ON THE DEPOLARIZATION OF DISCRETE RADIO SOURCES BY FARADAY DISPERSION

B. F. Burn

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Abstract

Summary A study is made of the implications of the recent polarization measurements for the structures of discrete radio sources and the source-observer media. Simple models of wavelength-dependent depolarizing mechanisms are investigated and it is found that most are incompatible with the observations of Gardner & Whiteoak. The models of internal Faraday dispersion predict a lower polarization at 30 cm than is observed. It is suggested that the depolarization of the Crab nebula is produced by Faraday rotation in the filamentary shell that surrounds the nebula. Such filaments could also exist in the outer regions of extragalactic sources. A complex number representation is used for the state of linear polarization and a Faraday dispersion function is defined to describe the distribution of polarized radiation with respect to Faraday depth. The persistence of polarization at 30 cm, after partial depolarization between 10 cm and 20 cm, implies that the radiation is spread over a large range of Faraday depths. The observed linearity of the plot of the angle of polarization against wavelength squared for most sources implies that it is justifiable to make an assumption which enables one to calculate the Faraday dispersion function of a source from the dependence of its polarization on wavelength. Estimates are given for upper limits to the densities of internal ionized gases in the sources for which we have polarization measurements.

Introduction. Within the last few years great interest has been shown in polarization measurements of discrete radio sources. Early observations ($1-4$) were made to test the theory of Alfven & Herlofsen (5) that the radiation from the sources is due to the synchrotron emission of relativistic electrons spiralling in magnetic fields. The unexpected results of Cooper & Price (6), showing the wavelength dependence of the state of polarization of Centaurus A, revealed the exciting prospect of obtaining from such observations much information about the sources and the media through which their radiation passes. Three further surveys (7-9) are now available and we have polarization measurements of about thirty sources at three or more wavelengths.

The first striking feature of the results to show up is the linearity of the plot of the angle of polarization against the square of the wavelength. The slope of this line for a given source is usually called the rotation measure of the source. Only three sources $_3\mathrm{C}-353$, Tauraus A and Pictor A) show reliable departures from such a law and in all cases these are small. Cooper & Price (6) suggested that this rotation of the plane of polarization is produced by the Faraday effect occurring in the vicinity of our Galaxy. This has been substantiated by the surveys of Gardner & Whiteoak (7) and Seielstad, Morris & Radhakrishnan (8) which showed that there is a correlation between the rotation measure and the galactic latitude of a source. The other important feature is that the degree of polarization of most sources decreases with increasing wavelength, and in no case is there a significant increase. Gardner &Whiteoak (7) suggested that the depolarization is produced by differential Faraday rotation of different lines of sight through the galaxy. However, Seielstad et al. (8) found that there is no correlation between the rate of depolarization and galactic latitude. It is shown in this paper that it is likely that the depolarization is due to Faraday rotation in the outskirts of the sources themselves.

I. Polarization of an extended source. Following the formulation of Burn & Sciama (I0), we define the complex linear polarization P as pe^{2ix} where p and χ are the degree and angle of polarization. For the radiation at wavelength λ from a point r we may write

$$\chi(\mathbf{r},\lambda)=\alpha(\mathbf{r})+\phi(\mathbf{r})\lambda^2,$$

where $\alpha(\mathbf{r})$ is the intrinsic angle of polarization and

$$\phi(\mathbf{r})=K\int_0^r n\mathbf{H}\cdot\mathbf{k}ds$$

is the Faraday depth of the point r with respect to an observer at the origin. In equation (2) n and $\mathbf{H}$ are the free electron density and magnetic field strength at the point sk where $\mathbf{k}$ is a unit vector in the direction of $\mathbf{r}$. When c.g.s. units are used throughout $K=2.62\times10^{-17}$. Assuming that the radiation is due to synchrotron emission by relativistic electrons whose energy distribution follows a power law of index γ, Le Roux (Ir) showed that the intrinsic degree of polarization is

$$p(\gamma)=\frac{3\gamma+3}{3\gamma+7}$$

We write $\epsilon(\mathbf{r},\lambda)$ for the power radiated at wavelength λ from unit volume at $\mathbf{r}$ per steradian in the direction of the observer. Neglecting the bandwidth, beamwidth and index effects (ro), it follows that the observed polarization of the integrated emission from a source is

$$P\left(\lambda^2\right)=\frac{\iint_{\text{source}}\ \epsilon(\mathbf{r},\lambda)p(\mathbf{r})e^{2i\left\{\alpha(\mathbf{r})+\phi(\mathbf{r})\lambda^2\right\}}dsd\Omega}{\iint_{\text{source}}\ \epsilon(\mathbf{r},\lambda)dsd\Omega},$$

where $d\Omega$ is an element of solid angle about $\mathbf{k}$.

2. Spectral effect. Apart from Faraday dispersion there is one further process which can produce variations of the observed polarization with wavelength. This effect is due to spatial variations of both the emission spectrum and the state of polarization. In order to estimate its importance we take the case where the source may be considered as consisting of two regions with different spectra and polarizations. Suppose that the polarizations of the two components are P_1 and P_2 and that the spectra are $k_1\nu^{-a_1}$ and $k_2\nu^{-a}$. The polarization of the whole source is

$$P = \frac{rP_1 + P_2}{r + \mathrm{I}},$$

where $r = (k_1/k_2)\,\nu^{-(a_1-a_3)}$ is the ratio of the fluxes from the two components. This situation has two main types.

(A). The first is the case where the degrees of polarization are the same and the interaction is due to differing angles of polarization. If the degrees of polarization are p and the difference in polarization angles is θ, then the minimum degree of polarization, $p\cos\theta$, occurs when $r = \mathrm{I}$. In passing from small to large wavelengths, the intrinsic angle of polarization is rotated by an angle θ, implying quite a significant departure from a λ^2 law of rotation. In general it seems unlikely that such deviations exist, though there are several sources for which this possibility cannot be ruled out.

The observed fact that the degree of polarization always decreases with λ increasing implies that it is always the component which is less intense at the frequencies of polarization observations which has the steeper spectrum. If the wavelength is increased still further until $r > \mathrm{I}$ the degree of polarization will rise again - this has not been observed. Fitting this model to the data, we find that for most sources the required difference in spectral indices is greater than 0.2 and that $r \approx \mathrm{I}$ at a wavelength in the range 30 cm to 60 cm. The superposition of two such spectra produces a concave spectrum at radio wavelengths-such spectra are very rare (15). Of the sources in references (**7**) and (**8**), only ${}_3\mathrm{C} - 273$ could be consistent with this model.

