第8讲 模拟角度调制

1. 角调制的基本概念
2. 窄带角度调制
3. 角度调制信号的频谱特性
4. 调角信号的产生与接收

1. 角调制的基本概念

$$s(t) = A_c \cos\left[2\pi f_c t + \theta(t)\right] = A_c \cos\left[\varphi(t)\right]$$

- 相位调制(phase modulation, PM)信号

瞬时相位偏移 $\theta(t) = K_p m(t)$

瞬时相位 $\varphi(t) = 2\pi f_c t + \theta(t)$

瞬时频率 $f_i(t) = \frac{1}{2\pi}\omega_i(t) = \frac{1}{2\pi}\left[\frac{d\varphi(t)}{dt}\right] = f_c + \frac{1}{2\pi}\left[\frac{d\theta(t)}{dt}\right]$

瞬时频率偏移 $f_d(t) = \frac{1}{2\pi}\left[\frac{d\theta(t)}{dt}\right]$

其中K_p为调相系数，单位为rad/V,表示调相器灵敏度。

1. 角调制的基本概念

$$s(t) = A_c \cos\left[2\pi f_c t + \theta(t)\right] = A_c \cos\left[\varphi(t)\right]$$

- 频率调制(frequency modulation, FM)信号

瞬时频率偏移 $$f_d(t) = f_i(t) - f_c = K_f m(t) = \frac{1}{2\pi}\frac{d\theta(t)}{dt}$$

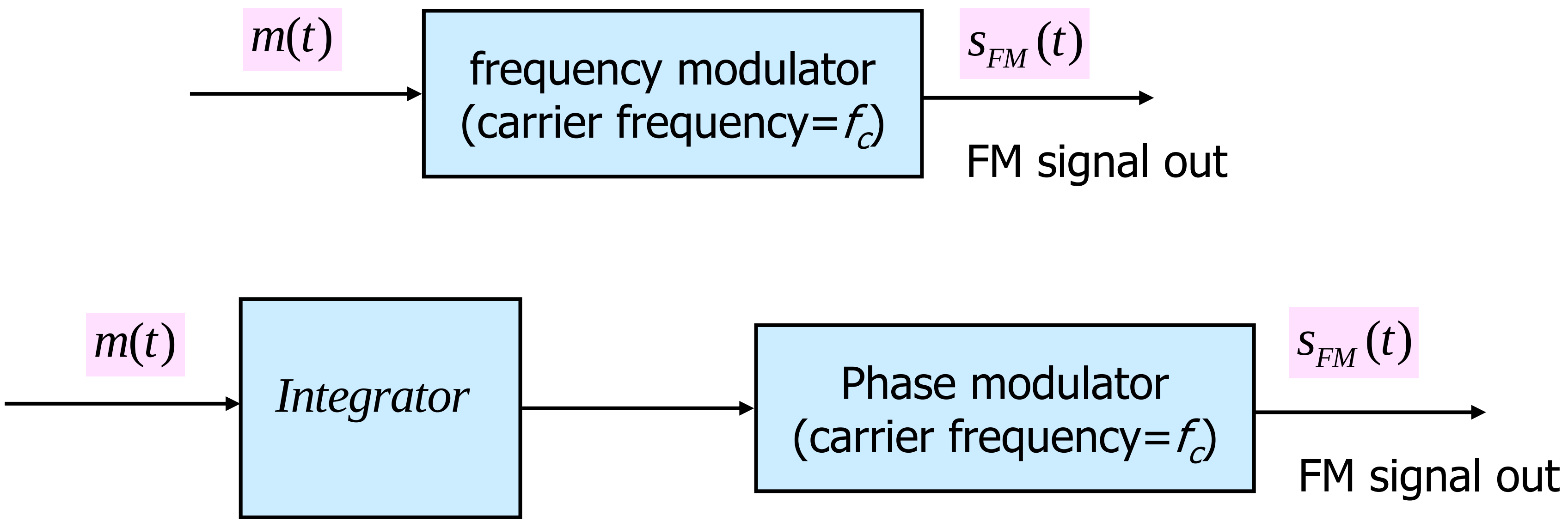
瞬时频率 $$f_i(t) = \frac{1}{2\pi}\omega_i(t) = \frac{1}{2\pi}\left[\frac{d\theta(t)}{dt}\right] = f_c + \frac{1}{2\pi}\left[\frac{d\varphi(t)}{dt}\right] = f_c + K_f m(t)$$

