Formulation of recurrence equation

1. 1000, 2000, 4000, 8000

$t_0 = 1000$

$t_1 = 2000 = 2 \times t_0$ $\Rightarrow t_n = 2 \times t_{n-1}$

$t_2 = 4000 = 2 \times t_1$

$t_3 = 8000 = 2 \times t_2$

2. $7, \frac{21}{4}, \frac{63}{16}, \frac{189}{64}$

$t_0 = 7$

$t_1 = t_0 \times \frac{3}{4}$

$\Rightarrow t_n = t_{n-1} * \frac{3}{4}$

$t_2 = t_1 \times \frac{3}{4}$

$t_3 = t_2 \times \frac{3}{4}$

3. Max possible edges among a graph

$n \geq 1$

~~t_0~~

$t_1 = 0$

$t_2 = 1$

$t_3 = 2$

$t_4 = 6$

$t_{n-1} = (n-1) + t_{n-1}$

4. Staircase Problem (1, 2 steps)

$t_1 = 1$

$t_2 = 2$

$t_3 = 3$

$t_4 = 5$

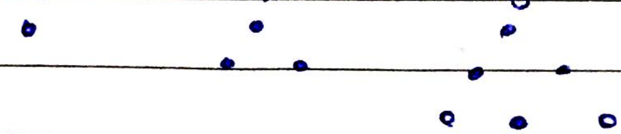
$t_n = t_{n-1} + t_{n-2}$

5. Towers of Hanoi

No. of disk	Moves
1	1
2	3
3	7
4	15
5	31

6. Triangular number

T_1 T_2 T_3



$t_n = n + t_{n-1}$

Techniques for solving recurrence equations.

→ must be non recursive.

→ such solution → general solution.

$$t_n = \frac{n(n+1)}{2}$$

closed form solution → particular solution

$$t_n = t_0 \pm * / (\quad)$$

→ sometimes there might not be a closed form solution.

① Guess & verify method.

1. $t_n = t_{n-1} + 2$

$t_0 = 1$

$t_0 = 1$

$t_1 = t_{n-1} + 2 = 1 + 2$

$t_2 = t_{n-1} + 2 = (1+2) + 2.$

$= 1 + 2*2.$

$t_3 = t_2 + 2 = 1 + 3*2.$

Guess

Guess: $t_n = n*2 + 1$

Verify :

$$\left.\begin{aligned} t_0 &= 2(0)+1 = 1 \\ t_1 &= 2(1)+1 = 3 \\ t_2 &= 2(2)+2 = 6 \end{aligned}\right\} \text{Part 1}$$

$$\cancel{t_{n+1} = 2(n+1)+2}$$
$$\cancel{= 2n}$$

$$\left.\begin{aligned} t_{n+1} &= t_n + 2 \\ &= (2n+1)+2 \\ &= 2n+2+1 \\ &= 2(n+1)+1 \quad \checkmark \end{aligned}\right\} \text{Part 2}$$

2. $t_n = t_{n-1} + n^2$

$t_1 = 1$.

Solution : $t_n = \dfrac{(n)(n+1)(2n+1)}{6}$.

3. $T(n) = 3T\left(\frac{n}{2}\right)$

$T(1) = \underline{1}$.

$T(1) = 1$

$T(2) = 3T(1) = 3$

$$T(4) = 3T(2)$$
$$= 3[T(1) * 3]$$
$$= 3^2$$

$$T(8) = 3T(4)$$
$$= 3^3$$

$$T(n) = 3^{\log_2 n} \rightarrow \text{Guess}$$

$$T(2n) = 3T\left(\frac{2n}{2}\right) \quad // \text{Verify}$$

$$= 3T(n)$$

$$= 3 * 3^{\log_2 n}$$

$$= 3^{\log_2 n + 1}$$

$$= 3^{\log_2 n + \log_2 2}$$

$$= 3^{\log_2 2n} \leftrightarrow$$

Substitution Method:

→ iteration method
→ Plug & Chug

From base to t_n
↳ forward substitution

from t_n to base
↳ backward substitution
(or) bactracking method.

1. $t_n = t_{n-1} + 3$
$t_1 = 4$

$$
\begin{aligned}
t_n &= t_{n-1} + 3 \\
&= (t_{n-2} + 3) + 3 \quad // \text{Plug} \\
&= t_{n-2} + 3 + 3 \\
&= t_{n-2} + 2*3 \quad // \text{Chug} \\
&= (t_{n-3} + 3) + 3*2 \quad // \text{P} \\
&= t_{n-3} + 3*3 \quad // \text{C}.
\end{aligned}
$$

$$
\begin{aligned}
t_n &= t_{n-(n-1)} + 3*(n-1) \\
&= t_1 + (n-1)*3 \\
&= 4 + 3n - 3 \\
&= 3n + 1.
\end{aligned}
$$

2. Compound interest
\$100 3%

[50th month.

$t_0 = 100$
$t_1 = t_0 + 0.03 t_0 = 1.03 t_0$
$t_2 = t_1 + 0.03 t_1 = 1.03 t_1$

$t_n = 1.03 t_n$ → recurrence sol.

$t_n = 1.03\, t_{n-1}$ //Plug

$= (1.03)^2 t_{n-2}$ //Chug

$= (1.03)^n t_0$.

$= 100\,(1.03)^n$

$t_{50} = 100\,(1.03)^{50}$.

3. $t_n = n t_{n-1}$

$t_0 = 1$

$t_n = n t_{n-1}$

$= n(n-1) t_{n-2}$

$= n(n-1)(n-2) t_{n-3}$

$= n!$.

