11-理想AWGN信道数字基带信号的接收

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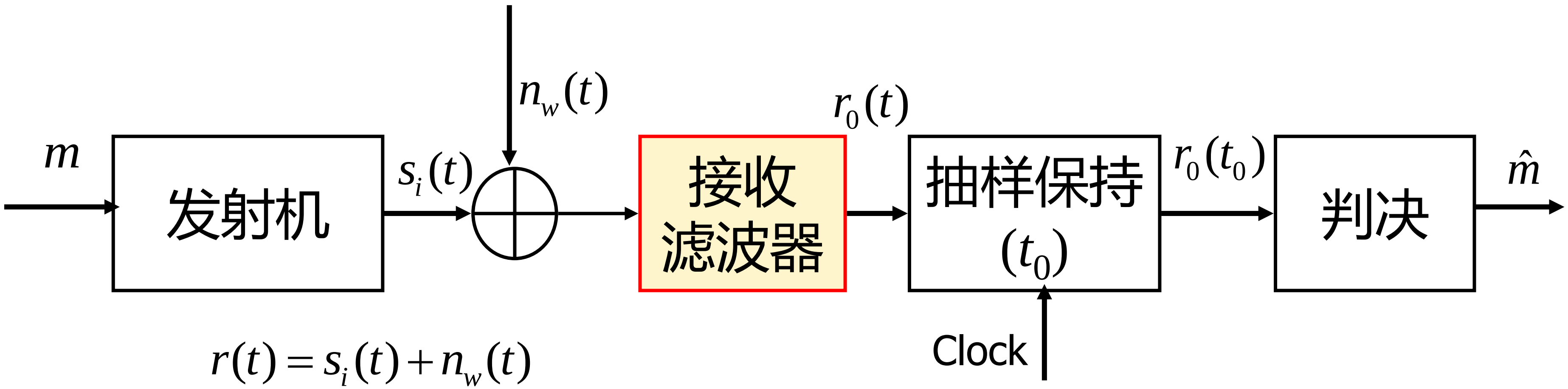
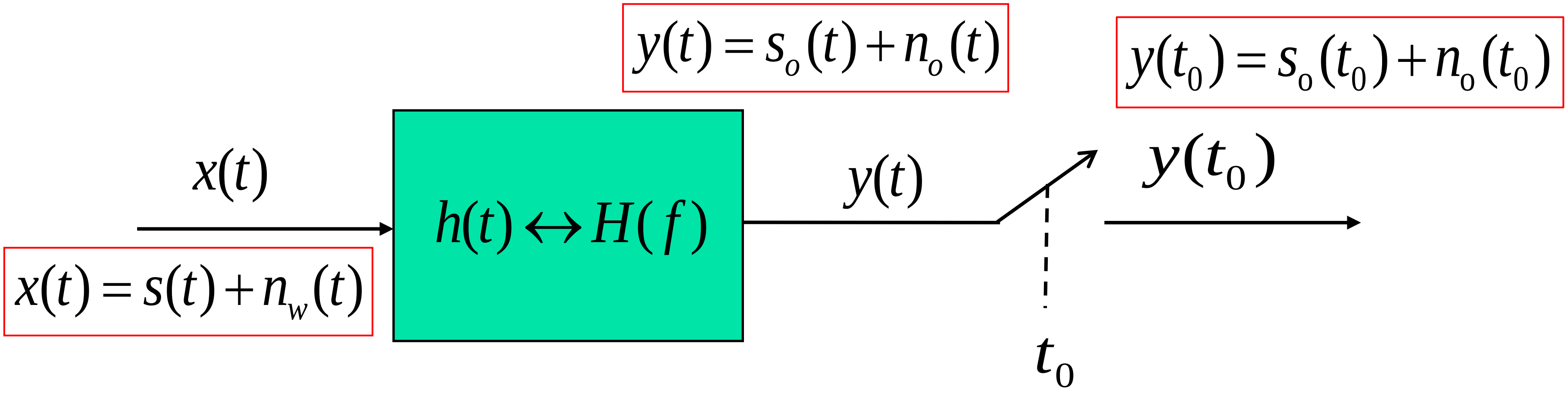
1.AWGN信道数字基带传输系统模型



