Z(f)=T_s$!



奈奎斯特第一准则讨论(2/3)

- 情况2：$r_s=2B$,

$$Z(f)=\sum_m H\left(f+\frac{m}{T_s}\right)$$



只有一种H(f)能导致Z(f)=T_s!

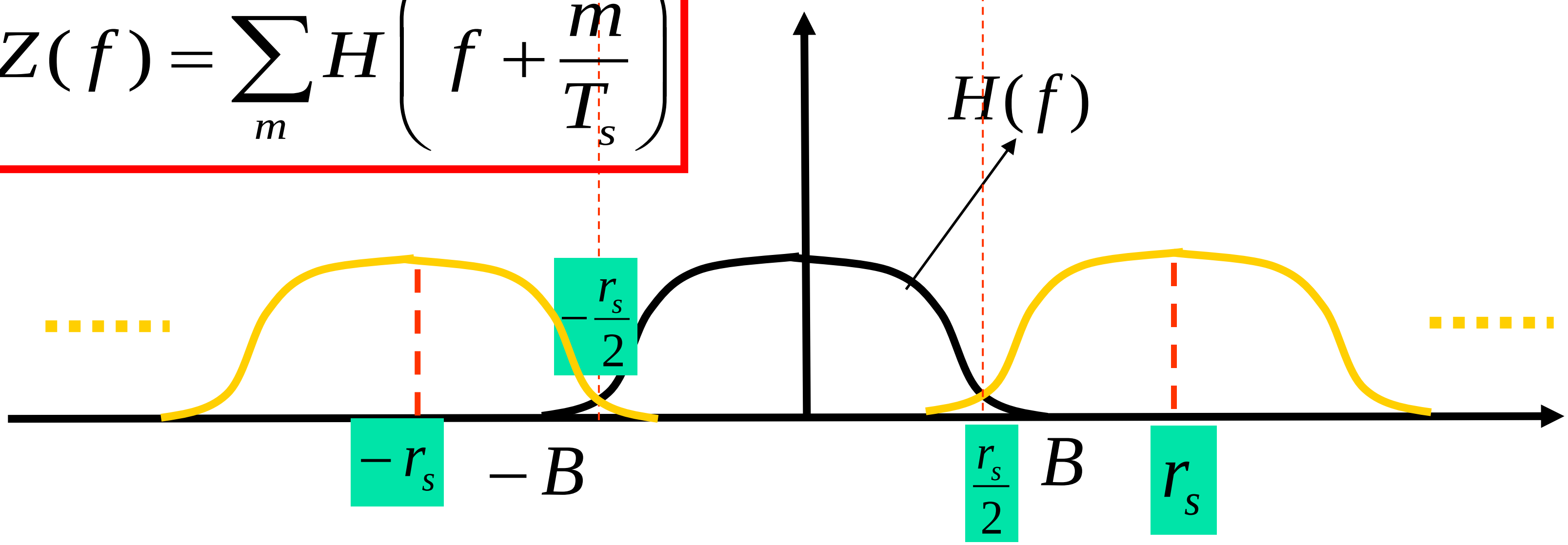
对应传输带宽最小的情况!

$$H(f)=\begin{cases}T_s, |f|\le B\\ 0, (其它)\end{cases}$$

奈奎斯特第一准则讨论(3/3)

