M.PHIL. / Ph.D. CET - JULY 2023 - MATHEMATICS - 22-07-2023

Total points 14/40

0 of 0 points

Subject *

Mathematics

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Multiple Choice Questions **14 of 40 points**

✓ * 1/1

If $f:[0,\infty)$ be a Lebesgue measurable function and E be a Lebesgue measurable subset of $\mathbb{R}$ such that $\int_E f\,dm = 0$, where m is the Lebesgue on $\mathbb{R}$. Then

a. $m(E) = 0$
b. $\{x \in \mathbb{R} : f(0) = 0\} = E$
c. $m(\{x \in E : f(x) \neq 0\}) = 0$
d. $m(\{x \in E : f(x) = 0\}) = 0$

- (●) Option a ✓
- ( ) Option b
- ( ) Option c
- ( ) Option d

✗ *

0/1

Which one of the following sets is compact?

a. $\left\{\frac{1}{n} : n \in \mathbb{N}\right\}$

b. $\{(x, y) : x + y \leq 2\}$

c. $\{(x, y) : x^2 + y^2 \geq 1\}$

d. $\{(x, y) : |x| \leq 1, |y| \leq 1\}$

- ○ Option a
- ◉ Option b ✗
- ○ Option c
- ○ Option d

Correct answer

- ◉ Option d

✗ *

0/1

Which of the following is a solution of the differential equation $\left(\frac{dy}{dx}\right)^2 + x\frac{dy}{dx} - y = 0$

a. $x^2 + 4y = 0$
b. $y = x + 1$
c. $y = x - 1$
d. $y^2 - 4x = 0$

- ○ Option a
- ◉ Option b ✗
- ○ Option c
- ○ Option d

Correct answer

- ◉ Option a

✗ *

0/1

If $f(z)$ is an entire function then

a. $f(\bar{z})$ is an entire function

b. $\overline{f(\bar{z})}$ is an entire function

c. $\overline{f(z)}$ is an entire function

d. $f(\bar{z}) + \overline{f(z)}$ is an entire function

- ○ Option a
- ○ Option b
- ○ Option c
- ◉ Option d ✗

Correct answer

- ◉ Option b

✗ *

0/1

Let $f(x,y) = \begin{cases} \frac{xy}{x^2+y^2}, & x^2+y^2 \neq 0 \\ 0, & x=y=0 \end{cases}$. Then

a. $f(x,y)$ is discontinuous at origin

b. $f(x,y)$ is continuous at origin

c. $f(x,y)$ is differentiable at origin

d. $f(x,y)$ has removable discontinuity at origin

- ○ Option a
- ◉ Option b ✗
- ○ Option c
- ○ Option d

Correct answer

- ◉ Option a

✗ *

0/1

The general solution of the second order differential equation $4u_{xx} - u_{yy} = 0$ is of the form $u(x, y)$ is

a. $f_1(x) + f_2(y)$
b. $f_1(x + 2y) + f_2(x - 2y)$
c. $f_1(x + 4y) + f_2(x - 4y)$
d. $f_1(4x + y) + f_2(4x - y)$

- (●) Option a ✗
- ( ) Option b
- ( ) Option c
- ( ) Option d

Correct answer

- (●) Option b

✗ *

0/1

Let V be a vector space of all sequences of real number such that $\sum 2^n|a_n|$ converges, define $\|.\|: V \to \mathbb{R}$ by $\|a\| = \sum_{n=1}^{\infty} 2^n|a_n|$, which of the following is true

a. V contains only the sequence $(0,0, \dots)$
b. V is finite dimensional
c. V has a countable linear basis
d. V is complete normed space

- ( ) Option a
- ( ) Option b
- (●) Option c ✗
- ( ) Option d

Correct answer

- (●) Option d

✓ *

1/1

Let $S_n = \left[\frac{1}{2^n}, 1\right]$ be the given interval. Then $\bigcup_{n=1}^{\infty} S_n$ is

a. An open set
b. A closed set
c. Neither open nor closed
d. Null set.

- ○ Option a
- ○ Option b
- ◉ Option c ✓
- ○ Option d

✓ *

1/1

Let A be a 3×3 matrix with real entries. Which of the following assertion is false ?

a. A must have a real eigenvalue.
b. If the determinant of A is 0, 0 is an eigenvalue of A.
c. If the determinant of A is negative and 3 is an eigenvalue of A, then A must have three real eigenvalues.
d. If the determinant of A is positive and 3 is an eigenvalue of A, then A must have three real eigenvalues.