2.匹配滤波器



匹配滤波器：在某时刻t_0使输出信号$s_o(t_0)$的瞬时功率与输出噪声$n_o(t_0)$的平均功率之比（称**输出信噪比**）**最大**的线性滤波器被称为信号$s(t)$的匹配滤波器。

输出信噪比:

$$\boxed{r_0=\frac{|s_o(t_0)|^2}{E\left[n_o^2(t_0)\right]}}$$

2.匹配滤波器

$$s_o(t) \leftrightarrow S_o(f) = S(f)H(f)$$

$$s_o(t) = \int_{-\infty}^{\infty} S_o(f) e^{j2\pi ft} df = \int_{-\infty}^{\infty} S(f)H(f) e^{j2\pi ft} df$$

$$s_o(t_0) = \int_{-\infty}^{\infty} S(f)H(f) e^{j2\pi ft_0} df$$

输出信噪比: $\boxed{r_0 = \dfrac{|s_o(t_0)|^2}{E\left[n_o^2(t_0)\right]}}$

$n_o(t)$ 功率谱密度: $P_{n_o}(f) = \dfrac{N_0}{2} |H(f)|^2$

$$r_o = \frac{\left| \int_{-\infty}^{\infty} S(f)H(f) e^{j2\pi ft_0} df \right|^2}{\int_{-\infty}^{\infty} \frac{N_0}{2} |H(f)|^2 df}$$

2.匹配滤波器

许瓦尔兹不等式:

$$\left|\int_{-\infty}^{\infty} X(f)H(f)df\right|^2 \le \int_{-\infty}^{\infty}|X(f)|^2 df \int_{-\infty}^{\infty}|Y(f)|^2 df$$

当且仅当 $X(f)=KY^*(f)$ 时，等式成立。

$$r_0=\frac{\left|\int_{-\infty}^{\infty} H(f)\left[S(f)e^{j2\pi f t_0}\right]df\right|^2}{\frac{N_0}{2}\int_{-\infty}^{\infty}|H(f)|^2 df}$$

$$\le\frac{\int_{-\infty}^{\infty}|H(f)|^2 df\cdot\int_{-\infty}^{\infty}|S(f)|^2 df}{\frac{N_0}{2}\int_{-\infty}^{\infty}|H(f)|^2 df}=\frac{\int_{-\infty}^{\infty}|S(f)|^2 df}{\frac{N_0}{2}}=\frac{2E_s}{N_0}$$

2.匹配滤波器

$$r_{o,\max} = \frac{\int_{-\infty}^{\infty} |S(f)|^2 df}{\frac{N_0}{2}} = \frac{2E_s}{N_0}, \qquad E_s = \int_{-\infty}^{\infty} s^2(t)dt$$

$$\boxed{H(f) = KS^*(f)e^{-j2\pi f t_0}}$$

$$h(t) = \int_{-\infty}^{\infty} H(f)e^{j2\pi ft}df = \int_{-\infty}^{\infty} KS^*(f)e^{-j2\pi ft_0}e^{j2\pi ft}df$$

$$= K\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} s(\tau)e^{-j2\pi f\tau}d\tau\right]^* e^{-j2\pi(t_0-t)}df$$

$$= K\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} e^{j2\pi(\tau-t_0+t)}df\right]s(\tau)d\tau$$

$$= K\int_{-\infty}^{\infty} \delta(\tau-t_0+t)s(\tau)d\tau = Ks(t_0-t)$$

$$\boxed{h(t) = Ks(t_0-t)}$$

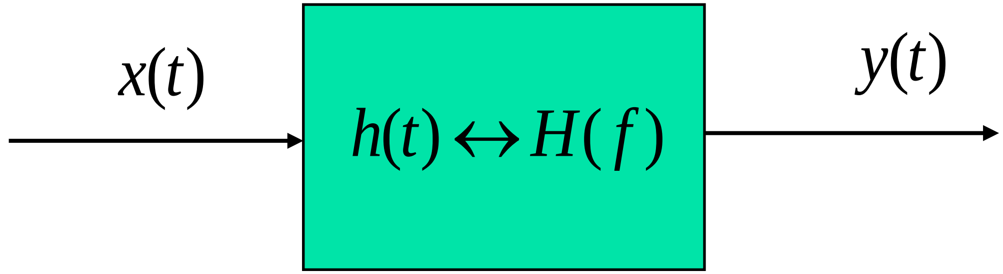
匹配滤波器的物理可实现性

物理可实现MF:

