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Trebuchet - Group 08

Modeling and Optimization of a Counterweight Trebuchet

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Contents

List of Symbols

Variable	Description	SI unit
l_1	Length of the longer (projectile side) arm part	[m]
l_2	Length of the shorter (counterweight side) arm part	[m]
l_3	Length of the counterweight c.g. offset	[m]
l_4	Length of the sling	[m]
m_{cwt}	Mass of the counterweight	[kg]
m_{prj}	Mass of the projectile	[kg]
ξ_{arm}	Arm ratio	[-]
ξ_{cwt}	Counterweight ratio	[-]
ξ_{mass}	Mass ratio	[-]
ξ_{sling}	Sling ratio	[-]
θ_1	Angle between arm and horizontal	[deg]
x_0	Initial x position projectile at detachment	[m]
z_0	Initial z position projectile at detachment	[m]
v_{x_0}	Initial x velocity projectile at detachment	[m/s]
v_{z_0}	Initial z velocity projectile at detachment	[m/s]
g	Gravity acceleration at sea level	[m/s^2]
R_e	Reynolds number	[-]
v	Magnitude of projectile velocity	[m/s]
ρ	Air density	[kg/m^3]
ν	Dynamic viscosity	[Ns/m^2]
C_d	Drag coefficient	[-]
A	Frontal area of projectile	[m^2]

1 | Introduction

The project's purpose is to simulate and perform an optimization of a medieval trebuchet. The chosen design is a counterweight trebuchet with sizes and dimensions taken from the Warwick Castle reconstruction of a medieval siege weapon, shown in fig. 1.1.

Figure 1.1: Warwick trebuchet

The goal is to achieve the longest range from a projectile throw. We begin by estimating the dimensions from visual footage of the trebuchet, the resulting data is reported in table 1.1

Table 1.1: Warwick trebuchet data

Variable	Value	Units
Vertical structure beam height	8	[m]
Total arm length	11.2	[m]
Length of the arm counterweight side	1.6	[m]
Length of the arm projectile side	9.6	[m]
Inclined structure beam length	9.6	[m]
Rope length	8.8	[m]
Counterweight mass	5000	[kg]
Projectile mass	36	[kg]
Range	300	[m]

For the initial models some of the values were partially changed to be more similar to the specification found in literature for an efficient trebuchet [4]. Nevertheless the dimensions remain similar to the Warwick trebuchet which will be our benchmark for evaluating the optimization of the models.

Some of the limitation and simplifying assumptions of the initial models (non flexibility, absence of air drag, absence of friction in the hinges) will be removed in more advanced ones, with an iterative modeling process, that improves the fidelity of the simulation.

2 | Modeling

Two distinct models were developed in ADAMS[1] and MBDyn[2] in order to validate the results. Given the intrinsic bi-dimensional nature of the mechanism, the third dimension was neglected. Given our objective, we are confident that our modeling choice will not substantially affect the outcome of the optimization.

2.1. Model Description

2.1.1. MBDyn

A bunch of increasingly complex models were implemented. The main elements are:

- **The projectile**: it is a rigid spherical body
- **The sling**: being always in tension while carrying the projectile, it is modeled as a rigid beam, which the projectile is clamped to. Converting it to a rod was a possibility we did not investigate
- **The arm**: it is a homogeneous wooden beam hinged at one end to the sling, at the other end to the counterweight and also hinged at ground at a certain height with respect to the horizontal. Initially it was modeled as a rigid body, then we converted it to a flexible object
- **The counterweight**: it is a rigid body weighting 5000 kg hinged at one end of the arm. It is not hinged in its centre of mass
- **The hinges**: the three revolute hinges that link the various elements of the trebuchet were implemented accounting for a simplified model of friction

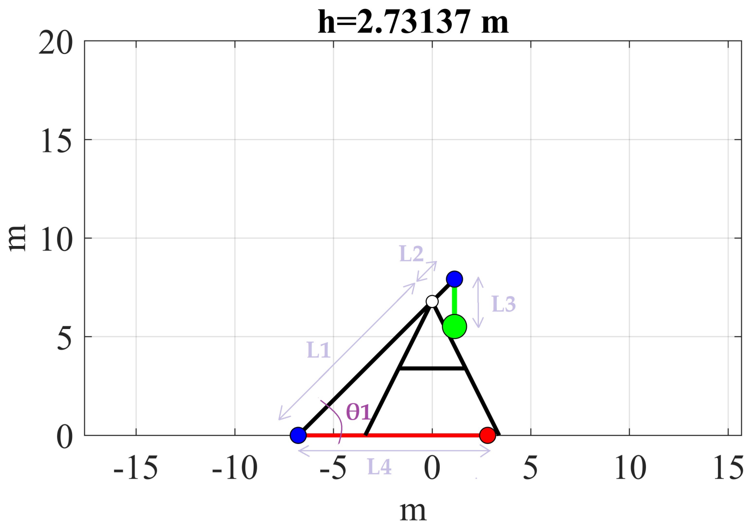

Figure 2.1: MBDyn model of the Warwick trebuchet at initial position; **h** is the maximal vertical displacement the counterweight can experience in this configuration

