Inverse Laplace Transform.

1. $L^{-1}\left[\frac{1}{s}\right] = 1.$

2. $L^{-1}\left[\frac{1}{s-a}\right] = e^{at},\ L^{-1}\left[\frac{1}{s+a}\right] = e^{-at}$

3. $L^{-1}\left[\frac{1}{s^2+a^2}\right] = \frac{\sin at}{a}.$

4. $L^{-1}\left[\frac{s}{s^2+a^2}\right] = \cos at.$

5. $L^{-1}\left[\frac{1}{s^2-a^2}\right] = \frac{\sinh at}{a}.$

6. $L^{-1}\left[\frac{s}{s^2-a^2}\right] = \cosh at.$

7. $L^{-1}\left[\frac{1}{s^n}\right] = \frac{t^{n-1}}{(n-1)!}.$

* $L^{-1}\left[\frac{1}{(s-a)^2+b^2}\right] = \frac{e^{at}\sin bt}{b}.$
* $L^{-1}\left[\frac{s-a}{(s-a)^2+b^2}\right] = e^{at}\cos bt.$

Completing the squares.

$x^2+bx+c.$

Add & Sub. $\left(\frac{b}{2}\right)^2.$ $\Rightarrow x^2+bx+\frac{b^2}{2^2}+c-\left(\frac{b}{2}\right)^2.$

$= \left(x+\frac{b}{2}\right)^2 + c - \frac{b^2}{4}.$

* $L^{-1}[sF(s)] = \frac{d}{dt} L^{-1}[F(s)].$

  $L^{-1}[s^2F(s)] = \frac{d^2}{dt^2} L^{-1}[F(s)].$

* $L^{-1}\left[\frac{1}{s}F(s)\right] = \int_0^t f(t)\,dt.$

* $L^{-1}[F(s)] = -\frac{1}{t} L^{-1}\left[\frac{d}{ds}F(s)\right].$

[Eg.: $\tan^{-1}s$, $\cot^{-1}(1+s)$, $\log\frac{s+1}{s+2}$]

* Partial Fractions method.

* **Convolution**: $f(t)*g(t) = \int_0^t f(u)g(t-u)\,du.$

Technique: $[L^{-1}[F(s)]]_{t\to u} = f(u).$

Shift: $g(t) \to g(t-u).$

Multiply: $f(u).g(t-u).$

Integrate: $\int_0^t f(u)g(t-u)\,du$

Application to solution of linear ODE of second order with constant co-efficients.

* $L[y'(t)] = sL[y(t)] - y(0).$
* $L[y''(t)] = s^2L[y(t)] - sy(0) - y'(0)$
* $L[y'''(t)] = s^3L[y(t)] - s^2y(0) - sy'(0) - y''(0).$

Transform of Periodic functions

$L[f(t)] = \frac{1}{1-e^{-ps}}\int_0^p e^{-st}f(t)\,dt.$

where $f(t)$ is a periodic function with period 'p'.

Identities

1. $$L^{-1}\left[\frac{1}{(s+a)(s+b)}\right] = \frac{e^{-at}-e^{-bt}}{b-a}.$$

2. $$L^{-1}\left[\frac{1}{(s^2+a^2)(s^2+b^2)}\right] = \frac{a\sin bt - b\sin at}{ab(a^2-b^2)}.$$

3. $$L^{-1}\left[\frac{s}{(s^2+a^2)(s^2+b^2)}\right] = \frac{\cos bt - \cos at}{a^2-b^2}.$$

4. $$L^{-1}\left[\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right] = \frac{a\sin at - b\sin bt}{a^2-b^2}.$$

5. $$L^{-1}\left[\frac{1}{(s^2+a^2)^2}\right] = \frac{\sin at - at\cos at}{2a^3}.$$

6. $$L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right] = \frac{t\sin at}{2a}.$$

7. $$L^{-1}\left[\frac{s^2}{(s^2+a^2)^2}\right] = \frac{at\cos at + \sin at}{2a}.$$