$$h(t)=0, t<0$$

$$h(t)=Ks(t_0-t) \Rightarrow \boxed{s(t')=0, t'>t_0}$$

$$设 t'=t_0-t, h(t_0-t')=Ks(t')$$



为了保证与$s(t)$匹配的滤波器$h(t)$的物理可实现性，信号$s(t)$应该在信噪比达到最大值的时刻点t_0之前结束。

用相关器来等效匹配滤波器

设匹配滤波器$h(t)=s(t_0-t)$

$$y(t)=x(t)*h(t)=\int_0^t x(\tau)h(t-\tau)d\tau$$

$$=\int_0^t x(\tau)s(t_0-t+\tau)d\tau$$

$$=\int_0^t [s(\tau)+n_w(\tau)]s(t_0-t+\tau)d\tau$$

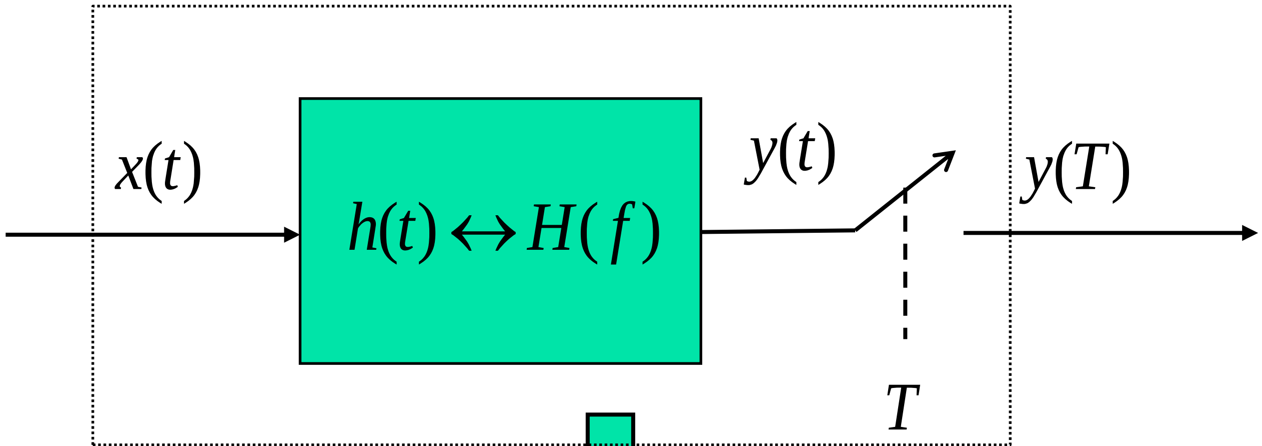
根据物理可实现条件，我们取$t_0=T$，这里T为信号$s(t)$结束的时刻，因此有

