```
clc    E         % Clears the Command Window
clear all       % Removes all variables from memory
syms x y lam real  % Creates symbolic variables: x, y, lambda (real numbers)
X^2
```

We want a clean workspace so old variables don't interfere.

syms is essential because we want to:

- Write f(x,y)f(x,y) symbolically.
- Differentiate exactly (not numerically).
- Solve equations symbolically.

2. User Inputs the Functions

```
f = input('Enter f(x,y) to be extremized : ');
g = input('Enter the constraint function g(x,y) : ');
```

f is the objective function — the one we want to maximize or minimize.

g is the constraint function — the one that must be equal to zero.

Example:

- f=x2+y2f=x2+y2
- g=x+y−1g=x+y−1

This is the core problem in constrained optimization:

"Find the maximum and minimum of f(x,y)f(x,y) subject to g(x,y)=0g(x,y)=0."

```
F = f+ lam*g
Fd = jacobian(F,[x y lam])
```

`jacobian(F, [x y lam])` returns:

∂F/∂x,∂F/∂x,∂F/∂λ

Explicitly, these are:

1. Fx=fx+λgx=0
2. Fy=fy+λgy=0
3. Fλ=−g(x,y)=0 (enforces the constraint)

We now have **3 equations** in **3 unknowns** (x,y,λ).

```
[ax, ay, alam] = solve(Fd, x, y, lam);

ax = double(ax);

ay = double(ay);
```

`solve(Fd, x, y, lam)` finds all solutions for:

Fx=0,Fy=0,g(x,y)=0

- Each solution is:
- ax(i) → x-coordinate of an extremum.
- ay(i) → y-coordinate of an extremum.
- alam(i) → the corresponding Lagrange multiplier value (not always needed for plotting).
- `double()` converts symbolic results to regular numbers for later plotting.

T = subs(f,{x,y},{ax,ay});

T = double(T);

Substitutes each (x,y) solution into f(x,y).

- This gives the **function value** at each extremum.

epxl = min(ax);

epxr = max(ax);

epyl = min(ay);

epyu = max(ay);

D = [epxl-1.5 epxr+1.5 epyl-1.5 epyu+1.5]

Finds min/max xx and yy among the solutions.

Expands range by 1.5 units in each direction for better visualization.

D = [xmin,xmax,ymin,ymax][xmin,xmax,ymin,ymax] is used by fcontour().

fcontour(f, D, 'LevelList', -12:1:12)

axis equal

hold on

fcontour() draws level curves (contour lines) where f(x,y)f(x,y) is constant.

'LevelList', -12:1:12 → draw contours for f=−12,−11,…,12f=−12,−11,…,12.

axis equal → same scale on x and y axes'

```
h = fimplicit(g);

set(h,'Color',[1,0.7,0.9])
```

fimplicit(g) draws the curve g(x,y)=0g(x,y)=0.

Sets its color to light pink.

hold on → allows us to add more plots without erasing this one.

```
for i = 1:length(T)
   fprintf('The function f(x,y) takes on its extreme value on the g(x,y) at  (%1.3f,%1.3f).', ax(i),
ay(i))
   fprintf('The value of the function is %1.3f\n', T(i))
   plot3(ax(i), ay(i), T(i), 'k.', 'markersize', 15)
end
```

Loops through each solution point.

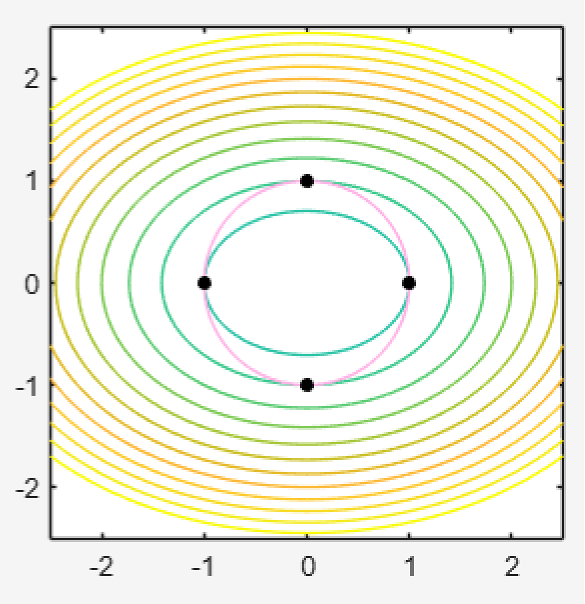
- `fprintf()` prints:
- The coordinates of the point.
- The value of f(x,y) there.
- `plot3()` plots the point in **3D space**:
- x = ax(i)
- y = ay(i)
- z = f(x,y) value.