- 情况3：$r_s<2B$,

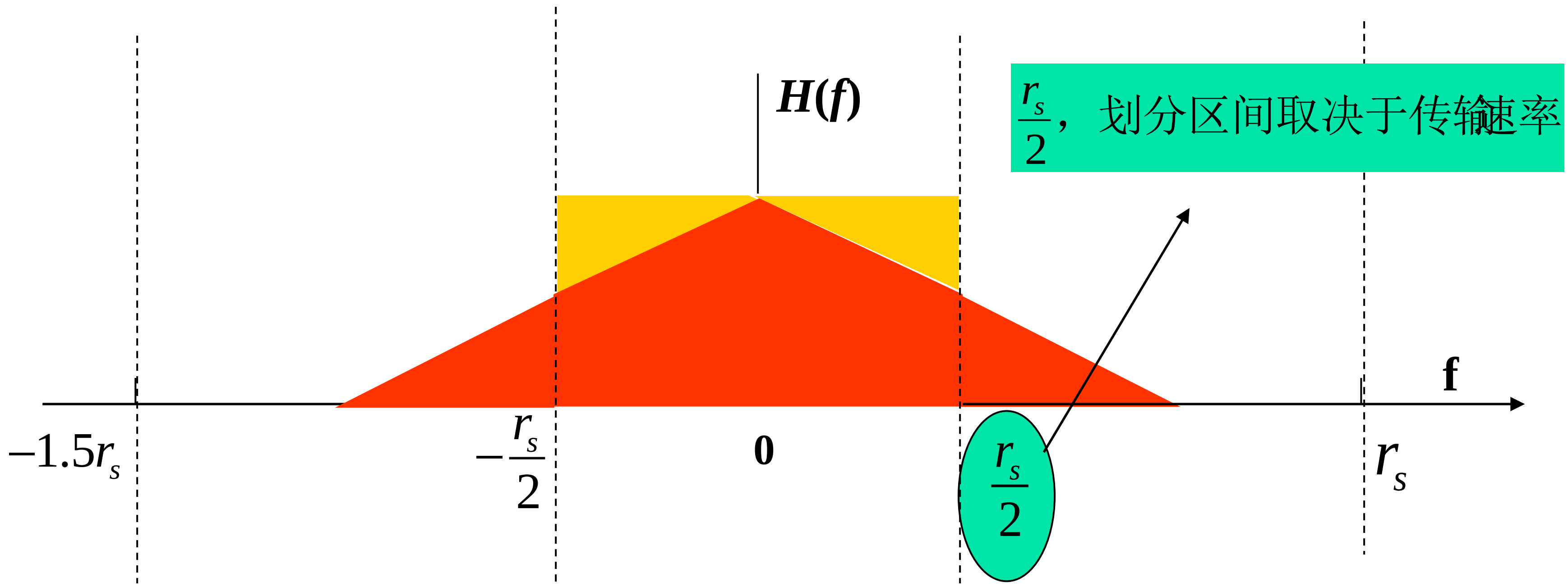
$$Z(f)=\sum_m H\left(f+\frac{m}{T_s}\right)$$



有无数种H(f)的选择能导致$Z(f)=T_s$!