(B). The second case is where the polarization angles are about the same, but the two regions have considerably different degrees of polarization. This is perhaps quite feasible in a source where relativistic particles are being continually produced in the central regions and energy loss mechanisms are important enough to modify their spectrum by the time they have diffused to the outer regions. Fitting this model to the data and taking care the total spectrum is not concave at radio wavelengths, we find that for most sources the required difference in spectral indices is between 0.6 and 2.5 . As it is the less intense component that has the flatter spectrum, in most cases its spectral index must be negative. Although this possibility cannot be ruled out it would seem that the required modifications of the spectra by energy losses are excessive.

3. Random fluctuations of the magnetic field. Before studying Faraday dispersion in greater detail, we should consider the effect of fluctuations of

the magnetic field within a source on the intrinsic polarization of the whole source. It has been noted for some time that the energy associated with an extragalactic radio source is so large as to indicate that some catastrophic event involving a whole galaxy or perhaps a super-star has taken place. It is to be expected then that the non-relativistic gas will be in a state of turbulent motion, this motion perhaps amplifying the magnetic field and/or accelerating the relativistic particles. The turbulent motion of the gas in which the field lines are frozen (assuming infinite conductivity) will produce random fluctuations of the magnetic field about any overall structure. We assume that the magnetic field consists of two components, one uniform $\left(H_x{}^0, H_y{}^0, H_s{}^0\right)$ and the other an isotropic random field which we represent by a Gaussian of variance $\frac{2}{3}H_r^2$. The probability that the total field at a given point lies in the range $\mathbf{H}$ to $\mathbf{H} + d\mathbf{H}$ is,

$$\text{Prob}\left(H_x, H_y, H_z\right) = \pi^{-3/2} H_r {-3} \exp\left\{-\frac{\left(H_x - H_x^0\right)^2 + \left(H_y - H_y{}^0\right)^2 + \left(H_z - H_z^0\right)^2}{H_r^2}\right\}.$$

The complex polarization of the radiation from such a point is,

$$P\left(H_x, H_y, H_z\right) = -p(\gamma)\frac{H_x^2 - H_y^2 + 2H_xH_yi}{H_x^2 + H_y^2}.$$

We assume that the emissivity at such a point is proportional to

$$\left(H_x^2 + H_y^2\right)^{(a+1)/2}$$

where a is a constant (the emission spectral index). If the scale of the random

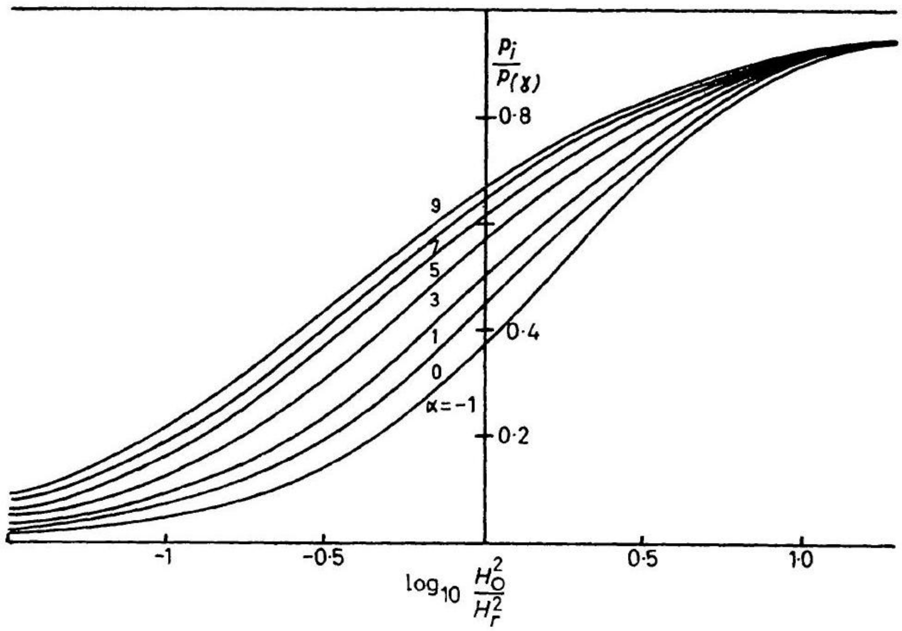


Fig. I. Polarization of a source containing a small scale random magnetic field.

component is much less than the dimensions of the source, then the expression for the intrinsic polarization of the whole source reduces to

where

$$P_i = \frac{a+3}{4} p(\gamma) \frac{{}_1F_1\left(\frac{a+5}{2}, 3, \frac{H_0^2}{H_r^2}\right) H_0^2}{{}_1F_1\left(\frac{a+3}{2}, \mathrm{I}, \frac{H_0^2}{H_r^2}\right) H_r^2} e^{(2\beta+\pi)i},$$

$$\left.\begin{aligned} {H_x}^0 &= H_0 \cos\beta \\ {H_y}^0 &= H_0 \sin\beta \end{aligned}\right\}$$

and the functions ${}_1F_1(.,.,.) aremodifiedhypergeometricfunctions$. Fig. I shows that $p_i \mid p(\gamma)$ is not critically dependent on a, so that one can usefully make the approximation $a \approx 1$. Equation (8) then gives the simple result

$$\begin{aligned} p_i &= p(\gamma) \frac{H_0^2}{H_0^2 + H_r^2} \\ &\approx p(\gamma) \frac{\text{Energy in uniform field}}{\text{Energy in total field}}. \end{aligned}$$

The latter expression follows if we assume that ${H_x}^0 \approx {H_y}^0 \approx {H_z}^0$.

4. Faraday dispersion-general remarks. Before considering some simple models we shall make a few remarks on the Faraday effect in general. It is

quite obvious from the observed linear dependence of χ on λ^2 that Faraday rotation is occurring, and it is natural to ask whether the depolarization is due to Faraday dispersion. Clearly there must be some restrictions on the nature of the dispersion if it is not to disturb the λ^2 law of rotation.

As in reference (10) we superpose all the radiation from the same Faraday depth and write $E(\phi)$ for the fraction of the radiation with Faraday depth ϕ and $\boldsymbol{P}(\phi)$ for its intrinsic polarization. Defining the 'Faraday dispersion function' as $F(\phi) = E(\phi)P(\phi)$, we obtain the Fourier transform relation

$$P\left(\lambda^2\right) = \int_{-\infty}^{\infty} F(\phi) e^{2i\phi\lambda^2} d\phi.$$

It would be very convenient to be able to invert this transform and so obtain the Faraday dispersion function from the relation

$$F(\phi) = \pi^{-1} \int_{-\infty}^{\infty} P\left(\lambda^2\right) e^{-2i\phi\lambda^2} d\left(\lambda^2\right).$$

However, to evaluate this integral we must know $P\left(\lambda^2\right)$ for $\lambda^2 < 0$, and this is not an observable quantity. It is readily seen from equation (II) that this is the polarization we would observe if all of the Faraday rotation were in the opposite sense (i.e. if all the magnetic fields were reversed).

