Problems from Engineering Mathematics 1 - Veerarajan.

1.

1. Verify that $\int_1^2 \int_0^1 (x^2+y^2)\,dx\,dy = \int_0^1 \int_1^2 (x^2+y^2)\,dy\,dx$.

2. Evaluate $\int_0^1 \int_x^{\sqrt{x}} xy(x+y)\,dx\,dy$.

3. Evaluate $\iint_R xy\,dx\,dy$, where R is the region bounded by the line $x+2y=2$, lying in the first quadrant.

4. Evaluate $\iint_R \frac{e^{-y}}{y}\,dx\,dy$ where R is the region bounded by the lines $x=0$, $x=y$ and $y=\infty$.
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5 Evaluate $\iint_R xy\,dx\,dy$, where R is the region bounded by the parabola $y^2=x$ and the lines $y=0$, $x+y=2$ lying in the first quadrant.
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Part A.

1. Evaluate $\int_0^2 \int_0^1 4xy\,dx\,dy$

2. $\int_1^a \int_1^b \frac{dx\,dy}{xy}$

3. $\int_0^{\pi/2} \int_0^{\pi/2} \sin(\theta+\phi)\,d\theta\,d\phi$.

4. $\int_0^1 \int_0^x dx\,dy$.

5. $\int_0^{\pi} \int_0^{\sin\theta} r\,dr\,d\theta$.

6. f

II Sketch the region of integration for the following double integrals.

1. $\int_{-b}^{b} \int_{-a}^{a} f(x,y)\,dx\,dy$

2. $\int_0^1 \int_0^x f(x,y)\,dx\,dy$.

3. $\int_0^a \int_0^{\sqrt{a^2-x^2}} f(x,y)\,dx\,dy$

4 $\int_0^b \int_0^{\frac{a}{b}(b-y)} f(x,y)\,dx\,dy$.

III Find the limits of integration in the double integral $\iint_R f(x,y)\,dx\,dy$ where R is in the I quadrant and bounded by

1. $x=0,\ y=0,\ x+y=1.$
2. $x=0,\ y=0,\ \frac{x^2}{a^2}+\frac{y^2}{b^2}=1.$
3. $x=0,\ x=y,\ y=1.$
4. $x=1,\ y=0,\ y^2=4x$

IV 1. Evaluate $\int_0^4 \int_{y^2/4}^{y} \frac{y\,dx\,dy}{x^2+y^2}$ and sketch the region of integration.

2. Evaluate $\int_0^a \int_{a-x}^{\sqrt{a^2-x^2}} y\,dx\,dy$ and sketch the region of integration.

3. Evaluate $\int_0^1 \int_x^1 \frac{y\,dx\,dy}{x^2+y^2}$ and sketch the region of integration.

V

1. Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{a^2-x^2-y^2}\,dx\,dy.$

I. Change of order of integration

1. Change the order of integration in $\int_0^a \int_x^a f(x,y)\,dy\,dx$.

2. Change the order of integration in $\int_0^a \int_y^a f(x,y)\,dx\,dy$.

3. Change the order of integration $\int_0^1 \int_0^x f(x,y)\,dy\,dx$.

II. Change the order of integration and hence evaluate

1) $\int_{-a}^{a} \int_0^{\sqrt{a^2-y^2}} x\,dx\,dy$. [Ans.: $\frac{2}{3}a^3$]

2) $\int_0^a \int_{a-\sqrt{a^2-y^2}}^{a+\sqrt{a^2-y^2}} dx\,dy$. [Ans. $\frac{\pi a^2}{2}$].

3) $\int_0^a \int_{a-\sqrt{a^2-y^2}}^{a+\sqrt{a^2-y^2}} xy\,dx\,dy$. [Ans. $\frac{2}{3}a^4$]

4) $\int_0^1 \int_{x^2}^{2-x} xy\,dy\,dx$. [Ans.: $\frac{3}{8}$].

5) $\int_0^3 \int_1^{\sqrt{4-y}} (x+y)\,dx\,dy$. [Ans.: $\frac{241}{60}$].