- ○ Option a
- ○ Option b
- ○ Option c
- ◉ Option d ✓

✗ *

0/1

The value of the complex integration $\int_C \frac{1}{z^2(e^z - e^{-z})} dz$ is

a. $\frac{\pi i}{6}$

b. $-\frac{\pi i}{6}$

c. $\frac{\pi i}{2}$

d. $-\frac{\pi i}{2}$

- ○ Option a
- ○ Option b
- ○ Option c
- ◉ Option d ✗

Correct answer

- ◉ Option a

✗ *

0/1

Let A be a 3×3 real matrix whose characteristic polynomial $p(T)$ is divisible by T^2. Which of the following statements is true ?

a. The eigenspace of A for the eigenvalue 0 is two dimensional
b. All the eigenvalues of A sre real
c. $A^3 = 0$
d. A is diagonalizable

- ○ Option a
- ○ Option b
- ○ Option c
- ◉ Option d ✗

Correct answer

- ◉ Option b

✗ *

0/1

Let $X = \mathbb{R}$ with cofinite topology $\tau_f = \{A \subseteq X : A^c \text{ is finite or } A = \emptyset\}$. Then (X, τ_f) is a

a. first countable space
b. T_1 - space
c. regular space
d. normal space

- ○ Option a
- ○ Option b
- ◉ Option c ✗
- ○ Option d

Correct answer

- ◉ Option b

✗ *

0/1

The number of conjugacy classes in the permutation group S_6 is

a. 12
b. 11
c. 10
d. 6

◉ Option a ✗

○ Option b

○ Option c

○ Option d

Correct answer

◉ Option b

✓ *

1/1

. The number of group homomorphism from $\mathbb{Z}_{10}$ to $\mathbb{Z}_{20}$ is

a. 0
b. 1
c. 5
d. 10

○ Option a

○ Option b

○ Option c

◉ Option d ✓

✓ *

1/1

Which of the following is a class equation of a finite group ?

a. $1+1+2+3+3+3+13+13=39$
b. $1+1+2+2+2+2+2+2=14$
c. $1+3+3+7+7=21$
d. $1+1+1+2+5+5+5+=15$

- ○ Option a
- ○ Option b
- ◉ Option c ✓
- ○ Option d

✗ *

0/1

The second order PDE $u_{yy}-yu_{xx}+x^3u=0$ is

a. Elliptic for all $x, y \in \mathbb{R}$
b. Parabolic for all $x, y \in \mathbb{R}$
c. Elliptic for all $x \in \mathbb{R}, y<0$
d. Hyperbolic for all $x \in \mathbb{R}, y<0$

- ○ Option a
- ◉ Option b ✗
- ○ Option c
- ○ Option d

Correct answer

- ◉ Option c

✓ *

1/1

Consider the following quadratic forms over $\mathbb{R}$

I. $6x^2 - 13xy + 6y^2$,

II. $x^2 - xy + 2y^2$,

III. $x^2 - xy - 2y^2$.

Which of the following statements is true ?

a. Quadratic forms (I) and (II) are equivalent

b. Quadratic forms (I) and (III) are not equivalent

c. Quadratic forms (II) is positive definite

d. Quadratic forms (III) is positive definite

- ○ Option a
- ○ Option b
- ◉ Option c ✓
- ○ Option d

✓ *

1/1

Let V be the vector space of polynomials of degree almost 10 over $\mathbb{R}$. Let $T: V \to V$ be a linear transformation defined by $T[p(x)] = p'(x)$ for all $p(x) \in V$, where $p'(x)$ is derivative of $p(x)$. Then which of the following is false ?

a. $\det(T) = 0$
b. $trace(T) = 0$
c. $rank(T) = 9$
d. All the eigenvalues are zero

- Option a
- Option b
- (●) Option c ✓
- Option d

✗ *

0/1

The number of subgroups of a cyclic group of order 100 is

a. 10
b. 25
c. 9
d. 4

- Option a
- Option b
- Option c
- (●) Option d ✗

Correct answer

- (●) Option c

✓ *

1/1

Let $u(x,y)$ be the solution of the Cauchy problem

$$uu_x + u_y = 0, x \in \mathbb{R}, y > 0, u(x,0) = x, x \in \mathbb{R}.$$

Which of the following is the value of $u(2,3)$?

a. 2
b. 3
c. $1/2$
d. $1/3$

- ○ Option a
- ○ Option b
- ◉ Option c ✓
- ○ Option d

✗ *

0/1

Which of the following is an example of a norm that satisfies the parallelogram law?