To make use of equation (12), we must assume some property of the source which will enable us to predict the behaviour of $P\left(-\lambda^2\right)$ from that of $P\left(\lambda^2\right)$. The simplest assumption to make is that $\alpha(\phi)$ is the same for all Faraday depths, that is

$$\alpha(\phi) = \text{ constant.}$$

It should be stressed that this does not require all points of the source to have the same intrinsic angle of polarization, but only that the superposition of all the radiation with the same Faraday depth has a polarization direction that is independent of the Faraday depth. Examples of situations where this assumption is valid are :

(i), the direction of the magnetic field is the same for all parts of the source;

(ii), the field of the source has random variations about a mean direction;

(iii), the rotation is external, the Faraday depths of different lines of sight varying at random.

It is possible to justify assumption (13) by the following argument. We have already pointed out that $P\left(-\lambda^2\right)$ is just the polarization that we would observe at wavelength λ if all of the magnetic fields had the opposite sense. A source would be dynamically similar if it contained such fields which are, in fact, those which would have been produced if the primary field had been in the opposite sense. It does not seem likely that the sense of the primary field could be in any way related to the occurrence of the phenomena leading to the production of the radio source. Hence in a large sample of similar sources, there should be roughly equal numbers with either sense. But it is observed that for almost all sources χ is proportional to λ^2 for $\lambda^2 > 0$. We may therefore deduce that for most sources this proportionality also holds for $\lambda^2 < 0$, and we may write

$$\chi\left(\lambda^2\right) = \rho\lambda^2,$$

for all λ^2 (with the appropriate choice of coordinates). The constant ρ is the rotation measure of the source. Substituting (14) in (12), it follows that

$$F(\phi - \rho) = F^*(\rho - \phi).$$

The value of $|F(\phi)|$ represents the flux of linearly polarized radiation at Faraday depth ϕ, expressed as a fraction of the total flux. This function is symmetrical about $\phi = \rho$, while the angles of polarization of these fractions are skew-symmetrical about the same Faraday depth ρ. This latter property would imply rather special relationships between the transverse and parallel components of the source's magnetic field and also a rather special orientation of the source relative to the solar system. It is therefore probable that the structure is such that $\alpha(\phi) =$ constant, as in the case for random fluctuations about a mean direction. It should perhaps be stressed that the author does not wish to imply that all sources with χ proportional to λ^2 must have these properties. The argument is that as nearly all sources have χ proportional to λ^2, nearly all will have the above properties.

If we accept assumption (13), it is possible to calculate the Faraday dispersion function directly from the observed polarization through the equation

$$F(\phi) = \frac{2}{\pi}\int_0^\infty \text{real}\left\{P\left(\lambda^2\right)e^{-2i\phi\lambda^2}\right\} d\left(\lambda^2\right).$$

Unfortunately, this procedure cannot yet be usefully applied to the present data because of the large errors and the small number of wavelengths covered. The one exception is Taurus A which will be discussed in greater detail in section 7 .

5. Internal Faraday dispersion. In this section we shall assume that the external depolarization is negligible and the only change in polarization possible after the radiation leaves the source is rotation of the angle of polarization by an amount proportional to λ^2. We assume that the magnetic field of a source is of the type discussed in section 3 and that the scale of the fluctuations is d. Let $f(x)dx$ be the fraction of the source's radiation which traverses a pathlength between x and $x + dx$ within the source. The distribution of Faraday depths of this fraction may be represented by a Gaussian with mean $KnH_z{}^0x$ and variance $\left(KnH_rd\right)^2 x/2d$. Writing $m = knH_z{}^0$ and $v^2 = \left(knH_r\right)^2 d/2$, the observed polarization at wavelength λ is

$$
\begin{aligned}
P\left(\lambda^2\right) &= p_i \int_0^\infty \int_{-\infty}^\infty \frac{f(x)}{\sqrt{(2\pi v^2 x)}} \exp\left\{-\frac{(\phi - mx)^2}{2v^2 x} + 2i\phi\lambda^2\right\} d\phi dx \\
&= p_i \int_0^\infty f(x) e^{-2sx} dx.
\end{aligned}
$$

where $s = v^2\lambda^4 - im\lambda^2$ and we have chosen the coordinate system such that the intrinsic angle of polarization is zero.

The function $f(x)$ depends on the geometry of the source. The simplest function we can take is $f(x) = L^{-1}$ in $\circ < x < L$, and $\circ$ otherwise. This represents a slab such that the linear depth of each line of sight through the source is L. Equation (17) then yields

$$
P\left(\lambda^2\right) = p_i \frac{\mathrm{I} - e^{-S}}{S},
$$

where

$$
S = \left(KnH_r\right)^2 dL\lambda^4 - 2iKnH_z{}^0 L\lambda^2.
$$

It is perhaps more realistic to assume that the source is a uniform sphere of diameter L, when $f(x) = (3/2L)\left\{\mathrm{I} - (x/L)^2\right\}$ in $\circ < x < L$, and o otherwise.

Equation (I7) then yields

$$
P\left(\lambda^2\right) = p_i \frac{3\left[(S+\mathrm{I})e^{-S} + \frac{1}{2}S^2 - \mathrm{I}\right]}{S^3}.
$$

Fig. 2 plots the polarizations of these models against a variable u which is chosen to be proportional to λ and such that $p \mid p_i = 0.5$ at $u = \mathrm{I}$. The parameter μ is the ratio of the real and imaginary parts of S at $u = \mathrm{r}$.

