Q9

October 26, 2024

[118]:
```
import sympy as sym
from IPython.display import display, Math, Latex
```

[119]:
```
r,theta,z = sym.symbols('r,theta,z')
```

[120]:
```
#sigma_rr,sigma_tt,sigma_zz,E,nu,epsilon_rr,epsilon_tt,epsilon_zz = sym.
↪Funtion('sigma_rr,sigma_tt,sigma_zz,E,nu,epsilon_rr,epsilon_tt,epsilon_zz')(r)
sigma_rr = sym.Function('sigma_rr')(r)
sigma_tt = sym.Function('sigma_tt')(r)
sigma_zz = sym.Function('sigma_zz')(r)
epsilon_rr = sym.Function('epsilon_rr')(r)
epsilon_tt = sym.Function('epsilon_tt')(r)
epsilon_zz = sym.Function('epsilon_zz')(r)
E,nu = sym.symbols('E,nu')
```

[121]:
```
eq1 = sym.Eq(epsilon_rr, 1/E*(sigma_rr - nu*(sigma_tt + sigma_zz)))
eq2 = sym.Eq(epsilon_tt, 1/E*(sigma_tt - nu*(sigma_rr + sigma_zz)))
eq3 = sym.Eq(0, 1/E*(sigma_zz - nu*(sigma_rr + sigma_tt)))
display(eq1,eq2,eq3)
```

$$\epsilon_{rr}(r) = \frac{-\nu\left(\sigma_{tt}(r) + \sigma_{zz}(r)\right) + \sigma_{rr}(r)}{E}$$

$$\epsilon_{tt}(r) = \frac{-\nu\left(\sigma_{rr}(r) + \sigma_{zz}(r)\right) + \sigma_{tt}(r)}{E}$$

$$0 = \frac{-\nu\left(\sigma_{rr}(r) + \sigma_{tt}(r)\right) + \sigma_{zz}(r)}{E}$$

[122]:
```
soln = sym.solve([eq1, eq2, eq3], [sigma_zz, sigma_rr, sigma_tt])
soln = sym.factor(soln)
display(soln)
```

$$\left\{\sigma_{rr}(r) : \frac{E\left(\nu\epsilon_{rr}(r) - \nu\epsilon_{tt}(r) - \epsilon_{rr}(r)\right)}{(\nu+1)(2\nu-1)},\ \sigma_{tt}(r) : -\frac{E\left(\nu\epsilon_{rr}(r) - \nu\epsilon_{tt}(r) + \epsilon_{tt}(r)\right)}{(\nu+1)(2\nu-1)},\ \sigma_{zz}(r) : -\frac{E\nu\left(\epsilon_{rr}(r) + \epsilon_{tt}(r)\right)}{(\nu+1)(2\nu-1)}\right\}$$

[123]:
```
u = sym.Function('u')(r)
sym.diff(u,r)
```

[123]:
$$\frac{d}{dr}u(r)$$

[124]:
```
eq4 = sym.Eq(epsilon_rr, sym.diff(u,r))
eq5 = sym.Eq(epsilon_tt, u/r)
soln2 = soln.subs({epsilon_rr: sym.diff(u,r), epsilon_tt: u/r})
sigma_rr = soln2[sigma_rr]
sigma_tt = soln2[sigma_tt]
sigma_zz = soln2[sigma_zz]
display(sigma_rr,sigma_tt,sigma_zz)
```

$$\frac{E\left(\nu\frac{d}{dr}u(r)-\frac{\nu u(r)}{r}-\frac{d}{dr}u(r)\right)}{(\nu+1)(2\nu-1)}$$

$$-\frac{E\left(\nu\frac{d}{dr}u(r)-\frac{\nu u(r)}{r}+\frac{u(r)}{r}\right)}{(\nu+1)(2\nu-1)}$$

$$-\frac{E\nu\left(\frac{d}{dr}u(r)+\frac{u(r)}{r}\right)}{(\nu+1)(2\nu-1)}$$

[125]:
```
cond = (sym.diff(sigma_rr,r) + (sigma_rr - sigma_tt)/r).simplify()
eq6 = sym.Eq(cond,0)
display(eq6)
```

$$\frac{E\left(-\nu u(r)+r^2(\nu-1)\frac{d^2}{dr^2}u(r)-r(1-\nu)\frac{d}{dr}u(r)+u(r)\right)}{r^2(\nu+1)(2\nu-1)}=0$$

