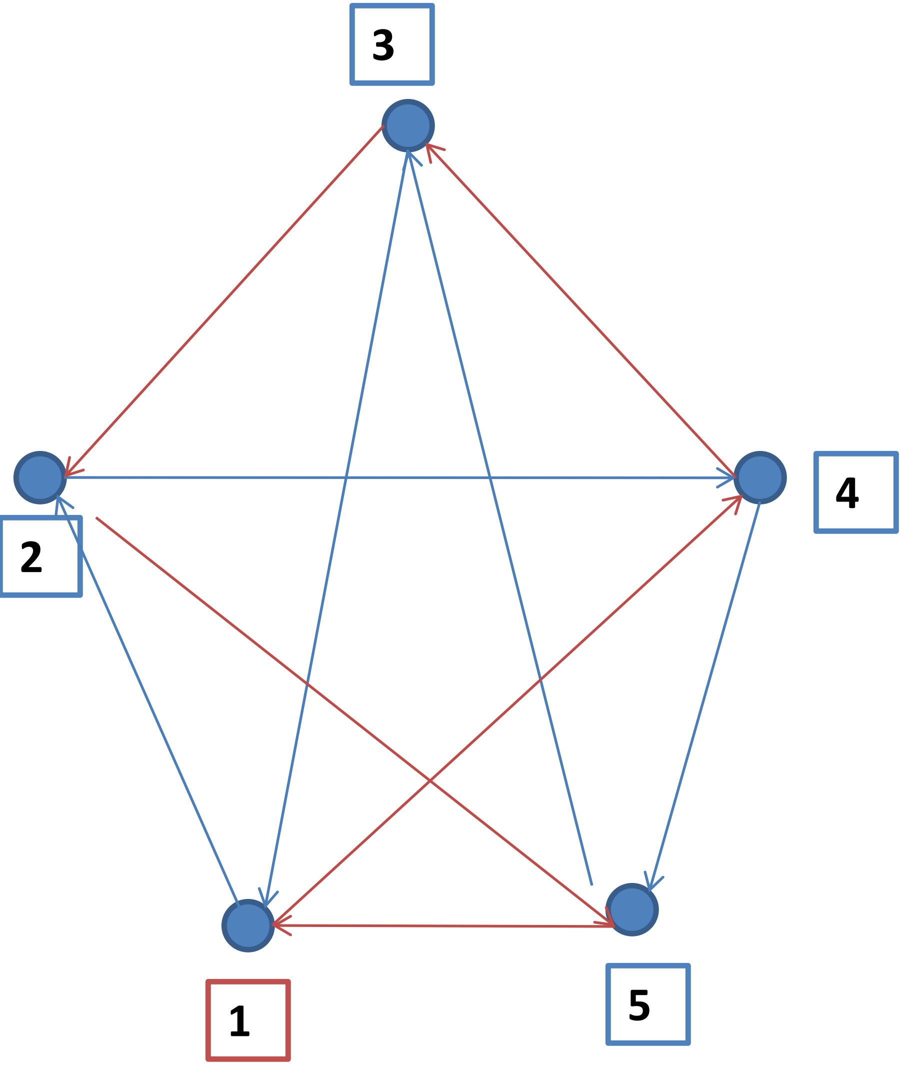
Travelling Salesman Problem

Travelling salesman Problem-Definition



- Let us look at a situation that there are 5 cities, Which are represented as NODES
- There is a Person at NODE-1
- This **PERSON HAS TO REACH EACH NODES ONE AND ONLY ONCE AND COME BACK TO ORIGINAL (STARTING)POSITION.**
- This **process has to occur with minimum cost or minimum distance travelled.**
- Note that starting point can start with any Node. For Example:

1-5-2-3-4-1

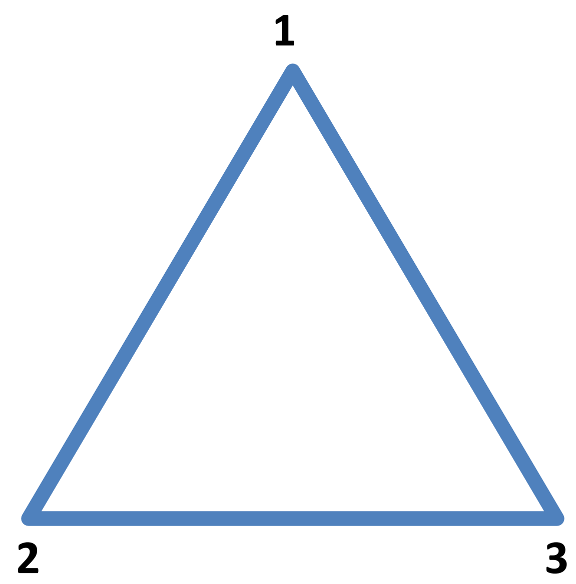
2-3-4-1-5-2

Travelling salesman Problem-Definition

- If there are ‘n’ nodes there are (n-1)! Feasible solutions
- From these (n-1)! Feasible solutions we have to find OPTIMAL SOLUTION.
- This can be related to GRAPH THEORY.
- Graph is a collection of Nodes and Arcs(Edges).

