Hybrid Image Creation and Fourier Property Verification

Abstract

This report presents the creation of a hybrid image using the 2D Discrete Fourier Transform (DFT) by fusing low-frequency content from one image and high-frequency content from another. The project also includes experimental verification of fundamental Fourier Transform properties such as rotation, translation, and flipping. These experiments confirm the behavior of the Fourier domain under basic spatial transformations using visual inspection of magnitude spectra.

1 Introduction

The 2D Fourier Transform is a powerful tool for frequency analysis and filtering in image processing. In this project, it is used to create a hybrid image by combining low-frequency information from a cat image and high-frequency information from a dog image. Additionally, the properties of the 2D DFT are experimentally verified through spatial transformations applied to the source images and inspection of their corresponding FFT magnitude spectra.

2 Theory: 2D Discrete Fourier Transform

The 2D DFT and its inverse are defined as:

$$F(u,v) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y) \cdot e^{-j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

$$f(x,y) = \frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v) \cdot e^{j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

where $f(x,y)$ is the spatial domain image and $F(u,v)$ is its frequency domain representation.

3 Hybrid Image Construction

To create the hybrid image:

1. The cat image was blurred using a Gaussian filter (large sigma) to retain only low-frequency components.

2. The dog image was high-pass filtered by subtracting a blurred version (small sigma) from the original.
3. The FFTs of both filtered images were computed, and frequency-domain masks were used to extract desired components.
4. A low-pass mask was applied to the cat image and a band-pass mask to the dog image.
5. The two filtered FFTs were added and inverse transformed to get the hybrid image.

