Introduction to Time Complexity

TLE on online Ide

Algo1	Algo2
5 sec	1 min

Prateek
Judge

Sort data
arrange

Vamsi	Viresh
Optimus sort	Hello Sort
execution time 5 secs	10 secs
c/c++	python

Language	c++	c++
exect	5 secs	2 sec
	samsung phone	mac pro
processor	mac	mac
execution	1 sec	2 sec
	10^5	10^4

Lot of factors come change → execution

⟹ Check the no. of operations.

single operation

No. of operation

Arithmetic op ⇒ +, −, *, ÷, % etc

Comparion op ⇒ ==, <=, >= etc

Assign op ⇒ a=5, b=1000

Bitwise op ⇒ |, &, ^, ~

logical op ⇒ &&, ||

fun^n call ⇒ add(a,b), ret a;

op^n on arr ⇒ arr[i], a[i][j][k]

Algo 1:

```
for ( i=0 ; i<N ; i++)      // 1 , 1*N , 1*N
{
    print a[i] ;            // 1*N , 1*N
}
```

$1 + N + N + N + N \Rightarrow (4N+1)$ ⇒ $f(N)$

Algo 2

No. of operations

```
for ( i=0 ; i<N ; i++)      // 1 , N , N
{
    for ( j=0 ; j<N ; j++)  // 1 , N , N
    {
        print a[i][j]       // N , N
    }
}
```

$4N+1$

$(4N+1)N + 2N + 1 \Rightarrow (4N^2+3N+$ ⇒ $h(N)$

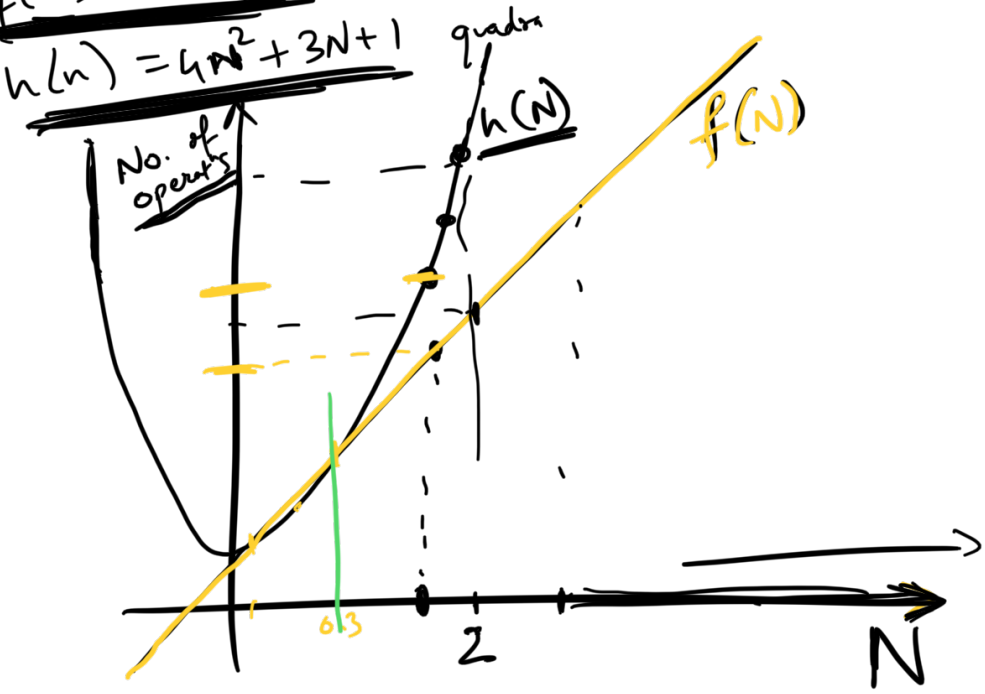
$f(N)$ & $h(N)$

How to compare $f(n)$ and $h(n)$

plot a graph

$f(n) = 4N + 1$ (linear)

$h(n) = 4N^2 + 3N + 1$ quadratic



For $n > 0.3$, $h(N)$ is always greater than $f(N)$

Time $\propto$ operation

Growth in time compared with growth in input

n	T_1 (linear)	T_2 (quadratic)
1	1	1
2	2	4
3	3	9
⋮	⋮	⋮
100	100	10000

50 kms ← (Approximate)

10 mins ← 1000 meters.

Order

Distance is of the order of 50 kms.

speed of light $\sim 3 \times 10^8$ <

$108953 \rightarrow (10^5)$

Asymptotic notation → O (Big O) ← (interested)
→ Ω (omega)
→ θ (theta)

$f(n) = 4N+1$ → order?