2.1.2. ADAMS

The configuration and initial dimensions of the model are shown in fig. 2.2

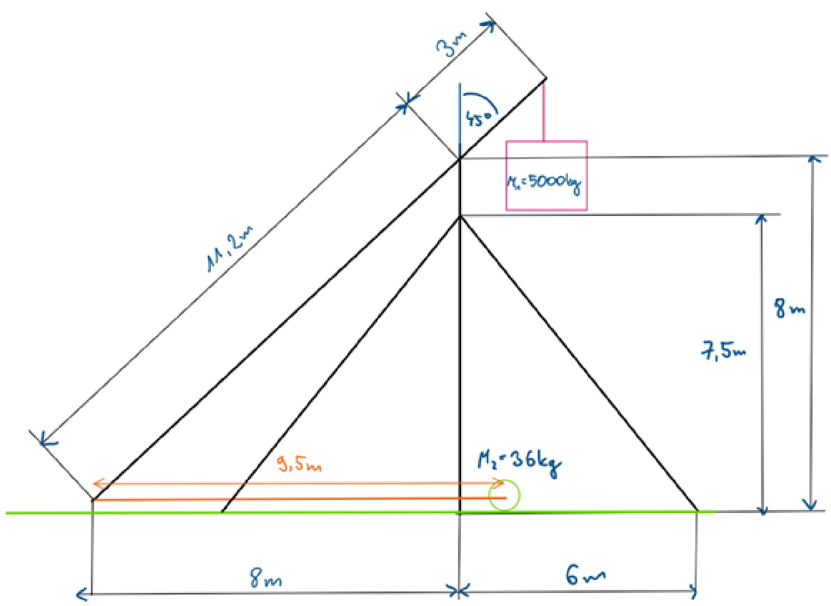

Figure 2.2: ADAMS initial model

The model is rigidly attached to the ground by fixed joints. The movement of the projectile is modelled by Contact Force (Type: Solid to Solid). All materials for the parts, except the sling, are modelled as wood. The mass of the arm is 644 kg while the one of the rope is 1.1 kg. The measure, which is calculating the angle between the rope and the arm is needed to simulate the release of the projectile at the optimal point. The written script allows us to perform the detachment of the projectile from the sling. There is also a sensor, which reads the Y position of the projectile and stops the simulation if the ball touch the ground (Earth). The release angle before optimization was measured as 150° and the achieved distance, before adding air resistance was 478m.

2.2. Simulation phases - only MBDyn

The projectile throw can be divided in three different phases:

1. **The projectile is in contact with the guide**: the projectile is initially on the guide and as the counterweight begins to fall, the projectile starts moving sliding on the guide until the pull exerted by the sling overcomes the projectile weight. The contact with the guide is imposed by means of a total joint constraining only the vertical displacement of the projectile: the joint is deactivated whenever the vertical component of the reaction force vanishes. In the following will be briefly explained how we modeled the contact friction between the two objects.

2. **The projectile is pulled by the sling through the air**: after the take-off, the projectile is still attached to the sling till it is released at a prescribed release angle dictated by a passive sling-release mechanism. In the implementation the angle controlling the release is the angle formed between the projectile absolute velocity vector and the horizontal. The real mechanism is composed by a finger and a ring: the friction/contact between these parts will keep the projectile in position until a certain angle between sling and arm is reached, usually determined by trial and error. To simplify the analysis we decided to not consider the mechanism in detail, assuming that for an equivalent release velocity angle a mechanism with the properties to achieve the aforementioned release condition with a certain precision can always be created. In the simulation, the release of the projectile is achieved by the deactivation of the clamping between the projectile and the sling.

3. **The projectile freely flies through the atmosphere**: now the projectile is free to fly in the air and to reach the objective. The simulation is thus ended through the activation of an intentionally wrongly implemented joint (we called it ABORT) whose only scope is to prevent the convergence of the integration scheme (which means interrupting the simulation) at the step right after the vertical component of

the absolute position vector of the projectile becomes negative. Initially no air drag was considered, afterwards a simplified 2D air drag resistance model was added.

Our original intention was to simulate all the three phases consecutively and without interrupting the simulation in between them, using two *driven elements* (one for each joint to be deactivated) with string drives directly referring to the Private Data generated during the simulation. This approach resulted in being very problematic, making us unable to run robustly the simulation without incurring in convergence issues. Nevertheless we noticed that using the *driven elements* with string drives referring to the simulation time did not produce any of the just mentioned issues: the immediate drawback of this approach is the need of running MBDyn thrice for each throw since using the simulation time means providing the time step at which the joints will be deactivated as input and clearly the instants of time at which the conditions of deactivation will be met cannot be known in advance, so every time we need to run the simulation a couple of times just to collect them before being sure to run the full simulation without any problem.

2.3. Flexible arm - only MBDyn

Given the high level of loads exerted on the arm, its deformation is expected to be non negligible and this could significantly change the behaviour and performances of the trebuchet with respect to the ideal rigid case. For this reason the arm was converted to a three node beam element.

An orthotropic linear elastic constitutive law was adopted, retrieving the values typical for red oak from [3].

2.4. Air drag

2.4.1. ADAMS modeling

To add realism and complexity to the Adams model, aerodynamic drag forces were added. In order to know which formula should be used to simulate such forces, one must evaluate the Reynolds number, calculated as eq. (2.1) linked to the trajectory of the projectile through the air.

$$R_e = \frac{v\rho L}{\nu} \tag{2.1}$$

With v being the velocity of the projectile, ρ the density of the air equal to 1.293 kg/m^3 at

STP (Standard Temperature and Pressure), L a characteristic length and ν the dynamic viscosity of the air equal to $1.7 \cdot 10^{-5} Ns/m^2$ also at STP.

In our case, the characteristic length of the system is its diameter, thus 2R. In simulations without drag forces included, the projectile would leave the sling with a velocity as high as 70 m/s. The Reynolds number of the trajectory can thus be evaluated at Re=$2.1 \cdot 10^6$. At such a high Reynolds number, drag forces can be modeled as quadratic, with eq. (2.2)

$$\boldsymbol{F} = -\frac{1}{2}\rho v C_d A v \boldsymbol{v} \tag{2.2}$$

The force thus increases with the square of the velocity. C_d corresponds to the drag coefficient and A is the reference area. According to the literature [5], for a sphere, a common value to use is 0.47 for the drag coefficient and a reference area equal to πR^2. Such values were used in our model.