最重要的$B>r_s/2$的基带传输系统为升余弦滚降特性

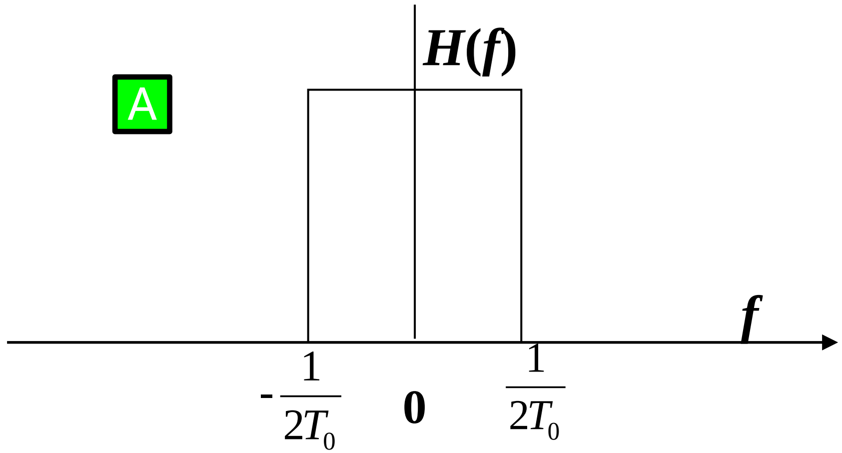
奈奎斯特第一准则物理解释



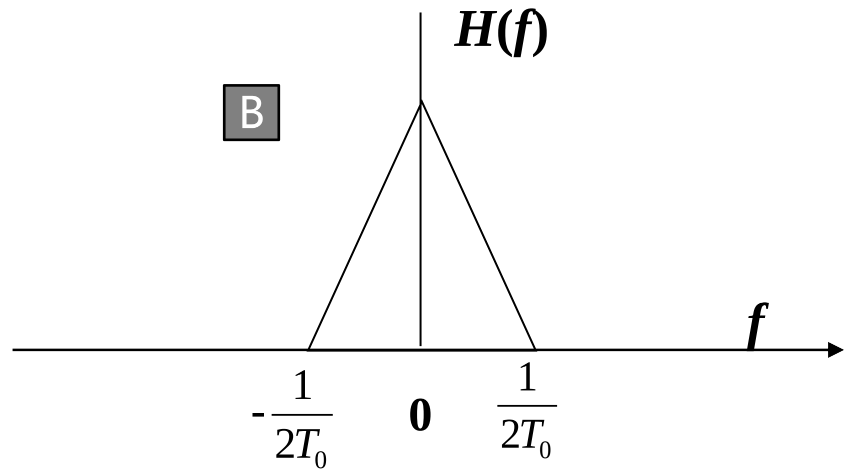
$\sum_m H(f+m\cdot r_s)$ 是H(f)移位mr_s再相加而成的，只要检查在区间（$-r_s/2$，$r_s/2$）上能否叠加出一根水平直线即可判断有无码间干扰。

