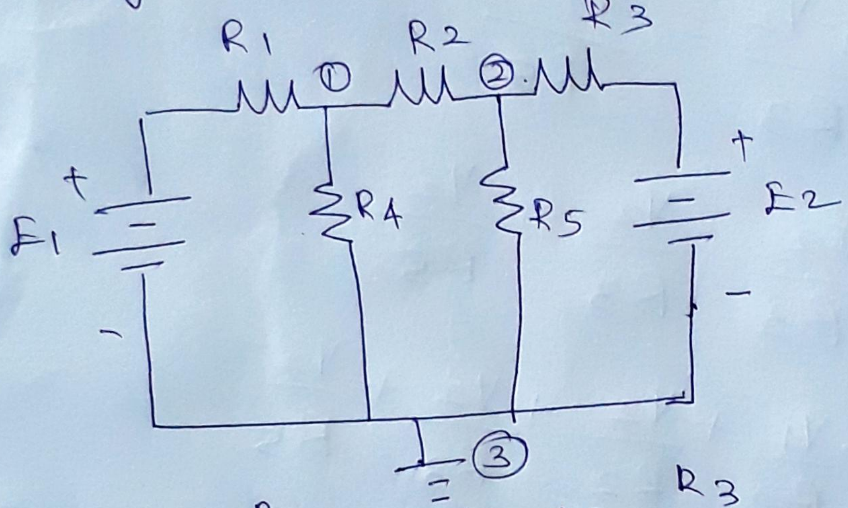
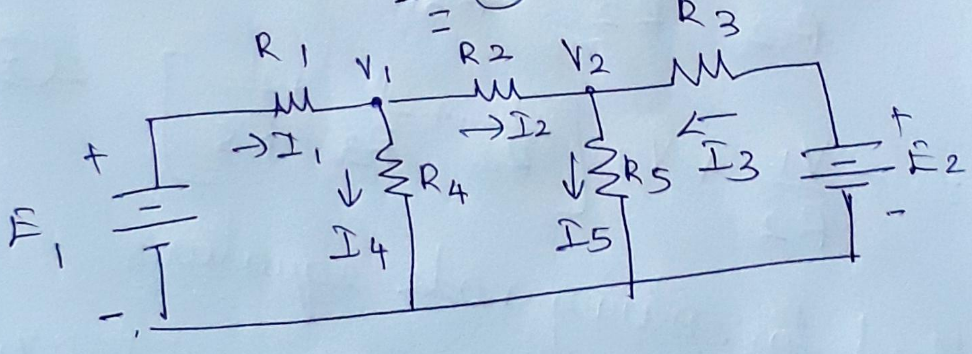
Nodal Analysis :-

This is a technique used to find the voltages at each node of electric circuit.





At node 1, Apply KCL

$$I_1 = I_2 + I_4.$$

$$\frac{E_1 - V_1}{R_1} = \frac{V_1 - V_2}{R_2} + \frac{V_1 - 0}{R_4}$$

$$\frac{E_1}{R_1} - \frac{V_1}{R_1} = \frac{V_1}{R_2} - \frac{V_2}{R_2} + \frac{V_1}{R_4}.$$

$$\frac{E_1}{R_1} = V_1\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4}\right) - \frac{V_2}{R_2}. \quad \text{—(1)}$$

At node 2, by KCL

$$I_2 + I_3 = I_5$$

$$\frac{V_1 - V_2}{R_2} + \frac{E_2 - V_2}{R_3} = \frac{V_2 - 0}{R_5}$$

$$\frac{V_1}{R_2} - \frac{V_2}{R_2} + \frac{E_2}{R_3} - \frac{V_2}{R_3} = \frac{V_2}{R_5}$$

$$\frac{E_2}{R_3} = -\frac{V_1}{R_2} + \frac{V_2}{R_2} + \frac{V_2}{R_3} + \frac{V_2}{R_5}$$

$$\frac{E_2}{R_3} = -\frac{V_1}{R_2} + V_2\left[\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5}\right] \quad -(2)$$

(iii) Matrix method for solving node equations

$$\begin{bmatrix} \frac{E_1}{R_1} \\ \frac{E_2}{R_3} \end{bmatrix} = \begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5} \end{bmatrix}$$

