**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans: To solve this problem,we can use the Z-score formula for a normal distribution

Z = X-µ/σ

In this case, the mean is 45 minutes, the standard deviation is 8 minutes,and the service manager wants to find the probability that the car will not be ready within 1 hr.

First, we have to determine the time beyond which the service manager cannot meet his commitment.

X=60 minutes-10 minutes=50 minutes

Now, calculating the Z-score

Z = X-µ/σ

= 50-45/8

=5/8 => 0.625

Looking up the z-scores of 0.625 in the standard normal distribution table gives us corresponding probability is approximately 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Ans:To determine this,we need to compare the probabilities associated with the two age ranges:older than 44 and between 38 and 44.we can use the z-score formula to find the probabilities.

1.For x>44:

Z = X-µ/σ=44-38/6=1

From the standard normal distribution table,the probability of Z>1 is approximately

0.1587

2. For 38<X<44:

Z1=38-38/6=0

Z2=44-38/6=1

The probability of 0<Z<1 is approximately 0.3413 from the table

Comparing the probabilities,0.3413>0.1587,so more employees are between 38 and 44 than older than 44. Therefore,statement A is false

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans:To determine this,we can find the z-score for X=30 and use it to find the probability.

Z=30-38/6=-1.33

From the standard normal distribution table,the probability of Z<-1.33 is approximately 0.0912

Therefore, the probability of attracting employees under 30 is not very high,and attracting about 36 employees might be too optimistic.Thus,statement B is likely false.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans:Lets consider two independent and identically distributed normal distribution variables,

*X1* ~ *N*(μ, σ2)

*X*2~ *N*(μ, σ2)

1.Distribution of 2X1:

If Y=2X1,then the mean and variance of Y can be calculated as follows:

* Mean of Y:

E(Y)=E(2X1)=2E(X1)=2 μ

* Variance of Y:

Var(Y)=var(2X1)=4Var(X1)=4 σ2

So,2X1 follows a normal distribution with mean 2 μ and variance 4 σ2.

2.Distribution of X1+X2:

If Z=X1+X2 ,then the mean and variance of Z can be calculated as follows:

* Mean of Z:

E(Z)=E(X1+X2)=E(X1)+E(X2)= μ + μ=2 μ

* Variance of Z:

Var(Z)=Var(X1+X2)=Var(X1)+Var(X2)=2σ2

So,X1+X2 follows a normal distribution with mean 2 μ and variance 2 σ2..

Therefore,

2X1 follows a normal distribution with mean 2.μ and variance 4 σ2.

X1+X2 follows a normal distribution with mean 2μ. and variance 2 σ2..

4. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

1. 90.5, 105.9
2. 80.2, 119.8
3. 22, 78
4. 48.5, 151.5
5. 90.1, 109.9

Ans: The probability of getting value between a and b is 0.99

So, the proability of getting value outside a and b is 1-0.99=0.01

The probability towards left of a=-0.01/2=-0.05

The probability towards right of b=0.01/2=0.05

Since we have probabilities of a and b,we need calculate the probability of X-random

Variable at a and b which has these probabilities

By finding standard normal variable (z),need to calculate X:

Z=(X- μ)/ ,σ

For a probability of 0.005,z values is -2.57

Z\*σ+μ=x

-(-2.57)\*20+100=151.4

(-2.57)\*20+100=48.6

Therefore ,the option is D.

5.Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45

1. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans: Range is Rs(99.00810 , 980.991) in Millions

1. Specify the 5th percentile of profit (in Rupees) for the company

Ans:The 5th percentile of profit for the company is 17 crore rupees

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans:The division 1 has the larger probability of making a loss.