Computational geometry

Homework, 15 points

1. For the convex hull algorithm we have to be able to test whether a point r lies left or right of the directed line through two points p and q. Let $p = (p_x, p_y)$, $q = (q_x, q_y)$, and $r = (r_x, r_y)$. Show that the sign of the determinant $D = \begin{vmatrix} 1 & p_x & p_y \\ 1 & q_x & q_y \\ 1 & r_x & r_y \end{vmatrix}$ determines whether r lies left or right of the line.

2. Let E be an unsorted set of n segments that are the edges of a convex polygon. Describe an $O(nlogn)$ algorithm that computes from E a list containing all vertices of the polygon, sorted in clockwise order.

3. Let S be a set of n triangles in the plane. The boundaries of the triangles are disjoint, but it is possible that a triangle lies completely inside another triangle. Let P be a set of n points in the plane. Give an $O(nlogn)$ algorithm that reports each point in P lying outside all triangles.

4. Let S be a set of n circles in the plane. Describe a plane sweep algorithm to compute all intersection points between the circles. (Because we deal with circles, not discs, two circles do not intersect if one lies entirely inside the other.) Your algorithm should run in $O((n + k)logn)$ time, where k is the number of intersection points.

5. Let P be a set of n points in the plane. Give an $O(nlogn)$ time algorithm to find two points in P that are closest together. Show that your algorithm is correct.