POLITECNICO MILANO 1863

DIPARTIMENTO DI
SCIENZE E TECNOLOGIE AEROSPAZIALI

Orbital mechanics project

Group 2146

Authors: *(name, surname, immatriculation #, person code)*

Boiroux Baptiste - 988958 - 10844243

Saison Quentin - 986119 - 10853302

Tognola Paolo - 911172 - 10618360

Eng. Zamblera Davide - 994071 - 10612897

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This paper contains a preliminary analysis of an interplanetary transfer with a singular flyby, the mission must preserve the following constraints:

Starting planet: Neptune

Flyby planet: Venus

Ending planet: Mercury

Time range of the mission: from 01/05/2026 to 01/05/2066

Radius of the fly-by: $r_{pmin} > radVen + altAtm \Leftrightarrow r_{pmin} > 6.4018e3\ km$

Preliminary Calculation

An initial estimation of time windows for the departure, flyby and arrival was obtained from:

1) a simplified Hohmann transfer time of flight calculation

During this calculation all the planets are assumed to have a circular orbit in the ecliptic plane with radius equal to the semi-major axis of the planet in orbit around the Sun.

2) synodic periods between the planets, to obtain an estimation of how the characteristic of the single transfer arc will repeat in time.

We also compute the deltaV required for an ideal Hohmann transfer from Neptune to Mercury and we will try to get close to it.

Simplified Hohmann transfers

To obtain TOF1 and TOF2 of the 2 interplanetary arc:

$$a_{t1} = (a_{Ven} + a_{Nep})/2$$

$$TOF1 = \pi\sqrt{\frac{a_{t1}^3}{muS}}$$

$$a_{t2} = (a_{Ven} + a_{Merc})/2$$

$$TOF2 = \pi\sqrt{\frac{a_{t2}^3}{muS}}$$

We get TOF1 = 1.1055e4 days and TOF2 = 75.55 days.

Calculation of Synodic periods

Knowledge of the synodic periods, which represent the minimum time when the relative position of 2 planets will be the same, can be quite useful to decide time windows and interpret data from porkchop plots. The calculation is the following:

$$T_{syn} = \frac{2\pi}{\left|n_{p1} - n_{p2}\right|}$$

We get T_{syn1} = 225.54 days and T_{syn2} = 144.56 days.

Calculation of the Hohmann deltaV

Here, the planets have circular orbit and are assumed to be in the same plane.

$$V_{Nep} = \sqrt{\frac{muS}{a_{Nep}}} = 5.4280\ km/s$$

$$V_{Merc} = \sqrt{\frac{muS}{a_{Merc}}} = 47.8721\ km/s$$

$$\Delta V = \sqrt{muS \times \left(\frac{2}{a_{Nep}} - \frac{1}{a_{trans}}\right)} = 0.8648\ km/s$$

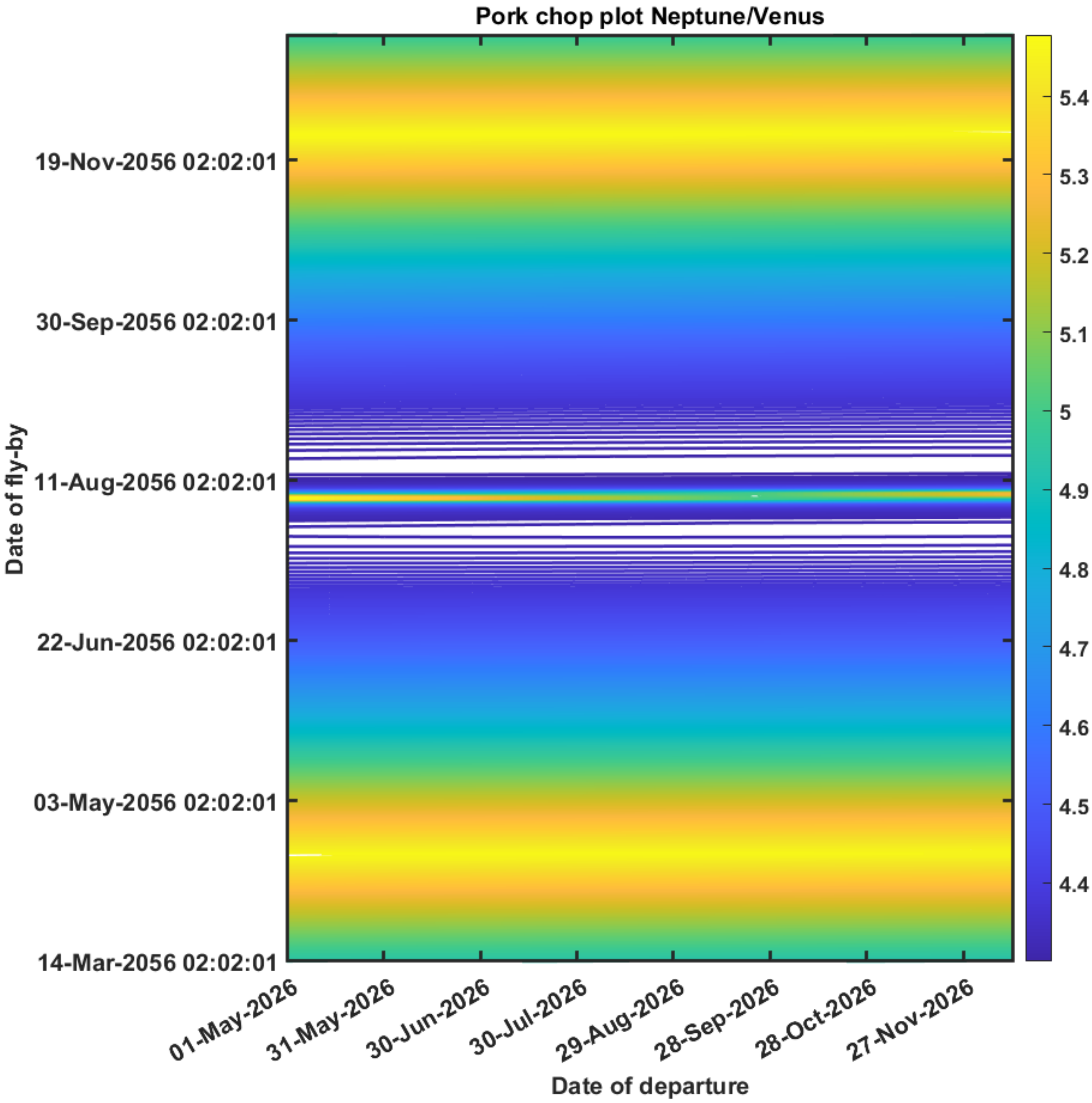
$$V_{arrMerc} = \sqrt{muS \times \left(\frac{2}{a_{Merc}} - \frac{1}{a_{trans}}\right)} = 67.2703\ km/s$$