6) $\int_0^a \int_y^a \frac{x}{x^2+y^2}\,dx\,dy$. [Ans.: $\frac{\pi a}{4}$].

7) $\int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} xy\,dy\,dx$. [Ans.: $\frac{64}{3}a^4$]

8) $\int_0^{\infty} \int_0^{y} y\,e^{-y^2/x}\,dx\,dy$. [Ans. $\frac{1}{2}$].

9) $\int_0^1 \int_y^{2-y} xy\,dx\,dy$. [Ans.: $\frac{1}{3}$]

10) $\int_0^b \int_0^{\frac{a}{b}\sqrt{b^2-y^2}} xy\,dx\,dy$. [Ans.: $\frac{a^2b^2}{8}$]

11). Evaluate by changing the order of integration.

$$\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{a^2-x^2-y^2}\, dy\, dx. \qquad \left\{ \text{Ans.: } \frac{\pi a^3}{6} \right\}$$

12). Change the order of integration and hence evaluate $\int_0^1 \int_x^{2-x} \frac{x}{y}\, dy\, dx$. $\{ \text{Ans.: } \log 4 - 1 \}$.

13) Change the order of integration in

$$\int_0^a \int_{x^2/a}^{2a-x} xy\, dx\, dy$$ and hence evaluate. $\left\{ \text{Ans.: } \frac{3}{8} a^4 \right\}$.

14). Change the order of integration and hence evaluate

$$\int_1^3 \int_0^{6/x} x^2\, dy\, dx. \qquad \{ \text{Ans.: } 24 \}.$$

15) Change the order of integration and then evaluate $\int_0^a \int_{a-y}^{\sqrt{a^2-y^2}} y\, dx\, dy$ $\left\{ \text{Ans. } \frac{a^3}{6} \right\}$.

16) $$\int_0^b \int_0^{\frac{a}{b}(b-y)} xy\, dx\, dy.$$

$\left\{ \text{Ans. } \frac{a^2 b^2}{24} \right\}$

17) $$\int_0^a \int_y^a \frac{x}{\sqrt{x^2+y^2}}\, dx\, dy$$

$\left\{ \text{Ans. } \frac{a^2}{2} \log(1+\sqrt{2}) \right\}$

18) $$\int_0^1 \int_x^1 \frac{x}{x^2+y^2}\, dx\, dy$$

$\left\{ \text{Ans.: } \frac{1}{2} \log 2 \right\}$.

19) $$\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2+y^2}}\, dy\, dx.$$

$\left\{ \text{Ans.: } \frac{2-\sqrt{2}}{2} \right\}$.

20. $$\int_0^a \int_0^{\frac{b}{a}\sqrt{a^2-x^2}} x^2\, dy\, dx.$$

$\left\{ \text{Ans.: } \frac{\pi}{16} a^3 b \right\}$

21). $$\int_0^a \int_0^x \frac{\cos y}{\sqrt{(a-x)(a-y)}}\, dx\, dy.$$

$\{ \text{Ans.: } 2 \sin a \}$.

22) $\int_1^2 \int_0^{4-x^2} (x+y)\, dy\, dx$

$\left\{ \text{Ans.: } \frac{241}{60} \right\}$

23) $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2\, dx\, dy$ $\left\{ \text{Ans. } \frac{\pi}{16} \right\}$

~~24)~~ $\int_1^2 \int_0^{4-x^2} \cancel{(x+y)}\, dy\, dx$

{ ~~Ans.:~~

25) $\int_0^a \int_{x/a}^{\sqrt{x/a}} (x^2+y^2)\, dx\, dy$

$\left\{ \text{Ans. } \frac{a^2}{28} + \frac{a}{20} \right\}$

24 $\int_0^{a/\sqrt{2}} \int_y^{\sqrt{a^2-y^2}} \log(x^2+y^2)\, dx\, dy$ $(a>0)$.