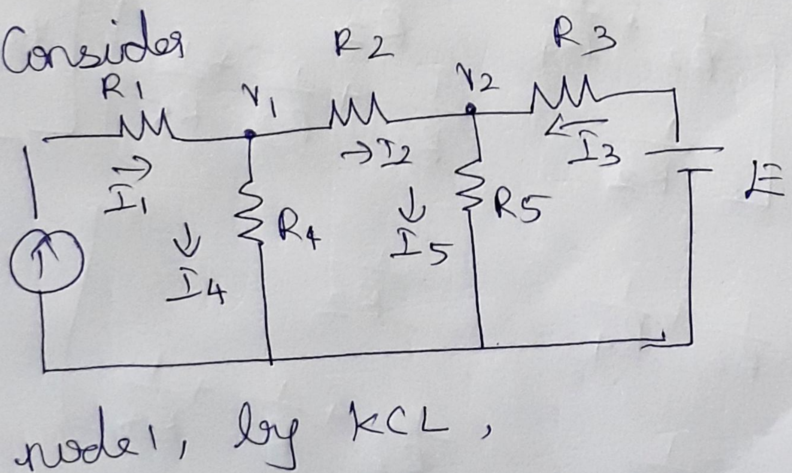
$$\Delta_1 = \begin{bmatrix} \frac{E1}{R_1} & -\frac{1}{R2} \\ \frac{E2}{R3} & \left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R5}\right) \end{bmatrix}.$$

$$\Delta_2 = \begin{bmatrix} \frac{1}{R1}+\frac{1}{R2}+\frac{1}{R4} & \frac{E1}{R_1} \\ -\frac{1}{R_2} & \frac{E2}{R3} \end{bmatrix}$$

The node voltages are given by,

$$V_1 = \frac{\Delta_1}{\Delta}, \quad V_2 = \frac{\Delta_2}{\Delta}.$$

(b) Consider



Let V_1 & V_2 be voltage at node 1 & 2.

At node 1, by KCL,

$$I = \frac{V_1}{R4} + \frac{V_1 - V2}{R2}$$

$$= \frac{V_1}{R4} + \frac{V_1}{R2} - \frac{V_2}{R2}$$

$$I = \left(\frac{1}{R_2} + \frac{1}{R_4}\right)V_1 - \frac{V_2}{R_2} \quad — (1)$$

At node 2, by KCL,

$$\frac{V_1 - V_2}{R_2} + \frac{E - V_2}{R_3} = \frac{V_2}{R_5}$$

$$\frac{E}{R_3} = \frac{V_2}{R_5} - \frac{V_1}{R_2} + \frac{V_2}{R_2} + \frac{V_2}{R_3}$$

$$\frac{E}{R_3} = -\frac{V_1}{R_2} + \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5}\right)V_2$$

— (2)

Equations (1) & (2) in matrix form

$$\begin{bmatrix} I \\ \frac{E}{R_3} \end{bmatrix} = \begin{bmatrix} \frac{1}{R_2} + \frac{1}{R_4} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} \frac{1}{R_2} + \frac{1}{R_4} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5} \end{bmatrix}$$

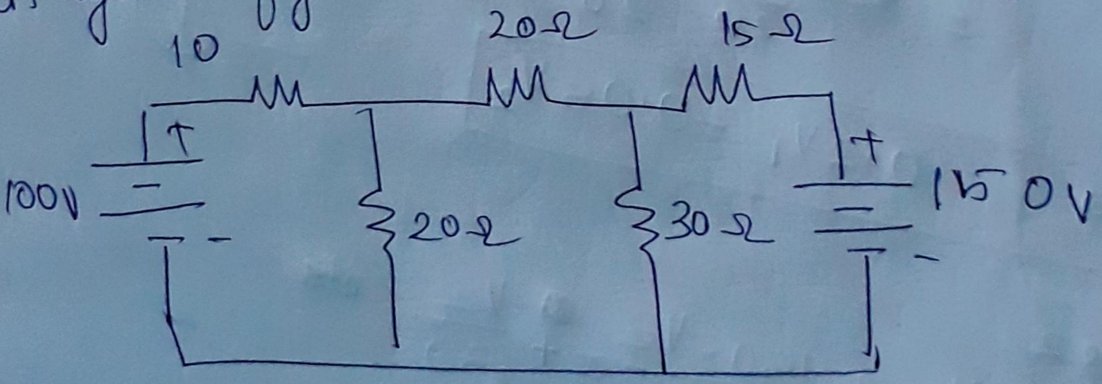
$$\Delta_1 = \begin{bmatrix} I & -\frac{1}{R_2} \\ \frac{E}{R_3} & \frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_5} \end{bmatrix}$$