[126]:
```
soln3 = sym.dsolve(eq6,u)
display(soln3)
```

$$u(r)=\frac{C_1}{r}+C_2 r$$

[127]:
```
C1,C2,C3,C4 = sym.symbols('C1,C2,C3,C4')
u_soln1 = C1/r + C2*r
u_soln2 = C3/r + C4*r
E_1,E_2,nu_1,nu_2 = sym.symbols('E_1,E_2,nu_1,nu_2')
sigma_rr1 = sigma_rr.subs([[E,E_1],[nu,nu_1]])
sigma_tt1 = sigma_tt.subs([[E,E_1],[nu,nu_1]])
sigma_zz1 = sigma_zz.subs([[E,E_1],[nu,nu_1]])
sigma_rr2 = sigma_rr.subs([[E,E_2],[nu,nu_2]])
sigma_tt2 = sigma_tt.subs([[E,E_2],[nu,nu_2]])
sigma_zz2 = sigma_zz.subs([[E,E_2],[nu,nu_2]])
```

[128]:
```
sigma_rr1 = sym.factor(sigma_rr1.replace(u,u_soln1).doit().simplify())
sigma_tt1 = sym.factor(sigma_tt1.replace(u,u_soln1).doit().simplify())
sigma_zz1 = sym.factor(sigma_zz1.replace(u,u_soln1).doit().simplify())
sigma_rr2 = sym.factor(sigma_rr2.replace(u,u_soln2).doit().simplify())
sigma_tt2 = sym.factor(sigma_tt2.replace(u,u_soln2).doit().simplify())
sigma_zz2 = sym.factor(sigma_zz2.replace(u,u_soln2).doit().simplify())
```

```
display(sigma_rr1,sigma_tt1,sigma_zz1,sigma_rr2,sigma_tt2,sigma_zz2)
```

$$-\frac{E_1 \cdot (2C_1\nu_1 - C_1 + C_2r^2)}{r^2\,(\nu_1+1)\,(2\nu_1-1)}$$

$$-\frac{E_1\,(-2C_1\nu_1 + C_1 + C_2r^2)}{r^2\,(\nu_1+1)\,(2\nu_1-1)}$$

$$-\frac{2C_2E_1\nu_1}{(\nu_1+1)\,(2\nu_1-1)}$$

$$-\frac{E_2 \cdot (2C_3\nu_2 - C_3 + C_4r^2)}{r^2\,(\nu_2+1)\,(2\nu_2-1)}$$

$$-\frac{E_2\,(-2C_3\nu_2 + C_3 + C_4r^2)}{r^2\,(\nu_2+1)\,(2\nu_2-1)}$$

$$-\frac{2C_4E_2\nu_2}{(\nu_2+1)\,(2\nu_2-1)}$$

[129]:
```
r_o,r_i,p_o,p_i,r_m = sym.symbols('r_o,r_i,p_o,p_i,r_m')
```

[130]:
```
bc1 = sym.Eq(sigma_rr1.subs(r,r_i),-p_i)
bc2 = sym.Eq(sigma_rr2.subs(r,r_o),-p_o)
bc3 = sym.Eq(sigma_rr1.subs(r,r_m),sigma_rr2.subs(r,r_m))
epsilon_rr1 = sym.diff(u_soln1,r)
epsilon_rr2 = sym.diff(u_soln2,r)
bc4 = sym.Eq(epsilon_rr1.subs(r,r_m),epsilon_rr2.subs(r,r_m))
display(bc1,bc2,bc3,bc4)
```

