Politecnico di Milano

Prova Finale: Introduzione all'Analisi di Missioni Spaziali
AA 2020/2021

Docente: Topputo Francesco

Elaborato
C11

Autori:

912308	Tognolini	Federico Silvio
911281	Wu	Christian XianYang
911098	Zamblera	Davide

Data di consegna: 03/11/20

Table of Contents

1 Introduction

The problem at hand is the analysis of different designs of orbital transfers, given the initial position on an initial orbit and the final position on a final orbit.

In the following sections we will describe the two assigned orbits to have a comprehensive view of the viable transfers. There are countless possibilities, some of them require a higher Δv budget and some of them require longer times of flight.
At first we will discuss about four types of standard transfer designs which were examined in the laboratories. Since these transfers might not be efficient, we will also discuss about two other alternative designs: the first one focuses on how changing the order of the manoeuvres can affect the total costs and the time of flight; the second one follows a different approach to achieve a cut in budget, which will be analysed in detail in the sections below.

Our team worked together to write MATLAB® scripts and functions in order to get the results needed for the report. Each member of the group was given some tasks to accomplish, followed by a discussion of the outcomes. Often this resulted in finding errors or flaws in our mental processes of solving the problem, which were consequently revised and restructured in a different way. In the end, after testing some non-standard designs, we came up with a transfer which is balanced both in costs and in times.

2 Characterisation of the initial orbit

2.1 Initial orbital parameters

For the initial orbit we are given the following set of data as Cartesian coordinates.

X	Y	Z
-7314.6175 km	-4835.0432 km	2185.2627 km
v_X	v_Y	v_Z
1.566 km/s	-4.935 km/s	-3.840 km/s

Applying the transformation from Cartesian coordinates to Keplerian parameters, we are able to obtain the wanted values to fully describe the orbit. In order to do so, we implemented an algorithm in MATLAB® called car2par.m which returns as outputs the following set of values:

a	e	i	Ω	ω	θ
8541.0515 km	0.08999	0.7205 rad	0.8719 rad	0.4282 rad	2.3381 rad

2.2 Discussion of the results

From the data obtained in the previous subsection, we assess that the eccentricity value is low and so we expect it to be similar to a circumference. Using the formulas discussed in the lectures, we can determine the values of the radius of perigee and apogee and these results, show that the orbit is in the range of Low Earth Orbits.

$$r_p = 7772.4377\ km \qquad r_a = 9309.6652\ km$$

One more relevant parameter is the specific orbital energy, the value shown below is negative, which proves that the orbit is an ellipse.

$$\epsilon = -23.3344\ kJ/kg$$

Finally, the initial orbital period is 2.18 hours which is an expected result from an orbit which is in the LEOs range.

2.3 Graphical representation

Using the previously discussed parameters, it is possible to plot in MATLAB® the initial orbit in a 2-dimensional perifocal coordinate system, as shown in Figure 2.1.

