# Rayleigh Scattering by Molecules

**V1.3**

## 1 General Rayleigh scattering

The general equation to compute Rayleigh scattering cross sections for molecules is given by:

$$\sigma_{\text{rayleigh}} = \frac{24\pi^3\nu^4}{n_{\text{ref}}^2} \cdot \left(\frac{n(\nu)^2 - 1}{n(\nu)^2 + 2}\right)^2 \cdot K(\nu) \ , \tag{1}$$

where $\nu$ is the wavenumber, $n$ the refractive index, $n_{\text{ref}}$ a reference particle number density, and $K$ the King factor. The King factor describes the non-sphericity of the molecules (Rayleigh scattering is only exact for spherical scatterers). It can be also written as a function of the depolarisation factor $D$

$$K(\nu) = \frac{6 + 3D(\nu)}{6 - 7D(\nu)} \tag{2}$$

Note that even though the cross section appears to depend on a particle number density, the product of

$$n_{\text{ref}}^{-1} \cdot \left(\frac{n(\nu)^2 - 1}{n(\nu)^2 + 2}\right) \tag{3}$$

itself is independent of the particle density. The refractive index actually implicitly depends on the number of particles present. The reference density is a particle density at a given standard reference pressure and temperature. In many cases this is the Avogadro constant.

## 2 Data for molecules in planetary atmospheres

### 2.1 $H_2$

- King factor: 1.0
- refractive index:

  $$n = 13.58 \cdot 10^{-5} \cdot \left(1. + 7.52 \cdot 10^{-3} \cdot \lambda^{-2}\right) + 1$$

  with $[\lambda] = \mu\text{m}$

- reference density: $n_{\text{ref}} = 2.65163 \cdot 10^{19}$ cm$^{-3}$
- reference: Cox (2000)

alternative cross section:

- cross section in cm$^{-2}$:

$$\sigma_{\text{rayleigh}} = 8.4909 \cdot 10^{-45} \cdot \lambda^{-4}$$

  with $[\lambda] = \text{cm}$
- reference: Vardya (1962)

## 2.2 $H_2O$

- depolarisation factor:

$$D = 3 \cdot 10^{-4}$$

- refractive index:

$$n = \left(\frac{2A+1}{1-A}\right)^{0.5}$$

  with

$$A = \delta\left(a_0 + a_1\delta + a_2\theta + a_3\Lambda^2\theta + a_4\Lambda^{-2} + \frac{a_5}{\Lambda^2 - \Lambda_{\text{UV}}^2} + \frac{a_6}{\Lambda^2 - \Lambda_{\text{IR}}^2} + a_7\delta^2\right)$$

- dimensionless ratios are given by:

$$\begin{aligned}\delta &= \rho/\rho^*,\ \rho^* = 1000\ \text{kg}\,\text{m}^{-3}\\ \Lambda &= \lambda/\lambda^*,\ \lambda^* = 0.589\ \mu\text{m}\\ \theta &= T/T^*,\ T^* = 273.15\ \text{K}\end{aligned}$$

- effective infrared and ultraviolet resonances:

$$\begin{aligned}\Lambda_{\text{IR}} &= 5.432937\\ \Lambda_{\text{UV}} &= 0.229202\end{aligned}$$

- values of $a_i$ given in Table 1
- reference density: actual number density $n(\rho, T)$
- note: the refractive index depends on temperature and density and is valid for both, liquid and gaseous water

- suggested validity range:

$$261.15\ \mathrm{K} \leq T \leq 773.15\ \mathrm{K}$$
$$0\ \mathrm{kg\,m^{-3}} \leq \rho \leq 1060\ \mathrm{kg\,m^{-3}}$$
$$0.2\ \mathrm{\mu m} \leq \lambda \leq 2.5\ \mathrm{\mu m}$$

- note: has singularities at 0.135 µm and 3.2 µm
- reference: Murphy (1977), Schiebener et al. (1990), Wagner & Kretzschmar (2008),

Table 1: Coefficients for refractive index of water

| $i$ | $a_i$ | $i$ | $a_i$ |
|---|---|---|---|
| 0 | 0.244257733 | 4 | $0.158920570 \cdot 10^{-2}$ |
| 1 | $0.974634476 \cdot 10^{-2}$ | 5 | $0.245934259 \cdot 10^{-2}$ |
| 2 | $-0.373234996 \cdot 10^{-2}$ | 6 | 0.900704920 |
| 3 | $0.268678472 \cdot 10^{-3}$ | 7 | $-0.166626219 \cdot 10^{-1}$ |

## 2.3 CO

- King factor: 1.0
- refractive index:

$$n = \left(22851 + \frac{0.456 \cdot 10^{14}}{71427^2 - \nu^2}\right) \cdot 10^{-8} + 1$$

  with $[\nu] = \mathrm{cm}^{-1}$

- reference density: $n_{\mathrm{ref}} = 2.546899 \cdot 10^{19}\ \mathrm{cm}^{-3}$
- reference: Sneep & Ubachs (2005)
- note: fixed a typo from Sneep & Ubachs (2005)

