S K THAMILVANAN CLA T3-18MAB102T-Advanced Calculus & Complex Analysis (C-Slot)

Date: 30/07/ 2021 | Max. Marks: 50 Marks
Time: 9.00 AM-10.40 AM | Max. Time: 100 Mins

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18MAB102T- CLAT-3-C SLOT- Advanced Calculus and Complex Analysis

Answer ALL Questions (50*1=50 Marks)

5. *

1 point

A certain function u(x,y) can be the real part of an analytic function if

(a) u satisfies C-R equations (b) u is a harmonic function

(b) u need not be a harmonic function (d) u is a continuous function

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

6. *

1 point

The function $f(z) = z\overline{z}$ is analytic

(a) At the origin (b) at infinity (c) nowhere (d) at all points of z-plane

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

7. * 1 point

If $f(z) = u(x, y) + iv(x, y)$ is analytic then u = constant, v = constant are

(a) Parallel (b) not cutting (c) straight lines (d) cutting orthogonally

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

8. * 1 point

If $f(z) = u + iv$ is analytic then $f(z) = -v + iu$ is

(a) analytic (b) not analytic

(c) analytic only at the origin (d) analytic except at the origin

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

9. * 1 point

If w = f (z) is an analytic function of z, then

(a) $\frac{\partial w}{\partial z} = i\frac{\partial w}{\partial x}$ (b) $\frac{\partial w}{\partial z} = \frac{\partial w}{\partial y}$ (c) $\frac{\partial^2 w}{\partial z \partial \bar{z}} \neq 0$ (d) $\frac{\partial w}{\partial \bar{z}} = 0$

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

10. * 1 point

$f(z) = \bar{z}$ is differentiable

(a) At all points (b) nowhere (c) at all points except at the origin (d) Only at the origin

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

11. * 1 point

The mapping $w = \frac{1}{z}$ gives

(a) Inversion only (b) reflection only (c) inversion and reflection (d) rotation

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

12. * 1 point

The map $w = \frac{1}{z}$ maps the totality of circles and lines as

(a) Circles or lines (b) circles and lines respectively

(c) Only circles (d) only straight lines

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

13. * 1 point

The critical points of $w = z + \frac{1}{z}$ are

(a) -1 (b) $+1$ (c) ± 1 (d) $0, 1, -1$

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

14. * 1 point

The image of the rectangular region bounded by the lines x =0, y = 0,x =2 and y = 1 under the transformation w =2z is a

(a) parabola (b) circle (c) straight line (d) rectangle is magnified twice.

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

15. * 1 point

The harmonic conjugate of $u = \frac{1}{2}\log(x^2 + y^2)$ is

(a) $\cos^{-1}(\frac{x}{y})$ (b) $\cot^{-1}(\frac{y}{x})$ (c) $\tan^{-1}(\frac{x}{y})$ (d) $\tan^{-1}(\frac{y}{x})$

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

16. * 1 point

What is the value of c_1 and c_2 such that the function $f(z) = c_1xy + i(c_2x^2 + y^2)$ is analytic ?

(a) $c_1 = 2, c_2 = -1$ (b) $c_1 = -1, c_2 = 2$ (c) $c_1 = -2, c_2 = 1$ (d) $c_1 = -2, c_2 = -1$

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

17. * 1 point

The function w = sinx coshy +i cosx sinhy is

(a) need not be analytic (b) analytic (c) discontinuous (d) differentiable only at origin

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

18. * 1 point

Find a function f(z) with a given $u = e^x\, cosy$

(a) $e^z + c$ (b) $-e^z + c$ (c) $-(1+i)e^z + c$ (d) $-ie^z + c$

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

19. * 1 point

Cauchy -Reimann equation in polar co-ordinates are

(a) $ru_r = v_\theta, -rv_r = u_\theta$ (b) $-ru_r = v_\theta, rv_r = u_\theta$

(c) $ru_r = v_\theta, rv_r = u_\theta$ (d) $u_r = rv_\theta, ru_\theta = v_r$

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

20. * 1 point

If u and v are harmonic, then $u + iv$ is

(a) harmonic (b) need not be analytic (c) analytic (d) continuous

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

21. * 1 point

The fixed points for the transformation $w = \frac{z-1}{z+1}$ are

(a) $z = \pm 1$ (b) $z = \pm i$ (c) $z = \pm 2$ (d) $z = 1$

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

22. * 1 point

The transformation w = cz where c is a real constant represents

(a) rotation (b)reflection (c)magnification and rotation (d)magnification

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

23. * 1 point

If $w = e^{2z}$, the real part of f(z) is

(a) $e^{y} sinx$ (b) $e^{x} cosy$ (c) $e^{y} cosy$ (d) $e^{2x} cos2y$

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

24. * 1 point

The bilinear transformation which maps the points ∞, **i** ,**0** **into** **0, i,** ∞ respectively is

(a) $w = 2 + z$ (b) $w = 2z$ (c) $\mathrm{w} = -\frac{1}{z}$ (d) $w = \frac{1+z}{1-z}$

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

25. *

1 point

The invariant points of the transformation $w = \frac{1-zi}{z-i}$

(a) $z = \pm 1$ (b) $z = \pm i$ (c) $z = \pm 2$ (d) $z = 1$

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

26. *

1 point

The function $|z|^2$ is

(a) differentiable at the origin (b) analytic (c) constant (d) differentiable everywhere

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

27. * 1 point

Choose the correct answer

$f(z) = \frac{1}{z^2+1}$ is analytic everywhere except at

(a) $z = 1$ (b) $z = i$ (c) $z = \pm i$ (d) $z = \pm 1$

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

28. * 1 point

Which of the following is not true with respect to a bilinear transformation?

a) It is conformal at all points

b) It maps circle into circle in general

c) It has two invariant points in general

d)It preserves the cross ratio of the four points

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

29. * 1 point

The relation between **a** and **b** if $ax^2 + by^2$ can be the real part of an analytic function is

(a) $a + b = 0$ (b) $a - b = 0$ (c) $ab = 0$ (d) $a = 2b$

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

30. * 1 point

A contour integral is an integral along a --------------- curve.

a. Open Curve
b. Closed curve
c. Simple closed curve
d. Multiple curve

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

31. * 1 point

If f(z) is analytic inside and on C , the value of $\oint_C f(z)\, dz$,where C is the simple closed curve is

a. f(a)
b. $2\pi i$f(a)
c. πif(a)
d. 0

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

32. * 1 point

If f(z) is analytic inside and on C , the value of $\oint_C \frac{f(z)}{(z-a)^n}\, dz$,where C is the simple closed curve and a is any point within c is

a. $2\pi i\, \frac{f^n(a)}{n!}$
b. $2\pi i\, f(a)$
c. $2\pi i\, \frac{f^{n-1}(a)}{(n-1)!}$
d. 0

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

33. * 1 point

The value of $\oint_C \frac{\sin z}{z+1}\, dz$ where C is the circle $|z| = \frac{1}{3}$ is

a. 0
b. $2\pi i$
c. $\frac{\pi}{2} i$
d. πi

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

34. * 1 point

The value of $\oint_C \frac{e^z}{(z-2)^2}\, dz$ where C is the circle $|z| = 3$ is

a. 0
b. $2\pi i e^{-2}$
c. $2\pi i e^{2}$
d. $4\pi i e^{-2}$

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

35. * 1 point

The value of $\oint_C \frac{z}{2z-1}\,dz$ where C is the circle $|z| = 1$ *is*

a. 0
b. $2\pi i$
c. $\frac{\pi}{2}i$
d. πi

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

36. * 1 point

The value of $\oint_C \frac{1}{(z-3)^2}\,dz$ where C is the circle $|z| = 1$ *is*

a. 0
b. 2π
c. $\frac{\pi}{2}i$
d. πi

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

37. * 1 point

Let $C_1: |z - a| = R_1$ and $C_2: |z - a| = R_2$ be two concentric circles($R_2 > R_1$), the annular region is defined as

a. Within C_1
b. Within C_2
c. Within C_2 and outside C_1
d. Within C_1 and outside C_2

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

38. * 1 point

The part $\sum_{n=0}^{\infty} a_n (z - a)^n$ consisting of positive integral powers of (z - a) is called as

a. The analytic part of the Laurent's series
b. The principal part of the Laurent's series
c. The real part of the Laurent's series
d. The imaginary part of the Laurent's series

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

39. * 1 point

.Let $C_1: |z-a| = R_1$ and $C_2: |z-a| = R_2$ be two concentric circles ($R_2 < R_1$), the f(z) can be expanded as a Laurent's series if

a. f(z) is analytic within C_2
b. f(z) is not analytic within C_2
c. f(z) is analytic in the annular region
d. f(z) is not analytic in the annular region

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

40. * 1 point

Expansion of $\frac{1-\cos z}{z}$ in Laurent's series about $z = 0$ is

a. $\frac{z}{2!} - \frac{z^3}{4!} + \frac{z^5}{6!} - \cdots$
b. $\frac{z^2}{2!} - \frac{z^4}{4!} + \frac{z^6}{6!} - \cdots$
c. $\frac{z}{1!} - \frac{z^3}{3!} + \frac{z^5}{5!} - \cdots$
d. $\frac{z}{2!} + \frac{z^3}{4!} - \frac{z^5}{6!} + \cdots$

