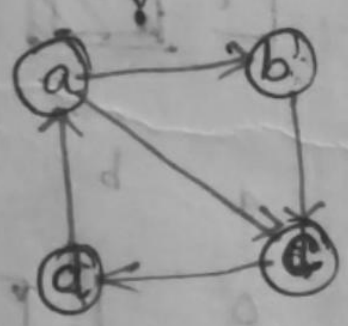
Analysis:

Time complexity is $O(n^3)$. This Operation is located within 3 Nested for loops.

→ Eg:-

Obtain the transitive closure for the following digraph using Warshall's Algthm.



Note: $R^{(0)}$ => initial nodes ie) Adj Matrix
$R^{(1)}$ => To reach, 'a' is the intermedia
$R^{(2)}$ => a+b, $R^{(3)}$ => a,b,c
$R^{(4)}$ = a, b, c, d

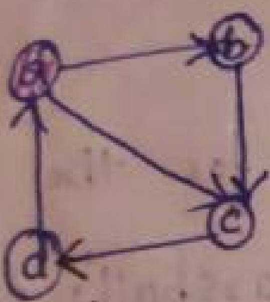
① Adjacency Matrix ie) $R^{(0)}$

$$R^{(0)} = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 1 & 1 & 0 \\ b & 0 & 0 & 1 & 0 \\ c & 0 & 0 & 0 & 1 \\ d & 1 & 0 & 0 & 0 \end{array}$$

=> Edge is present w
else 0.

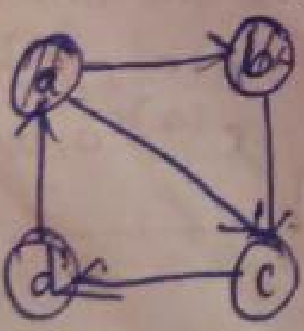
$$R^{(0)} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

* If there is a direct connection use 1 else 0. Final Ans is same as transitive the



$$R^{(1)} = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 1 & 1 & 0 \\ b & 0 & 0 & 1 & 0 \\ c & 0 & 0 & 0 & 1 \\ d & 1 & 1 & 0 & 0 \end{array}$$

In $R^{(1)} \Rightarrow$ Source-dest (d-b)
intermediate node 'a'.
2 ways, 1) directly d-b
2) d-a-b
iii) $R^{(k)}_{(i,j)} \leftarrow R^{(k-1)}_{(i,j)}$



$$R^{(2)} = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 1 & 1 & 0 \\ b & 0 & 0 & 1 & 0 \\ c & 0 & 0 & 0 & 1 \\ d & 1 & 1 & 1 & 0 \end{array}$$

$$R^{(0)} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

* if there is a direct connection use 1 else 0.
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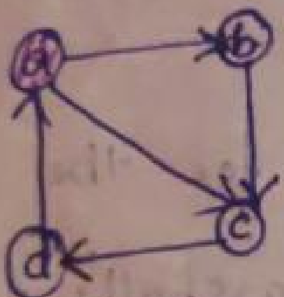

$$R^{(1)} = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 1 & 1 & 0 \\ b & 0 & 0 & 1 & 0 \\ c & 0 & 0 & 0 & 1 \\ d & 1 & 1 & 0 & 0 \end{array}$$

In $R^{(1)}$ ⇒ Source - dest (d-b)
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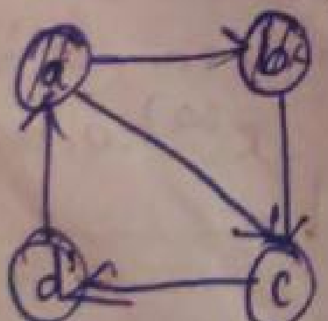
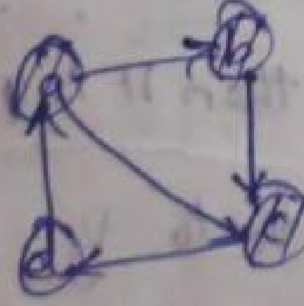


$$R^{(2)} = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 1 & 1 & 0 \\ b & 0 & 0 & 1 & 0 \\ c & 0 & 0 & 0 & 1 \\ d & 1 & 1 & 1 & 0 \end{array}$$

Here intermediate vertices are a and b. We have to find a path length of upto 3 edges going through intermediate vertices a and b. We will keep adjacency matrix of $R^{(1)}$ as it is and add more 1's for path length 3 with intermediate vertices a and b.

For instance: We get d-a-b-c.

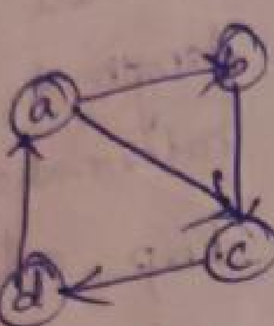
Hence matrix [d][c] = 1.



$$R^{(3)} = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

Here intermediate vertices a, b,

Matrix [a][d] = 1.



$$R^{(4)} = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & \boxed{1} & 1 & 1 & 1 \\ b & \boxed{1} & \boxed{1} & 1 & 1 \\ c & \boxed{1} & \boxed{1} & \boxed{1} & 1 \\ d & 1 & 1 & 1 & 1 \end{array}$$

We can compute all the el each matrix $R^{(k)}$ from previous R

Let $r_{ij}^{(k)}$ → is the element in row & j^{th} column of matrix $R^{(k)}$, if th is a path from v_i to v_j then it is 1 else

the path from v_i to v_j can be computed in 2 cases.

* List of vertices that doesnot contain k^{th} vertex is noted. Then

paths from v_i to v_j with intermediate Vertices numbered not higher than $k-1$.

Then set $r_{ij}^{k-1} = 1$.

* The path not containing k^{th} Vertex v_k in intermediate vertices.

This indicates path from v_i to v_k with each intermediate vertex numbered not higher than $(k-1)$. $\therefore r_{ik}^{(k-1)} = 1$.

This indicates path from v_k to v_j with each intermediate vertex numbered not higher than $(k-1)$.

$\therefore r_{kj}^{(k-1)} = 1$.

We can Generate r^k from r^{k-1} as.

$$r_{ij}^{(k)} = \boxed{r_{ij}^{(k-1)} \text{ or } r_{ik}^{(k-1)} \text{ and } r_{kj}^{(k-1)}}$$

Assume,

$i = 1$ to 4

$j = 1$ to 4

$k = 1$

$r_{11}^{1} = r_{ij}^{(k-1)}$ or $r_{ik}^{(k-1)}$ & $r_{kj}^{(k-1)} \leftarrow i=1, j=1,$

$= r_{11}^{0}$ or r_{11}^{0} and r_{11}^{0}

$= 0$ or $0 \& 0$

$= 0.$

$r_{12}^{1} = r_{12}^{0}$ or r_{11}^{0} and r_{12}^{0}

$= 1$ or 0 and 1

$= 1$

⋮

$r_{42}^{1} = r_{42}^{0}$ or r_{41}^{0} and r_{12}^{0}

$= 0$ or 1 and 1

$= 0$ OR $1 = 1.$