Since ADAMS does not allow for an easy way to implement drag forces, a force was added acting on the projectile with a direction opposed to that of its velocity and a magnitude corresponding to the formula above. This force is considered to act on the projectile only once it leaves the sling. Before that, it is considered negligible with respect to the reaction force between the two objects and would not impact its trajectory. The magnitude of the drag force with respect to time is shown in fig. 2.3.

We can observe that it is equal to zero at the beginning of the simulation for the reason listed above and then reaches a maximum value of around 300 N once the projectile leaves the sling. Its magnitude then decreases as the projectile slows down but increases again since the simulation does not end once the projectile hits the ground and keeps on accelerating through the ground.

2.4.2. MBDyn modeling

In the MBDyn model, the same logic and coefficients were used in the addition of the air drag contribution to the projectile free flight.

2.5. Friction

Friction was inserted to add realism to the action of the hinges and to the contact between the projectile and the guide. The adopted models are not sophisticated but we are confident they are able to grasp the main effect of the physical phenomenon.

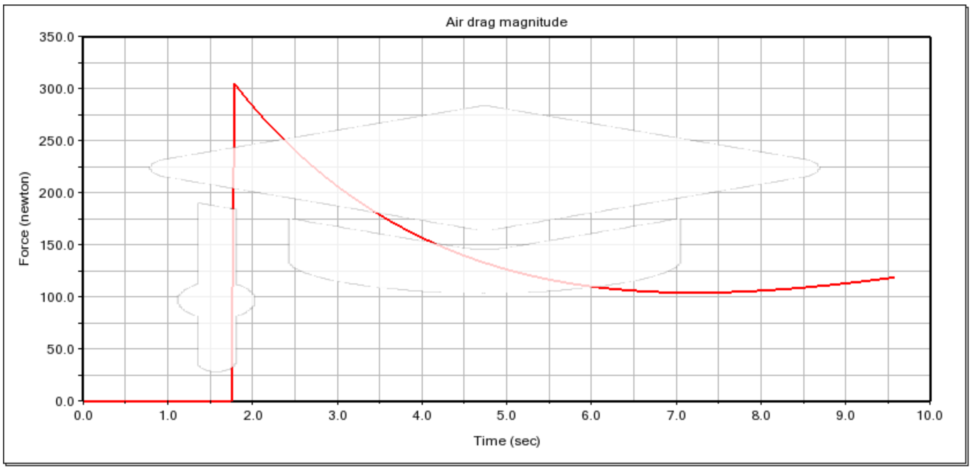

Figure 2.3: Air Drag force

2.5.1. Guide and projectile

Concerning the contact between the projectile and the guide, we modeled it distinguishing between static and dynamic conditions, basically adopting a simplified Coulomb friction model; the projectile is made of stone, while the guide is made of wood, so after a research in the literature we finally ended up with a static coefficient equal to 0.5 and a dynamic one equal to 0.4.

The static condition was implemented by clamping the projectile at ground till the horizontal pulling action exerted by the sling overcomes the static limit equal to the product between the normal reaction force and the static coefficient. It is worth noticing that after having observed that the pulling action was always much higher (from the very first step of simulation) than the static limit, we actually removed this additional unused joint to simplify the model.

In the dynamic case we applied an horizontal resistance force proportional to the normal reaction force, scaled through the dynamic coefficient.

2.5.2. Hinges

In each hinge we simply applied a resistance couple constant in module and opposite with respect to the relative angular velocity between the hinged parts. The values of the couples were tuned such that the range obtained with the model including air drag and flexibility was similar to the real one (around 300 m).

2.6. Results

2.6.1. Comparison MBDyn - ADAMS

The parallel modeling both in MBDyn and ADAMS allows us to double check the correctness of our models. The main comparison was done in the case without flexibility and without friction in the hinges (with the configuration parameters estimated for the Warwick trebuchet). The obtained ranges are 477m (312m) for ADAMS and 486m (323m) for MBDyn, the parenthesis containing the values with drag. So we have differences around 1.9% without drag and 3.5% with drag, low enough to validate our models. The trajectories are shown in fig. 2.4 for ADAMS and in fig. 2.5 for MBDyn.

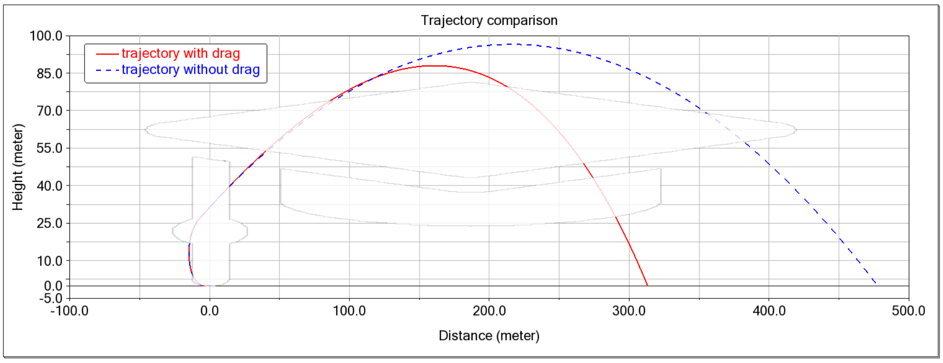

Figure 2.4: ADAMS results

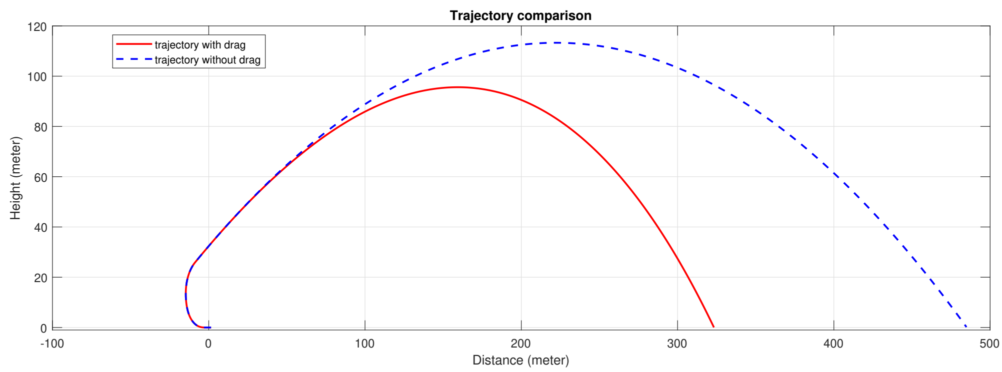

Figure 2.5: MBDyn results

2.6.2. Comparison rigid-flexible

In fig. 2.6 it is presented the comparison between the rigid and flexible models (friction and air drag is included in both).

