Tutorial Sheet No. 2B(2)

1. Find bases for row and column spaces of the following matrix

$$\underset{\sim}{A} = \begin{bmatrix} 1 & -3 & 4 & -2 & 5 & 4 \\ 2 & -6 & 9 & -1 & 8 & 2 \\ 2 & -6 & 9 & -1 & 9 & 7 \\ -1 & 3 & -4 & 2 & -5 & -4 \end{bmatrix}.$$

2. Find a basis for the subspace of $\mathbb{R}^5$ spanned by the vectors:

$\underset{\sim}{v_1} = (1, -2, 0, 0, 3)$, $\underset{\sim}{v_2} = (2, -5, -3, -2, 6)$

$\underset{\sim}{v_3} = (0, 5, 15, 10, 0)$, $\underset{\sim}{v_4} = (2, 6, 18, 8, 6)$.

(Hint: Consider the matrix $\underset{\sim}{A}$ whose rows are the given vectors. The required basis is a basis of the row space of $\underset{\sim}{A}$.)

3. Find a basis for the row space of the following matrix

$$\underset{\sim}{A} = \begin{bmatrix} 1 & -2 & 0 & 0 & 3 \\ 2 & -5 & -3 & -2 & 6 \\ 0 & 5 & 15 & 10 & 0 \\ 2 & 6 & 18 & 8 & 6 \end{bmatrix}$$

Consisting entirely of the row vectors of $\underset{\sim}{A}$.

(Hint: consider the row-echelon form of the transpose $\underset{\sim}{A}^t$ of $\underset{\sim}{A}$ and look for the basis of its column space.)

Q.1 Row-echelon form of $\underset{\sim}{A}$ is

$$\hat{\underset{\sim}{A}} = \begin{bmatrix} 1 & -3 & 4 & -2 & 5 & 4 \\ 0 & 0 & 1 & 3 & -2 & -6 \\ 0 & 0 & 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Basis for the row space of $\underset{\sim}{A}$ are the nonzero rows of $\hat{\underset{\sim}{A}}$

$[1 \quad -3 \quad 4 \quad -2 \quad 5 \quad 4]$

$[0 \quad 0 \quad 1 \quad 3 \quad -2 \quad -6]$

$[0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 5]$ (Remember: $r(\underset{\sim}{A}) = r(\hat{\underset{\sim}{A}})$)

The basis for Column space of the row-echelon form $\hat{\underset{\sim}{A}}$ of $\underset{\sim}{A}$

Consists of the columns $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 4 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 5 \\ -2 \\ 1 \\ 0 \end{bmatrix}$

The corresponding column vectors of $\underset{\sim}{A}$ ~~are~~

$$\begin{bmatrix} 1 \\ 2 \\ 2 \\ -1 \end{bmatrix} \quad \begin{bmatrix} 4 \\ 9 \\ 9 \\ -4 \end{bmatrix} \quad \begin{bmatrix} 5 \\ 8 \\ 9 \\ -5 \end{bmatrix}$$

form a basis for the column space $c(\underset{\sim}{A})$ of $\underset{\sim}{A}$.

Q. 2 The space spanned by the vectors is the row space $r(\underset{\sim}{A})$ of the matrix

$$\underset{\sim}{A} = \begin{bmatrix} 1 & -2 & 0 & 0 & 3 \\ 2 & -5 & -3 & -2 & 6 \\ 0 & 5 & 15 & 10 & 0 \\ 2 & 6 & 18 & 8 & 6 \end{bmatrix}$$

whose row echelon form is

$$\underset{\sim}{R} = \begin{bmatrix} 1 & -2 & 0 & 0 & 3 \\ 0 & 1 & 3 & 2 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Basis for the row space $r(\underset{\sim}{R})$ consists of the non-zero rows

$(1, -2, 0, 0, 3)$, $(0, 1, 3, 2, 0)$ and $(0, 0, 1, 1, 0)$

Since the row space $r(\underset{\sim}{A})$ is the same as the row space $r(\underset{\sim}{R})$ of $\underset{\sim}{R}$., The basis for the space spanned by the given vectors consists of the three vectors as above.

3. We have

$$\underset{\sim}{A}^t = \begin{bmatrix} 1 & 2 & 0 & 2 \\ -2 & -5 & 5 & 6 \\ 0 & -3 & 15 & 18 \\ 0 & -2 & 10 & 8 \\ 3 & 6 & 0 & 6 \end{bmatrix}$$

Its row-echelon form is

$$\begin{bmatrix} 1 & 2 & 0 & 2 \\ 0 & 1 & -5 & -10 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The columns that are linearly independent ~~are~~ in the row-echelon form are 1st, 2nd and 4Th. The corresponding columns in $\underset{\sim}{A}^t$

$$\underset{\sim}{c}_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \\ 0 \\ 3 \end{bmatrix} \qquad \underset{\sim}{c}_2 = \begin{bmatrix} 2 \\ -5 \\ -3 \\ -2 \\ 6 \end{bmatrix} \qquad \underset{\sim}{c}_4 = \begin{bmatrix} 2 \\ 6 \\ 18 \\ 8 \\ 6 \end{bmatrix}$$

form a basis for the column space of $\underset{\sim}{A}^t$ = the row space of $\underset{\sim}{A}$