Frequency and Phase Modulation

Principles of Frequency and Phase Modulation (Frenzel, 2022)

Frequency Modulation (FM)

A modulation process that puts the message in a carrier wave by changing the instantaneous frequency of the wave.

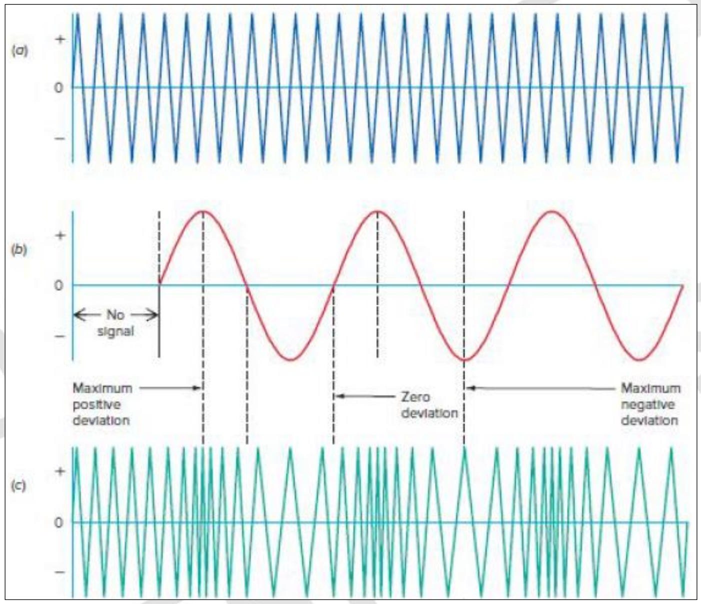

Figure 1. Retrieved from Frenzel, L. (2022). Principles of electronic communication systems: Fifth edition. McGraw Hill

In *Figure 1*, a) is the carrier signal, b) is the modulating signal, and c) is the FM signal. There is an observable difference between Amplitude Modulation and Frequency Modulation wherein the carrier amplitude remains constant, and the carrier frequency is changed by the modulating signal based on its troughs.

Here are other characteristics observed when FM is used:

- The carrier frequency shifts proportionately as the modulating signal's amplitude varies.
- The carrier frequency increases as the modulating signal amplitude increases.
- If a reverse relationship is implemented, a decreasing modulating signal will increase the carrier frequency above its center value, while an increasing modulating signal will decrease the carrier frequency below the center value.

For clearer visualization, see *Figure 2,* which shows the modulating signal and the FM signal together.

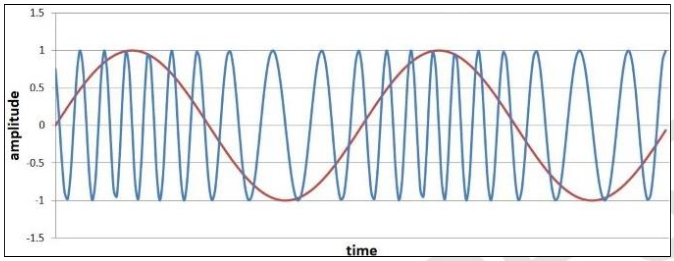

Figure 2. Frequency modulation. Retrieved from https://www.allaboutcircuits.com

Frequency Deviation (f_d)
It is the amount of change in the carrier frequency caused by the modulating signal. The maximum frequency deviation happens at the maximum amplitude of the modulating signal.

$$Maximum\ Deviation = carrier\ frequency + maximum\ frequency$$
$$Minimun\ Deviation = carrier\ frequency - maximum\ frequency$$
$$f_d = Maximum\ deviation - minimum\ deviation$$

For example, c) signal has a 150 MHz carrier frequency. If the peak amplitude of the modulating signal (b) causes a maximum frequency shift of 40 kHz. What are the frequency deviations?