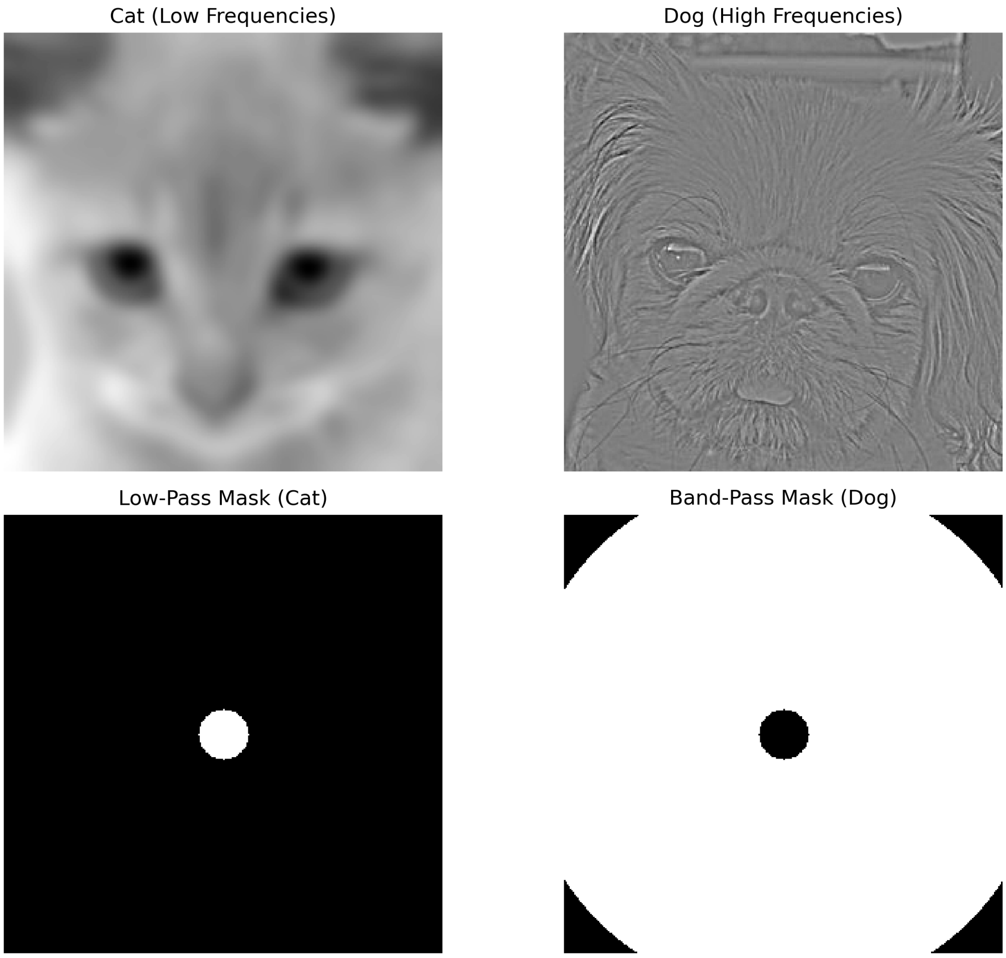

Figure 1: Pre-processed components for hybrid image generation. Top: Low-pass filtered cat and high-pass filtered dog images. Bottom: Corresponding frequency-domain masks.

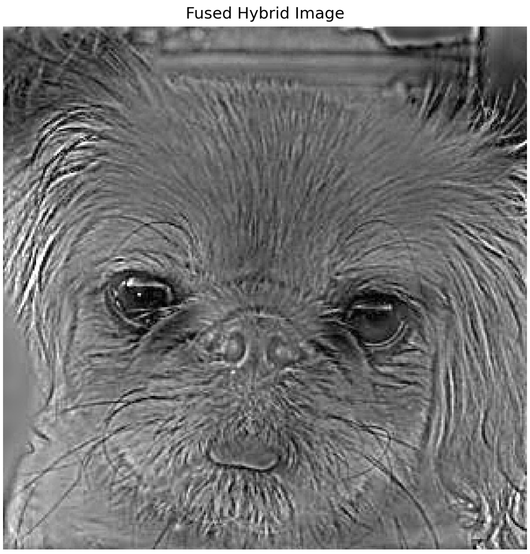

Figure 2: Final fused hybrid image. From a distance, the cat (low frequencies) dominates perception; up close, the dog (high frequencies) becomes more visible.

Observation: When viewed from afar, the hybrid resembles the cat (low frequencies). When viewed closely, the details of the dog (high frequencies) dominate — validating successful hybrid construction.

4 Verification of Fourier Transform Properties

4.1 Experiment 1: Rotation

The low-frequency cat image was rotated by 90°, and its FFT magnitude spectrum was plotted.

