Department of Engineering/Informatics, King's College London
Pattern Recognition, Neural Networks and Deep Learning

(7CCSMPNN)
Assignment: Support Vector Machines (SVMs)

Q1. Write down the first 7 digits of your student ID as s1s2s3s4s5s6s7.

2012475

Q2. Find R1 which is the remainder of $\frac{s1+s2+s3+s4+s5+s6+s7}{4}$. Table 1 shows the multiclass methods to be used corresponding to the value of R1 obtained.

R1 = remainder of $\frac{2+0+1+2+4+7+5}{4}$

= remainder of $\frac{21}{4}$

= 1

Therefore the method is "**one against all**".

Q3. Create a linearly separable two-dimensional dataset of your own, which consists of 3 classes. List the dataset in the format as shown in Table 2. Each class should contain at least 10 samples and all three classes have the same number of samples. Note: This is your own created dataset. The chance of having the same dataset in other submissions is slim. Do not share your dataset with others to avoid any plagiarism/collusion issues.

Presented a synthetic dataset of flowers with two-dimensions: (petal length, petal width) in millimeters.

Table 2

Sample of Class 1 (Daffodils)	Sample of Class 2 (Tulip)	Sample of Class 3 (Lotus)
(8, 4.5) (5, 3.8) (3, 4.5) (5, 3) (3, 4.1)	(13, 1.7) (10, 2) (18, 1.8) (17, 0.2) (12, 1.2)	(21, 4.5) (23, 3) (25, 3.5) (27, 3.8) (23, 4.5)

(6, 4.8)	(11, 1.5)	(24, 3.5)
(3, 4.8)	(19, 1)	(25, 4.5)
(6, 4)	(17, 1.6)	(29, 4.5)
(7, 4.5)	(14, 1.2)	(22, 3.8)
(5, 4)	(13, 1.5)	(26, 3.7)

Q4. Plot the dataset in Q3 to show that the samples are linearly separable. Explain why your dataset is linearly separable. Hint: the Matlab built-in function plot can be used and show some example hyperplanes which can linearly separable the datasets. Identify which hyperplane is for which classes. (20 Marks)

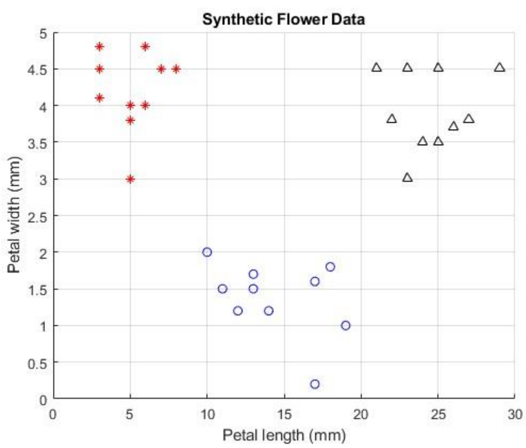

Fig.3.a. Sample Synthetic Data

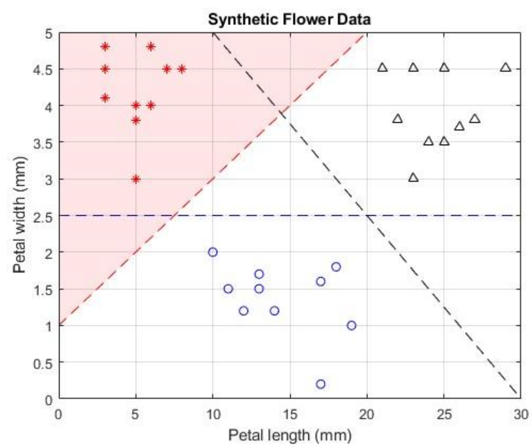

Fig.3.b. Linearly separating Hyperplanes

As shown in Fig.3.a. The synthetic flower data set created for the purpose of this assignment can be classified into 3 linearly separable classes (Daffodils (Class 1), Tulip (Class 2) and Lotus (Class 3)).

Fig.3.b. Shows an example hyperplane of linear kernel which can be used to classify new unseen data. For e.g. if we get a new data set with (petal length = 7, petal width = 4.5), this can be classified into "Daffodils (Class 1)" using the hyperplane as it is fitting into the position of the shaded area for "one against all" classifier method.

Q5. According to the method obtained in Q2, draw a block diagram at SVM level to show the structure of the multi-class classifier constructed by linear SVMs. Explain the design (e.g., number of inputs, number of outputs, number of SVMs used, class label assignment, etc.) and describe how this multi-class classifier works.
Remark: A blocking diagram is a diagram which is used to, say, show a concept or a structure, etc. Here in this question, a diagram is used to show the structure of the multi-class SVM classifier, i.e., how to put binary SVM classifiers together to work as a multi-class SVM classifier. For example, Q5 of tutorial 8 is an example of a block diagram at SVM level. Neural network diagram is a kind of diagram to show its structure at the neuron level. The block diagrams in lecture 9 are to show the architecture of ensemble classifiers, etc. (20 Marks)

