

$$x[n] = (0.5)^n \cos\left(\frac{\pi n}{3}\right) u[n]$$

$$(0.5)^n u[n] \longrightarrow \frac{1}{1 - 0.5z^{-1}}$$

$$\cos(\omega n) x[n] \longrightarrow \frac{1}{2} X\left(\frac{z}{z_0}\right) + \frac{1}{2} X\left(\frac{z}{z_1}\right), \quad z_0 = e^{j\omega}, z_1 = e^{-j\omega}$$

$$x[n] \longrightarrow X(z) = \frac{0.5}{1 - 0.5\left(ze^{\frac{j\pi}{3}}\right)^{-1}} + \frac{0.5}{1 - 0.5\left(ze^{-\frac{j\pi}{3}}\right)^{-1}}$$

```

1 import numpy as np
2
3 #part 1a
4 N = 8
5 n = np.arange(N)
6 x = ((0.5)**n)*np.cos(np.pi*n/3)

```

```

1 #part 1b
2 delta = np.zeros(N)
3 delta[0] = 1

```

```

1 #part 1c
2 z = np.arange(1, N+1)
3 X = (0.5/(1-0.5*(z*np.exp(1j*np.pi/3))**(-1))) + (0.5/(1-0.5*(z*np.exp(-1j*np.pi/3))**(-1)))

```

```

1 import matplotlib.pyplot as plt
2 from scipy import signal
3
4 y = signal.lfilter(np.array([1]), X, delta)
5
6 plt.figure()
7 plt.subplot(211)
8 plt.stem(x)
9 plt.subplot(212)
10 plt.stem(y)

```

part e :

در حقیقت تبدیل زد سیگنال، یک سیگنال مختلط است و چون خود زد مختلط است به همین دلیل خروجی شبیه به سیگنال ورودی نشده است

```
return array(a.dtype, copy=False, order=order)
```

Part 2:

 $H(z) :$

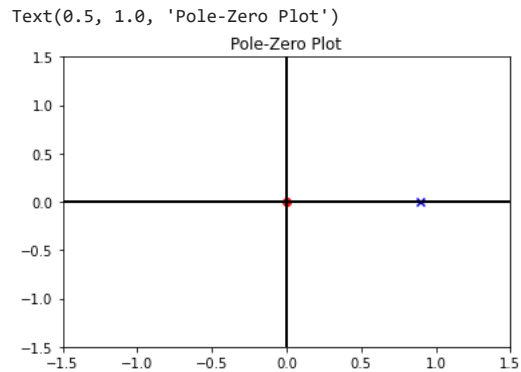
7. כ.ט.

$$y[n] - 0.9 y[n - 1] = x[n] \quad \text{ztrans} \rightarrow Y(z) - 0.9 Y(z)z^{-1} = X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - 0.9z^{-1}}$$

1

```
1 #part 2a
2 import matplotlib.pyplot as plt
3
4 a = [1, -0.9]
5 b = [1, 0]
6 plt.figure()
7 z,p,_ = signal.tf2zpk(b, a)
8 plt.scatter(np.real(p), np.imag(p), c='b', marker='x')
9 plt.scatter(np.real(z), np.imag(z), c='r', marker='o')
10 plt.axhline(0, color='black', lw=2)
11 plt.axvline(0, color='black', lw=2)
12 plt.xlim([-1.5, 1.5])
13 plt.ylim([-1.5, 1.5])
14 plt.title("Pole-Zero Plot")
15
```

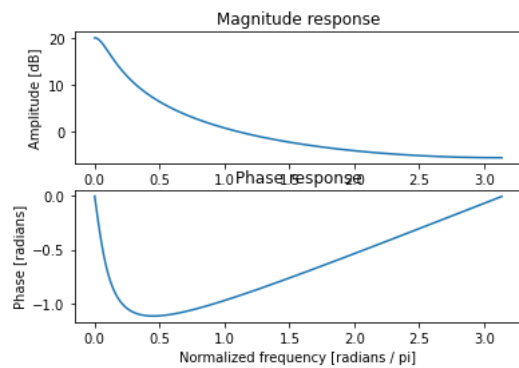


```
1 #part 2b
2 N = 8
3 delta = np.zeros(N)
4 delta[0] = 1
5
6 a = [1, -0.9]
7 b = [1, 0]
8
9 fig, axs = plt.subplots(2,1)
10 w, h = signal.freqz(b, a)
11 axs[0].plot(w, 20 * np.log10(abs(h)))
12 axs[0].set_title('Magnitude response')
13 axs[0].set_xlabel('Normalized frequency [radians / pi]')
14 axs[1].set_ylabel('Amplitude [dB]')
15
```

```

16 axs[1].plot(w, np.angle(h))
17 axs[1].set_title('Phase response')
18 axs[1].set_xlabel('Normalized frequency [radians / pi]')
19 axs[1].set_ylabel('Phase [radians]')
20 plt.show()
21

```



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