XIAMEN UNIVERSITY MALAYSIA

Online Practical Test 1

Course Code:	BSC128		
Course Name:	Numerical Methods		
Question Paper Setter:	Liu Meifeng		
Academic Session:	2023/09	Question Paper:	A ☐ B ■
Total No. of Pages:	2	Time Allocated:	2 hours
Additional Materials:	Algorithm 3.1, Algorithm 3.3		
Apparatus Allowed:	Computer, Laptop		

INSTRUCTIONS TO CANDIDATES

1. This paper consists of Three questions. Please answer ALL questions.
2. Read the above information carefully to ensure you have the correct and complete question paper.
3. Please follow the requirement of each section and write down all the corresponding answers on the answer book provided.
4. Please note that presentation of solutions (clarity, coherence, conciseness, and completeness) is important. Show your work and organize your solution. Answer without proper justification may receive less, or even zero credit.
5. Communication between candidates in any means is forbidden. Answers must be entirely individual candidate’s independent effort. If you are found sharing your solutions with other candidates, or suspected of doing so, you would be penalized accordingly.

DO NOT TURN OVER THIS PAGE UNTIL INSTRUCTED TO DO SO.

(Student ID: _______________ Full Name: ___________________________________)

Question 1 (30 marks)

A car traveling along a straight road is clocked at three points. The data from the observations are given in the following table, where the time is in seconds, the distance is in feet, and the speed is in miles per second.

Time t (in seconds)	0	8	13
Distance d (in feet)	0	623	993
Speed (miles per hour)	75	74	72

(a) Go to website Octave Online, create three arrays named `T`, `Dist`, `Speed` to store the given data: Time, Distance and Speed, respectively, find the length of the arrays. In addition, convert miles per hour to feet per second and create an array named `convertSpeed` to store converted speed. [5 marks]

(b) Apply Neville's method (Algorithm 3.1) to the given data by constructing a recursive table `myQ1` to determine the position of the ball when $t = 6.5$ seconds. [5 marks]

(c) Use the Hermite Interpolating Method (Algorithm 3.3) by constructing a recursive table `myQ2` to determine the position of the ball when $t = 6.5$ second. [5 marks]

(d) According to Newton interpolatory divided-difference formula, the Hermite polynomial is then given by

$$H_{2n+1}(x) = f[z_0] + \sum_{k=1}^{n} f[z_0, \quad z_1, \quad \cdots, \quad z_k]\,(x - z_0)\cdots(x - z_{k-1}).$$

However, referring to the Hermite Interpolating Method (Algorithm 3.3), the Hermite interpolating polynomial $H(x)$ is given by

$$\begin{aligned} H(x) = Q_{0,0} + Q_{1,1}(x - x_0) + Q_{2,2}(x - x_0)^2 + Q_{3,3}(x - x_0)^2(x - x_1) \\ + Q_{4,4}(x - x_0)^2(x - x_1)^2 + \cdots \\ + Q_{2n+1,2n+1}(x - x_0)^2(x - x_1)^2 \cdots (x - x_{n-1})^2(x - x_n). \end{aligned}$$

Determine the n, x_i and z_i in this problem, state the formula for both the Hermite polynomial $H(x)$ and its derivative $H'(x)$ in the comment line of your MatLab / Octave code. [5 marks]

(e) Use the formula for the derivative of the Hermite polynomial to estimate the speed of the travelling car in miles per hour at $t = 6.5$ seconds. [Hint: Used the converted speed to solve the problem and then convert back to miles per hour to give the answers.] [5 marks]

(f) Does the maximum velocity of the car occur at $t = 0$, or does the derivative of the Hermite polynomial have a maximum exceeding 75 miles per hour? If so, when does the maximum velocity occurs? Does this seem reasonable? [5 marks]

Instructions:

- Save your script in Octave Online as **Practical1_ID.m**, download it and print it as **Practical1_ID.pdf**.
- Take a screenshot on your code and results, save the screenshot as **Practical1_ID.png**.
- Share your script as Octave Bucket, copy the link and paste it on the comment field.
- Remember to attach the **PDF** file, the **M** file and the **PNG** file of your code in the Moodle system in order to complete your submission.

- END OF PAPER -

In the preceding example, we have $f(2.1) = \ln 2.1 = 0.7419$ to four decimal places, so the absolute error is

$$|f(2.1) - P_2(2.1)| = |0.7419 - 0.7420| = 10^{-4}.$$

However, $f'(x) = 1/x$, $f''(x) = -1/x^2$, and $f'''(x) = 2/x^3$, so the Lagrange error formula (3.3) in Theorem 3.3 gives the error bound

$$|f(2.1) - P_2(2.1)| = \left| \frac{f'''(\xi(2.1))}{3!}(x - x_0)(x - x_1)(x - x_2) \right|$$

$$= \left| \frac{1}{3\,(\xi(2.1))^3}(0.1)(-0.1)(-0.2) \right| \le \frac{0.002}{3(2)^3} = 8.\overline{3} \times 10^{-5}.$$

Notice that the actual error, 10^{-4}, exceeds the error bound, $8.\overline{3} \times 10^{-5}$. This apparent contradiction is a consequence of finite-digit computations. We used four-digit rounding arithmetic, and the Lagrange error formula (3.3) assumes infinite-digit arithmetic. This caused our actual errors to exceed the theoretical error estimate.

- Remember: You cannot expect more accuracy than the arithmetic provides.