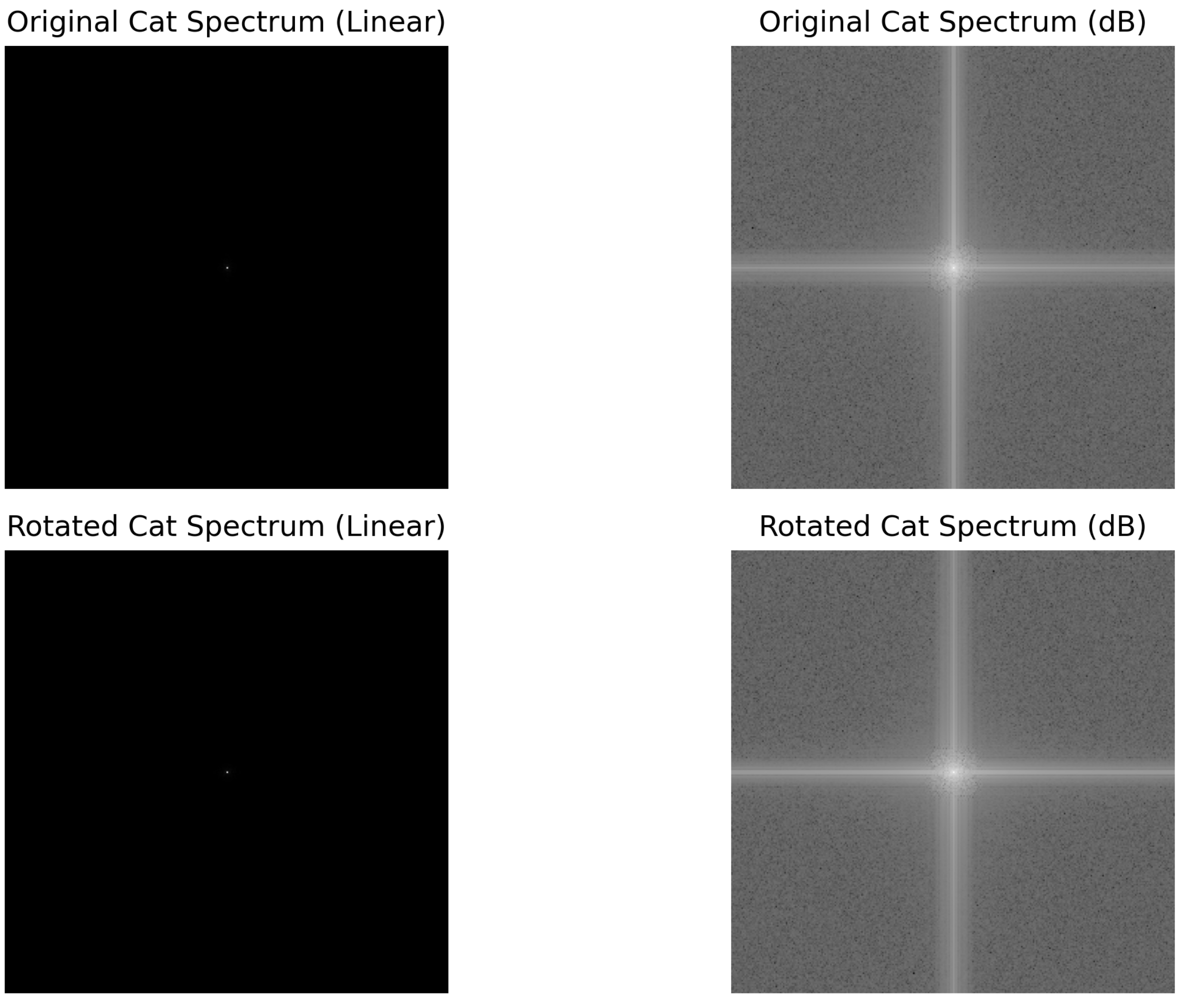

Figure 3: FFT magnitude of original (top) and 90° rotated cat image (bottom)

Inference: The FFT spectrum of the rotated image is also rotated by 90°. **Conclusion:** Rotation in the spatial domain causes an equivalent rotation in the frequency domain.

4.2 Experiment 2: Translation

The low-frequency cat image was translated (shifted) by 30 pixels down and 50 pixels right.