For maximum and minimum deviation:

$$Maximum\ Deviation = carrier\ frequency + maximum\ frequency$$
$$Maximum\ Deviation = 150\ MHz + 40\ kHz$$
$$Maximum\ Deviation = \mathbf{150.04\ MHz}$$

$$Minimun\ Deviation = carrier\ frequency - maximum\ frequency$$
$$Maximum\ Deviation = 150\ MHz - 30\ kHz$$
$$Maximum\ Deviation = \mathbf{149.96\ MHz}$$

For total frequency deviation:

$$f_d = Maximum\ deviation - minimum\ deviation$$
$$f_d = 150.04\ MHz - 149.96\ MHz$$
$$\boldsymbol{f_d = 0.08\ MHz\ or\ 80\ kHz}$$

The total frequency deviation of this problem could also be expressed as $\pm 40\ kHz$ as frequency deviation is expressed as the amount of frequency shift of the carrier above and below the center frequency. The $\pm$ sign in $\pm 40\ kHz$ indicates that the modulating signal varies the carrier above and below its center frequency by $40\ kHz$.

Frequency-shift Keying (FSK)
In FM, a series of rectangular waves, such as serial binary data, can represent the modulating signal. This kind of modulating signal will only have two amplitudes and will also make the carrier frequency have two values (1 and 0), such as this modulating signal:

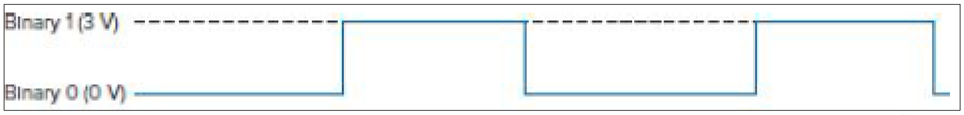

Figure 3. Retrieved from Frenzel, L. (2022). Principles of electronic communication systems: Fifth edition. McGraw Hill.

In *Figure 3*, the modulating signal starts at binary 0, making the carrier frequency the center frequency value. When the modulating signal is at binary 1, the carrier frequency rises to a higher frequency level. The amount of shift depends on the amplitude of the binary signal. This modulation is called **frequency-shift keying**.

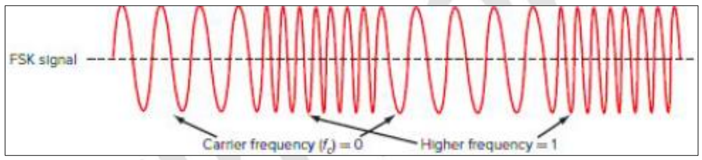

Figure 4. Retrieved from Frenzel, L. (2022). Principles of electronic communication systems: Fifth edition. McGraw Hill.

In *Figure 4*, by translating the binary code signal into an FSK signal, it can be observed that the carrier frequency stretches if it is at binary 0 and it compresses when it is at a higher frequency or binary 1.

Phase Modulation (PM)

A modulation process wherein the phase of the carrier signal varies based on the amplitude variation of the message signal or modulating signal.