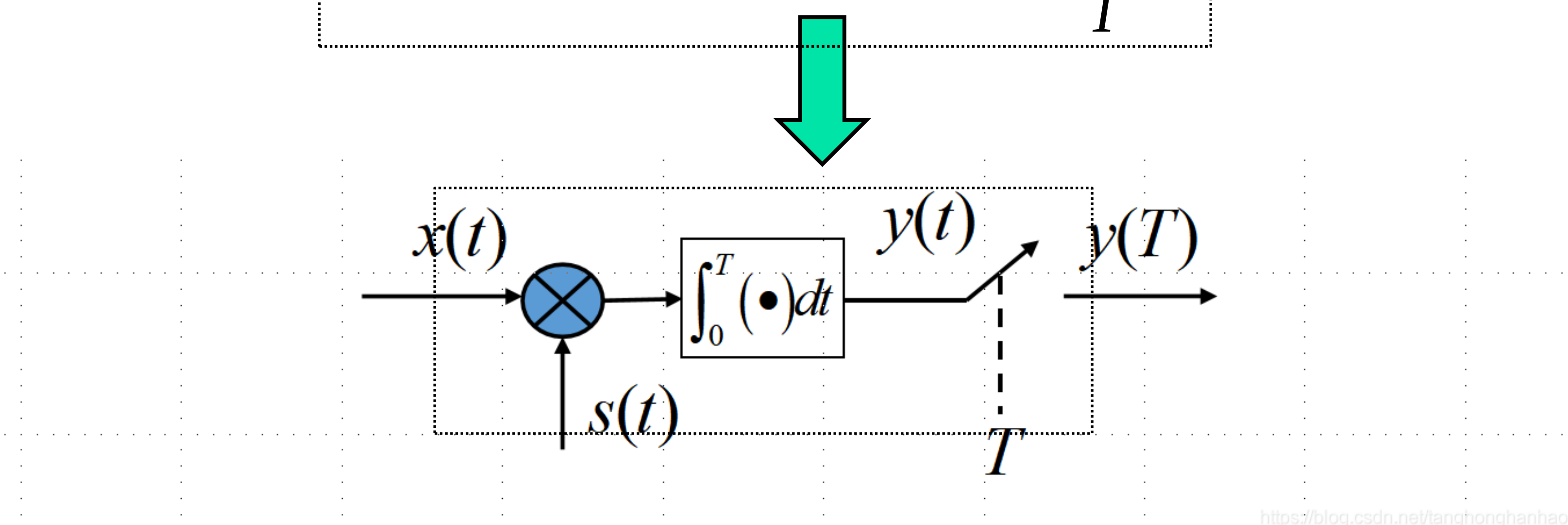
$$y(t)=\int_0^t [s(\tau)+n_w(\tau)]s(T-t+\tau)d\tau$$

进一步，$t=T$时刻$y(t)$的抽样值为

$$y(T)=\int_0^T [s(\tau)+n_w(\tau)]s(\tau)d\tau$$

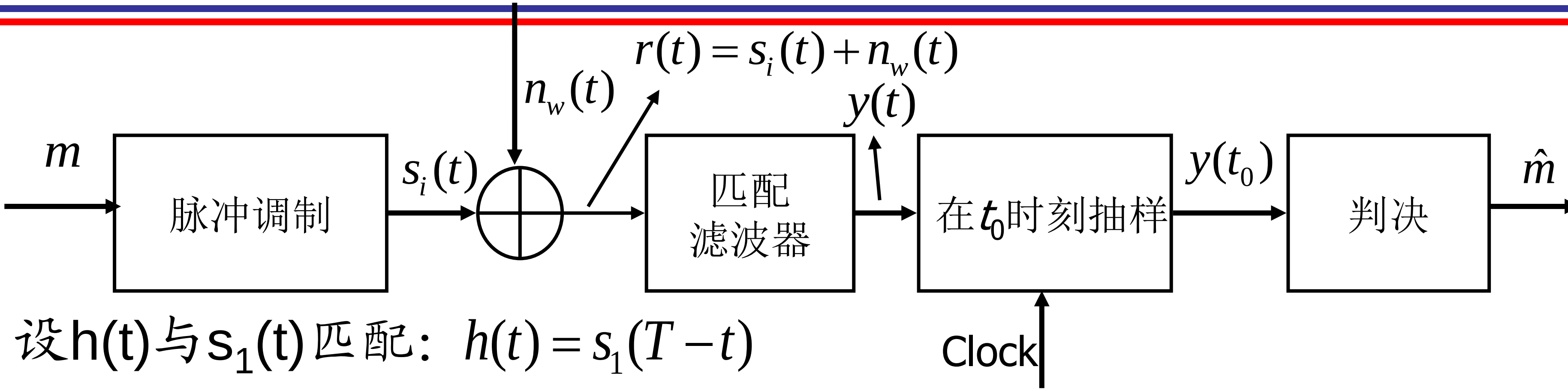
用相关器来等效匹配滤波器





$$y(T) = \int_0^T [s(\tau) + n_w(\tau)] s(\tau) d\tau$$

3.通过MF接收



设h(t)与s_1(t)匹配：$h(t)=s_1(T-t)$

发 “1”→$s_1(t)$ $r(t)=s_1(t)+n_w(t), 0\le t\le T$

$$y(t)=r(t)*h(t)=\int_0^t r(\tau)h(t-\tau)d\tau$$

$$=\int_0^t r(\tau)s_1(T-t+\tau)d\tau=\int_0^t\left[s_1(\tau)+n_w(\tau)\right]s_1(T-t+\tau)d\tau$$