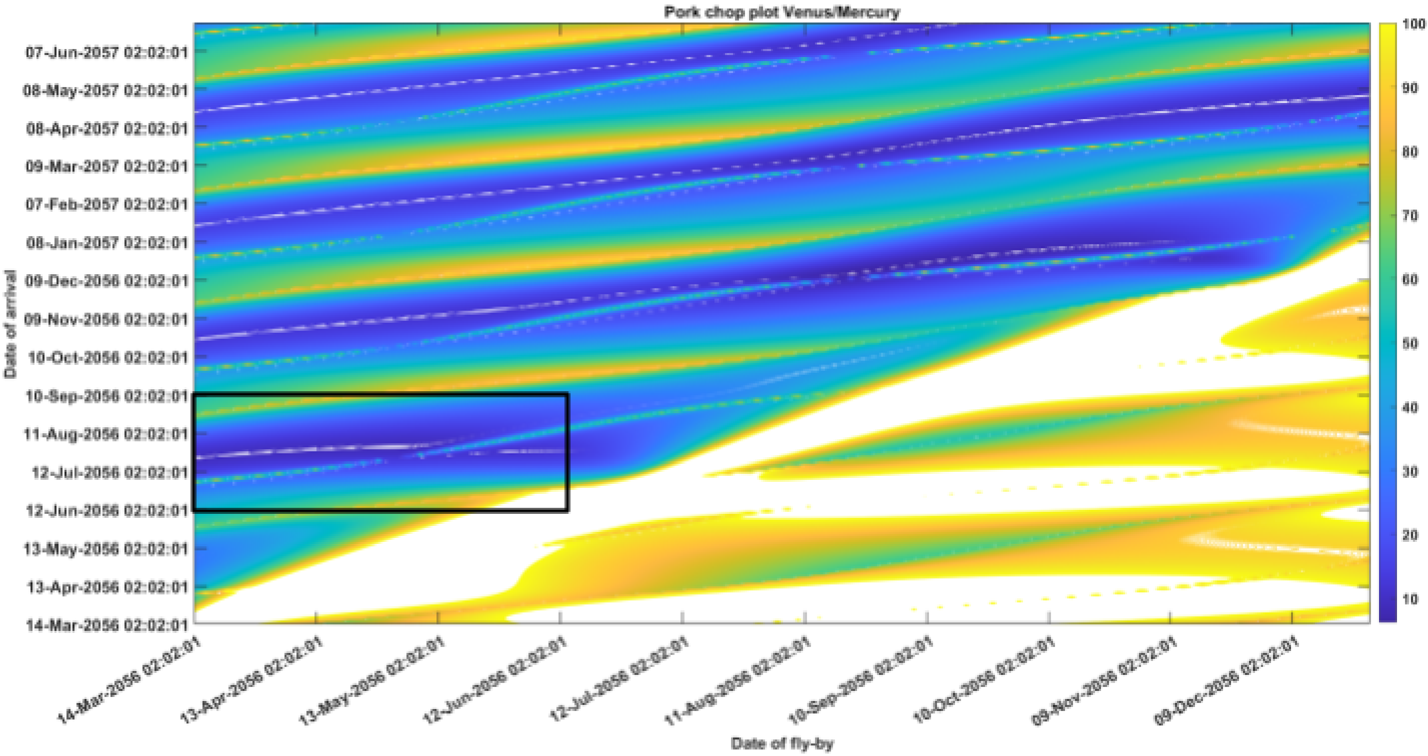
$$\Delta V_{hohm} = \left|V_{depVen} - V_{Nep}\right| + \left|V_{Merc} - V_{arrMerc}\right| = 23.9614\ km/s$$

We now have a target value of a good ΔV_{tot} for the mission we have to perform.

Choice of the time windows

To choose the time windows, we are using the following pork chop plots:





It can be seen that the ideal Δv_{dep} is obtained around the ideal TOF1 we calculated. Moreover, we can see that the departure time does not matter. Consequently, the departure window is chosen as a single point: the first day of the mission. This choice allows less computational time.

From the second pork chop plot, we can see some ideal windows of time which leads us to a minimal Δv_{arr}.

The windows are:

- departure window = 01/05/2026
- gravity assist window = 14/03/2056 to 11/06/2056
- arrival window = 12/06/2056 to 10/09/2056

First solution

The choice of these windows gives us a first look of the ΔV_{tot} needed for this mission:

Δv_{dep}	Δv_{ga}	Δv_{arr}	ΔV_{tot}
4.62 km/s	18.4km/s	12.57km/s	35.6082 km/s

The solution already seems good for the distance we have to cover. Nevertheless, Δv_{ga} seems too high as it represents 51% of the total dV while the aim of a gravity assist is to reduce the total ΔV of a mission.

Δv_{ga} is the ΔV that we input at the pericenter of the hyperbolic flyby.

Second choice of the windows

Consequently, we decided to create two new windows for fbWin and arrWin. For fbWin, we take a centered window around the ideal ToF1 of the first arc. Its length is 2 synodic periods Venus/Mercury. For arrWin we take a window from the ideal ToF2 for the second arc and we go till the end of the mission window to explore more solutions. The length of this window should not impact too much the duration of the computation as the small length of the departure is compensating. Moreover, to reduce the computation time, we first take 1 point per week.

