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CSE 321 - Introduction to Algorithm Design

Homework 1

1) $T_1(n) = 3n^4 + 3n^3 + 1$ $\rightarrow O(n^a)$ $a > 1$ polynomial

$T_2(n) = 3^n$ $\rightarrow O(a^n)$ exponential

$T_3(n) = (n-2)!$ $\rightarrow O(n!)$ exponential

$T_4(n) = \ln^2 n$ $\rightarrow O(\ln^2 n)$ polynomial

$T_5(n) = 2^{2n}$ $\rightarrow O(a^n)$ exponential

$T_6(n) = \sqrt[3]{n}$ $\rightarrow O(n^{1/3})$ $a < 1$ polynomial

Hızları:

$T_4 > T_6 > T_1 > T_2 > T_5 > T_3$

$\hookrightarrow T_4 \subset T_6 \subset T_1 \subset T_2 \subset T_5 \subset T_3$

$\Rightarrow$ T_4 ve T_6'yı limit teoremi kullanarak kanıtlayalım;

$$\lim_{n\to\infty} \frac{T_4(n)}{T_6(n)} = \lim_{n\to\infty} \frac{\ln^2 n}{n^{1/3}} = \frac{\infty}{\infty}$$ belirsizliği, türev alacağız!

$$\lim_{n\to\infty} \frac{\ln^2 n}{n^{1/3}} = \lim_{n\to\infty} \frac{2 . \ln(n) . \frac{1}{n}}{\frac{1}{3} . n^{-2/3}} = \lim_{n\to\infty} \frac{6 \ln(n)}{n^{1/3}}$$ yine türev alacağız.

$$\lim_{n\to\infty} \frac{6\ln(n)}{n^{1/3}} = \lim_{n\to\infty} \frac{6 . \frac{1}{n}}{\frac{1}{3} . n^{-2/3}} = \lim_{n\to\infty} \frac{18}{n^{1/3}} = \frac{\text{sayı}}{\infty} = 0$$

$T_4 > T_6$, $T_4 \subset T_6$

$\Rightarrow T_6$ ve T_1

$$\lim_{n\to\infty} \frac{T_6(n)}{T_1(n)} = \lim_{n\to\infty} \frac{n^{1/3}}{3n^4+3n^3+1} = \lim_{n\to\infty} \frac{\cancel{n^{1/3}}(1)}{\cancel{n^{1/3}}(3.n^{11/3}+3.n^{8/3}+n^{-2/3})}$$

$$\lim_{n\to\infty} \frac{1}{3n^{11/3}+3.n^{8/3}+n^{-2/3}} = \frac{\text{sayı}}{\infty} = \underline{\underline{0}}$$

$T_6 > T_1$, $T_6 \subset T_1$

$$\Rightarrow \lim_{n\to\infty} \frac{T_1(n)}{T_2(n)} = \lim_{n\to\infty} \frac{3n^4+3n^3+1}{3^n} = \frac{\infty}{\infty}$$, türev alacağız!

$$\lim_{n\to\infty} \frac{12n^3+9n^2}{3^n . \ln 3} \overset{\text{türev}}{=} \lim_{n\to\infty} \frac{36n^2+18n}{3^n . \ln^2 3} \overset{\text{türev}}{=} \lim_{n\to\infty} \frac{72n+18}{3^n . \ln^3 3}$$

$$\overset{\text{türev}}{=} \lim_{n\to\infty} \frac{72}{3^n . \ln^4 3} = \frac{\text{sayı}}{\infty} = \underline{\underline{0}}$$

$T_1 > T_2$, $T_1 \subset T_2$

$$\Rightarrow \lim_{n\to\infty} \frac{T_2(n)}{T_5(n)} = \lim_{n\to\infty} \frac{3^n}{4^n} = \lim_{n\to\infty} \left(\frac{3}{4}\right)^n = 0$$

$T_2 > T_5$, $T_2 \subset T_5$

$$\Rightarrow \lim_{n\to\infty} \frac{T_5(n)}{T_3(n)} = \lim_{n\to\infty} \frac{2^{2n}}{(n-2)!}$$

$(n-2)!$ $\rightarrow$ $n!$ gibi düşünelim;

$$\lim_{n\to\infty} \frac{4^n}{n!}$$

$n!$ için Stirlings formülünü uygulayalım;

$$\lim_{n\to\infty} \frac{4^n}{\sqrt{2\pi n}\cdot\left(\frac{n}{e}\right)^n} = \lim_{n\to\infty} \frac{1}{\sqrt{2\pi n}} \cdot \left(\frac{4e}{n}\right)^n = \frac{\text{sayı}}{\infty} = 0$$

$$T_5 > T_3 \,, \quad T_5 \subset T_3$$

* $T_4 > T_6 > T_1 > T_2 > T_5 > T_3$
* $T_4 \subset T_6 \subset T_1 \subset T_2 \subset T_5 \subset T_3$

kanıtlanmış oldu!