$$y\equiv y(T)=\int_0^T\left[s_1(\tau)+n_w(\tau)\right]s_1(\tau)d\tau$$

$$=\int_0^T s_1^2(\tau)d\tau+\int_0^T s_1(\tau)n_w(\tau)d\tau=E_s+Z$$

3.通过MF接收

$$Z=\int_0^T s_1(\tau)n_w(\tau)d\tau$$

$$E(Z\,|\,s_1)=0$$

$$D(Z\,|\,s_1)=E\left\{\left[Z-E(Z)\right]^2\Big|\,s_1\right\}$$

$$=E\left[\int_0^T\int_0^T n_w(t_1)n_w(t_2)s_1(t_1)s_1(t_2)dt_1dt_2\right]$$

$$=\int_0^T\int_0^T E\left[n_w(t_1)n_w(t_2)\right]s_1(t_1)s_1(t_2)dt_1dt_2$$

$$E\left[n_w(t_1)n_w(t_2)\right]=R_w(\tau)=\frac{N_0}{2}\delta(\tau),\tau=t_2-t_1$$

$$D(Z\,|\,s_1)=\frac{N_0}{2}E_s=\sigma^2$$

3.通过MF接收

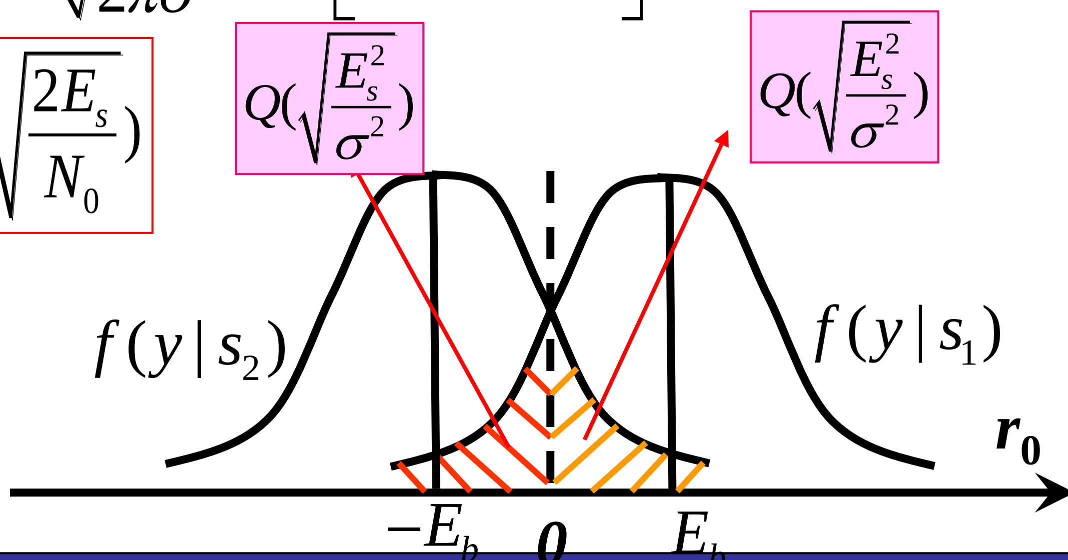
$$p(y|s_1)=\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left[-\frac{(y-E_s)^2}{2\sigma^2}\right]\Rightarrow P_{e1}=Q\left(\sqrt{\frac{E_s^2}{\sigma^2}}\right)=Q\left(\sqrt{\frac{E_s^2}{E_sN_0/2}}\right)=Q\left(\sqrt{\frac{2E_s}{N_0}}\right)$$

发 “0”→$s_2(t)$　双极性:

$$y\equiv y(T)=\int_0^T\left[s_2(\tau)+n_w(\tau)\right]s_1(\tau)d\tau=-E_s+Z$$

$$p(y|s_1)=\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left[-\frac{(y+E_s)^2}{2\sigma^2}\right]$$

