ECE 211 Workshop: Thevenin’s and Norton’s Theorems

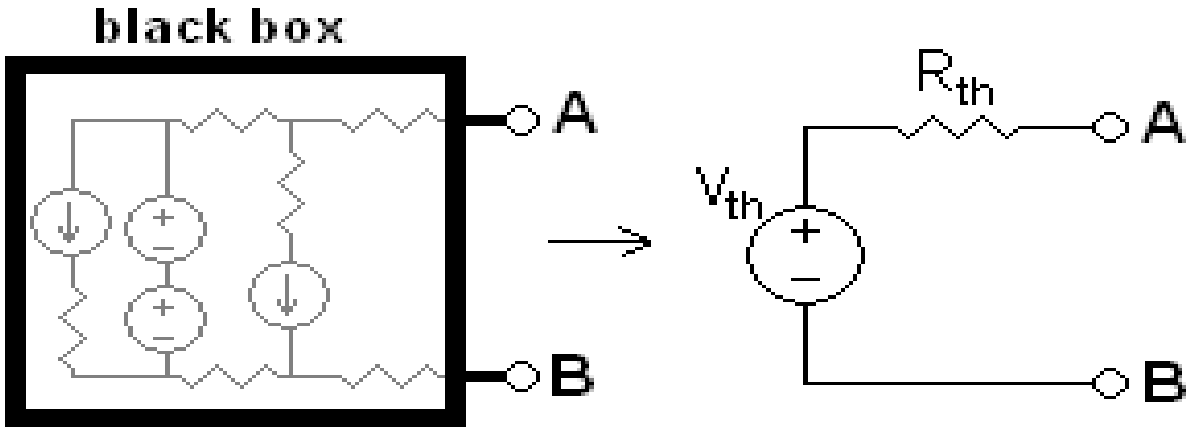
Academic Resource Center

Agenda

- Background: Thevenin’s Theorems Review
- Thevenin's Analysis
  - How to find Equivalent Thevenin's Voltage Source and relating problems
  - How to find Equivalent Thevenin's Resistor and relating problems
- Transformation between two Theorems
- Practice Problems and Solutions

Thevenin's Theorem Review

General Idea: In circuit theory, Thévenin's theorem for linear electrical networks states that any combination of voltage sources, current sources, and resistors with two terminals is electrically equivalent to a single voltage source V in series with a single series resistor R. Those sources mentioned above can be either independent or dependent.



Thevenin's Theorem Review

Analyze Procedure:

1.Calculate the output voltage, V, when in open circuit condition (no load resistor—meaning infinite resistance). This is V_{Th}.

2.Calculate the output current, I_{AB}, when the output terminals are short circuited (load resistance is 0). R_{Th} equals V_{Th} divided by this I_{AB}.

Step 2 could also be thought of as:

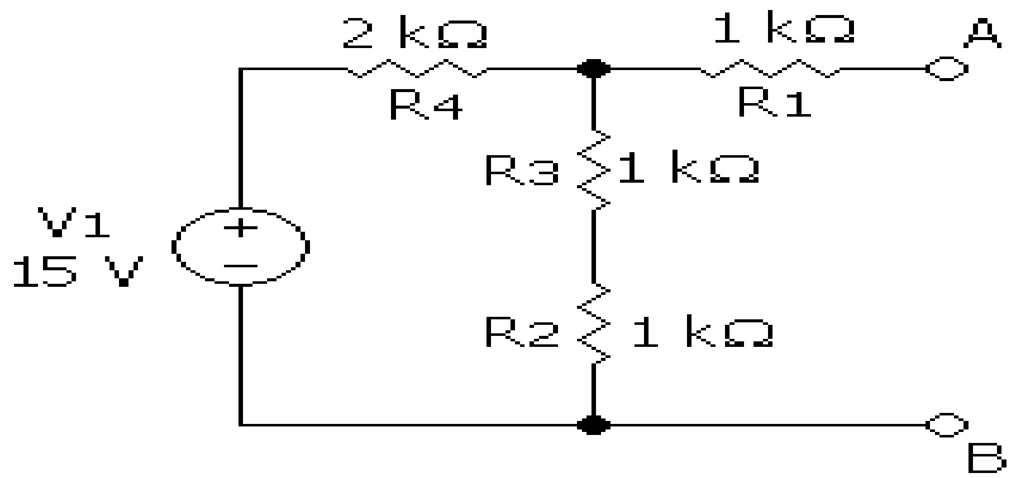
2a. Replace voltage sources with short circuits(wires), and current sources with open circuits(disconnections).

2b. Calculate the resistance between terminals A and B. This is R_{Th}.