瞬时相位 $$\theta(t) = 2\pi\int_{-\infty}^{t} f_d(\tau)d\tau = 2\pi K_f \int_{-\infty}^{t} m(\tau)d\tau$$

瞬时相位偏移 $$\varphi(t) = 2\pi f_c t + \theta(t)$$

其中K_p为调频系数，单位为**Hz/V**,表示调频器灵敏度。

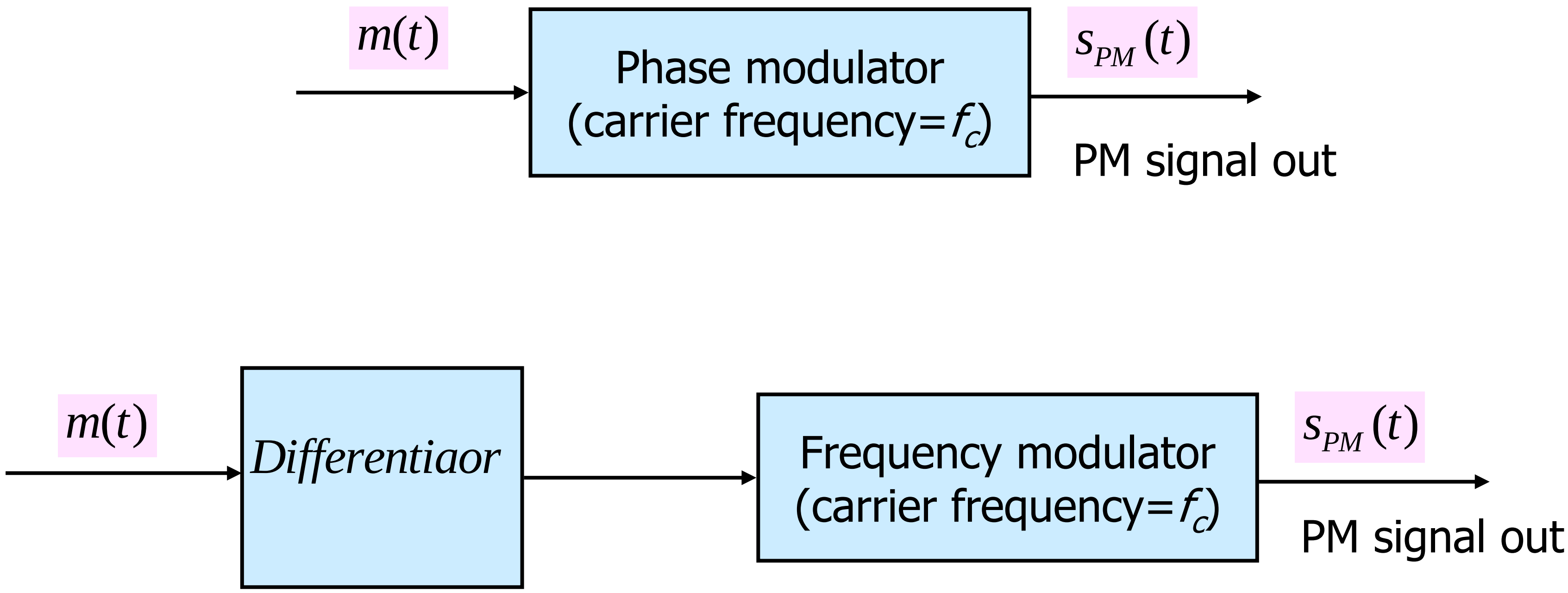
PM与FM信号的等效关系



(a) Generation of FM signal

Relationship between frequency and phase modulation

PM与FM信号的等效关系



(b) Generation of PM signal

Relationship between frequency and phase modulation

频率与相位偏移

- 频率偏移

$$f_d(t) = f_i(t) - f_c = \frac{1}{2\pi}\left[\frac{d\theta(t)}{dt}\right]$$

- 峰值频偏

$$\Delta f_{\max} = \max\left\{\frac{1}{2\pi}\left[\frac{d\theta(t)}{dt}\right]\right\}$$