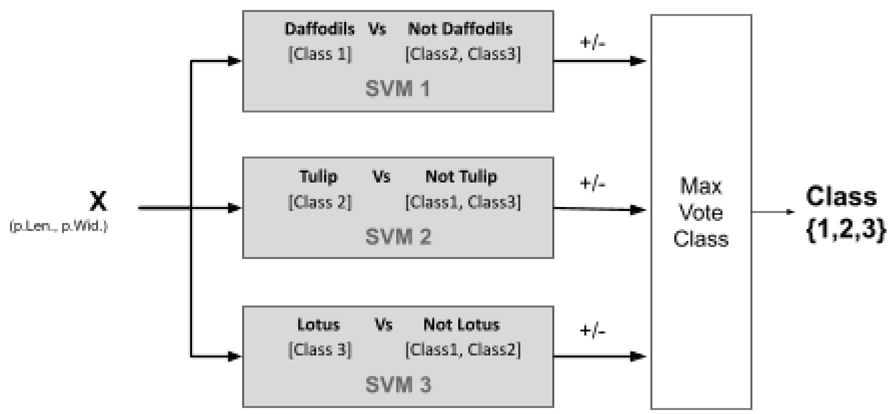

Fig.5. “one against all” SVM classifier block diagram

Fig.5. Shows the block diagram of our “one against all” SVM classifier. It takes an input “X” of 2 dimensions (petal length, petal width) and binary classify using each SVM machines designed to classify the particular class or not. The classifier has 3 “one against all” SVM machines as we target to classify data into three distinct classes (Daffodils, Tulip and Lotus). The majority votes (+) obtained for the class is chosen to be the final predicted class. Output from the classifier is an index {1:Daffodils, 2:Tulip and 3:Lotus} for the predicted class.

Q6. According to your dataset in Q3 and the design of your multi-class classifier in Q5, identify the support vectors of the linear SVMs by "inspection" and design their hyperplanes by hand. Show the calculations and explain the details of your design. (20 Marks)

Class 1: Daffodils SVM

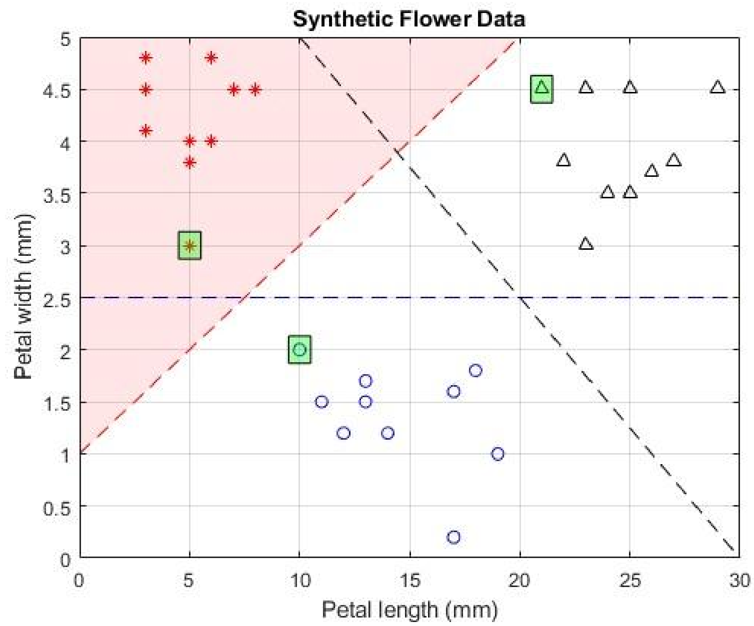

Fig.6.1.a. "One against all" support vectors for class 1: Daffodils

Fig.6.1.a. shows the identified support vectors marked in "green" by inpection for the SVM classifier of Daffodils class.

Define the labels for class 1: Daffodils as '+1' and not class 1: not Daffodils as '-1':

Identified Support Vectors (Class 1: Daffodils)	Label (Class 1: Daffodils)
$x_1 = (5,3)$ $x_2 = (21,4.5)$ $x_3 = (10,2)$	$y_1 = +1$ $y_2 = -1$ $y_3 = -1$

Define the linear hyperplane for the class 1: Daffodils SVM:

$$w^T x + w_0 = 0$$

According to the $\frac{\partial \mathscr{L}(\mathbf{w}, w_0, \lambda)}{\partial \mathbf{w}} = \mathbf{0} \quad \Rightarrow \quad \mathbf{w} = \sum_{i=1}^{N} \lambda_i y_i \mathbf{x}_i$

$$w = \lambda_1 y_1 x_1 + \lambda_2 y_2 x_2 + \lambda_3 y_3 x_3$$

$$w = \lambda_1 \begin{pmatrix} 5 \\ 3 \end{pmatrix} - \lambda_2 \begin{pmatrix} 21 \\ 4.5 \end{pmatrix} - \lambda_3 \begin{pmatrix} 10 \\ 2 \end{pmatrix} \tag{1.1}$$

According to $\lambda_i\left(y_i(\mathbf{w}^T\mathbf{x}_i+w_0)-1\right)=0,\quad i=1,2,\ldots,N$, recall that for support vectors

$$y_i\left(w^T x_i + w_0\right) = 1$$

$$y_i\left(\left(\lambda_1\begin{pmatrix}5\\3\end{pmatrix} - \lambda_2\begin{pmatrix}21\\4.5\end{pmatrix} - \lambda_3\begin{pmatrix}10\\2\end{pmatrix}\right)^T x_i + w_0\right) = 1$$

i = 1:

$$\left(\left(\lambda_1\begin{pmatrix}5\\3\end{pmatrix} - \lambda_2\begin{pmatrix}21\\4.5\end{pmatrix} - \lambda_3\begin{pmatrix}10\\2\end{pmatrix}\right)^T \begin{pmatrix}5\\3\end{pmatrix} + w_0\right) = 1$$

$$34\lambda_1 - 118.5\lambda_2 - 56\lambda_3 + w_0 = 1 \tag{1.2}$$

i = 2:

$$-\left(\left(\lambda_1\begin{pmatrix}5\\3\end{pmatrix} - \lambda_2\begin{pmatrix}21\\4.5\end{pmatrix} - \lambda_3\begin{pmatrix}10\\2\end{pmatrix}\right)^T \begin{pmatrix}21\\4.5\end{pmatrix} + w_0\right) = 1$$