Algorithm 3.1 constructs the entries in Neville's method by rows.

ALGORITHM 3.1

Neville's Iterated Interpolation

To evaluate the interpolating polynomial P on the $n+1$ distinct numbers $x_0, \ldots, x_n$ at the number x for the function f:

INPUT numbers $x, x_0, x_1, \ldots, x_n$; values $f(x_0), f(x_1), \ldots, f(x_n)$ as the first column $Q_{0,0}, Q_{1,0}, \ldots, Q_{n,0}$ of Q.

OUTPUT the table Q with $P(x) = Q_{n,n}$.

Step 1 For $i = 1, 2, \ldots, n$
for $j = 1, 2, \ldots, i$

$$\text{set } Q_{i,j} = \frac{(x - x_{i-j})Q_{i,j-1} - (x - x_i)Q_{i-1,j-1}}{x_i - x_{i-j}}.$$

Step 2 OUTPUT (Q);
STOP. ▪

EXERCISE SET 3.2

1. Use Neville's method to obtain the approximations for Lagrange interpolating polynomials of degrees one, two, and three to approximate each of the following:
   a. $f(8.4)$ if $f(8.1) = 16.94410$, $f(8.3) = 17.56492$, $f(8.6) = 18.50515$, $f(8.7) = 18.82091$
   b. $f\left(-\frac{1}{3}\right)$ if $f(-0.75) = -0.07181250$, $f(-0.5) = -0.02475000$, $f(-0.25) = 0.33493750$, $f(0) = 1.10100000$
   c. $f(0.25)$ if $f(0.1) = 0.62049958$, $f(0.2) = -0.28398668$, $f(0.3) = 0.00660095$, $f(0.4) = 0.24842440$
   d. $f(0.9)$ if $f(0.6) = -0.17694460$, $f(0.7) = 0.01375227$, $f(0.8) = 0.22363362$, $f(1.0) = 0.65809197$

ALGORITHM 3.3

Hermite Interpolation

To obtain the coefficients of the Hermite interpolating polynomial $H(x)$ on the $(n+1)$ distinct numbers $x_0, \ldots, x_n$ for the function f:

INPUT numbers $x_0, x_1, \ldots, x_n$; values $f(x_0), \ldots, f(x_n)$ and $f'(x_0), \ldots, f'(x_n)$.

OUTPUT the numbers $Q_{0,0}, Q_{1,1}, \ldots, Q_{2n+1,2n+1}$ where

$$\begin{aligned} H(x) = Q_{0,0} &+ Q_{1,1}(x-x_0) + Q_{2,2}(x-x_0)^2 + Q_{3,3}(x-x_0)^2(x-x_1) \\ &+ Q_{4,4}(x-x_0)^2(x-x_1)^2 + \cdots \\ &+ Q_{2n+1,2n+1}(x-x_0)^2(x-x_1)^2 \cdots (x-x_{n-1})^2(x-x_n). \end{aligned}$$

Step 1 For $i = 0, 1, \ldots, n$ do Steps 2 and 3.

Step 2 Set $z_{2i} = x_i$;
$z_{2i+1} = x_i$;
$Q_{2i,0} = f(x_i)$;
$Q_{2i+1,0} = f(x_i)$;
$Q_{2i+1,1} = f'(x_i)$.

Step 3 If $i \neq 0$ then set

$$Q_{2i,1} = \frac{Q_{2i,0} - Q_{2i-1,0}}{z_{2i} - z_{2i-1}}.$$

Step 4 For $i = 2, 3, \ldots, 2n+1$
for $j = 2, 3, \ldots, i$ set $Q_{i,j} = \dfrac{Q_{i,j-1} - Q_{i-1,j-1}}{z_i - z_{i-j}}$.

Step 5 OUTPUT $(Q_{0,0}, Q_{1,1}, \ldots, Q_{2n+1,2n+1})$;
STOP. ■

EXERCISE SET 3.4

1. Use Theorem 3.9 or Algorithm 3.3 to construct an approximating polynomial for the following data.

a.

x	$f(x)$	$f'(x)$
8.3	17.56492	3.116256
8.6	18.50515	3.151762

b.

x	$f(x)$	$f'(x)$
0.8	0.22363362	2.1691753
1.0	0.65809197	2.0466965

c.

x	$f(x)$	$f'(x)$
−0.5	−0.0247500	0.7510000
−0.25	0.3349375	2.1890000
0	1.1010000	4.0020000

d.

x	$f(x)$	$f'(x)$
0.1	−0.62049958	3.58502082
0.2	−0.28398668	3.14033271
0.3	0.00660095	2.66668043
0.4	0.24842440	2.16529366

2. Use Theorem 3.9 or Algorithm 3.3 to construct an approximating polynomial for the following data.

a.

x	$f(x)$	$f'(x)$
0	1.00000	2.00000
0.5	2.71828	5.43656

b.

x	$f(x)$	$f'(x)$
−0.25	1.33203	0.437500
0.25	0.800781	−0.625000

c.

x	$f(x)$	$f'(x)$
0.1	−0.29004996	−2.8019975
0.2	−0.56079734	−2.6159201
0.3	−0.81401972	−2.9734038

d.

x	$f(x)$	$f'(x)$
−1	0.86199480	0.15536240
−0.5	0.95802009	0.23269654
0	1.0986123	0.33333333
0.5	1.2943767	0.45186776