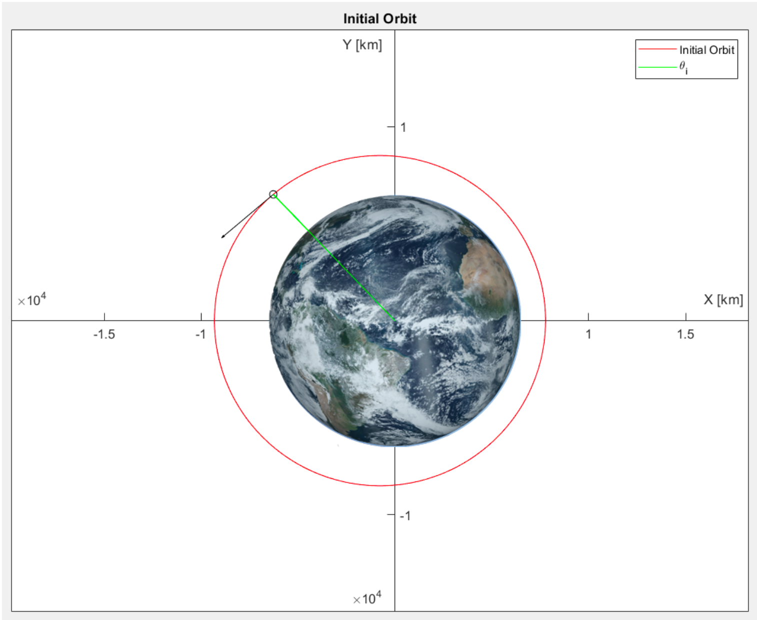


Figure 2.1: Plot of the initial orbit on a X-Y plane with the Earth in the origin.

3 Characterisation of the final orbit

3.1 Final orbital coordinates

In this case, we are given the Keplerian parameters, as followed:

a	e	i	Ω	ω	θ
13930 km	0.3192	0.6282 rad	1.764 rad	2.054 rad	0.1929 rad

We implemented an algorithm in MATLAB®, par2car.m, for the transformation from classical orbital parameters to position and velocity vectors in the Cartesian frame of reference, obtaining the values reported in the table below.

X	Y	Z
-4755.6311 km	-7004.5881 km	4366.9265 km
v_X	v_Y	v_Z
4.621 km/s	-5.208 km/s	-2.568 km/s

3.2 Discussion of the results

The final orbit has a higher eccentricity value, compared to the initial one, and so it is expected to have a more marked elliptic shape.

In the same way as in the previous section, we can calculate the radius of perigee and apogee and the specific orbital energy of this orbit. The values of the radii are significantly higher which means that the transfer will be done from an initial orbit, similar to a LEO, to a final moderately elliptical orbit.

$r_p =$ 9843.5440 km $\quad$ $r_a =$ = 18376.4560 km $\quad$ ϵ = -14.3073 kJ/kg

In this case the orbital period is 4.55 hours which is also larger than the previous one and it is in the range of Medium Earth Orbits.

3.3 Graphical representation

In the same way as in section 2, we are able to plot the final orbit in a 2-dimensional perifocal coordinate system, as shown in Figure 3.1.

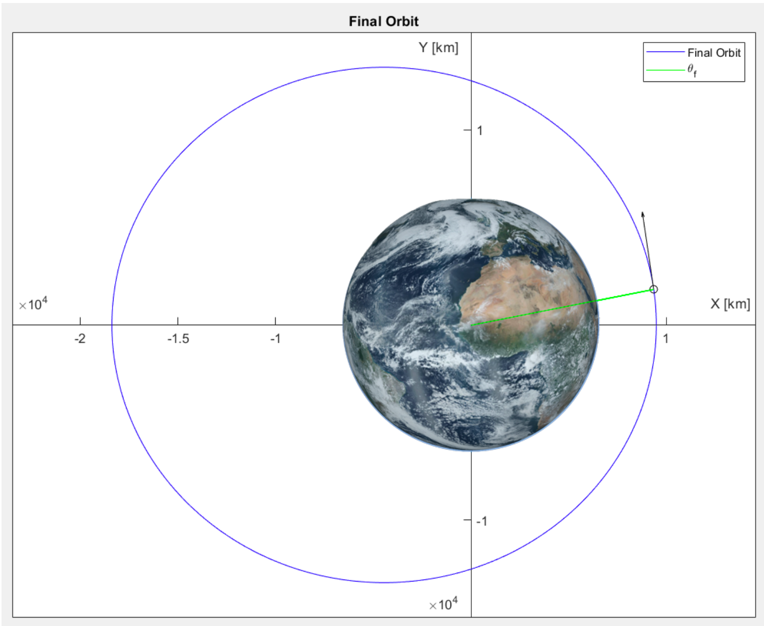


Figure 3.1: Plot of the final orbit on a X-Y plane with the Earth in the origin.