$\left\{ \text{Ans.: } \frac{\pi a^2}{2}\left(\log a - \frac{1}{2}\right) \right\}$

26) $\int_0^a \int_{\sqrt{ax}}^a \frac{y^2}{\sqrt{y^4 - a^2x^2}}\, dy\, dx$

$\left\{ \text{Ans. } \frac{\pi a^2}{6} \right\}$

27) $\int_0^\infty \int_0^x x e^{-x^2/y}\, dy\, dx$

$\left\{ \text{Ans. } \frac{1}{2} \right\}$

28) $\int_0^4 \int_{\sqrt{y}}^2 e^{y/x}\, dx\, dy$

$\left\{ \text{Ans. } e^2 - 1 \right\}$

29) $\int_0^1 \int_y^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2+y^2}}\, dx\, dy$

$\left\{ \text{Ans.: } 1 - \frac{1}{\sqrt{2}} \right\}$

30) $\int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} 3y\, dx\, dy$

$\left\{ \text{Ans.: } 2 \right\}$

31) $\int_0^a \int_y^a \frac{x^2}{\sqrt{x^2+y^2}}\, dx\, dy$

$\left\{ \text{Ans.: } \frac{a^3}{3} \log(1+\sqrt{2}) \right\}$

32) $\int_0^a \int_y^a \frac{x+y}{x^2+y^2}\, dx\, dy$

$\left\{ \frac{\pi a}{4} + \frac{a}{2} \log 2 \right\}$

33) $\int_0^a \int_0^{x^2} x(x^2+y^2)\, dy\, dx$

$\left\{ \frac{a^6}{24}(a^2+4) \right\}$

34) $\int_0^4 \int_{x^2/4}^{2\sqrt{x}} dy\, dx$

$\left\{ \text{Ans. } \frac{16}{3} \right\}$

35) $\int_0^1 \int_{x^3}^{\sqrt{x}} x^2\, dx\, dy$

$\left\{ \text{Ans. } \frac{5}{42} \right\}$

36) $\int_0^4 \int_x^{2\sqrt{x}} (x^2+y^2)\, dy\, dx$

$\left\{ \text{Ans. } \frac{768}{35} \right\}$

37) $$\int_0^a \int_0^{\sqrt{a^2-x^2}} xy\,dx\,dy$$

$\left\{ \text{Ans.}\ \frac{a^4}{8} \right\}$

38) $$\int_{-a}^{a} \int_0^{\sqrt{a^2-x^2}} (x^2+y^2)\,dx\,dy.$$

$\left\{ \text{Ans.}\ \frac{\pi a^4}{4} \right\}$

39) $$\int_0^{a/\sqrt{2}} \int_x^{\sqrt{a^2-x^2}} y^2\,dx\,dy.$$

$\left\{ \frac{a^4}{16} + \frac{\pi}{32} a^4 \right\}$

40) $$\int_0^{2a} \int_{x^2/4a}^{3a-x} xy\,dy\,dx$$

$\left\{ \frac{8}{3} a^4 \right\}$

41) $$\int_0^1 \int_{x^2}^{2-x} xy\,dy\,dx$$

$\left\{ \frac{3}{8} \right\}$

42) $$\int_0^a \int_{a-\sqrt{a^2-y^2}}^{a+\sqrt{a^2-y^2}} xy\,dx\,dy$$

$\left\{ \frac{2}{3} a^4 \right\}$

43) $$\int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} dy\,dx.$$

$\left\{ \frac{16}{3} a^2 \right\}$

44) $$\int_0^2 \int_{2-y}^{\sqrt{4-y^2}} y\,dx\,dy$$

45) $$\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x}{x^2+y^2}\,dx\,dy$$

$\left\{ 1 - \frac{1}{\sqrt{2}} \right\}$

46) $$\int_0^1 \int_0^{\sqrt{1-y^2}} \frac{dx\,dy}{1+x^2+y^2}.$$

$\left\{ \frac{\pi}{4} \log(1+\sqrt{2}) \right\}$

47) $$\int_0^{2a} \int_{x^2/4a}^{3a-x} (x^2+y^2)\,dy\,dx.$$

$\left(\frac{314}{35} a^4 \right)$

48) $$\int_1^4 \int_{2/y}^{2\sqrt{y}} dx\,dy.$$

$\left\{ \frac{28}{3} - 2\log 4 \right\}$