a. $\|x\| = \sum_{i=1}^{n} |x_i|$ on $\mathbb{R}^n$
b. $\|x\| = \sup_{i \in \mathbb{N}} |x_i|$ where $x = (x_i)$ satisfies $\sup_{i \in \mathbb{N}} |x_i| < \infty$
c. $\|z\| = \sqrt{x^2 + y^2}$ on $\mathbb{C}$ where $z = x + iy$
d. $\|x\| = (x_1{}^3 + x_2{}^3)^{1/3}$ on $\mathbb{R}^2$

- ○ Option a
- ○ Option b
- ○ Option c
- ◉ Option d ✗

Correct answer

- ◉ Option c

✓ *

1/1

Let H be a Hilbert space and $K \subseteq H$ be a closed linear space. Let $x \in H - K$. Let $x_0 \in K$ be such that $\|x - x_0\| = \inf\{\|x - y\| : y \in K\}$. Then

a. Such a x_0 is not unique
b. $x \perp K$
c. $x_0 \perp K$
d. $x - x_0 \perp K$

- ○ Option a
- ○ Option b
- ○ Option c
- ◉ Option d ✓

✗ *

0/1

Let C be the counter-clockwise oriented circle of radius $1/2$ centered at $i = \sqrt{-1}$. Then the value of the contour integral $\int_C \frac{dz}{z^4-1}$ is

a. $\frac{-\pi}{2}$
b. $\frac{\pi}{2}$
c. $-\pi$
d. π

- ○ Option a
- ◉ Option b ✗
- ○ Option c
- ○ Option d

Correct answer

- ◉ Option a

✗ *

0/1

Let $\mathbb{R}, \mathbb{R}_l, \mathbb{R}_d$ be denote the set of all real numbers with usual topology, lower limit topology and discrete topology respectively. Then which one of the following identity functions is not continuous?

a. $f : \mathbb{R} \to \mathbb{R}_l$ by $f(x) = x$ for all $x \in \mathbb{R}$
b. $f : \mathbb{R}_l \to \mathbb{R}$ by $f(x) = x$ for all $x \in \mathbb{R}$
c. $f : \mathbb{R}_d \to \mathbb{R}$ by $f(x) = x$ for all $x \in \mathbb{R}$
d. $f : \mathbb{R}_d \to \mathbb{R}_l$ by $f(x) = x$ for all $x \in \mathbb{R}$

- ○ Option a
- ◉ Option b ✗
- ○ Option c
- ○ Option d

Correct answer

- ◉ Option a

✗ *

0/1

Consider a function $f(x) = x|x|, x \in \mathbb{R}$. Then the function $f(x)$ is

a. Continuous and differentiable only at $x = 0$
b. Differentiable for all $x \in \mathbb{R}$ except at $x = 0$
c. Differentiable for all $x \in \mathbb{R}$
d. Continuous but not differentiable

- (●) Option a ✗
- ( ) Option b
- ( ) Option c
- ( ) Option d

Correct answer

- (●) Option c

✗ *

0/1

The order of element 3 in $(\mathbb{Z}_4, \otimes)$ is

a. 2
b. 3
c. 4
d. 6

- ○ Option a
- ◉ Option b ✗
- ○ Option c
- ○ Option d

Correct answer

- ◉ Option a

✓ *

1/1

Let $u = u(x, y)$ be the solution of the following Cauchy problem $u_x + u_y = e^u$ for $(x, y) \in \mathbb{R} \times (0, 1/e)$ and $u(x, 0) = 1$ for $\in \mathbb{R}$. Which of the following statements is true ?

a. $u\left(\frac{1}{2e}, \frac{1}{2e}\right) = 1$

b. $u_x\left(\frac{1}{2e}, \frac{1}{2e}\right) = 0$

c. $u_y\left(\frac{1}{4e}, \frac{1}{4e}\right) = \log 4$

d. $u_y\left(0, \frac{1}{4e}\right) = 0$

- ○ Option a
- ◉ Option b ✓
- ○ Option c
- ○ Option d

✗ *

0/1

For $z \in \mathbb{C}$, define $f(z) = e^z/(e^z - 1)$. Then

a. f is entire

b. the only singularities of f are poles

c. f has finitely poles on the imaginary axis

d. Each pole of f is simple

◉ Option a ✗

○ Option b

○ Option c

○ Option d

Correct answer

◉ Option d

✗ *

0/1

Let V be an inner product space and S be a subset of V. Let $\bar{S}$ be the closure of S in V with respect to the topology induced by the inner product. Which of the following statement are true

a. $S = (S^{\perp})^{\perp}$
b. $\bar{S} = (S^{\perp})^{\perp}$
c. $\overline{span(S)} = (S^{\perp})$
d. $S^{\perp} = S^{\perp\perp\perp}$

