Chapter 7

Magnetic circuits

At the end of this chapter you should be able to:

- appreciate some applications of magnets
- describe the magnetic field around a permanent magnet
- state the laws of magnetic attraction and repulsion for two magnets in close proximity
- define magnetic flux, Φ, and magnetic flux density, B, and state their units
- perform simple calculations involving $B = \Phi/A$
- define magnetomotive force, F_m, and magnetic field strength, H, and state their units
- perform simple calculations involving $F_m = NI$ and $H = NI/l$
- define permeability, distinguishing between μ_0, μ_r and μ
- understand the B–H curves for different magnetic materials
- appreciate typical values of μ_r
- perform calculations involving $B = \mu_0\mu_r H$
- define reluctance, S, and state its units
- perform calculations involving

$$S = \frac{\text{m.m.f.}}{\Phi} = \frac{l}{\mu_0\mu_r A}$$

- perform calculations on composite series magnetic circuits
- compare electrical and magnetic quantities
- appreciate how a hysteresis loop is obtained and that hysteresis loss is proportional to its area

7.1 Introduction to magnetism and magnetic circuits

The study of magnetism began in the thirteenth century with many eminent scientists and physicists such as William Gilbert, Hans Christian Oersted, Michael Faraday, James Maxwell, André Ampère and Wilhelm Weber all having some input on the subject since. The association between electricity and magnetism is a fairly recent finding in comparison with the very first understanding of basic magnetism.

Today, magnets have **many varied practical applications**. For example, they are used in motors and generators, telephones, relays, loudspeakers, computer hard drives and floppy disks, anti-lock brakes, cameras, fishing reels, electronic ignition systems, keyboards, t.v. and radio components and in transmission equipment.

The full theory of magnetism is one of the most complex of subjects; this chapter provides an introduction to the topic.

7.2 Magnetic fields

A **permanent magnet** is a piece of ferromagnetic material (such as iron, nickel or cobalt) which has properties of attracting other pieces of these materials. A permanent magnet will position itself in a north and south direction when freely suspended. The north-seeking end of the magnet is called the **north pole, N**, and the south-seeking end the **south pole, S**.

The area around a magnet is called the **magnetic field** and it is in this area that the effects of the **magnetic force** produced by the magnet can be detected. A magnetic field cannot be seen, felt, smelt or heard and therefore is difficult to represent. Michael Faraday suggested that the magnetic field could be represented pictorially, by imagining the field to consist of **lines of magnetic flux**, which enables investigation of the distribution and density of the field to be carried out.

The distribution of a magnetic field can be investigated by using some iron filings. A bar magnet is placed on a flat surface covered by, say, cardboard, upon which is sprinkled some iron filings. If the cardboard is gently tapped the filings will assume a pattern similar to that shown in Fig. 7.1. If a number of magnets of different strength are used, it is found that the stronger the field the closer are the lines of magnetic flux and vice versa. Thus a magnetic field has the property of exerting a force, demonstrated in this case by causing the iron filings to move into the pattern shown. The strength of the magnetic field decreases as we move away from the magnet. It should be realized, of course, that the magnetic field is three dimensional in its effect, and not acting in one plane as appears to be the case in this experiment.
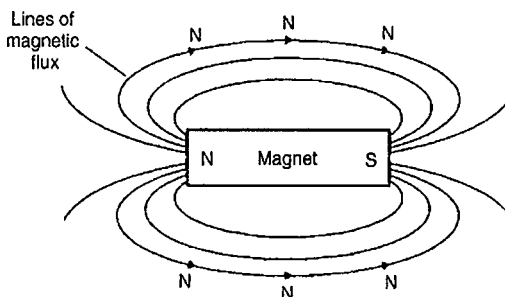

Figure 7.1

If a compass is placed in the magnetic field in various positions, the direction of the lines of flux may be determined by noting the direction of the compass pointer. The direction of a magnetic field at any point is taken as that in which the north-seeking pole of a compass needle points when suspended in the field. The direction of a line of flux is from the north pole to the south pole on the outside of the magnet and is then assumed to continue through the magnet back to the point at which it emerged at the north pole. Thus such lines of flux always form complete closed loops or paths, they never intersect and always have a definite direction.

The laws of magnetic attraction and repulsion can be demonstrated by using two bar magnets. In Fig. 7.2(a), **with unlike poles adjacent, attraction takes place**. Lines of flux are imagined to contract and the magnets try to pull together. The magnetic field is strongest in between the two magnets, shown by the lines of flux being close together. In Fig. 7.2(b), **with similar poles adjacent (i.e. two north poles), repulsion occurs**, i.e. the two north poles try to push each other apart, since magnetic flux lines running side by side in the same direction repel.
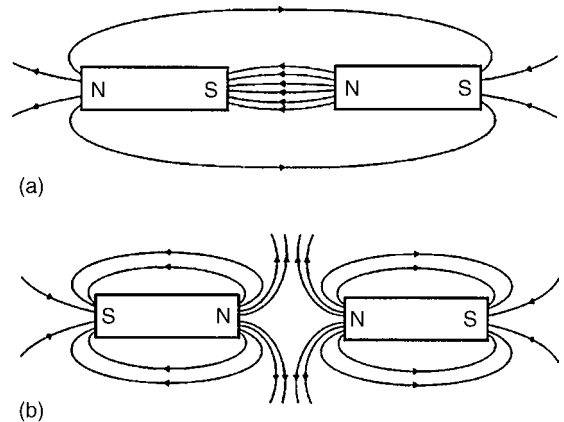

(a)

(b)

Figure 7.2

7.3 Magnetic flux and flux density

Magnetic flux is the amount of magnetic field (or the number of lines of force) produced by a magnetic source. The symbol for magnetic flux is Φ (Greek letter 'phi'). The unit of magnetic flux is the **weber, Wb**.