$$118.5\lambda_1 - 461.25\lambda_2 - 219\lambda_3 + w_0 = -1 \tag{1.3}$$

i = 3:

$$-\left(\left(\lambda_1\begin{pmatrix}5\\3\end{pmatrix} - \lambda_2\begin{pmatrix}21\\4.5\end{pmatrix} - \lambda_3\begin{pmatrix}10\\2\end{pmatrix}\right)^T \begin{pmatrix}10\\2\end{pmatrix} + w_0\right) = 1$$

$$56\lambda_1 - 219\lambda_2 - 104\lambda_3 + w_0 = -1 \tag{1.4}$$

According to $\frac{\partial \mathscr{L}(\mathbf{w}, w_0, \lambda)}{\partial w_0} = 0 \Rightarrow \sum_{i=1}^{N} \lambda_i y_i = 0$

$$\lambda_1 - \lambda_2 - \lambda_3 = 0 \tag{1.5}$$

Eqn. (1.2), (1.3), (1.4) and (1.5),

$$\begin{bmatrix} 34 & -118.5 & -56 & 1 \\ 118.5 & -461.25 & -219 & 1 \\ 56 & -219 & -104 & 1 \\ 1 & -1 & -1 & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ w_0 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 0 \end{bmatrix}$$

λ_1 = 0.46, λ_2 = -0.19, λ_3 = 0.65, and w_0 = -0.75

Eqn. (1.1),

$$w = 0.46\begin{pmatrix}5\\3\end{pmatrix} + 0.19\begin{pmatrix}21\\4.5\end{pmatrix} - 0.65\begin{pmatrix}10\\2\end{pmatrix}$$

$$w = \begin{pmatrix}2.3\\1.38\end{pmatrix} + \begin{pmatrix}3.99\\0.855\end{pmatrix} - \begin{pmatrix}6.5\\1.3\end{pmatrix}$$

$$w = \begin{pmatrix} -0.21 \\ 0.935 \end{pmatrix}$$

Therefore hyperplane equation is,

$$\begin{pmatrix} -0.21 \\ 0.935 \end{pmatrix}^T x - 0.75 = 0\,, \qquad where\ x = \begin{pmatrix} petal\,length \\ petal\,width \end{pmatrix}$$

-0.21 * (petal length) + 0.935* (petal width) - 0.75 = 0

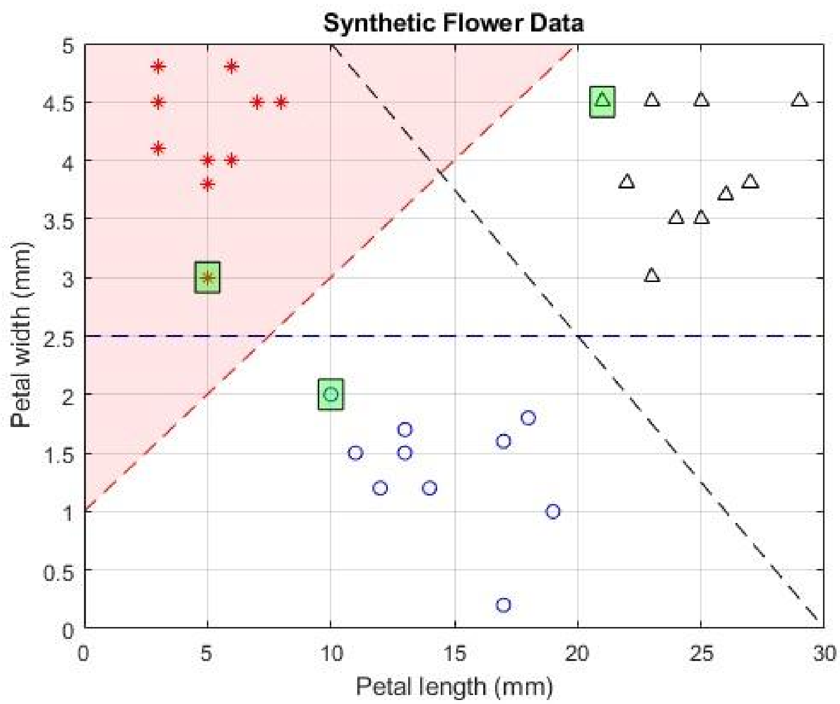

Fig.6.1.b. Derived “One against all” support vectors for class 1: Daffodils

Class 2: Tulip SVM

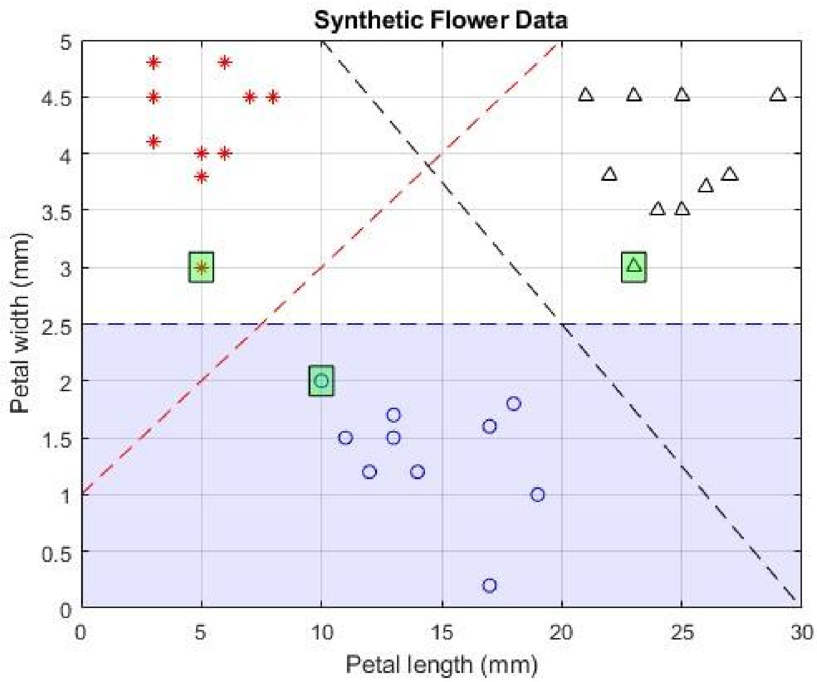

Fig.6.2.a. “One against all” support vectors for class 2: Tulip

Fig.6.2.a. shows the identified support vectors marked in “green” by inpection for the SVM classifier of Tulip class.

