I. BLACKJACK BASIC STRATEGY

We test the Basic Strategy for Blackjack, given in Edward Thorp's *Beat the Dealer* [1]. Our goal is to test this strategy in a simulation, so first we review precisely what is prescribed in [1]. In accordance to [1] we refer to this prescription as the Basic Strategy. The figures 1, 2, 3, 4 are taken from [1].

The Basic Strategy reviewed here is designed and tested for the game of Blackjack with one player and a single 52-card deck. The dealer stands on soft 17. [2] Player can double down after receiving its hand (the first two cards). If doubled down, the played is dealt exactly only one more card. Player can split any pair. If aces are split player receives up to 3 cards total on each of the split hands. Split hands cannot be split again. Player can double down on split hands, apart from split aces. Natural plays 3 : 2, split's 21 are not counted as natural.

Basic strategy doesn't take insurance on dealer's upcard being an ace. Decisions made by the player are taken in the following order. First the player decides on splitting, using the prescription given in figure 1. If split, the player cannot split again, so it proceeds to the doubling down prescription, see figure 2 for the hard hand, and figure 3 for the soft hand. Then the player proceeds to the holding vs standing prescription on each of the split hands, see figure 4. If not split, the player decides on doubling down depending on its hand being hard, see figure 2, or soft, see figure 3. After these decisions are made the player decides on holding vs standing using figure 4.

We run $N = 10000$ rounds of game with \$2 bets each round, starting with \$1000 bankroll for the player, and calculate the number of wins W and the number of losses L for the player. We then estimate the probability of win, $p_w = W/N$, and the probability of loss, $p_l = L/N$. We repeat this over $M = 1000$ simulations, and plot the histogram of the player's edge $p_w - p_l$, see figure 5, and histogram of the player's resulting (after $N = 10000$ rounds of game with \$2 bets) bankroll, see figure 6. A generic bankroll time series is given in figure 7. From figure 6 it looks like the mean edge is -1.8%. In fact the edge is about $+0.275\%$, because each \$2 the player bets results in $\simeq 55/10000 = 0.0055$ win, according to the $\simeq +55$ outcome for the bankroll after 10000 rounds shown in figure 6. The edge is then $0.0055/2 = 0.00275$. The disagreement from figure 5 is due to the double down win probabilities, see figure 9. In blackjack the player can actually win less hands but still end

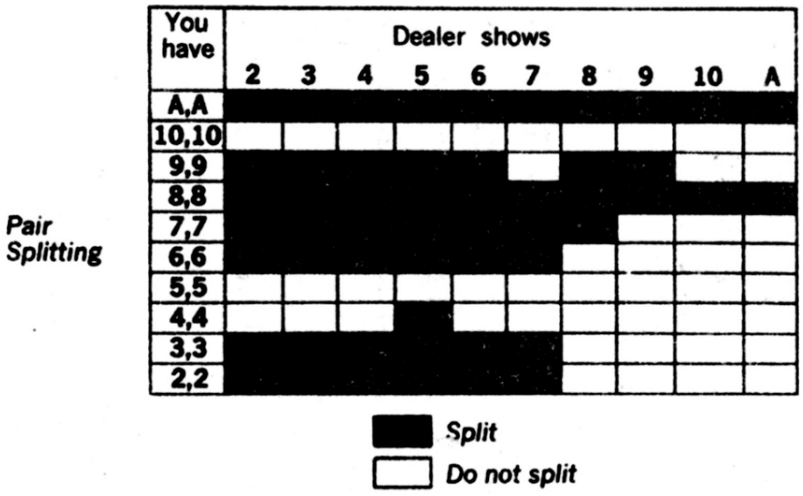

FIG. 1: Edward Thorp's Basic Strategy, pair splitting prescription, from *Beat The Dealer* [1].

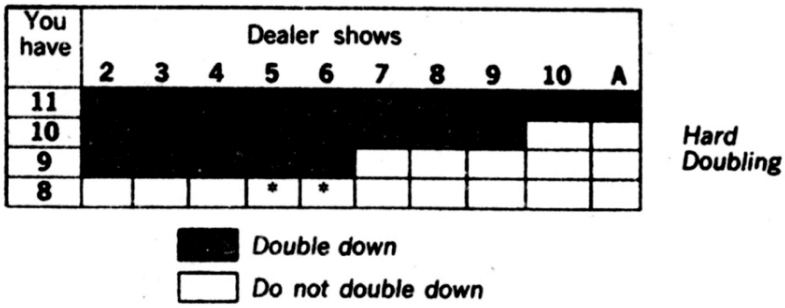

FIG. 2: Edward Thorp's Basic Strategy, hard doubling prescription, from *Beat The Dealer* [1].

up ahead, under certain conditions when it wins most of the hands on which it doubles down. We can also analyze how profitable is the splitting prescription of the basic strategy, see figure 8. We reiterated the calculation used to obtain figure 6 over 100 simulations, the results are plotted in figure 10.

