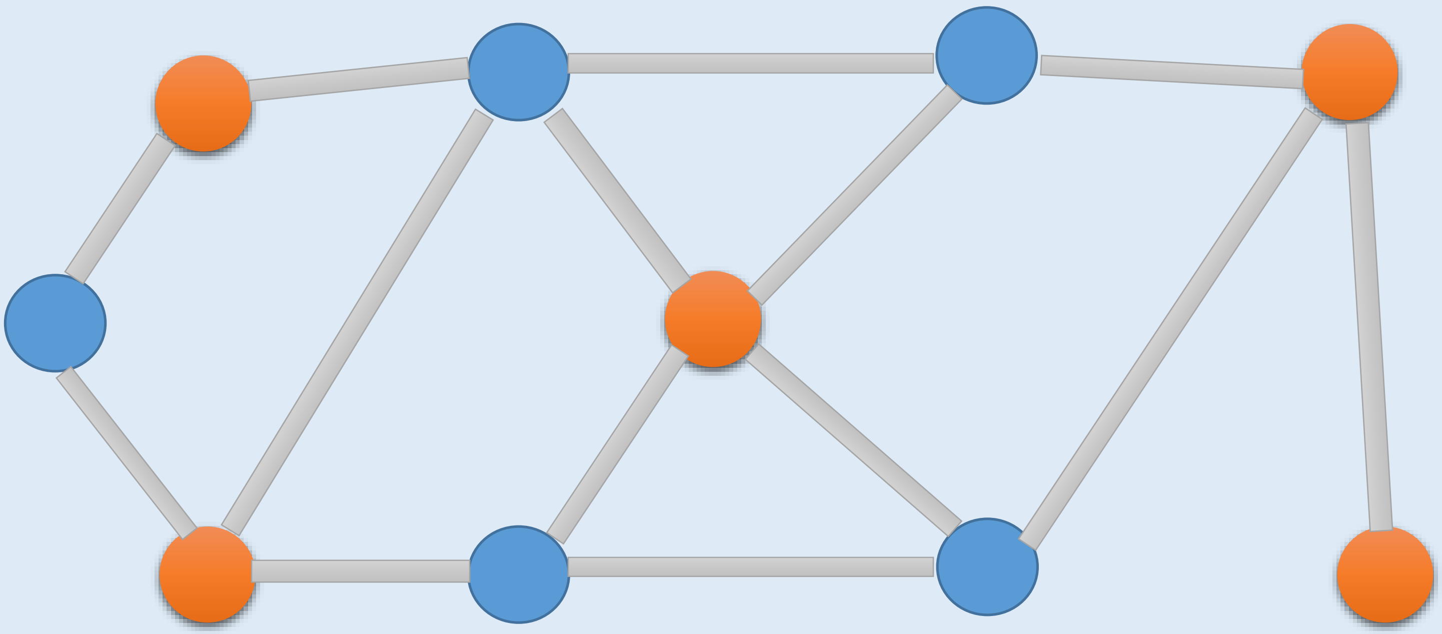
VERTEX COVER PROBLEM



-Gajanand Sharma

APPROXIMATION ALGORITHMS

- Definition: Approximation algorithm
  - An approximation algorithm for a problem is a *polynomial-time algorithm* that, when given input *i*, outputs an element of FS(*i*).
- Feasible solution set
  - A feasible solution is an object of the right type but not necessarily an optimal one.

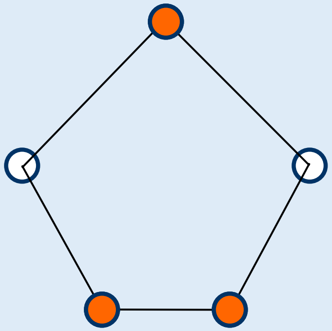
    FS(*i*) is the set of feasible solutions for *i*.

Vertex Cover Problem

- In the mathematical discipline of graph theory, "A *vertex cover* (sometimes node cover) of a graph is a subset of vertices which "*covers*" every edge.
- An edge is *covered* if one of its endpoint is chosen.
- In other words "A *vertex cover* for a graph G is a set of vertices incident to every edge in G."
- The *vertex cover problem*: What is the minimum size vertex cover in G?

Vertex Cover Problem

Problem: Given graph G = (V, E), find *smallest* $V' \subseteq V$ *s. t.* if (u, v) $\in$ E, then u $\in$ V′ or v $\in$ V′ or both.