Magnetic flux density is the amount of flux passing through a defined area that is perpendicular to the direction of the flux:

$$\text{Magnetic flux density} = \frac{\text{magnetic flux}}{\text{area}}$$

The symbol for magnetic flux density is B. The unit of magnetic flux density is the tesla, T, where $1\,T = 1\,Wb/m^2$. Hence

$$B = \frac{\Phi}{A} \text{ tesla}$$

where $A(m^2)$ is the area

Problem 1. A magnetic pole face has a rectangular section having dimensions 200 mm by 100 mm. If the total flux emerging from the pole is 150 µWb, calculate the flux density.

Flux $\Phi = 150\,µWb = 150 \times 10^{-6}\,Wb$
Cross sectional area $A = 200 \times 100 = 20\,000\,mm^2$
$$= 20\,000 \times 10^{-6}\,m^2.$$
$$\text{Flux density, } B = \frac{\Phi}{A} = \frac{150 \times 10^{-6}}{20\,000 \times 10^{-6}}$$
$$= \mathbf{0.0075\,T} \text{ or } \mathbf{7.5\,mT}$$

Problem 2. The maximum working flux density of a lifting electromagnet is 1.8 T and the effective area of a pole face is circular in cross-section. If the total magnetic flux produced is 353 mWb, determine the radius of the pole face.

Flux density $B = 1.8\,T$ and
flux $\Phi = 353\,mWb = 353 \times 10^{-3}\,Wb$.
Since $B = \Phi/A$, cross-sectional area $A = \Phi/B$

i.e. $$A = \frac{353 \times 10^{-3}}{1.8}\,m^2 = 0.1961\,m^2$$

The pole face is circular, hence area $= \pi r^2$, where r is the radius. Hence $\pi r^2 = 0.1961$ from which, $r^2 = 0.1961/\pi$ and radius $r = \sqrt{(0.1961/\pi)} = 0.250\,m$ i.e. **the radius of the pole face is 250 mm**.

7.4 Magnetomotive force and magnetic field strength

Magnetomotive force (m.m.f.) is the cause of the existence of a magnetic flux in a magnetic circuit,

$$\text{m.m.f. } \mathbf{F_m = NI} \text{ amperes}$$

where N is the number of conductors (or turns) and I is the current in amperes. The unit of m.m.f is sometimes expressed as 'ampere-turns'. However since 'turns' have no dimensions, the S.I. unit of m.m.f. is the ampere.

Magnetic field strength (or **magnetising force**),

$$H = \frac{NI}{l} \text{ ampere per metre}$$

where l is the mean length of the flux path in metres. Thus

$$\text{m.m.f. } = NI = Hl \text{ amperes}$$

Problem 3. A magnetising force of 8000 A/m is applied to a circular magnetic circuit of mean diameter 30 cm by passing a current through a coil wound on the circuit. If the coil is uniformly wound around the circuit and has 750 turns, find the current in the coil.

$H = 8000\,A/m$, $l = \pi d = \pi \times 30 \times 10^{-2}\,m$ and $N = 750$ turns. Since $H = NI/l$, then

$$I = \frac{Hl}{N} = \frac{8000 \times \pi \times 30 \times 10^{-2}}{750}$$

Thus, **current $I = 10.05\,A$**

Now try the following exercise

Exercise 32 Further problems on magnetic circuits

1. What is the flux density in a magnetic field of cross-sectional area $20\,cm^2$ having a flux of 3 mWb? [1.5 T]

2. Determine the total flux emerging from a magnetic pole face having dimensions 5 cm by 6 cm, if the flux density is 0.9 T
[2.7 mWb]

3. The maximum working flux density of a lifting electromagnet is 1.9 T and the effective area of a pole face is circular in cross-section. If the total magnetic flux produced is 611 mWb determine the radius of the pole face.
[32 cm]

4. A current of 5 A is passed through a 1000-turn coil wound on a circular magnetic circuit of radius 120 mm. Calculate (a) the magnetomotive force, and (b) the magnetic field strength.
[(a) 5000 A (b) 6631 A/m]

5. An electromagnet of square cross-section produces a flux density of 0.45 T. If the magnetic

flux is $720\,\mu\text{Wb}$ find the dimensions of the electromagnet cross-section.

[4 cm by 4 cm]

6. Find the magnetic field strength applied to a magnetic circuit of mean length 50 cm when a coil of 400 turns is applied to it carrying a current of 1.2 A [960 A/m]

7. A solenoid 20 cm long is wound with 500 turns of wire. Find the current required to establish a magnetising force of 2500 A/m inside the solenoid. [1 A]

8. A magnetic field strength of 5000 A/m is applied to a circular magnetic circuit of mean diameter 250 mm. If the coil has 500 turns find the current in the coil. [7.85 A]

7.5 Permeability and B–H curves

For air, or any non-magnetic medium, the ratio of magnetic flux density to magnetising force is a constant, i.e. $B/H =$ a constant. This constant is μ_0, the **permeability of free space** (or the magnetic space constant) and is equal to $4\pi \times 10^{-7}$ H/m, i.e. **for air, or any non-magnetic medium**, the ratio

$$\frac{B}{H} = \mu_0$$

(Although all non-magnetic materials, including air, exhibit slight magnetic properties, these can effectively be neglected.)

For all media other than free space,

$$\frac{B}{H} = \mu_0 \mu_r$$

where μ_r is the relative permeability, and is defined as

$$\mu_r = \frac{\textbf{flux density in material}}{\textbf{flux density in a vacuum}}$$

μ_r varies with the type of magnetic material and, since it is a ratio of flux densities, it has no unit. From its definition, μ_r for a vacuum is 1.