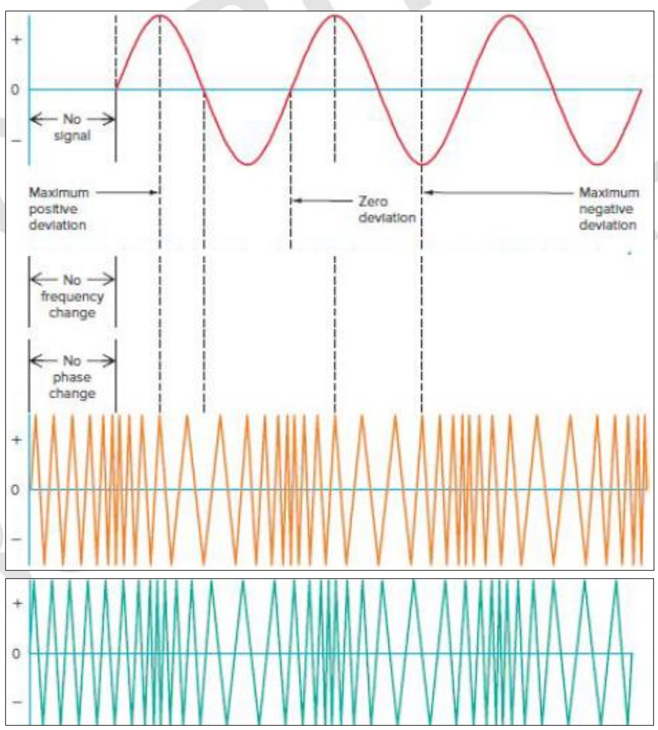

Figure 5. Retrieved from Frenzel, L. (2022). Principles of electronic communication systems: Fifth edition. McGraw Hill.

In *Figure 5*, the sine wave is the modulating signal, the orange triangular wave is the PM, and the blue triangular signal is the FM. There is a noticeable lagging and leading between FM and PM.

In the FM signal, the frequency compresses in accordance with the positive modulating signal while it stretches when at zero deviation. But for the PM signal, it starts to lag or stretch at the positive peak of the modulating signal until the second zero deviation. It starts to lead or compress at the second zero deviation until the next positive peak of the modulating signal. These are called **lagging phase shifts** and **leading phase shifts**.

To further visualize, *Figure 6* shows the modulating signal and the phase modulation signal together.

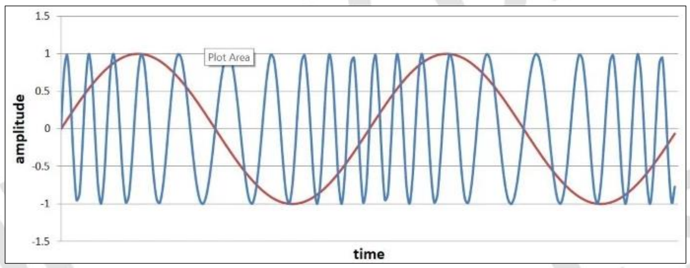

Figure 6. Phase modulation. Retrieved from https://www.allaboutcircuits.com

Phase-shift Keying (PSK)
In PM, the modulating signal can also be used with the binary signal. When the binary 0 is $0\ V$, the PM signal is simply the carrier frequency, and when a binary 1 voltage level occurs ($3\ V$), the modulator, or phase shifter, changes the phase of the carrier, and not its frequency.

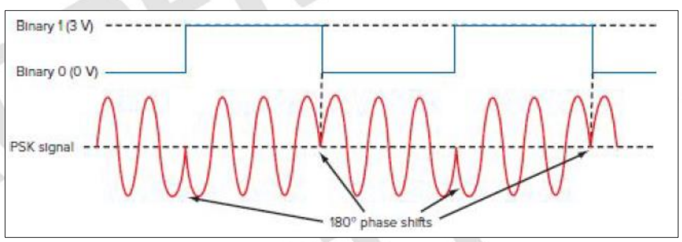

Figure 7. Retrieved from Frenzel, L. (2022). Principles of electronic communication systems: Fifth edition. McGraw Hill.

In *Figure 7*, the phase shift is 180°. Each time the signal changes from 0 to 1 or 1 to 0, a 180°phase shift happens. Notice how the PSK signal does a 180°phase shift when it hits binary 1 or 0. The shift happens whether the signal is positive or negative. This process is called **phase-shift keying** or **binary phase-shift keying**.

Sidebands and Modulation Index (Frenzel, 2022)

When a constant-frequency sine wave modulates a carrier, two side frequencies or sidebands are produced. In FM and PM, just like with Amplitude Modulation (AM), it is the sum and difference of the carrier and the modulating frequency. The spectrum of an FM and a PM signal is wider than AM because of sideband pairs.