Travelling salesman Problem-Definition

- Let us say there are Nodes Connected as shown
- We can find a Sub graph as 1-3-2-1.Hence this **GRAPH IS HAMILTONIAN**

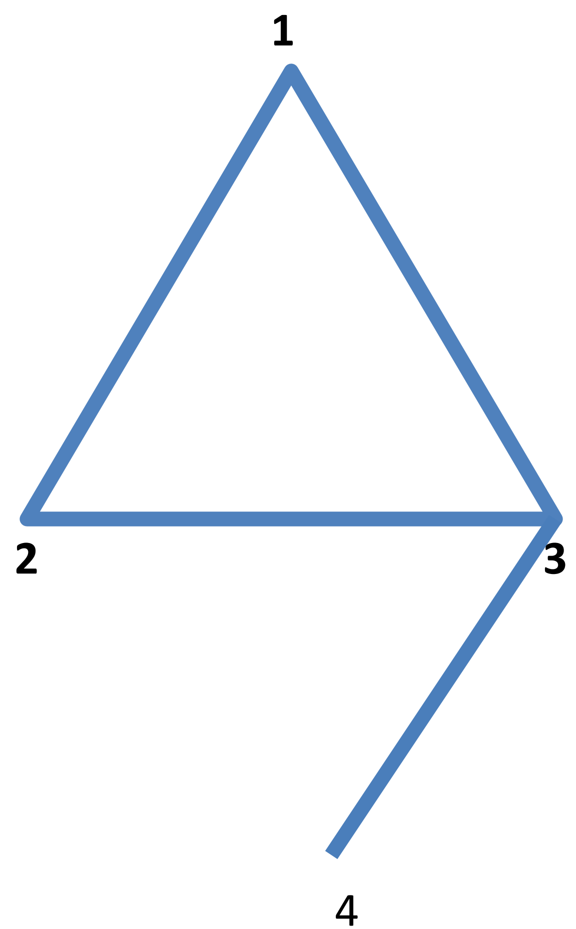


Travelling salesman Problem-Definition

- But let us consider this graph
- We can go to

1-3-4-3-2-1

But we are reaching 3 again to make a cycle. HENCE THIS GRAPH IS NOT HAMILTONIAN

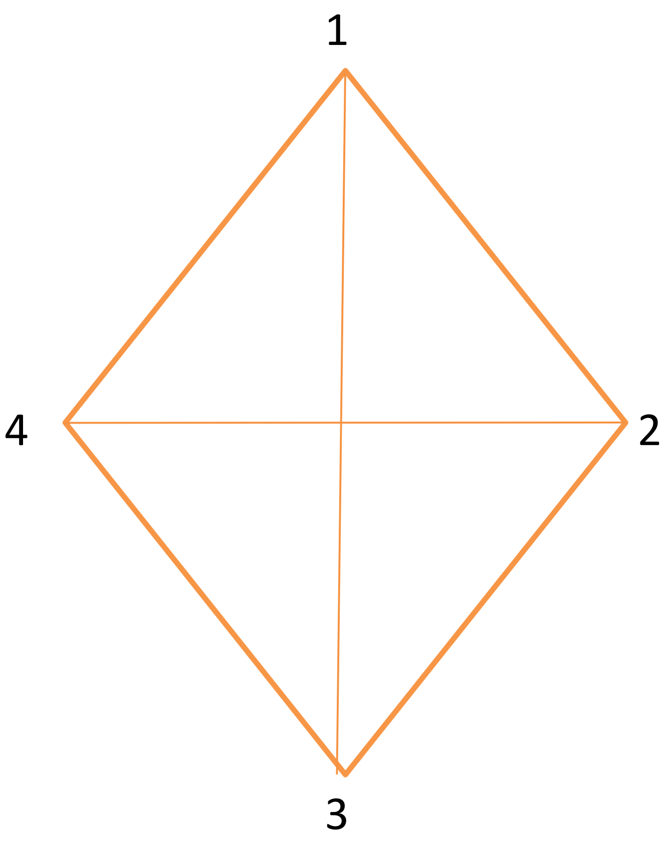


HAMILTONIAN GRAPHS

- The Given Graph is Hamiltonian
- If a graph is Hamiltonian, it may have more than one Hamiltonian Circuits.
- For eg:

1-4-2-3-1

1-2-3-4-1 etc.,



Hamiltonian Graphs And Travelling Salesman Problem

- Graphs Which are Completely Connected i.e., if we have Graphs with every vertex connected to every other vertex, then Clearly That graph is HAMILTONIAN.
- So Travelling Salesman Problem is nothing but finding out LEAST COST HAMILTONIAN CIRCUIT

Travelling salesman Problem Example

	1	2	3	4	5
1	-	10	8	9	7
2	10	-	10	5	6
3	8	10	-	8	9
4	9	5	8	-	6
5	7	6	9	6	-

Here Every Node is connected to every other Node. But the cost of reaching the same node from that node is Nil. So only a DASH is put over there.