$\mu_0 \mu_r = \mu$, called the **absolute permeability**

By plotting measured values of flux density B against magnetic field strength H, a **magnetisation curve** (or **B–H curve**) is produced. For non-magnetic materials this is a straight line. Typical curves for four magnetic materials are shown in Fig. 7.3
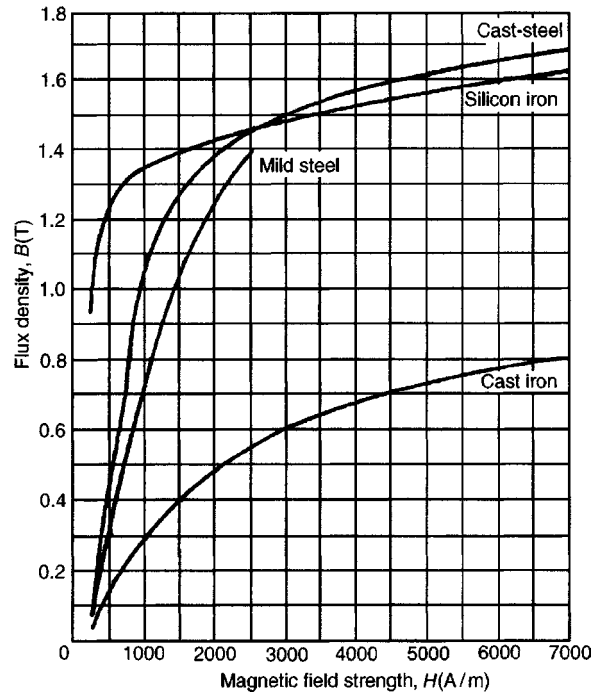

Figure 7.3

The **relative permeability** of a ferromagnetic material is proportional to the slope of the B–H curve and thus varies with the magnetic field strength. The approximate range of values of relative permeability μ_r for some common magnetic materials are:

Cast iron $\mu_r = 100$–250
Mild steel $\mu_r = 200$–800
Silicon iron $\mu_r = 1000$–5000
Cast steel $\mu_r = 300$–900
Mumetal $\mu_r = 200$–5000
Stalloy $\mu_r = 500$–6000

Problem 4. A flux density of 1.2 T is produced in a piece of cast steel by a magnetising force of 1250 A/m. Find the relative permeability of the steel under these conditions.

For a magnetic material: $B = \mu_0 \mu_r H$

i.e. $\quad \mu_r = \dfrac{B}{\mu_0 H} = \dfrac{1.2}{(4\pi \times 10^{-7})(1250)} = \textbf{764}$

Problem 5. Determine the magnetic field strength and the m.m.f. required to produce a flux density of 0.25 T in an air gap of length 12 mm.

For air: $B = \mu_0 H$ (since $\mu_r = 1$)

Magnetic field strength,

$$H = \frac{B}{\mu_0} = \frac{0.25}{4\pi \times 10^{-7}} = \textbf{198\,940\,A/m}$$

m.m.f. $= Hl = 198\,940 \times 12 \times 10^{-3} = \textbf{2387\,A}$

Problem 6. A coil of 300 turns is wound uniformly on a ring of non-magnetic material. The ring has a mean circumference of 40 cm and a uniform cross-sectional area of 4 cm^2. If the current in the coil is 5 A, calculate (a) the magnetic field strength, (b) the flux density and (c) the total magnetic flux in the ring.

(a) Magnetic field strength

$$H = \frac{NI}{l} = \frac{300 \times 5}{40 \times 10^{-2}}$$

$$= \textbf{3750\,A/m}$$

(b) For a non-magnetic material $\mu_r = 1$, thus flux density $B = \mu_0 H$

i.e $\quad B = 4\pi \times 10^{-7} \times 3750$

$$= \textbf{4.712\,mT}$$

(c) Flux $\Phi = BA = (4.712 \times 10^{-3})(4 \times 10^{-4})$

$$= \textbf{1.885\,\mu Wb}$$

Problem 7. An iron ring of mean diameter 10 cm is uniformly wound with 2000 turns of wire. When a current of 0.25 A is passed through the coil a flux density of 0.4 T is set up in the iron. Find (a) the magnetising force and (b) the relative permeability of the iron under these conditions.

$l = \pi d = \pi \times 10\,\text{cm} = \pi \times 10 \times 10^{-2}\,\text{m}$,
$N = 2000$ turns, $I = 0.25$ A and $B = 0.4$ T

(a) $\quad H = \dfrac{NI}{l} = \dfrac{2000 \times 0.25}{\pi \times 10 \times 10^{-2}}$

$$= \textbf{1592\,A/m}$$

(b) $\quad B = \mu_0 \mu_r H$, hence

$$\mu_r = \frac{B}{\mu_0 H} = \frac{0.4}{(4\pi \times 10^{-7})(1592)} = \textbf{200}$$

Problem 8. A uniform ring of cast iron has a cross-sectional area of 10 cm^2 and a mean circumference of 20 cm. Determine the m.m.f. necessary to produce a flux of 0.3 mWb in the ring. The magnetisation curve for cast iron is shown on page 74.

$A = 10\,\text{cm}^2 = 10 \times 10^{-4}\,\text{m}^2$, $\quad l = 20\,\text{cm} = 0.2\,\text{m}$ and
$\Phi = 0.3 \times 10^{-3}$ Wb.

$$\text{Flux density } B = \frac{\Phi}{A} = \frac{0.3 \times 10^{-3}}{10 \times 10^{-4}} = 0.3\,\text{T}$$

From the magnetisation curve for cast iron on page 74, when $B = 0.3$ T, $H = 1000$ A/m, hence **m.m.f.** $= Hl = 1000 \times 0.2 = \textbf{200\,A}$

A tabular method could have been used in this problem. Such a solution is shown below in Table 7.1.

Problem 9. From the magnetisation curve for cast iron, shown on page 74, derive the curve of μ_r against H.

$B = \mu_0 \mu_r H$, hence

$$\mu_r = \frac{B}{\mu_0 H} = \frac{1}{\mu_0} \times \frac{B}{H}$$

$$= \frac{10^7}{4\pi} \times \frac{B}{H}$$

A number of co-ordinates are selected from the B–H curve and μ_r is calculated for each as shown in Table 7.2.

μ_r is plotted against H as shown in Fig. 7.4. The curve demonstrates the change that occurs in the relative permeability as the magnetising force increases.