4 Transfer trajectory analysis

4.1 Reaching the final position and final velocity

Having discussed the initial and final orbits, there are many different strategies to reach the final position starting from the initial one.
The first to be analysed is the standard design, the second is a variant of the standard transfers and the last one is a redesign of some manoeuvres.

4.2 Transfer strategies

4.2.1 Standard orbital transfer

The standard orbital transfer is made of 3 manoeuvres: at first a change of orbital plane in order to make the initial and final orbit co-planar, then a change of pericenter anomaly, so

that the two eccentricities vectors are aligned and finally a bitangent manoeuvre to move the satellite onto the final orbit.
The cases of bitangent transfer from pericenter to pericenter or apocenter to apocenter slightly differ from the previous description, because to be able to do those two manoeuvres, the two pericenters have to be in opposition. Since the two given orbits do not satisfy this condition, we have to rotate the anomaly of pericenter by 180°.

Due to the different Δv impulses for each type of bitangent manoeuvre, we considered them as complete separate designs. The Δv budgets and the times of transfer to do the complete standard orbital transfers are shown in the table below, where there is also a distinction regarding the two positions $\theta_{1,2}$ where it is possible to begin the change of pericenter anomaly.

The differences in costs and times are summarised in the table below, in which it is visible that the first two strategies are less efficient than the last two.

	Pericenter - Apocenter	Apocenter - Pericenter	Pericenter - Pericenter	Apocenter - Apocenter
Total Δv	6.1136 km/s	6.1731 km/s	5.5164 km/s	5.5125 km/s
Total Δt	32176.74 s (θ_1)	17161.39 s (θ_1)	24212.14 s (θ_1)	32582.02 s (θ_1)
	31348.17 s (θ_2)	24188.40 s (θ_2)	16004.85 s (θ_2)	32230.31 s (θ_2)

Figure 4.1 shows all the four standard strategies which are plotted in MATLAB® with the view angle set to (250, 175). These plots are made using one of the true anomalies between $\theta_{1,2}$, in particular the one associated with the least total Δt.