Since Every Node is connected to every other Node various Hamiltonian Circuits are Possible.

Travelling salesman Problem Example

	1	2	3	4	5
1	-	10	8	9	7
2	10	-	10	5	6
3	8	10	-	8	9
4	9	5	8	-	6
5	7	6	9	6	-

We can have various Feasible Solutions.
For Example
1-2-4-5-3-1
2-5-1-4-3-2
Etc…

But From these Feasible Solutions We want to find the optimal Solution.
We should not have SUBTOURS.
It should comprise of TOURS.

Travelling salesman Problem Example- Formulations

- X_{ij} = 1,if person moves IMMEDIATELY from I to j.
- Objective Function is to minimize the total distance travelled which is given by

$$\sum\sum C_{ij} X_{ij}$$

Where C_{ij} is given by Cost incurred or Distance Travelled

$$\text{For } j=1 \text{ to } n, \sum X_{ij}=1, \quad \forall i$$

$$\text{For } i= 1 \text{ to } n, \sum X_{ij}=1, \quad \forall j$$

$$X_{ij}=0 \text{ or } 1$$

Sub Tour Elimination Constraints

- We can have Sub tours of length n-1
- We eliminate sub tour of length 1 By making Cost to travel from j to j as infinity.

$$C_{jj}=\infty$$

- To eliminate Sub tour of Length 2 we have

$$X_{ij}+X_{ji}<=1$$

- To eliminate Sub tour of Length 3 we have

$$X_{ij}+X_{jk}+X_{ki}<=2$$

- If there are n nodes Then we have the following constraints
- nc_2 for length 2
- nc_3 for length 3
- ……
- nc_{n-1} for length n-1

Travelling salesman Problem Example
Sub tour elimination

	1	2	3	4	5
1	-	10	8	9	7
2	10	-	10	5	6
3	8	10	-	8	9
4	9	5	8	-	6
5	7	6	9	6	-

We can eliminate Sub tours by a formidable method as

$$U_i - U_j + nX_{ij} <= n-1$$

For i=1 to n-1
And j=2 to n

TSP - SOLUTIONS

- Branch and Bound Algorithm
- Heuristic Techniques

Travelling salesman Problem Example
Row Minimum

	1	2	3	4	5
1	-	10	8	9	7
2	10	-	10	5	6
3	8	10	-	8	9
4	9	5	8	-	6
5	7	6	9	6	-

Total Minimum Distance =Sum of Row Minima

Here Total Minimum Distance =31

Lower Bound=31 that a person should surely travel. Our cost of optimal Solution should be surely greater than or equal to 31

Travelling salesman Problem Example
Column Minimum

	1	2	3	4	5
1	-	10	8	9	7
2	10	-	10	5	6
3	8	10	-	8	9
4	9	5	8	-	6
5	7	6	9	6	-

Total Minimum Distance =Sum of Column Minima

Here Total Minimum Distance also =31

Hence the Problem Matrices is Symmetric.