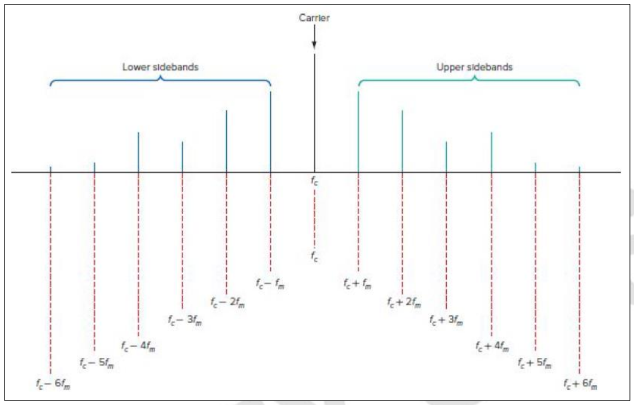

Figure 8. Retrieved from Frenzel, L. (2022). Principles of electronic communication systems: Fifth edition. McGraw Hill.

Figure 8 shows a frequency spectrum of an FM signal with sample carrier and sideband amplitudes. Sidebands can be spaced in two ways; one is from the carrier (f_c) and the second is from one another by a frequency equal to the modulating frequency (f_m). The values of the lower ($\boldsymbol{f_{LSB}}$) and upper ($\boldsymbol{f_{USB}}$) sidebands are computed as:

$\boldsymbol{f_{LSB1}} = f_c - f_m$ for the first sideband from the carrier, $\boldsymbol{f_{LSB2}} = f_c - 2f_m$ for the second, $\boldsymbol{f_{LSB3}} = f_c - 3f_m$ for third, and so on.

$\boldsymbol{f_{USB1}} = f_c + f_m$ for the first sideband from the carrier, $\boldsymbol{f_{USB2}} = f_c + 2f_m$ for the second, $\boldsymbol{f_{USB3}} = f_c + 3f_m$ for third, and so on.

If *Figure 8* has a modulating frequency of $1\ kHz$ and a carrier by $1\ MHz$, the first pair, upper and lower of sidebands is:

$$f_{USB1} = f_c + f_m$$
$$f_{USB1} = 1{,}000{,}000\ Hz + 1000\ Hz$$
$$\boldsymbol{f_{USB1} = 1{,}001{,}000\ Hz\ or\ 1.001\ MHz}$$

$$f_{LSB} = f_c - f_m$$
$$f_{LSB1} = 1{,}000{,}000\ Hz - 1000\ Hz$$
$$\boldsymbol{f_{LSB1} = 999{,}000\ Hz\ or\ \ 0.999\ MHz}$$

For the second pair, upper and lower, of the sidebands is:

$$f_{USB2} = f_c + 2f_m$$
$$f_{USB2} = 1{,}000{,}000\ Hz + (2)1000\ Hz$$
$$f_{USB2} = 1{,}000{,}000\ Hz + 2000\ Hz$$
$$\boldsymbol{f_{USB2} = 1{,}002{,}000\ Hz\ or\ 1.002\ MHz}$$

$$f_{LSB2} = f_c - 2f_m$$
$$f_{LSB2} = 1{,}000{,}000\ Hz - (2)1000\ Hz$$
$$f_{LSB2} = 1{,}000{,}000\ Hz - 2000\ Hz$$
$$\boldsymbol{f_{LSB2} = 998{,}000\ Hz\ or\ 0.998\ MHz}$$

For the third pair, upper and lower, of the sidebands is:

$$f_{USB3} = f_c + 3f_m$$
$$f_{USB3} = 1{,}000{,}000\ Hz + (3)1000\ Hz$$
$$f_{USB3} = 1{,}000{,}000\ Hz + 3000\ Hz$$
$$\boldsymbol{f_{USB3} = 1{,}003{,}000\ Hz\ or\ 1.003\ MHz}$$

$$f_{LSB3} = f_c - 3f_m$$
$$f_{LSB3} = 1{,}000{,}000\ Hz - (3)1000\ Hz$$
$$f_{LSB3} = 1{,}000{,}00\ Hz - 3000\ Hz$$
$$\boldsymbol{f_{LSB3} = 997{,}000\ Hz\ or\ 0.997\ MHz}$$

The same process applies until the last available pair of sidebands.