若系统的码元传输速率$r_s=1/T_0$，下面基带传输系统中满足无码间干扰传输的有________________

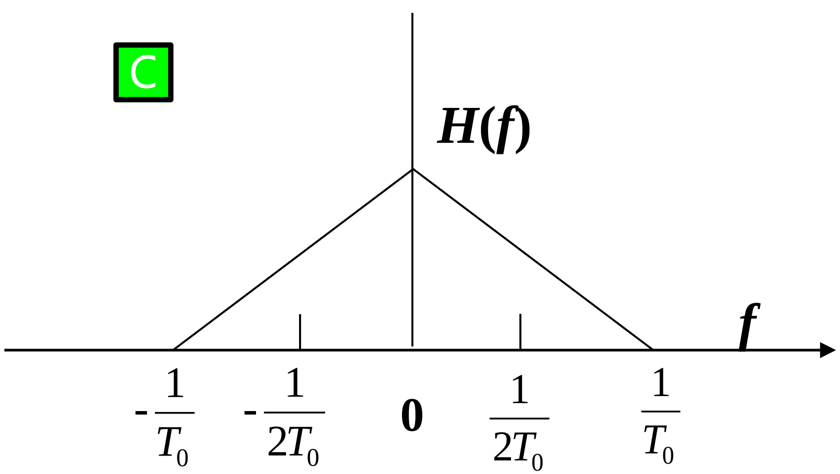
A



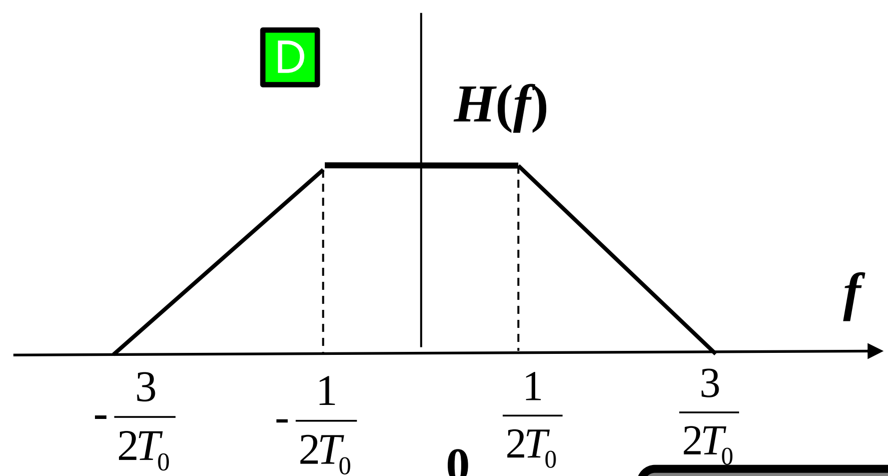
B



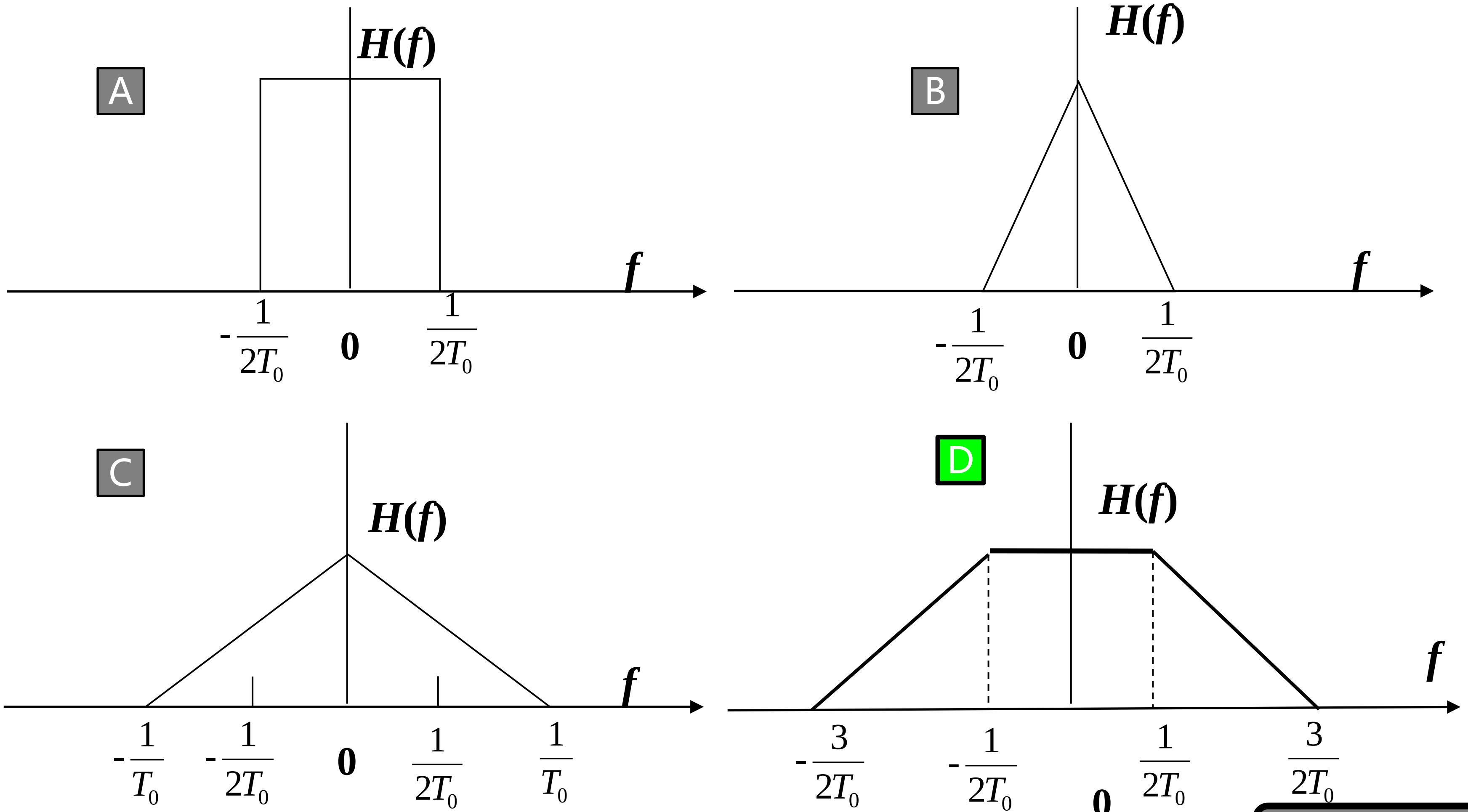
C



D

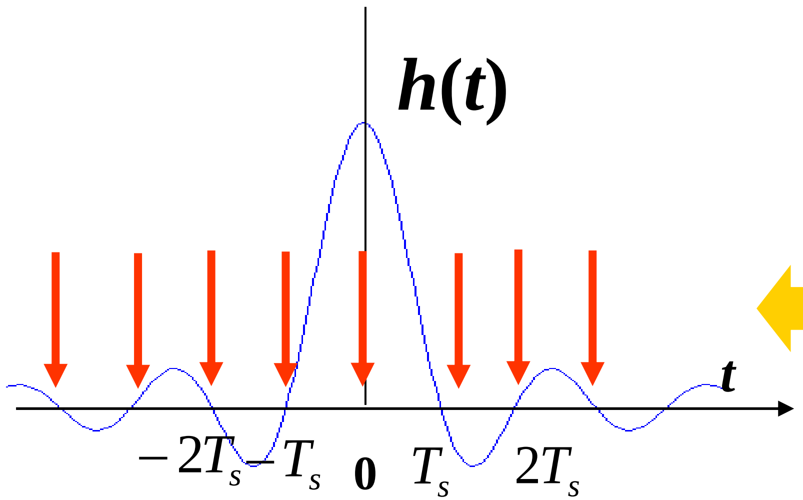


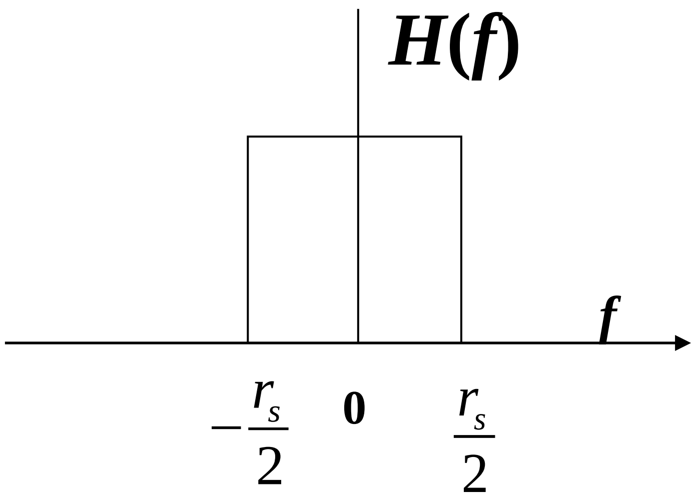
若系统的码元传输速率$r_s=2/T_0$，下面基带传输系统中满足无码间干扰传输的有________________



提交

信号带宽最小的波形(1/3)





$$h(t) = Sa\left(\frac{\pi t}{T_s}\right) \leftrightarrow H(f) = T_s\,\mathrm{Rect}\left(\frac{f}{r_s}\right)$$