- (●) Option a ✗
- ( ) Option b
- ( ) Option c
- ( ) Option d

Correct answer

- (●) Option d

✓ *

1/1

The integrating factor of the differential equation $xp - y = x\cos x$ is

a. $e^{1/x}$
b. x
c. $1/x$
d. e^{-1/x^2}

- (●) Option a ✓
- ( ) Option b
- ( ) Option c
- ( ) Option d

✓ *

1/1

Let f be a rational function of a complex variable z given by $f(z) = \frac{z^3+2z-4}{z}$. The radius of convergence of the Taylor series of f at $z = 1$ is

a. 0
b. 1
c. 2
d. ∞

- ○ Option a
- ◉ Option b ✓
- ○ Option c
- ○ Option d

✗ *

0/1

Consider the set of sequences $X = \{(x_n): x_n \in \{0,1\}, n \in \mathbb{N}\}$. Then

a. X is finite and countable
b. X is infinite and countable
c. X is infinite and uncountable
d. None of these

- ◉ Option a ✗
- ○ Option b
- ○ Option c
- ○ Option d

Correct answer

- ◉ Option c

✗ *

0/1

Consider the initial value problem $\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0, u(0,y) = 4e^{-2y}$. Then the value of $u(1,1)$ is

a. $4e^{-2}$
b. $4e^{2}$
c. $2e^{-4}$
d. $4e^{4}$

◉ Option a ✗

○ Option b

○ Option c

○ Option d

Correct answer

◉ Option b

✗ *

0/1

In a group $(G, *)$, a is an element of order 30, then the order of a^{18} is

a. 5
b. 6
c. 18
d. 30

- ○ Option a
- ○ Option b
- ◉ Option c ✗
- ○ Option d

Correct answer

- ◉ Option a

✗ *

0/1

The number of solution of the equation $x^2 = 1$ in the ring $\mathbb{Z}/105\mathbb{Z}$ is

a. 0
b. 2
c. 4
d. 8

- ○ Option a
- ○ Option b
- ○ Option c
- ◉ Option d ✗

Correct answer

- ◉ Option c

✗ *

0/1

Consider the system of ordinary differential equation $\frac{dy}{dx} + Ay, y(0) = \binom{2}{-1}$, where $A = \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}$ and $y = \begin{pmatrix} y_1(x) \\ y_2(x) \end{pmatrix}$. Then which of the following is correct ?

a. $y_1(x) \to 0$ and $y_2(x) \to \infty$ as $x \to \infty$
b. $y_1(x) \to 0$ and $y_2(x) \to 0$ as $x \to \infty$
c. $y_1(x) \to -\infty$, $y_2(x) \to -\infty$ as $x \to -\infty$
d. $y_1(x) \to \infty$ and $y_2(x) \to -\infty$ as $x \to -\infty$

- ○ Option a
- ○ Option b
- ◉ Option c ✗
- ○ Option d

Correct answer

- ◉ Option d

✗ *

0/1

Which of the following is not an integrating factor of the equation $xdy - ydx = 0$.

a. $\frac{1}{x}$

b. $\frac{1}{xy}$

c. $\frac{1}{x^2}$

d. $\frac{1}{(x^2+y^2)}$

- ○ Option a
- ◉ Option b ✗
- ○ Option c
- ○ Option d

Correct answer

- ◉ Option a

✓ *

1/1

If $A = \begin{pmatrix} \alpha & 2 \\ 2 & \alpha \end{pmatrix}$ and $|A|^3 = 125$, then the value of α is _____.

a. $\neq 1$

b. $\neq 2$

c. $\neq 3$

d. $\neq 5$

- ○ Option a
- ○ Option b
- ○ Option c
- ◉ Option d ✓

✗ *

0/1

Which of the following subsets of $\mathbb{R}^2$ is a convex set?

a. $\{(x,y)|y < x^2\}$

b. $\left\{(x,y)|x^2 + \frac{y^2}{4} < 1\right\}$

c. $\{(x,y)|y = x^2\}$

d. $\left\{(x,y)|x^2 + \frac{y^2}{4} = 1\right\}$

- Option a
- Option b
- Option c
- Option d ✗

Correct answer

- Option b

✓ *

1/1

The number of generator of a cyclic group of order 3^n, where $n \in \mathbb{Z}^+$ is

a. 3^{n-1}

b. 3^n

c. $2 \cdot 3^{n-1}$

d. $2 \cdot 3^n$

- Option a
- Option b
- Option c ✓
- Option d

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127

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