Modulation Index

The amplitudes of the carrier and sidebands of the frequency spectrum of an FM signal still depend on the modulation index m_f, still unitless, such as:

$$m_f = \frac{f_d}{f_m}$$

Wherein:
f_d = frequency deviation
f_m = modulating frequency

For example, if the maximum modulating frequency is 3.5 kHz and the maximum frequency deviation of the carrier is $\pm 14\ kHz$ and :

$$m_f = \frac{f_d}{f_m}$$
$$m_f = \frac{14\ kHz}{3.5\ kHz}$$
$$\boldsymbol{m_f = 4}$$

Bessel Functions

The number and amplitudes of the sidebands are obtained by solving the FM signal equation:

$$V_{FM} = V_c \sin\left[2\pi\ f_c t + m_f \sin\left(2\pi f_m t\right)\right]$$

This equation is solved with a complex mathematical process, *Bessel functions*.

For the amplitudes of the sidebands, Bessel functions have a widely available table for different carrier and sideband amplitudes for different modulation indexes of FM signals.

Modulation Index	Carrier	Sidebands (Pairs) 1st	2d	3d	4th	5th	6th	7th	8th	9th	10th	11th	12th	13th	14th	15th	16th
0.00	1.00	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
0.25	0.98	0.12	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
0.5	0.94	0.24	0.03	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1.0	0.77	0.44	0.11	0.02	—	—	—	—	—	—	—	—	—	—	—	—	—
1.5	0.51	0.56	0.23	0.06	0.01	—	—	—	—	—	—	—	—	—	—	—	—
2.0	0.22	0.58	0.35	0.13	0.03	—	—	—	—	—	—	—	—	—	—	—	—
2.5	−0.05	0.50	0.45	0.22	0.07	0.02	—	—	—	—	—	—	—	—	—	—	—
3.0	−0.26	0.34	0.49	0.31	0.13	0.04	0.01	—	—	—	—	—	—	—	—	—	—
4.0	−0.40	−0.07	0.36	0.43	0.28	0.13	0.05	0.02	—	—	—	—	—	—	—	—	—
5.0	−0.18	−0.33	0.05	0.36	0.39	0.26	0.13	0.05	0.02	—	—	—	—	—	—	—	—
6.0	0.15	−0.28	−0.24	0.11	0.36	0.36	0.25	0.13	0.06	0.02	—	—	—	—	—	—	—
7.0	0.30	0.00	−0.30	−0.17	0.16	0.35	0.34	0.23	0.13	0.06	0.02	—	—	—	—	—	—
8.0	0.17	0.23	−0.11	−0.29	−0.10	0.19	0.34	0.32	0.22	0.13	0.06	0.03	—	—	—	—	—
9.0	−0.09	0.24	0.14	−0.18	−0.27	−0.06	0.20	0.33	0.30	0.21	0.12	0.06	0.03	0.01	—	—	—
10.0	−0.25	0.04	0.25	0.06	−0.22	−0.23	−0.01	0.22	0.31	0.29	0.20	0.12	0.06	0.03	0.01	—	—
12.0	−0.05	−0.22	−0.08	0.20	0.18	−0.07	−0.24	−0.17	0.05	0.23	0.30	0.27	0.20	0.12	0.07	0.03	0.01
15.0	−0.01	0.21	0.04	0.19	−0.12	0.13	0.21	0.03	−0.17	−0.22	−0.09	0.10	0.24	0.28	0.25	0.18	0.12

Table 1. Bessel functions. Retrieved from https://www.coursehero.com/

Table 1 has various modulation indices for relative amplitudes of the carrier and the sideband pairs. The carrier and sideband amplitudes with negative signs mean the signal represented by that amplitude is shifted in phase 180°.

References:

Frenzel, L. (2022). *Principles of electronic communication systems: 5th ed.* McGraw Hill.

Shaik, A. (2023). *Phase modulation.* [Web Article]. Retrieved on August 3, 2023, from https://www.physics-and-radio-electronics.com/blog/phase-modulation/