As can be noticed, flexibility sensitively increases the range. This is thought to be mainly

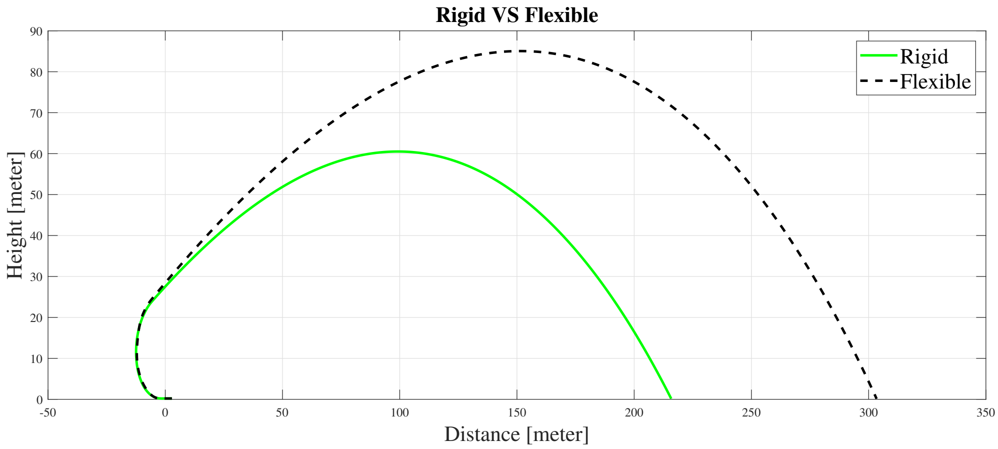

Figure 2.6: Flexibility effects over basic model

due to the additional energy provided by the flexural deformation of the beam.

3 | Parametric analysis

As an initial step towards an optimal design a preliminary parametric analysis of the model was considered. This analysis has the objectives of gaining insight on the geometric and physical parameters governing the simulation results and hopefully obtain a simpler optimization problem. The main step in this analysis were the choice of parameters and objective, these should be general enough to explore well the design space but not excessively to avoid unnecessary complexities.

3.1. Choice of the parameters

In general the parameters were chosen to be adimensional, even if there are exceptions, this makes the result more general as it is plausible that even for trebuchet of various dimensions, the optimal ratios will remain similar. The parameters that were chosen for the parametric analysis are:

1. The mass ratio ξ_{mass} which is the ratio between mass of the counterweight m_{cwt} and the mass of the projectile m_{prj}
2. $\theta_1(t_0)$, the initial angle between the arm and the horizontal
3. The arm ratio ξ_{arm} which is the ratio between the length of the longer (projectile side) arm part l_1 and the shorter (counterweight side) arm part l_2.
4. The counterweight ratio ξ_{cwt} which is the ratio between the length of the counterweight c.g. offset l_3 and the shorter (counterweight side) arm part l_2.
5. The sling ratio ξ_{sling} which is the ratio between the length of the sling l_4 and the longer (projectile side) arm part l_1

To keep the analysis consistent from an energetically point of view it is necessary to reason about some of these parameters [4]. That is because including configurations with different initial potential energy will favour some of those, despite how the simulation and the energy exchange will be carried out. In particular it is necessary to deal with the cases in which the initial angle is varied. If no changes were made to the other parameters greater angles would provide an higher initial height to the counterweight. To avoid this we define a constant height h which is defined in eq. (3.1), this is the maximum height difference that the counterweight c.g. can achieve. Then once θ_1 is fixed also the shorter arm part length is also fixed.

$$h = l_2(1 + \sin\theta_1) \tag{3.1}$$

3.2. Choice of the objective

We consider as objective of our study the maximization of the range, other kind of objectives that would be equally valid are maximization of the range efficiency or energy efficiency with respect to a theoretically computed one, in which all potential energy is transformed in kinetic energy.

To avoid long simulation time it was opted to simulate the launch without releasing the projectile until the first full sling. Then we retrieve the velocity (fig. 3.1) of the projectile at all times and we use eq. (3.2) to compute the resulting range if the air drag is not considered. This strategy allows to avoid picking a release time/angle as optimization parameter, because the optimal one is obtained as the best, function of the release conditions.

$$R = x_0 + v_{x_0}\left[\frac{v_{z_0}}{g} + \sqrt{\left(\frac{v_{z_0}}{g}\right)^2 + \frac{2z_0}{g}}\right] \tag{3.2}$$

When air drag for the projectile is added, instead of making use of an analytical formula, we had to write a simple forward Euler integration scheme to search for the optimal (velocity) release angle, which in any case does its job faster than running a further simulation for any possible release angle and does it with acceptable precision.