Define the labels for class 2: Tulip as ‘+1’ and not class 2: not Tulip as ‘-1’:

Identified Support Vectors (Class 2: Tulip)	Label (Class 2: Tulip)
x_1 = (5,3) x_2 = (23,3) x_3 = (10,2)	y_1 = -1 y_2 = -1 y_3 = +1

Define the linear hyperplane for the class 2: Tulip SVM:

$$w^T x + w_0 = 0$$

According to the $\frac{\partial \mathscr{L}(\mathbf{w}, w_0, \lambda)}{\partial \mathbf{w}} = \mathbf{0} \quad \Rightarrow \quad \mathbf{w} = \sum_{i=1}^{N} \lambda_i y_i \mathbf{x}_i$

$$w = \lambda_1 y_1 x_1 + \lambda_2 y_2 x_2 + \lambda_3 y_3 x_3$$

$$w = -\lambda_1 \begin{pmatrix} 5 \\ 3 \end{pmatrix} - \lambda_2 \begin{pmatrix} 23 \\ 3 \end{pmatrix} + \lambda_3 \begin{pmatrix} 10 \\ 2 \end{pmatrix} \tag{2.1}$$

According to $\lambda_i\left(y_i(\mathbf{w}^T\mathbf{x}_i + w_0) - 1\right) = 0, \quad i = 1,2,\ldots,N$, recall that for support vectors

$$y_i\left(w^T x_i + w_0\right) = 1$$

$$y_i\left(\left(-\lambda_1\begin{pmatrix}5\\3\end{pmatrix} - \lambda_2\begin{pmatrix}23\\3\end{pmatrix} + \lambda_3\begin{pmatrix}10\\2\end{pmatrix}\right)^T x_i + w_0\right) = 1$$

i = 1:

$$-\left(\left(-\lambda_1\begin{pmatrix}5\\3\end{pmatrix} - \lambda_2\begin{pmatrix}23\\3\end{pmatrix} + \lambda_3\begin{pmatrix}10\\2\end{pmatrix}\right)^T \begin{pmatrix}5\\3\end{pmatrix} + w_0\right) = 1$$

$$34\lambda_1 + 124\lambda_2 - 56\lambda_3 - w_0 = 1 \quad (2.2)$$

i = 2:

$$-\left(\left(-\lambda_1\begin{pmatrix}5\\3\end{pmatrix} - \lambda_2\begin{pmatrix}23\\3\end{pmatrix} + \lambda_3\begin{pmatrix}10\\2\end{pmatrix}\right)^T \begin{pmatrix}23\\3\end{pmatrix} + w_0\right) = 1$$

$$124\lambda_1 + 538\lambda_2 - 236\lambda_3 - w_0 = 1 \quad (2.3)$$

i = 3:

$$\left(\left(-\lambda_1\begin{pmatrix}5\\3\end{pmatrix} - \lambda_2\begin{pmatrix}23\\3\end{pmatrix} + \lambda_3\begin{pmatrix}10\\2\end{pmatrix}\right)^T \begin{pmatrix}10\\2\end{pmatrix} + w_0\right) = 1$$

$$-56\lambda_1 - 236\lambda_2 + 104\lambda_3 + w_0 = 1 \quad (2.4)$$

According to $\frac{\partial \mathscr{L}(\mathbf{w}, w_0, \lambda)}{\partial w_0} = 0 \Rightarrow \sum_{i=1}^{N} \lambda_i y_i = 0$

$$-\lambda_1 - \lambda_2 + \lambda_3 = 0 \quad (2.5)$$

Eqn. (2.2), (2.3), (2.4) and (2.5),

$$\begin{bmatrix} 34 & 124 & -56 & -1 \\ 124 & 538 & -236 & -1 \\ -56 & -236 & 104 & 1 \\ -1 & -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ w_0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

$\lambda_1 = 1.4444$, $\lambda_2 = 0.5556$, $\lambda_3 = 2$, and $w_0 = 5$

Eqn. (2.1),

$$w = -1.4444\begin{pmatrix}5\\3\end{pmatrix} - 0.5556\begin{pmatrix}23\\3\end{pmatrix} + 2\begin{pmatrix}10\\2\end{pmatrix}$$

$$w = -\begin{pmatrix}7.222\\4.3332\end{pmatrix} - \begin{pmatrix}12.7788\\1.6668\end{pmatrix} + \begin{pmatrix}20\\4\end{pmatrix}$$

$$w = \begin{pmatrix} -0.008 \\ -2 \end{pmatrix}$$

Therefore hyperplane equation is,

$$\begin{pmatrix} -0.008 \\ -2 \end{pmatrix}^T x + 5 = 0\,, \qquad where\ x = \begin{pmatrix} petal\,length \\ petal\,width \end{pmatrix}$$