Thevenin's Voltage Example

- Find equivalent voltage source in new circuit
- <u>Solution:</u>

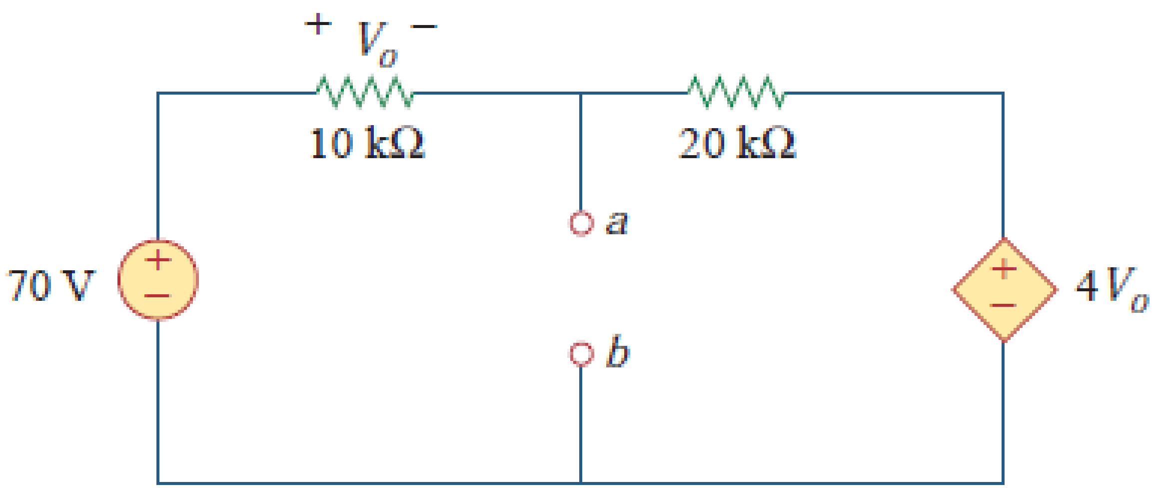
Between terminals A and B, we need to find out V. Since it's open circuit and there is no current going through R_1.Treat R_1 as wire. ciucuit become simple three series resistor and a voltage source.
Secondly, find the current.
Thirdly, find the sum voltage across R_3 and R_2. That's the answer we're looking for.



Using Ohm’s law, we find:
I = V/R, where V = 15V
R = R4+R3+R2 = 4Ohm
I = 15/4 = 3.75A
Ohm's law again,
Vab = 2*3.75 = 7.5V

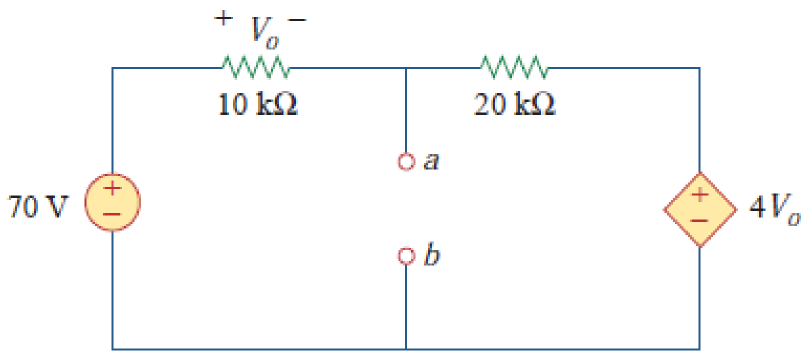
Practice Problems 1A

Find the Thevenin equivalent at terminals *a*-*b* of the circuit in Fig. 4.107.



Practice Problems 1B

4.40 Find the Thevenin equivalent at terminals *a*-*b* of the circuit in Fig. 4.107.



There is only one path in the circuit, so the ratio of the two resistors are the ratio of their voltages(same current).

V_0 + $2V_0$(20K Ohms) +$4V_0$ = 70V

V_0 = 10V

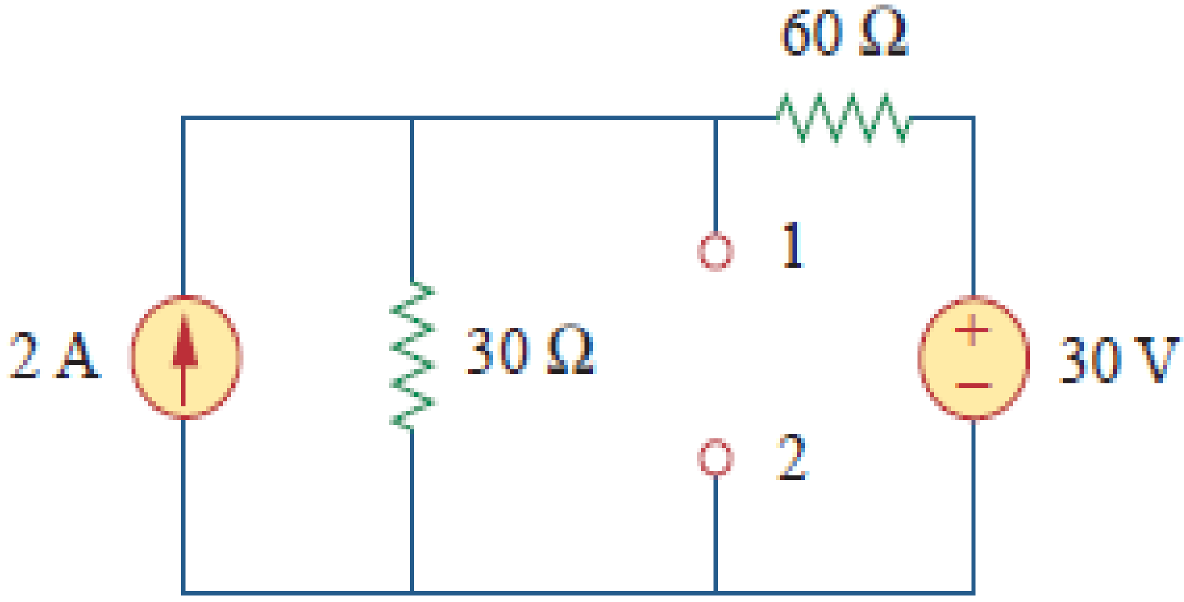
Two ways finding Vab:

V_{ab} = 70V- V_0

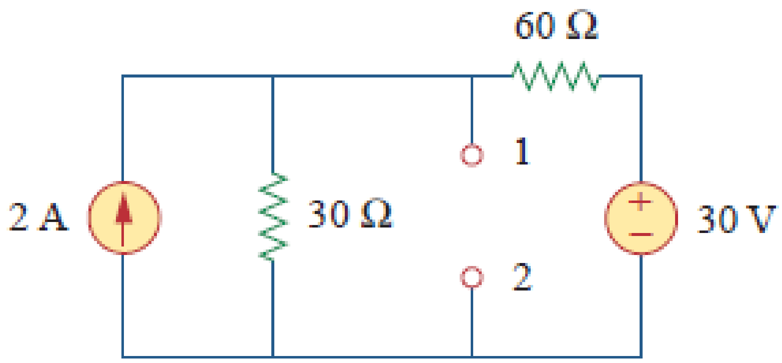
=$2V_0$ + $4V_0$

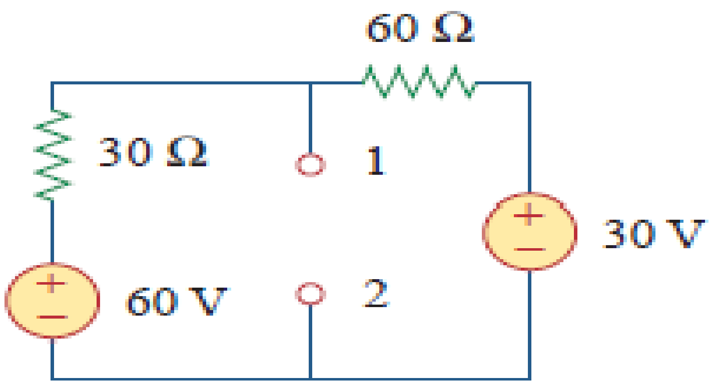
= 60V

Practice Problems 2A



Practice Problems 2B





Firstly, if we perform the source transformation, the original circuit changes to a simple series one. Notice the two source are in the opposite direction.

$V_{sigma} = 60V - 30V = 30V$

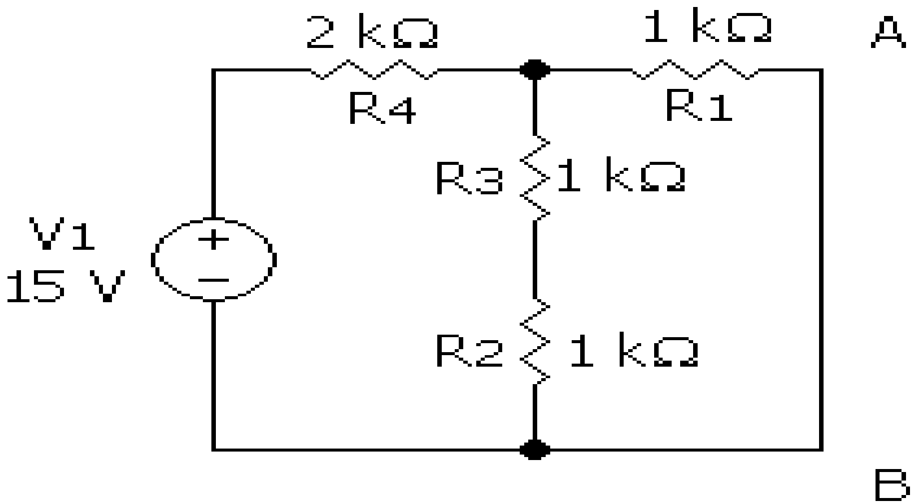
$R_{sigma} = 30 \text{ Ohm} + 60 \text{ Ohm} = 90 \text{ Ohm}$

$I_{sigma} = V_{simga}/R_{sigma} = 1/3 \text{ A}$

$V_{12} = V_{60} + 30V$

$= 1/3 * 60 + 30V = 50V$

Thevenin's Resistor Example A

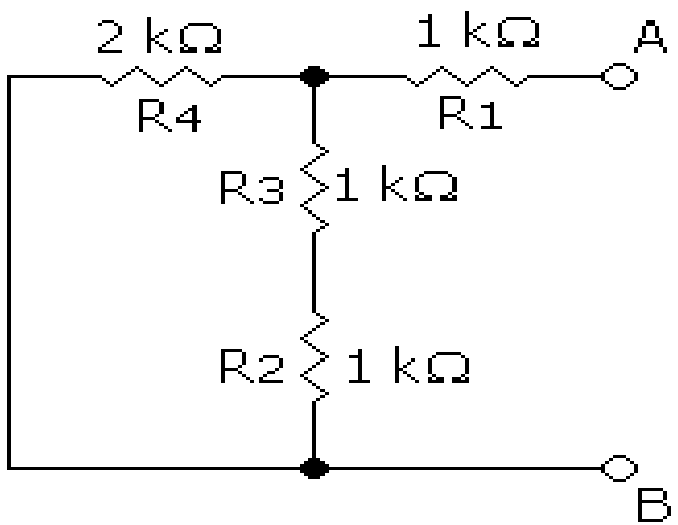


- Find equivalent resistor in new circuit
- Solution:

Original method: short terminals A and B as shown in the picture. Find the current I going through A to B. R_{th} can be found by V/I,where V is the voltage we get from last problem.