Table 7.1

Part of circuit	Material	Φ(Wb)	$A(m^2)$	$B = \dfrac{\Phi}{A}(T)$	H from graph	$l(m)$	m.m.f. $= Hl(A)$
Ring	Cast iron	0.3×10^{-3}	10×10^{-4}	0.3	1000	0.2	200

Table 7.2

$B(T)$		0.04	0.13	0.17	0.30	0.41	0.49	0.60	0.68	0.73	0.76	0.79
$H(A/m)$		200	400	500	1000	1500	2000	3000	4000	5000	6000	7000
$\mu_r = \dfrac{10^7}{4\pi} \times \dfrac{B}{H}$		159	259	271	239	218	195	159	135	116	101	90
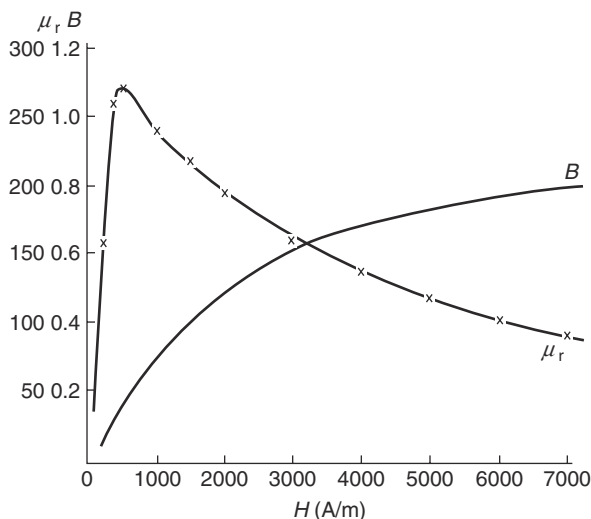
Figure 7.4

Now try the following exercise

Exercise 33   Further problems on magnetic circuits

(Where appropriate, assume $\mu_0 = 4\pi \times 10^{-7}$ H/m)

1. Find the magnetic field strength and the magnetomotive force needed to produce a flux density of 0.33 T in an air-gap of length 15 mm.
    [(a) 262 600 A/m (b) 3939 A]

2. An air-gap between two pole pieces is 20 mm in length and the area of the flux path across the gap is 5 cm². If the flux required in the air-gap is 0.75 mWb find the m.m.f. necessary.
    [23 870 A]

3. (a) Determine the flux density produced in an air-cored solenoid due to a uniform magnetic field strength of 8000 A/m (b) Iron having a relative permeability of 150 at 8000 A/m is inserted into the solenoid of part (a). Find the flux density now in the solenoid.
    [(a) 10.05 mT (b) 1.508 T]

4. Find the relative permeability of a material if the absolute permeability is 4.084×10^{-4} H/m.   [325]

5. Find the relative permeability of a piece of silicon iron if a flux density of 1.3 T is produced by a magnetic field strength of 700 A/m.
    [1478]

6. A steel ring of mean diameter 120 mm is uniformly wound with 1500 turns of wire. When a current of 0.30 A is passed through the coil a flux density of 1.5 T is set up in the steel. Find the relative permeability of the steel under these conditions.   [1000]

7. A uniform ring of cast steel has a cross-sectional area of 5 cm² and a mean circumference of 15 cm. Find the current required in a coil of 1200 turns wound on the ring to produce a flux of 0.8 mWb. (Use the magnetisation curve for cast steel shown on page 74)
    [0.60 A]

8. (a) A uniform mild steel ring has a diameter of 50 mm and a cross-sectional area of 1 cm². Determine the m.m.f. necessary to produce a flux of 50 μWb in the ring. (Use the B–H curve for mild steel shown on page 74) (b) If a coil of 440 turns is wound uniformly around the ring in Part (a) what current would be required to produce the flux?   [(a) 110 A (b) 0.25 A]

9. From the magnetisation curve for mild steel shown on page 74, derive the curve of relative permeability against magnetic field strength. From your graph determine (a) the value of μ_r when the magnetic field strength is 1200 A/m, and (b) the value of the magnetic field strength when μ_r is 500.   [(a) 590–600 (b) 2000]

7.6 Reluctance

Reluctance S (or R_M) is the 'magnetic resistance' of a magnetic circuit to the presence of magnetic flux. **Reluctance**,

$$S = \frac{F_M}{\Phi} = \frac{NI}{\Phi} = \frac{Hl}{BA} = \frac{l}{(B/H)A} = \frac{l}{\mu_0 \mu_r A}$$

The unit of reluctance is $1/H$ (or H^{-1}) or A/Wb.
Ferromagnetic materials have a low reluctance and can be used as **magnetic screens** to prevent magnetic fields affecting materials within the screen.

Problem 10. Determine the reluctance of a piece of mumetal of length 150 mm and cross-sectional area 1800 mm² when the relative permeability is 4000. Find also the absolute permeability of the mumetal.

Reluctance,

$$S = \frac{l}{\mu_0 \mu_r A}$$

$$= \frac{150 \times 10^{-3}}{(4\pi \times 10^{-7})(4000)(1800 \times 10^{-6})}$$

$$= \mathbf{16\,580/H}$$

Absolute permeability,

$$\mu = \mu_0 \mu_r = (4\pi \times 10^{-7})(4000)$$

$$= \mathbf{5.027 \times 10^{-3}\,H/m}$$

Problem 11. A mild steel ring has a radius of 50 mm and a cross-sectional area of 400 mm². A current of 0.5 A flows in a coil wound uniformly around the ring and the flux produced is 0.1 mWb. If the relative permeability at this value of current is 200 find (a) the reluctance of the mild steel and (b) the number of turns on the coil.