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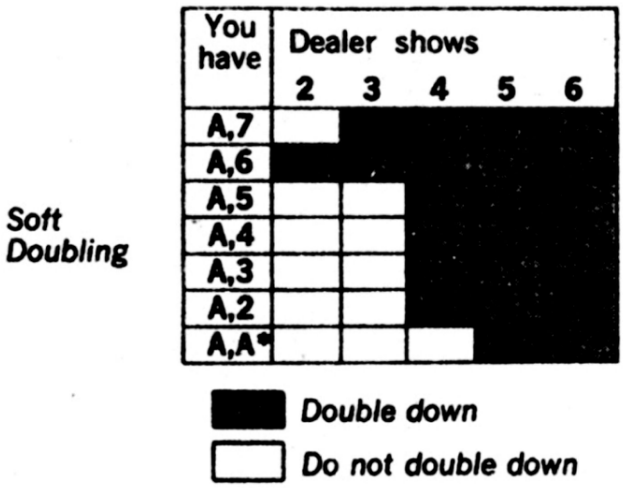
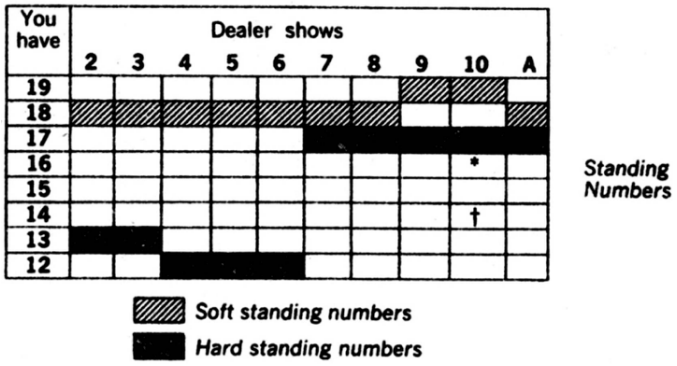
[1] Edward O. Thorp, *Beat the Dealer. A Winning Strategy for the Game of 21.* Vintage Books, Penguin Random House LLC, New York, 2016.

[2] A hand is called soft if it contains an ace which can be counted as 11 without the total value of the hand exceeding 21. Otherwise the hand is called hard.



*Double down with (A,A) only if Aces cannot be split.

FIG. 3: Edward Thorp's Basic Strategy, soft doubling prescription, from *Beat The Dealer* [1].



* Holding hard 16, draw if you hold two cards, namely (10,6) or (9,7), and stand if you hold three or more cards, for example (6,4,4,2).

† Stand holding (7,7) against 10.

FIG. 4: Edward Thorp's Basic Strategy, holding versus standing prescription, from *Beat The Dealer* [1].

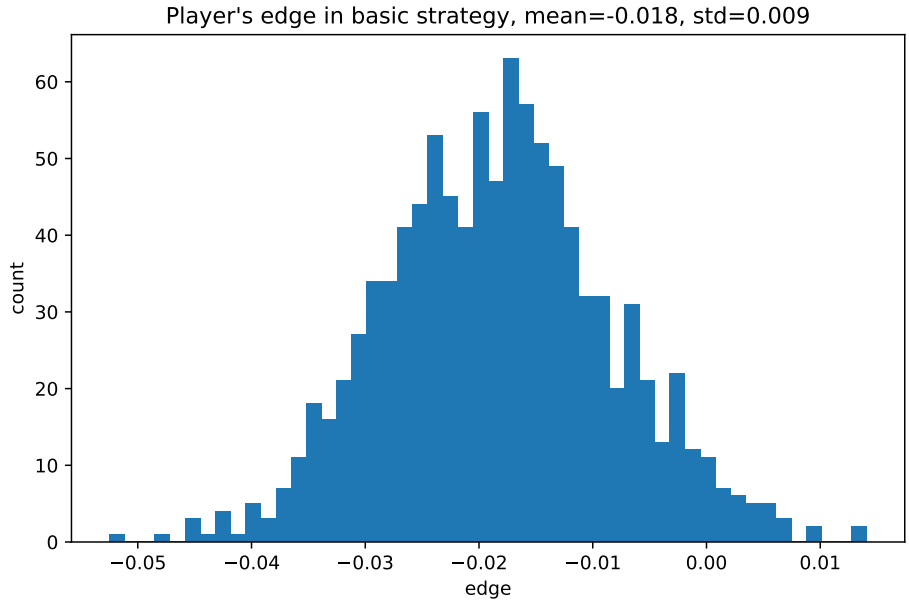


FIG. 5: Player's edge when using the Basic Strategy, defined as $p_w - p_l$, where the probabilities of player's win and loss p_w and p_l are calculated using the total counts of win and loss.

