18MAB102T-CLAT-2 - B2 SLOT - Advanced Calculus and Complex Analysis

Date of Examination: 23/06/2021
Time: 9.00 am - 10.30 am
Max. Mark: 50 Max. time:90 Min

The respondent's email (**null**) was recorded on submission of this form.

* Required

1. Email *

2. Name of the Student *

3. Registration Number *

4. Department *

18MAB102T-CT-2 - B SLOT -Adanced Calculus and Complex Analysis

MCQ Questions
Each question carry one mark

5. *

The divergence of $\vec{F} = xyz\vec{i} + 3x^2y\vec{j} + (xz^2 - y^2z)\vec{k}$ at (2, -1, 1) is

A. 12

B. 14

C. 11

D. 13

Mark only one oval.

- A
- B
- C
- D

6. *

If $\vec{F} = (axy - z^3)\vec{i} + (a-2)x^2\vec{j} + (1-a)xz^2\vec{k}$ is irrotational, then the value of a is

A. 4

B. 2

C. -1

D. 0

Mark only one oval.

- A
- B
- C
- D

7. *

The unit normal vector to the surface $x^3 - xyz + z^3 = 1$ at the point $(1,1,1)$ is

A. $\dfrac{\vec{i}+\vec{j}+\vec{k}}{3}$

B. $\dfrac{2\vec{i}+2\vec{j}+2\vec{k}}{3}$

C. $\dfrac{2\vec{i}-\vec{j}+2\vec{k}}{3}$

D. $\dfrac{2\vec{i}-2\vec{j}+2\vec{k}}{3}$

Mark only one oval.

- A
- B
- C
- D

8. *

The directional derivative of $\phi = 3x^2 + 2y - 3z$ at the point (1, 1, 1) in the direction of the vector $2\vec{i} + 2\vec{j} - \vec{k}$ is

A. $\dfrac{17}{3}$

B. $\dfrac{13}{3}$

C. $\dfrac{11}{3}$

D. $\dfrac{19}{3}$

Mark only one oval.

- A
- B
- C
- D

9. *

Evaluate the line integral $\int_C \vec{r}.d\vec{r}$ where C is the line $y = x$ in XY plane from (1, 1) to (2,2)

A. 2
B. 3
C. 4
D. 1

Mark only one oval.

- A
- B
- C
- D

10. *

Find the constant a, if the vector $\vec{F} = (2x^2y+yz)\vec{i}+(xy^2-xz^2)\vec{j}+(axyz-2x^2y^2)\vec{k}$ is solenoidal

A. -4
B. -2
C. -5
D. -6

Mark only one oval.

- A
- B
- C
- D

11. *

If $\phi = x^2y + y^2x + z^2$ then $\nabla\phi$ at the point (1, 1, 1) is

A. $2\vec{i} + 2\vec{j} + \vec{k}$
B. $3\vec{j} + 2\vec{k} + 2\vec{k}$
C. $3\vec{i} + 3\vec{j} + 2\vec{k}$
D. $\vec{i} + 3\vec{j} + 2\vec{k}$

Mark only one oval.

- A
- B
- C
- D

12. *

If $\vec{F} = (y^2 + 2xz^2)\vec{i} + (2xy - z)\vec{j} + (2x^2z - y + 2z)\vec{k}$ then $\vec{F}$ is

A. Solenoidal
B. Irrotational
C. Neither Solenoidal Nor Irrotational
D. Both Solenoidal and Irrotational

Mark only one oval.

- A
- B
- C
- D

13. *

The maximum directional derivative of $\phi = xyz^2$ at $(1, 0, 3)$ is

A. 8

B. 4

C. 9

D. 6

Mark only one oval.

- A
- B
- C
- D

14. *

If $\vec{r}$ is the position vector of the point (x, y, z) with respect to origin, then div$\vec{r}$ is

A. 0

B. 3

C. 2

D. 1

Mark only one oval.

- A
- B
- C
- D

15. *

According to Green's theorem $\int\limits_C (M\,dx + N\,dy) =$

A. $\iint\limits_R \left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) dx\,dy$

B. $\iint\limits_R \left(\frac{\partial M}{\partial x} - \frac{\partial N}{\partial y}\right) dx\,dy$

C. $\iint\limits_R \left(\frac{\partial N}{\partial y} - \frac{\partial M}{\partial x}\right) dx\,dy$

D. $\iint\limits_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy$

Mark only one oval.