Second solution

These windows lead to the result whose the characteristics are summed up in the following table:

Date of fly-by	Date of arrival	ΔV_{tot}	Δv_{ga}	1st arc TOF	2nd arc TOF
25/09/2056	11/09/2065	24.1107 km/s	0.0688 km/s	11,106 days	3,273 days

Refining the windows

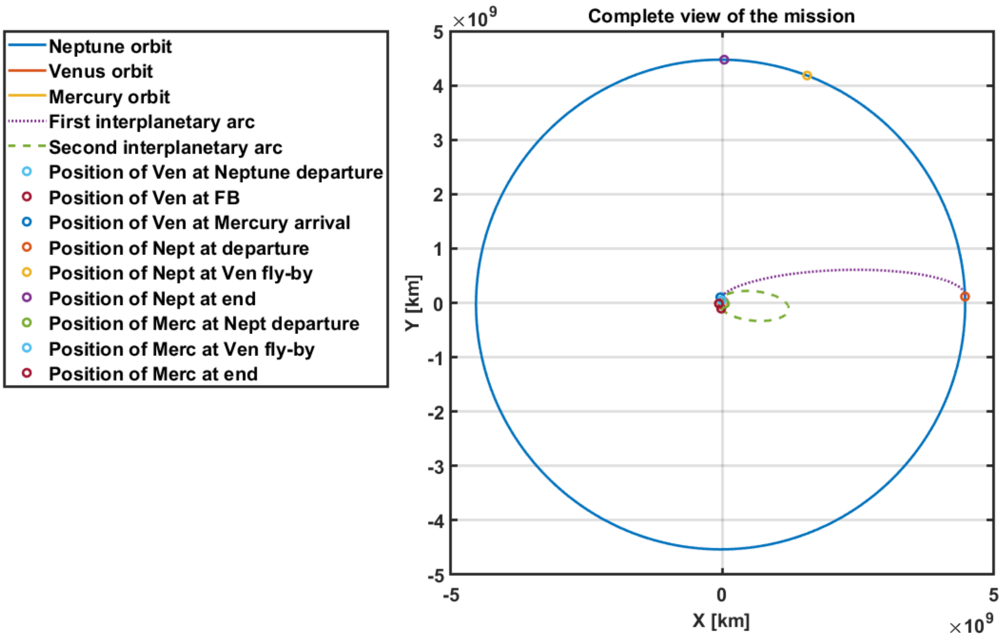
After getting these results, we decide to be more precise and get down to one point per hour around the 3 best days which were found in the previous calculations. The new windows are:

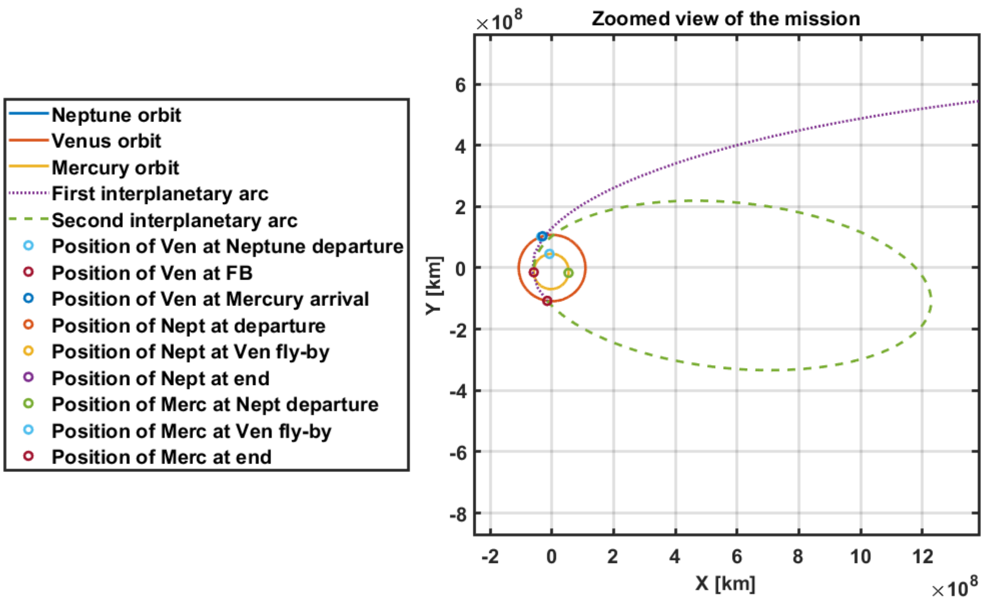
departure window = 1st day of mission

gravity assist window = 25/09/2056 +/- 3 days

arrival window =11/09/2065+/- 3 days

Final solution

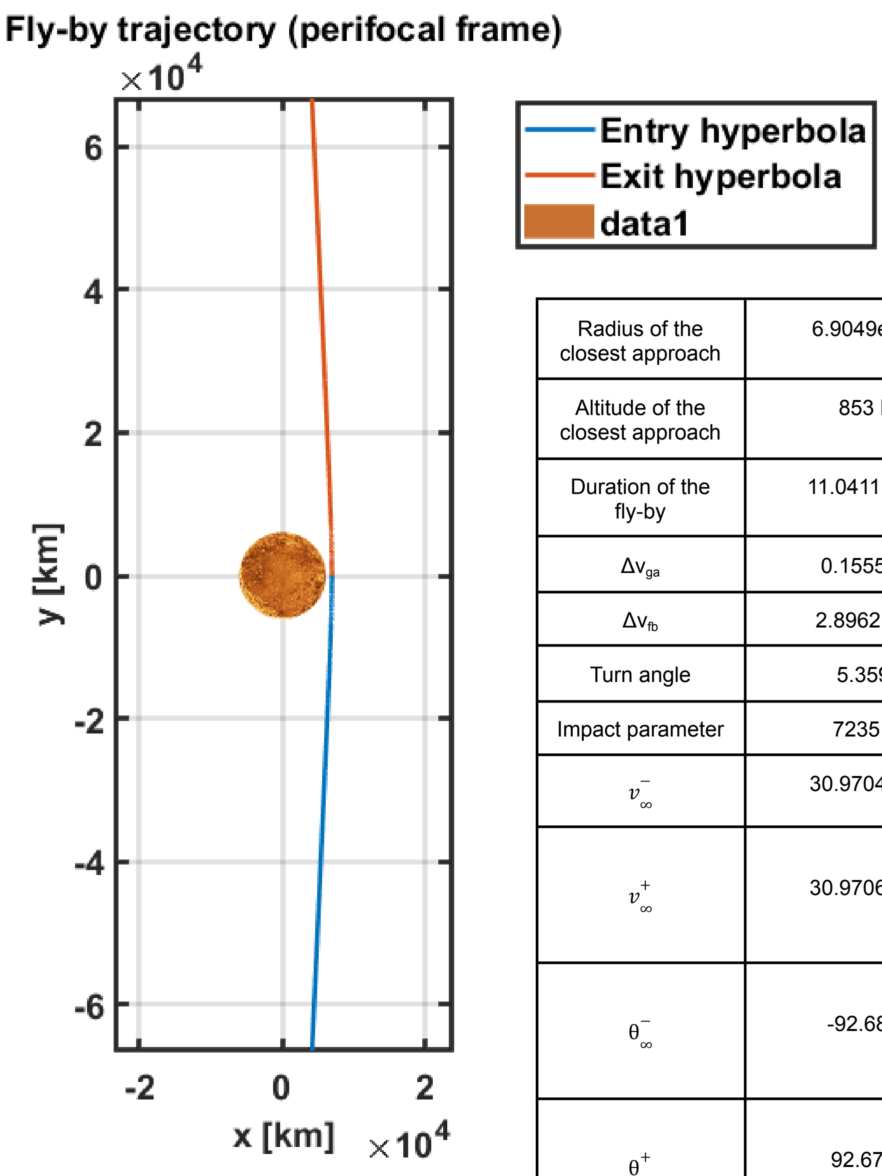




The refined windows lead to our final results which can be visualized on the following plots:

The characteristics of the interplanetary arcs:

	Arc 1	Arc 2
Semi-major axis	2.2713e9 km	6.4837e8 km
Eccentricity	0.9717	0.9088
Travel time	11,105 days	3,240 days



Radius of the closest approach	6.9049e3 km
Altitude of the closest approach	853 km
Duration of the fly-by	11.0411 hours
Δv_{ga}	0.1555 m/s
Δv_{fb}	2.8962 km/s
Turn angle	5.3599°
Impact parameter	7235 km
v_{∞}^{-}	30.9704 km/s
v_{∞}^{+}	30.9706 km/s
θ_{∞}^{-}	-92.6800°
θ_{∞}^{+}	92.6799°

This fly-by is much more interesting than the one given by the first windows. Indeed Δv_{ga} of this maneuver is now representing only 0.0054% of the whole Δv_{fb} of the gravity assist.

Characteristic of the mission:

Date of departure	Date of fly-by	Date of arrival	Total ΔV	Δv_{dep}	Δv_{ga}	Δv_{arr}
01/05/2026 00:00	24/09/2056 00:26 21 sec	23/11/2065 23:33 39 sec	23.9569 km/s	4.5501 km/s	0.1555 m/s	19.4067 km/s

This solution is far more interesting than the first one using the naive windows. Indeed, Δv_{ga} is no longer the bigger Δv of the mission and the objective of getting close to the ΔV_{hohm} is even exceeded.

SECOND TASK

Assignment Data

The Planetary Explorer Mission orbit has the following initial orbital elements:

a = 8123 km   e = 0.1789   i = 50.3529°   **Ω** = 180°   **ω** = 180°   **ϴ** = 0°

$T=2\pi\sqrt{\frac{a^3}{\mu}}$ = 7285.9s = 121.43min

The orbit has a radius of pericenter h_p = 298.75 km and it easily falls into the definition of LEO orbit, which can be defined as an orbit with period T<128min and eccentricity less than 0.25; the inclination doesn't characterize any standard orbit.

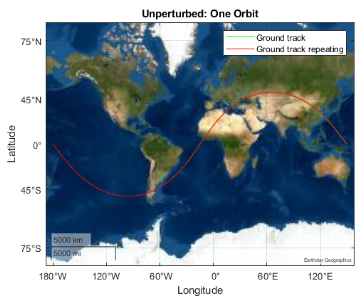
This satellite will complete 11.8256 orbits in a single day. The main perturbations acting on the satellite can be found in J2 and atmospheric drag.

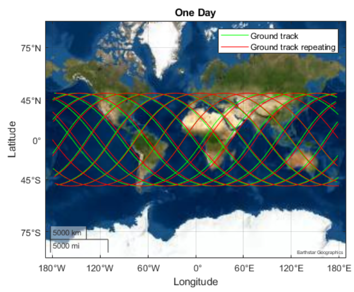
Ground Track

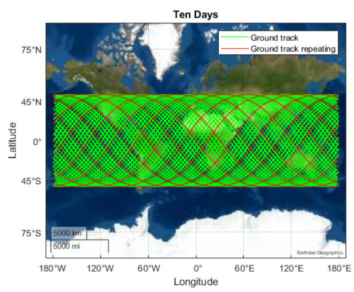
The assigned mission requires for the satellite to complete k=12 orbit as the Earth completes m=1 rotations around its axis. Then we can use the proportion of rotations (k,m) and the angular velocities (orbital and of the Earth rotation) to obtain the required orbital period of the new orbit, we found this to be:

$n_{new} = m/k * \omega_{Earth}$ $\quad T_{new} = 7180.9s$ =119.68min

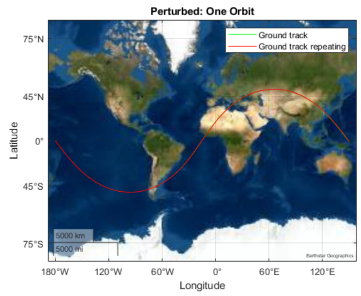
Then the semi-major axis of the new orbit will be a_{new} = 8044.7 km, by propagating the orbit via ode45 for the unperturbed two body problem, and by converting the coordinates into longitude λ and latitude ϕ, the plots of the ground track were created. This was done for the assigned orbit and for the repeating orbit, for both the evolution can be seen in graphs of 1 orbit (considering the different periods), 1 day and 10 days.

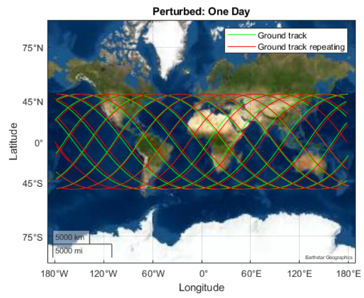


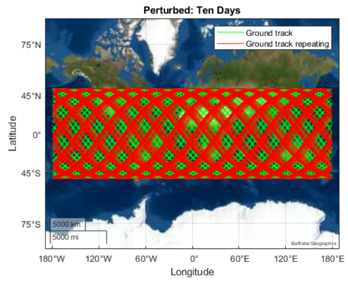




Adding perturbations to the 2 body problem generates changes in the orbital elements, this in turns means that the repetition of the orbit will not happen with a_{new} computed for the unperturbed problem. This is clearly visible in the following images.







Perturbed Two body problem

The knowledge of a satellite position at various instant of time in the future is of crucial importance for various types of operations, such as standard check and orbit control operations or flight safety and collision avoidance. For a satellite orbiting a planet this means solving a 2 body problem with superposed orbital perturbation, much smaller than the main spherically symmetric gravitational forces, which can be modeled as accelerations. For a satellite in LEO orbit the initial analysis can include J2 effect and atmospheric drag. We detail these forces and their effects in the next paragraphs.

