

String Matching Using The **Rabin-Karp** Algorithm

Outline

- Definition of the Rabin-Karp algorithm
- How Rabin-Karp works
- A Rabin-Karp example
- Complexity
- Real Life applications

String Matching Problem

- The idea of the string matching problem is that we want to find all occurrences of the pattern P in the given text T .
- We could use the brute force method for string matching, which utilizes iteration over T . At each letter, we compare the sequence against P until all letters match or until the end of the alphabet is reached.
- The worst case scenario can reach $O(N*M)$

Definition of Rabin-Karp

- A string search algorithm which compares a string's **hash** values, rather than the strings themselves. For efficiency, the hash value of the next position in the text is easily computed from the hash value of the current position.

How Rabin-Karp works

- Let characters in both arrays T and P be digits in radix- Σ notation. ($\Sigma = (0,1,...,9)$)
- Let p be the value of the characters in P
- Choose a prime number q such that fits within a computer word to speed computations.
- Compute $(p \bmod q)$
 - The value of $p \bmod q$ is what we will be using to find all matches of the pattern P in T .

How Rabin-Karp works (continued)

- The Rabin-Karp string searching algorithm calculates a **hash value** for the pattern, and for each M-character subsequence of text to be compared.
- If the hash values are unequal, the algorithm will calculate the hash value for next M-character sequence.
- If the hash values are equal, the algorithm will do a **Brute Force** comparison between the pattern and the M-character sequence.
- In this way, there is only one comparison per text subsequence, and Brute Force is only needed when hash values match.

A Rabin-Karp example

- Given $T = 31415926535$ and $P = 26$
- We choose $q = 11$
- $P \bmod q = 26 \bmod 11 = 4$

3	1	4	1	5	9	2	6	5	3	5
---	---	---	---	---	---	---	---	---	---	---

$31 \bmod 11 = 9$ not equal to 4

3	1	4	1	5	9	2	6	5	3	5
---	---	---	---	---	---	---	---	---	---	---

$14 \bmod 11 = 3$ not equal to 4

3	1	4	1	5	9	2	6	5	3	5
---	---	---	---	---	---	---	---	---	---	---

$41 \bmod 11 = 8$ not equal to 4

Rabin-Karp example continued

3	1	4	1	5	9	2	6	5	3	5
---	---	---	---	---	---	---	---	---	---	---

$15 \bmod 11 = 4$ equal to 4 \rightarrow spurious hit

3	1	4	1	5	9	2	6	5	3	5
---	---	---	---	---	---	---	---	---	---	---

$59 \bmod 11 = 4$ equal to 4 \rightarrow spurious hit

3	1	4	1	5	9	2	6	5	3	5
---	---	---	---	---	---	---	---	---	---	---

$92 \bmod 11 = 4$ equal to 4 \rightarrow spurious hit

3	1	4	1	5	9	2	6	5	3	5
---	---	---	---	---	---	---	---	---	---	---

$26 \bmod 11 = 4$ equal to 4 \rightarrow an exact match!!

3	1	4	1	5	9	2	6	5	3	5
---	---	---	---	---	---	---	---	---	---	---

$65 \bmod 11 = 10$ not equal to 4

Rabin-Karp example continued

3	1	4	1	5	9	2	6	5	3	5
---	---	---	---	---	---	---	---	---	---	---

$53 \bmod 11 = 9$ not equal to 4

3	1	4	1	5	9	2	6	5	3	5
---	---	---	---	---	---	---	---	---	---	---

$35 \bmod 11 = 2$ not equal to 4

As we can see, when a match is found, further testing is done to insure that a match has indeed been found.

Rabin-Karp example (Alphabets)

- Let's say that our alphabet consists of 10 letters.
- our alphabet = **a, b, c, d, e, f, g, h, i, j**
- Let's say that “a” corresponds to 1, “b” corresponds to **2** and so on.

The hash value for string “cah” would be ...

$$3*100 + 1*10 + 8*1 = 318$$

Complexity

- The running time of the Rabin-Karp algorithm in the worst-case scenario is $O((n-m+1)m)$ but it has a good average-case running time.
- If a sufficiently **large prime** number is used for the *hash function*, the hashed values of two different patterns will usually be distinct.
- If the expected number of valid shifts is small $O(1)$ then the Rabin-Karp algorithm can be expected to run in time $O(n+m)$ plus the time required to process spurious hits.

Applications

- Bioinformatics
 - Used in looking for similarities of two or more proteins; i.e. high sequence similarity usually implies significant structural or functional similarity.