$l = 2\pi r = 2 \times \pi \times 50 \times 10^{-3}$ m, $\quad A = 400 \times 10^{-6}$ m²,
$I = 0.5$ A, $\Phi = 0.1 \times 10^{-3}$ Wb and $\mu_r = 200$

(a) **Reluctance**,

$$S = \frac{l}{\mu_0 \mu_r A}$$

$$= \frac{2 \times \pi \times 50 \times 10^{-3}}{(4\pi \times 10^{-7})(200)(400 \times 10^{-6})}$$

$$= \mathbf{3.125 \times 10^6/H}$$

(b) $S = \dfrac{\text{m.m.f.}}{\Phi}$ from which

m.m.f. $= S\Phi \quad$ i.e. $\quad NI = S\Phi$

Hence, number of terms

$$N = \frac{S\Phi}{I} = \frac{3.125 \times 10^6 \times 0.1 \times 10^{-3}}{0.5}$$

$$= \mathbf{625\,turns}$$

Now try the following exercise

7.7 Composite series magnetic circuits

For a series magnetic circuit having n parts, the **total reluctance** S is given by: $S = S_1 + S_2 + \cdots + S_n$ (This is similar to resistors connected in series in an electrical circuit).

Problem 12. A closed magnetic circuit of cast steel contains a 6 cm long path of cross-sectional area 1 cm² and a 2 cm path of cross-sectional area 0.5 cm². A coil of 200 turns is wound around the 6 cm length of the circuit and a current of 0.4 A flows. Determine the flux density in the 2 cm path, if the relative permeability of the cast steel is 750.

For the 6 cm long path:

Reluctance $S_1 = \dfrac{l_1}{\mu_0 \mu_r A_1}$

$$= \frac{6 \times 10^{-2}}{(4\pi \times 10^{-7})(750)(1 \times 10^{-4})}$$

$$= 6.366 \times 10^5 /\text{H}$$

For the 2 cm long path:

Reluctance $S_2 = \dfrac{l_2}{\mu_0 \mu_r A_2}$

$$= \frac{2 \times 10^{-2}}{(4\pi \times 10^{-7})(750)(0.5 \times 10^{-4})}$$

$$= 4.244 \times 10^5 /\text{H}$$

Total circuit reluctance $S = S_1 + S_2$

$$= (6.366 + 4.244) \times 10^5$$

$$= 10.61 \times 10^5 /\text{H}$$

$S = \dfrac{\text{m.m.f.}}{\Phi}$ i.e. $\Phi = \dfrac{\text{m.m.f.}}{S} = \dfrac{NI}{S}$

$$= \frac{200 \times 0.4}{10.61 \times 10^5} = 7.54 \times 10^{-5} \text{ Wb}$$

Flux density in the 2 cm path,

$$B = \frac{\Phi}{A} = \frac{7.54 \times 10^{-5}}{0.5 \times 10^{-4}} = \textbf{1.51 T}$$

> **Problem 13.** A silicon iron ring of cross-sectional area 5 cm^2 has a radial air gap of 2 mm cut into it. If the mean length of the silicon iron path is 40 cm calculate the magnetomotive force to produce a flux of 0.7 mWb. The magnetisation curve for silicon is shown on page 74.

There are two parts to the circuit — the silicon iron and the air gap. The total m.m.f. will be the sum of the m.m.f.'s of each part.

For the silicon iron:

$$B = \frac{\Phi}{A} = \frac{0.7 \times 10^{-3}}{5 \times 10^{-4}} = 1.4 \text{ T}$$

From the B–H curve for silicon iron on page 74, when $B = 1.4$ T, $H = 1650$ A/m. Hence the m.m.f. for the iron path $= Hl = 1650 \times 0.4 = 660$ A

For the air gap:

The flux density will be the same in the air gap as in the iron, i.e. 1.4 T (This assumes no leakage or fringing occurring). For air,

$$H = \frac{B}{\mu_0} = \frac{1.4}{4\pi \times 10^{-7}} = 1\,114\,000 \text{ A/m}$$

Hence the m.m.f. for the air gap
$= Hl = 1\,114\,000 \times 2 \times 10^{-3} = 2228$ A.
Total m.m.f. to produce a flux of 0.6 mWb
$= 660 + 2228 = \textbf{2888 A}$.

A tabular method could have been used as shown in Table 7.3 at top of next page.

> **Problem 14.** Figure 7.5 shows a ring formed with two different materials — cast steel and mild steel.
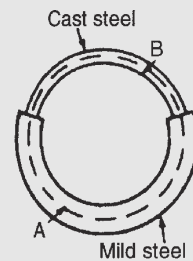
Figure 7.5

The dimensions are:

	mean length	cross-sectional area
Mild steel	400 mm	500 mm^2
Cast steel	300 mm	312.5 mm^2

Find the total m.m.f. required to cause a flux of 500 μWb in the magnetic circuit. Determine also the total circuit reluctance.

Table 7.3

Part of circuit	Material	Φ(Wb)	A(m^2)	B(T)	H(A/m)	l(m)	m.m.f. = Hl(A)
Ring	Silicon iorn	0.7×10^{-3}	5×10^{-4}	1.4	1650 (from graph)	0.4	660
Air-gap	Air	0.7×10^{-3}	5×10^{-4}	1.4	$\dfrac{1.4}{4\pi \times 10^{-7}}$ $= 1\,114\,000$	2×10^{-3}	2228
						Total:	**2888 A**

Table 7.4

Part of circuit	Material	Φ(Wb)	A(m^2)	B(T) ($=\Phi/A$)	H(A/m) (from graphs page 74)	l(m)	m.m.f. $= Hl$(A)
A	Mild steel	500×10^{-6}	500×10^{-6}	1.0	1400	400×10^{-3}	560
B	Cast steel	500×10^{-6}	312.5×10^{-6}	1.6	4800	300×10^{-3}	1440
						Total:	**2000 A**

A tabular solution is shown in Table 7.4 above.

$$\left.\begin{array}{l}\textbf{Total circuit}\\\textbf{reluctance}\end{array}\right\} S = \frac{\text{m.m.f.}}{\Phi}$$

$$= \frac{2000}{500 \times 10^{-6}} = \mathbf{4 \times 10^6 /H}$$

Problem 15. A section through a magnetic circuit of uniform cross-sectional area 2 cm^2 is shown in Fig. 7.6. The cast steel core has a mean length of 25 cm. The air gap is 1 mm wide and the coil has 5000 turns. The B–H curve for cast steel is shown on page 74. Determine the current in the coil to produce a flux density of 0.80 T in the air gap, assuming that all the flux passes through both parts of the magnetic circuit.
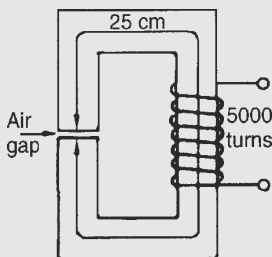

25 cm

Air gap

5000 turns

Figure 7.6

For the cast steel core, when $B = 0.80$ T, $H = 750$ A/m (from page 74).