- A
- B
- C
- D

16. *

If $\vec{a}$ is a constant vector and $\vec{r}$ is the position vector of the point (x, y, z) w.r.to the origin then div$(\vec{a} \times \vec{r})$ is

A. 0

B. 1

C. $\vec{r}$

D. $\vec{a}$

Mark only one oval.

- A
- B
- C
- D

17. *

If $\phi(x, y, z) = x^3 + y^3 + z^3 - 3xyz$ then the value of $\nabla.\nabla\phi$ at the point $(1, 2, 3)$ is

A. 18

B. 42

C. 36

D. 24

Mark only one oval.

- A
- B
- C
- D

18. *

If ϕ is a scalar function, then curl(gradϕ) is

A. 0

B. 1

C. -1

D. 2

Mark only one oval.

- A
- B
- C
- D

19. *

If $\vec{F} = (x^2 - y^2 + 2xz)\vec{i} + (xz - xy + yz)\vec{j} + (z^2 + x^2)\vec{k}$ **then** $\nabla.(\nabla \times \vec{F}) =$

A. 1

B. 0

C. 2

D. 4

Mark only one oval.

- A
- B
- C
- D

20. *

Angle between two level surfaces $\phi_1 = c_1$ and $\phi_2 = c_2$ is given by

A. $\cos\theta = \dfrac{\nabla\phi_1 \times \nabla\phi_2}{|\nabla\phi_1||\nabla\phi_2|}$

B. $\cos\theta = \dfrac{\nabla\phi_1 - \nabla\phi_2}{|\nabla\phi_1||\nabla\phi_2|}$

C. $\cos\theta = \dfrac{\nabla\phi_1 + \nabla\phi_2}{|\nabla\phi_1||\nabla\phi_2|}$

D. $\cos\theta = \dfrac{\nabla\phi_1 . \nabla\phi_2}{|\nabla\phi_1||\nabla\phi_2|}$

Mark only one oval.

- A
- B
- C
- D

21. *

If $\vec{r}$ is the position vector of the point (x, y, z) w.r.to the origin, then $\nabla \times \vec{r}$ is

A. 1

B. $\vec{i} - \vec{j} - \vec{k} = 0$

C. $x\vec{i} - y\vec{j} - z\vec{k} = 0$

D. 0

Mark only one oval.

- A
- B
- C
- D

22. *

If $\vec{a}$ is a constant vector and $\vec{r}$ is the position vector of the point (x, y, z) with respect to the origin, then grad$(\vec{a}.\vec{r})$ is

A. $2\vec{a}$

B. $\vec{a}$

C. 1

D. 0

Mark only one oval.

- A
- B
- C
- D

23. *

The condition for the surfaces $\phi_1(x, y, z) = 0$ cut orthogonally to $\phi_2(x, y, z) = 0$ is

A. $\dfrac{\nabla\phi_1}{\nabla\phi_2} = 0$

B. $\nabla\phi_1 \cdot \nabla\phi_2 = 0$

C. $\nabla\phi_1 - \nabla\phi_2 = 0$

D. $\nabla\phi_1 + \nabla\phi_2 = 0$

Mark only one oval.

- A
- B
- C
- D

24. *

Applying Stoke's theorem, the value of $\int_C (e^x dx + 2y dy - dz)$, where C is the curve $x^2 + y^2 = 4, z = 2$ is

A. $\frac{1}{2}$

B. $\frac{1}{3}$

C. $\frac{4}{3}$

D. 0

Mark only one oval.

- A
- B
- C
- D

25. *

The condition for $\vec{F}$ to be conservative is, $\vec{F}$ should be

A. rotational

B. solenoidal

C. irrotational

D. neither solenoidal nor irrotational

Mark only one oval.

- A
- B
- C
- D

26. *

The relation between line integral and a surface integral is known as

A. Green's theorem

B. Residue theorem

C. Stokes theorem

D. Divergence theorem

Mark only one oval.