2)
```
def delicious(fruits):
    plum = sys.maxsize
    watermelon = 0
    orange = 0
    orangeTime = False

    while not orangeTime:
1.for       for fruit in fruits:
                if fruit > watermelon
                    watermelon=fruit
                if fruit <plum:
                    plum = fruit
                    break
        else:
            orangeTime = True

2.for   for fruit in fruits:
            if abs(fruit - (watermelon + plum)//2
                < abs(orange - (watermelon + plum) //2 )
                orange = fruit

    return orange.
```

a)

4 Bu algoritma verilen arrayin sıraladığı zaman ortaya denk gelen elemanı yani medyanı (ortancayı) buluyor.

watermelon: minumum elemanın tutulduğu değişken.
plum: maximum elemanın tutulduğu değişken.
orange: ortanca (medyan) elemanın tutulduğu değişken
orangeTime: max ve min elemanlar bulunduğu zaman, while döngüsünü durdurmak için kullanılan değişken. (max veya min ilk eleman olsaydı array bir daha dolaşılırdı)

b) worst case :

En büyük eleman ya da en küçük eleman arrayin sonundaysa bütün array aranır! (1. for döngüsü n defa döner)

En büyük ya da en küçük eleman arrayin başındaysa while döngüsü 2 defa döner, 1. for döngüsü n defa döner. (2n)

2. for döngüsü her şekilde n defa döner.

Bu durumda;

$2n + n = 3n \Rightarrow W(n) = \Omega(n)$

best case :

Arraydaki bütün elemanlar aynıysa while döngüsü 1 defa, 1. for döngüsü 2 defa, 2. for döngüsü n defa döner.

Bu durumda ;

$2 + n \Rightarrow B(n) = O(n)$

Avarage case :

best ve worst case ler eşit çıktığına göre, bu problemin avarage case karmaşıklığı ;

$A(n) = \Theta(n)$

3)

a) $\sum_{i=0}^{n-1} (i^2+1)^2 = \sum_{i=0}^{n-1} (i^4+2i^2+1)$

$= \sum_{i=0}^{n-1} i^4 + 2\sum_{i=0}^{n-1} i^2 + n-1$

upper bound:

$$\sum_{i=0}^{n-1} (i^4+2i^2+1) = \underbrace{(0^4+2.0^2+1)+(1^4+2.1^2+1)+(2^4+2.2^2+1) + (3^4+2.3^2+1) + \dots + ((n-1)^4+2.(n-1)^2+1)}_{n-1+1 \,=\, \text{n tane terim var!}}$$

$$\sum_{i=0}^{n-1} (i^4+2i^2+1) \leq n.[(n-1)^4+2.(n-1)^2+1]$$

$$\sum_{i=0}^{n-1} (i^4+2i^2+1) \in O(n^5)$$

lower bound:

$$\sum_{i=0}^{n-1} (i^4+2i^2+1) = \sum_{i=0}^{n-1} i^4 + 2\sum_{i=0}^{n-1} i^2 + \sum_{i=0}^{n-1} 1 \geq 2\sum_{i=0}^{n-1} i^2$$

$$2\sum_{i=0}^{n-1} i^2 = 2[\underbrace{0^2+1^2+2^2+\dots+(n-1)^2}_{\text{n tane terim var!}}]$$ her terim $\geq (n-1)^2$

$$\sum_{i=0}^{n-1} (i^4+2i^2+1) \geq n.(n-1)^2 \Rightarrow \sum_{i=0}^{n-1} (i^4+2i^2+1) \in \Omega(n^3)$$

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b) upper bound :

$$\sum_{i=2}^{n-1} \log i^2 = \sum_{i=2}^{n-1} 2.\log i = 2\sum_{i=2}^{n-1} \log i$$

$$= 2\Big[\underbrace{\log 2 + \log 3 + \log 4 + \cdots + \log(n-1)}_{n-1-2+1 \; = n-2 \text{ tane terim var!}}\Big]$$

$$\sum_{i=2}^{n-1} \log i^2 \leq (n-2).2.\log(n-1) \Rightarrow \sum_{i=2}^{n-1} \log i^2 \in O(n\log n)$$

lower bound :

$$2\sum_{i=2}^{n-1} \log i \Rightarrow 2\Bigg[\sum_{i=2}^{\lfloor \frac{n-1}{2} \rfloor} \log i + \sum_{i=\lfloor \frac{n-1}{2} \rfloor+1}^{n-1} \log i\Bigg] \geq 2.\sum_{i=\lfloor \frac{n-1}{2} \rfloor+1}^{n-1} \log i$$