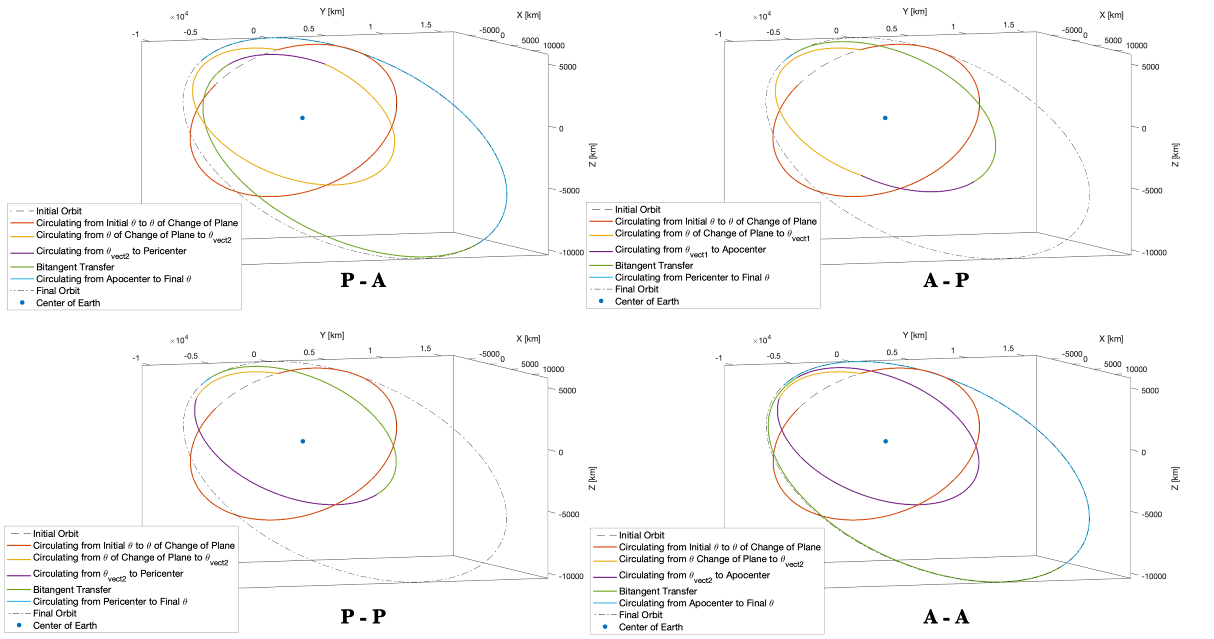


Figure 4.1: Plot of the standard transfer method sorted by case of bitangent manoeuvre.

4.2.2 First alternative orbital transfer

After the standard strategies, in this alternative design we focused on lowering the budget by changing the order of the manoeuvres.

From the lectures, we know that going far from the main body and doing the change of orbital plane there, the cost of the manoeuvre is decreased. At first we change the shape of the initial orbit with a bitangent transfer, which is done in the same way as in the standard strategy. Afterwards, the change of orbital plane is carried out in the farthest point between the two positions where it is possible to begin this manoeuvre. Finally, we change the anomaly of pericenter in order to align the current orbit with the final one.

The total costs and total times of transfer of this alternative design, considering each case of bitangent manoeuvre, are reported in the table below:

	Pericenter - Apocenter	Apocenter - Pericenter	Pericenter - Pericenter	Apocenter - Apocenter
Δv	$7.5798\ km/s$	$7.6393\ km/s$	$5.7234\ km/s$	$5.7195\ km/s$
Δt	$46750.30\ s\ (\theta_1)$	$31734.95\ s\ (\theta_1)$	$41028.22\ s\ (\theta_1)$	$49398.10\ s\ (\theta_1)$
	$40669.23\ s\ (\theta_2)$	$25653.88\ s\ (\theta_2)$	$21964.07\ s\ (\theta_2)$	$30333.96\ s\ (\theta_2)$

In this case as well, the view angle of the plots is set to (250, 175) and the θ used is the one which cuts down the total time of transfer. Similarly to the standards strategies, the first two cases are worse than the last two, both in terms of costs and in terms of times.