## 2.4 $CH_4$

- King factor: 1.0
- refractive index:

$$n = \left(46662. + 4.02 \cdot 10^{-6} \cdot \nu^2\right) \cdot 10^{-8} + 1$$

  with $[\nu] = \mathrm{cm}^{-1}$

- reference density: $n_{\mathrm{ref}} = 2.546899 \cdot 10^{19}\ \mathrm{cm}^{-3}$
- reference: Sneep & Ubachs (2005), Thalman et al. (2014)

### 2.5 $CO_2$

- King factor:

$$K = 1.1364 + 25.3 \cdot 10^{-12} \nu^2$$

- refractive index:

$$n = \left( \frac{5799.25}{128908.9^2 - \nu^2} + \frac{120.05}{89223.8^2 - \nu^2} + \frac{5.3334}{75037.5^2 - \nu^2} \right. \\ \left. + \frac{4.3244}{67837.7^2 - \nu^2} + \frac{0.1218145 \cdot 10^{-6}}{2418.136^2 - \nu^2} \right) \cdot 1.1427 \cdot 10^3 + 1$$

  with $[\nu] = \mathrm{cm}^{-1}$

- reference density: $n_{\mathrm{ref}} = 2.546899 \cdot 10^{19}\ \mathrm{cm}^{-3}$
- reference: Sneep & Ubachs (2005), Thalman et al. (2014)
- note: fixed a typo from Sneep & Ubachs (2005)

### 2.6 He

- King factor: 1.0
- refractive index:

$$n = \left( 2283.0 + \frac{1.8102 \cdot 10^{13}}{1.5342 \cdot 10^{10} - \nu^2} \right) \cdot 10^{-8} + 1$$

  with $[\nu] = \mathrm{cm}^{-1}$

- reference density: $n_{\mathrm{ref}} = 2.546899 \cdot 10^{19}\ \mathrm{cm}^{-3}$
- reference: Sneep & Ubachs (2005), Thalman et al. (2014)

### 2.7 Ar

- King factor: 1.0
- refractive index:

$$n = \left( 6432.135 + \frac{286.06021 \cdot 10^{12}}{14.4 \cdot 10^9 - \nu^2} \right) \cdot 10^{-8} + 1$$

  with $[\nu] = \mathrm{cm}^{-1}$

- reference density: $n_{\mathrm{ref}} = 2.546899 \cdot 10^{19}\ \mathrm{cm}^{-3}$
- reference: Sneep & Ubachs (2005), Thalman et al. (2014)

### 2.8 $O_2$

- King factor:

$$K = 1.09 + 1.385 \cdot 10^{-11} \cdot \nu^2 + 1.448 \cdot 10^{-20} \cdot \nu^4$$

- refractive index:

$$n = \left(20564.8 + \frac{2.480899 \cdot 10^{13}}{4.09 \cdot 10^9 - \nu^2}\right) \cdot 10^{-8} + 1$$

  with $[\nu] = \mathrm{cm}^{-1}$

- reference density: $n_{\mathrm{ref}} = 2.68678 \cdot 10^{19}\ \mathrm{cm}^{-3}$
- reference: Sneep & Ubachs (2005), Thalman et al. (2014)

### 2.9 $N_2$

- King factor:

$$K = 1.034 + 3.17 \cdot 10^{-12} \cdot \nu$$

- refractive index ($\nu < 21360\ \mathrm{cm}^{-1}$):

$$n = \left(6498.2 + \frac{307.4335 \cdot 10^{12}}{14.4 \cdot 10^9 - \nu^2}\right) \cdot 10^{-8} + 1$$

- refractive index ($\nu > 21360\ \mathrm{cm}^{-1}$):

$$n = \left(5677.465 + \frac{318.81874 \cdot 10^{12}}{14.4 \cdot 10^9 - \nu^2}\right) \cdot 10^{-8} + 1$$

  with $[\nu] = \mathrm{cm}$

- reference density: $n_{\mathrm{ref}} = 2.546899 \cdot 10^{19}\ \mathrm{cm}^{-3}$
- reference: Sneep & Ubachs (2005), Thalman et al. (2014)

## References


Cox, A. N. 2000, Allen's astrophysical quantities

Murphy, W. F. 1977, The Journal of Chemical Physics, 67, 5877

Schiebener, P., Straub, J., Sengers, J. M. H. L., & Gallagher, J. S. 1990, J. Phys. Ch. R., 19, 677

Sneep, M. & Ubachs, W. 2005, J. Quant. Spec. Radiat. Transf., 92, 293

Thalman, R., Zarzana, K. J., Tolbert, M. A., & Volkamer, R. 2014, J. Quant. Spec. Radiat. Transf., 147, 171

Vardya, M. S. 1962, ApJ, 135, 303

Wagner, W. & Kretzschmar, H.-J. 2008, International Steam Tables: Properties of Water and Steam Based on the Industrial Formulation IAPWS-IF97 (Springer-Verlag Berlin Heidelberg)