- 峰值相偏

$$\Delta\theta_{\max} = \max\{\theta(t)\}$$

- 调相指数

$$\beta_p = \Delta\theta_{\max}$$

- 调频指数

$$\beta_f = \frac{\Delta f_{\max}}{B}$$

PM信号的相位偏移

- 相位偏移

$$\theta(t) = K_p m(t)$$

- 峰值相位偏移

$$\Delta\theta_{\max} = K_p \max\{|m(t)|\}$$

- 调相指数

$$\beta_p = \Delta\theta_{\max} = K_p \max\{|m(t)|\}$$

FM信号的频率偏移

- 频率偏移

$$f_d(t) = f_i(t) - f_c = \frac{1}{2\pi}\left[\frac{d\theta(t)}{dt}\right] = K_f m(t)$$

- 峰值频偏

$$\Delta f_{\max} = K_f \max\left\{\left|m(t)\right|\right\}$$

- 调频指数

$$\beta_f = \frac{\Delta f_{\max}}{B} = K_f \frac{\max\left\{\left|m(t)\right|\right\}}{B}$$

例1 基带信号 $m(t)=a\cos(2\pi f_m t)$，载波为 $A_c\cos(2\pi f_c t)$，请写出调频以及调相信号表示式，并求调制指数。

【解答】

1）PM

$$s_{PM}(t)=A_c\cos\left[2\pi f_c t+K_p a\cos(2\pi f_m t)\right]$$

$$\beta_p=\Delta\varphi_{\max}=K_p\max\left\{\left|m(t)\right|\right\}=aK_p$$

2）FM

$$s(t)=A_c\cos\left[2\pi f_c t+K_f\int_{-\infty}^{t}m(\tau)d\tau\right]$$

$$=A_c\cos\left[2\pi f_c t+\frac{aK_f}{f_m}\sin(2\pi f_m t)\right]$$

$$\beta_f=\frac{aK_f}{f_m}$$

填空题 6分

设置

此题未设答案

已知角度调制信号，$s(t)=10\cos\left[\left(2\pi\times10^{6}\right)t+4\cos\left(2\pi\times10^{3}t\right)\right]$，则该角度调制信号的归一化功率为 [填空1] ;该角度调制信号的瞬时相位偏移为 [填空2]； 最大相位偏移为 [填空3]； 瞬时频率偏移为 [填空4]； 最大频率偏移为 [填空5]； 带宽为 [填空6] 。

正常使用填空题需3.0以上版本雨课堂

作答

2. 窄带角度调制

- 若 $\varphi(t)$ 取值较小：

$$s(t)=A_c\cos\left[2\pi f_c t+\varphi(t)\right]=A_c\cos\left(2\pi f_c t\right)\cos\varphi(t)-A_c\sin\left(2\pi f_c t\right)\sin\varphi(t)$$