-0.008 * (petal length) - 2* (petal width) + 5 = 0

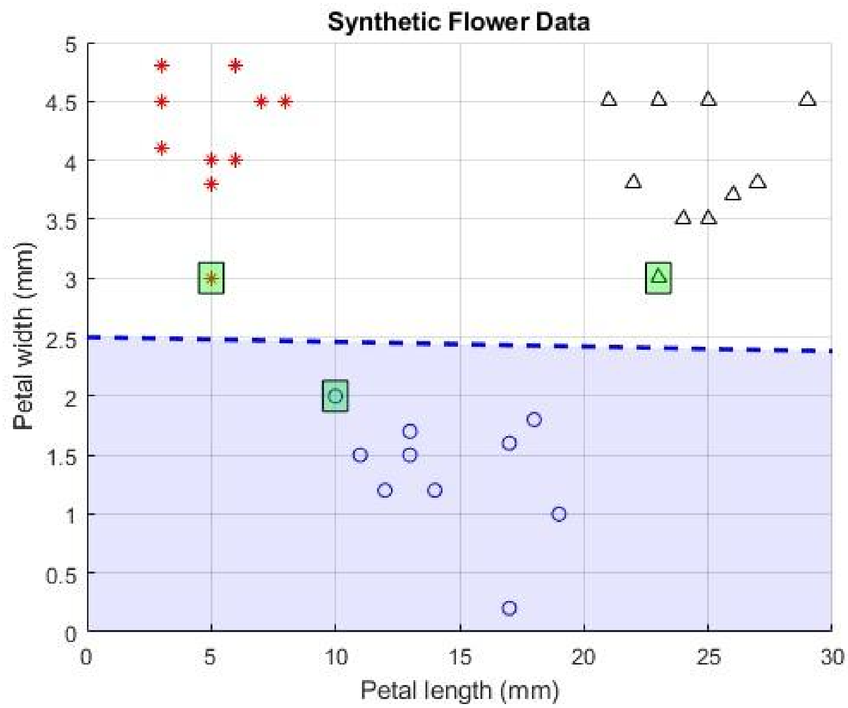

Fig.6.2.b. Derived “One against all” support vectors for class 2: Tulip

Class 3: Lotus SVM

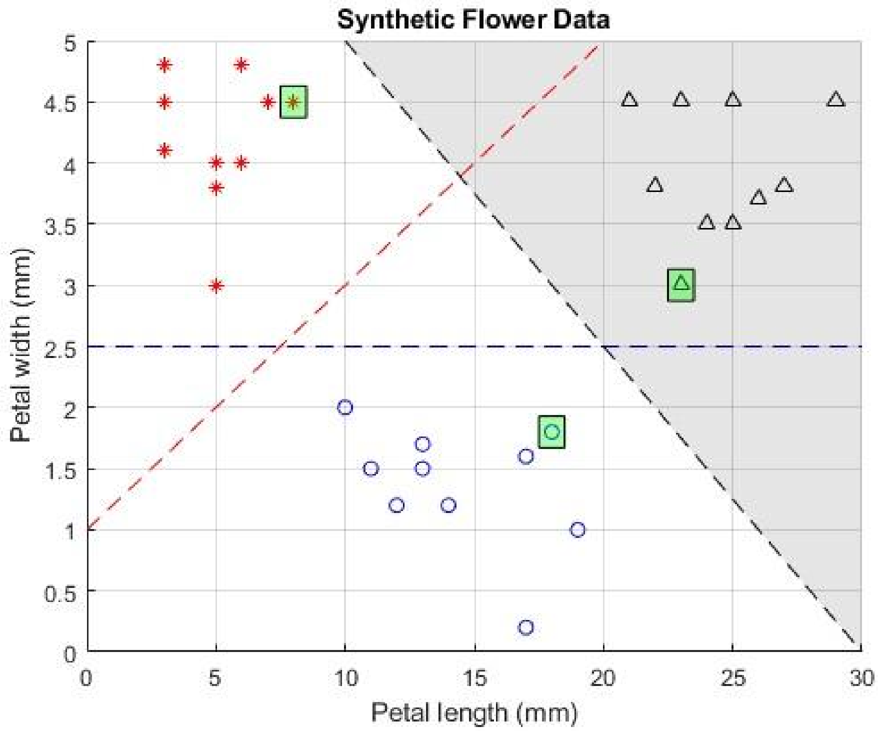

Fig.6.3.a. "One against all" support vectors for class 3: Lotus

Fig.6.3.a. shows the identified support vectors marked in "green" by inpection for the SVM classifier of Lotus class.

Define the labels for class 3: Lotus as '+1' and not class 3: not Lotus as '-1':

Identified Support Vectors (Class 2: Tulip)	Label (Class 2: Tulip)
x_1 = (8,4.5) x_2 = (23,3) x_3 = (18,1.8)	y_1 = -1 y_2 = +1 y_3 = -1

Define the linear hyperplane for the class 2: Tulip SVM:

$$w^T x + w_0 = 0$$

According to the $\frac{\partial \mathscr{L}(\mathbf{w}, w_0, \lambda)}{\partial \mathbf{w}} = \mathbf{0} \quad \Rightarrow \quad \mathbf{w} = \sum_{i=1}^{N} \lambda_i y_i \mathbf{x}_i$

$$w = \lambda_1 y_1 x_1 + \lambda_2 y_2 x_2 + \lambda_3 y_3 x_3$$

$$w = -\lambda_1\begin{pmatrix}8\\4.5\end{pmatrix} + \lambda_2\begin{pmatrix}23\\3\end{pmatrix} - \lambda_3\begin{pmatrix}18\\1.8\end{pmatrix} \tag{3.1}$$

