SRM INSTITUTE OF SCIENCE AND TECHNOLOGY, RAMAPURAM CAMPUS

COMPUTER SCIENCE AND ENGINEERING

Continuous Learning Assessment-II

SET B

18CSC204J / Design and Analysis of Algorithms

Answer Key

Year/ Sem: II/IV

Date: 05/04/2021

Duration: 90 mins

Marks:50

PART-A (30*1=30)

1. Division Pattern of Problems in Divide and Conquer approach
Select one:
a) Iterative **b) Recursive** c) Parallel d) Random

2. The given array is arr = {2, 3, 4, 5}. Bubble sort is used to sort the array elements. How many iterations will be done to sort the array?
a) 4 b) 2 c) 1 d) 0

3. Given an array arr = {1, 2, 3, 4, 5, 6} and key = 6; what are the mid values (corresponding array elements) in the first and second levels of recursion?
a) 4 and 6 b) 4 and 5 c) 3 and 6 d) 3 and 5

4. Which of the following is not an application of binary search?
a) To find the lower/upper bound in an ordered sequence
b) Union of intervals
c) Debugging
d) to search in unordered list

5. What will be the time Complexity when binary search is applied on a linked list
a. O(1) **b.O(n)** c. $O(n^2)$ d. $O(n^3)$

6. Find the maximum sub-array sum for the given elements.
{2, -1, 3, -4, 1, -2, -1, 5, -4}
a) 3 **b) 5** c) 8 d) 6

7. Which is the worst method of choosing the pivot
a) First element
b) Last element
c) Median of three partitioning
d) Random

8. How many recursive calls are there in Recursive matrix multiplication through simple divide and conquer method
a. 2 b. 6 c. 9 **d. 8**

9. How many cases are there under Master's theorem?
a. 2 **b) 3** c) 4 d) 5

10. What is the result of the recurrences which fall under second case of Master's theorem (let the recurrence be given by T(n)=aT(n/b)+f(n) and $f(n)=n^c$?
a) $T(n) = O(n \log_b a)$
b) $T(n) = O(n^c \log n)$
c) $T(n) = O(f(n))$
d) $T(n) = O(n^2)$

11. Which one of the following is the recurrence equation for the worst case time complexity of the Quicksort algorithm for sorting n (≥2) numbers? In the recurrence equations given in the options below, c is a constant.
a) T (n) =2T (n/2) +cn
b) T (n) =T (n−1) +T (1)+cn
c) T(n)=2T(n−1)+cn
d) T(n)=T(n/2)+cn

12. Which of the following algorithm is the fastest
a) Quick Sort
b) Merge Sort
c) Insertion Sort
d) Bubble Sort

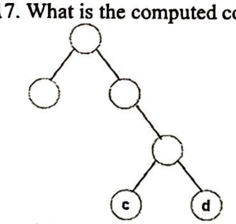
13. Which Approach is based on computing the distance between each pair of distinct points and finding a pair with the smallest distance?
a) Brute force
b) Exhaustive Search
c) Divide and Conquer
d) Branch and Bound

14. Which of the following algorithm is similar to quick hull algorithm?
a) Quick Sort
b) Merge Sort
c) Insertion Sort
d) Bubble Sort

15. What is the average case complexity of a quick hull algorithm?
a) O(N)
b) O(N log N)
c) $O(N^2)$
d) O(log N)

16. The matrix chain multiplication problem can be solved using?
a) Dynamic programming
b) Brute force
c) Recursion
d) Dynamic Programming, Brute force, Recursion

17. What is the computed codeword for node'c'?



a) 111 b) 101 **c) 110** d) 011

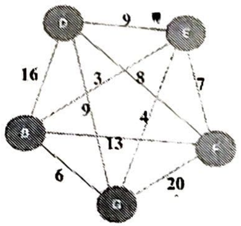
18. Purpose of Kruskal's algorithm is ______
a) find all pair shortest path algorithm
b)find single source shortest path
c) find minimum spanning tree
d) traverse the graph

19. Time complexity of Kruskal's algorithm?
a) O(log V) **b) O(E log V)** c) $O(E^2)$ d) O(V log E)

20. What is the time complexity of the brute force algorithm used to find the longest common subsequence?
a) O(n) b) $O(n^2)$ c) $O(n^3)$ **d) $O(2^n)$**

