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18CSC204J - Design and Analysis Of Algorithms.

Assignment - 1.

Backward substitution.

Q) $x(n) = x(n-1)+5$, $n>1$, $x(1)=0$.

$x(n) = x(n-1)+5$ — ①

$x(n-1) = x(n-2)+5$ — ②

② in ①,

$x(n) = x(n-2)+10$. — ③

$x(n-2) = x(n-3)+5$ — ④.

④ in ③,

$x(n) = x(n-3)+15$.

In terms of k,

$x(n) = x(n-k) + k.5$

Halting condition, $x(1)=0$

$n-k = 1$.

$k = n-1$.

$x(n) = x(1) + (n-1)5$

$x(n) = 5n - 5$

$\therefore x(n) = O(n)$.

Q) $x(n) = 3x(n-1)$, $n>1$, $x(1) = 4$.

$x(n) = 3x(n-1)$. — ①

$x(n-1) = 3x(n-2)$ — ②.

② in ①,

$x(n) = 9x(n-2)$ — ③

$x(n-2) = 3x(n-3)$ — ④

④ in ③,

$x(n) = 27\, x(n-3)$.

In terms of k,

$x(n) = 3^{k+1}\, x(n-k)$.

Halting condition, $x(1) = 4$.

$n - k = 1$

$k = n-1$.

$x(n) = 3^{n}\, x(1)$

$x(n) = 4 \cdot 3^{n}$.

$\therefore\ x(n) = O(3^{n})$.

Q). $x(n) = x(n-1) + n, \quad n > 0, \quad x(0) = 0$

$x(n) = x(n-1) + n$ — ①.

$x(n-1) = x(n-2) + n - 1$ — ②.

② in ①,

$x(n) = x(n-2) + 2n - 1$ — ③

$x(n-2) = x(n-3) + n - 2$. — ④

④ in ③,

$x(n) = x(n-3) + 3n - 3$.

In terms of k,

$x(n) = x(n-k) + k \cdot n - \frac{k(k-1)}{2}$

Halting condition, $x(0) = 0$.

$n - k = 0$.

$k = n$.

$x(n) = x(0) + n^2 - \frac{n(n-1)}{2}$.

$x(n) = n^2 - \frac{n^2 - n}{2}$.

$x(n) = \frac{2n^2 - n^2 + n}{2} = \frac{n^2 + n}{2}$

$x(n) = O(n^2)$.

Q) $x(n) = x(n/2) + n \quad , n > 1 \quad , x(1) = 1.$

$x(n) = x(n/2) + n \quad -①$

$x(n/2) = x(n/4) + n/2 \quad -②$

② in ①,

$x(n) = x(n/4) + \frac{3n}{2} \quad -③$

$x(n/4) = x(n/8) + \frac{n}{4}. \quad -④$

③ in ④,

$x(n) = x(n/8) + \frac{7n}{4}$

In terms of k,

$x(n) = x\left(\frac{n}{2^k}\right) + \left(\frac{2^k - 1}{2^{k-1}}\right) n.$

Halting condition, $x(1) = 1$

$n/2^k = 1 \rightarrow k = \log_2 n.$

$x(n) = x(1) + \left(\frac{2^{\log_2 n} - 1}{2^{\log_2 n - 1}}\right) n = 1 + \frac{2(n-1)\cancel{n}}{\cancel{n}} = 2n - 1$

$x(n) = O(n).$

Q) $x(n) = x(n/3) + 1. \quad , n > 1, \; x(1) = 1.$

$x(n) = x(n/3) + 1 \quad -①$

$x(n/3) = x(n/9) + 1 \quad -②$

② in ①,

$x(n) = x(n/9) + 2 \quad -③$

$x(n/9) = x(n/27) + 1 \quad -④$

④ in ③,

$x(n) = x(n/27) + 3$

In terms of k,

$x(n) = x\left(\frac{n}{3^k}\right) + k.$

Halting condition, $x(1) = 1$

$\frac{n}{3^k} = 1 \rightarrow k = \log_3 n$

$x(n) = x(1) + \log_3 n$

$x(n) = \log_3 n + 1.$

~~x(O)~~ $x(n) = O(\log_3 n)$

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