20. Let X be a problem that belongs to the class NP. Then which one of the following is true?
(A) If X is NP-hard, then it is NP-complete
(B) If X can be solved deterministically in polynomial time, then P = NP
(C) There is no polynomial time algorithm for X
(D) X may be undecidable

**PART – B (5 × 4 = 20 Marks)**
Answer **ANY FIVE** Questions

21. Prove the equations using mathematical induction $\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$.

22. Solve the following recurrence equation using substitution method $t_n = nt_{n-1}$ with initial condition $t_0 = 1$.

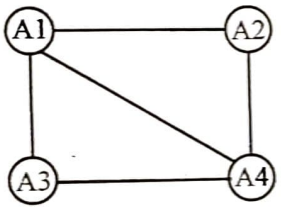
23. Write the general algorithm for divide and conquer method.

24. Multiply the following two matrices using strassen's multiplication method

$$A = \begin{bmatrix} 1 & 3 \\ 4 & 7 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

25. Find an optimal solution to the knapsack instance $n = 7$, $m = 15$,
$(p_1, p_2, \ldots\ldots p_7) = (10,5,15,7,6,18,3)$ and $(w_1, w_2, \ldots\ldots w_7) = (2,3,5,7,1,4,1)$.

26. Colour the following graph using graph colouring algorithm. What is the minimum number of colour required?




27. Distinguish between randomized and deterministic algorithms.

**PART – C (5 × 12 = 60 Marks)**
Answer **ALL** Questions

28. a. Define master theorem and its cases. Apply master theorem to solve the following equations
(i) $T(n) = 3T(n/4) + n\log n$
(ii) $T(n) = 9T(n/3) + n$
(iii) $T(n) = T(2n/3) + 1$

**(OR)**

b.i. Devise an algorithm for generating 'n' terms of Fibonacci series.

ii. Calculate the computing time of the above algorithm using frequency count method and analyze the time complexity using Big-oh notation.

iii. Write the recurrence equation for Fibonacci series and solve the equation using substitution method.

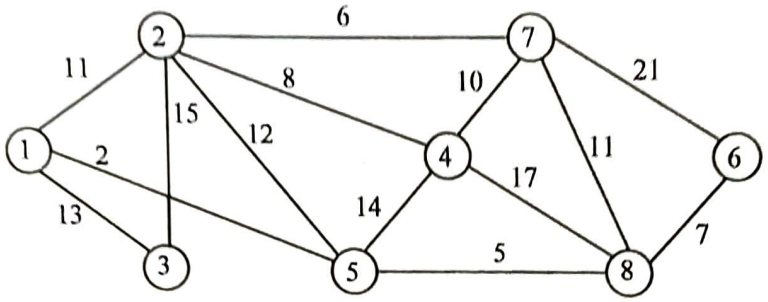
29. a.i. Devise an algorithm for quicksort using divide and conquer method. Also sort the following sequence of characters in non-decreasing order using quicksort. "EXPONENTIAL". (7 Marks)

ii. Analyze the best, average and worst case complexity of quick sort algorithm. (5 Marks)

**(OR)**

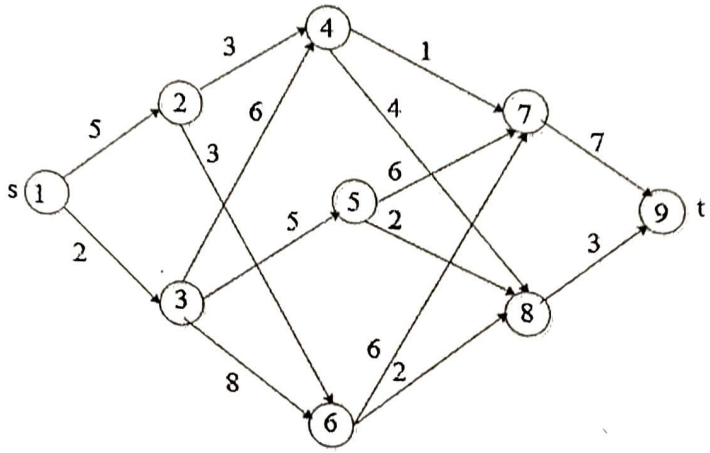
b. Given a set of points in the plane. Write an algorithm to find the smallest convex polygon that contains all the points of it. Also analyse the time complexity of your algorithm.

30. a. Compute a minimum cost spanning tree for the graph of the following figure using
(i) Kruskals algorithm
(ii) Prim's algorithm




**(OR)**

b. Find a minimum cost path from 's' to ' t' in the multistage graph of the following figures. Do this using
(i) Forward approach and
(ii) Backward approach




31. a. Write a backtracking algorithm for the sum of subsets problem using the state space tree corresponding to the variable tuple size formulation.

**(OR)**

b. Develop an algorithm to find all the Hamilton cycles of a graph. The graph is stored as an adjacency matrix G[1:n] [1:n] and all cycles begin at node 1.

32. a. Consider the travelling salesperson problem instance defined by the cost matrix

$$\begin{bmatrix} \alpha & 2 & 3 & 4 \\ 1 & \alpha & 4 & 3 \\ 2 & 3 & \alpha & 4 \\ 4 & 3 & 2 & \alpha \end{bmatrix}$$

Solve using branch and bound technique and generate the state space tree.

**(OR)**

b. Write a randomized algorithm for 'Hiring problem' and analyse the time complexity.

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