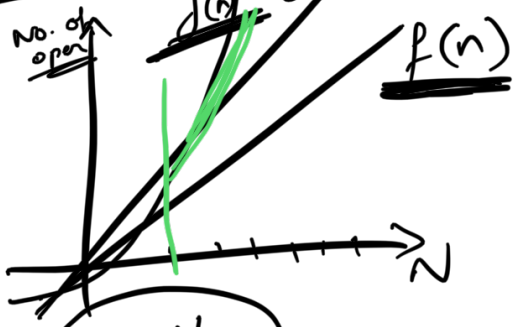
$h(n) = 4N^2+3N+1$ → order?

→ order of $g(n)$

$g(n) \geq f(n)$

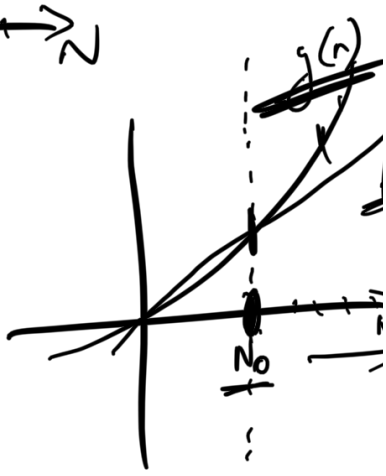
$2f(N)$

$< 3f(n)$



constant $c\,g(n) \geq fn \quad \forall \; N > N_0$ (for all)

Approximating $f(N)$ with other func^n $g(N)$ which has value greater than $f(N)$ for given input size more than N_0.



$h(N) = 4(N^2+3N+1)$

$< 4N^2 + 4N + 4$

$< 4(N^2+N+1)$

$< 4(N^2+N^2+N^2)$

$< (12N^2)$

$N > N_0$

$N_0 = 1$

$N > 1$

$< 2^N$

$< 4 (\text{sin} \cdots$

$< (12) N^2$

$O(N^2)$

$< 5 N^{10}$

$C N^2 + C$

$F(N) = 4N+1 \Rightarrow O(N)$

$\Sigma N^2 +$

$10 N$

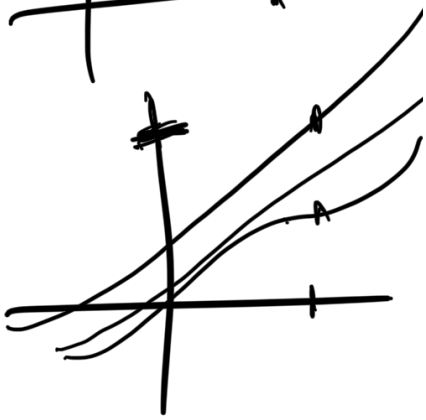
$100 N^3$

$g(n) \; 4N^3 + 8N^2 + 3 \log N \rightarrow O(N^3)$

$K(n) \; N^2 + 2N^4 \rightarrow O(N^4)$

after some N_0,

$O(N^4) > O(N^3)$



$f(n) = O(g(n))$

$cg(n) \geq f(n) \quad \forall \; N > N_0$

$f = \frac{N}{2} \Rightarrow O(N)$

guess

print half of elements in matrix

total $\frac{N}{2}$

$O(N^2)$!

N

Quiz 1

i=-2

for (i=0 ; i<N ; i+=2)

0 → 2 → 4 → 6 → 8 → 10 → 12 . 14 . . . N

N/2 ⇒ O(N)

N/2 + 1

O(N/2)

O(N) → 4N → N/2 execution time

Quiz 2

int i = N → 1

a = 0 → 1 ⇒ O(log N)

while (i > 0)

a += i → 1

i = i/2 → 1

log N

No. of operation:

Value of i:

N → N/2 → N/4 → N/8 . . . . N/2^k → 1 → 0

N/2^(k+1)

k steps

N/2^k = 1 ⇒ N = 2^k

log2 N = log2 2^k

$\log_2 N = K$

$\log_K K = 1$

$\log_b X$

↘

To what power b
Should be raised to get x

$\log_2 32 = 5$

$2^5 = 32$

$2 \log N + 2 \Rightarrow O(\log N)$

AP Linear

0 → 2 → 4 → 6 → 8 → 10 → 12 → 14 → 16

GP

20 → 10 → 5 → 2 → 1 → 0

while ( ) $O(\log_3 N) < O(N)$

{

$i = i/3$

}

$\frac{1}{N}$ $(\log \log N)$

$(N \log N$

Quiz 3 Which is least effective

A) for ($i=0$; $i<N$; $i++$) → $O(N)$

B) for ($i=0$; $i<N$; $i+=2$) → $O(N)$

C) for ($i=1$; $i<N$; $i*=2$) → $O(\log N$

~~D)~~ for ($i=N$; $i>-1$; $i=i/2$) → (infinite

1 → 0 > -1

$N \to N/2 \to N/4 \to \cdots$

$1 \to 2 \to 4 \to 8 \to 16 \cdots N \; (2^K)$

K steps.