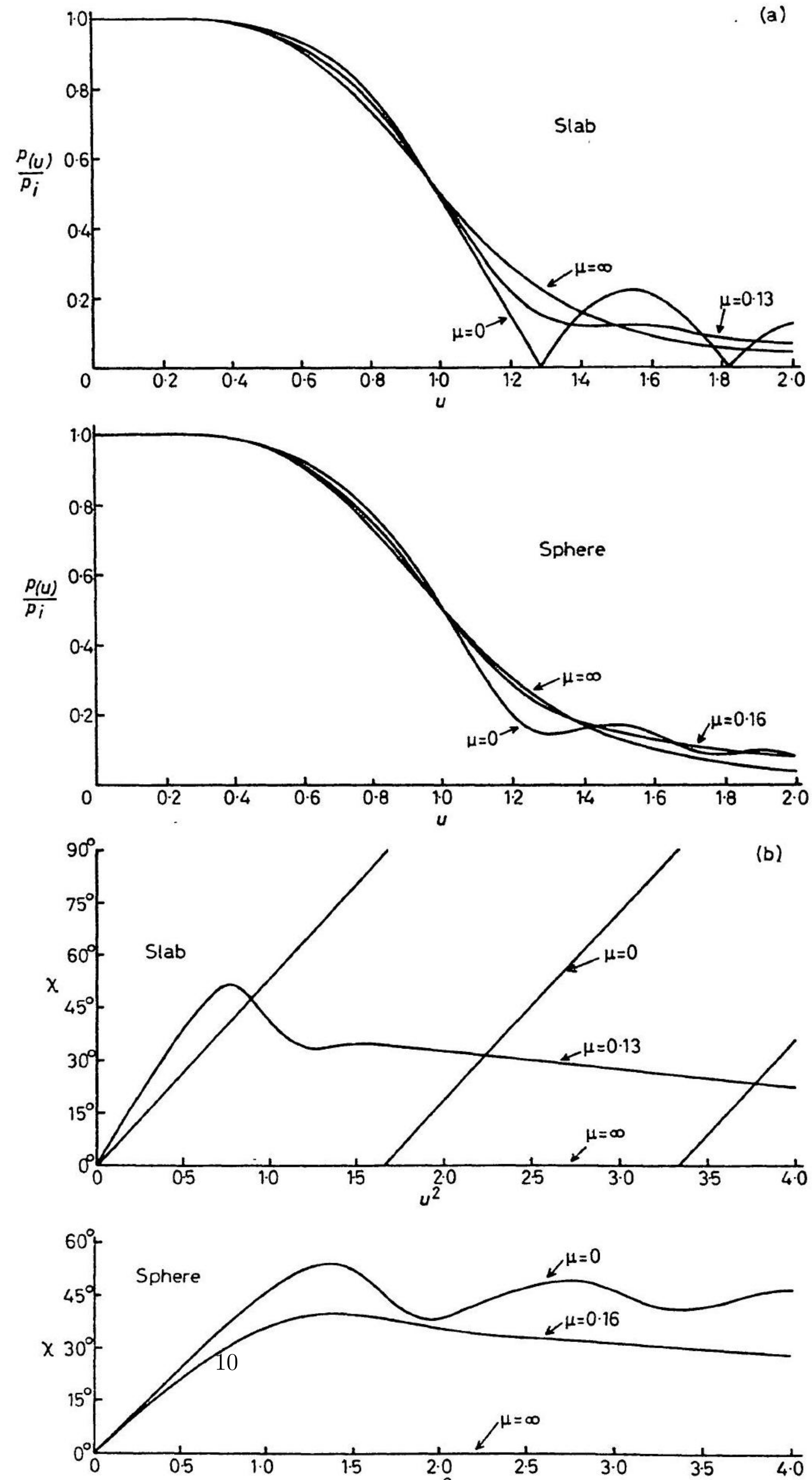
The properties of these models depend on the ratio of the real and imaginary parts of S in the wavelength range where most of the depolarization occurs; (i.e. $|S| \sim \mathrm{I}$). It is readily seen that this is determined by the relative magnitudes of the ratios $\left(H_r/2H_z^0\right)^2$ and L/d (the number of cells, N say, cut by the longest line of sight through the source).

(I) $N \gg \left(H_r/2H_z^0\right)^2$. The imaginary part dominates; the spread in Faraday depths at the appropriate wavelengths is due to the z component of the uniform magnetic field. This corresponds to $\mu = 0$ in Fig. 2. Hence for the slab the depolarization follows the familiar $p_i(\sin\delta/\delta)$ law $\left(\delta = KnH_z L\lambda^2\right)$ with the constant factor p_i due to the intrinsic depolarization produced by the random component of the magnetic field. The angle of polarization follows a λ^2 law of rotation except for discontinuities of $\pi/2$ at $\delta = n\pi$, where the polarization falls to zero. At much longer wavelengths the real part will become more important and finally dominate. However, at these wavelengths, the source is essentially completely depolarized. For a spherical source the Faraday dispersion function is asymmetric and, as a result, there are departures from a λ^2 law of rotation. Such a law is followed very closely for $u < \mathrm{I}$. As the wavelength increases beyond this range, the angle of rotation steadies down with damped oscillations about $\chi = \pi/4$.

(2) $N \ll \left(H_r/2{H_z}^0\right)^2$. The real part dominates at the appropriate wavelengths, corresponding to the case $\mu = \infty$ in Fig. 2. The Faraday dispersion function is essentially symmetric so there are no deviations from a λ^2 law of rotation. For $u <$ I the polarization falls in the same manner as the previous case. At larger wavelengths, the polarization falls more quickly $\left(\propto \lambda^{-4}\right.$ rather than $\left.\lambda^{-2}\right)$ and without the fluctuations present in the previous case.

Assuming that the 10 cm degree of polarization is close to the zero wavelength value, we seen that in most sources the random field has reduced the intrinsic polarization to about (I/IO)$p(\gamma)$. Assuming that ${H_x}^0 \approx {H_y}^0 \approx {H_z}^0$ it follows that $\left(H_r/2H_z^0\right)^2 \sim$ I, and hence the former condition is normally satisfied. Faraday dispersion due to the random component of the magnetic field can only be important if the steady magnetic field is nearly perpendicular to the line of sight.

As we know the polarization of eighteen sources at three or more wavelengths, it is possible to test if any of the above models are consistent with the observations. Assuming a model, we take as given the polarization at two wavelengths, predict the polarization at another wavelength, and test for agreement



with the observed

Fig. 2. Polarization of models of internal Faraday dispersion. (a) Degree of polarization; (b) angle of polarization. value. The three wavelengths chosen for each source are those which appear to give the least errors coupled with maximum spread in wavelength. The polarizations at the two shortest wavelengths were used to predict that at the longest. To allow for the large errors, three values are taken at each wavelength; the observed value and the extremities of the error range. Table I gives the number of combinations out of the twenty-seven possible for which the predicted polarization is greater than the assumed observed value. If the data are consistent with the model, the number of high predictions may be expected to be in the range nine to eighteen, less than nine indicating that the polarization falls at a slower rate than the model, and more than eighteen, that it falls faster. The observations used are those of Gardner & Whiteoak (7) and Cooper & Price (6).