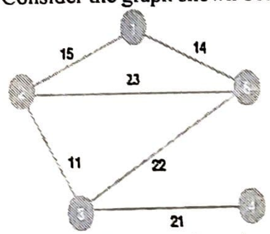
21. What is the running time of the Huffman encoding algorithm?
a) O(C) b) O(log C) **c) O(C log C)** d) O(N log C)

22. Using Kruskal's algorithm, which edge will be selected first?



a) GF b) DE **c) BE** d) BG

23. Consider the graph shown below.



Which of the following edges form the MST of the given graph using Prim'a algorithm, starting from vertex 4.
a) (4-3)(5-3)(2-3)(1-2)
b) (4-3)(3-5)(5-1)(1-2)
c) (4-3)(3-5)(5-2)(1-5)
d) (4-3)(3-2)(2-1)(1-5)

23. Kruskal solution for the MST follows?
a) **Greedy technique**
b) divide-and-conquer technique
c) dynamic programming technique
d) The algorithm combines more than one of the above techniques

24. ------------is an algorithm design method that can be used when the solution to a problem can be viewed as the result of a sequence of decisions.
a) **Dynamic Programming**
b) Greedy method
c) Huffman coding
d) Tree traversal

26. Which of the following is false about the Kruskal's algorithm?
a) It is a greedy algorithm
b) It constructs MST by selecting edges in increasing order of their weights
c) It can accept cycles in the MST
d) It uses union-find data structure

27. Mention the time complexity of the brute force algorithm used to find the longest common subsequence?
a) O(n) b) O(n^2) c) O(n^3) **d) O(2^n)**

28. The travelling salesman problem can be solved using ________
a) A spanning tree
b) A minimum spanning tree
c) Max-Min algorithm
d) DFS traversal

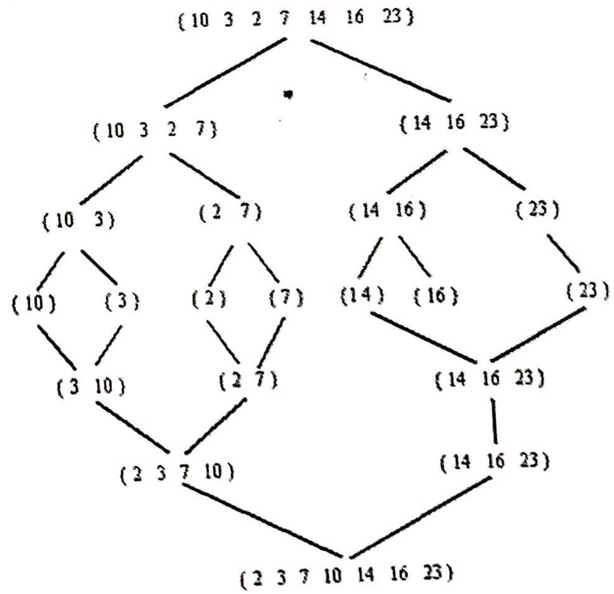
29. Which of the following is true?
a) Prim's algorithm initialises with a vertex
b) Prim's algorithm initialises with a edge
c) Prim's algorithm initialises with a vertex which has smallest edge
d) Prim's algorithm initialises with a forest

30. What is the length of the longest common subsequence of the strings "PQRSTPQRS" and "PRATPBRQRPS?
a) 9
b) 8
c) 7
d) 6

PART-B (2*10=20)

1. a. Sort the following sequence using merge sort algorithm in increasing order. Also explain the algorithm and discuss its time complexity
{10, 3, 2, 7, 14, 16, 23}

PROBLEM (4 m)
Solution:



ALGORITHM (4 M)
MergeSort(arr, left, right):
if left > right

```
return
mid = (left+right)/2
  mergeSort(arr, left, mid)
   mergeSort(arr, mid+1, right)
  merge(arr, left, mid, right)
  end
```

Time Complexity of Merge sort (2 M)

- In the worst case, in every iteration, we are dividing the problem into further 2 subproblems. Hence this will perform log n operations and this has to be done for n iteration resulting in n log n operations total.
- In the best case that is sorted array, we can do some modification by using a flag to check whether the lament is already sorted or not
- Best Time Complexity: O(nlogn)
- Average Time Complexity: O(nlogn)
- Worst Time Complexity: O(nlogn)

OR

b. Find the maximum and minimum of the given array A= {22, 17, 18, 3, 4, 7, 9, 30} using divide and conquer method. Explain the algorithm with its time complexity.