The general integration for non-conservative perturbed problem can be done:

1) Adding the perturbed accelerations directly in the reduced 2 body problem and integrating numerically these equations, the results obtained will be vector r and v, if the purpose is to analyze the change in orbital elements then at every time a conversion will be operated (using the function car2kep)

$$r_{dotdot} = -\ \mu \frac{r_{vec}}{r3} + a_{per}$$

2) An alternative method is to directly integrate the Gauss form of the planetary equations, this will result in a time series of the orbital elements directly. With respect to the Lagrange form they maintain less insight in the problem considering the acceleration of the perturbation instead of a potential and its derivatives but in turn can be used also for non-conservative forces.

Perturbation due to J2 effect

The J2 disturbance, also called first zonal harmonics, is a conservative perturbation that arises from Earth oblateness, the radius of Earth in fact is greater at the equator, due to its spinning rotation. This causes the gravity field to deviate from the spherical symmetry. Other deviations from the spherical symmetry are much smaller than J2 and usually are not considered in initial analysis.

The perturbation secular effects on a,e,i are null, what is instead greatly affected on the long term is the variation on OM, om and M. In particular the effect on OM is called nodal regression and it's stronger the smaller the orbit. The argument of perigee instead has a drift that doesn't manifest only in the case for which i=63.4°.

Perturbation due to atmospheric drag

The atmospheric drag can be modeled using the following formula:

$$\bar{a}_{Drag} = -\frac{1}{2}\frac{A_{Cross}c_D}{m}\rho(h,t)v_{rel}^2\frac{\bar{v}_{rel}}{\left\|\bar{v}_{rel}\right\|}$$

The velocity at which we are referring is the relative velocity with respect to the atmosphere, which is turning because of Earth rotation. In the formula it can be identified a ballistic coefficient as A*Cd/M which needs to be determined sperimentally, for this work it is assumed with: A/M=0.0171m^2/kg and Cd=2.1. Lastly it's important to highlight that most of the uncertainty comes from the difficulty in modeling the atmosphere density rho, the model used in this paper consists in an exponential interpolatory model with data which uses US Standard Atmosphere (1976) for 0 km, CIRA-72 for 25-500 km, and CIRA-72 with Tinf = 1000 K for 500-1000 km. For more advanced uses, models like Jacchia-Bowman 2008 could be used to obtain more precise, date dependent densities.

The main effect of this perturbation is to subtract energy to the orbit as energy ε is related to the semi-major axis via $\varepsilon = \frac{-\mu}{2a}$, then this means the perturbation will determine a diminishing of *a*. As the action is greater closer the spacecraft is to the planet because the density is higher, the effect can be considered concentrated on the pericenter, this means that also the eccentricity *e* will decrease. A secular variation of *i* can be attributed to a small out of plane component of the drag. All the other effects on the remaining parameters are only long period variations

Evolution of the Keplerian elements

The perturbed model has then been propagated for 100 initial periods (periods related to the first value of *a*), this time of propagation provides a clear visualization of both the local effects and the secular effects of the perturbation. The elements then are compared between the two methods and are analyzed through the theoretical knowledge of J2 and drag effects.

From a computational point of view, for an initial implementation without the moving atmosphere, it can be appreciated that the Gaussian propagation is much faster, and provides directly the orbital elements. Adding the moving atmosphere requires computing r,v from the keplerian elements, this slows down the process to similar values of cartesian propagation and conversion.

Cartesian propagation: Elapsed time is 6.053138 seconds.

Gaussian propagation: Elapsed time is 3.842621 seconds.

Conversion from cartesian to keplerian: Elapsed time is 2.012784 seconds.

With moving atmosphere:

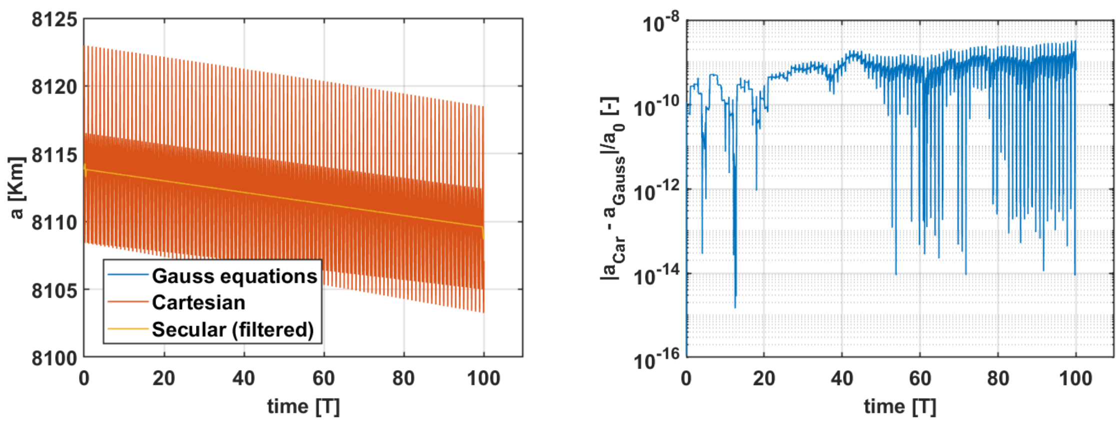
Cartesian propagation: Elapsed time is 6.115574 seconds.

Gaussian propagation: Elapsed time is 9.301830 seconds.

Conversion from cartesian to keplerian: Elapsed time is 2.004369 seconds.

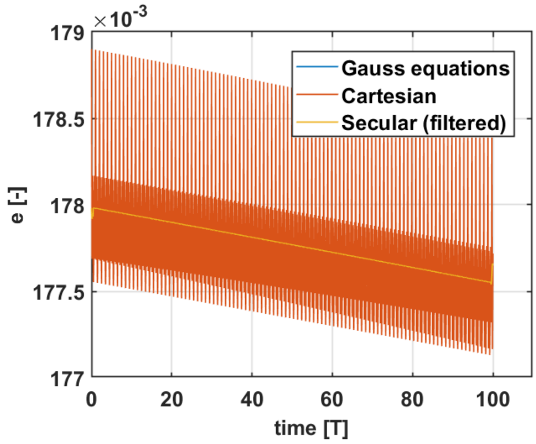
Numerical propagation of semi-major axis a

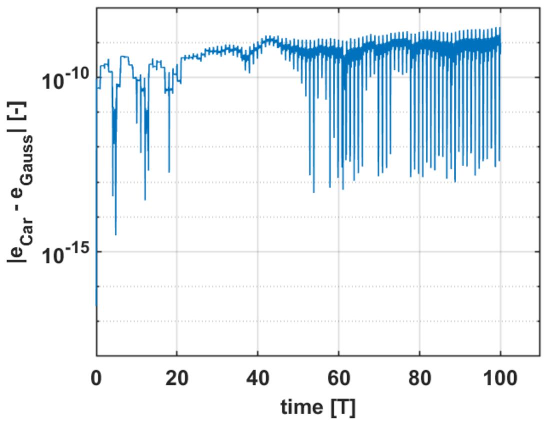
The semi-major axis exhibits a secular linear regression effect, this is due to the atmospheric drag, on top of this there are oscillations from period to period due to the J2 perturbation.