$R_{2,3} = R_2 + R_3 = 2$ Ohm
$R_{2,3,1} = R_{2,3} \parallel R_1 = 2/3$ ohm
$R_{sigma} = R_{1,2,3,4} = R_{2,3,1} + R_4 = 8/3$ ohm
$I_{sigma} = V_1/R_{sigma} = 45/8$ A
$I_{AB} = I_{sigma} * (R_2+R_3) / (R_1+R_2+R_3) = 15/4$ A
$R_{th} = V_{AB}/I_{AB} = 2$ Ohm

Thevenin's Resistor Example B



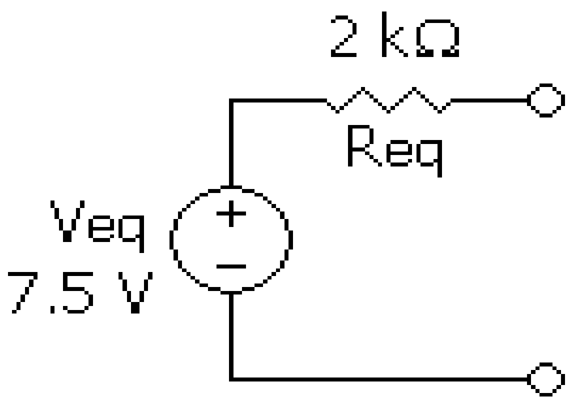
- Find equivalent resistor in new circuit
- Solution:

Alternatively method: leave terminals A and B open,instead of short Volage source V1, shown in the picture. Since no dependent source apprearing in the graph, we just need to find Rth by series and parallel theory.

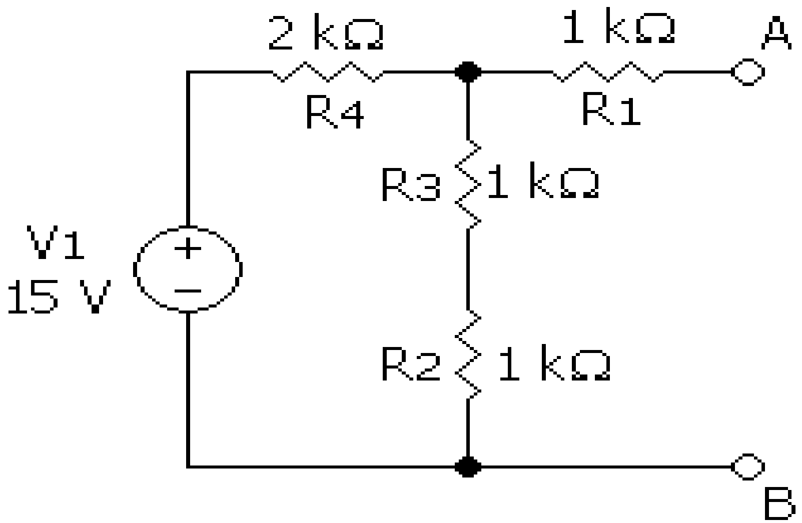
$R_{2,3} = R_2 + R_3 = 2$ Ohm
$R_{2,3,4} = R_{2,3} \,||\, R_4 = 1$ ohm
$R_{sigma} = R_{1,2,3,4} = R_{2,3,4} + R_1 = 2$ ohm
Which matches the previous results.

Thevenin's Example summary

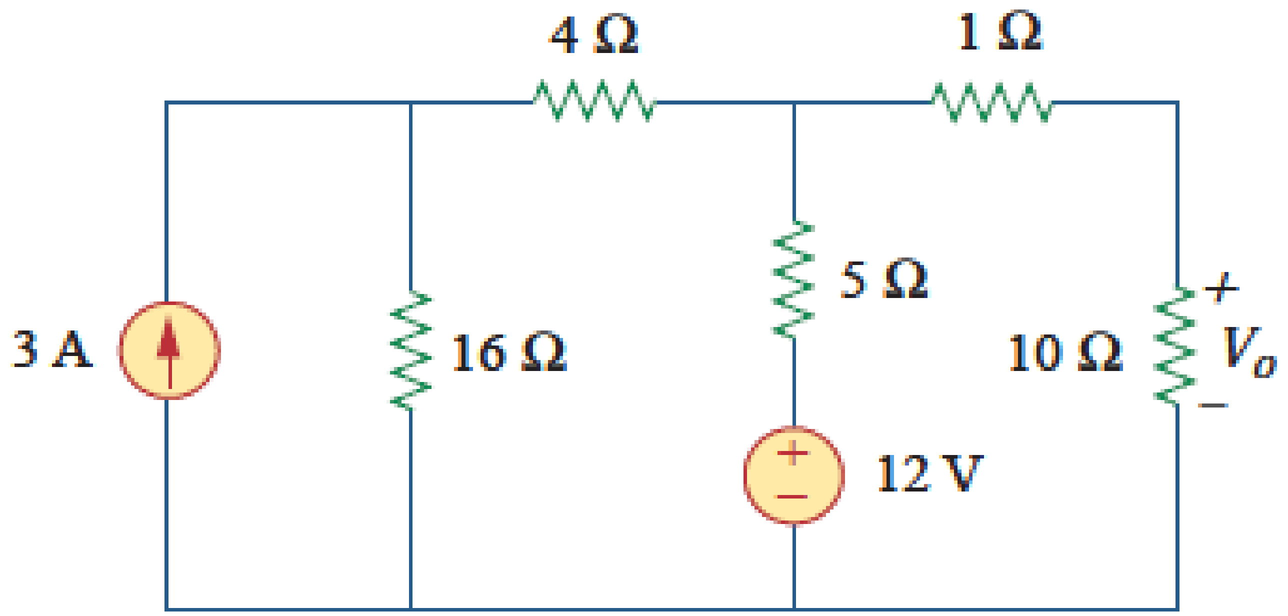
1 The graph shown in the right hand side gives the final result for Thevenin's theory. compared with original circuit, it looks a lot easier to further analyze.