According to $\lambda_i(y_i(\mathbf{w}^T\mathbf{x}_i + w_0) - 1) = 0, \quad i = 1, 2, \ldots, N$, recall that for support vectors

$$y_i\left(w^T x_i + w_0\right) = 1$$

$$y_i\left(\left(-\lambda_1\begin{pmatrix}8\\4.5\end{pmatrix} + \lambda_2\begin{pmatrix}23\\3\end{pmatrix} - \lambda_3\begin{pmatrix}18\\1.8\end{pmatrix}\right)^T x_i + w_0\right) = 1$$

i = 1:

$$-\left(\left(-\lambda_1\begin{pmatrix}8\\4.5\end{pmatrix} + \lambda_2\begin{pmatrix}23\\3\end{pmatrix} - \lambda_3\begin{pmatrix}18\\1.8\end{pmatrix}\right)^T \begin{pmatrix}8\\4.5\end{pmatrix} + w_0\right) = 1$$

$$84.25\lambda_1 - 197.5\lambda_2 + 152.1\lambda_3 - w_0 = 1 \tag{3.2}$$

i = 2:

$$\left(\left(-\lambda_1\begin{pmatrix}8\\4.5\end{pmatrix} + \lambda_2\begin{pmatrix}23\\3\end{pmatrix} - \lambda_3\begin{pmatrix}18\\1.8\end{pmatrix}\right)^T \begin{pmatrix}23\\3\end{pmatrix} + w_0\right) = 1$$

$$-197.5\lambda_1 + 538\lambda_2 - 419.4\lambda_3 + w_0 = 1 \tag{3.3}$$

i = 3:

$$-\left(\left(-\lambda_1\begin{pmatrix}8\\4.5\end{pmatrix} + \lambda_2\begin{pmatrix}23\\3\end{pmatrix} - \lambda_3\begin{pmatrix}18\\1.8\end{pmatrix}\right)^T \begin{pmatrix}18\\1.8\end{pmatrix} + w_0\right) = 1$$

$$152.1\lambda_1 - 419.4\lambda_2 + 327.24\lambda_3 - w_0 = 1 \tag{3.4}$$

According to $\frac{\partial \mathscr{L}(\mathbf{w}, w_0, \lambda)}{\partial w_0} = 0 \Rightarrow \sum_{i=1}^{N} \lambda_i y_i = 0$

$$-\lambda_1 + \lambda_2 - \lambda_3 = 0 \tag{3.5}$$

Eqn. (3.2), (3.3), (3.4) and (3.5),

$$\begin{bmatrix}84.25 & -197.5 & 152.1 & -1\\ -197.5 & 538 & -419.4 & 1\\ 152.1 & -419.4 & 327.24 & -1\\ -1 & 1 & -1 & 0\end{bmatrix}\begin{bmatrix}\lambda_1\\ \lambda_2\\ \lambda_3\\ w_0\end{bmatrix} = \begin{bmatrix}1\\1\\1\\0\end{bmatrix}$$

λ_1 = -0.1438, λ_2 = 0.33, λ_3 = 0.4738, and w_0 = -6.2235

Eqn. (3.1),

$$w = 0.1438\begin{pmatrix}8\\4.5\end{pmatrix} + 0.33\begin{pmatrix}23\\3\end{pmatrix} - 0.4738\begin{pmatrix}18\\1.8\end{pmatrix}$$

$$w = \begin{pmatrix}1.1504\\0.6471\end{pmatrix} + \begin{pmatrix}7.59\\0.99\end{pmatrix} - \begin{pmatrix}8.5284\\0.85284\end{pmatrix}$$

$$w = \begin{pmatrix} 0.212 \\ 0.78426 \end{pmatrix}$$

Therefore hyperplane equation is,

$$\begin{pmatrix} 0.212 \\ 0.78426 \end{pmatrix}^T x - 6.2235 = 0\,, \qquad where\ x = \begin{pmatrix} petal\ length \\ petal\ width \end{pmatrix}$$

0.212 * (petal length) + 0.78426* (petal width) - 6.2235 = 0

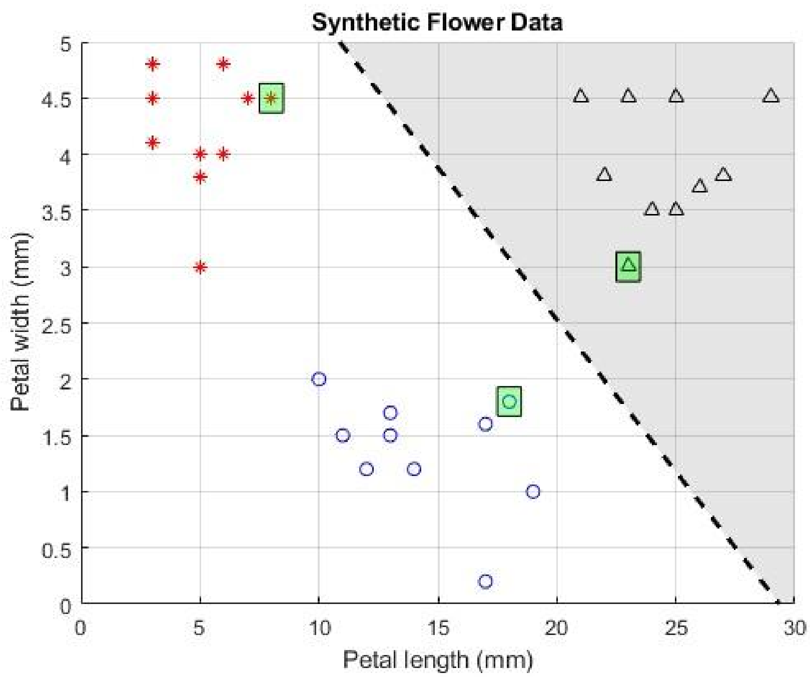

Fig.6.3.b. Derived “One against all” support vectors for class 3: Lotus

Final SVM classifier for three classes is shown below in Fig.6.4.