$$\approx A_c\cos 2\pi f_c t-A_c\varphi(t)\sin 2\pi f_c t$$

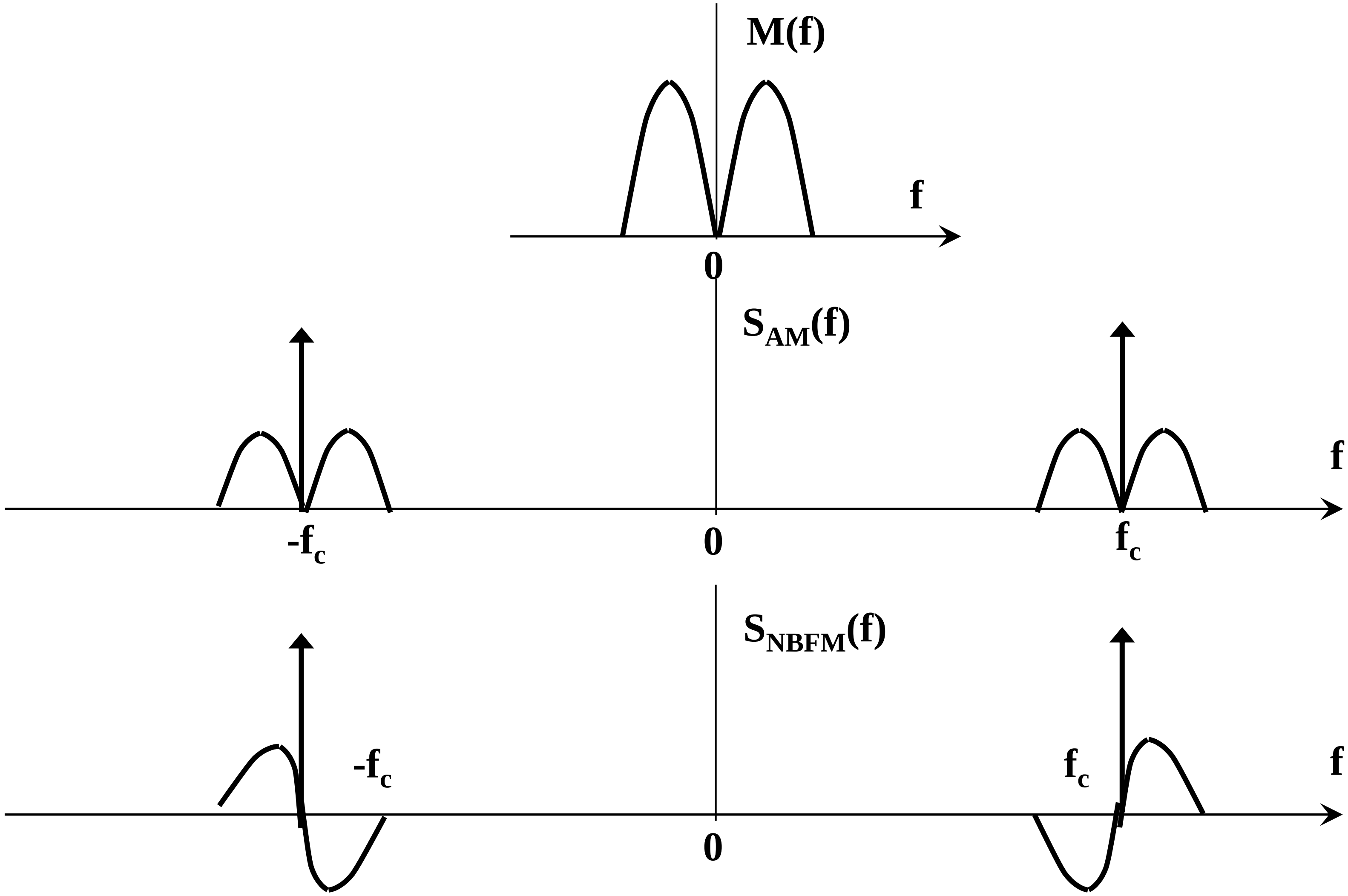
$$S(f)=\frac{A_c}{2}\left[\delta(f-f_c)+\delta(f+f_c)\right]-\frac{A_c}{j2}\left[\Phi(f-f_c)-\Phi(f+f_c)\right]$$

- FM:

$$\varphi(t)=2\pi K_f\int_{-\infty}^{t}m(\tau)d\tau\leftrightarrow\Phi(f)=K_f\frac{M(f)}{jf}$$

$$S(f)=\frac{A_c}{2}\left[\delta(f-f_c)+\delta(f+f_c)\right]+\frac{A_c}{2}K_f\left[\frac{M(f-f_c)}{f-f_c}-\frac{M(f+f_c)}{f+f_c}\right]$$