$$-\frac{E_1 \cdot (2C_1\nu_1 - C_1 + C_2r_i^2)}{r_i^2\,(\nu_1+1)\,(2\nu_1-1)} = -p_i$$

$$-\frac{E_2 \cdot (2C_3\nu_2 - C_3 + C_4r_o^2)}{r_o^2\,(\nu_2+1)\,(2\nu_2-1)} = -p_o$$

$$-\frac{E_1 \cdot (2C_1\nu_1 - C_1 + C_2r_m^2)}{r_m^2\,(\nu_1+1)\,(2\nu_1-1)} = -\frac{E_2 \cdot (2C_3\nu_2 - C_3 + C_4r_m^2)}{r_m^2\,(\nu_2+1)\,(2\nu_2-1)}$$

$$-\frac{C_1}{r_m^2} + C_2 = -\frac{C_3}{r_m^2} + C_4$$

[131]:
```
soln4 = sym.factor(sym.solve([bc1,bc2,bc3,bc4],[C1,C2,C3,C4]))
soln4 = soln4.simplify()

sigma_rr1_soln = sym.factor(sigma_rr1.
 ↪subs([(C1,soln4[C1]),(C2,soln4[C2]),(C3,soln4[C3]),(C4,soln4[C4])]).
 ↪simplify())
sigma_tt1_soln = sym.factor(sigma_tt1.
 ↪subs([(C1,soln4[C1]),(C2,soln4[C2]),(C3,soln4[C3]),(C4,soln4[C4])]).
 ↪simplify())
```

```
sigma_zz1_soln = sym.factor(sigma_zz1.
 ↪subs([(C1,soln4[C1]),(C2,soln4[C2]),(C3,soln4[C3]),(C4,soln4[C4])]).
 ↪simplify())
sigma_rr2_soln = sym.factor(sigma_rr2.
 ↪subs([(C1,soln4[C1]),(C2,soln4[C2]),(C3,soln4[C3]),(C4,soln4[C4])]).
 ↪simplify())
sigma_tt2_soln = sym.factor(sigma_tt2.
 ↪subs([(C1,soln4[C1]),(C2,soln4[C2]),(C3,soln4[C3]),(C4,soln4[C4])]).
 ↪simplify())
sigma_zz2_soln = sym.factor(sigma_zz2.
 ↪subs([(C1,soln4[C1]),(C2,soln4[C2]),(C3,soln4[C3]),(C4,soln4[C4])]).
 ↪simplify())

display(sigma_rr1_soln,sigma_tt1_soln,sigma_zz1_soln,sigma_rr2_soln,sigma_tt2_soln,sigma_zz2_s
```

$$-\frac{2E_1\nu_2^2p_ir^2r_i^2r_m^2 - 2E_1\nu_2^2p_ir_i^2r_m^4 - 2E_1\nu_2^2p_or^2r_m^2r_o^2 + 2E_1\nu_2^2p_or_i^2r_m^2r_o^2 + E_1\nu_2p_ir^2r_i^2r_m^2 + E_1\nu_2p_ir^2r_i^2r_o^2 - E_1\nu_2p_ir_i^2r}{r^2\cdot(2E_1\nu_2^2r_i^2r_m^2 - 2E_1\nu_2^2r_m^4 + E_1\nu_2r_i^2r_}$$

$$-\frac{2E_1\nu_2^2p_ir^2r_i^2r_m^2 + 2E_1\nu_2^2p_ir_i^2r_m^4 - 2E_1\nu_2^2p_or^2r_m^2r_o^2 - 2E_1\nu_2^2p_or_i^2r_m^2r_o^2 + E_1\nu_2p_ir^2r_i^2r_m^2 + E_1\nu_2p_ir^2r_i^2r_o^2 + E_1\nu_2p_ir_i^2r}{r^2\cdot(2E_1\nu_2^2r_i^2r_m^2 - 2E_1\nu_2^2r_m^4 + E_1\nu_2r_i^2r_}$$