$$\sum_{i=\lfloor \frac{n-1}{2} \rfloor+1}^{n-1} \log i = \log\left(\left\lfloor \tfrac{n-1}{2} \right\rfloor+1\right) + \log\left(\left\lfloor \tfrac{n-1}{2} \right\rfloor+2\right) + \cdots + \log(n-1)$$

terim sayısı $= (n-1) - \left\lfloor \frac{n-1}{2} \right\rfloor = \left\lceil \frac{n-1}{2} \right\rceil \Rightarrow \left\lceil \frac{n-1}{2} \right\rceil \geq \frac{n-1}{2}$

her terim $\geq \log(n-1)$

$$\sum_{i=2}^{n-1} \log i^2 \geq \frac{n-1}{2}.\log(n-1) \Rightarrow \sum_{i=2}^{n-1} \log i^2 \in \Omega(n\log n)$$

$$\circledast \sum_{i=2}^{n-1} \log i^2 \in \Theta(n\log n)$$

c) upper bound:

$$\sum_{i=1}^{n} (i+1).2^{i-1} = \underbrace{(1+1).2^{1-1} + (2+1).2^{2-1} + \dots + (n+1).2^{n-1}}_{n \text{ tane terim var!}}$$

$$\sum_{i=1}^{n} (i+1).2^{i-1} \leq n.\left[(n+1).2^{n-1}\right]$$

$$\sum_{i=1}^{n} (i+1).2^{i-1} \in O(n^2.2^n)$$

lower bound:

$$\sum_{i=1}^{n} (i+1).2^{i-1} = \frac{1}{2}\left(\sum_{i=1}^{n} i.2^i + \sum_{i=1}^{n} 2^i\right) \geq \sum_{i=1}^{n} 2^i$$

$$\sum_{i=1}^{n} 2^i = \underbrace{2^1 + 2^2 + \dots + 2^n}_{n \text{ tane terim var.}}$$ her terim $\geq 2^n$

$$\sum_{i=1}^{n} (i+1)2^{i-1} \geq n.2^n$$

$$\sum_{i=1}^{n} (i+1).2^{i-1} \in \Omega(n.2^n)$$

d) upper bound:

$$\sum_{i=0}^{n-1}\sum_{j=0}^{i-1}(i+j) = \sum_{i=0}^{n-1}\left[i.(i-1) + \frac{(i-1).(i)}{2}\right] = \frac{3}{2}\sum_{i=0}^{n-1} i(i-1)$$

$$\frac{3}{2}\sum_{i=0}^{n-1} i(i-1) = \frac{3}{2}\Big[\underbrace{0.(0-1) + 1.(1-1) + 2(2-1) + \dots + (n-1).(n-2)}_{n \text{ tane terim var!}}\Big]$$

$$\sum_{i=0}^{n-1}\sum_{j=0}^{i-1}(i+j) \leq n.(n-1)(n-2)$$

$$\sum_{i=0}^{n-1}\sum_{j=0}^{i-1}(i+j) \in O(n^3)$$

lower bound:

$$\sum_{i=0}^{n-1}\sum_{j=0}^{i-1}(i+j) = \frac{3}{2}\sum_{i=0}^{n-1} i(i-1) = \frac{3}{2}\left[\sum_{i=0}^{\lfloor\frac{n-1}{2}\rfloor} i(i-1) + \sum_{i=\lfloor\frac{n-1}{2}\rfloor+1}^{n-1} i(i-1)\right] \geq \sum_{i=\lfloor\frac{n-1}{2}\rfloor+1}^{n-1} i(i-1)$$

$$\sum_{i=\lfloor\frac{n-1}{2}\rfloor+1}^{n-1} i(i-1) = \left(\left\lfloor\frac{n-1}{2}\right\rfloor+1\right)\cdot\left\lfloor\frac{n-1}{2}\right\rfloor + \left(\left\lfloor\frac{n-1}{2}\right\rfloor+2\right)\left(\left\lfloor\frac{n-1}{2}\right\rfloor+1\right) + \dots + (n-1)(n-2)$$

$n-1-\left\lfloor\frac{n-1}{2}\right\rfloor - 1 + 1 = \left\lceil\frac{n-1}{2}\right\rceil - 1$ terim var $\qquad \left\lceil\frac{n-1}{2}\right\rceil - 1 \geq \frac{n-1}{2}$

her terim $\geq (n-1)(n-2)$

$$\sum_{i=0}^{n-1}\sum_{j=0}^{i-1}(i+j) \geq \frac{n-1}{2}.(n-1).(n-2) \in \Omega(n^3)$$