TSP USUALLY SATISFIES

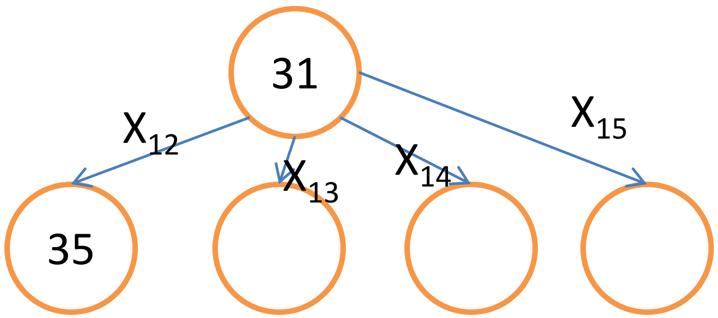
1.SQUARE

2.SYMMETRIC

3.TRIANGLE INEQUALITY

$$d_{ij}+d_{jk}>=d_{ik}$$

Branch and Bound-Step

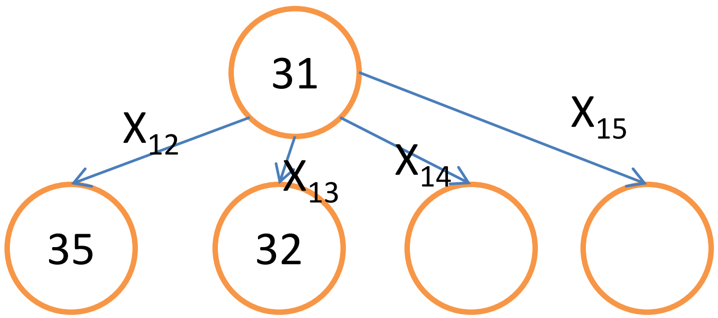


	1	2	3	4	5
1		10	8	9	7
2	10		10	5	6
3	8	10	-	8	9
4	9	5	8	-	6
5	7	6	9	6	-

	1	3	4	5
2	-	10	5	6
3	8	-	8	9
4	9	8	-	6
5	7	9	6	-

For X_{12}
10+5+8+6+6=35

Branch and Bound-Step



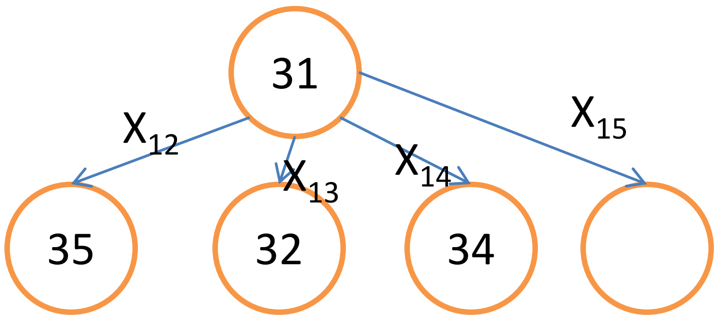
	1	2	4	5
2	10	-	5	6
3	-	10	8	9
4	9	5	-	6
5	7	6	6	-

	1	2	3	4	5
1		10	8	9	7
2	10	-	10	5	6
3	8	10		8	9
4	9	5	8	-	6
5	7	6	9	6	-

For X_{13}

$8+5+8+5+6=32$

Branch and Bound-Step



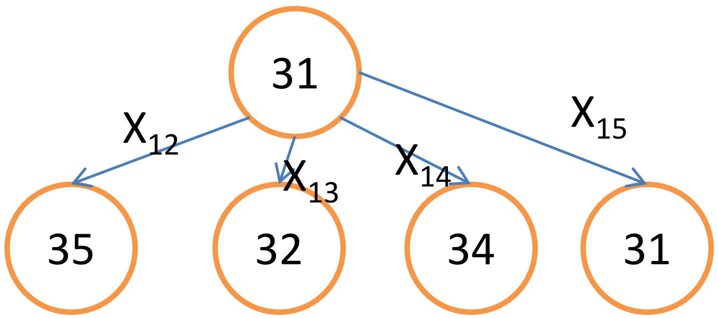
	1	2	3	5
2	10	-	10	6
3	8	10	-	9
4	-	5	8	6
5	7	6	9	-

	1	2	3	4	5
1		10	8	9	7
2	10	-	10	5	6
3	8	10	-	8	9
4	9	5	8	-	6
5	7	6	9	6	-

For X_{14}

$9+6+8+5+6=34$

Branch and Bound-Step



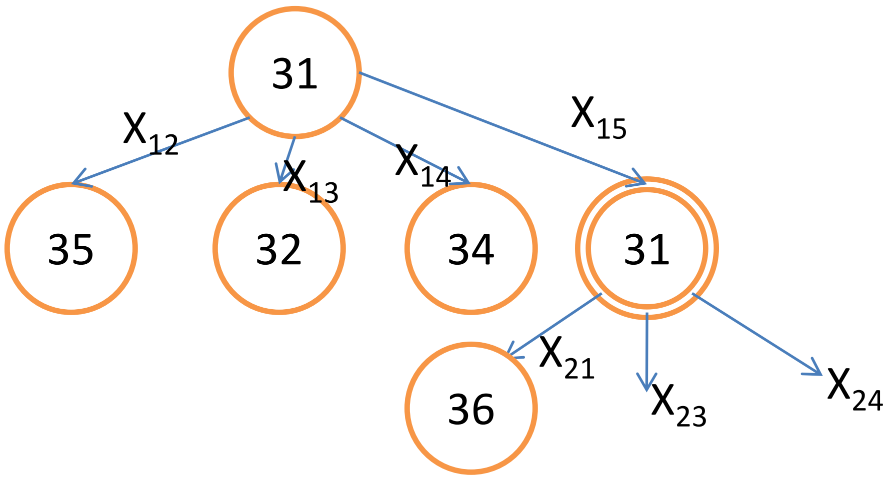
	1	2	3	4
2	10	-	10	5
3	8	10	-	8
4	9	5	8	-
5	-	6	9	6

	1	2	3	4	5
1		10	8	9	7
2	10	-	10	5	6
3	8	10	-	8	9
4	9	5	8	-	6
5	7	6	9	6	-

For X_{15}

7+5+8+5+6=31

Branch and Bound-Step

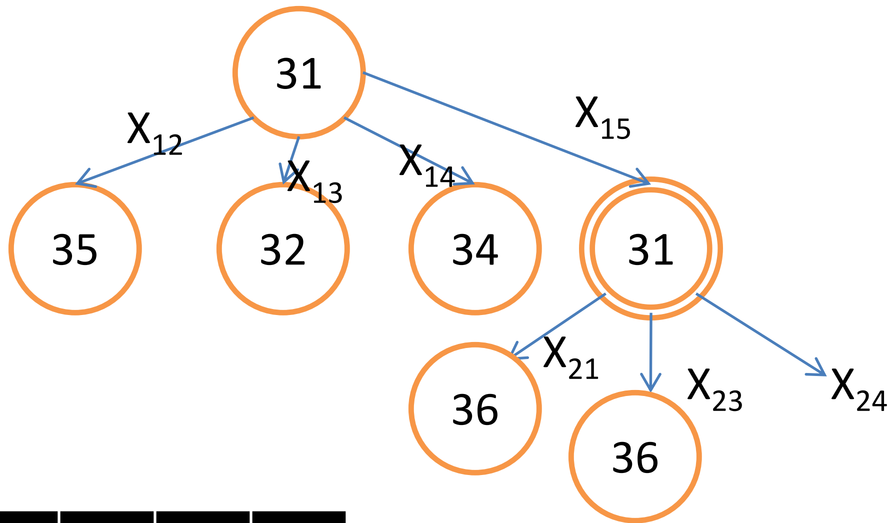


	2	3	4
3	10	-	8
4	5	8	-
5	-	9	6

	1	2	3	4	5
1	[illegible]	10	8	9	[illegible]
2	10	-	10	5	[illegible]
3	8	10	-	8	[illegible]
4	9	5	8	-	[illegible]
5	7	6	9	6	[illegible]

For X_{15}
And X_{21}
7+10+8+5+6=36

Branch and Bound-Step

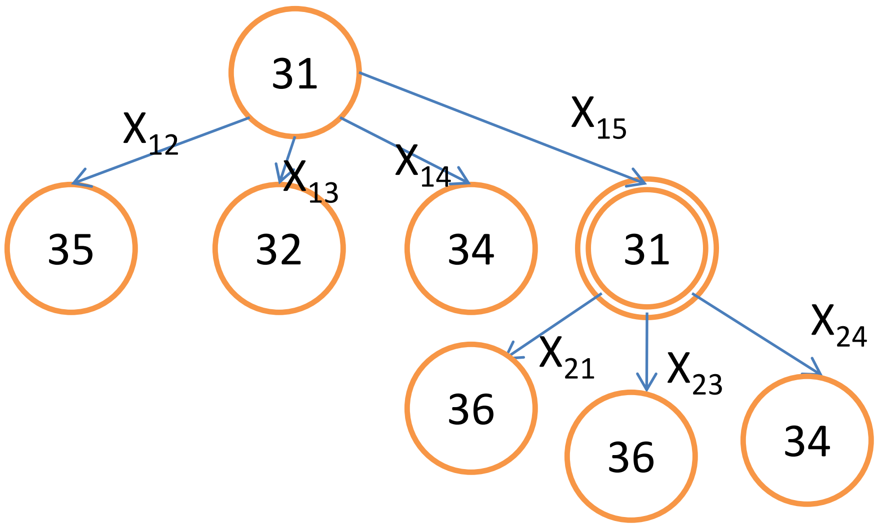


	1	2	[illegible]	4	[illegible]
1	[illegible]	10	[illegible]	9	[illegible]
2	10	-	[illegible]	5	[illegible]
3	8	10	[illegible]	8	[illegible]
4	9	5	[illegible]	-	[illegible]
5	7	6	[illegible]	6	[illegible]

	1	2	4
3	-	10	8
4	9	5	-
5	7	6	6

For X_{15}
And X_{23}
7+10+8+5+6=36

Branch and Bound-Step

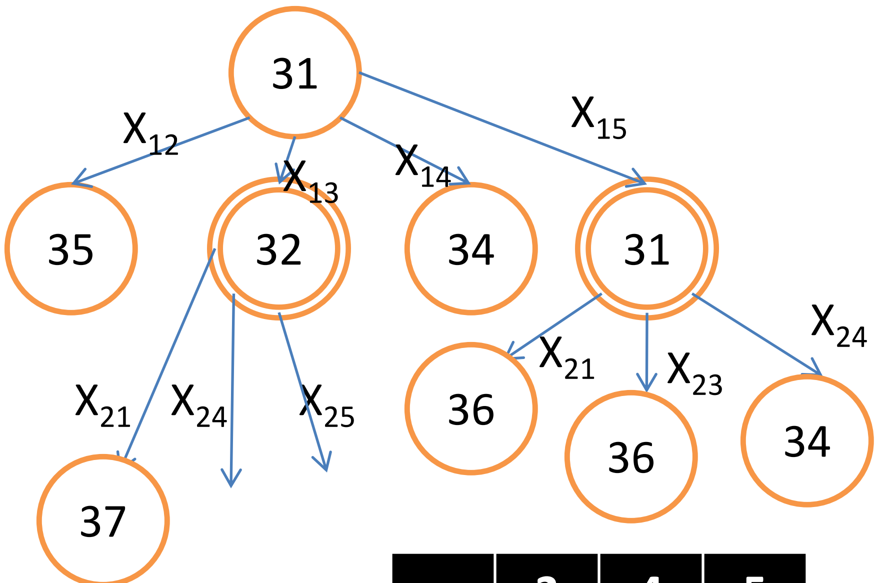


	1	2	3	4	5
1		10	8	[illegible]	[illegible]
2	10	-	10	[illegible]	[illegible]
3	8	10	-	[illegible]	[illegible]
4	9	5	8	[illegible]	[illegible]
5	7	6	9	[illegible]	

	1	2	3
3	8	10	-
4	9	-	8
5	7	6	9

For X_{15}
And X_{24}
7+5+8+8+6=34

Branch and Bound-Step

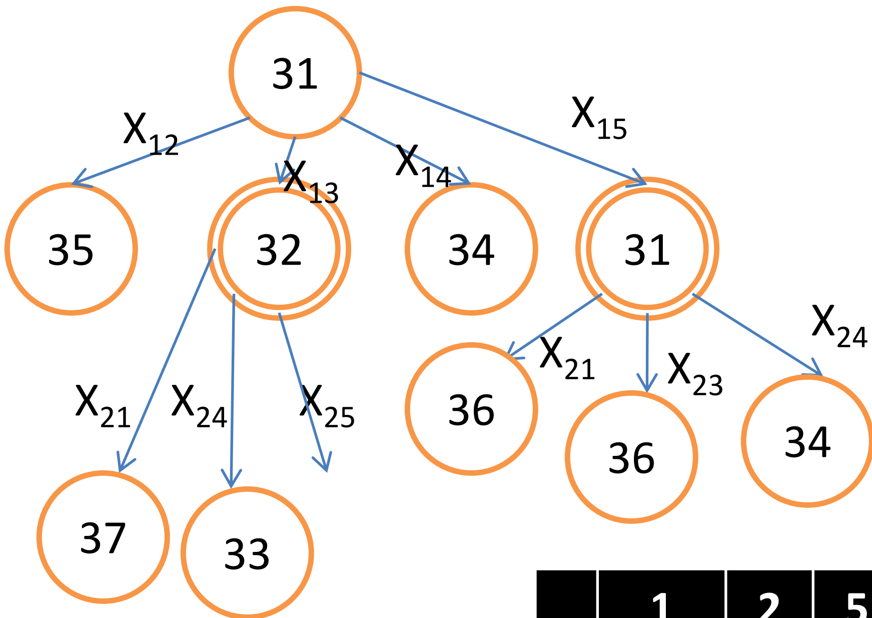


	1	2	3	4	5
1	-	10	8	9	7
2	10	-	10	5	6
3	8	10		8	9
4	9	5	8	-	6
5	7	6	9	6	-

	2	4	5
3	-	8	9
4	5	-	6
5	6	6	-

For X_{13}
And X_{21}
8+10+8+5+6=37

Branch and Bound-Step



	1	2	5
3	8	10	9
4	9	-	6
5	7	6	-

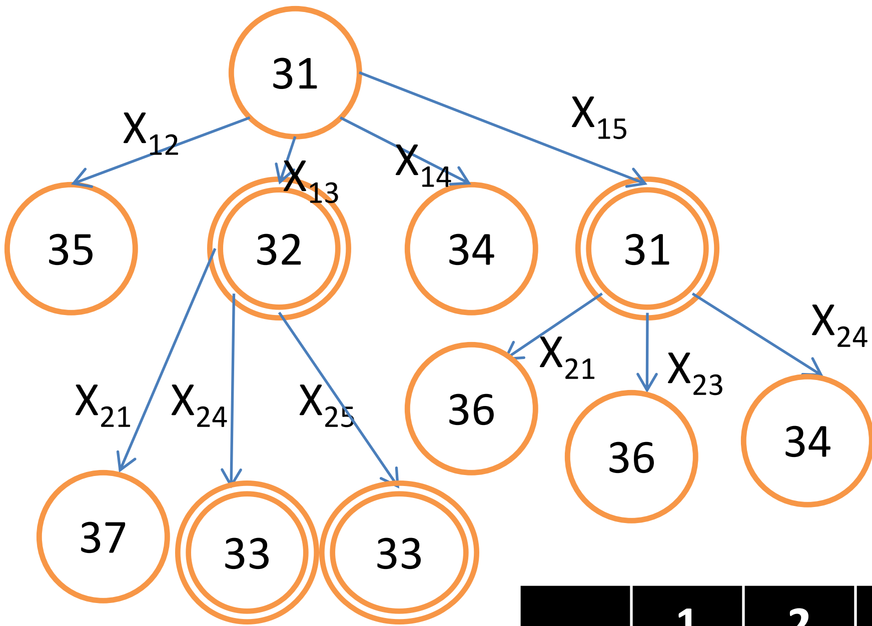
	1	2	[illegible]	4	5
1	[illegible]	10	[illegible]	[illegible]	7
2	10	-	[illegible]	[illegible]	6
3	8	10		8	9
4	9	5	8	-	6
5	7	6	9	6	-

For X_{13}

And X_{24}

8+5+8+6+6=33

Branch and Bound-Steps

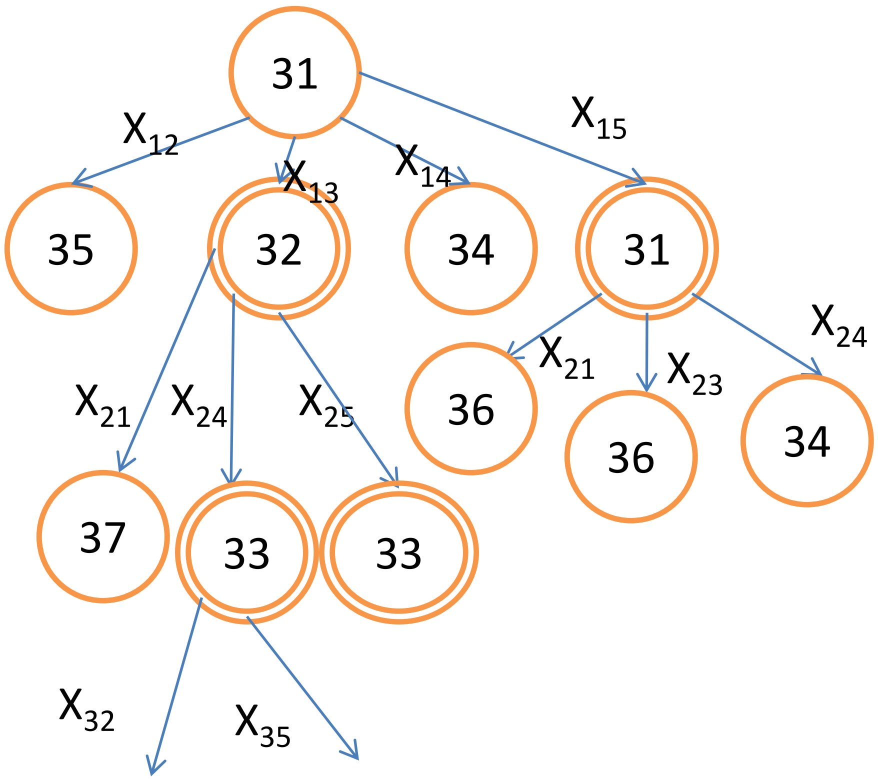


	1	2	[illegible]	4	[illegible]
1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2	[illegible]	-	[illegible]	[illegible]	[illegible]
3	8	10	[illegible]	8	[illegible]
4	9	5	[illegible]	-	[illegible]
5	7	6	[illegible]	6	[illegible]