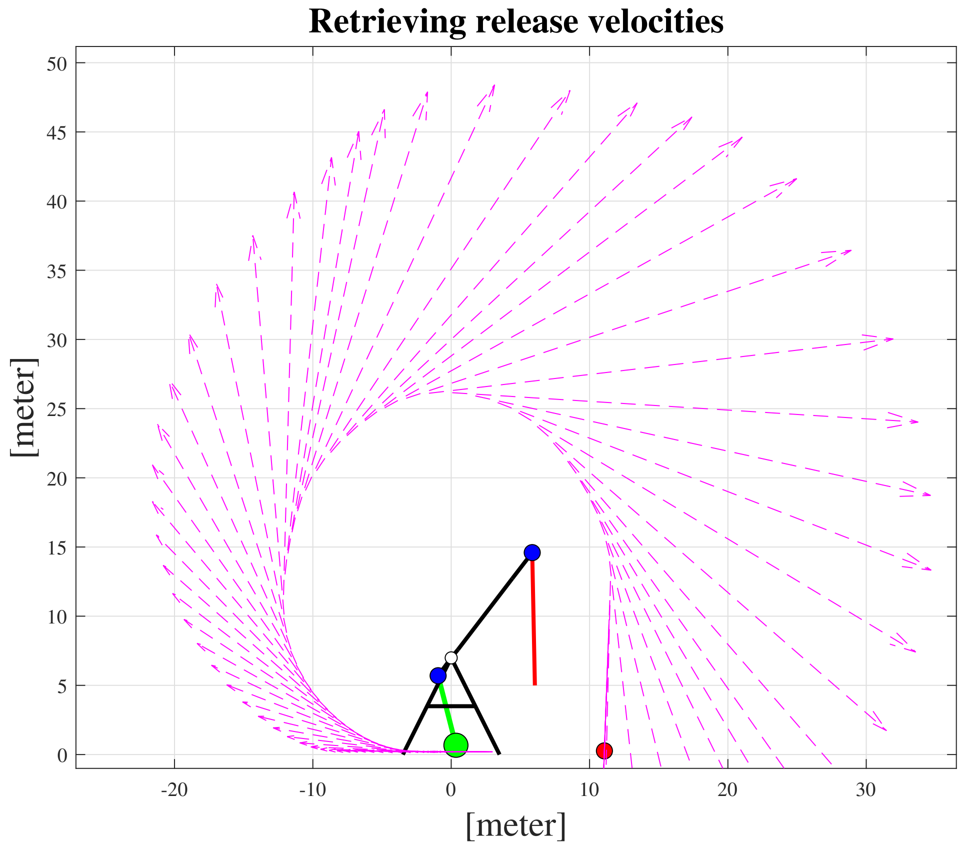

Figure 3.1: The image shows the velocity vectors of the projectile (for clarity, they have been printed every 5 steps of simulation)

3.3. Results

After these initial definition, each individual parameter was varied keeping the others fixed. The exploratory bounds in which the parameters can vary are obtained around the Warwick trebuchet values.

In fig. 3.2 are showed the results obtained with the complete model (the rigid model investigation is not presented for brevity); they clearly show maxima for $\theta_1(t_0)$, ξ_{sling} and ξ_{arm}. Concerning the ξ_{mass} in absence of air drag, the lighter the projectile the better it is: this is reasonable considering we are optimizing range, but probably would change in a efficiency optimization setting. Finally for the ξ_{cwt} we can see that increasing it lead to better results (obviously a check on the height of the counterweight c.g. was made to avoid hitting the terrain). This can be explained by noting that with greater counterweight offset the cwt drop more vertically and has much less oscillation, meaning less energy trapped in the trebuchet motion and more added to the projectile.

Considering instead the plots with the addition of drag we can see that the mass ratio has now a maxima as lighter projectiles will go faster and will be more impeded by the air drag.