$2^K > N$

$K > \log N$

$O(\log N)$

Quiz 4:

```
K = 0
for( i = 0; K < n; i++)
{
    K = K + i
}
```

step

$O(\sqrt{N})$

100

1000

N

K = 0

K = 0	i = 0	0
K = 1	i = 1	0+1
K = 3	i = 2	1+2
K = 6	i = 3	1+2+3
K = 10	i = 4	1+2+3+4
	i = x	1+2+3+...+x

x steps

K

$1+2+3+\cdots+x = \frac{x(x+1)}{2}$

K = N

$O(2^N)$

fibonacci

$\frac{1}{2}(x^2+x) \geq N$

$\frac{1}{2}(x^2+x^2) \geq N$

$\frac{1}{2}(2x^2) \geq N$

$x \geq \sqrt{N}$

$O(\sqrt{N})$

```
K=0
for (i=0 ; K<N ; i++)
{
  K = K + i ⟹ 1
}
```

x steps

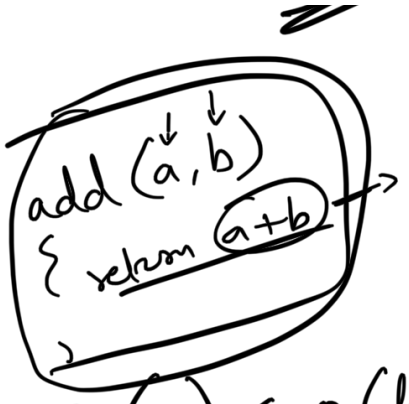
K ≥ N

K=0	i=0	cal
K=0	i=1	0+0
K=1	i=2	0+1
K=3	i=3	1+2
K=6	i=4	1+2+3
K=10	⋮	1+2+3+4
K=?	i=x	1+2+3+⋯ x ≥ N
	i=√N	$\frac{x(x+1)}{2} \geq N$

$O(\sqrt{N})$

$\frac{1}{2}(2x^2+x) \geq N$

$\frac{1}{2}(x^2+x^2) \geq N$



$x^2 \geq N$

$x \; O(\sqrt{N})$

$\frac{1}{2}(x^2 + x^2) \geq N$

$\frac{1}{2}(2x^2) \geq N$

$x^2 \geq N$

$x \geq \sqrt{N}$

$O(1) < O(\log N) < O(N) < O(N \log N)$

$< O(N^2) < O(N^3) \ldots$

$< O(2^N)$

$< O(N!)$

$2^N \; (\geq ?) \; N!$

$2 \times 2 \times 2 \times \cdots$ (N) — N times

$2^{10} \Rightarrow 1024$

$1 \times 2 \times 3 \times 4 \times 5 \times \cdots \times N$ — N numbers

$N > 2$

$10!$

$2^N \quad N^2 \quad N \times N$

$2 \times 2 \times 2 \times \cdots$ (N)

$2^{10} \Rightarrow 1000$

$N = 10 \quad 100$

$x \longleftrightarrow N$

$x \stackrel{?}{=} \sqrt{N - x}$

2 secs

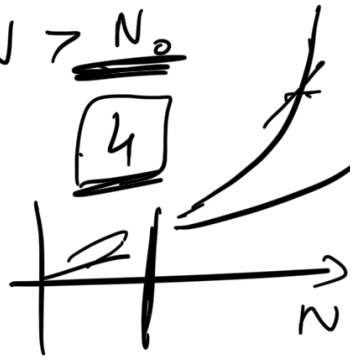
10^8

$O(N^2)$

```
for (i=0; i<N; i++)
{
  for(j=0; j<N; j++)
    a = i+j
}
```

$N = 10$

for all $N > N_0$




```
for(i=0 ; i<N ; i*=2)
{
}
```

$i = 5$

$i = 10$

$fib(N) = fib(N-1) + fib(N-2)$ recursive

$O(2^N)$