信号带宽最小的波形(2/3)

- 传输带宽

$$B_T = \frac{r_s}{2}$$

  - 奈奎斯特带宽:码元传输速率一定时的最窄传输带宽
  - 奈奎斯特速率:信道频带一定时的最大传输速率
  - 奈奎斯特间隔:信道频带一定时的最小码元间隔

- 频带利用率

$$\eta_s = \frac{r_s}{B_T} = 2(Baud/Hz)$$

$$\eta_b = \frac{r_b}{B_T} = \frac{r_s \log_2(M)}{B_T} = 2\log_2(M)(bit/s/Hz)$$

信号带宽最小的波形(3/3)

- 最小带宽波形的主要缺点
  - 理想低通特性物理不可实现;
  - 冲激响应h(t)波形收敛速度较慢，拖尾以1/t速率衰减，当存在定时误差时会带来比较大的干扰

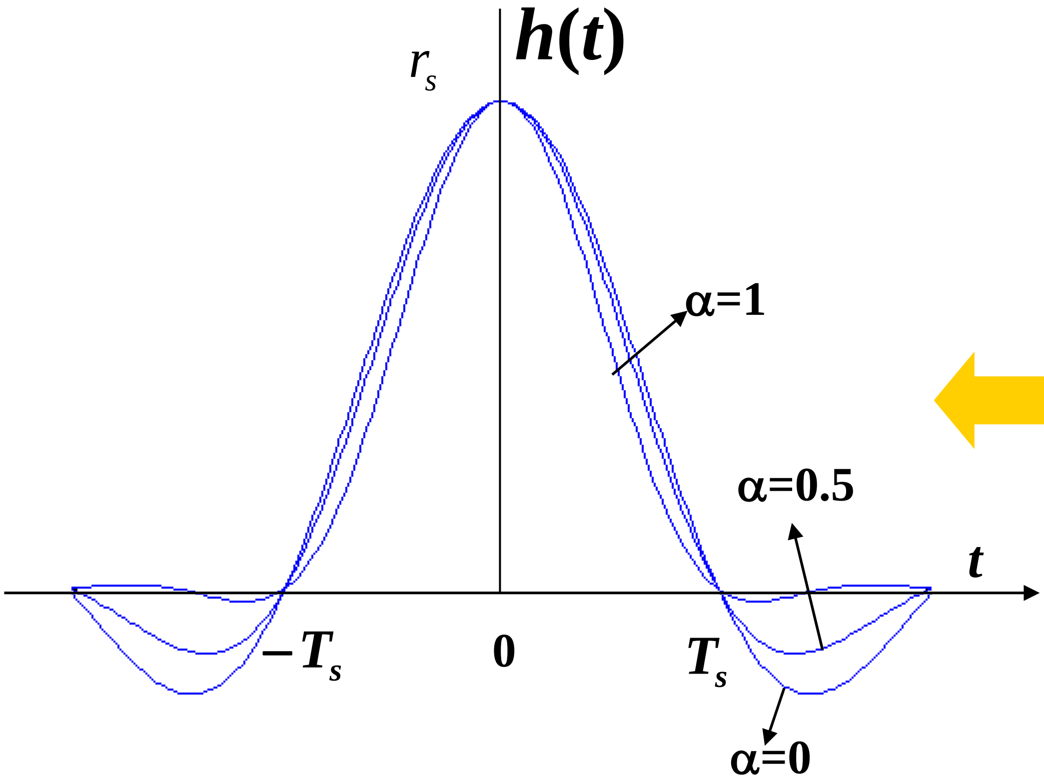
因此实际系统中通常采用升余弦谱!