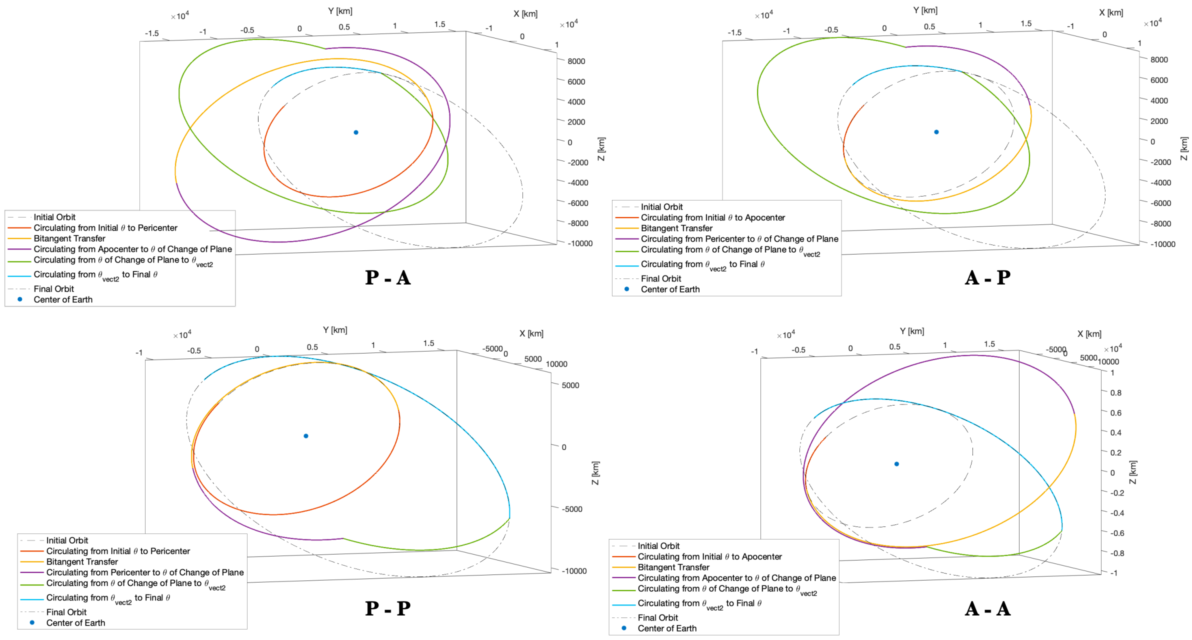


Figure 4.2: Plot of the first alternative transfers sorted by case of bitangent manoeuvre.

4.2.3 Second alternative orbital transfer

The transfer starts with a bitangent manoeuvre from the initial orbit to an auxiliary circular one. On this orbit we change the orbital plane to make it co-planar to the final orbit. At last one more bitangent manoeuvre which starts from the auxiliary orbit and ends in the final one. Unlike the standard orbits, there is no change of the anomaly of pericenter.

The auxiliary orbit is circular and its orbital parameters are reported in the table below:

a	e	i	Ω
9483.544 km	0	0.6282 rad	1.764 rad

We implemented an iterative script in MATLAB® in which we calculated the overall cost of the transfer, as described in the previous paragraph, with the semi-major axis as a variable parameter. The minimum overall cost[1] was obtained with the value reported above.

The main reason we chose this design over the standard ones is because we achieve a lower Δv, with a tradeoff regarding the total time. The results are summarised in the following table:

	Pericenter - Pericenter
Δv	4.8584 km/s
Δt	17357.08 s (θ_1)

The values reported above and the plot of Figure 4.3 are referred to the alternative design that makes use of the pericenter to pericenter bitangent manoeuvres. The reason behind this choice is that the others cases are not as efficient as this one in terms of total Δt and Δv. One exception is the alternative with the first bitangent transfer done with a pericenter to pericenter and the second one with an apocenter to apocenter. The difference in cost is in the order of 1 mm/s, which is not a remarkable value, given the fact that it takes longer to complete the transfer.

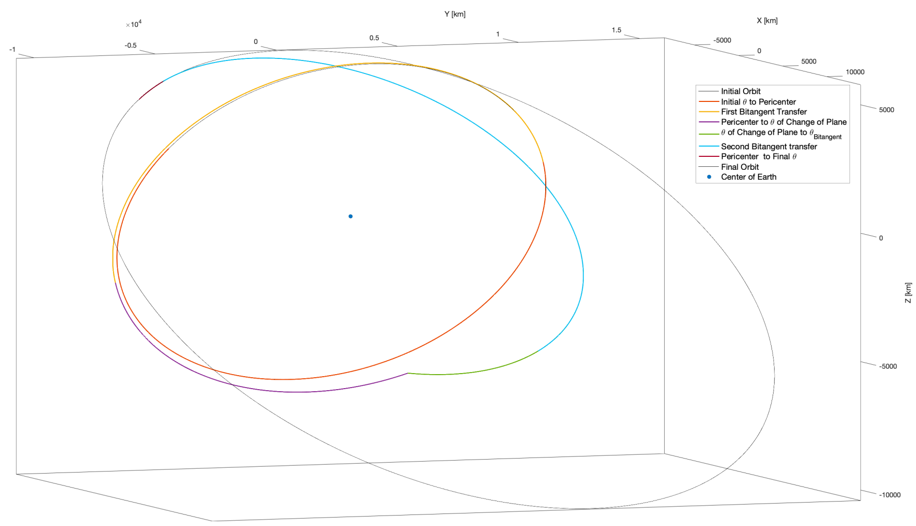


Figure 4.3: Plot of the second alternative design.