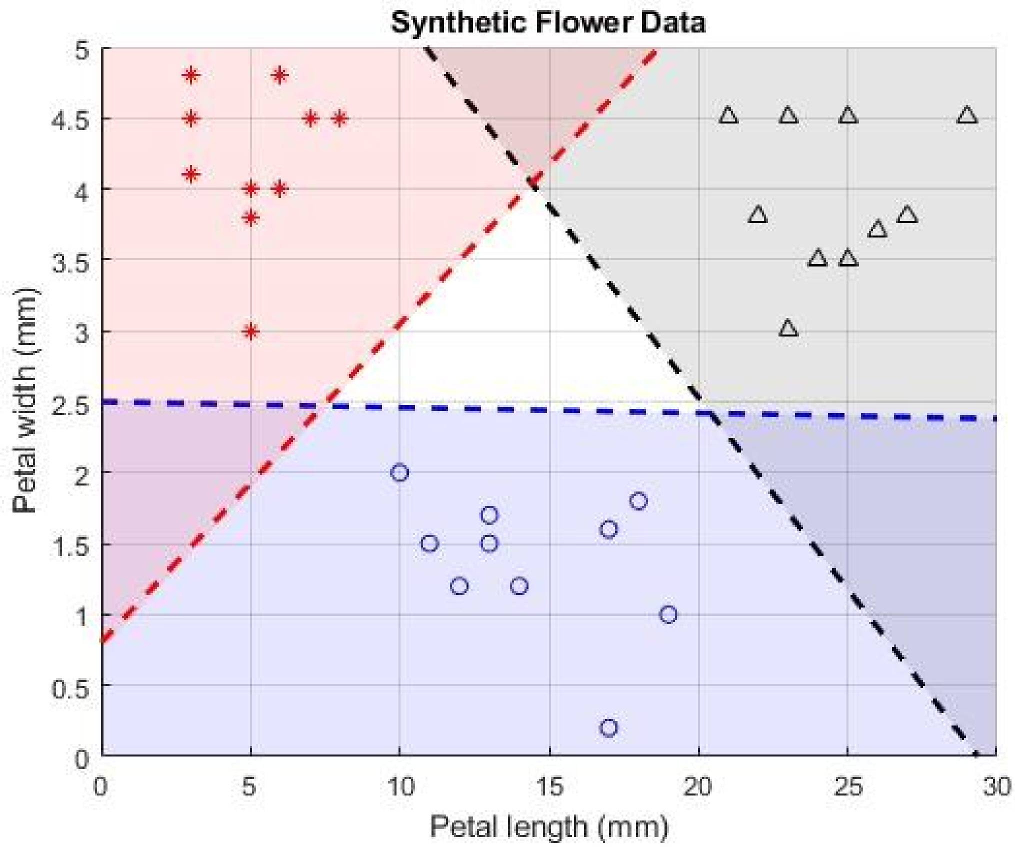

Fig.6.4. Derived “One against all” SVM classifier

Q7. Produce a test dataset by averaging the samples for each row in Table 2, i.e., (sample of class 1 + sample of class 2 + sample of class 3)/3. Summarize the results in the form of Table 3, where N is the number of SVMs in your design and "Classification" is the class determined by your multi-class classifier. Explain how to get the "Classification" column using one test sample. Show the calculations for one or two samples to demonstrate how to get the contents in the table. (20 Marks)

Generate Test data from the sample data (Table 2)

Sample of Class 1 (Daffodils)	Sample of Class 2 (Tulip)	Sample of Class 3 (Lotus)	Test Data
(8, 4.5)	(13, 1.7)	(21, 4.5)	(14, 3.5667)
(5, 3.8)	(10, 2)	(23, 3)	(12.6667, 2.9333)
(3, 4.5)	(18, 1.8)	(25, 3.5)	(15.3333, 3.2667)
(5, 3)	(17, 0.2)	(27, 3.8)	(16.333, 2.3333)
(3, 4.1)	(12, 1.2)	(23, 4.5)	(12.6667, 3.2667)
(6, 4.8)	(11, 1.5)	(24, 3.5)	(13.6667, 3.2667)
(3, 4.8)	(19, 1)	(25, 4.5)	(15.6667, 3.4333)
(6, 4)	(17, 1.6)	(29, 4.5)	(17.3333, 3.3667)
(7, 4.5)	(14, 1.2)	(22, 3.8)	(14.3333, 3.1667)
(5, 4)	(13, 1.5)	(26, 3.7)	(14.6667, 3.0667)