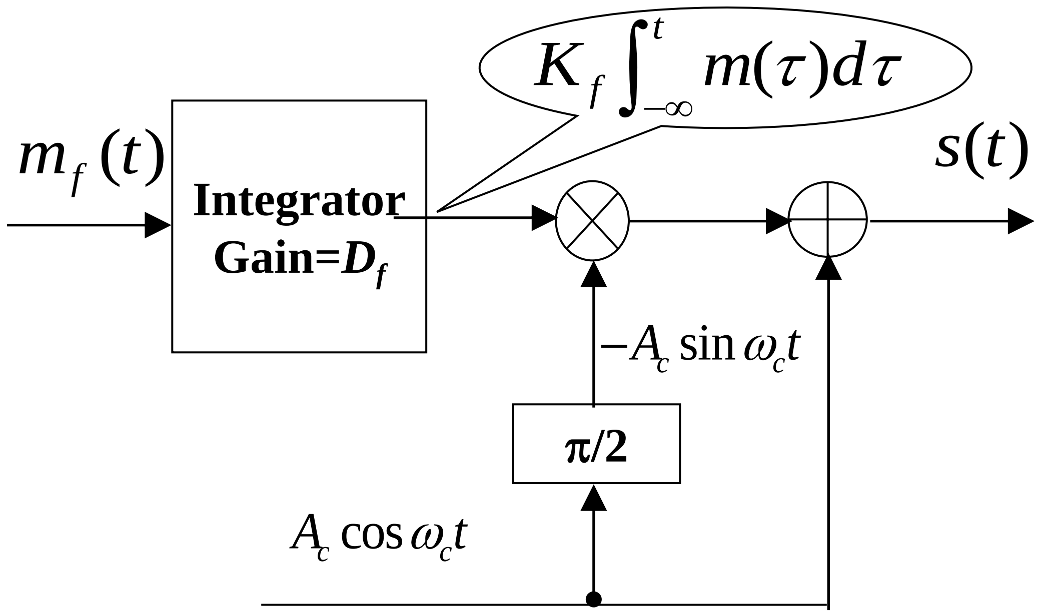
AM与NBFM信号频谱比较



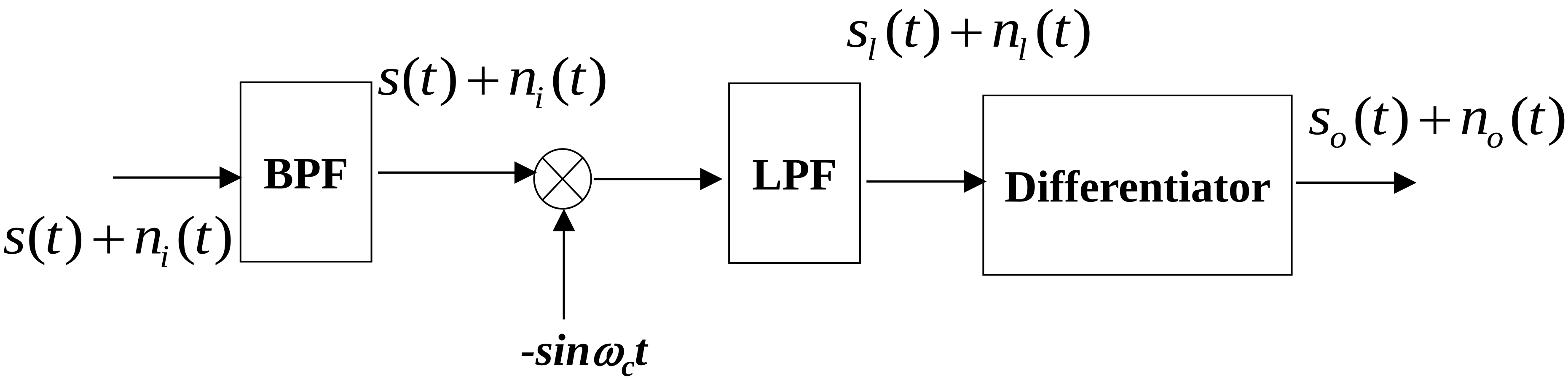
NBFM信号调制器

$$s(t) \approx A_c \cos 2\pi f_c t - A_c \theta(t) \sin 2\pi f_c t$$

$$= A_c \cos 2\pi f_c t - A_c \left[K_f \int_{-\infty}^{t} m(\tau) d\tau \right] \sin 2\pi f_c t$$



NBFM相干解调器



$$s_l(t)=\frac{A_c}{2}K_f\int_{-\infty}^{t}m(\tau)d\tau$$

$$s_d(t)=\frac{A_c}{2}K_f m(t)$$

3. 角度调制信号的频谱特性（单音信号）

$$s(t) = A_c \cos\left[2\pi f_c t + \theta(t)\right] = \mathrm{Re}\left[A_c e^{j2\pi f_c t} e^{j\theta(t)}\right] = \mathrm{Re}\left[g(t) e^{j2\pi f_c t}\right]$$

$g(t) = A_c e^{j\theta(t)}$：$s(t)$的复包络

$$s(t) = \frac{1}{2}\left[g(t) e^{j2\pi f_c t} + g^*(t) e^{-j2\pi f_c t}\right] \leftrightarrow S(f) = \frac{1}{2}\left[G(f - f_c) + G^*(-f - f_c)\right]$$

$$g(t) = A_c e^{j\theta(t)} \leftrightarrow G(f) = A_c \mathcal{F}\left[e^{j\theta(t)}\right]$$