Reluctance of core $S_1 = \dfrac{l_1}{\mu_0 \mu_r A_1}$ and

since $B = \mu_0 \, \mu_r \, H$, then $\mu_r = \dfrac{B}{\mu_0 H}$.

$$S_1 = \frac{l_1}{\mu_0 \left(\dfrac{B}{\mu_0 H}\right) A_1} = \frac{l_1 H}{B A_1}$$

$$= \frac{(25 \times 10^{-2})(750)}{(0.8)(2 \times 10^{-4})} = 1\,172\,000/\text{H}$$

For the air gap:

$$\text{Reluctance, } S_2 = \frac{l_2}{\mu_0 \mu_r A_2}$$

$$= \frac{l_2}{\mu_0 A_2} \quad (\text{since } \mu_r = 1 \text{ for air})$$

$$= \frac{1 \times 10^{-3}}{(4\pi \times 10^{-7})(2 \times 10^{-4})}$$

$$= 3\,979\,000/\text{H}$$

Total circuit reluctance

$$S = S_1 + S_2 = 1\,172\,000 + 3\,979\,000$$

$$= 5\,151\,000/\text{H}$$

Flux $\Phi = BA = 0.80 \times 2 \times 10^{-4} = 1.6 \times 10^{-4}$ Wb

$$S = \frac{m.m.f.}{\Phi}$$

thus

$$m.m.f. = S\Phi \text{ hence } NI = S\Phi$$

and

$$\textbf{current } I = \frac{S\Phi}{N} = \frac{(5\,151\,000)(1.6 \times 10^{-4})}{5000}$$

$$= \textbf{0.165\,A}$$

Now try the following exercise

Exercise 35 Further problems on composite series magnetic circuits

1. A magnetic circuit of cross-sectional area $0.4\,cm^2$ consists of one part $3\,cm$ long, of material having relative permeability 1200, and a second part $2\,cm$ long of material having relative permeability 750. With a 100 turn coil carrying $2\,A$, find the value of flux existing in the circuit. [0.195 mWb]

2. (a) A cast steel ring has a cross-sectional area of $600\,mm^2$ and a radius of $25\,mm$. Determine the m.m.f. necessary to establish a flux of $0.8\,mWb$ in the ring. Use the B–H curve for cast steel shown on page 74. (b) If a radial air gap $1.5\,mm$ wide is cut in the ring of part (a) find the m.m.f. now necessary to maintain the same flux in the ring.

   [(a) 270 A (b)1860 A]

3. A closed magnetic circuit made of silicon iron consists of a $40\,mm$ long path of cross-sectional area $90\,mm^2$ and a $15\,mm$ long path of cross-sectional area $70\,mm^2$. A coil of 50 turns is wound around the $40\,mm$ length of the circuit and a current of $0.39\,A$ flows. Find the flux density in the $15\,mm$ length path if the relative permeability of the silicon iron at this value of magnetising force is 3000.

   [1.59 T]

4. For the magnetic circuit shown in Fig. 7.7 find the current I in the coil needed to produce a flux of $0.45\,mWb$ in the air-gap. The silicon iron magnetic circuit has a uniform cross-sectional area of $3\,cm^2$ and its magnetisation curve is as shown on page 74. [0.83 A]
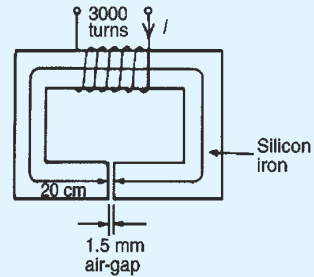

Figure 7.7

5. A ring forming a magnetic circuit is made from two materials; one part is mild steel of mean length $25\,cm$ and cross-sectional area $4\,cm^2$, and the remainder is cast iron of mean length $20\,cm$ and cross-sectional area $7.5\,cm^2$. Use a tabular approach to determine the total m.m.f. required to cause a flux of $0.30\,mWb$ in the magnetic circuit. Find also the total reluctance of the circuit. Use the magnetisation curves shown on page 74.

   [550 A, 1.83×10^6/H]

6. Figure 7.8 shows the magnetic circuit of a relay. When each of the air gaps are $1.5\,mm$ wide find the m.m.f. required to produce a flux density of $0.75\,T$ in the air gaps. Use the B–H curves shown on page 74. [2970 A]
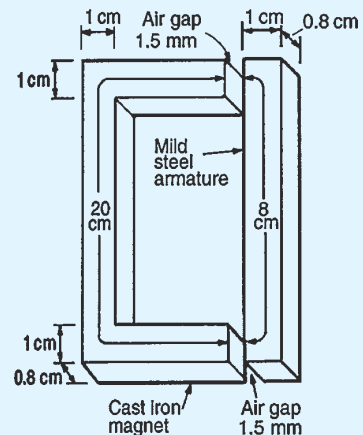

Figure 7.8

7.8 Comparison between electrical and magnetic quantities

Electrical circuit	Magnetic circuit
e.m.f. E (V)	m.m.f. F_m (A)
current I (A)	flux Φ (Wb)
resistance R (Ω)	reluctance S (H^{-1})
$I = \dfrac{E}{R}$	$\Phi = \dfrac{\text{m.m.f.}}{S}$
$R = \dfrac{\rho l}{A}$	$S = \dfrac{l}{\mu_0 \mu_r A}$