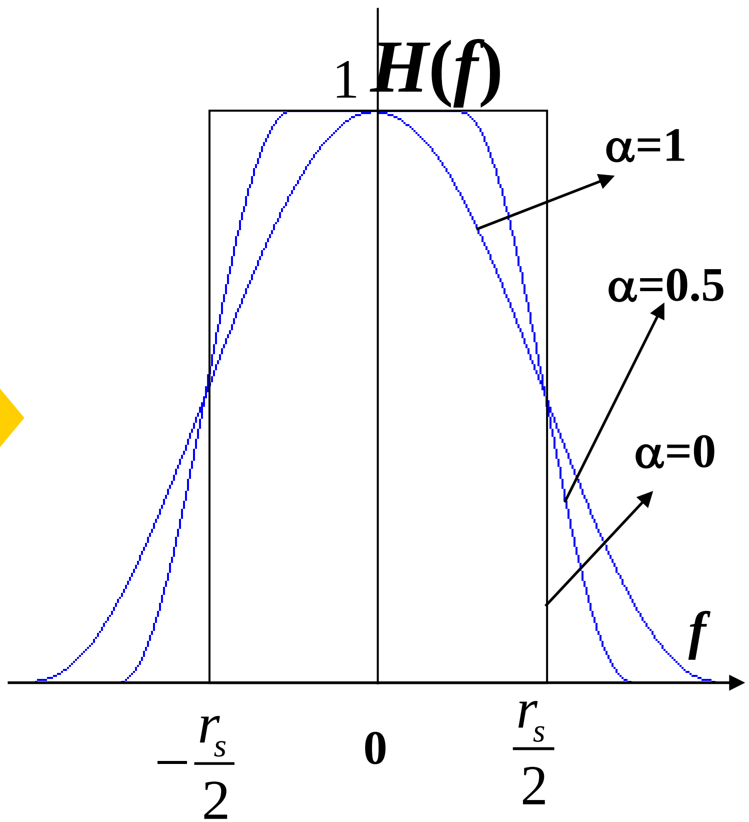
升余弦谱及其波形(1/3)

$$h(t) = sa(\frac{\pi t}{T_s})\frac{\cos(\alpha\pi t/T_s)}{1-4\alpha^2 t^2/T_s^2}$$

$$H(f) = \begin{cases} T_s, |f| < \frac{1}{2T_s}(1-\alpha) \\ \frac{T_s}{2}\{1-\sin[\frac{T_s}{\alpha}(f-\frac{1}{2T_s})]\}, |\frac{1}{2T_s}(1-\alpha) \le |f| \le \frac{1}{2T_s}(1+\alpha) \\ 0, |f| > \frac{1}{2T_s}(1+\alpha) \end{cases}$$

升余弦谱及其波形(2/3)





滚降因子 $0 \le \alpha \le 1$

升余弦谱及其波形(3/3)

- 传输带宽 $B=\frac{1+\alpha}{2}r_s$
- 频带利用率 $\eta_s=\frac{r_s}{B}=\frac{2}{1+\alpha}(Baud/Hz)$
- 全升余弦系统 $\alpha=1$

牺牲频带利用率换取频谱的滚降特性和时域更大的拖尾衰减速率！