Examples:
Find the extreme values of the function $f(x, y) = x^2 + 2y^2$,
on the circle $x^2 + y^2 = 1$
Command window: *

```
Enter f(x,y) to be extremized : x^2+2*y^2
Enter the constraint function g(x,y) : x^2+y^2-1

The function f(x,y) takes on its extreme value on the g(x,y) at
 (-1.000,0.000).The value of the function is 1.000
The function f(x,y) takes on its extreme value on the g(x,y) at
 (1.000,0.000).The value of the function is 1.000
The function f(x,y) takes on its extreme value on the g(x,y) at
 (0.000,-1.000).The value of the function is 2.000
The function f(x,y) takes on its extreme value on the g(x,y) at
 (0.000,1.000).The value of the function is 2.000
```

Figure:



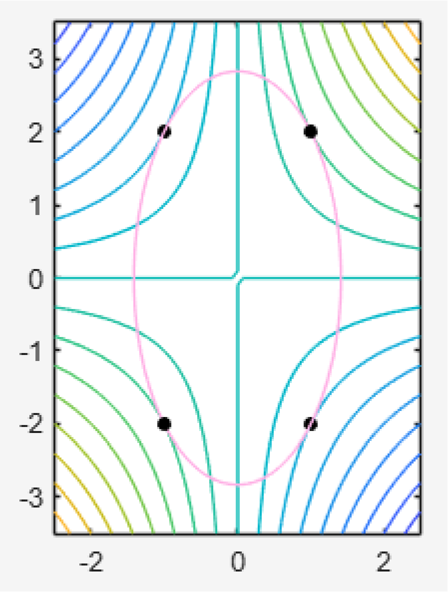
The extreme values of $f(x,y) = x^2 + 2y^2$, correspond to the level curves that touch the circle $x^2 + y^2 = 1$

Example 2:

Find the extreme values of the function $f(x,y) = xy$, on the circle $4x^2 + y^2 = 8$

Command window

```
Enter f(x,y) to be extremized : x*y
Enter the constraint function g(x,y) : 4*x^2+y^2-8
The function f(x,y) takes on its extreme value on the g(x,y) at  (1.000,-2.000).The
value of the function is -2.000
The function f(x,y) takes on its extreme value on the g(x,y) at  (-1.000,2.000).The
value of the function is -2.000
The function f(x,y) takes on its extreme value on the g(x,y) at  (-1.000,-2.000).The
value of the function is 2.000
The function f(x,y) takes on its extreme value on the g(x,y) at  (1.000,2.000).The
value of the function is 2.000
```



Exercise 1

Find the extreme values of the function $f(x,y) = 3x + 4y$, on the circle $x^2+y^2=1$

```
Lagrange multiplier method (three variables)
clc
clear all
syms x y z lam real
f= input('Enter f(x,y,z) to be extremized : ');
g= input('Enter the constraint function g(x,y,z) : ');
F=f-lam*g
Fd=jacobian(F,[x y z lam])
[ax,ay,az,alam]=solve(Fd,x,y,z,lam);
ax=double(ax)
ay=double(ay)
az=double(az)
T = subs(f,{x,y,z},{ax,ay,az})
T=double(T);
 for i = 1:length(T);
fprintf('The function f(x,y,z) takes on its extreme value on the g(x,y,z) at
(%1.3f,%1.3f,%1.3f).',ax(i),ay(i),az(i))
fprintf('The value of the function is %1.3f\n',T(i))
end
```

Example:
Find the points on the sphere $x^2+y^2+z^2=4$ that are closest to and farthest from the point (3, 1, -1).

Command window:

```
Enter f(x,y,z) to be extremized : (x-3)^2+(y-1)^2+(z+1)^2
Enter the constraint function g(x,y,z) : x^2+y^2+z^2-4
The function f(x,y,z) takes on its extreme value on the g(x,y,z) at  (1.809,0.603,-
0.603).The value of the function is 1.734
The function f(x,y,z) takes on its extreme value on the g(x,y,z) at  (-1.809,-
0.603,0.603).The value of the function is 28.266
```

<u>The following code considers restriction on the variables (x,y,z) >=0 .</u>

Example: A rectangular box without a lid is to be made from $12m^2$ of cardboard. Find the maximum volume of such a box. (Dimensions positive)

```
clc
clear all
syms x y z lam real
f= input('Enter f(x,y,z) to be extremized : ');
g= input('Enter the constraint function g(x,y,z) : ');
F=f-lam*g
Fd=jacobian(F,[x y z lam])
equ1=x>=0
equ2=y>=0
equ3=z>=0
eqns=[equ1,equ2,equ3,Fd]
[ax,ay,az,alam]=solve(eqns,x,y,z,lam);
ax=double(ax)
ay=double(ay)
az=double(az)
T = subs(f,{x,y,z},{ax,ay,az})
T=double(T);
 for i = 1:length(T);
fprintf('The function f(x,y,z) takes on its extreme value on the g(x,y,z) at
```

```
(%1.3f,%1.3f,%1.3f).',ax(i),ay(i),az(i))
fprintf('The value of the function is %1.3f\n',T(i))
end
```

Example

A rectangular box without a lid is to be made from $12m^2$ of cardboard. Find the maximum volume of such a box.

Command window:

Enter f(x,y,z) to be extremized : x*y*z

Enter the constraint function g(x,y,z) : x*y+2*y*z+2*x*z-12

The function f(x,y,z) takes on its extreme value on the g(x,y,z) at (2.000,2.000,1.000).The valuex of the function is 4.000.

Exercise 1

A rectangular box, open at the top, is to have a volume of 32 c.c. Find the dimensions of the box, that requires the least material for its construction