Vertex Cover : Greedy Algorithm(1)

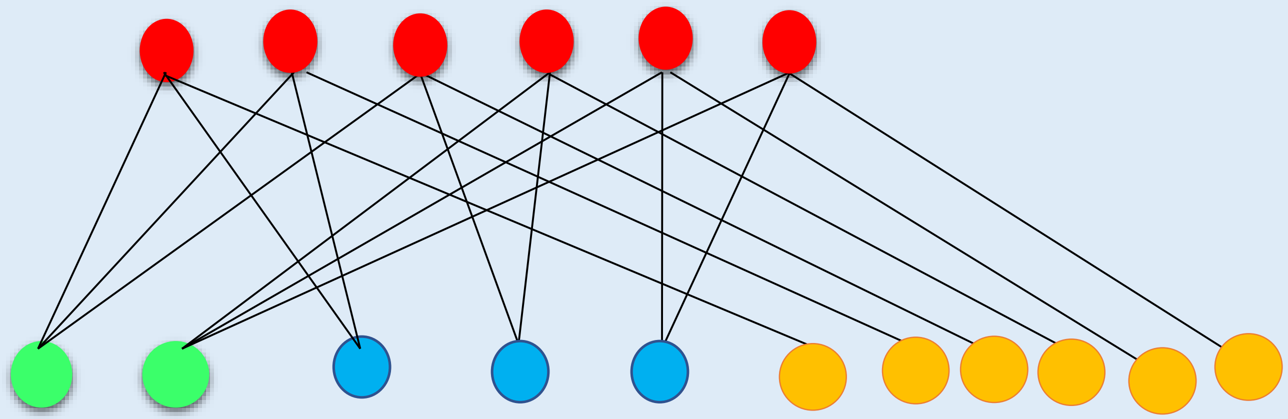
- **Idea:** Keep finding a vertex which covers the maximum number of edges.

  Step 1: Find a vertex v with maximum degree.

  Step 2: Add v to the solution and remove v and all its incident edges from the graph.

  Step 3: Repeat until all the edges are covered.

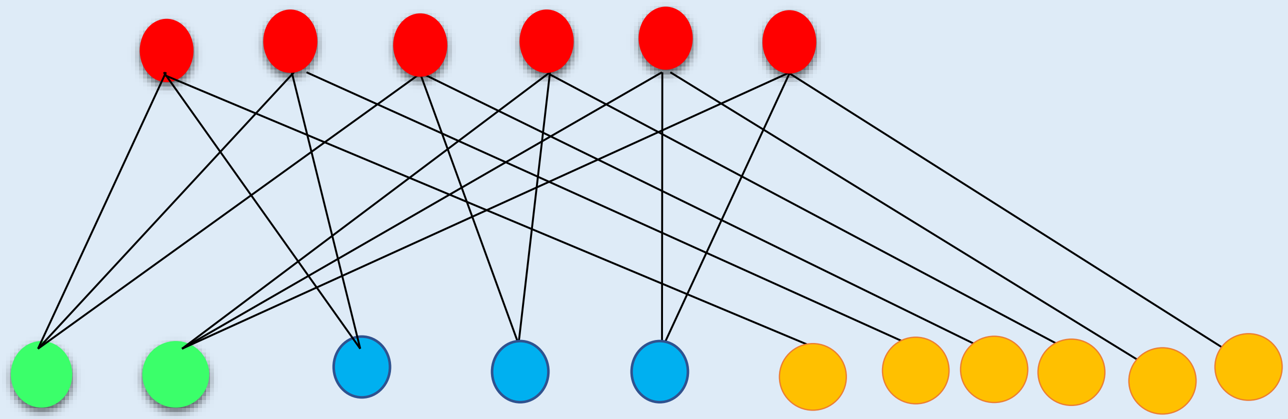
Greedy Algorithm(1): Analysis



Optimal Solution = 6, select all red vertices.

- Greedy approach does not always lead to the best approximation algorithm.

Greedy Algorithm(1): Analysis



Unfortunately if we select the vertices in following order, then we will get worst solution for this vertex cover problem-

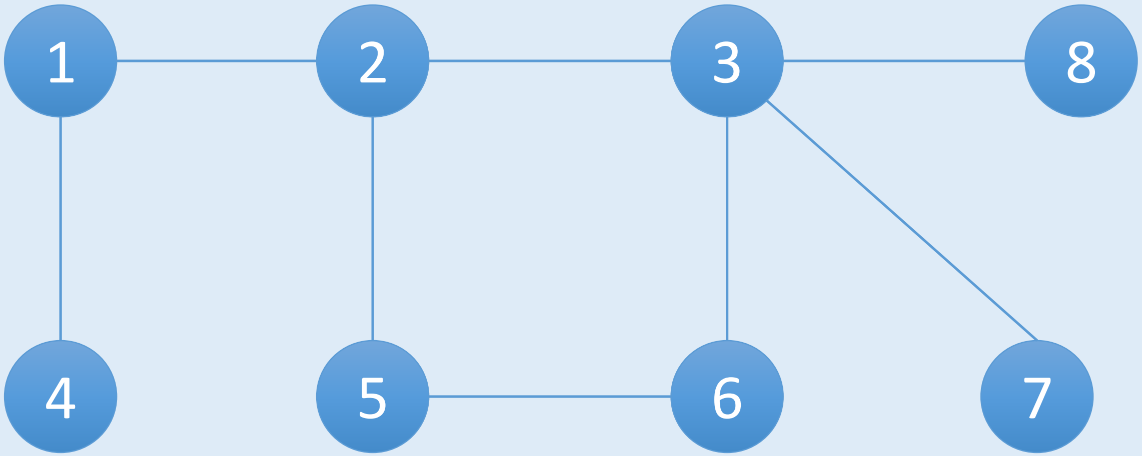
- First we might choose all the green vertices.
- Then we might choose all the blue vertices.
- And then we might choose all the orange vertices.

Solution=11;

Vertex Cover : Algorithm(2)

```
APPROX-VERTEX-COVER
C ← Ø ;
E′ ← E
while E′ ≠ Ø; do
        let (u, v) be an arbitrary edge of E′
        C ← C ∪ {(u, v)}
        remove from E′ all edges incident on either u or v
end while
```