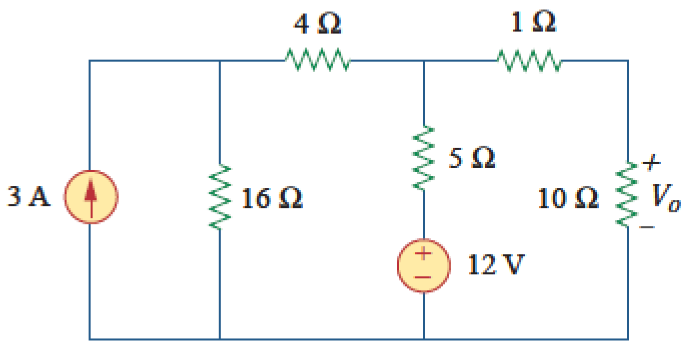
2 Generally speaking, out of the two ways in findng equivalent resistor. A is more suitable for graph containing denpendent source. B is more useful in the situation of simply parallel and series resistors.

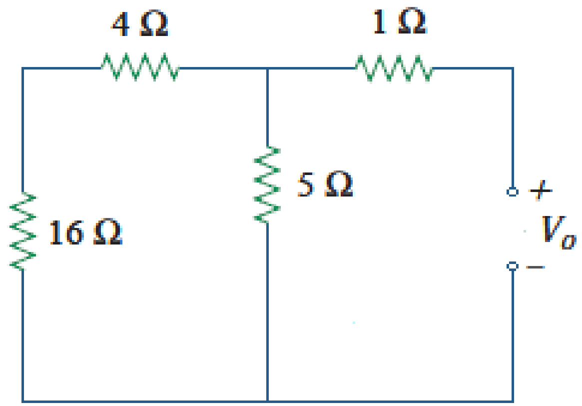


Practice Problems 3A



Practice Problems 3B





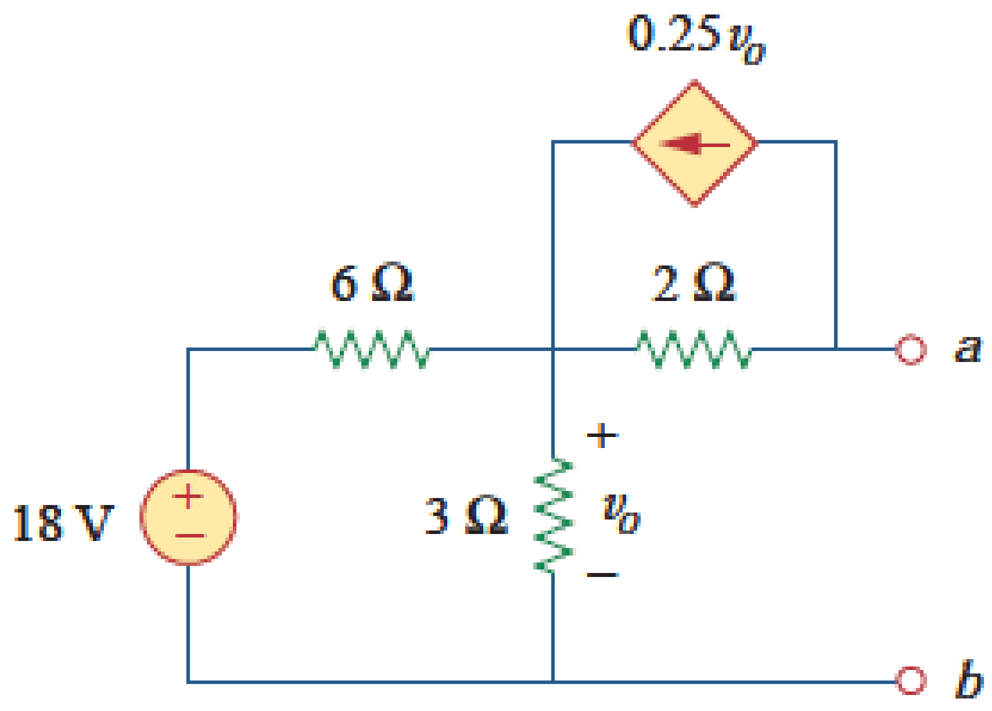
When we try to calculate the quivalent resistor, simply short indenpendet voltage source and open independent current source. Then, the lower left circuit is derived.

R_1 = 16 Ohm + 4 Ohm = 20 Ohm

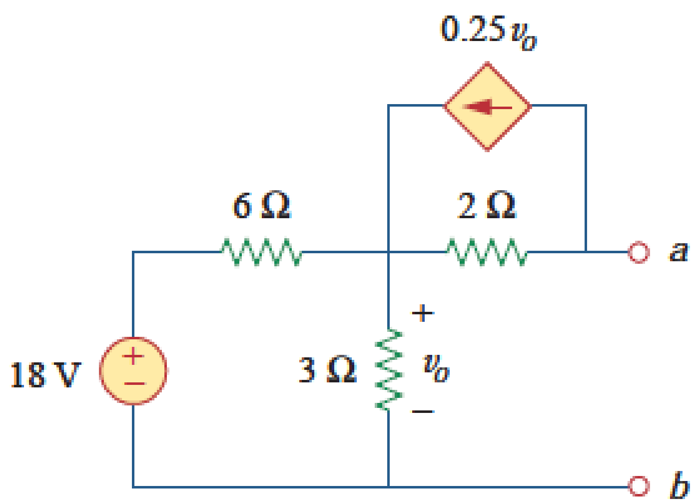
R_2 = R1 || 5 Ohm = 20 Ohm || 5 Ohm = 4 Ohm