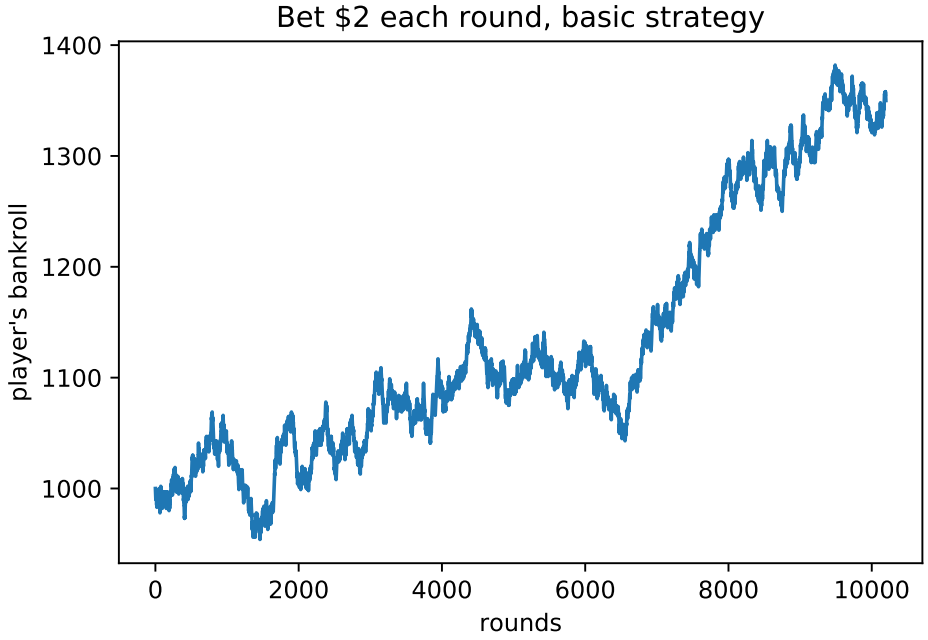


FIG. 6: Time series of the player's bankroll in a simulation with 10000 rounds of $2 bets, starting with $1000.

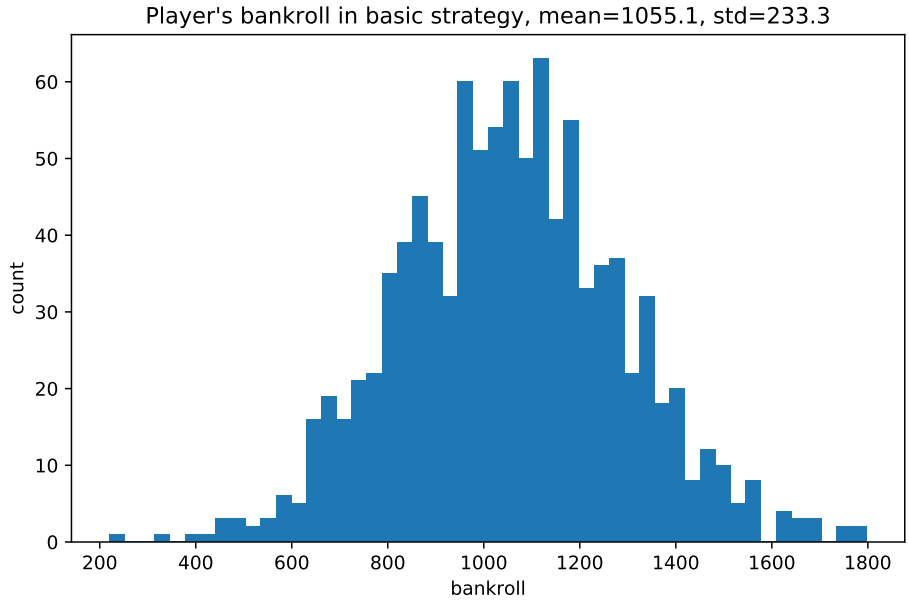


FIG. 7: Player's bankroll after 10000 rounds of play with \$2 bet, starting with \$1000, over 1000 simulations.