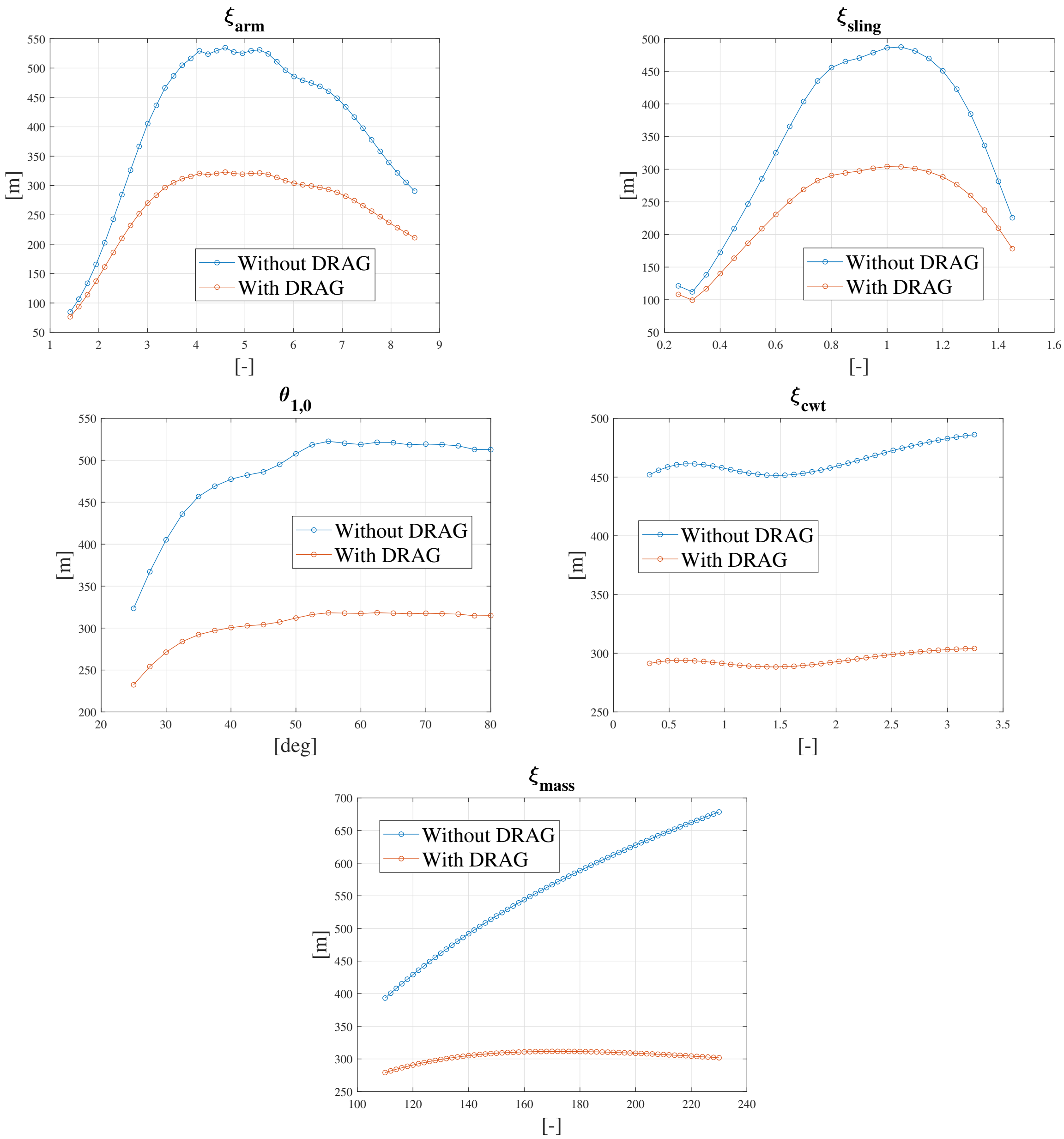

Figure 3.2: Results of parametric analysis

4 | Optimization

4.1. Initial grid search

Starting from the result obtained in the parameter analysis we can follow up with a grid search. To speed up this phase, the search was done only on ξ_{arm} and $\theta_1(t_0)$. The ξ_{sling} (=0.95) and ξ_{mass} (=165) are taken as the best from the parametric analysis, while ξ_{cwt} was taken as the maximum geometrically possible. The result of this grid search is depicted in fig. 4.1.

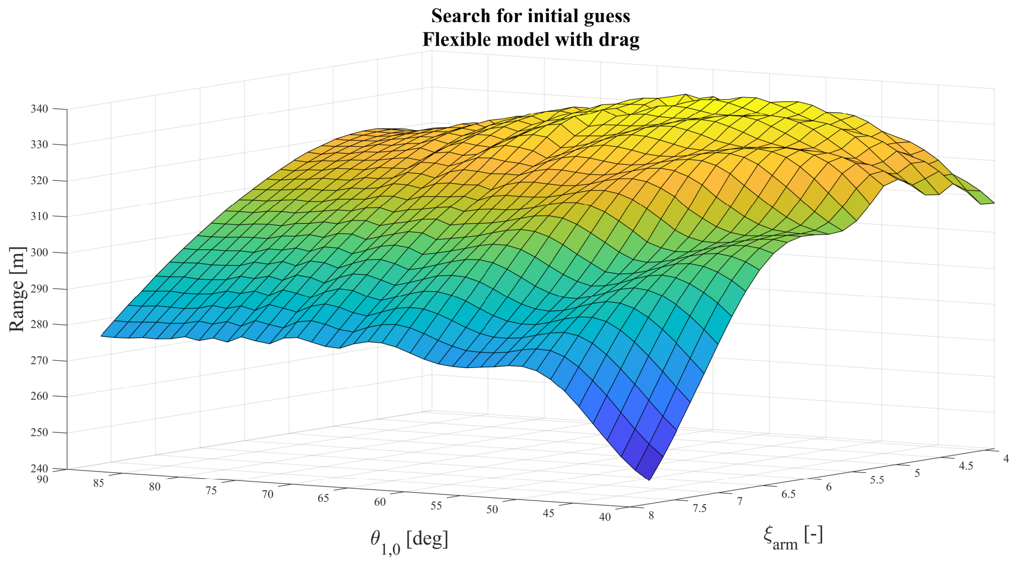

Figure 4.1: Initial guess determination

4.2. Optimal configuration via optimization

4.2.1. MBDyn

For the optimization the Matlab function `fmincon` was used, constrained only by upper and lower limits on the parameters assigned around the initial conditions. To increase robustness and convergence, the central finite difference were used instead of the default setting; moreover, the decision variables were normalized before being passed to the function in order to make them drive the optimization in a comparable manner. The results are contained in table 4.1, these are similar to values found in literature and values of the Warwick trebuchet, but slightly higher confirming that the original design is not fully optimized.

Table 4.1: Optimization results

Variable	Initial Value	Optimal Value	Warwick
$\xi mass$	165	**162.8317**	138
$\theta_1(t_0)$	56.25°	**57.953°**	45°
ξarm	4.5116	**4.4894**	6
ξ_{sling}	0.95	**0.98**	1
Range	336.56 m	**338.26 m**	circa 300 m

4.2.2. ADAMS

4.2.3. Optimization varying release angle

The main purpose of the design study was to find the best release angle for the throw of the projectile to achieve the longest distance. Air resistance has been taken into account. The performed optimization for the angle showed that it should be 150° (with respect of MBDyn here the release angle is actually the angle between sling and arm), as it was initially calculated, and it allows us to obtain a distance of 313m. The results of this study are also shown in fig. 4.2