R_{sigma} = R2 + 1 Ohm = 5 Ohm

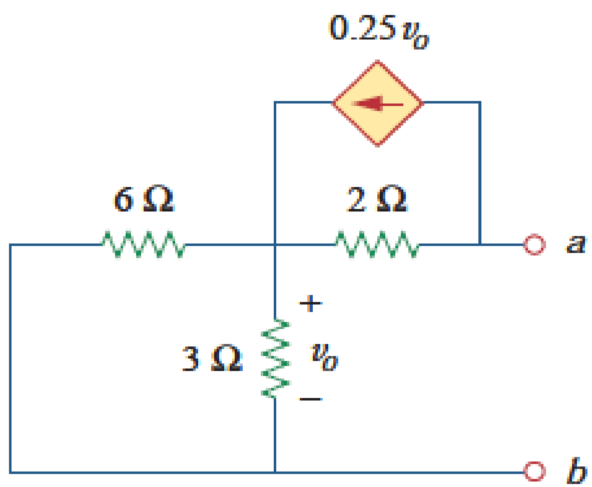
Practice Problems 4A

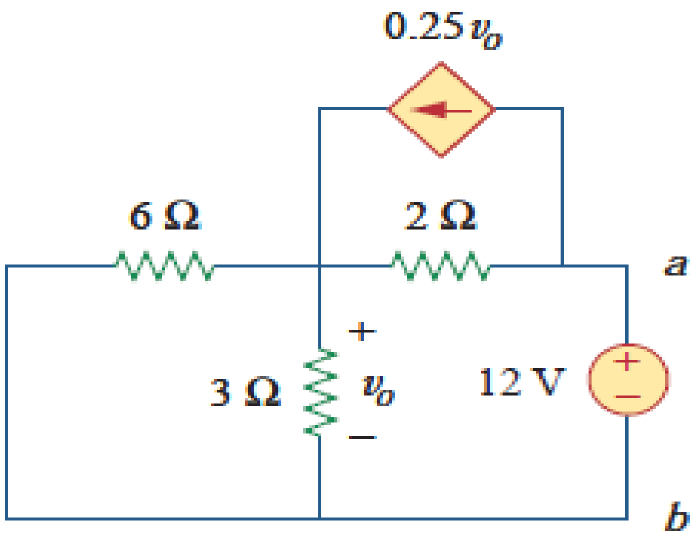


Practice Problems 4B

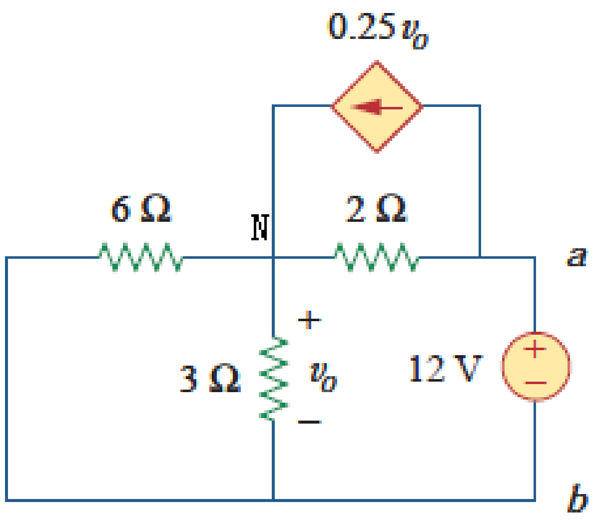


Short indenpendet voltage source, then you have lower left circuit.
In order to find out the equivalent resistor, we need an additional indentpent source apply to terminnals a and b. Figure show lower.





Practice Problems 4B



Since we assume the voltage source has a value of 12V, we need one more parameter, current I(through voltage source) to figure out resistance using equation R= U/I.

Assumption: current direction in 2 Ohm is a to N. 6 Ohm and 3 Ohm share the same voltage(V_0). For node N, current going out are ($V_0/6 + V_0/3$).
Assumption: current direction in 2 Ohm is a to N. Total current going in: ($0.25v_0$ + i). From KCL we equal ($V_0/6 + V_0/3$) with ($0.25v_0$ + i), we get i = $v_0/4$.
From KVL $v_0/4 * 2 + v_0 = 12$ V
$v_0 = 8$ V
Iab = $0.25v_0 + v_0/4$(2 Ohm) = $0.5\ v_0$ = 4A
$R_{th} = V_{ab}/I_{ab} = 12/4 = 3$ Ohm

ARC.

Norton's Theorem Review

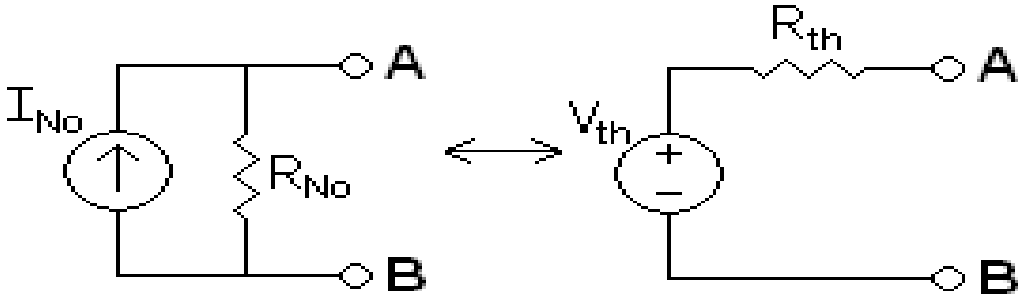
General Idea: Norton's theorem for linear electrical networks, known in Europe as the Mayer–Norton theorem, states that any collection of voltage sources, current sources, and resistors with two terminals is electrically equivalent to an ideal current source, I, in parallel with a single resistor, R. Those sources mentioned above can also either be dependent or independent sources.