2. a. Explain in detail about greedy knapsack problem. Find an optimal solution to the knapsack instance n=7,m=15,$(P_1,P_2,P_3,P_4,P_5,P_6,P_7)$=(10,5,15,7,6,18,3) and $(W_1,W_2,W_3,W_4,W_5,W_6,W_7)$= (2,3,5,7,1,4,1)

Knapsack Problem **5 Marks**

- There are n items in a store.
- For i =1,2, . . . , n, item i has weight $w_i > 0$ and worth $v_i > 0$. Thief can carry a maximum weight of W pounds in a knapsack.
- In this version of a problem the items can be broken into smaller piece, so the thief may decide to carry only a fraction x_i of object i, where $0 \le x_i \le 1$. Item i contributes $x_i w_i$ to the total weight in the knapsack, and $x_i v_i$ to the value of the load.

Algorithm

Greedy-fractional-knapsack (*w, v, W*)

```
For i =1 to n
        do x[i] =0

weight = 0

while weight < W
        do i = best remaining item
                If weight + w[i] ≤ W then
                        x[i] = 1
                        weight = weight + w[i]
                else
                        x[i] = (w - weight) / w[i]
                        weight = W

return x
```

Problem **5 Marks**

Solution:

Step(1): Given $n=7$, $m=15$, $(P_1,P_2,\ldots P_7)=(10,5,15,7,6,18,3)$

$(w_1,w_2,\ldots w_7)=(2,3,5,7,1,4,1)$.

Find The Profit & weight ratio.

$P_1/w_1 = 10/2 = 5.$ $\quad P_6/w_6 = 18/4 = 4.5$

$P_2/w_2 = 5/3 = 1.66$ $\quad P_7/w_7 = 3/1 = 3.$

$P_3/w_3 = 15/5 = 3$

$P_4/w_4 = 7/7 = 1$

$P_5/w_5 = 6/1 = 6$

Step(2): Arrange the object in the increasing order of P/w ratios, we get the order as $x_5, x_1, x_6, x_3, x_7, x_2, x_4$.

Consider $U = \underline{m = 15}$

$W_5 < U$ so $x_5 = 1 \Rightarrow U = 15 - W_5 = 15 - 1 = \underline{14}$

$W_1 < U$ so $x_1 = 1 \Rightarrow U = 14 - W_1 = 14 - 2 = 12$

$W_6 < U$ so $x_6 = 1 \Rightarrow U = 12 - W_6 = 12 - 4 = 8$

$W_3 < U$ so $x_3 = 1 \Rightarrow U = 8 - W_3 = 8 - 5 = 3$

$W_7 < U$ so $x_7 = 1 \Rightarrow U = 3 - W_7 = 3 - 1 = 2$

$W_2 < U$ is not true $\Rightarrow 3 < 2$ So $X_2 = U/w_2 = \underline{2/3} \Rightarrow U = 2 - 2 = 0$

$\therefore (x_1, x_2, x_3, x_4, x_5, x_6, x_7) = (\underline{1, 2/3, 1, 0, 1, 1, 1})$

Step(3): Find the optimum solution (profit)

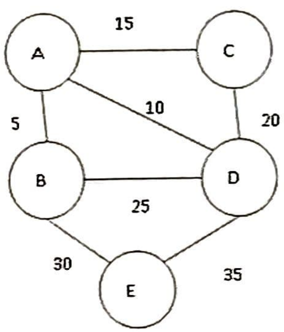
$$\sum p_i x_i = (1\times 10) + (2/3\times 5) + (1\times 15) + (0\times 7) + (1\times 6) + (1\times 18) + (1\times 3)$$

$$= 10 + 3.33 + 15 + 0 + 6 + 18 + 3$$

$$\boxed{P = 55.33}$$

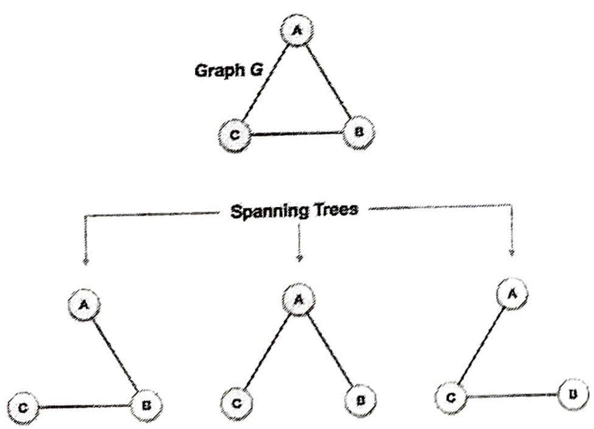
OR

b. Explain MST using Prim's Algorithm for



MST Explanation **3 Marks**

- A spanning tree is a subset of Graph G, which has all the vertices covered with minimum possible number of edges.
- A complete undirected graph can have maximum n^{n-2} number of spanning trees, where n is number of nodes.