The first three models are those of a spherical source. We see that nine of the fourteen sources showing depolarization are more polarized at large wavelengths than this model predicts. One noticeable exception is $2\mathrm{I}-64$ which is

1 TABLE I

Source	Wavelength (cm)	$\mu = 0$	$\mu = 0 \cdot 16$	$\mu = \infty$	Gaussian
Fornax A (a)	10, 21, 30	0	0	1	1
Pictor A	10, 21, 30	6	6	6	6
3C-161	10, 21, 30	0	0	0	0
3C-270	10, 21, 30	10	9	9	10
3C-273	10, 15, 21	2	2	0	0
Hercules A	10, 21, 30	0	0	0	0
3C-327 (a)	10, 21, 30	6	6	6	6
3C-353	10, 21, 30	9	18	15	1
21-64	10, 21, 30	25	27	27	18
Tauras A	10, 21, 30,	2	0	0	0
Centaurus A (a)	10, 21, 30	17	17	17	3
Centaurus A (b)	15, 19, 30	0	0	0	0
Centaurus A (c)	15, 21, 30	2	2	1	1
13S6 A	10, 19, 21	14	15	13	12

at the other extreme. The remaining sources can be fitted to all three cases. The source ${}_3\mathrm{C}-270$ has a very large error in the 10 cm measurement, and so the fit is not convincing. ${}_{13}$ S6 A is also not significant as we used measurements at 19 and 21 cm, and this spacing is not really large enough to differentiate between models. For Centaurus A (a), the errors are small, but we must bear in mind that this is a double source with only one component polarized (16). The beamwidth effect is operating as the spacing of the components is of the same order as the beamwidth. This will tend to depolarize the 30 cm measurement and so give a false fit with these models. The only source that does give a reliable agreement is ${}_3\mathrm{C}-353$ which is satisfied by all three models. It should be noted that the variations of the angle of polarization show deviations from a

λ^2 law that are consistent with the case $\mu \ll \mathrm{I}$.

In all of the above models most of the radiation comes from regions with Faraday depths of the same order of magnitude. In such models, if there is significant depolarization between 10 cm and 20 cm, then the polarization at 30 cm should be very much lower. However, the 30 cm observations of Gardner & Whiteoak showed that there is still quite appreciable polarization at this wavelength. This would imply that a considerable fraction of the radiation comes from a range of Faraday depths much smaller than the rest. To obtain such a Faraday dispersion function, the author has calculated several models in which there were systematic variations of the emissivity, the magnetic field strength, or the electron density. It was found that very large deviations were required and that these produced a strong asymmetry in the Faraday dispersion function. Hence there should be very significant departures from a λ^2 law of rotation. As such departures have not been found, we must look for other mechanisms which can produce the required Faraday dispersion functions without such asymmetry.

6. External Faraday dispersion. Although there is, as yet, no evidence for a dependence of the rate of depolarization on galactic coordinates, we should not overlook the possibility that the depolarization is produced in either the disk or the halo of our galaxy. In this section we consider a few models of such depolarization, showing that it is unlikely to be significant.

We first investigate the effects of random fluctuations in the magnetic field and/or electron density in a region extending for a distance R from the observer. If the scale of the fluctuations $d \ll \alpha R$, where α is the angular dimension of a radio source, the Faraday dispersion function of the source is well represented by a Gaussian with variance $K^2 \left(nH_{\|}\right)_f^2 dR$, where $\left(nH_{\|}\right)_f{}^2$ is the variance of the product of the electron density and the line of sight magnetic field of a cell. The degree of polarization at wavelength λ is therefore

$$p\left(\lambda^2\right) = p_i \exp\left\{-2K^2 \left(nH_{\|}\right)_f{}^2 dR\lambda^4\right\}.$$

There are three objections to such a model.

(i) The dependence of the depolarization on R should produce a correlation with galactic coordinates. This objection is perhaps not quite so serious for fluctuations in the halo as for fluctuations in the disk.

(ii) Qualitatively, the polarization falls off much faster at large u than do the models of the previous section. The results of an attempt to fit the Gardner & Whiteoak results to this model are shown in the last column of Table I. The only source to give satisfactory agreement is $2\mathrm{I} - 64$.

(iii) For there to be significant depolarization at $\lambda = 20$ cm, we require

$$2K^2 \left(nH_{\|}\right)_f^2 dR \approx 20^{-4}.$$

Applying the condition $d \ll \alpha R$ and taking $\alpha \approx \mathrm{I}'$, we obtain

$$\left(nH_{\|}\right)_f > 4 \times \ \mathrm{I}^{\ -6} \mathrm{Gauss cm}^{-3},$$

in the disk $\left(R \approx \mathrm{IO}^{21}\ \mathrm{cm}\right)$, and $\left(nH_{\|}\right)_f > 4 \times 1\mathrm{I}^{-8}$Gausscm^{-3} in the halo $\left(R \approx \mathrm{IO}^{23}\ \mathrm{cm}\right)$. If $n \approx 0 \cdot I$ for the disk, the magnetic field required is $H_f > 4 \times 10^{-5}$ Gauss which is much too large. This objection is even more serious for the halo.

Objection (iii) above is weaker for large d so we consider the other extreme $d \gg \alpha R$. In this case nearly all the lines of sight to the source pass through the same cells. The spread of Faraday depth across the source produced by one cell will be of the order $\alpha RK\left(nH_{\|}\right)_f$. The polarization at wavelength λ is therefore well represented by

$$p\left(\lambda^2\right) \approx p_i \exp\left\{-2K^2\left(nH_{\|}\right)_f{}^2 \frac{\alpha^2 R^3}{d} \lambda^4\right\}$$

The objection (i) above is now even stronger due to the R^3 dependence of the variance. Objection (ii) is still valid as the qualitative features of the depolarization are identical. The inequalities for $\left(nH_{\|}\right)_f$ in (iii) are still true and now become more marked for larger d. The above models assume that the dispersing cells fill the entire region. We now suppose that they are discrete clouds of dimension d with average spacing $D, \left(nH_{\|}\right)_c^2$ being the variance of $nH_{\|}$for the clouds. The probability of there being m clouds on a line of sight is

$$\mathrm{Prob}(m) = \frac{\eta^m e^{-\eta}}{m!}$$

where $\eta = d^2 R/D^3$ is the average number of cells on a line of sight. If $\eta \gg \mathrm{I}$ the depolarization is qualitatively identical to the above models, and objections (i) and (ii) hold. When $\eta \ll I$ most lines of sight will not pass through any clouds and there will be almost no depolarization.