[1] The trend of Δv with radius r as variable is shown in the subsection 6.2 in the Appendix.

5 Conclusions

Considering the results shown in the subsection 4.2.1 about the standard strategies costs and transfer times, we can infer that there are convenient choices for both time and cost. The most convenient in terms of cost is the apocenter to apocenter strategy with a $\Delta v = 5.5125\ km/s$, whereas the most convenient in terms of transfer time[2] $\Delta t = 16004.85\ s$ is the pericenter to pericenter strategy, choosing θ_2 as starting point for the change of pericenter anomaly.

According to the lectures, it could be possible to obtain a lowering of the total cost by changing the order of the manoeuvres, especially in the change of orbital plane. We tried this solution in the subsection 4.2.2, however the results are both in terms of cost and time worse than the best standard strategies. Based on the results obtained in each manoeuvre, we realised that the cost of change of pericenter and bitangent transfer increases more than what it is saved from the change of orbital plane far from the main body.

In the light of the above considerations, we decided to embrace another type of design which we fully described in the subsection 4.2.3.
This strategy makes use of an auxiliary circular orbit, since the initial orbit is already similar to a circular one due to its low eccentricity and because of this the cost of the bitangent transfer is significantly lowered. Moreover, on this auxiliary orbit the cost of the change of pericenter is nullified. In addition, it can be noticed that the second bitangent is moderate in cost because the first impulse is near to zero, due to the similarity between the auxiliary orbit and the transfer orbit. In the Appendix section it is reported a graph which shows the trend of the total cost of transfer depending on the radius of the auxiliary orbit. This kind of analysis led us to the choice of the radius which minimises the overall cost of the transfer strategy and it is the one used in the subsection 4.2.3.

Our preferred design is the last one described, it saves us a $\Delta v = 0.658\ km/s$ compared to the standard pericenter to pericenter, despite having an additional $\Delta t = 1352.23\ s$. Although it is the best strategy described so far in terms of cost, there is a downside which is the added probability of error, induced by the two more impulses of the additional bitangent transfer.

[2] In all the previous strategies the true anomaly used for the change of orbital plane is the one which diminish the total cost, without taking into consideration the added time of flight.

6 Appendix

6.1 Characterisation values

The three tables below contain all the orbital parameters required to characterise all the orbits used in the strategies. We choose to include only the transfers that we consider as best among the others in their respective group of transfers.

Standard Transfer (Pericenter - Pericenter)

t (s)	a (km)	e (-)	i (rad)	Ω (rad)	ω (rad)	θ (rad)	Δv (km/s)
0	8541.051	0.0899	0.7205	0.8719	0.4282	2.3381	-
6944.47	8541.051	0.0899	0.7205	0.8719	0.4282	1.6537	3.7112
	8541.051	0.0899	0.6282	1.7640	5.9957	1.6537	
8436.31	8541.051	0.0899	0.6282	1.7640	5.9957	2.7415	0.4808
	8541.051	0.0899	0.6282	1.7640	5.1956	3.5417	
11770.52	8541.051	0.0899	0.6282	1.7640	5.1956	0	0.0314
	8627.991	0.0992	0.6282	1.7640	5.1956	0	
15758.43	8627.991	0.0992	0.6282	1.7640	5.1956	3.1416	1.2930
	13930	0.3192	0.6282	1.7640	2.0540	0	
16004.85	13930	0.3192	0.6282	1.7640	2.0540	0.1922	-