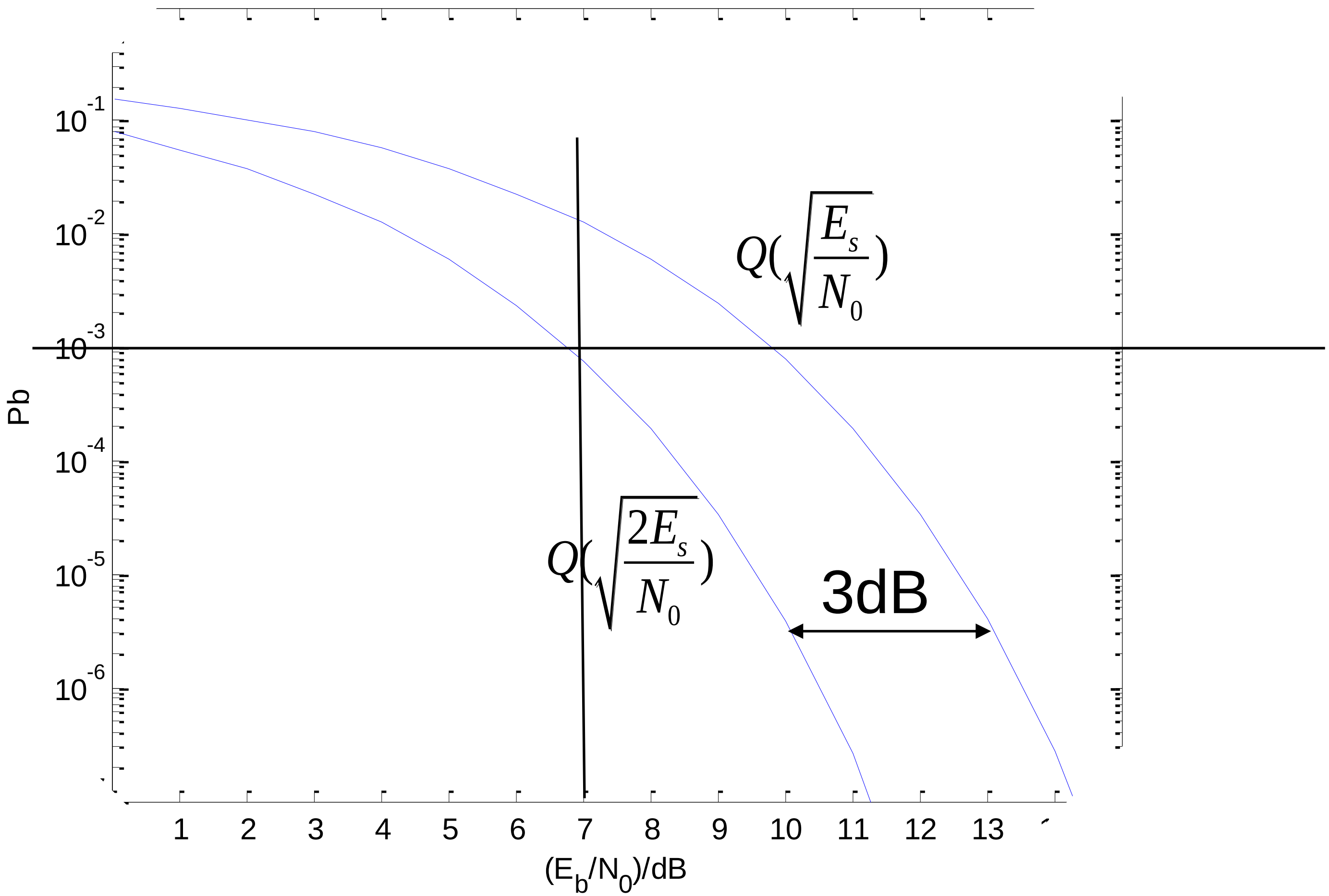
$$\boxed{P_b=Q\left(\sqrt{\frac{2E_s}{N_0}}\right)}$$



单极性:

$$\boxed{P_b=Q\left(\sqrt{\frac{E_s}{N_0}}\right)}$$

3.通过MF接收



10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
Pb
1 2 3 4 5 6 7 8 9 10 11 12 13
(E_b/N_0)/dB
$Q(\sqrt{\frac{E_s}{N_0}})$
$Q(\sqrt{\frac{2E_s}{N_0}})$
3dB