# Revision

1. Gradient : $\nabla\phi = \vec{i}\frac{\partial\phi}{\partial x} + \vec{j}\frac{\partial\phi}{\partial y} + \vec{k}\frac{\partial\phi}{\partial z}$.

2. Gradient of a constant is a null vector $\vec{0}$.

3. Directional derivative : $\nabla\phi \cdot \frac{\vec{a}}{|\vec{a}|}$

4. Directional derivative is maximum in the direction and magnitude of the maximum is $|\nabla\phi|$.

5. Unit tangent vector : $\dfrac{\frac{d\vec{r}}{dt}}{\left|\frac{d\vec{r}}{dt}\right|}$.

6. Unit normal vector : $\hat{n} = \frac{\nabla\phi}{|\nabla\phi|}$.

7. Angle between surfaces : $\cos\theta = \frac{\nabla\phi_1 \cdot \nabla\phi_2}{|\nabla\phi_1||\nabla\phi_2|}$.

   surfaces cut orthogonally if $\nabla\phi_1 \cdot \nabla\phi_2 = 0$.

8. Divergence $\operatorname{div}\vec{F} = \nabla\cdot\vec{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$.

   * $\nabla\cdot\vec{F}$ is scalar.
   * $\nabla\cdot\vec{F} = 0$ then $\vec{F}$ is solenoidal

9. Curl $\operatorname{curl}\vec{F} = \nabla\times\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$.

   * $\nabla\times\vec{F}$ is a vector
   * $\nabla\times\vec{F} = \vec{0}$ then $\vec{F}$ is irrotational.

10. If $\vec{r} = x\vec{i}+y\vec{j}+z\vec{k}$ then $\nabla\cdot\vec{r} = 3$ and $\nabla\times\vec{r} = \vec{0}$.

**11. $\operatorname{curl}(\operatorname{grad}\phi) = \vec{0}$ i.e. $\nabla\times(\nabla\phi) = \vec{0}$

**12. $\nabla\cdot(\nabla\times\vec{F}) = 0$ i.e. $\operatorname{div}(\operatorname{curl}\vec{F}) = 0$.

**13 If $\vec{A}$ and $\vec{B}$ are irrotational then $\vec{A}\times\vec{B}$ is solenoidal.

Solution:

$\nabla \times \vec{A} = \vec{0}$ , $\nabla \times \vec{B} = \vec{0}$

Consider $\nabla \cdot (\vec{A} \times \vec{B}) = (\nabla \times \vec{A}) \cdot \vec{B} - (\nabla \times \vec{B}) \cdot \vec{A}$

$= \vec{0} \cdot \vec{B} - \vec{0} \cdot \vec{A} = 0 - 0 = 0.$

**14. If $\nabla \phi$ and $\nabla \psi$ are irrotational, then $\nabla \phi \times \nabla \psi$ is solenoidal.

Solution: $\nabla \times \nabla \phi = \vec{0}$ , $\nabla \times \nabla \psi = \vec{0}$

$\nabla \cdot (\nabla \phi \times \nabla \psi) = (\nabla \times \nabla \phi) \cdot \nabla \psi - (\nabla \times \nabla \psi) \cdot \nabla \phi.$

$= \vec{0} \cdot \nabla \psi - \vec{0} \cdot \nabla \phi = 0 \cdot 0 = 0.$

15. Conservative vector field.

Let $\vec{F}$ be a vector field in space and let AB be a curve described from A to B. then

$\int_A^B \vec{F} \cdot d\vec{r}$ depends not only on the path C but also on the terminal points A and B.

If the integral depends only on the end points but not on the path C, then $\vec{F}$ is called a conservative vector field.

If $\vec{F}$ is conservative, then curl $\vec{F}$ = curl (grad $\phi$) = $\vec{0}$.

$\Rightarrow \vec{F}$ is irrotational.

Equivalently, the work done is moving a particle., by the conservative force around the closed curve is $\nabla \times \vec{F} = \vec{0}$: