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Spacecraft attitude dynamics project

Group 24

Project 460

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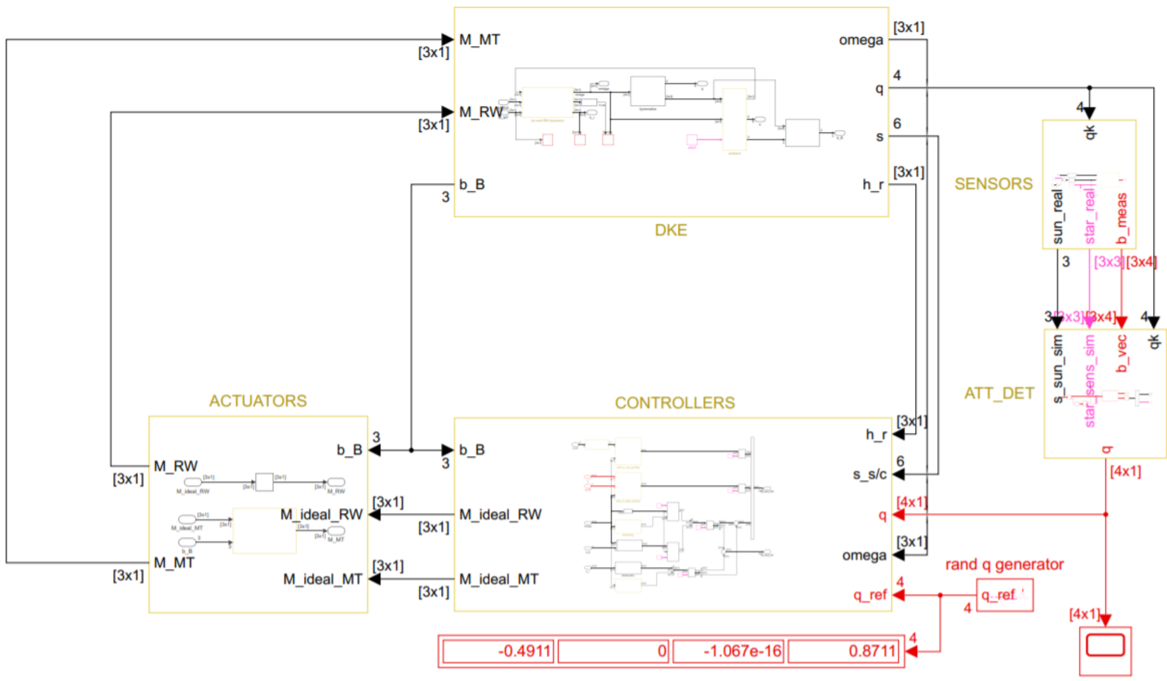


Introduction

Parametrization

The mission consists of controlling an orbiting microsat (10~100kg) around the Earth using at least:

- quaternions as attitude parameters
- a Sun sensor
- 3 reaction wheels as actuators

The satellite chosen has the same geometry as SDS-4, a small satellite developed by Jaxa.

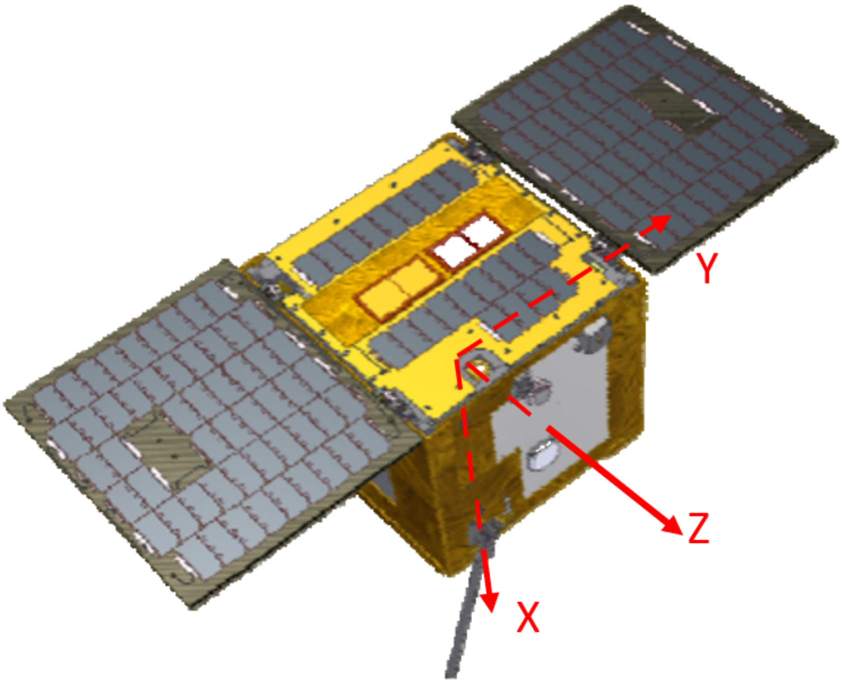


Figure 1 : Geometry of the spacecraft

Its dimensions are 500 mm * 500 mm * 500 mm and it weighs roughly 54 kg.

Its inertia matrix is J = $\begin{matrix} 2.5000 & 0 & 0 \\ 0 & 2.2500 & 0 \\ 0 & 0 & 2.7500 \end{matrix}$

The principal inertia moments have been roughly estimated considering the geometry of the s/c and the fact that the inertia moment of a cube with constant density is equal to:

$$I = \frac{1}{6} m^2 l^2$$

*In this case I_{jj} of the cube would be equal to 2.25 kg*m^2*

The chosen orbit has :

- semi-major axis a = 8123 m
- eccentricity e = 0.1789
- inclination i = 1.0266 rad
- longitude of the ascending node Ω = π rad
- argument of periapsis ω = π rad
- mean anomaly M = π rad
- We can represent it as follows :

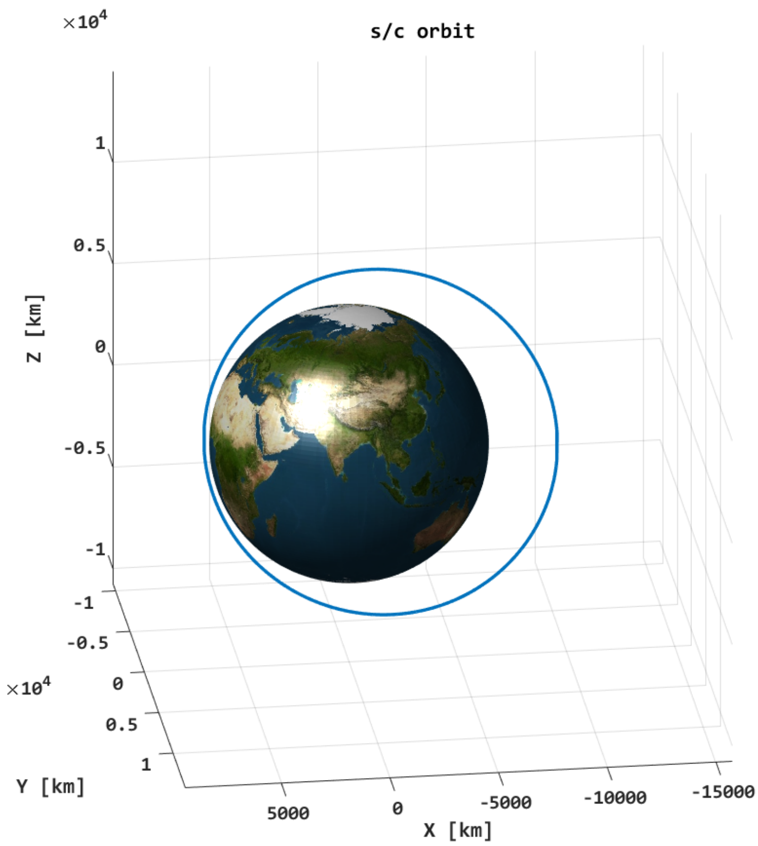


Figure 2 : Spacecraft orbit around the Earth

Dynamics and Actuators

The dynamics of the s/c are described by the Euler equations in matricial form:

$$\frac{d}{dt}\bar{\bar{J}}\,\bar{\omega} = (\bar{\bar{J}}\,\bar{\omega}) \wedge \bar{\omega} + \bar{M}_{ext}$$

J is the 3x3 matrix of inertia of the s/c expressed in it's principal axis (Ixy = 0, Izy = 0, Izx = 0)
ω is the rotational speed of the s/c with reference to the inertial perifocal earth centered frame [e,p,h]
M_{ext} is an external torque vector

This relation can be found using the d/dt transport theorem between two rotating frames of reference:

$$\frac{d^I}{dt}\bar{x} = \frac{d^B}{dt}\bar{x} + \bar{\omega}_{B/I} \wedge \bar{x}$$

x is a random vector variable, while the the apices I,B define if the derivative is referred to the inertial frame of reference of to the body frame of reference
B/I instead defines the value of the variable from the body frame to the inertial frame

Actuators

Our s/c is controlled through 3 reaction wheels (or RW), that for simplicity have their own momentum pointed in the same direction as the body frame axis [x,y,z], and 3 magnetic torquers.

Magnetic torques are actuators that apply a certain external force on the s/c by interacting with the Earth's magnetic field (they are optimal for LEO orbits, where the field is strong.

$$\bar{M}_{MT} = \bar{d} \wedge \bar{b} \qquad \bar{d} = \mu n S \bar{I}$$

d is the magnetic dipole of the magnetic torquers
b is the magnetic field of the earth
mu is the magnetic permeability
n is the number of coil windings
S is the surface area of the coil
I is the vector of the current intensity applied to the 3 different magnetic rods

We are using magnetic rods because they are highly efficient, require little energy and are very lightweight. Plus, they don't use any fuel and can be used indefinitely as long as you have electricity. The cons are that they provide low torque (between 10^{-2} and 10^{-6}), can only be used in LEO orbits and they can't produce torque independently in all 3 axis.
The dipole of the magnetic rods can be moduled between the max and the negative max by varying the intensity of the current.
When they are shut off with no current they have a negligible magnetic dipole, approximated to 0.
They are easily introduced into the dynamics since they can be considered as external forces.

We have chosen a NMTR-X [Custom] magnetic rod.
By our calculations it will be 45cm long and require 4W of power for a nominal maximum 200Am magnetic dipole, so 3 of them can perfectly fit our s/c.
https://www.cubesatshop.com/wp-content/uploads/2018/05/NewSpace-Magnetorquer-Rod_2021-10c.pdf

Meanwhile RW are actuators that don't apply any external force on the s/c, they rely instead on the internal exchange of momentum to make the s/c rotate.

By definition, (in the absence of external torques):

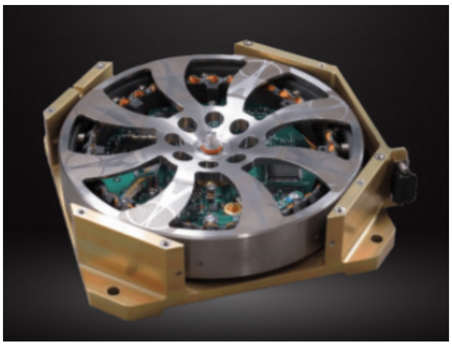
$$\overline{h}_{tot} = \overline{\overline{J}}\,\overline{\omega} + \overline{\overline{J}}_R\,\overline{\omega}_R \qquad \frac{d}{dt}\overline{h}_{tot} = 0$$

h is the angular momentum
J_R is the 3x3 matrix in principal axis of the inertia of the RW
ω_R is the vector angular velocity of the RW

We can consider the RW as applying a certain torque (M_{RW}) to the s/c and an opposite one to themselves. That M_{RW} will be determined by the controller.
from now on for simplicity I'll refer to $\omega_{B/I}$ as ω

The dynamics of the RW are pretty simple:

$$\frac{d}{dt}\overline{\overline{J}}_R\,\overline{\omega}_R = -\,\overline{M}_{RW}$$



We have chosen the RL-RW-1.0 reaction wheel.
It produces a maximum torque of 0.1 Nm and has a nominal max angular momentum of +-1 Nms.
https://www.rocketlabusa.com/assets/Uploads/RL-RW-1.0-Data-Sheet.pdf

Dynamics

Using all these equations, and applying the d/dt transport theorem to the momentum, we can easily find the dynamics of the s/c with RW:

$$\frac{d}{dt}\overline{\overline{J}}\,\overline{\omega} = (\overline{\overline{J}}\,\overline{\omega} + \overline{\overline{J}}_R\,\overline{\omega}_R) \wedge \overline{\omega} + \overline{M}_{RW} + \overline{M}_{MT} + \overline{M}_{ext}$$

$$\frac{d}{dt}\overline{h}_r = -\overline{M}_{RW}$$

$$\overline{h}_r = \overline{\overline{J}}_R\,\overline{\omega}_R$$

$$\overline{M}_{MT} = \overline{d} \wedge \overline{b}$$

M_{MT} is the torque applied by the magnetic torquers
M_{ext} will be defined better in the Environment section by Davide.

Validation

The dynamics has been validated in various ways.
First of all it behaves as expected in a periodic way following Euler's law when the initial ω of s/c is different from 0.
Then, with no external torques and while applying a varying M_{RW}, the total angular momentum remains constant.
And when we finally apply some external torques it varies as expected.

Kinematics

We will use quaternions for the kinematics.
A quaternion can be used to describe the attitude or the rotation of a rigid body with respect to a reference frame (we will be using them in the ICE inertial frame of reference).
A quaternion is defined this way:

$$\underline{q} = [e_1 sin(\theta/2),\ e_2 sin(\theta/2),\ e_3 sin(\theta/2),\ cos(\theta/2)]^T$$
$$|\underline{e}|^2 = 1$$
$$q_1^2 + q_2^2 + q_3^2 + q_4^2 = 1$$

The first 3 components of the quaternion are the vectorial components while the fourth one is the scalar one.
They have the pros of not having discontinuities and singularities, of being less prone to error propagation when normalized compared to DCMs and Euler's angles and being less computationally heavy, but they don't have a direct physical meaning and can be hard to grasp at first sight.
They can be calculated from the ω received from the dynamics (or from the sensors):

$$\frac{d}{dt}\overline{q} = \frac{1}{2}\overline{\overline{\Omega}}(\omega)\,\overline{q} \qquad [\Omega] = \begin{bmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 \end{bmatrix}$$

Environment

Initial estimates

The satellite attitude motion, besides internal dynamics, is determined by the torque applied to it. A part of this torque is defined by the environment forces applied in a different point than the center of gravity. The implementation of these torques to be used in the simulation can be decided by a preliminary computation of the maximum they will apply on the spacecraft, and by considering the order of magnitude of this initial result.

The formula used to do these preliminary estimates are:

1) Gravity Gradient

$$T_{max} = \frac{3Gm_t}{2R^3} |I_M - I_m|$$

2) Sun Radiation Pressure torque

$$T_{max} = P_s A_s (1 + q)(c_{ps} - c_g)$$

3) Magnetic torque

$$T_{max} = D_s B_{max}$$

4) Aerodynamic torque

$$T_{max} = \frac{1}{2} \rho V^2 A_s C_D (c_{pa} - c_g)$$

These initial formulas are derived from the general ones via some assumption, they include parameters from the spacecraft and its orbit. The values of the parameters can be found in the introduction part of this paper.

The order of magnitude of the maximal disturbance torque are the following:

$T^{gg}_{max} = 10^{-6}$Nm $T^{SRP}_{max} = 10^{-7}$Nm $T^{mag}_{max} = 10^{-6}$Nm $T^{drag}_{max} = 10^{-4}$Nm

*greatest center of mass, center of pressure distance (cps-cg)=(cpa-cg)=0.559m

From these results it was decided to develop a model for the gravity gradient torque, the aerodynamic torque, and the magnetic torque.

Gravity Gradient

The mass and geometry of the spacecraft are made such that it can't be modeled as a simple point when considering the gravity interaction with Earth. This means that on the spacecraft the gravity force acts in a different way, closer points experience a higher force, further points a lower force. The used formulation was made considering a first order expansion and approximating the gravity field as spherically symmetric.

The overall effect is equivalent to a torque that tends to align the minor axis of inertia with the nadir direction.

Aerodynamic Torque

As the spacecraft is in LEO orbit, this type of disturbance torque must be accounted for. It will change in magnitude periodically as it increases passing near the pericenter and lower through the apocenter, due to density and velocity variation.It is also dependent on the difference between center of pressure and barycenter, and coefficient of drag. These parameters should be obtained experimentally, for this work they were chosen arbitrarily and assumed constant. The aerodynamic torque should consider the relative velocity of the spacecraft with respect to the rotating atmosphere and so the orbital velocity was modified accordingly.

As for the density it was chosen an exponential interpolation model with data points taken from an hybrid model, which uses US Standard Atmosphere (1976) for 0 km, CIRA-72 for 25-500 km, and CIRA-72 with Tinf = 1000 K for 500-1000 km[6].

Magnetic Torque

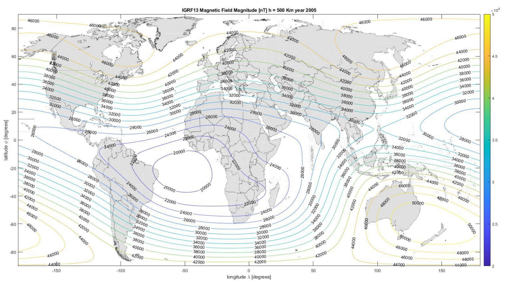
The model used for computing the magnetic field is the IGRF model[5], their last iteration utilizes 13 degree g,h parameters obtained from measures every five years, the secular variation can be used to extrapolate the g,h parameter in the 5 years wait.

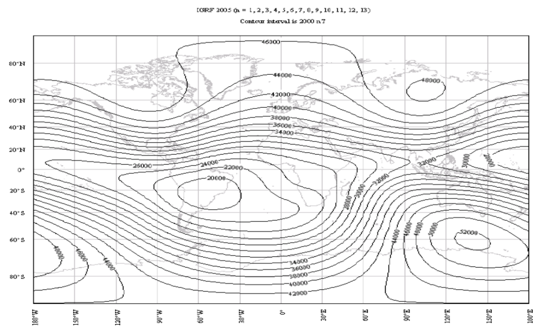
The IGRF is a spherical harmonic model, the available parameters are Schmidt semi-normalized spherical harmonic coefficients. The implementation in this work is done using a Gaussian normalization with recursive formulas so the coefficient before being used must be transformed using a proper Snm function.

The validity of the model was established by confronting the magnitude of the field on an available chart with altitude 500 Km and year 2005, it can be seen that the graphs have the same contour lines, and the Brazilian Anomaly is found in the correct position.

The g,h parameters have a secular variation between 2 of the IGRF measurements, in this mission case they were approximated as constant as the time window of the mission is relatively small with respect to the years needed to obtain a significant change of the parameters.

The dipole generated from the spacecraft was decided with a value 0.05 Am^2, since it uses magnetorquers that will result in a residual and considering the dipole generated from the electronics.





Modeling of Sun direction from Earth

Another way the spacecraft interacts with the environment is by sensing data, this is modeled by generating a mathematical representation of the motion of different bodies in the celestial sphere.

A sun sensor is used on the spacecraft, so the sun direction with respect to the Earth must be obtained. If there is a need to work with high precision the model should use known ephemerides, this is not the case for the satellite mission, what was used then is an analytic technique which results have an accuracy of 0.01°[4]. It's validity goes from 1950 to 2050 because of the truncation of the expansions.

Another important task is to determine when the spacecraft is shadowed by the Earth (Moon shadowing was not accounted for). For LEO orbits a reasonable approximation is considering that the Earth shadow is a cylindrical projection of the Earth's diameter along the direction of the Sun to the Earth. The condition when the spacecraft is in the shadow is:

$$r \cdot e_{\oplus\odot} < -\sqrt{r^2 - R_{\oplus}^2}$$

Sensors and attitude determination

Sensors

For the simulation, we are using a sun sensor and a star sensor. To perform the attitude determination, we need to know at least two directions. The star sensor gives us additional directions. It is even more important when the sun sensor can not be used because the spacecraft is in the shadow of the Earth. To model the sensor, we proceed as follows: we get the true direction of the sun and the stars given by the simulation and we add to them an error. After that we add a Quantizer block to model the accuracy of the sensor.

The error of the sun sensor is made of a rotation matrix. It is created with a random number generator which gives us a vector of rotation and another RNG parametrized with the angle of the greatest angular error the sensor can have. The given rotation matrix is then multiplied to the real measurement to obtain the sun direction given by the sensor. We also add in the simulation the passage of the satellite in the shadow of the Earth. During this time, only the star sensor is working but we know it is enough as it measures three independent positions.

The star sensor we are using measures the position of three virtual stars so it outputs a 3x3 matrix. It has 2 types of errors. The first one is a rotation matrix whose characteristics are fixed as opposed to the sun sensor. The other source or error is a white noise modeled by a random number generator. This white noise is applied by adding it to the matrix given by the real measurement matrix multiplied by the first error matrix.

Attitude determination

The attitude determination model used in the simulation is Davenport's quaternion method. It is based on the Wahba's problem for spacecraft attitude determination. This problem is to find the orthogonal matrix A such that det(A) = 1 and which minimize the loss function:

$$L(A) = \frac{1}{2}\sum_{i=1}^{N} a_i \left\| b_i - Ar_i \right\|^2$$

where b_i is a set of N unit vectors measured in the body frame of the spacecraft and r_i are the corresponding unit vectors in the reference frame. a_i are non-negative weights. These weights are chosen to be the inverse measurement variance.

In Davenport's method, the problem is rewrite to find the largest value of the following matrix:

$$K(B) = \sum_{i=1}^{N} a_i [\mathbf{b}_i \otimes]^T [\mathbf{r}_i \odot] = \begin{bmatrix} B + B^T - (\mathrm{tr}B) I_3 & \mathbf{z} \\ \mathbf{z}^T & \mathrm{tr}B \end{bmatrix}$$

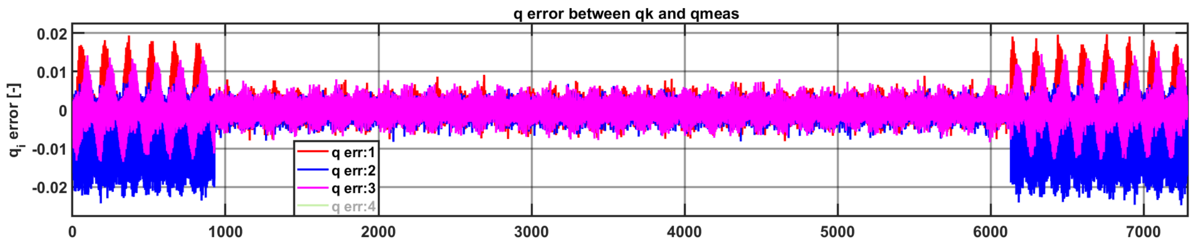
Where:

$$B \equiv \sum_{i=1}^{N} a_i \mathbf{b}_i \mathbf{r}_i^T \qquad \mathbf{z} \equiv \begin{bmatrix} B_{23} - B_{32} \\ B_{31} - B_{13} \\ B_{12} - B_{21} \end{bmatrix} = \sum_{i=1}^{N} a_i (\mathbf{b}_i \times \mathbf{r}_i)$$

To find the attitude quaternion, we look for the eigenvector associated with the greatest eigenvalue of the matrix K(B). In the simulation, it is implemented with a matlab function including the *eig* function, which looks for the eigenvalues and the eigenvectors. So:

$$q_{meas} = eigenvector\ of\ max(eigenvalue(K(B)))$$

There are different reasons to prefer this algorithm of attitude determination to the others. First, with respect to the TRIAD method, the choice of the weights scalar is better. Indeed, in the Davenport's method, the more precise a sensor is, the more it is taken into account in the attitude determination. For example, the star sensor has more impact in our simulation than the sun sensor. Then, as we are using quaternions in all the simulation, this method is ideal because it gives the attitude in a quaternion form so we do not have to perform a conversion from a matrix to a quaternion.



Example: this is the error between the true q from kinematics and the q measured from the sensors when the s/c is left uncontrolled. Initial q_0 = [0 0 0 1].
It behaves as expected, since the error is larger when the s/c is behind the Earth (eclipse) and we have less data since we aren't getting any from the Sun sensor. omega0 = $[0.2\ 0.3\ 0.1]^T$

Actuator block

Before introducing the maneuvers and the algorithms to evaluate the ideal closed loop response (the controller block), we need to introduce a block that actually converts these ideal outputs (in our case, the magnetic torques and the torques related to the RW) to the ones that we can actually use, given our real actuators, in our real s/c.

Each RW has a maximum torque of 0.1 Nm.
Let's call T_i the ideal output calculated by our controller and T_r the real one that will be applied to our s/c.
The actuator block for the RW is defined like this (for each component of the 3x1 input vector):

$$T_r = 0.1,\ if\ T_i \geq 0.1,\ \forall\ component$$
$$T_r = T_i\ if\ -0.1 < T_i < 0.1,\ \forall\ component$$
$$T_r = -0.1,\ if\ T_i \leq -0.1,\ \forall\ component$$

The magnetic torquers are instead limited by the magnetic dipole that we can generate in our torque rods (max 200 Am), so:

$$\overline{d}_i = \frac{\overline{b} \wedge \overline{ui}}{||\overline{b}||^2}$$

$$d_r = 200,\ if\ d_i \geq 200,\ \forall\ component$$
$$d_r = d_i\ if\ -200 < d_i < 200,\ \forall\ component$$
$$d_r = -200,\ if\ d_i < -200,\ \forall\ component$$

$$\overline{T}_r = \overline{d}_r \wedge \overline{b}$$

Maneuvers

For all the maneuvers the IDEAL output will be presented. It will later go into the actuator block to calculate the REAL one.

1. Detumbling

The detumble is a maneuver that has the objective of stopping the rotation of the s/c, starting from any initial rotational speed.
The RW aren't suited for this maneuver since they will easily saturate and then become unusable. We have decided to implement an hybrid method that will first decrease the ω of the s/c to an acceptably low level using the magnetic torquers and then finally stabilize it in a precise way with the RW when the residual momentum of the s/c is very low.

We have decided that the cutoff between the two parts will be determined when $||\omega|| < 0.1$.

Using the same function that is present in the environment we can get the magnetic field b in the inertial frame. By using the rotation matrix A_B/I that we have derived in the kinematics from the quaternions we can calculate b in our body frame.

$$\overline{b}_B = \overline{A}_I^B \, \overline{b}_I$$

The control law is then:

$$\overline{T}_{MT} = -k_\omega (I^{3x3} - \frac{\overline{b}\,\overline{b}^T}{||\overline{b}||^2})\overline{e}_\omega$$
$$\overline{e}_\omega = \overline{\omega} - \overline{\omega}_{ref}$$

Where ω_{ref} is obviously $[0, 0, 0]^T$

k_ω has been tuned to 0.01.

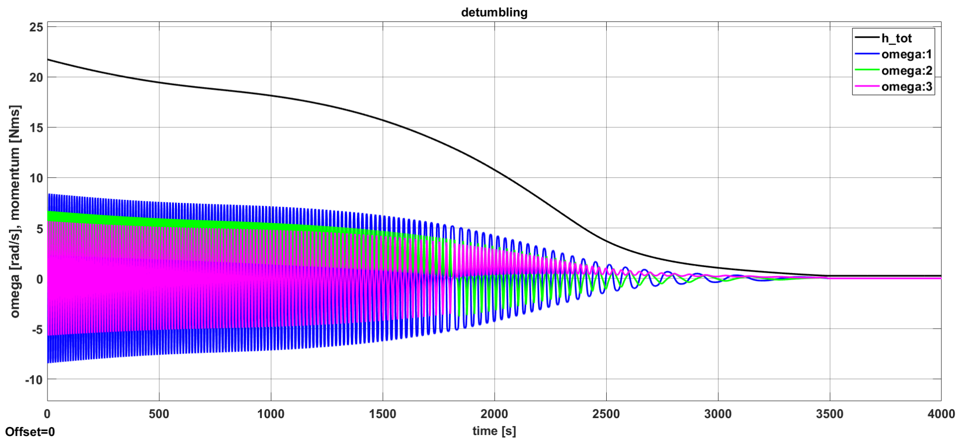
Then when $||\omega|| < 0.1$ we switch to a simple PID method based on the error of the angular velocity, where;

$$\overline{T}_{RW} = -kp * e_\omega$$

kp has been tuned to 20

The resulting method is very stable even for very high starting ω and then uses the RWs for very little, transferring to them just a very small amount of momentum that keeps them far away from saturation.

Example using ω0 = [6,5,4] rad/s. A detumbling time of approx 1 hour is acceptable.



2. Slew (can also be used for fixed inertial pointing)

The slew maneuver is intended to bring a static non rotating s/c from an attitude to another different static attitude, performing a rotation.

We have implemented a nonlinear control law that both uses the angular velocity error and the quaternion error (attitude error) to give us the intended output. This maneuver uses only RW since its cost is very limited (from our tests for every kind of rotation it doesn't use more than 10% of the maximum momentum capacity of the wheel before returning to its static position).
The quaternion error is modeled this way:

$$\overline{q}_e = conjugate(\overline{q}_{ref}) \otimes \overline{q}$$

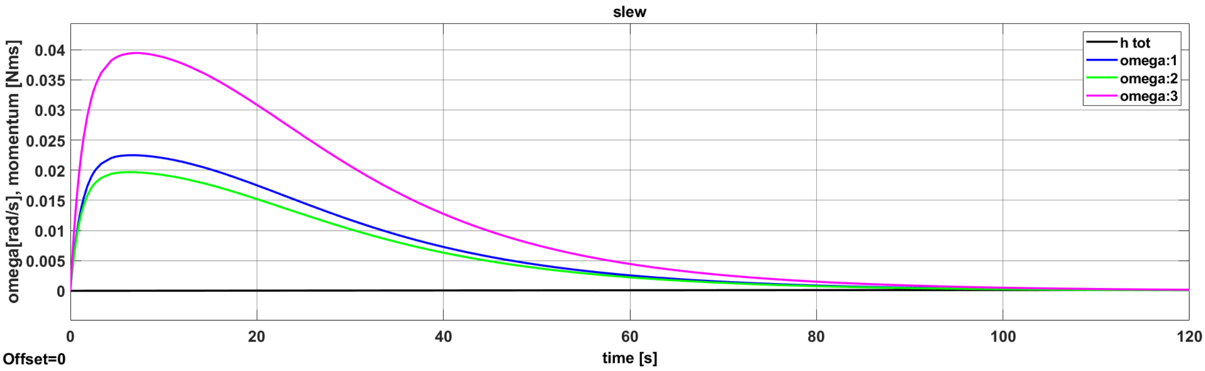
$\otimes$ is the quaternion product
the conjugate of a quaternion (q = ai + bj + ck + d) is (q_{conj} = -ai -bj -ck +d)
the q error is 0 when q_e = [0, 0, 0, +-1]

The control law is the following:

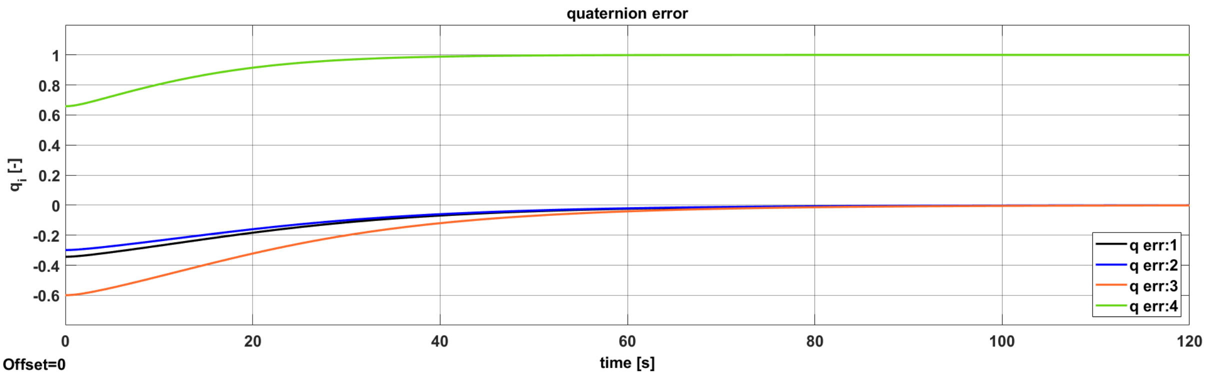
$$\overline{T}_{RW} = -\ kp1\ *\ \overline{qv}_e * qs_e\ -\ kp2\ * \overline{e}_\omega$$
$$\overline{e}_\omega = \overline{\omega} - \overline{\omega}_{ref}$$

where ω_{ref} is equal to [0, 0, 0]
qv_e is the vectorial part of q_e while qs_e is the scalare part of q_e
kp1 has been tuned to 0.2 and kp2 to 2

Ex. for random quaternion q_{ref} = [0.3517, 0.3058, 0.6136, 0,674] starting from q = [0, 0, 0, 1]



As we can see the algorithm is very smooth, as the angular velocities are always low.
The RWs give a push at the beginning in the direction of the rotation and then slowly stop the s/c before it reaches its desired attitude in approx 2 minutes (these have been verified to be on the long end of the average time for this kind of maneuver).



The quaternion error smoothly goes to 0 in the same way as the graphic precedently shown.

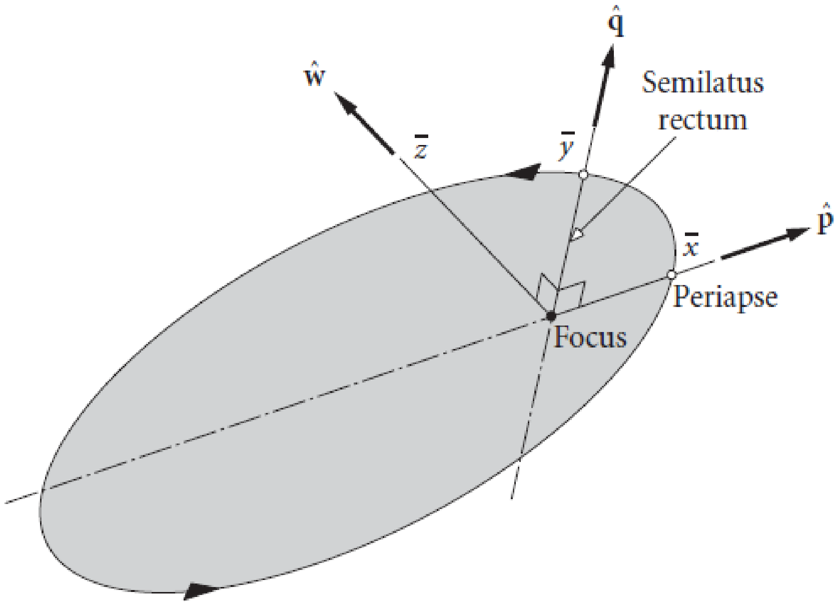
Notes

This algorithm could also be used for **fixed inertial pointing,** as it keeps the s/c oriented in that q_{ref} direction for as long as we like (all of this considering that we are always in the ECEI frame).

3. Earth pointing

(note: we aren't propagating the Earth's orbit with drag and J2 perturbation since it's not the scope of this course, and we are considering a constant orbit over time)

Considering our very fast (period wise) and little orbit (radius wise) we have considered that the most optimal pointing for this mission would be an Earth one (even though a fixed inertial pointing has already been implemented with the slew algorithm).



Our Earth pointing algorithm works considering the inertial perifocal frame of reference.

So we have first to find the rotation matrix to go from ECI to perifocal.

That's pretty easy using rotation matrices. We will define R_1 as the rotation matrix around the first axis, and etc for the other two with the pedix 2 and 3.

A_perifocal_ECI = $R_3(\omega)$ * $R_1(i)$ * $R_3(\Omega)$

since we are rotating between two inertial frame and all the constants involved in the rotation matrices are constant, so the resulting A matrix will be constant

We have considered that to keep a s/c pointed towards the Earth it's enough to first point it (or slew it) in the wanted direction (the one were the perifocal

and body frame are the same) and then rotate it around the perifocal frame of the same angle as the s/c rotates around the Earth.
The initial attitude in ECI will then be the one calculated with the aforementioned A matrix, from which we can easily get the initial attitude in quaternions (for simplicity and to not write all the calculations, command dcm2quat(A) in MATLAB and then convert to our quaternion notation).
The initial attitude for the pointing will then be:

$$q0 = [0 \quad 0 \quad 0.8711 \quad -0.4911]^T$$

The only rotation will be around the h (or z in body frame) axis, since it's the one perpendicular to the orbital plane. The other two angular speeds will have to be kept to 0.
Considering that the angular momentum of the orbit is constant, we can see that:

$$h = \sqrt{\mu * a(1 - e^2)}$$

$$h = r^2 * \frac{d}{dt}\theta$$

We can then easily get the variation of the true anomaly by inverting the last formula. That is exactly the angular speed by which we need to rotate the s/c.
(Let's remind ourselves that while the momentum of the orbit is constant the radius is not, so the angular speed will vary).
We can calculate the radius through the basic orbit ODE.

$$\frac{d^2}{dt^2}\overline{r} = -\mu * \frac{\overline{r}^{-3}}{||\overline{r}||}$$

where μ *is the planetary constant of Earth = 3.986*10*^{5} *km*^{3}*/s*^{2}

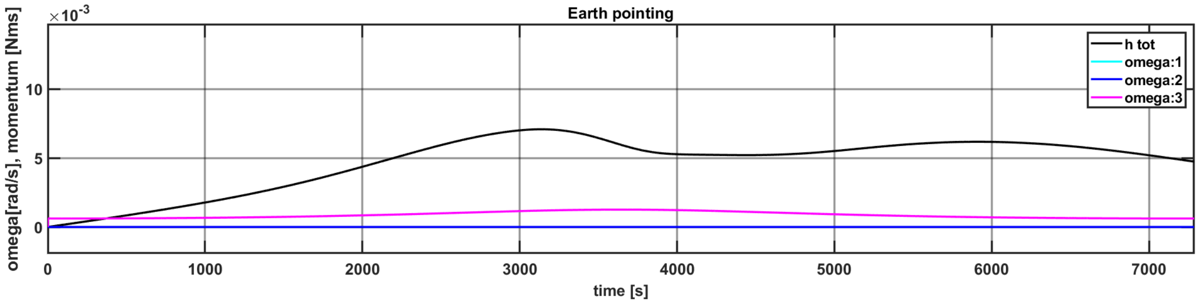
So we finally have $\omega_{ref} = [0, 0, \frac{d}{dt}\theta]$.
The control law will now be a simple PID that controls the error on the angular velocity.

$$\overline{e}_\omega = \overline{\omega} - \overline{\omega}_{ref}$$

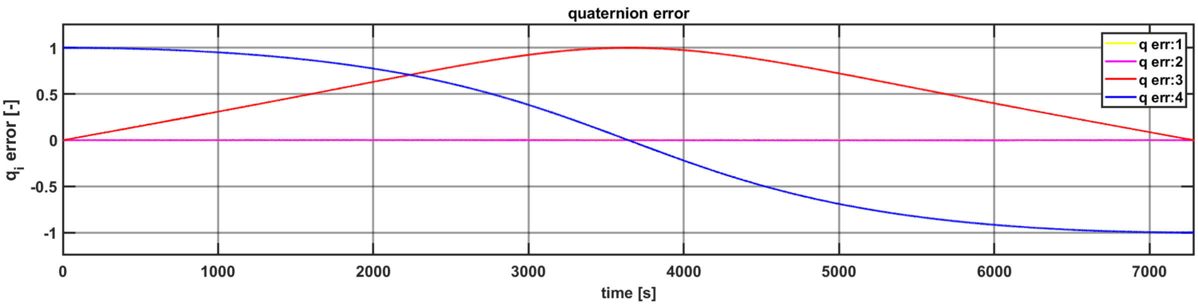
$$\overline{T}_{RW} = - kpp * \overline{e}_\omega - kip * \int_0^t \overline{e}_\omega dt$$

where kpp has been tuned to 10 and kip to 1

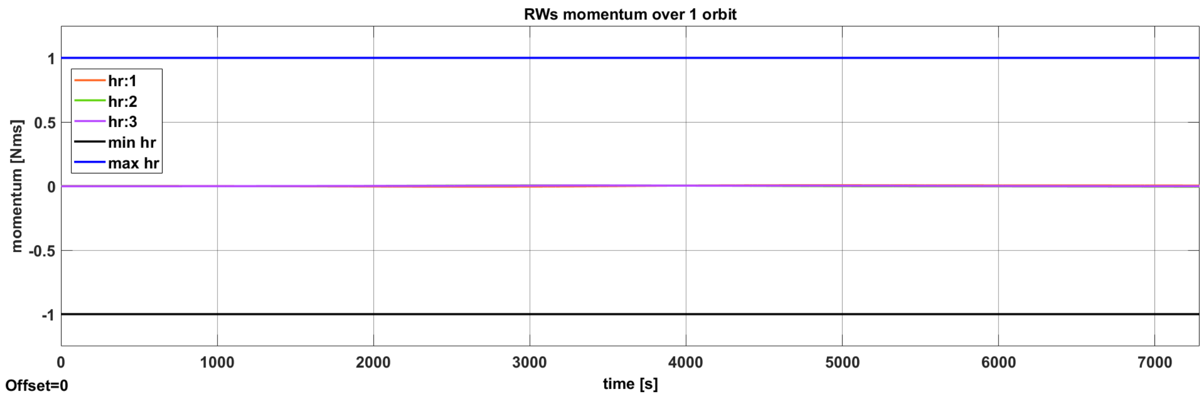
The orbital period is equal to T = 2*pi*sqrt(a^3/mu) = $7.2859*10^3$ s = 2.0239 h



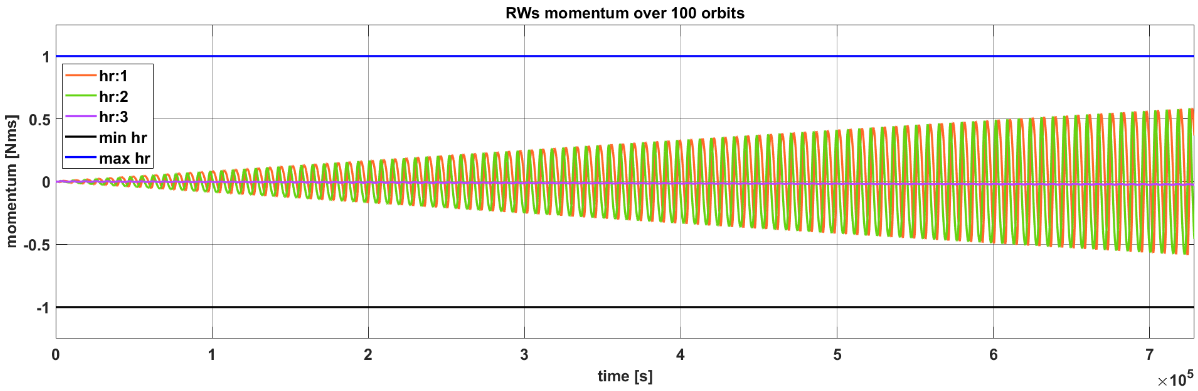
We can see that it's performing as expected by knowing that our s/c started the simulation at a true anomaly of pi, at the periapsis (where the rotational speed on the third axis would be the lowest) and gets its highest point exactly in the middle where the apoapsis is located.
All of this while responding to the external perturbations that afflict our s/c, as we can see by the varying h_tot.
But let's run another check. If the Earth pointing is successful, then the quaternion error (set at 0 at the beginning) should come back to its initial value at the end of the period. Where q_{ref} is the q0 at the beginning of the orbit/pointing (that coincides with the perifocal frame).



And it happens (because the q [0, 0, 0, 1] represents the same rotation as the q [0, 0, 0, -1]).
So we have confirmed that our Earth pointing system is working.
But how much are the reaction wheels working to do so? How much will it take to saturate them?



We can see that over a single orbit the RWs barely have to store any momentum.
But what happens when they are at their limit and can no longer properly work?



We have calculated that it will take approx 2 weeks to saturate at least one RW.

4. Desaturation

The desaturation maneuver is intended to keep the sat static (non rotating) while expelling excess momentum from the RW to get them again into a working position and away from the saturation.
We can do so by using both the magnetic torquers and the RWs at the same time, by creating a force with the MTs that will make the RWs react in a way that they have to start spinning and decreasing their speed to keep the s/c static.

We have create our very own control law for this, by imposing first the torque created by the magnetometers (ideally equal to the max one of the RW, realistically after the actuator block way lower) and then applying a simple PID over the angular speed error (with omega_ref = [0,0,0]).

In about an hour we are able to dump more or less 80% of the energy from the reaction wheels, while starting with all 3 of them fully saturated.

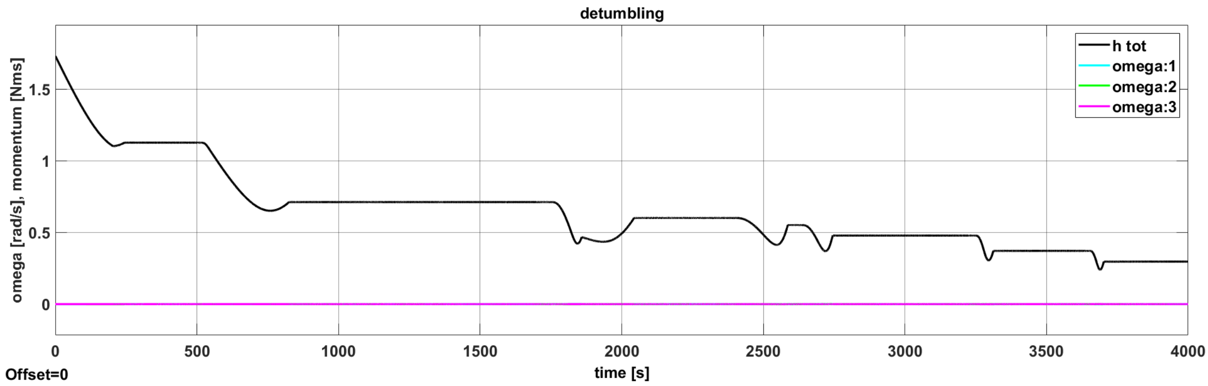
This is the control law:

$$\overline{T}_{MT} = \frac{1}{10} * \left(\overline{h}_{RW}\,\overline{\overline{I}}^{3x3}\right)^{-1} |\overline{h}_{RW}^{\,T}|$$