- A
- B
- C
- D

27. *

Suppose $\vec{F} = (2xy + z^3)\vec{i} + x^2\vec{j} + 3xz^2\vec{k}$ is irrotational and if $\nabla\phi = \vec{F}$, then the scalar potential function ϕ is

A. $xy^2 + xz^2 + C$

B. $x^2y + xz^3 + C$

C. $xy^3 + xz^2 + C$

D. $xy + xz^3 + C$

Mark only one oval.

- A
- B
- C
- D

28. *

Using Green's theorem, the value of $\int_C [(x-2y)\,dx + x\,dy]$, where C is the circle $x^2+y^2=4$ is

A. 4π
B. 6π
C. 8π
D. 12π

Mark only one oval.

A
B
C
D

29. *

If S is any closed surface enclosing the volume V and if $\vec{F} = ax\vec{i} + by\vec{j} + cz\vec{k}$ then the value of $\iint_S \vec{F}.\vec{n}\,dS$ is

A. $abcV$
B. $abc(a+b+c)V$
C. $(a+b+c)V$
D. $(ab+bc+ca)V$

Mark only one oval.

A
B
C
D

30. *

The Laplace transform of a function $f(t)$ exists if

A. it is piecewise continuous of exponential order

B. it is uniformly continuous of exponential order

C. it is continuous

D. it is not continuous

Mark only one oval.

- A
- B
- C
- D

31. *

If $L[f(t)] = F(s)$, then $L[f(at)] =$

A. $\frac{1}{a} F\left(\frac{s}{a}\right)$

B. $\frac{1}{a} F(sa)$

C. $F\left(\frac{s}{a}\right)$

D. $F(sa)$

Mark only one oval.

- A
- B
- C
- D

32. *

The Laplace transform of $t^4 e^{-at}$ is

A. $\dfrac{4!}{(s+a)^4}$

B. $\dfrac{4!}{(s-a)^4}$

C. $\dfrac{4!}{(s+a)^5}$

D. $\dfrac{5!}{(s-a)^5}$

Mark only one oval.

- A
- B
- C
- D

33. *

$$L\left[\cos\left(\frac{t}{2}\right)\right] =$$

A. $\dfrac{4s}{4s^2+1}$

B. $\dfrac{2s}{s^2+4}$

C. $\dfrac{s}{s^2+4}$

D. $\dfrac{4s}{4s^2-1}$

Mark only one oval.

- A
- B
- C
- D

34. *

If $f(t) = \sin t$ then $L\left[f'(t)\right] =$

A. $\dfrac{s}{s^2-1}$

B. $\dfrac{s}{s^2+1}$

C. $\dfrac{1}{s^2+1}$

D. $\dfrac{1}{s^2-1}$

Mark only one oval.

- A
- B
- C
- D

35. *

What is the Laplace transform of $e^{-t}\cos 6t$?

A. $\dfrac{s+1}{(s+1)^2-36}$

B. $\dfrac{s+1}{(s+1)^2+36}$

C. $\dfrac{s-1}{(s-1)^2-36}$

D. $\dfrac{s-1}{(s-1)^2+36}$

Mark only one oval.

- A
- B
- C
- D

36. *

$$L\left(te^{t}\right) =$$

A. $\dfrac{1}{s^2-1}$

B. $\dfrac{1}{s-1}$

C. $\dfrac{1}{(s-1)^2}$

D. $\dfrac{1}{(s+1)^2}$

Mark only one oval.

- A
- B
- C
- D

37. *

$L(4^t)$ **is**

A. $\dfrac{1}{s+4}$

B. $\dfrac{1}{s+\log 4}$

C. $\dfrac{1}{s-4}$

D. $\dfrac{1}{s-\log 4}$

Mark only one oval.

- A
- B
- C
- D

38. *

Find $\lim_{t \to 0} f(t)$ **where** $f(t) = 1 + e^{-t} + t^2$

A. 1

B. 2

C. 0

D. 3

Mark only one oval.

- A
- B
- C
- D

39. *

$$L\left[(1+e^{-2t})^2\right] =$$

A. $\frac{1}{s} + \frac{2}{s+2}$

B. $\frac{1}{s} - \frac{2}{s+2} + \frac{1}{s+4}$

C. $\frac{1}{s} + \frac{2}{s-2} + \frac{1}{s-4}$

D. $\frac{1}{s} + \frac{2}{s+2} + \frac{1}{s+4}$

Mark only one oval.