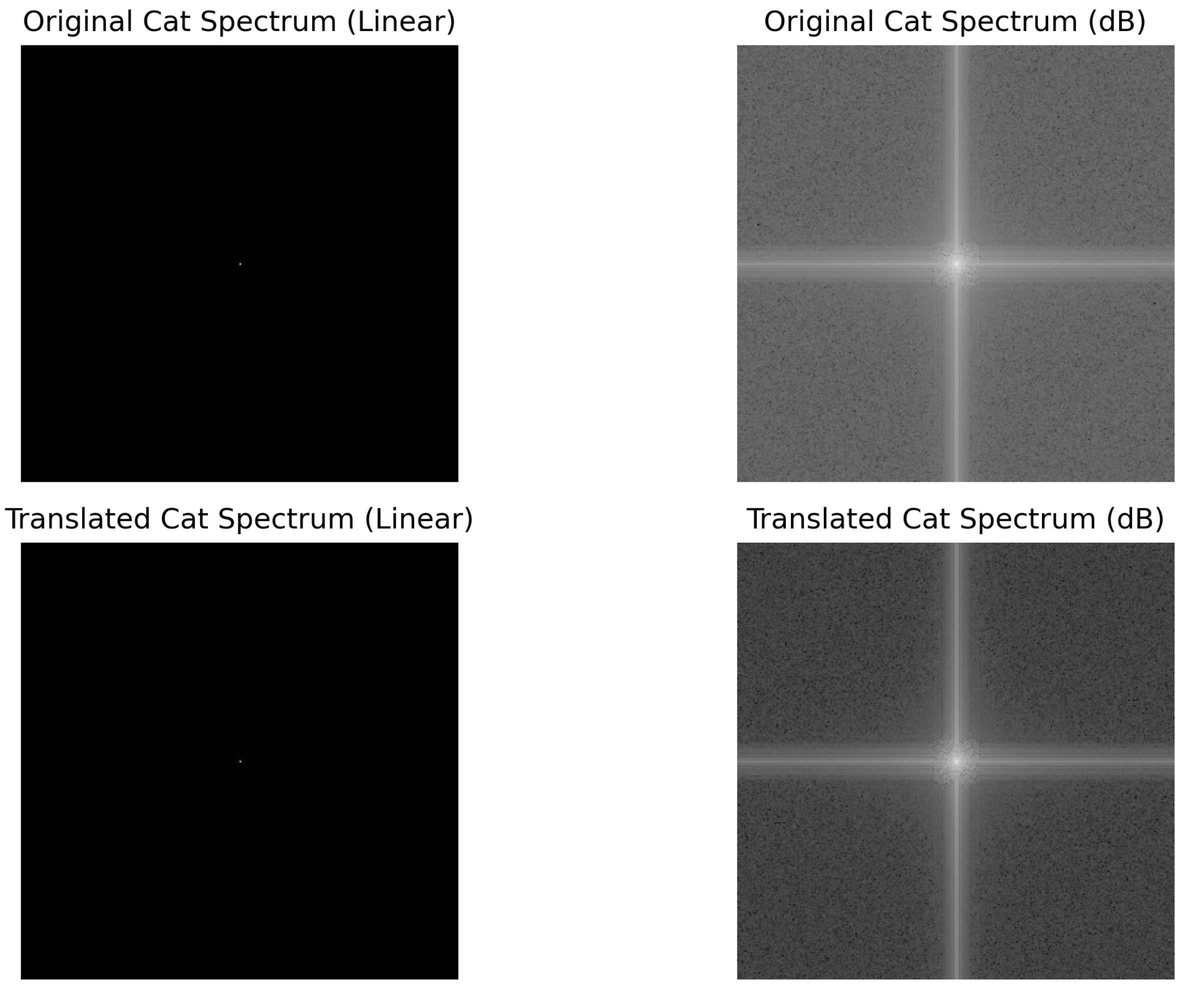

Figure 4: FFT magnitude of original (top) and translated cat image (bottom)

Inference: There is no visible change in the FFT magnitude after translation. **Conclusion:** Spatial translation results in a phase shift in the frequency domain but leaves the magnitude unchanged.

4.3 Experiment 3: Flipping

The low-frequency cat image was flipped horizontally.