7.9 Hysteresis and hysteresis loss

Hysteresis loop

Let a ferromagnetic material which is completely demagnetised, i.e. one in which $B = H = 0$ be subjected to increasing values of magnetic field strength H and the corresponding flux density B measured. The resulting relationship between B and H is shown by the curve Oab in Fig. 7.9. At a particular value of H, shown as Oy, it becomes difficult to increase the flux density any further. The material is said to be saturated. Thus **by** is the **saturation flux density**.
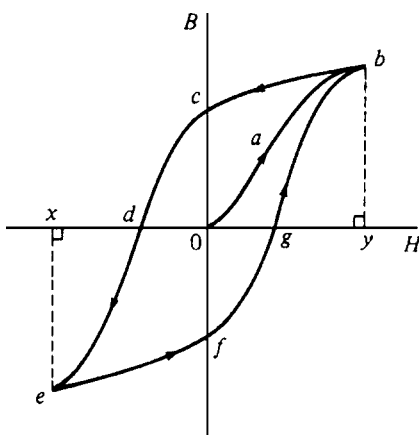

Figure 7.9

If the value of H is now reduced it is found that the flux density follows curve **bc**. When H is reduced to zero, flux remains in the iron. This **remanent flux density** or **remanence** is shown as **Oc** in Fig. 7.9. When H is increased in the opposite direction, the flux density decreases until, at a value shown as **Od**, the flux density has been reduced to zero. The magnetic field strength **Od** required to remove the residual magnetism, i.e. reduce B to zero, is called the **coercive force**.

Further increase of H in the reverse direction causes the flux density to increase in the reverse direction until saturation is reached, as shown by curve **de**. If H is varied backwards from **Ox** to **Oy**, the flux density follows the curve **efgb**, similar to curve **bcde**.

It is seen from Fig. 7.9 that the flux density changes lag behind the changes in the magnetic field strength. This effect is called **hysteresis.** The closed figure **bcdefgb** is called the **hysteresis loop** (or the B/H loop).

Hysteresis loss

A disturbance in the alignment of the domains (i.e. groups of atoms) of a ferromagnetic material causes energy to be expended in taking it through a cycle of magnetisation. This energy appears as heat in the specimen and is called the **hysteresis loss**.

The energy loss associated with hysteresis is proportional to the area of the hysteresis loop.

The area of a hysteresis loop varies with the type of material. The area, and thus the energy loss, is much greater for hard materials than for soft materials.

Figure 7.10 shows typical hysteresis loops for:

(a) **hard material**, which has a high remanence Oc and a large coercivity **Od**

(b) **soft steel**, which has a large remanence and small coercivity

(c) **ferrite**, this being a ceramic-like magnetic substance made from oxides of iron, nickel, cobalt, magnesium, aluminium and mangenese; the hysteresis of ferrite is very small.

For a.c.-excited devices the hysteresis loop is repeated every cycle of alternating current. Thus a hysteresis loop with a large area (as with hard steel) is often unsuitable since the energy loss would be considerable. Silicon steel has a narrow hysteresis loop, and thus small hysteresis loss, and is suitable for transformer cores and rotating machine armatures.
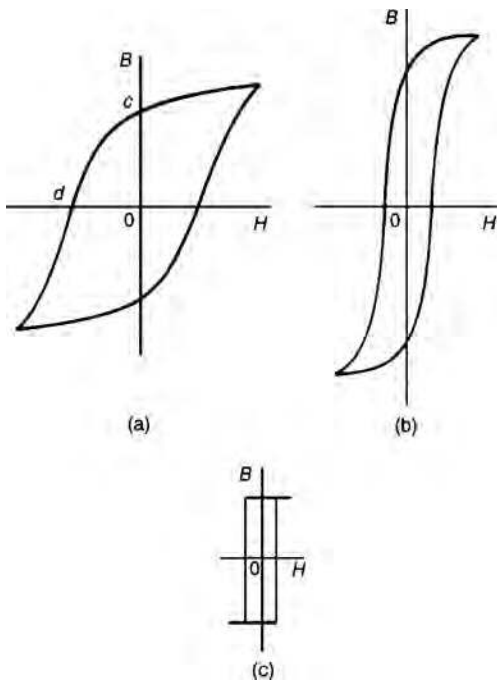
Figure 7.10

Now try the following exercise

Exercise 36   Short answer questions on magnetic circuits

1. State six practical applications of magnets

2. What is a permanent magnet?

3. Sketch the pattern of the magnetic field associated with a bar magnet. Mark the direction of the field.

4. Define magnetic flux

5. The symbol for magnetic flux is . . . and the unit of flux is the . . .

6. Define magnetic flux density

7. The symbol for magnetic flux density is . . . and the unit of flux density is . . .

8. The symbol for m.m.f. is . . . and the unit of m.m.f. is the . . .

9. Another name for the magnetising force is . . . . . . ; its symbol is . . . and its unit is . . .

10. Complete the statement:
$$\frac{\text{flux density}}{\text{magnetic field strength}} = \ldots$$

11. What is absolute permeability?

12. The value of the permeability of free space is . . .

13. What is a magnetisation curve?

14. The symbol for reluctance is . . . and the unit of reluctance is . . .

15. Make a comparison between magnetic and electrical quantities

16. What is hysteresis?

17. Draw a typical hysteresis loop and on it identify:
    (a) saturation flux density
    (b) remanence
    (c) coercive force

18. State the units of (a) remanence (b) coercive force

19. How is magnetic screening achieved?

20. Complete the statement: magnetic materials have a . . . reluctance; non-magnetic materials have a . . . . reluctance

21. What loss is associated with hysteresis?

Now try the following exercise

Exercise 37   Multi-choice questions on magnetic circuits
(Answers on page 398)

1. The unit of magnetic flux density is the:
   (a) weber   (b) weber per metre
   (c) ampere per metre   (d) tesla

2. The total flux in the core of an electrical machine is 20 mWb and its flux density is 1 T. The cross-sectional area of the core is:
   (a) $0.05\,\text{m}^2$ (b) $0.02\,\text{m}^2$
   (c) $20\,\text{m}^2$ (d) $50\,\text{m}^2$

3. If the total flux in a magnetic circuit is 2 mWb and the cross-sectional area of the circuit is $10\,\text{cm}^2$, the flux density is:
   (a) 0.2 T   (b) 2 T
   (c) 20 T   (d) 20 mT

   Questions 4 to 8 refer to the following data:
   A coil of 100 turns is wound uniformly on a wooden ring. The ring has a mean circumference of 1 m and a uniform cross-sectional area of $10\,\text{cm}^2$. The current in the coil is 1 A.