- A
- B
- C
- D

40. *

Find $\lim\limits_{s \to \infty} sF(s)$ where $f(t) = \cos t$

A. 1
B. 0
C. 2
D. -1

Mark only one oval.

- A
- B
- C
- D

41. *

Find $L[f(t)]$ where $f(t) = 2$

A. $\frac{1}{s}$
B. $\frac{2}{s}$
C. 2
D. $\frac{1}{2}$

Mark only one oval.

- A
- B
- C
- D

42. *

Laplace transform of $\sin^2 3t$ is

A. $\dfrac{3}{s^2+36}$

B. $\dfrac{6}{s^2+36}$

C. $\dfrac{18}{s(s^2+36)}$

D. $\dfrac{18}{s^2+36}$

Mark only one oval.

- A
- B
- C
- D

43. *

$$L\left(t^2 e^{-3t}\right) =$$

A. $\dfrac{1}{(s+3)^3}$

B. $\dfrac{2}{(s+3)^2}$

C. $\dfrac{3}{(s+3)^3}$

D. $\dfrac{2}{(s+3)^3}$

Mark only one oval.

- A
- B
- C
- D

44. *

$$L[\frac{1-e^{-t}}{t}] =$$

A. $log\left(\frac{s+1}{s}\right)$

B. $log\left(\frac{s-1}{s}\right)$

C. $log\left(\frac{s}{s+1}\right)$

D. $log\left(\frac{s}{s-1}\right)$

Mark only one oval.

- A
- B
- C
- D

45. *

If $f(t+T) = f(t)$, then the function $f(t)$ is called as

A. anti-periodic with period T

B. periodic with period T

C. unit-step with period T

D. double-valued with period T

Mark only one oval.

- A
- B
- C
- D

46. *

Find $\lim_{s \to 0} sF(s)$ where $f(t) = t^2 e^{-3t}$

A. 0

B. 1

C. -1

D. ∞

Mark only one oval.

- A
- B
- C
- D

47. *

$$L^{-1}\left[\frac{s}{s^2+25}\right] =$$

A. $\cos 5t$

B. $\dfrac{\cos 5t}{5}$

C. $\left(\dfrac{\cos 5t}{5}\right)^2$

D. $\dfrac{\sin 5t}{5}$

Mark only one oval.

- A
- B
- C
- D

48. *

$$L^{-1}\left[\frac{1}{(s+a)^2}\right] =$$

A. e^{at}

B. e^{-at}

C. te^{-at}

D. te^{at}

Mark only one oval.

- A
- B
- C
- D

49. *

$$L^{-1}\left[\frac{s-1}{(s-1)^2+4}\right]=$$

A. $e^{-t}\sin 2t$

B. $e^{-t}\cos 2t$

C. $e^{t}\sin 2t$

D. $e^{t}\cos 2t$

Mark only one oval.

- A
- B
- C
- D

50. *

Find $L^{-1}\left[\frac{1}{s+2}\right]$

A. $-e^{-2t}$

B. e^{2t}

C. e^{-2t}

D. $-e^{2t}$

Mark only one oval.

- A
- B
- C
- D

51. *

Find $L^{-1}\left[\frac{1}{s^2-2}\right]$

A. $\frac{1}{\sqrt{2}}\sinh\sqrt{2}t$

B. $\sinh\sqrt{2}t$

C. $\cosh\sqrt{2}t$

D. $\frac{1}{\sqrt{2}}\sin\sqrt{2}t$

Mark only one oval.

- A
- B
- C
- D

52. *

If $L[f(t)] = F(s)$ and $L[g(t)] = G(s)$, then $L[f(t) * g(t)] =$

A. $F(s) - G(s)$

B. $F(s) + G(s)$

C. $\frac{F(s)}{G(s)}$

D. $F(s)G(s)$

Mark only one oval.

- A
- B
- C
- D

53. *

What is the period of the function $f(t) = sint$ is?

A. 2π

B. 3π

C. π

D. $\frac{\pi}{2}$

Mark only one oval.

- A
- B
- C
- D

54. *

The convolution theorem gives a relationship between the inverse Laplace transform of the ———

A. product of two functions

B. sum of two functions

C. difference of two functions

D. quotient of two functions

Mark only one oval.

- A
- B
- C
- D