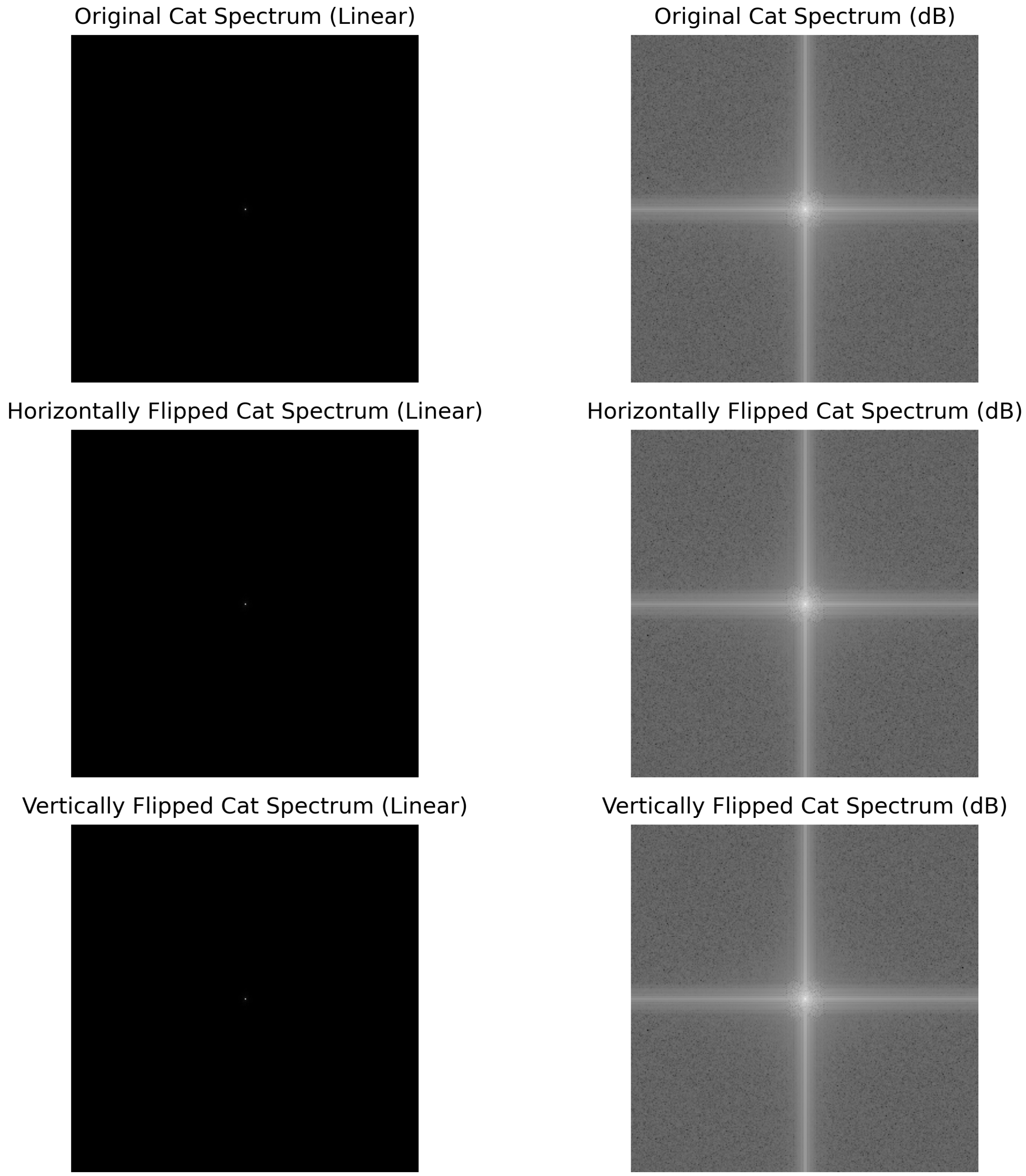

Figure 5: FFT magnitude of original (top) and horizontally flipped cat image (bottom)

Inference: The FFT spectrum is mirrored. Flipping an image mirrors its frequency spectrum along the same axis.

5 Conclusion

This project successfully demonstrates the creation of a hybrid image using 2D Fourier Transform techniques. The frequency content of two images was separated and combined to produce a perceptually dependent hybrid. Furthermore, the experiments verified key Fourier properties:

- **Rotation**: Spatial rotation results in frequency rotation.
- **Translation**: Translation affects phase, not magnitude.
- **Flipping**: Flipping mirrors the spectrum accordingly.

These results confirm theoretical properties of the 2D Fourier Transform and show their practical significance in image fusion and analysis.