Mathematical properties of spanning tree

- Spanning tree has n-1 edges, where n is number of nodes (vertices)
- A complete graph can have maximum n^{n-2} number of spanning trees.
- So we can conclude here that spanning trees are subset of a connected Graph G and disconnected Graphs do not have spanning tree.

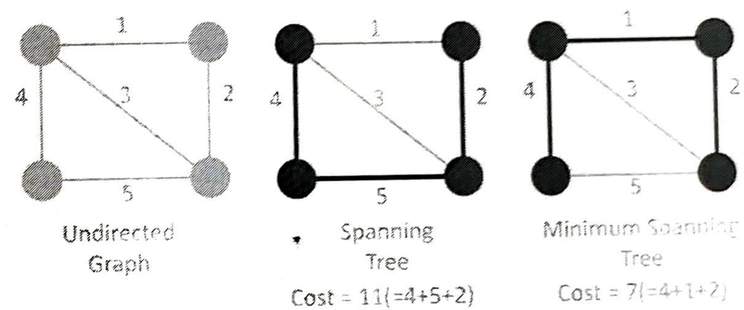
Minimum Spanning Tree (MST)

- In a weighted graph, a minimum spanning tree is a spanning tree that has minimum weight that all other spanning trees of the same graph.

MST Algorithm

- Kruskal's Algorithm
- Prim's Algorithm

Both are greedy algorithms.



Algorithm **3 Marks**

%% Input: Graph Prim (G)

%% Output: Minimum spanning tree T

Begin

S = pick any vertex of G

$V_T = \{S\}$

$E_T = \emptyset$

n = |V|

repeat |n| - 1 times

pick an edge (v,u) such that v $\in V_T$ and u $\in V - V_T$

$V_T = V \cup \{v\}$

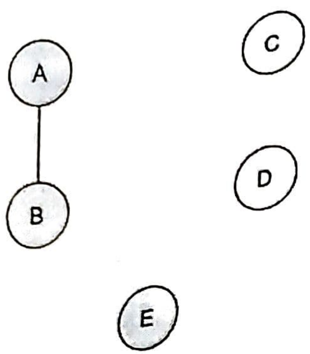
$E_T = U \{ v, u\}$

End repeat

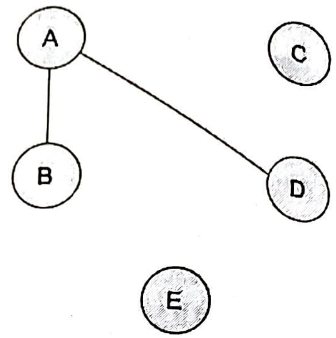
Return $T(V_T, E_T)$

End

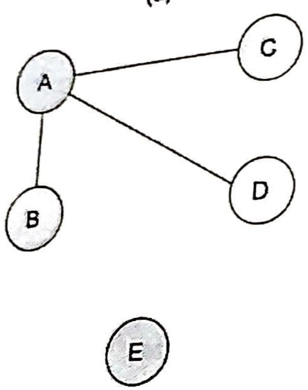
Step by step problem solution **4 Marks**



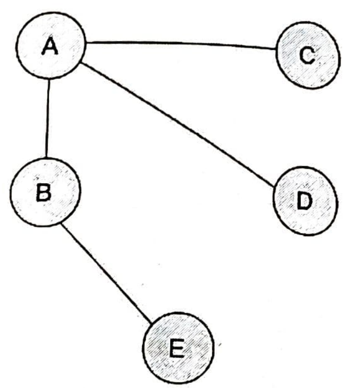
(a)



(b)



(c)



(d)

Prepared by

09/04/2021

Dr M.Azhagiri

AP/CSE

Verified by

Dr K.RAJA

HOD/CSE 9/4/21