When $\eta \sim \mathrm{I}$ we consider the two extremes for d. If $d \gg \alpha R$ all the lines of sight from a source pass through the same cloud which will produce a gradient in Faraday depth across the source. Simple models of this situation, e.g. a wedge-shaped cloud in front of a spherical source, exhibit rapid depolarization very similar to the Gaussian for random fluctuations. Therefore, objection (ii) still holds. If $d \ll \alpha R$ the degree of polarization at wavelength λ is

$$P\left(\lambda^2\right) = p_i \exp\left\{-\eta\left[\mathrm{I} - e^{-2K^2 (nH_{11})_c^2 d^2 \lambda^4}\right]\right\}.$$

The fraction $e^{-\eta}$ of the source which is not covered by clouds is not affected while the rest is depolarized rather quickly. This could get around objection (ii) and explain the high polarization observed at 30 cm. It should be pointed out that objection (i) will still hold as η depends on R. To satisfy the observations the parameters must be such that $\eta \approx \mathrm{I}$ and $K\left(nH_{\|}\right)_c d \approx 5 \times 10^{-3}\ \mathrm{cm}^{-2}$. Suppose that the clouds are within the disk. The following inequalities must hold.

$$d \lesssim 1\mathrm{IO}^{18} \text{ cm},$$
$$d/D \lesssim 3 \times 10^{-2},$$
$$(nH)_c \gtrsim 2 \times 10^{-4}\text{Gausscm}^{-3}.$$

In the absence of containing forces such clouds would soon disrupt under their own thermal and magnetic pressure which would be at least 2 orders of magnitude greater than in the surrounding interstellar gas. We might wonder if the containment could be due to the gravitational attraction of stars. The distance to which a star of one solar mass can contain an ionized gas of temperature $10^{4\circ}$K is about 2×10^{14} cm. This is much too small for clouds which would have the interstellar spacing of about 10^{19} cm. It is extremely unlikely that there can exist in the interstellar region clouds having properties that could produce the observed depolarization. It is even more unlikely that the clouds can exist in the halo. However, if future observations show that there is a correlation between depolarization and galactic latitude, we may have to take the existence of such clouds seriously.

7. The Crab nebula. Evidently the Faraday dispersion functions of most sources must have two properties in common. For each source it must be symmetric about a mean Faraday depth, which is the rotation measure of the source, otherwise there will be departures from a λ^2 rotation law. Also, comparable fractions of the radiation must be spread over ranges of Faraday depths of different orders of magnitude. The Faraday dispersion function may be calculated from equation (I6) provided we have a large number of polarization measurements over a large range of wavelengths. Such information is available on the Crab nebula and is summarized in Fig. 3 .

It is obvious that there are departures from a λ^2 law of rotation at short wavelengths. The best fit using all the radio data is $\chi = \left(150.5 - 0.147\lambda^2\right)^\circ$, giving a deviation at 3 cm of -6°, and at optical wavelengths of 9°. Ignoring the observations at wavelengths less than 9 cm, the best fit is $\chi = \left(154 - 0.155^2\right)^\circ$, giving a 3 cm deviation of -9.5°, and an optical deviation of 5.5°. It has been noted that the distributions of optical and radio emission differ (17), indicating that the spectral index is less in the outer regions by about 0.07 . This would allow a spectral effect between optical and radio frequencies which could possibly explain the discrepancy between the extrapolated and observed angle of polarization of the optical continuum. To estimate the magnitude of possible discrepancies we can use the surveys of Walraven (13) and Woltjer (14) to

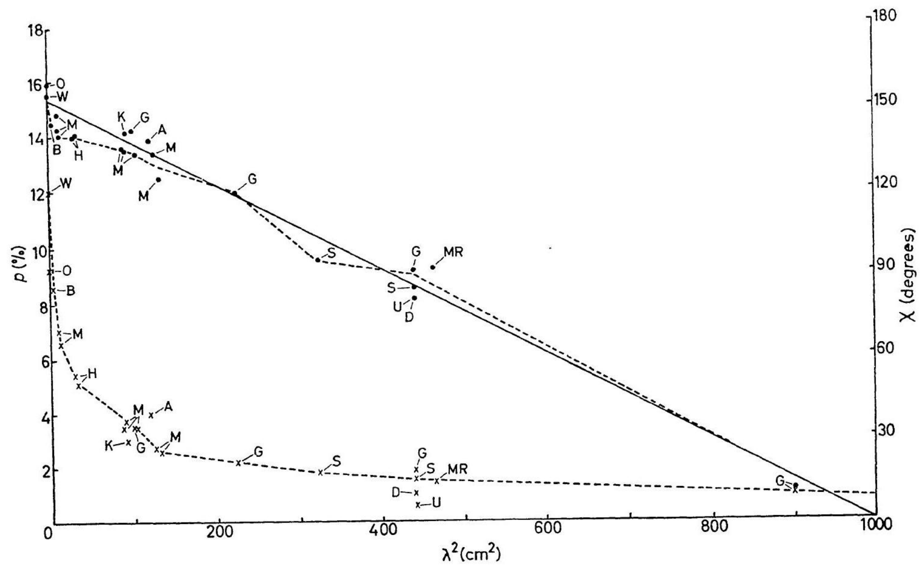


Fig. 3. Summary of the polarization data. The dots $(\cdot)$ and crosses (x) represent the observed angles and degrees of polarization. The solid line is the best fitting straight line through the dots for $\lambda > 9$. The broken lines show the assumed dependence of polarization on wavelength for the calculation of the Faraday dispersion function. O, Oort $\mathcal{E}^5$ Walraven (12); W, Woltjer (14); B, Boland et al. (21); M, Mayer et al. (9); H, Hollinger et al. (22); K, Kusmin E' Udal'tsov (23); A, Altenhoff et al. (24); G, Gardner &' Whiteoak (7); S, Seielstad et al. (25); D, Davies Ξ' Verschuur (26); U, Udal'tsov (27); MR, Morris & Radhakrishnan (28).

estimate the states of polarization of the central and outer regions. It is found that the degree of polarization is less in the outer regions, but the angle of polarization does not change significantly. This means that although the spectral effect probably reduces the degree of polarization by about 2%, it will not produce any noticeable change in the angle, $2^\circ - 3^\circ$ at most. It is significant that the results of Walraven give lower degrees of polarization in the outer region. This is perhaps because Woltjer subtracted the radiation due to the filaments, while Walraven did not make this correction. The polarization measurements of Oort & Walraven also did not allow for filamentary emission, and so it seems that the best value to take for the degree of polarization of the optical continuum is 14% and for the intrinsic polarization at radio frequencies 12%. There also appears to be a discrepancy between the results of Oort & Walraven and the later surveys in measurements of the angle of polarization, which according to the later data is about 155°, in good agreement with the straight line fitted ignoring the short wavelength results.