Numerical propagation of eccentricity e

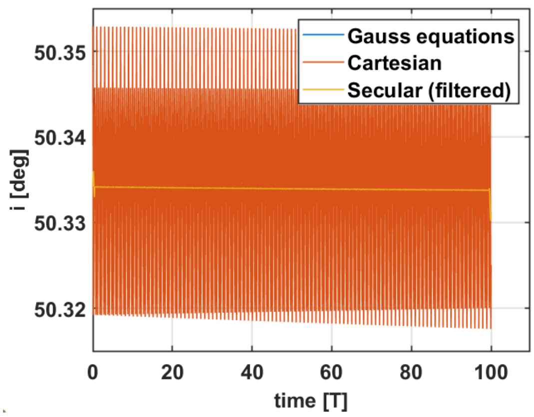
The eccentricity also exhibits a secular regression effect from the drag, on top of that it shows the typical oscillations of J2.

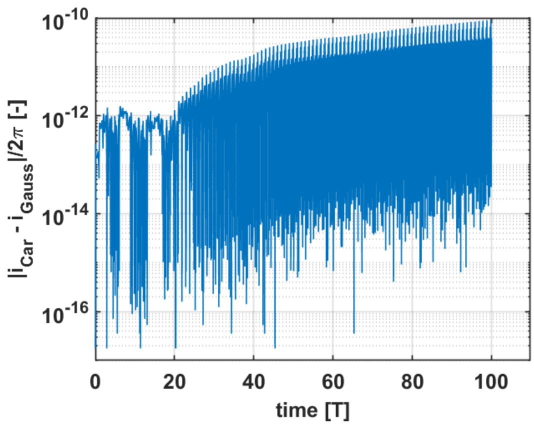




Numerical propagation of inclination i

While for *i* the periodic oscillation due to J2 can still be seen, secularly it varies very slowly and only due to atmospheric drag which is considered with a small out of plane component.



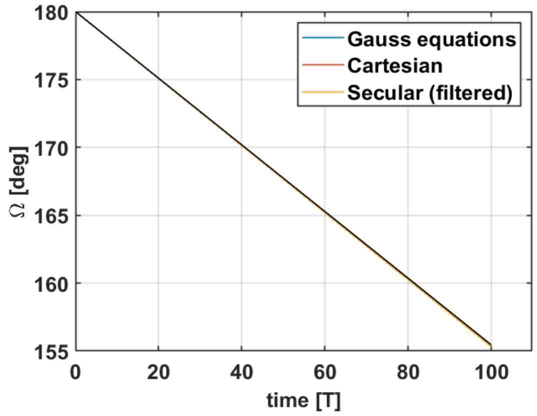


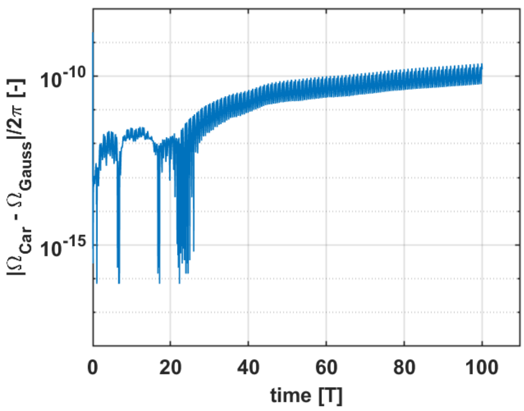
Numerical propagation of RAAN Ω

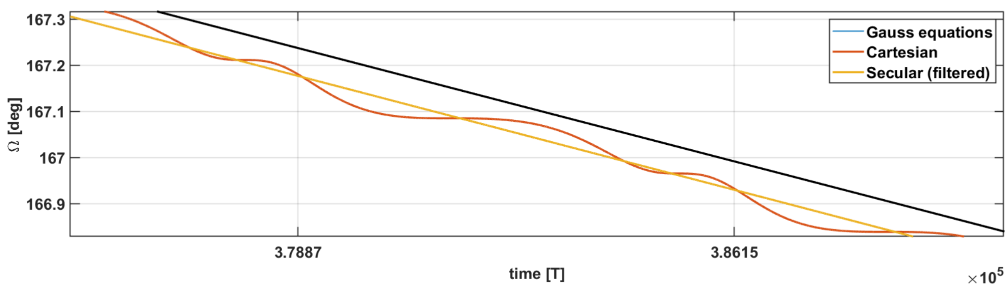
The J2 effect tends to shift this parameter secularly, as in 100 orbits the element diminishes linearly by nearly 25°. A long period vibration can also be highlighted, this is due also to drag because of the small out of plane component.

It can be seen that the secular J2 perturbation line follows the following analytical solution:

$$\Omega_{SEC}^{dot} =- \frac{3nR_{Earth}^{2}J_{2}}{2p^{2}} cos(i) =- 5.8794 \cdot 10^{-7} rad/s$$







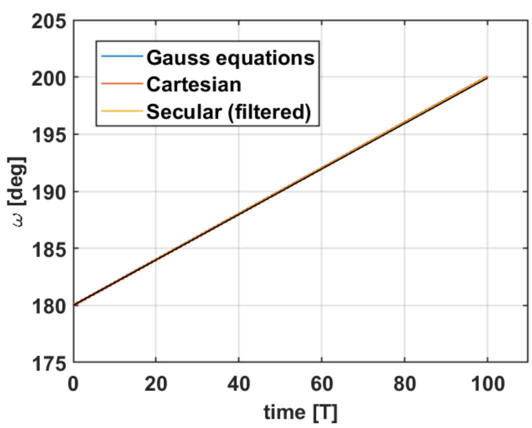
zoom over 1 period T, the black line is the analytical formula

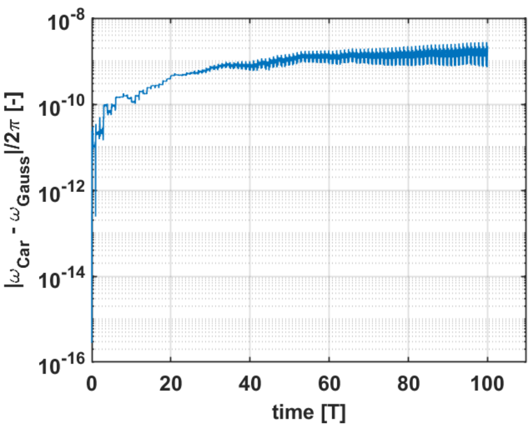
Numerical propagation of argument of pericenter ω