First Alternative Transfer (Pericenter - Pericenter)

t (s)	a (km)	e (-)	i (rad)	Ω (rad)	ω (rad)	θ (rad)	Δv (km/s)
0	8541.051	0.0899	0.7205	0.8719	0.4282	2.3381	-
5102.11	8541.051	0.0899	0.7205	0.8719	0.4282	6.2832	0.0314
	8627.991	0.0992	0.7205	0.8719	0.4282	0	
9090.02	8627.991	0.0992	0.7205	0.8719	0.4282	3.1416	1.2930
	13930	0.3192	0.7205	0.8719	3.5698	0	
11735.54	13930	0.3192	0.7205	0.8719	3.5698	1.6537	2.9956
	13930	0.3192	0.6282	1.7640	2.8541	1.6537	
15403.84	13930	0.3192	0.6282	1.7640	2.8541	2.7415	1.4035
	13930	0.3192	0.6282	1.7640	2.0540	3.5417	
21964.07	13930	0.3192	0.6282	1.7640	2.0540	0.1922	-

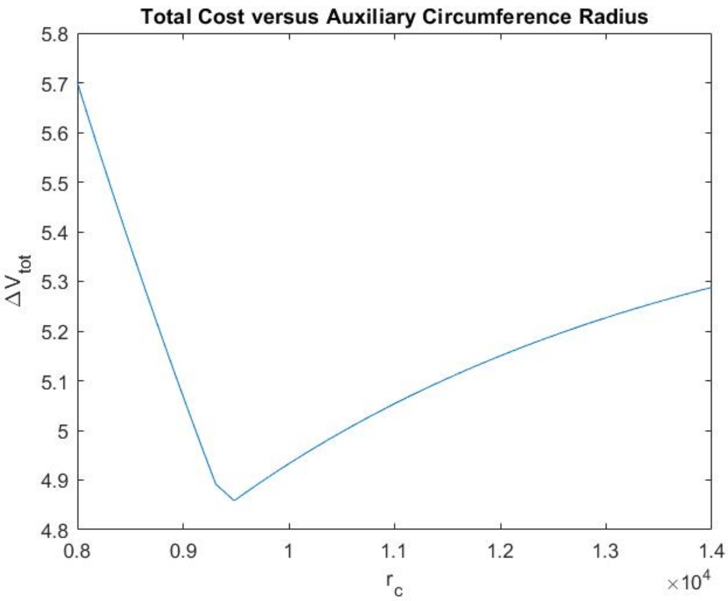
Second Alternative Transfer

t (s)	a (km)	e (-)	i (rad)	Ω (rad)	ω (rad)	θ (rad)	Δv (km/s)
0	8541.051	0.0899	0.7205	0.8719	0.4282	2.3381	-
5102.11	8541.051	0.0899	0.7205	0.8719	0.4282	6.2832	0.0314
	8627.991	0.0992	0.7205	0.8719	0.4282	0	
9090.02	8627.991	0.0992	0.7205	0.8719	0.4282	3.1416	0.3298
	9483.544	0	0.7205	0.8719	3.5698*	0	
11509.10	9483.544	0	0.7205	0.8719	3.5698*	1.6537	3.5340
	9483.544	0	0.6282	1.7640	2.8541*	1.6537	
12515.12	9483.544	0	0.6282	1.7640	2.8541*	2.3415	1.77e-15
	9483.544	2.22e-16	0.6282	1.7640	5.1956	0	
17110.67	9483.544	2.22e-16	0.6282	1.7640	5.1956	3.1416	0.9632
	13930	0.3192	0.6282	1.7640	2.0540	0	
17357.08	13930	0.3192	0.6282	1.7640	2.0540	0.1922	-

* The anomalies of pericenter in the circular orbit are priorly decided to easily determine the true anomalies.

6.2 Function graph

The function, discussed in the Conclusions section, has a minimum point because the Δv of the first bitangent has a minimum point, while the Δv of the change of orbital plane and of the second bitangent are monotonically decreasing.



The radius chosen for the alternative design corresponds to the minimum in the graph.

6.3 3-dimensional plot

This plot shows how the two given orbits are arranged in space and around the Earth. The two velocity vectors are not in scale with the axises, instead they serve as indicators of the direction of the motion.