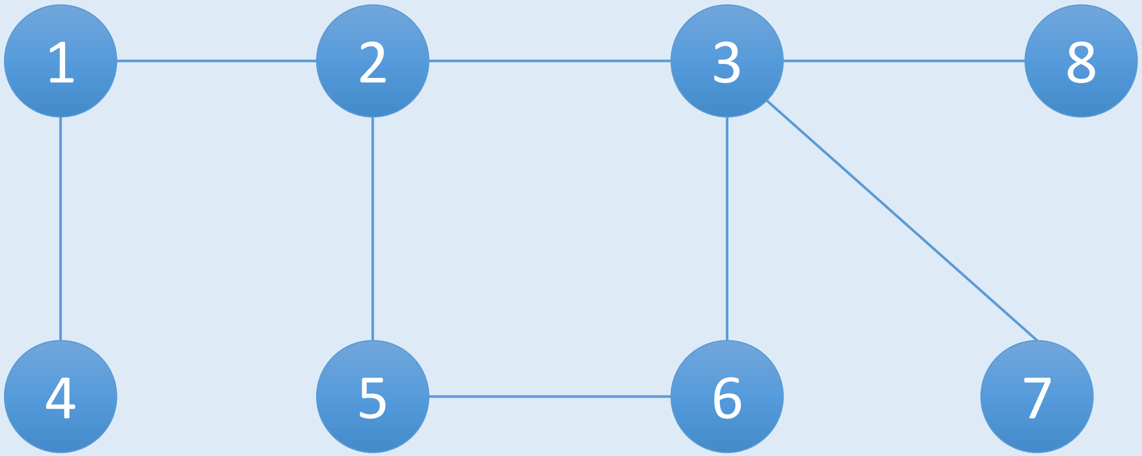
Algorithm(2): Example



Initially C = Ø

E′ = {(1,2) (2,3) (1,4) (2,5) (3,6) (5,6) (3,7) (3,8)}

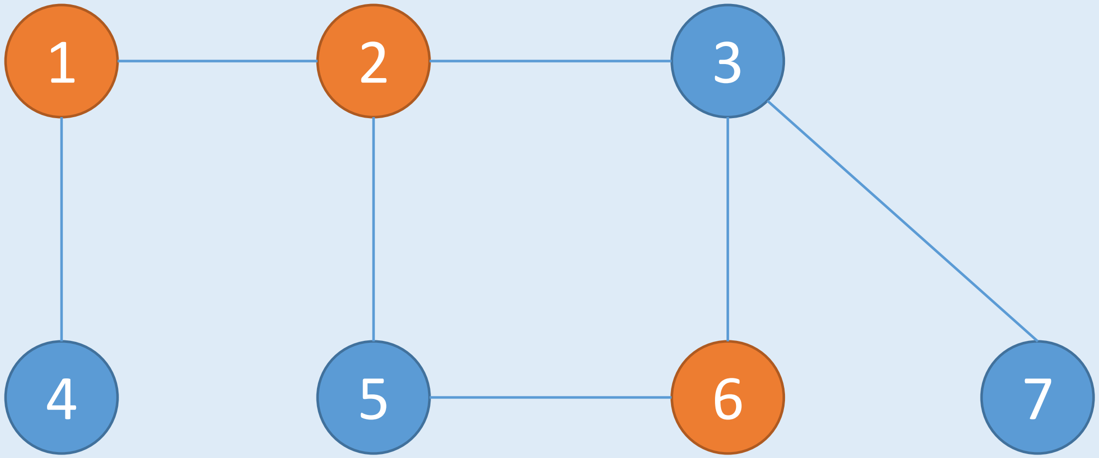
Algorithm(2): Example



C =	1	2	3	6

E′ = {~~(3,6) (5,6) (3,7) (3,8)~~}

Algorithm(2): Example (Cont…)

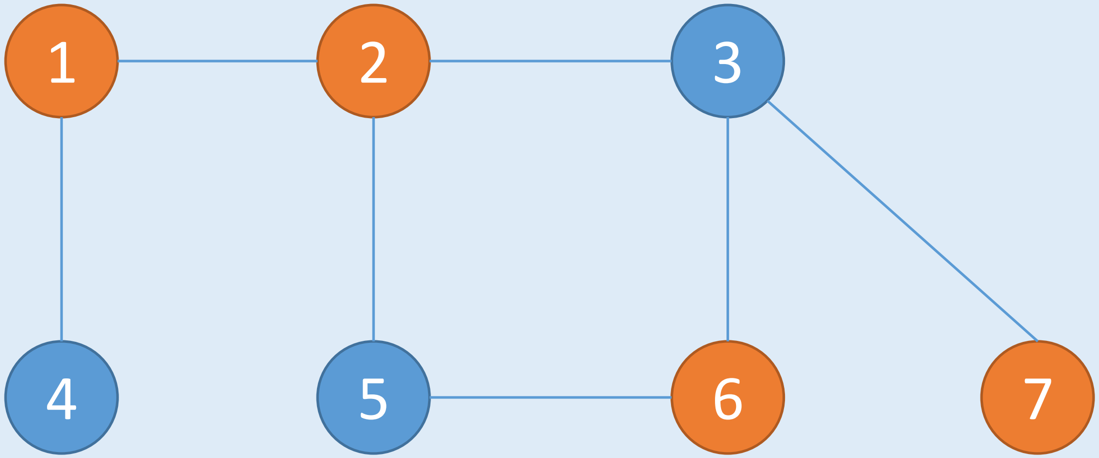


Are the red vertices a vertex-cover?

No………. why?

Edge (3, 7) is not covered by it.

Algorithm(2): Example (Cont…)



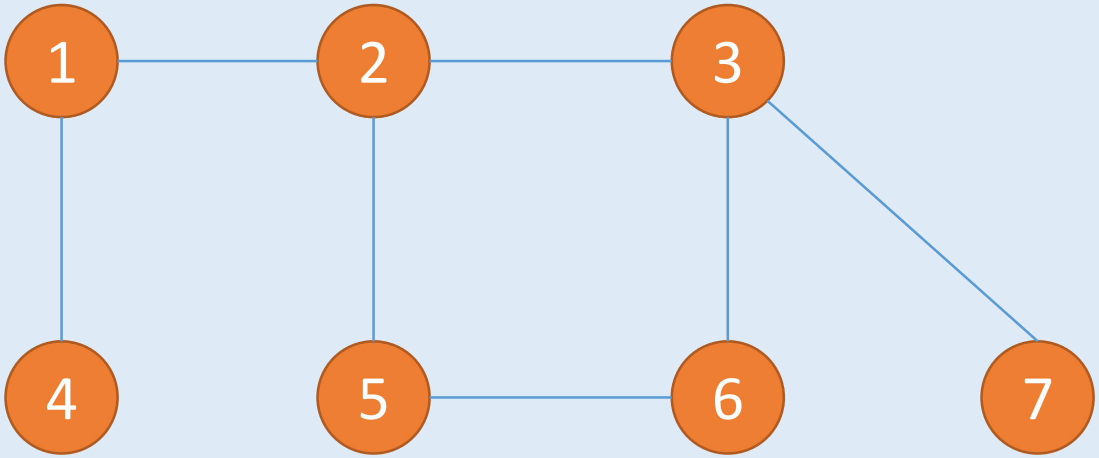
Are the red vertices a vertex-cover?