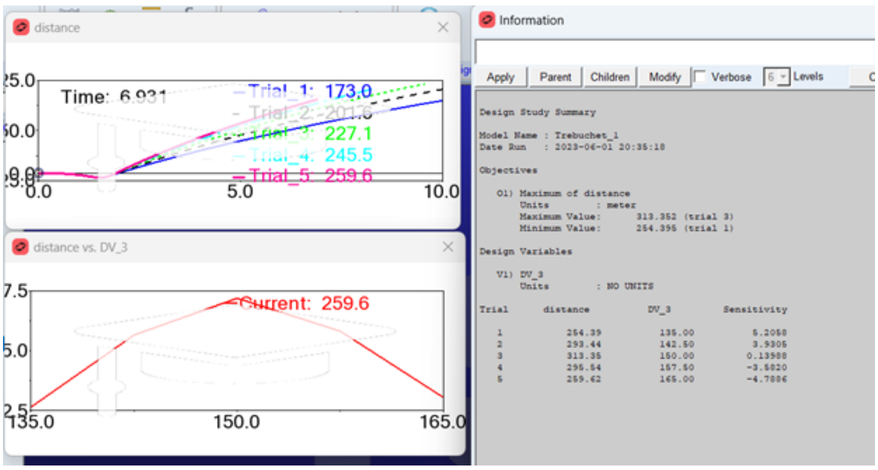

Figure 4.2: Range optimization over release angle ADAMS

4.2.4. Optimization varying sling length

Another important aspect that can be optimized in the trebuchet is the length of the sling. In order to achieve the best result (the highest range) we performed three iterations of optimization, where the varying parameters were the X coordinates of both ends of the sling. We were able to achieve a range of 314.1m from the previous result of 313.4m. The best results were obtained when we used the X coordinates of the left end as a "-8.213" and of the right end as a "1.557". The final length of the sling is 9.77m from 9.5m. The optimization results are shown in fig. 4.3.

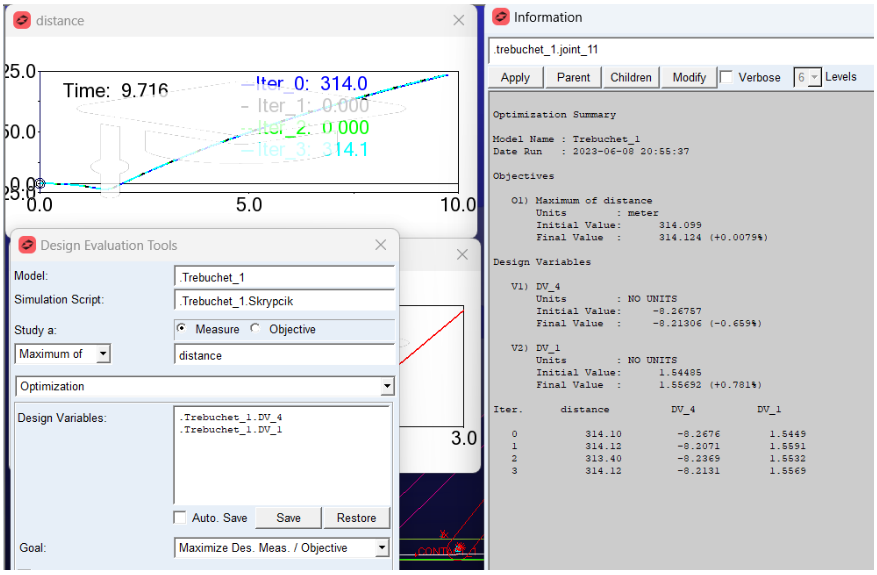

Figure 4.3: Range optimization over sling length ADAMS

4.2.5. Optimization varying projectile radius

A further optimization was performed to study the impact of the radius of the projectile. By considering the density of the object constant and equal to 1300 kg/m^3, this radius has an impact on both its mass and its surface area which have opposing effects in terms of aerodynamics. This radius was thus allowed to vary between 0.1m and 0.3m and the plot of the projectile's range with respect to this parameter is reported in fig. 4.5

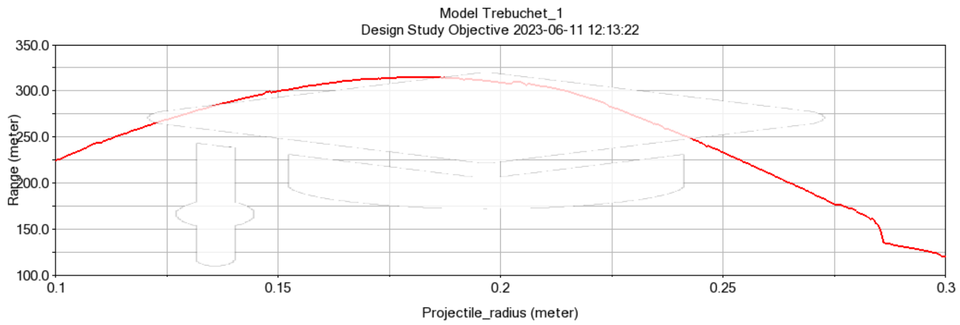

Figure 4.4: Range optimization over projectile radius ADAMS

As expected, a radius too small means the projectile is very light and is easily slowed down by air resistance. Meanwhile, a radius too large also means more drag as well as less acceleration from the movement of the counterweight and thus a smaller initial velocity for the ballistic part of the trajectory.

This variation of the radius can also be represented as a variation of ξ_{mass}, as visible in eq. (4.1) when the counterweight mass is fixed.

$$\xi_{mass} = \frac{m_{cwt}}{\frac{4}{3}\pi R^3 \rho_{prj}} \tag{4.1}$$

The radius associated to the maximum range is R = 18.34cm, which corresponds to a value for the mass ratio equal to 148.61, the results are reported in fig. 4.5. This configuration allows for the projectile to reach a distance of 314.83m.

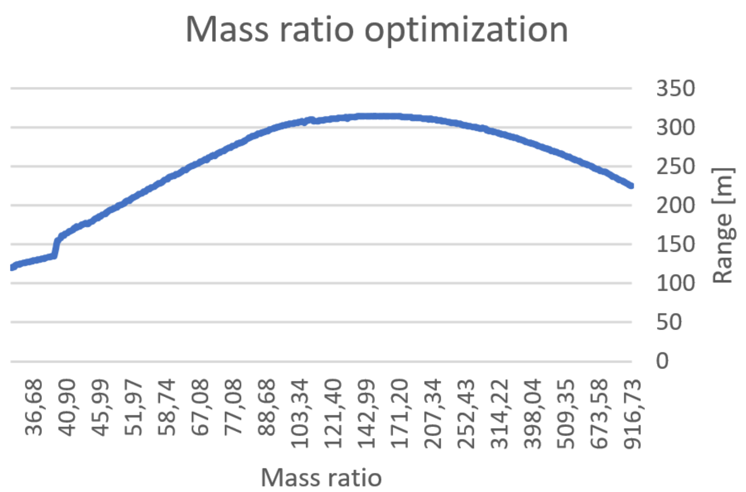

Figure 4.5: Range optimization over mass ratio ADAMS

5 | Further Improvements

5.1. Whipper Trebuchet

Another design that was experimented in recent times is the Whipper trebuchet. This design allows for a longer counterweight offset, counterweight which is initially blocked in its rotation by the main arm, this condition was modeled in MBDyn with a conditionally activated constraint. After the firing the counterweight will initiate a rotation, and once it has reached the vertical position can now fall freely, fully transferring its kinetic energy to the projectile. Some instants of the launch are shown in fig. 5.1.

Due to its design differences we cannot simply compare the ranges (as the initial potential energy of the Whipper Trebuchet is higher), then we report the range efficiency on the theoretical range. The results show that the whipper trebuchet in this current configuration achieves a range efficiency of 54% versus an efficiency of the traditional Warwick design of 62%, it must be noted though that the Warwick trebuchet is near an optimal configuration while the Whipper was not optimized.

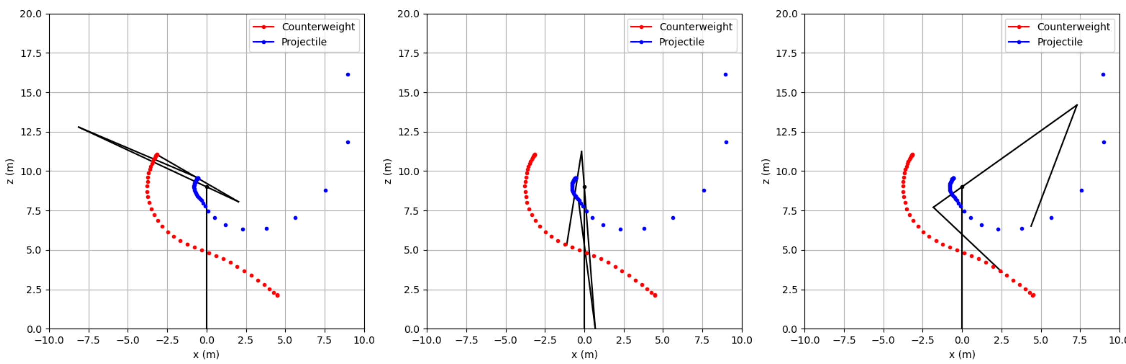

Figure 5.1: Whipper Trebuchet frames

Further investigations should compare an optimized Whipper to an optimized hinged trebuchet to tell exactly which is the best design. There are still concerns over the feasibility of this particular choice of parameters since the counterweight mass in a real siege weapon would probably be diminished as it would be at a higher height, thus more difficult to lift and would impose non indifferent stresses on the whole structure.

5.2. "Sliding" Trebuchet

Finally, we tried to see what happens if the trebuchet is left free to "slide" on the ground by means of wheels or a sliding platform. In reality, this would probably be very difficult to realise due to the really big weight of the machine; nevertheless, we attempted to understand if any advantage is gained allowing the trebuchet to move.

In MBDyn, given the complete (flexible) model, the sliding motion was simply obtained by relaxing the horizontal constraint clamping the main hinge of the arm to the ground: a new dynamic node - to which was attached a rigid body with a mass (around 10000 Kg) representing the whole supporting structure of the trebuchet - was added to the model, left free to slide and then clamped to the arm.

We observed that in general, given a certain configuration, allowing the trebuchet to slide does not increase the range, on the contrary it slightly reduces it. The idea was to determine if the liberty of movement somehow could increase the range by changing the synchronization between the reach of the optimal release angle and the stall condition of arm and cwt (the idea is that if the instant of time corresponding to the best release angle and the instant of time at which the kinetic energy of the arm and the cwt are at minimum coincide, the most kinetic energy can be successfully delivered to the launched projectile). Anyway, the big mass of the trebuchet does not allow the whole structure to move significantly, nullifying the previously mentioned hypothetical beneficiary effects of this design choice.

6 | Conclusions

This report presented two multibody models of a classical counterweight trebuchet, inspired by the Warwick Castle reconstruction. The result of the comparison between the models showed their validity.

The created models are vastly simplified versions of the real trebuchet. Further steps in the modeling process might be: improving the friction model in the hinges collecting some experimental data, taking a look at the reaction forces generated by the models to understand if the obtained dynamics could be really withstood by the trebuchet structure, refining the flexible model of the arm and adding flexibility also to the sling, developing a detailed model of the release mechanism including the contact between the release finger and the sling.

When it comes to optimization, a crucial aspect lies in selecting the appropriate parameters that enable the attainment of optimal outcomes. In the end for the ADAMS model we opted for a sequential optimization varying the lengths of the arm, sling, and the value for the release angle, while for MBDyn we conducted an optimization with multiple parameters varying in parallel.

The results of the optimization showed that the Warwick trebuchet is already near optimum, as only relatively small range improvements were achieved over the reported range of the original machine, thus demonstrating the quality of the craftsmanship of the Medieval siege weapon builders.

Concerning the proposed alternative configurations, they do not show a clear advantage, but further analysis should be conducted to determine the potential impacts of the design changes.

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