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

41. * 1 point

The annular region for the function $f(z) = \frac{1}{z^2-3z+2}$ is

a. $0 < |z| < 1$
b. $1 < |z| < 2$
c. $2 < |z| < 3$
d. $|z| < 3$

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

42. * 1 point

.The Laurent's series expansion $1 + \frac{3}{z}\sum\frac{(-1)^n 2^n}{z^n} - \sum\frac{(-1)^n 3^n}{z^n}$ for the function $f(z) = \frac{z^2-1}{(z+2)(z+3)}$ is valid in the region

a. $|z| < 3$
b. $|z| < 2$
c. $2 < |z| < 3$
d. $|z| > 3$

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

43. * 1 point

.If f(z) is not analytic at $z = z_o$ and there exists a neighborhood of $z = z_o$ containing no other singularity, then

a. The point $z = z_o$ is isolated singularity of f(z)
b. The point $z = z_o$ is a zero point of f(z)
c. The point $z = z_o$ is nonzero of f(z)
d. The point $z = z_o$ is non isolated singularity of f(z)

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

44. * 1 point

.If $f(z) = e^{\frac{1}{z+1}}$ then

a. z = -1 is removable singularity
b. z = -1 is pole of order 2
c. z = -1 is an essential singularity
d. z = -1 is zero of f(z)

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

45. * 1 point

.Let z = a is a simple pole for $f(z) = \frac{P(z)}{Q(z)}$, then the Residue of $f(z)$ is

a. $\frac{P'(a)}{Q(a)}$

b. $\frac{P(a)}{Q(a)}$

c. $\frac{P'(a)}{Q'(a)}$

d. $\frac{P(a)}{Q'(a)}$

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

46. * 1 point

Let z = a is a pole of order 3 for f(z), then the residue is

a. $\lim_{z \to a} [(z - a) f(z)]$

b. $\lim_{z \to a} [(z - a) f''(z)]$

c. $\lim_{z \to a} \frac{1}{2!} \frac{d^2}{dz^2} [(z - a)^3 f(z)]$

d. $\lim_{z \to a} \frac{1}{3!} \frac{d^3}{dz^3} [(z - a)^3 f(z)]$

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

47. * 1 point

The residue of f(z) $= \frac{z}{(z-2)}$ at $z = 2$ is

a. $2\pi i$

b. 1

c. 2

d. 0

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

48. * 1 point

The residue of $f(z) = \frac{1}{(z^2+1)^2}$ at $z = i$ is

a. 4i
b. 1/4i
c. 0
d. 1/2i

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

49. * 1 point

If $f(z) = \frac{\sin z - z}{z^3}$, then
a. z= 0 is a simple pole
b. z= 0 is a pole of order 2
c. z= 0 is a removable singularity
d. z= 0 is a zero of f(z)

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

50. * 1 point

The value of the integral $\oint_C \frac{1}{ze^z}dz$ where $|z| = 1$ is

a. $2\pi i$
b. $\frac{\pi}{2}i$
c. πi
d. 0

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

51. * 1 point

.If $f(z) = \frac{1}{z} + [2 + 3z + 4z^2 + \cdots]$ then the residue of $f(z)$ at z=0 is

a. 1
b. -1
c. 0
d. -2

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

52. * 1 point

.If the integral $\oint_0^{2\pi} \frac{d\theta}{13+5\cos\theta} = \oint_C f(z)dz$, C is $|z| = 1$, then
(A) $z = -i/5$ is pole of $f(z)$ lies inside C and
(B) $z = -5i$ is pole of $f(z)$ lies outside C. Which of the following is true?

a. Both A and B
b. Only A
c. Only B
d. Neither A nor B

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

53. * 1 point

If the integral $\oint_{-\infty}^{\infty} \frac{\cos mx}{(x^2+1)^2} dx = \oint_C \frac{e^{imz}}{(z^2+1)^2} dz$, $m > 0$ and where C is the part of the real axis from $-R\ to + R\ and$ semicircle $|z| = R$ above the real axis, then
(A) $z = i$ double pole lies in the upper half of the z-plane and
(B) $z = -i$ double pole does not lie in the upper half of the z-plane.
Which of the following is true?

a. Both A and B
b. Only A
c. Only B
d. Neither A nor B

Mark only one oval.

- (a)
- (b)
- (c)
- (d)

54. * 1 point

If $f(z)$ be continuous function such that $|f(z)| \to 0$ as $|z| \to \infty$, where C is the semicircle $|z| = R$ above the real axis, then

a. $\oint_C e^{-imz} f(z)dz \to \infty \; as \; R \to \infty$.

b. $\oint_C e^{imz} f(z)dz \to 0 \; as \; R \to \infty$.

c. $\oint_C e^{imz} f(z)dz \to 0 \; as \; R \to 0$.

d. $\oint_C f(z)dz \to \infty \; as \; R \to 0$.

Mark only one oval.

- (a)
- (b)
- (c)
- (d)