Yes

What is the size?

Size = 4

Algorithm(2): Example (Cont…)



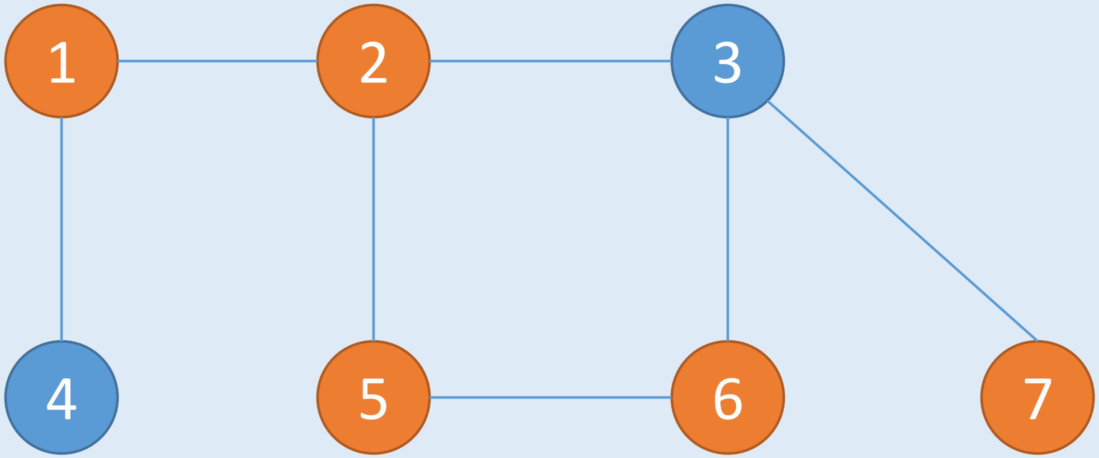
Are the red vertices a vertex-cover?

Of course……..

What is the size?

Size = 7

Algorithm(2): Example (Cont…)



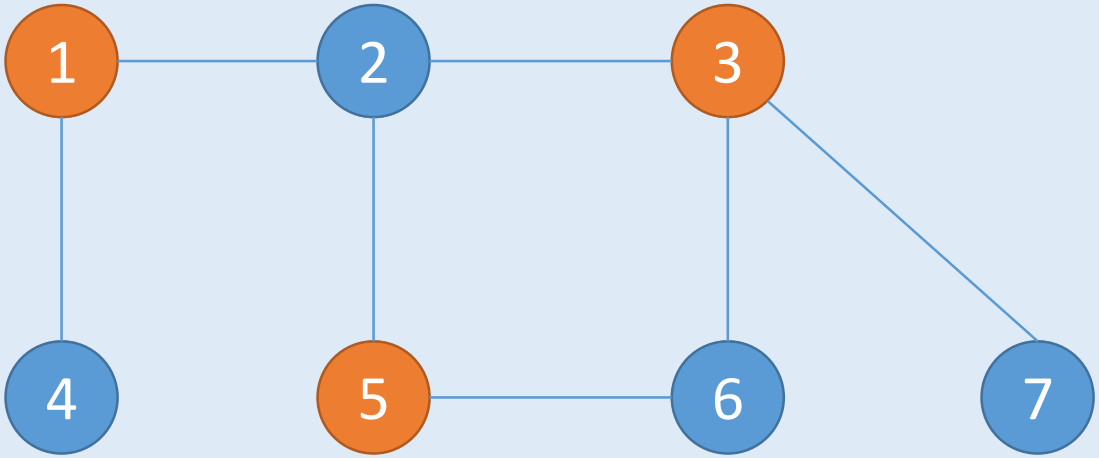
Are the red vertices a vertex-cover?

Yes

What is the size?

Size = 5

Algorithm(2): Example (Cont…)



Are the red vertices a vertex-cover?

Yes

What is the size?

Size = 3

Conclusion

- A set of **vertices** such that each edge of the graph is incident to at least one **vertex** of the set, is called the vertex cover.
- Greedy algorithm may or may not produce optimal solution.
- Approximation algorithm does not always guarantee optimal solution but it’s aim is to produce a solution which is as close as possible to the optimal solution.

Thank You……