It is quite easy to attribute the short wavelength deviations to Faraday rotation effects. The rotation measure at long wavelengths is the mean Faraday depth of the fraction of the source with a small spread of Faraday depths, as the radiation from regions with a large spread of Faraday depth is essentially unpolarized. If the average Faraday depth of the fraction with a large spread is not the same, there will be deviations from λ^2 dependence of the angle of polarization at short wavelengths. We are able to use equation (I6) to estimate the required Faraday dispersion function. The assumed $P\left(\lambda^2\right)$ is shown in Fig. 3. The results are shown in Fig. 4 where the fraction of the radiation in ranges of $(\phi - \rho)$ of half an order of magnitude are shown. The rotation measure at long wavelengths $\rho = 2.7 \times 10^{-3}\mathrm{radcm}^{-2}$, is probably due to regions between the source and the observer.

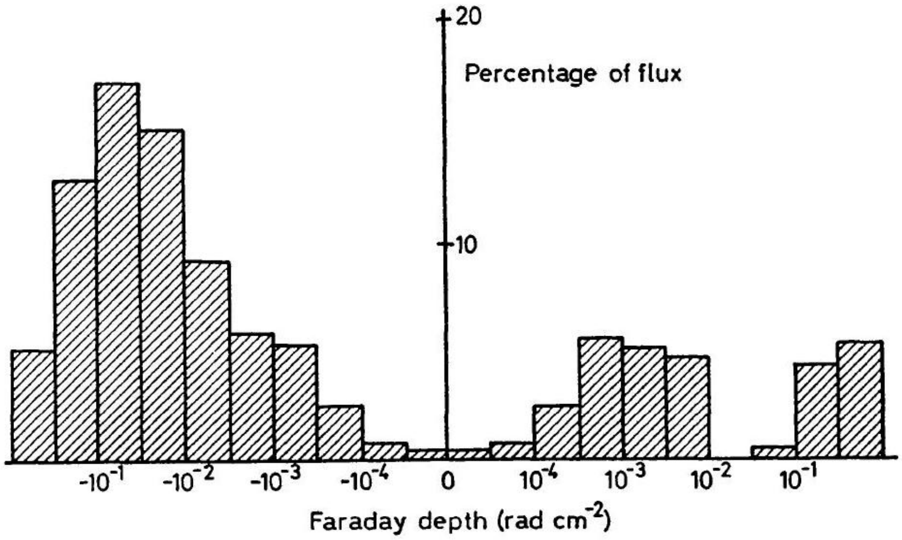


FIG. 4. Faraday dispersion function of the Crab nebula. The rotation measure is taken as the zero of Faraday depth.

A closer look at the structure of the nebula reveals the likely reason for this large range of Faraday depths. The most striking feature of the nebula is the filamentary shell which surrounds the central continuum. The filaments are threadlike dense, ionized regions which, as Woltjer (I7) pointed out, are probably due to the passage of electric currents on the surface of the nebula. These currents are necessary to match the force free field of the nebula to the interstellar field and are directed along the axes of the filaments. The magnetic fields due to these currents are circular about the filaments, and so large fluctuations are to be expected in the Faraday depths of different lines of sight through the filaments. Assuming that the filaments are held together by a pinch effect, we can estimate the magnetic field due to the currents (see reference 17). For a bright filament, they are of the order 3×10^{-4} Gauss, and so the maximum Faraday depth is of the order 0.2 cm^{-2} which is of the required magnitude. The extent of the filaments is difficult to estimate, but they are certainly more extensive than those visible in photographs taken in the light of emission lines (r8). Between the individual filaments there are gaps, and the radiation from behind these will have a small spread in Faraday depth, as both the density and the magnetic field due to the filament currents will be very small. The variation of the mean Faraday depths of the fractions with different spreads is also to be expected if there is a net flow of lines of force across the surface; that is, if the external field is partially linked to the source's field. If the linkage was present before the formation of the filaments, it would be expected that the normal field is also greater in the filaments, as the lines of force will have been crowded together when they condensed. The variations of the mean Faraday depth can therefore also be explained by the filamentary structure.

We can calculate the mass of the filamentary shell from $E(\phi)$. The density, the magnetic field and the thickness of the filaments (d) in front of regions with

TABLE II

Range of $\log\|\Phi\|$	Average Faraday depth $\left(\mathrm{cm}^{-2}\right)$	Fraction of radiation	Mass of filaments (Solar masses)
$-0.5\ldots0.0$	0.56	0.09	0.30
$-1.0\ldots-0.5$	0.18	0.16	0.17
$-1.5\ldots-1.0$	0.056	0.18	0.06
$-2.0\ldots-1.5$	0.018	0.14	0.02
$-2.5\ldots-2.0$	0.0056	0.14	0.01
$-3.0\ldots-2.5$	0.0018	0.11	–
$-3.5\ldots-3.0$	0.00056	0.11	–
$\ldots-3.5$	0.00018	0.07	–

Faraday depth in ϕ to $\phi+d\phi$ satisfy the relation $\phi = KnH_{\|}d$. Assuming that $H_{\|}$is constant and A is the apparent area of the source, then the mass dm in these filaments is

$$\begin{aligned} dm &= nAm_H d\phi \\ &= \frac{Am_H\phi E}{KH_{\|}}d\phi, \end{aligned}$$

where m_H is the mass of a hydrogen atom. Hence

$$m = \frac{Am_H}{K}\int\frac{\phi E}{H_{\|}}d\phi.$$

The results of this calculation are shown in Table II. The first three columns give the ranges of Faraday depth, the average Faraday depth and the fraction of the radiation in these ranges. Column 4 gives the masses of the filament: producing these Faraday depths assuming that $H_{\|}$is 10^{-4} Gauss and A i r.85 $\times$ IO^{37} cm^2. The total mass of the shell is twice the sum of the masses is column 4 as the filaments at the back have very little effect on $E(\phi)$. It is sees to be I $\cdot$ I$M\odot$ which is in quite reasonable agreement with the value of $0.64Mc$ that was obtained by O'Dell (18) from photoelectric observations of the flux i H_β emission.