3. 角度调制信号的频谱特性（单音信号）

- 瞬时频偏

$$\theta(t)=\beta\sin 2\pi f_m t$$

- 复包络信号

$$g(t)=A_c e^{j\theta(t)}=A_c e^{j\beta\sin 2\pi f_m t}\leftrightarrow G(f)=A_c\mathcal{F}\left[e^{j\theta(t)}\right]$$

$$g(t)=A_c e^{j\beta\sin 2\pi f_m t}=A_c\sum_{n=-\infty}^{\infty}c_n e^{jn2\pi f_m t}$$

周期信号，周期为$1/f_m$

$$c_n=f_m\int_0^{1/f_m}e^{j\beta\sin 2\pi f_m t}e^{-jn\cdot 2\pi f_m t}dt\overset{m=2\pi f_m t}{=}\frac{1}{2\pi}\int_0^{2\pi}e^{j(\beta\sin m-nm)}dm=J_n(\beta)$$

$$g(t)=A_c e^{j\beta\sin 2\pi f_m t}=A_c\sum_{n=-\infty}^{\infty}J_n(\beta)e^{jn2\pi f_m t}$$

3. 角度调制信号的频谱特性（单音信号）

- 频谱特性

$$s(t)=A_c\sum_{n=-\infty}^{\infty}J_n(\beta)\cos 2\pi\left(f_c+nf_m\right)t$$

$$\boxed{S(f)=A_c\sum_{n=-\infty}^{\infty}J_n(\beta)\left[\delta(f-f_c-nf_m)+\delta(f+f_c+nf_m)\right]}$$