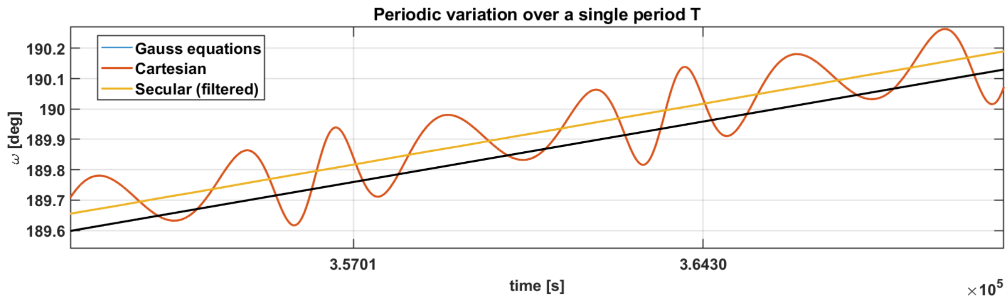
The secular shifting of the argument of pericenter of 20° is due to the J2 effect, the long period effect is due to both perturbations.

It can be seen that the secular J2 perturbation line follows the following analytical

solution:



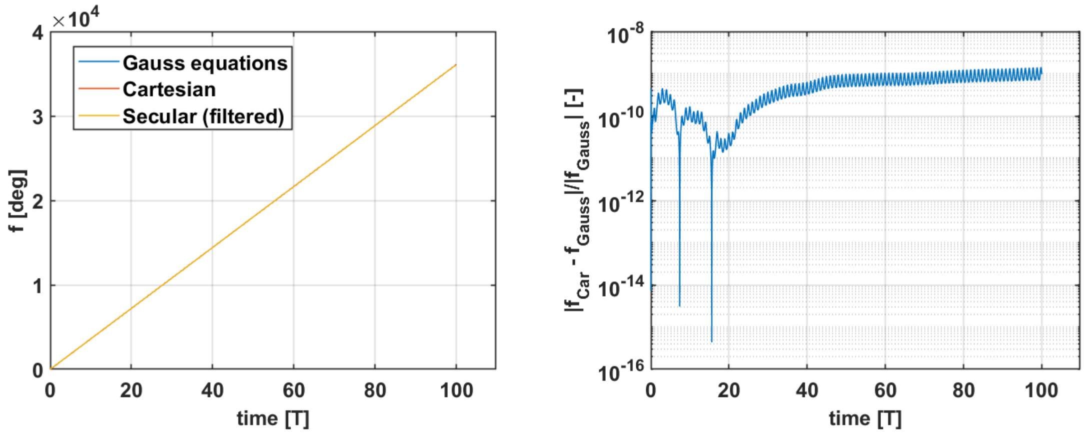


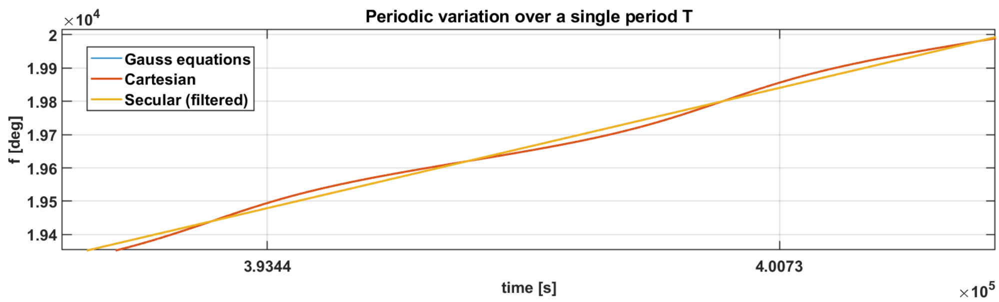


the black line is the analytical formula

Numerical propagation of true anomaly θ

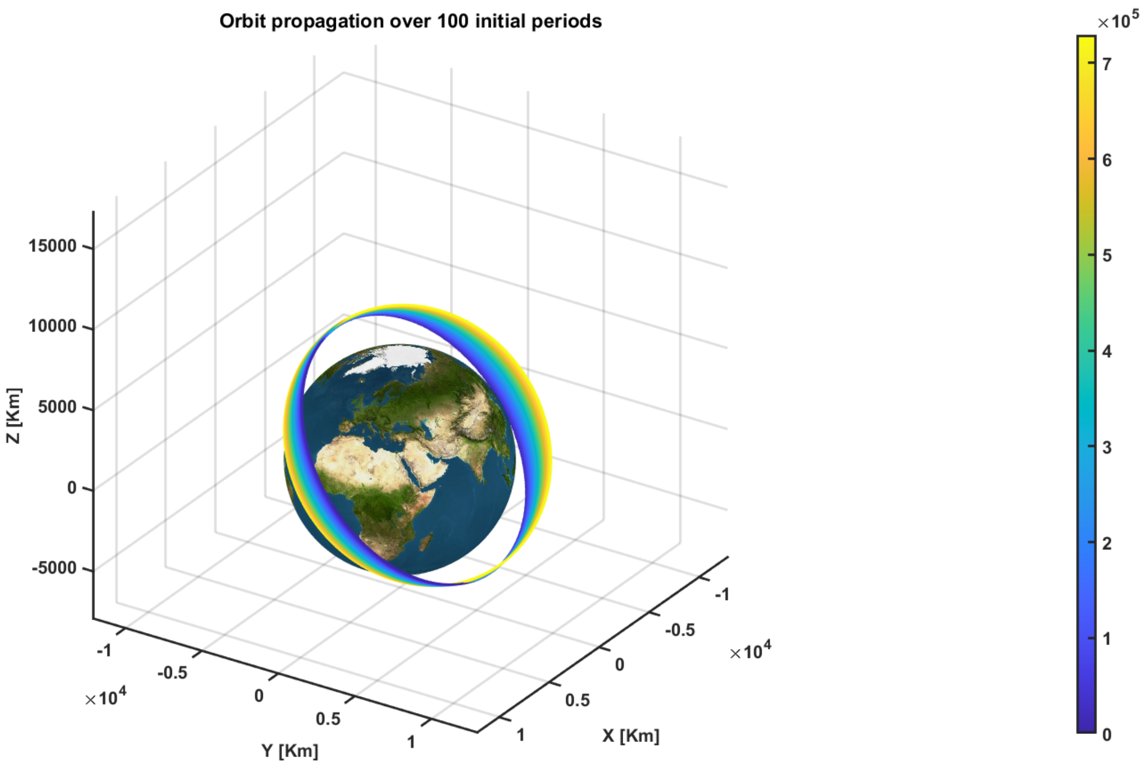
This is the only element that naturally varies in the unperturbed two body problem, still a long period and secular perturbation due to J2 can be seen.