$$\overline{e}_{\omega} = \overline{\omega} - \overline{\omega}_{ref}$$

$$\overline{T}_{RW} = -kp * \overline{e}_{\omega}$$

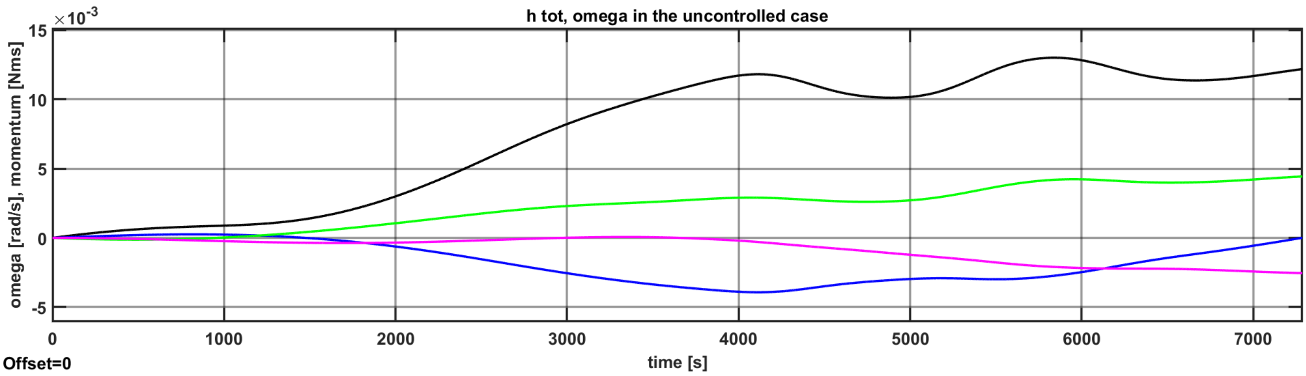
with kp again set to 20

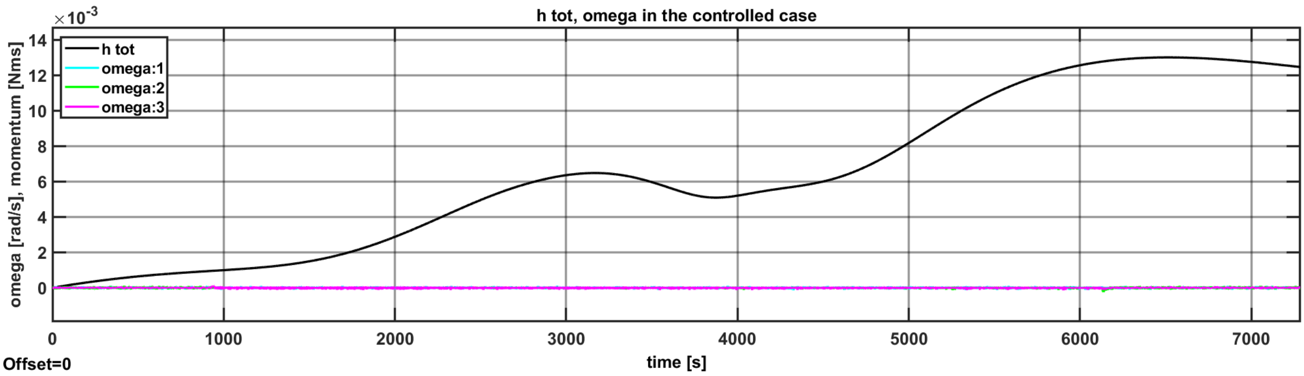


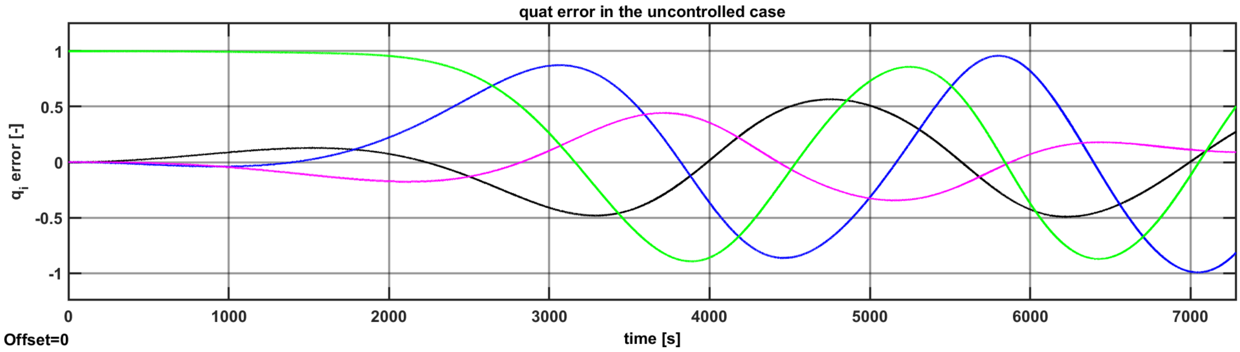
As we can see we are successfully keeping our s/c stable and non rotating while we are dumping energy from the reaction wheels. For this example the initial h_{RW} is [1, 1, -1] Nms.

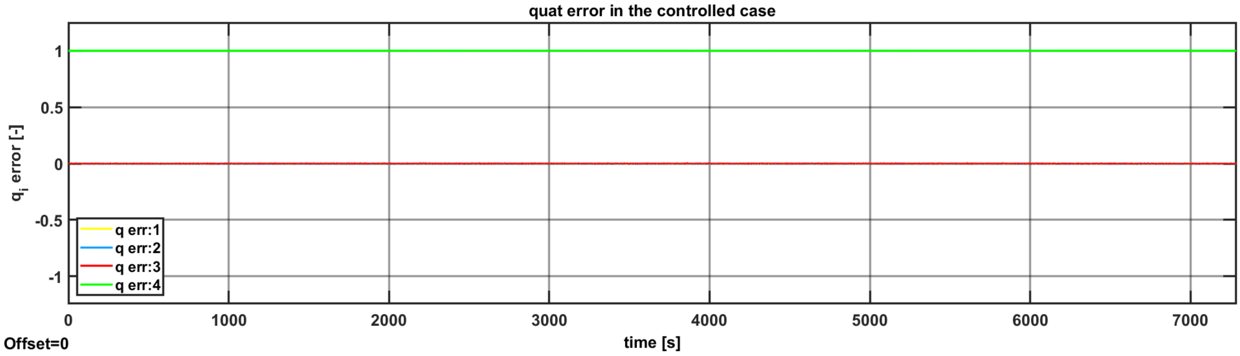
Statistical analysis on controlled vs uncontrolled inertial pointing

To show how a closed loop control system improves the pointing vs an uncontrolled system we'll point the s/c in a fixed direction (using the slew with the same initial condition as the reference). We will firstly observe how it behaves when controlled and then how it behaves when uncontrolled for an orbit (2 hours).
The pointing position will be q = [0 0 0 1]'









As we can see from the previous graphics in the uncontrolled case in not even under an hour the external torques start to heavily impact the attitude of our s/c. After an hour it's already completely out of control and not even remotely pointing in the right direction.
When we instead apply a control on the pointing we can see that the external torques still apply a disturbing element on our fixed inertial pointing, but our control system (and our RWs) are able to keep the s/c completely under control, stable, and always pointing in the wanted direction.

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