	1	2	4
3	8	10	8
4	9	5	-
5	7	-	6

For X_{13}
And X_{25}
8+6+8+5+6=33

Branch and Bound-Steps

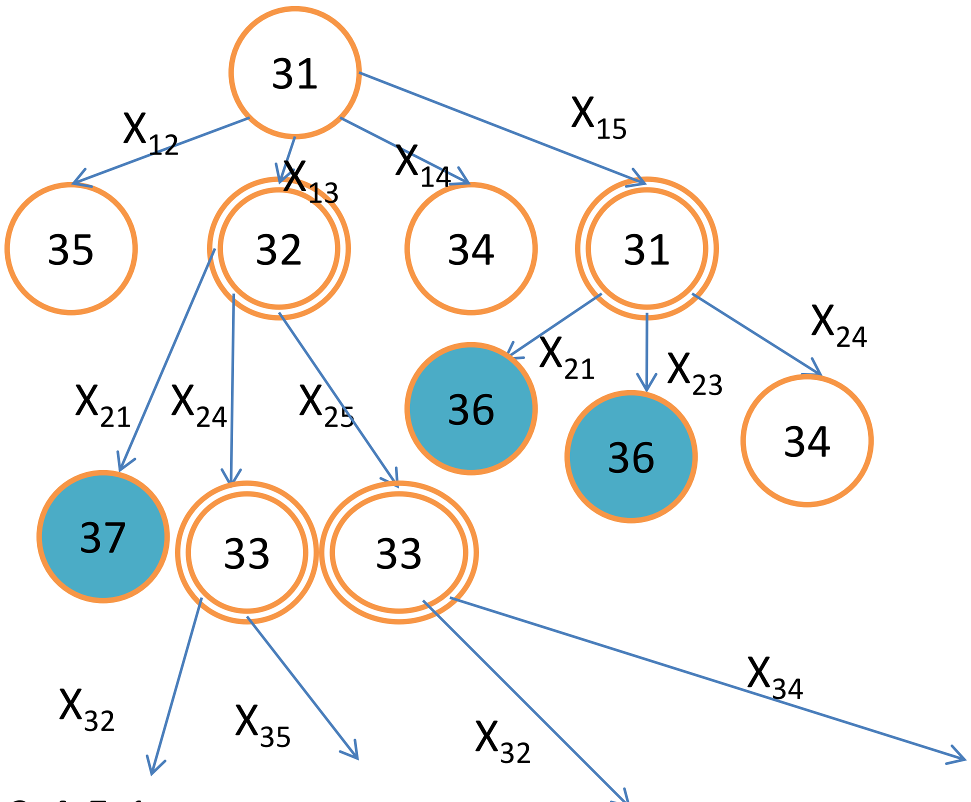


1-3-2-4-5-1
Distance=8+10+
5+6+7=36

1-3-5-2-4-1
Distance=8+9+6
+5+9=37

	1	2	3	4	5
1	-	10	8	9	7
2	10	-	10	5	6
3	8	10	-	8	9
4	9	5	8	-	6
5	7	6	9	6	-

Branch and Bound-Steps



1-3-2-4-5-1
Distance=8+10+5+6+7=36

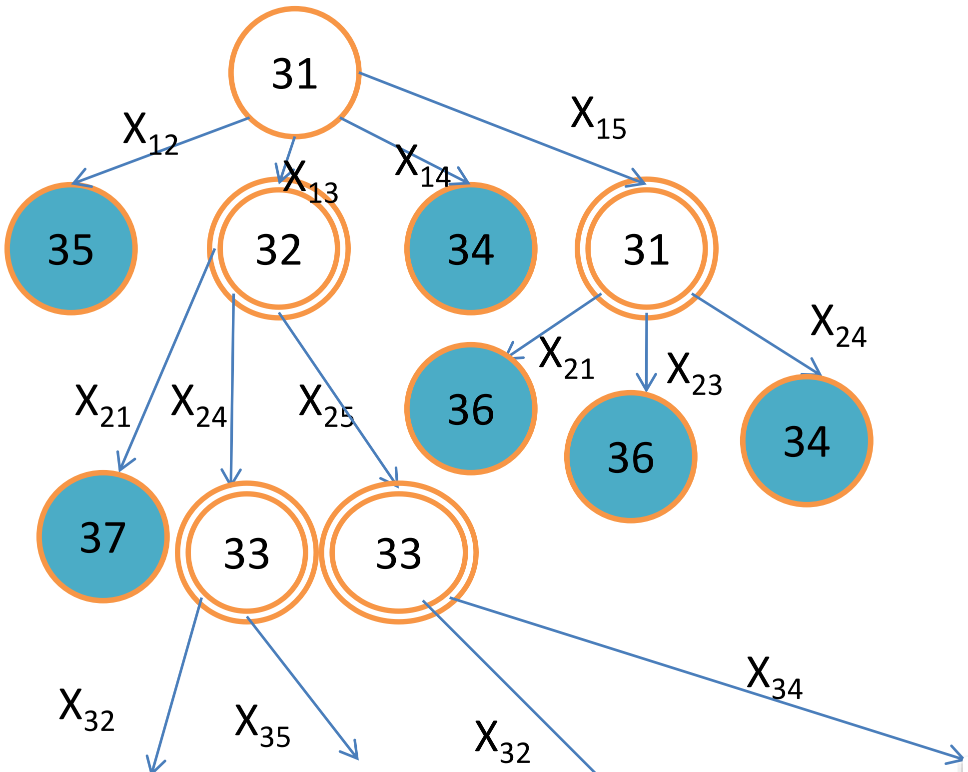
1-3-5-2-4-1
Distance=8+9+6+5+9=37

1-3-2-5-4-1
Distance=8+10+6+6+9=39

1-3-4-2-5-1
Distance=8+8+5+6+7=34

	1	2	3	4	5
1	-	10	8	9	7
2	10	-	10	5	6
3	8	10	-	8	9
4	9	5	8	-	6
5	7	6	9	6	-

Branch and Bound-Steps



	1	2	3	4	5
1	-	10	8	9	7
2	10	-	10	5	6
3	8	10	-	8	9
4	9	5	8	-	6
5	7	6	9	6	-

1-3-2-4-5-1
Distance=8+10+
5+6+7=36

1-3-5-2-4-1
Distance=8+9+6
+5+9=37

1-3-2-5-4-1
Distance=8+10+
6+6+9=39

1-3-4-2-5-1
Distance=8+8+5+6+7=34
This is the Optimal Solution.
This is same as
1-5-2-4-3-1