Bessel函数性质

$$J_n(\beta)=\frac{1}{2\pi}\int_0^{2\pi} e^{j(\beta\sin x-nx)}dx$$

$$J_n(\beta)\approx 0 \qquad (n>\beta+1)$$

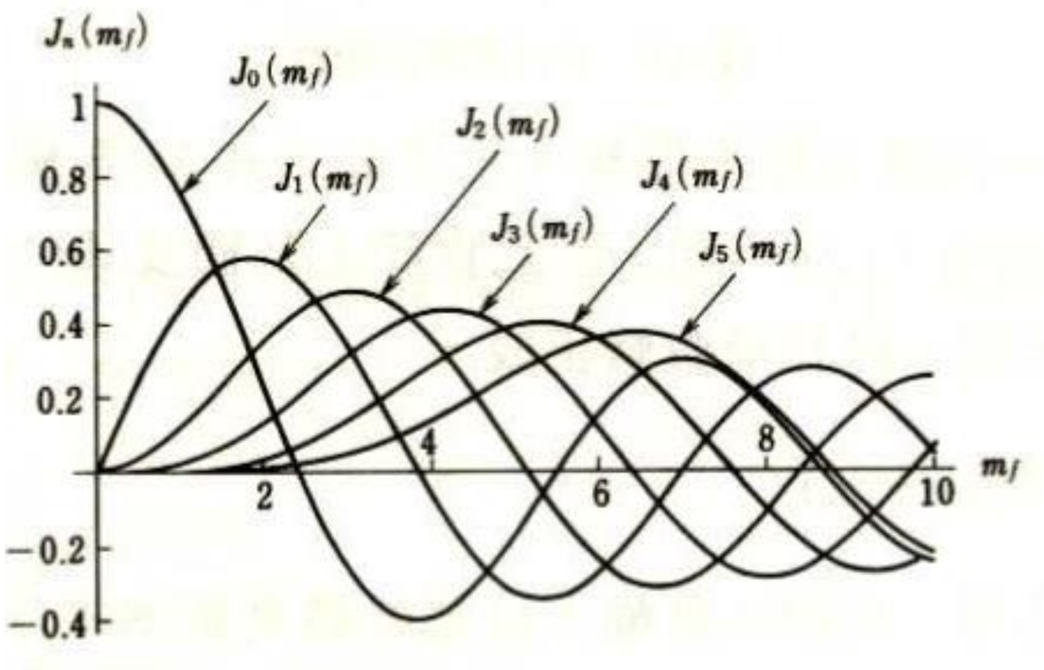
$$J_{-n}(\beta)=(-1)^n J_n(\beta)$$

$$\sum_{n=-\infty}^{\infty} J_n^2(\beta)=1$$

When β is very small

$$J_0(\beta)\approx 1 \qquad J_1(\beta)\approx\frac{1}{2}\beta$$

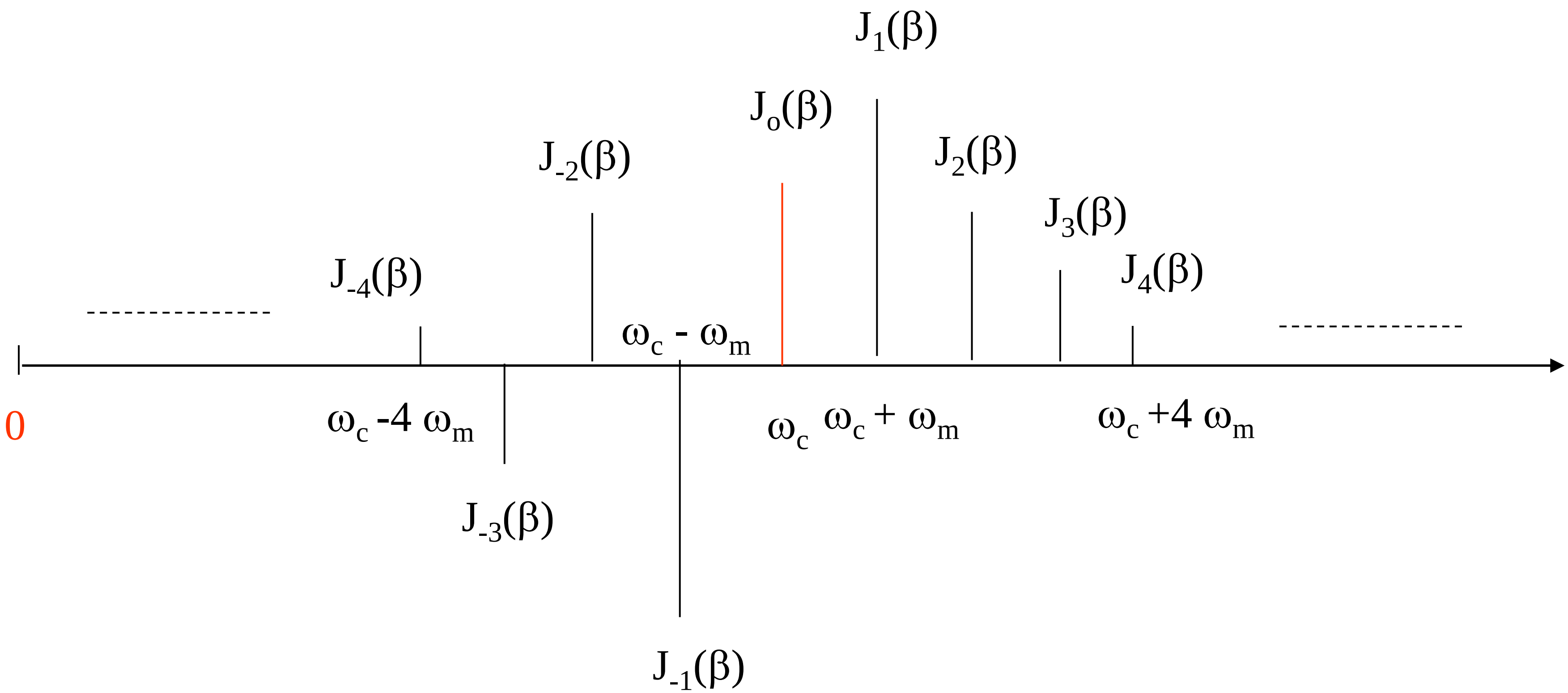
$$J_n(\beta)\approx 0 \qquad (n>1)$$



Bessel函数取值

m_f	$J_0(m_f)$	$J_1(m_f)$	$J_2(m_f)$	$J_3(m_f)$	$J_4(m_f)$	$J_5(m_f)$	$J_6(m_f)$	$J_7(m_f)$	$J_8(m_f)$	$J_9(m_f)$
0.01	1.00	0.005								
0.20	0.99	0.100								
0.50	0.94	0.24	0.03							
1.00	0.77	0.44	0.11	0.02						
2.00	0.22	0.58	0.35	0.13	0.03					
3.00	0.26	0.34	0.49	0.31	0.13	0.04	0.01			
4.00	0.39	0.06	0.36	0.43	0.28	0.13	0.05	0.01		
5.00	0.18	0.33	0.05	0.36	0.39	0.26	0.13	0.05	0.02	
6.00	0.15	0.28	0.24	0.11	0.36	0.36	0.25	0.13	0.06	0.02