4. The magnetomotive force is:
   (a) 1 A   (b) 10 A
   (c) 100 A   (d) 1000 A

5. The magnetic field strength is:
   (a) 1 A/m   (b) 10 A/m
   (c) 100 A/m   (d) 1000 A/m

6 The magnetic flux density is:
   (a) 800 T   (b) 8.85×10^{-10} T
   (c) $4\pi \times 10^{-7}$ T   (d) $40\pi\,\mu$T

7. The magnetic flux is:
   (a) $0.04\pi\,\mu$Wb   (b) 0.01 Wb
   (c) $8.85\,\mu$Wb   (d) $4\pi\,\mu$Wb

8. The reluctance is:
   (a) $\dfrac{10^8}{4\pi}\,\text{H}^{-1}$ (b) $1000\,\text{H}^{-1}$
   (c) $\dfrac{2.5}{\pi} \times 10^9\,\text{H}^{-1}$ (d) $\dfrac{10^8}{8.85}\,\text{H}^{-1}$

9. Which of the following statements is false?
   (a) For non-magnetic materials reluctance is high
   (b) Energy loss due to hysteresis is greater for harder magnetic materials than for softer magnetic materials
   (c) The remanence of a ferrous material is measured in ampere/metre
   (d) Absolute permeability is measured in henrys per metre

10. The current flowing in a 500 turn coil wound on an iron ring is 4 A. The reluctance of the circuit is 2×10^6 H. The flux produced is:
    (a) 1 Wb   (b) 1000 Wb
    (c) 1 mWb   (d) $62.5\,\mu$Wb

11. A comparison can be made between magnetic and electrical quantities. From the following list, match the magnetic quantities with their equivalent electrical quantities.
    (a) current   (b) reluctance
    (c) e.m.f.   (d) flux
    (e) m.m.f.   (f) resistance

12. The effect of an air gap in a magnetic circuit is to:
    (a) increase the reluctance
    (b) reduce the flux density
    (c) divide the flux
    (d) reduce the magnetomotive force

13. Two bar magnets are placed parallel to each other and about 2 cm apart, such that the south pole of one magnet is adjacent to the north pole of the other. With this arrangement, the magnets will:
    (a) attract each other
    (b) have no effect on each other
    (c) repel each other
    (d) lose their magnetism

Revision test 2

This revision test covers the material contained in Chapters 5 to 7. *The marks for each question are shown in brackets at the end of each question.*

1. Resistances of 5 Ω, 7 Ω, and 8 Ω are connected in series. If a 10 V supply voltage is connected across the arrangement determine the current flowing through and the p.d. across the 7 Ω resistor. Calculate also the power dissipated in the 8 Ω resistor. (6)

2. For the series-parallel network shown in Fig. R2.1, find (a) the supply current, (b) the current flowing through each resistor, (c) the p.d. across each resistor, (d) the total power dissipated in the circuit, (e) the cost of energy if the circuit is connected for 80 hours. Assume electrical energy costs 14p per unit. (15)

3. The charge on the plates of a capacitor is 8 mC when the potential between them is 4 kV. Determine the capacitance of the capacitor. (2)

4. Two parallel rectangular plates measuring 80 mm by 120 mm are separated by 4 mm of mica and carry an electric charge of 0.48 μC. The voltage between the plates is 500 V. Calculate (a) the electric flux density (b) the electric field strength, and (c) the

capacitance of the capacitor, in picofarads, if the relative permittivity of mica is 5. (7)

5. A 4 μF capacitor is connected in parallel with a 6 μF capacitor. This arrangement is then connected in series with a 10 μF capacitor. A supply p.d. of 250 V is connected across the circuit. Find (a) the equivalent capacitance of the circuit, (b) the voltage across the 10 μF capacitor, and (c) the charge on each capacitor. (7)

6. A coil of 600 turns is wound uniformly on a ring of non-magnetic material. The ring has a uniform cross-sectional area of 200 mm^2 and a mean circumference of 500 mm. If the current in the coil is 4 A, determine (a) the magnetic field strength, (b) the flux density, and (c) the total magnetic flux in the ring. (5)

7. A mild steel ring of cross-sectional area 4 cm^2 has a radial air-gap of 3 mm cut into it. If the mean length of the mild steel path is 300 mm, calculate the magnetomotive force to produce a flux of 0.48 mWb. (Use the B–H curve on page 74) (8)
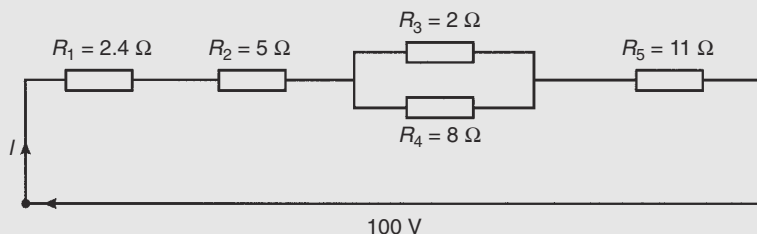

$R_1 = 2.4$ Ω $\quad R_2 = 5$ Ω $\quad R_3 = 2$ Ω $\quad R_5 = 11$ Ω

$R_4 = 8$ Ω

100 V

Figure R2.1