Analyze Procedure: 1.Find the Norton current I_{No}. Calculate the output current, I_{AB}, with a short circuit as the load (meaning 0 resistance between A and B). This is I_{No}.

2.Find the Norton resistance R_{No}. There are two methods of determining the Norton impedance R_{No}. (the same as Thevenin's Theorem)

Transformation between two methods

From the description we have seen at least two simiarities. Firstly, they both use load equaling to 0 finding current. Secondly, the way they finds quivalent resistors are precisely matched. Thus we conclude that those two are essentially the same.



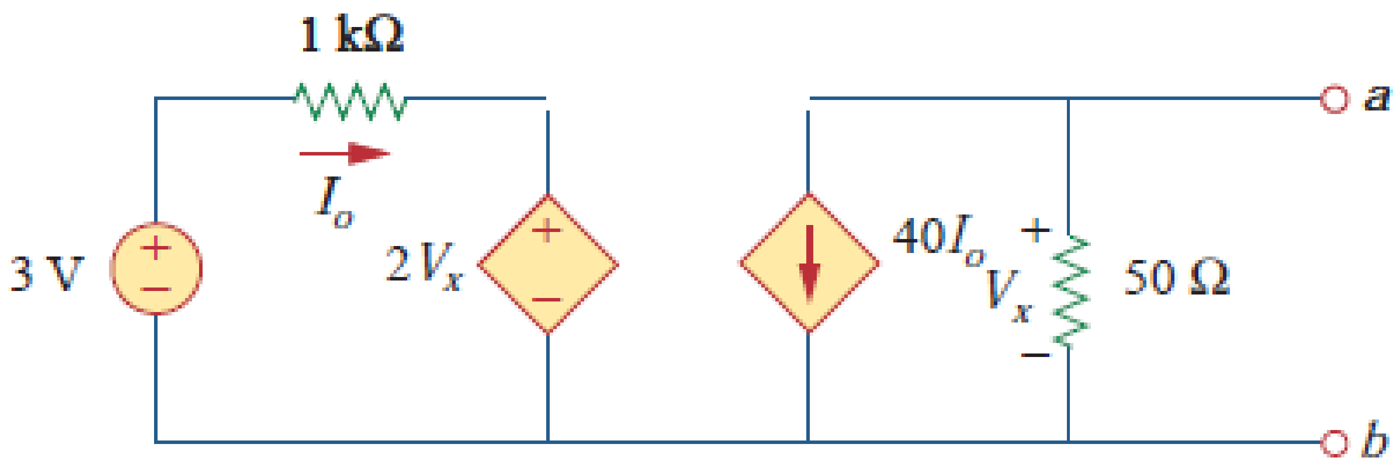
Transformation eqation are:

Rno = Rth

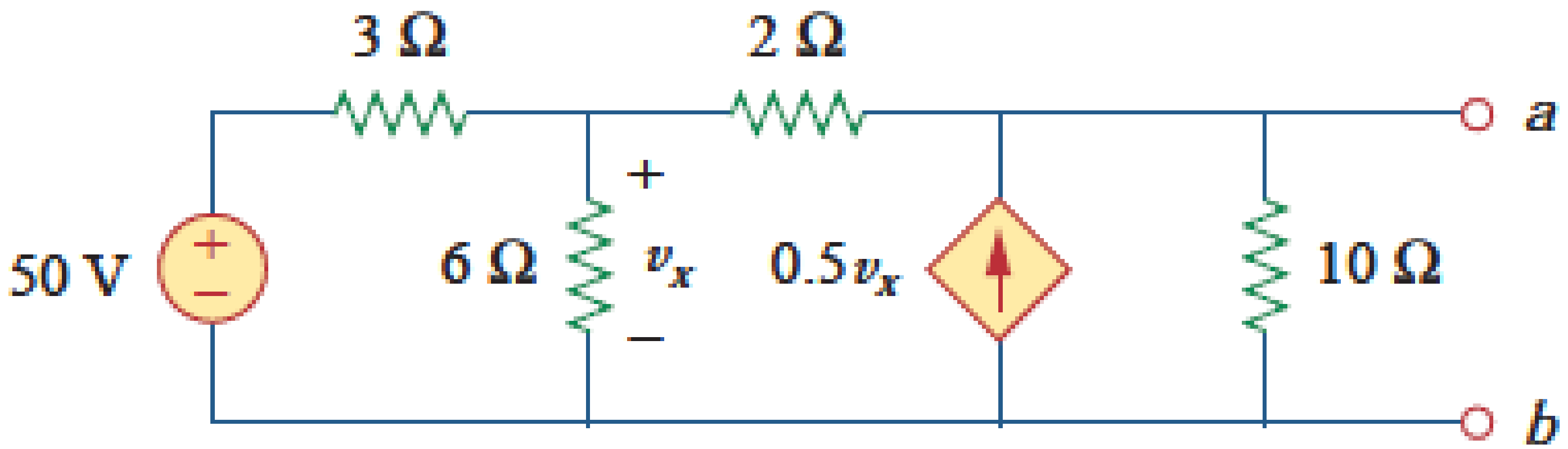
$V_{th} = I_{no} * R_{no}$

$I_{no} = V_{th} / R_{no}$

Practice Problems



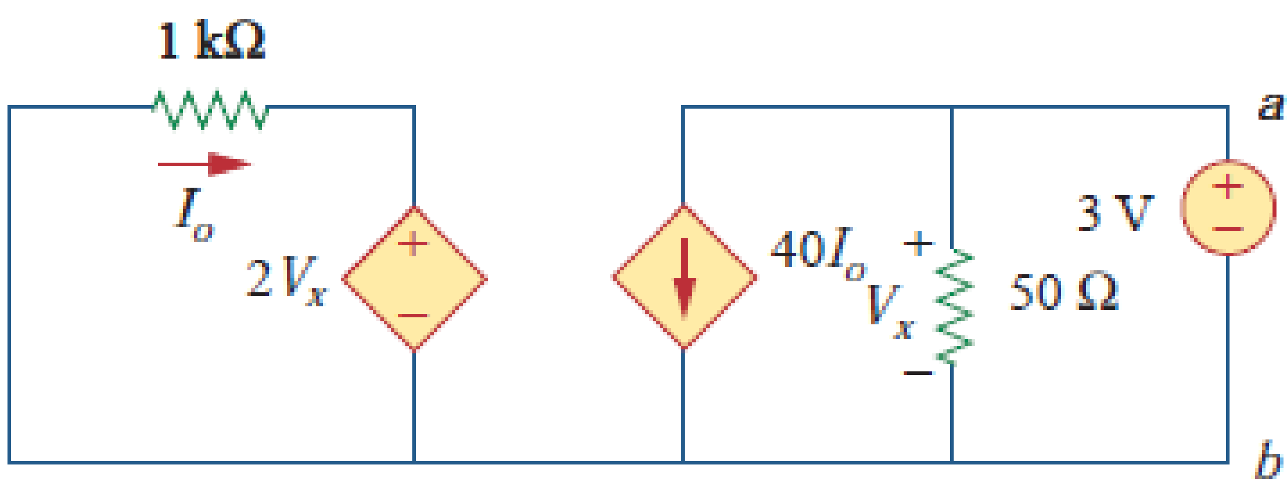
Practice Problems



Solutions to Practice Problems

V_{th}: (original figure)
$1k * I_0 + 2\,V_x = 3V$
$V_x = 40I_0 * 50$
solve the above:
$V_x = V_{th} = 1.2V$
$I_0 = 0.6$ mA
We short the votage source and add an additional votage source between a and b.(lower figure)

R_{th}:
So, we have: $1000 * I_0 = -2*Vx$
$V_x = 3V$
solve the above:
$I_0 = -6$ mA
$-40\,I_0 - V_x/50 = I_{ab} = 0.18A$
direction: a to b
$Rth = 3V/\,I_{ab} = -16.67$ Ohm



Solutions to Practice Problems

V_{th}:

$(2i + 0.5\ V_x)\ *10 + 2i = V_x$

$(V_x/6 + i)\ *\ 3 + V_x = 50$

solve the above:

$V_x = 100V, i = -33.3A$

$V_{th} = (0.5\ V_x + i\ ) = 166.67\ V$

We short the votage source and add an additional votage source between a and b.

R_{th}:

So, we have:

$V_x/6 + V_x/3 + i = 0$

$V_x - 2i = 50V$

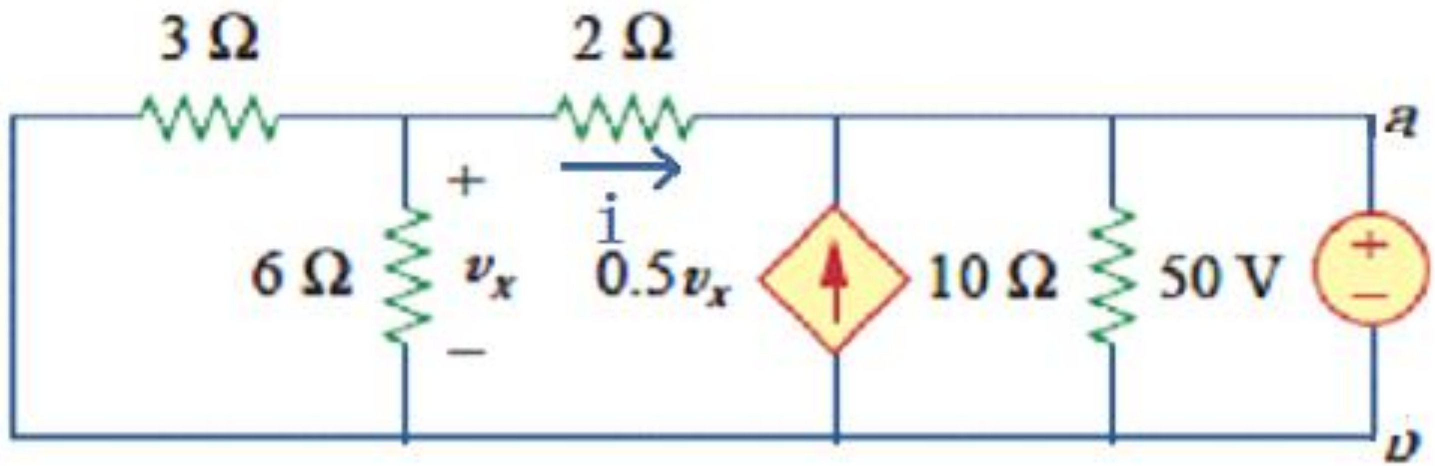
solve the above:

$i = -12.5A,\ V_x = 25V$

$I_{ab} = -i - 0.5V_x + 50V/10 = 5A$

direction: b to a

$Rth = 50V/\ I_{ab} = 10\ Ohm$



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