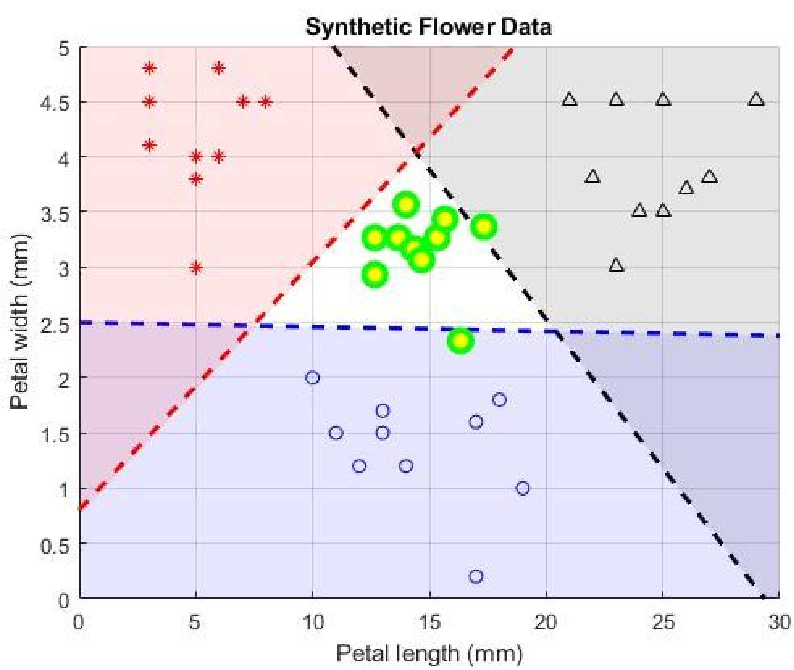

Fig.7.a. Derived Test Data

Using the hard classifier for **SVM 1: Daffodils**

$$f(x) \;=\; sgn\left(\begin{pmatrix}-0.21\\0.935\end{pmatrix}^T x \;-\; 0.75\right) \qquad\qquad where\; x \;=\; \begin{pmatrix}petal\,length\\petal\,width\end{pmatrix}$$

For testdata 4,

$$\begin{aligned}
f(x_{test,4}) \;&=\; sgn\left(\begin{pmatrix}-0.21\\0.935\end{pmatrix}^T \begin{pmatrix}16.333\\2.333\end{pmatrix} \;-\; 0.75\right)\\
f(x_{test,4}) \;&=\; sgn(-1.2485 \;-\; 0.75\,)\\
f(x_{test,4}) \;&=\; sgn(-1.998575)\\
f(x_{test,4}) \;&=\; -1
\end{aligned}$$

Similarly for all other test data points with SVM 1s,

Using the hard classifier for **SVM 2: Tulip**

$$f(x) \;=\; sgn\left(\begin{pmatrix}-0.008\\-2\end{pmatrix}^T x \;+\; 5\right) \qquad\qquad where\; x \;=\; \begin{pmatrix}petal\,length\\petal\,width\end{pmatrix}$$

For testdata 4,

$$\begin{aligned}
f(x_{test,4}) \;&=\; sgn\left(\begin{pmatrix}-0.008\\-2\end{pmatrix}^T \begin{pmatrix}16.333\\2.333\end{pmatrix} \;+\; 5\right)\\
f(x_{test,4}) \;&=\; sgn(-4.796664 \;+\; 5\,)\\
f(x_{test,4}) \;&=\; sgn(0.203336)\\
f(x_{test,4}) \;&=\; +1
\end{aligned}$$

Using the hard classifier for **SVM 3: Lotus**

$$f(x) \;=\; sgn\left(\begin{pmatrix}0.212\\0.78426\end{pmatrix}^T x \;-\; 6.2235\right) \qquad\qquad where\; x \;=\; \begin{pmatrix}petal\,length\\petal\,width\end{pmatrix}$$

For testdata 4,

$$\begin{aligned}
f(x_{test,4}) \;&=\; sgn\left(\begin{pmatrix}0.212\\0.78426\end{pmatrix}^T \begin{pmatrix}16.333\\2.333\end{pmatrix} \;-\; 6.2235\right)\\
f(x_{test,4}) \;&=\; sgn(5.29227458 - \;6.2235\,)\\
f(x_{test,4}) \;&=\; sgn(-0.93122542\,)\\
f(x_{test,4}) \;&=\; -1
\end{aligned}$$

Table 3

Test Data	Output of SVM 1: Daffodils	Output of SVM 2: Tulip	Output of SVM 3: Lotus	Classification
(14, 3.5667)	-1	-1	-1	Not belong to any classes
(12.6667, 2.9333)	-1	-1	-1	Not belong any to classes
(15.3333, 3.2667)	-1	-1	-1	Not belong to any classes
(16.333, 2.333)	**-1**	**+1**	**-1**	**Class 2: Tulip**
(12.6667, 3.2667)	-1	-1	-1	Not belong to any classes
(13.6667, 3.2667)	-1	-1	-1	Not belong any to classes
(15.6667, 3.4333)	-1	-1	-1	Not belong to any classes
(17.3333, 3.3667)	-1	-1	+1	Class 3: Lotus
(14.3333, 3.1667)	-1	-1	-1	Not belong to any classes
(14.6667, 3.0667)	-1	-1	-1	Not belong any to classes

Since the test data was created using averaging the synthetic class data of each, the resulting data has populated in the region where the class is undefined using our SVM classifier (Refer to Fig.7.a). In other words, it can be considered as a class of data which does not belong to any other classes.

Using a soft classifier, classified using the maximum output value (i.e. without the sgn component). The maximum value represents how closer the test data is to the hyperplanes (proximity to the selected class).

Test Data	Output of SVM 1: Daffodils	Output of SVM 2: Tulip	Output of SVM 3: Lotus	Classification
(14, 3.5667)	-0.3551355	-2.2453	-0.4583	Class 1: Daffodils
(12.6667, 2.9333)	-0.6673	-0.9680	-1.2377	Class 1: Daffodils
(15.3333, 3.2667)	-0.9157	-1.6560	-0.4109	Class 3: Lotus
(16.333, 2.333)	**-1.998575**	**0.203336**	**-0.931225**	**Class 2: Tulip**
(12.6667, 3.2667)	-0.3557	-1.6347	-0.9763	Class 1: Daffodils
(13.6667, 3.2667)	-0.5657	-1.6427	-0.7643	Class 1: Daffodils
(15.6667, 3.4333)	-0.8298	-1.9920	-0.2095	Class 3: Lotus
(17.3333, 3.3667)	-1.2422	-1.8720	0.0915	Class 3: Lotus
(14.3333, 3.1667)	-0.7992	-1.4480	-0.7013	Class 3: Lotus
(14.6667, 3.0667)	-0.9627	-1.2507	-0.7091	Class 3: Lotus