$$-\frac{2\nu_1\cdot(2E_1\nu_2^2p_ir_i^2r_m^2 - 2E_1\nu_2^2p_or_m^2r_o^2 + E_1\nu_2p_ir_i^2r_m^2 + E_1\nu_2p_ir_i^2r_o^2 - 2E_1\nu_2p_or_m^2r_o^2}{2E_1\nu_2^2r_i^2r_m^2 - 2E_1\nu_2^2r_m^4 + E_1\nu_2r_i^2r_m^2 + E_1\nu_2r_i^2r_o^2 - E_1\nu_2r_m^4 - E_1\nu_2r_m^2r_o^2 - E_1r_i^2r_m^2 + E_1r_i^2r_o^2 + E_1r_m^4 - E_1r_m^2r_o^2 +}$$

$$-\frac{2E_1\nu_2^2p_or_i^2r_m^2r_o^2 - 2E_1\nu_2^2p_or_m^4r_o^2 + E_1\nu_2p_or^2r_i^2r_o^2 - E_1\nu_2p_or^2r_m^2r_o^2 + E_1\nu_2p_or_i^2r_m^2r_o^2 - E_1\nu_2p_or_m^4r_o^2 + E_1p_or^2r_i^2r_o^2 -}{r^2\cdot(2E_1\nu_2^2r_i^2r_m^2 - 2E_1\nu_2^2r_m^4 + E_1\nu_2r_i^2r_}$$

$$-\frac{-2E_1\nu_2^2p_or_i^2r_m^2r_o^2 + 2E_1\nu_2^2p_or_m^4r_o^2 + E_1\nu_2p_or^2r_i^2r_o^2 - E_1\nu_2p_or^2r_m^2r_o^2 - E_1\nu_2p_or_i^2r_m^2r_o^2 + E_1\nu_2p_or_m^4r_o^2 + E_1p_or^2r_i^2r_o^2}{r^2\cdot(2E_1\nu_2^2r_i^2r_m^2 - 2E_1\nu_2^2r_m^4 + E_1\nu_2r}$$

$$-\frac{2\nu_2\left(E_1\nu_2p_or_i^2r_o^2 - E_1\nu_2p_or_m^2r_o^2 + E_1p_or_i^2r_o^2 - E_1p_or_m^2r_o^2 + 2E_2\nu_1^2p_ir_i^2r_m^2 - 2E_2\nu_1^2\right.}{2E_1\nu_2^2r_i^2r_m^2 - 2E_1\nu_2^2r_m^4 + E_1\nu_2r_i^2r_m^2 + E_1\nu_2r_i^2r_o^2 - E_1\nu_2r_m^4 - E_1\nu_2r_m^2r_o^2 - E_1r_i^2r_m^2 + E_1r_i^2r_o^2 + E_1r_m^4 - E_1r_m^2r_o^2 +}$$

[132]:
```
I1 = sym.factor((sigma_rr1_soln + sigma_tt1_soln+sigma_zz1_soln).simplify())
display(I1)
```

$$-\frac{2\left(\nu_1+1\right)\left(2E_1\nu_2^2p_ir_i^2r_m^2 - 2E_1\nu_2^2p_or_m^2r_o^2 + E_1\nu_2p_ir_i^2r_m^2 + E_1\nu_2p_ir_i^2r_o^2 - 2E_1\nu_2p_or_m^2\right.}{2E_1\nu_2^2r_i^2r_m^2 - 2E_1\nu_2^2r_m^4 + E_1\nu_2r_i^2r_m^2 + E_1\nu_2r_i^2r_o^2 - E_1\nu_2r_m^4 - E_1\nu_2r_m^2r_o^2 - E_1r_i^2r_m^2 + E_1r_i^2r_o^2 + E_1r_m^4 - E_1r_m^2r_o^2 +}$$

[133]:
```
sym.diff(I1,r)
```

[133]:
$$0$$

[ ]:
```

```