$$\Delta_2 = \begin{bmatrix} \frac{1}{R_2}+\frac{1}{R_4} & I \\ -\frac{1}{R_2} & \frac{E}{R_3} \end{bmatrix}$$

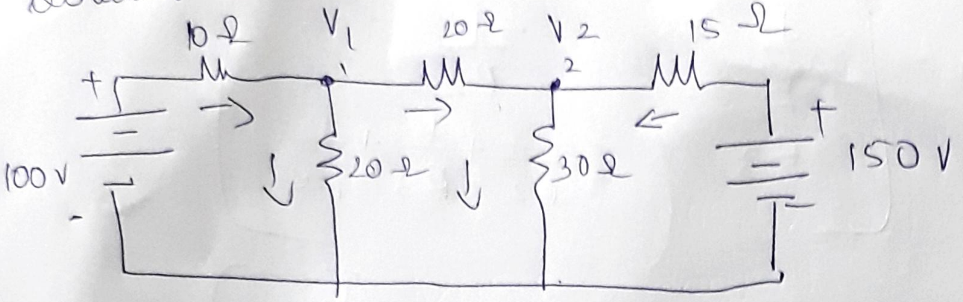
$$V_1 = \frac{\Delta_1}{\Delta} \; ; \; V_2 = \frac{\Delta_2}{\Delta}$$

Problems

Using nodal analysis, find all node voltages in given figure



Solution



Node equations

At node 1

$$\frac{100 - V_1}{10} = \frac{V_1}{20} + \frac{V_1 - V_2}{20}$$

$$\frac{100}{10} - \frac{V_1}{10} = \frac{V_1}{20} + \frac{V_1}{20} - \frac{V_2}{20}$$

$$10 - \frac{V_1}{10} = \frac{V_1}{20} + \frac{V_1}{20} - \frac{V_2}{20}$$

$$10 = \frac{V_1}{20} + \frac{V_1}{20} + \frac{V_1}{10} - \frac{V_2}{20}$$

$$\boxed{10 = 0.2V_1 - 0.05V_2} \quad \text{—} (1)$$

At node 2

$$\frac{V_1 - V_2}{20} + \frac{150 - V_2}{15} = + \frac{V_2}{30}$$

$$\frac{V_1}{20} - \frac{V_2}{20} + \frac{150}{15} - \frac{V_2}{15} - \frac{V_2}{30} = 0$$

$$\frac{150}{15} = -\frac{V_1}{20} + \frac{V_2}{20} + \frac{V_2}{15} + \frac{V_2}{30}$$

$$\boxed{10 = -0.05V_1 + 0.15V_2} \quad \text{—} (2)$$

(iii) equation

$$\begin{bmatrix} 10 \\ 10 \end{bmatrix} = \begin{bmatrix} 0.2 & -0.05 \\ -0.05 & +0.15 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 0.2 & -0.05 \\ -0.05 & +0.15 \end{bmatrix} = 0.2 \times 0.15 - (0.05) \times (-0.05)$$
$$= 0.0275$$

$$\Delta_1 = \begin{bmatrix} 10 & -0.05 \\ 10 & 0.15 \end{bmatrix} = 10 \times 0.15 - [(-0.05) \times 10]$$
$$= 2.$$

$$\Delta_2 = \begin{bmatrix} 0.2 & 10 \\ -0.05 & 10 \end{bmatrix} = 0.2 \times 10 + 10 \times 0.05$$
$$= 2.5$$

Final Sol

Voltages are

$$V_1 = \frac{\Delta_1}{\Delta} = \frac{2}{0.0275} = 72.73\,V,$$

$$V_2 = \frac{\Delta_2}{\Delta} = \frac{2.5}{0.0275} = 91\,V$$