Evolution of the orbit

A representation of the orbit for the simulated time shows the main long term effects of the perturbations, a,e and ω changes and more clearly the shift of Ω. The graph is then colored by a colormap that starts from cold colors to warm colors as the time of simulation advances.

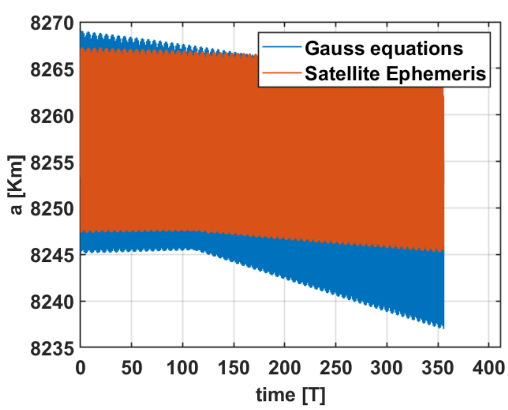


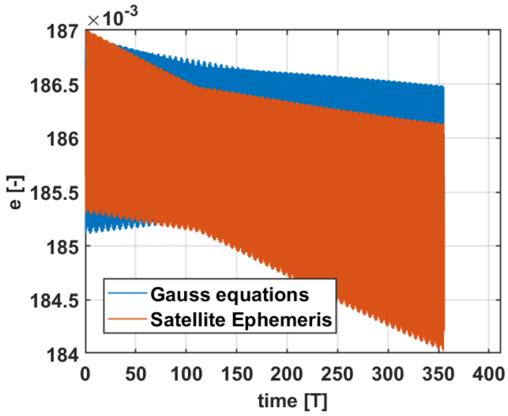
High Frequency Filters

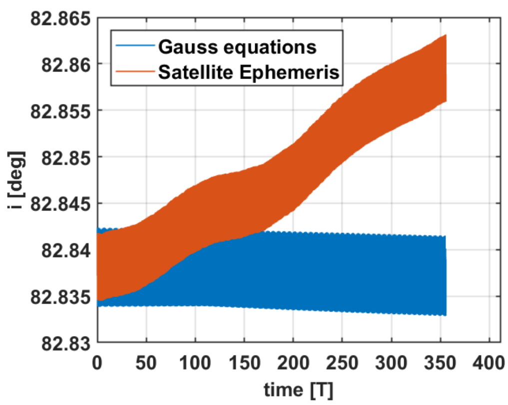
To observe the main secular effects on the various elements simple moving mean filters were used to cancel out the long term period oscillations. This was accomplished by using the movmean function available in Matlab, considering a window of a period, and the discontinuities at the extremes were partially solved using the parameter ‘shrink’ to let the filter use useful data even if a complete window cannot be seen by it. The results of the filter are reported in the plots regarding the evolution of the keplerian elements

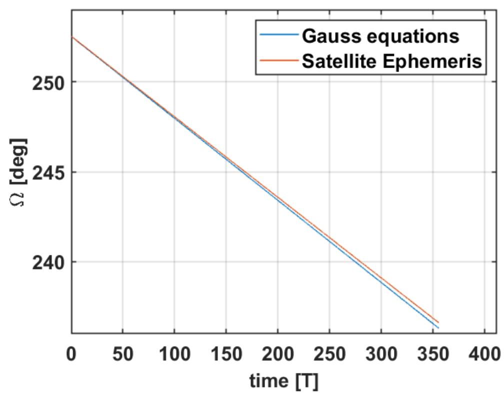
Comparison with Real Data

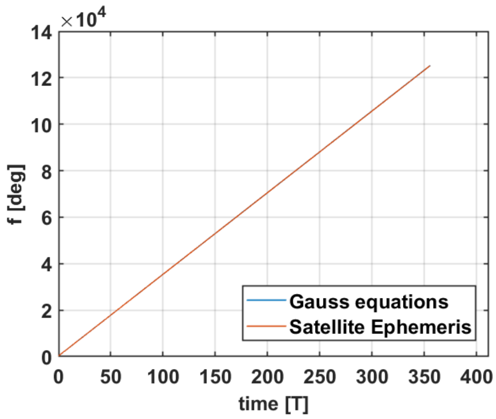
The orbital perturbation model built is then confronted with a series of observations from a real satellite in LEO. For the characteristic of the assigned orbit the best comparison was found from the Space-Track catalogue searching via orbital period and apogee-perigee, the chosen real satellite is FAST (NORAD 24285). Its orbit is similar to the assigned one except for inclination which is near polar instead of near 50°, the measured TLE elements obtained from 2021-10-01 to 2021-10-31 where then inputted in NASA JPL horizon system to find the orbital elements for 10000 equal intervals, using a ICRF reference frame with x-y axes equatorial-aligned. The elements at the first available time were then integrated using the perturbation model consisting of J2 and atmospheric drag. The confront between the model and the real observation is then reported in the following graphs:

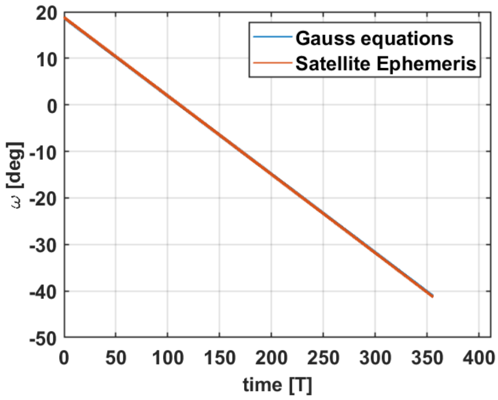












From the results it can be seen that the model follows the general shape of the real orbital elements, the relative error is still not trascurable as it reaches a 10^{-3} order of magnitude on many of the orbital elements, but the model could be used for some preliminary analysis. In particular the inclination exhibits a secular increment that our model is not capable of representing. The model has also a secular slope of Ω which is more inclined than the one of the observation.