4. $t_n = 7 t_{n-1}$ Solution: 7^n

$t_0 = 1$

5. $t_n = k t_{n-1}$ where $t_3 = 343$

$t_4 = 2401$

$t_0 = 1$

$t_n = k^n . t_0 = k^n$.

$t_3 = k^3$

$t_4 = k^4$

$\Rightarrow \frac{t_4}{t_3} = \frac{k^4}{k^3} = k = \frac{2401}{343} = 7$

Forward Substitution

1. $t_n = t_{n-1} + 3$

   $t_0 = 4.$

$t_0 = 4$

$t_1 = t_0 + 3$ plug.

$t_2 = t_1 + 3$ plug

$= t_0 + 3 + 3$

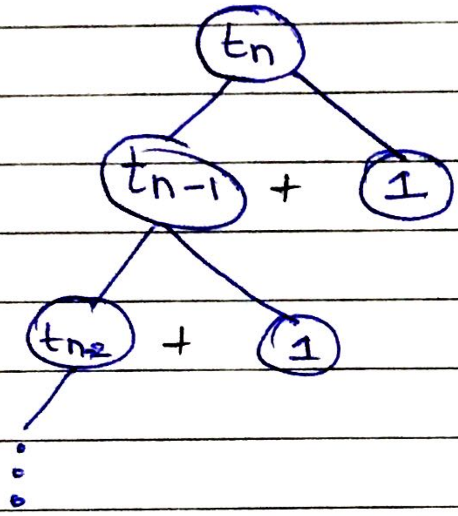
$= t_0 + 2 * 3$ chug.

$t_n = t_0 + n * 3.$

Recurrence - tree Method.

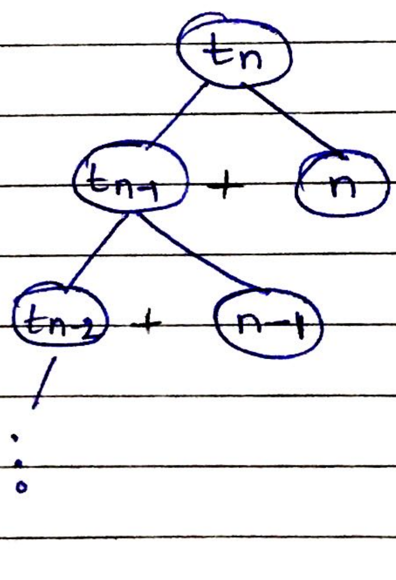
1. $t_n = \begin{cases} 1 & n=1 \\ t_{n-1}+a & n>1 \end{cases}$ $\quad a=1$, $a=n$.

When $a=1$



$$\sum_{i=1}^{n} 1 = n.$$

When $a=n$.

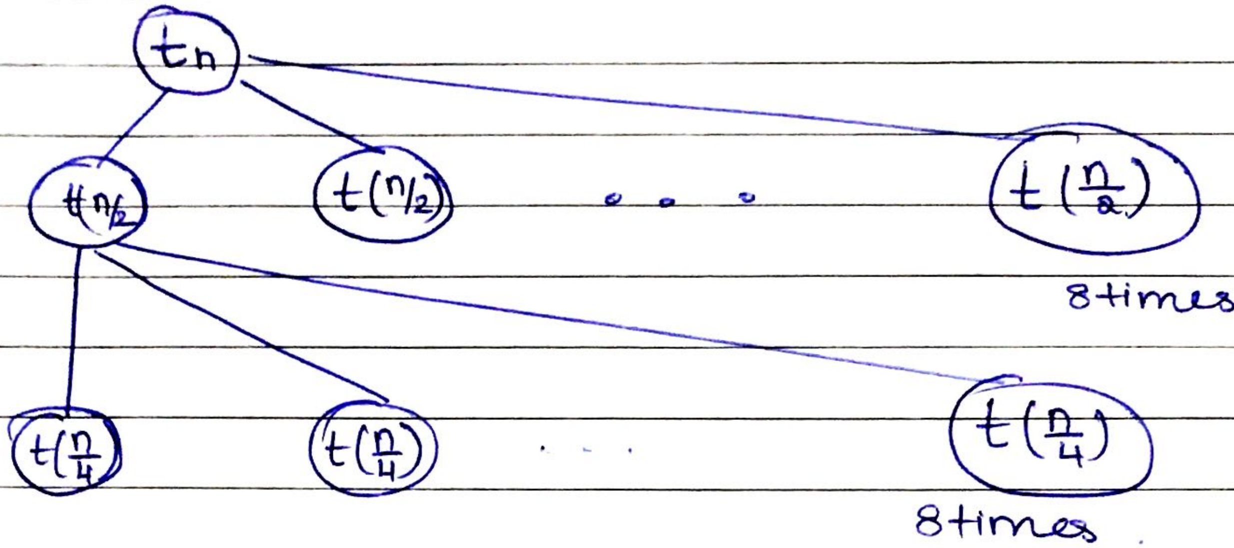


$\Rightarrow \cancel{n + (n-1) + (n-2) + \ldots + 1}$

$\Rightarrow n + (n-1) + (n-2) + \ldots + 1$

$$\Rightarrow \frac{n(n+1)}{2}.$$

2. $t_n = \begin{cases} 1 & n=1 \\ 8T\left(\frac{n}{2}\right) & n>1. \end{cases}$



Levels

$1, 8, 8^2, 8^3, \ldots 8^i$

$8^{\log n} * T(1)$

$= 8^{\log n}$

$= n^{\log_2 8}$

$= n^3 \rightarrow O(n^3)$

asymptotic upper bound.