8. Other sources. The absence of internal Faraday dispersion of the tyf of section 4 enables us to put an upper limit to the density of the thermal

g within a source once the magnetic field strength and the dimensions of the sour are known. Both of these parameters may be estimated if an optical identificati enables us to determine the distance of the source (19). If H_t is the total magnetic field strength, then $H_{\|}$may be estimated from equation (IO) as

$$H_{\|} \approx \left[\frac{p_i}{3p(\gamma)}\right]^{1/2} H_t$$

Assuming that the depolarization between the two longest wavelengths is due to internal dispersion in a spherical source, we can estimate upper limits for the density and total mass of the ionized gas in the body of the source. Columns 4 and 5 of Table III give these limits for all of the sources, in the polarization surveys, which have been optically identified. The volumes and diameters in columns 2 and 3 are taken from reference (8). In the case of a double source, we give the diameter of each component and the total volume.

TABLE III

Source	Volume (cm^3)	Diameter (kpc)	$B \times 10^5$ (Gauss)	Density (cm^{-3})	Mass $(\mathbf{M}_{\odot})$
$_3\mathrm{C}-33$	$I \times 10^{66}$	$4 \cdot 4, 4 \cdot 4$	30	6×10^{-5}	5×10^4
$_3\mathrm{C}-78$	4×10^{68}	29	$3 \cdot 0$	1×10^{-4}	4×10^7
Fornax A (a)	$I \times 10^{70}$	89	0.8	3×10^{-4}	3×10^9
Fornax A (b)	1×10^{70}	89	0.8	3×10^{-5}	3×10^8
$_3\mathrm{C}-98$	6×10^{68}	34	3	2×10^{-4}	9×10^7
Pictor A	2×10^{70}	103	$2 \cdot 4$	9×10^{-5}	$1 \times 10/$
Taurus A	6×10^{55}	0.0016	ro	0.9	0.04
$_3\mathrm{C}-270$	2×10^{67}	$8 \cdot 6, 8 \cdot 6$	3	6×10^{-4}	1×10^7
$_3\mathrm{C}-273$	3×10^{67}	Io, Io	I5	3×10^{-4}	9×10^6
Centaurus A (a)	2×10^{66}	$3 \cdot 5, 3 \cdot 5$	$8 \cdot 0$	5×10^{-4}	7×10^5
Centaurus A (b)	5×10^{70}	120,120	0.6	9×10^{-5}	4×10^9
Hercules A	3×10^{70}	100, roo,	$5^{\cdot 0}$	4×10^{-5}	9×10^8
$_3\mathrm{C}-327$	$1 \cdot 6 \times 10^{70}$	25,25	$4 \cdot 0$	3×10^{-4}	1×10^8
$_3\mathrm{C}-433$	3×10^{68}	26	$3 \cdot 0$	4×10^{-4}	1×10^8
$_3\mathrm{C}-353$	5×10^{68}	25	$4 \cdot 0$	$I \times 10^{-3}$	$4 \times 10^{8*}$

- This is an estimate of the density as model D does fit the data.

The depolarization features of other sources are similar to those of the Crab, giving the same difficulty in interpreting the large polarization at long wavelengths. It is therefore natural to wonder whether these features are also produced by filamentary structures in the outskirts of these extragalactic sources. The above estimates of upper limits to the internal densities show that for all the sources if the temperature is less than $10^{8\circ}$K the magnetic and cosmic ray pressure are very much greater than the thermal pressure. The internal magnetic field is therefore force free, and we may expect filamentary structure in the outskirts associated with surface currents.

To calculate the amount of radiation emitted by these filaments in emission lines, we must know their density, temperature, and dimensions. We tentatively assume that within the filaments the temperature T is $2 \times 10^{4\circ}$K as in the Crab and that the thermal and magnetic forces balance. Hence

$$2KnT \approx \frac{H^2}{8\pi}$$

A fraction $E(\phi)d\phi$ of the source is covered by filaments with Faraday depth ϕ, and these filaments have thickness d where $\phi \approx knH_{\|}d$. The emissivity in H_β is given by Burgess (20) as

$$j\left(H_\beta\right) = \mathrm{I} \cdot 2 \times \mathrm{IO}^{-25} n^2 \mathrm{ergcm}^{-3}\ \mathrm{s}^{-1}.$$

It follows that the total flux in H_β received from a source of angular diameter α' is

$$S\left(H_\beta\right) \approx 6 \times \mathrm{I}^{-7} \alpha^2 H \int |\phi| E(\phi) d\phi.$$

For the extragalactic sources, we have no knowledge of $E(\phi)$ for $|\phi| > 10^{-2}$, and the brightness could be down to $^{-3-3}$ of that of the Crab. However, if the filaments are dense enough to produce depolarization at cm wavelengths, there could be detectable line emission from the outskirts of these sources. It should be pointed out that the filaments are too small to be resolved. If the scale of the filaments were larger, the density would need to be correspondingly less and the flux reduced in the same ratio.

9. Conclusions. In this paper we have shown that it is possible to obtain interesting information concerning the internal structure of radio sources from low resolution polarization observations, provided a large range of wavelengths is covered. The linearity of the plot of the angle of polarization against wavelength squared implies that we may deduce the Faraday dispersion function of a source from the dependence of its polarization on wavelength. This calculation requires the assumption that the Faraday dispersion function may be represented by a real function and that any departures from a λ^2 law of rotation are due to its asymmetry. As infinitely many source structures can give rise to the same Faraday dispersion function, we must use other types of observation to determine the actual structure of the source.

There are sufficient data on the Crab nebula for its Faraday dispersion function to be calculated. It is found to be spread over a large range of Faraday depths with comparable fractions of the radiation being spread over ranges of different orders of magnitude. This is probably produced by the filamentary shell which surrounds the nebula, the fraction of the radiation from behind the filaments being depolarized at short wavelengths, while the radiation from behind the gaps remains polarized at much longer wavelengths. The high polarizations measured by Gardner & Whiteoak (7) and Cooper & Price (6) at 30 cm indicate that the Faraday dispersion functions of other sources are similar to that of the Crab. It should perhaps be mentioned at this point that there is tentative evidence (29) that there is still significant polarization at much longer wavelengths. It is therefore suggested that filamentary structures may also exist in the outskirts of extragalactic sources. It is interesting to note that there have recently been other indications of such structures in extragalactic objects (**30**, **3I**).

It is possible to estimate an upper limit to the density of the internal ionized gas of a source from its rate of depolarization once we know its dimensions and magnetic field strength.

In the near future the results of more complete surveys covering a larger range of wavelengths will be available. Such observation will be invaluable in determining the physical structure of the discrete radio sources and will perhaps also shed some light on the source observer media. Acknowledgments. I would like to thank Dr D. W. Sciama for suggesting the problem and for much valuable advice. Also, I am indebted to Dr C. H. Mayer for communicating the N.R.L. results prior to publication. The many helpful suggestions of the referee are gratefully acknowledged.

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