频谱与带宽



Carson’s rule: $B = 2(1+\beta) f_m$

卡松规则

$$B_T = 2(1+D)B = 2\Delta f_{\max} + 2B$$

频偏比 $D = \dfrac{\Delta f_{\max}}{B}$

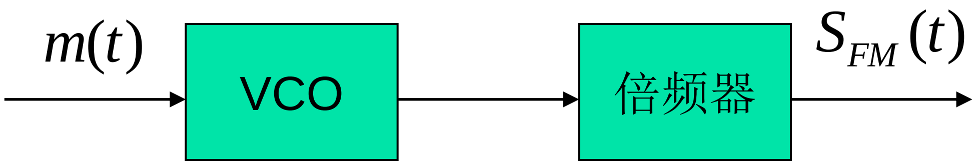
D>2时修正为：$B_T = 2(D+2)B$

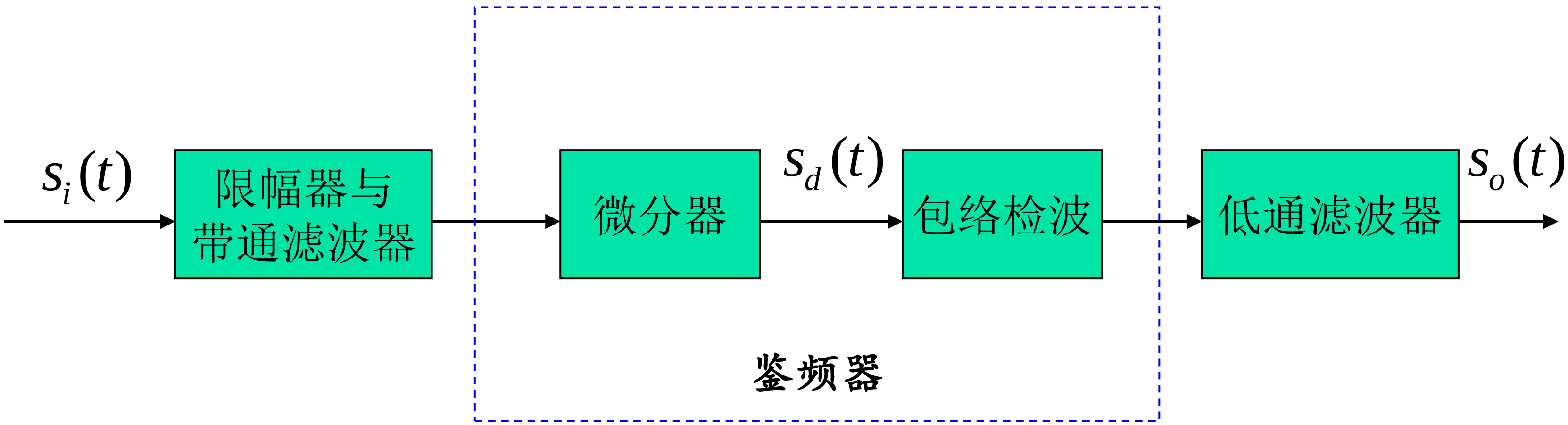
功率分配

$$P = \overline{s^2(t)} = \overline{[A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\cos(\omega_c t + n\omega_m t)]^2}$$

$$= \frac{A_c^2}{2} J_0^2(\beta) + \frac{A_c^2}{2} \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} J_n^2(\beta)$$

$$= P_c + P_s = \frac{A_c^2}{2}$$


$m(t)$
VCO
倍频器
$S_{FM}(t)$



$$s_d(t) = \frac{\mathrm{d}}{\mathrm{d}t} A_c \cos\left[2\pi f_c t + 2\pi k_{FM} \int m(t)dt\right]$$

$$= -A_c\left[2\pi f_c t + 2\pi k_{FM} m(t)\right]\sin\left[2\pi f_c t + 2\pi k_{FM} \int m(t)dt\right]$$

$$s_o(t) = Km(t)$$