(A) $$\sum_{i=0}^{n-1}\sum_{j=0}^{i-1}(i+j) \in \Theta(n^3)$$

d şıkkı için (c kodunda)

```c
int foo (int n) {
    int i,j,k;
    int count=0;

    for(i=0; i<n ; i++)
    {
        for (j=0; j<i; j++)
        {
            for(k=0; k<i; k++)
                count+=1;
            for (k=0; k<j; k++)
                count+=1;
        }
    }
    return count;
}
```

for(i=0; i<n; i++) $\rightarrow \sum_{i=0}^{n-1} \sum_{j=0}^{i-1} (i+j)$

for (j=0; j<i; j++) $\rightarrow \sum_{j=0}^{i-1} (i+j)$

for(k=0; k<i; k++) count+=1; $\sum_{k=0}^{i} 1 = i$

for (k=0; k<j; k++) count+=1; $\sum_{k=0}^{j} 1 = j$

4)
```
int fun (int n)
{
    int count=0;
    for (int i=n; i>0; i/=2)
        for (int j=0; j<i; j++)
            count+=1;
} return count;
```

içteki for'a her girildiğinde;

$n, n/2, n/4, n/8, \ldots 1$

zamanlarında yürütülür. (bu kadar dönerde diyebiliriz)

$$\sum_{i=0}^{\log n} \frac{n}{2^i} = \underbrace{n + n/2 + n/4 + \cdots + \frac{n}{2^{\log n}}}_{\text{n tane terim var!}}$$

$$\frac{n}{2^{\log n}} \cdot n = n$$

$$T(n) = \underline{\underline{O(n)}}$$

5)

a) $n^3 \in O(3^{2n})$

$$\lim_{n\to\infty} \frac{n^3}{3^{2n}} = \lim_{n\to\infty} \frac{n^3}{9^n} \overset{\text{türev}}{=} \lim_{n\to\infty} \frac{3n^2}{9^n \cdot \ln 9}$$

$$\overset{\text{türev}}{=} \lim_{n\to\infty} \frac{6n}{9^n \ln^2 9} \overset{\text{türev}}{=} \lim_{n\to\infty} \frac{6}{9^n \cdot \ln^3 9} = \frac{\text{sayı}}{\infty} = 0$$

$\underline{n^3 \in O(3^{2n})}$ ✓

b) $n \in o(\lg\lg n)$ ✗

$$\lim_{n\to\infty} \frac{n}{\lg\lg n} \overset{\text{türev}}{=} \lim_{n\to\infty} \frac{1}{\frac{\frac{1}{n}}{\lg n \cdot \ln 2}} = \lim_{n\to\infty} n \cdot \lg n \cdot \ln 2$$

$$\lim_{n\to\infty} n \cdot \lg n \cdot \ln 2 = \infty$$

$\underline{n \notin o(\lg\lg n)}$

c) $n^2\log^2 n \in O(n!)$

$$\lim_{n\to\infty} \frac{n^2\log^2 n}{n!} \Rightarrow$$ n! için Stirling's formülü $$\Rightarrow \lim_{n\to\infty} \frac{n^2\log^2 n}{\sqrt{2\pi n}\cdot\left(\frac{n}{e}\right)^n} = \lim_{n\to\infty} \frac{\cancel{n^2}\cdot\log^2 n\cdot e^n}{\cancel{n^2}(\sqrt{2\pi n}\cdot n^{n-2})}$$

$$= \lim_{n\to\infty} \frac{\log^2 n\cdot e^n}{\sqrt{2\pi n}\cdot n^{n-2}} = \lim_{n\to\infty} \frac{\log n\cdot\log n\cdot e^n}{\sqrt{2\pi n}\cdot n\cdot n\cdot n^{n-4}} = \lim_{n\to\infty} \frac{e^n}{\sqrt{2\pi n}}\cdot\frac{\log n}{n}\cdot\frac{\log n}{n}\cdot\frac{1}{n^{n-4}}$$

$$= \lim_{n\to\infty} \frac{e^n}{\sqrt{2\pi n}}\cdot\frac{1}{n^{n-4}} = \frac{\text{sayı}}{\infty} = 0$$

$\underline{n^2\log^2 n \in O(n!)}$ ✓

d) $\sqrt{10n^2+7n+3} \in \Theta(n)$

$$\lim_{n\to\infty} \frac{\sqrt{10n^2+7n+3}}{n} = 1$$ → sabit sayı çıktı. Bu durumda eşit artma hızına sahiptir deriz!

$\underline{\sqrt{10n^2+7n+3} \in \Theta(n)}$ ✓

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