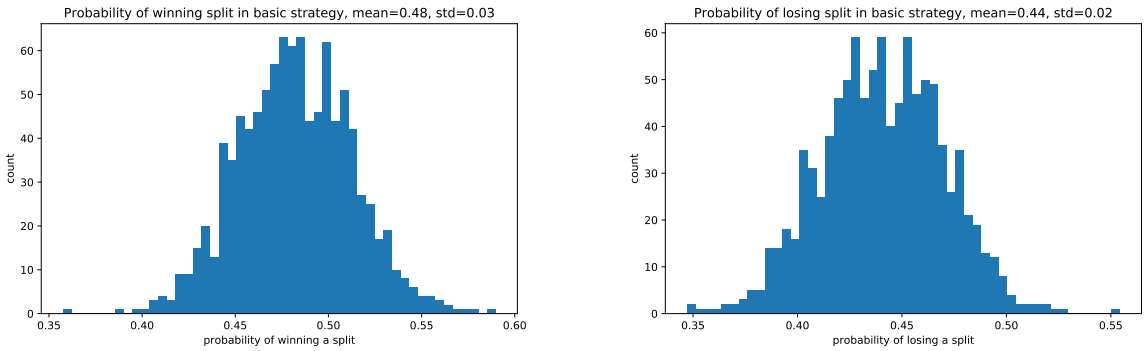


FIG. 8: Player's split win (**left**) and loss (**right**) probability (conditioned on split) after 10000 rounds of play, over 1000 simulations. The split probability is 0.02.

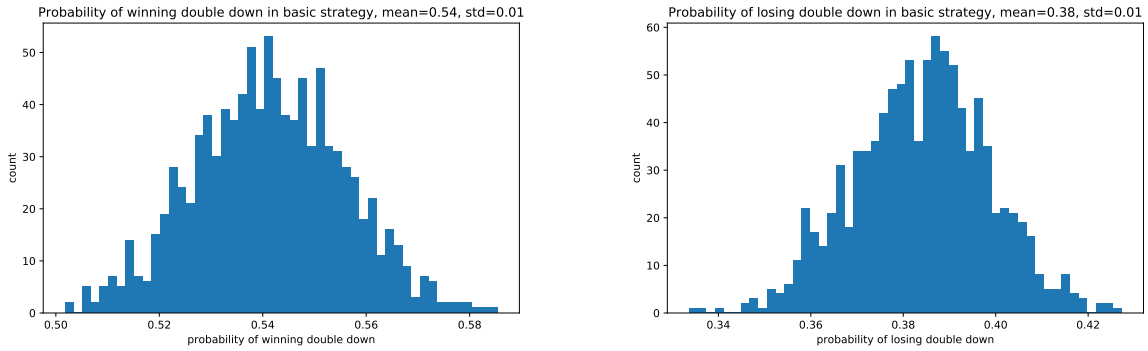


FIG. 9: Player's double down win (**left**) and loss (**right**) probability (conditioned on double down) after 10000 rounds of play, over 1000 simulations. The double down probability is 0.11.

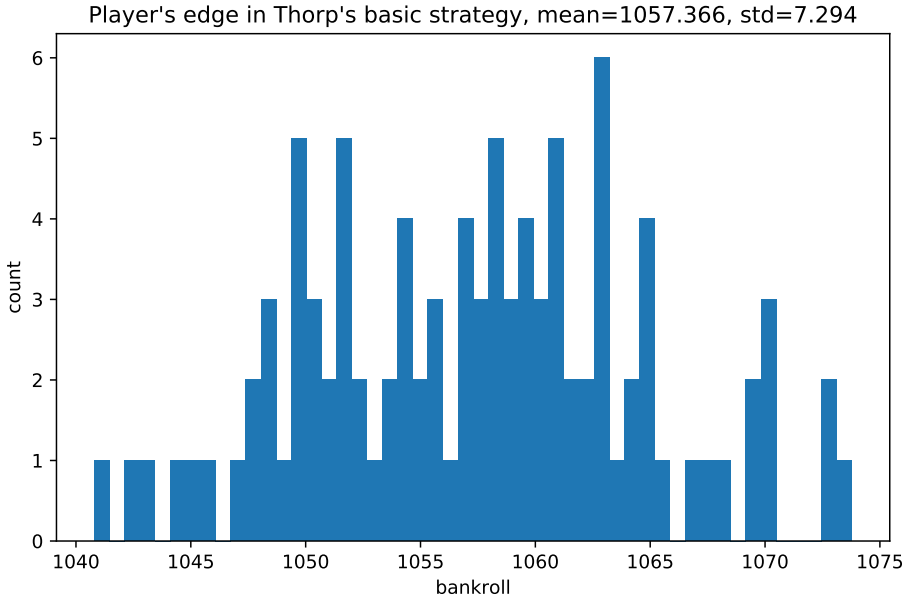


FIG. 10: Player's mean bankroll of 1000 games of 10000 rounds each, simulated 100 times.