

**SRM INSTITUTE OF SCIENCE AND TECHNOLOGY**  
**DEPARTMENT OF MATHEMATICS**  
**18MAB201T/Transforms and Boundary value problems**  
**UNIT II – FOURIER SERIES**  
**TUTORIAL SHEET -1**

**PART B Questions**

1. State Dirichlet condition's for a given function to expand in Fourier series.
2. Find  $a_1$  for the periodic function  $f(x) = \begin{cases} \sin x, 0 < x < \pi \\ 0, \pi < x < 2\pi \end{cases}$
3. Find  $a_0$  for the periodic function  $f(x) = e^{-x}, 0 < x < 2\pi$ .
4. Find  $a_n$  for the Fourier series of periodicity 3 for  $f(x) = 2x - x^2$  in  $0 < x < 3$
5. Find half –range cosine series for  $f(x) = x, 0 < x < \pi$

**PART C Questions**

6. Find the Fourier series to represent  $(x - x^2)$  in the interval  $[-\pi, \pi]$ . Deduce the value of  $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \dots$
7. Obtain the Fourier series expansion for  $f(x) = x^2$  in  $-\pi < x < \pi$  and hence the sum of the series  $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \dots$
8. If  $f(x) = \begin{cases} \sin x, 0 < x < \frac{\pi}{4} \\ \cos x, \frac{\pi}{4} < x < \frac{\pi}{2} \end{cases}$ . Express  $f(x)$  in a series of sines.
9. Find the Fourier series for  $f(x) = |\cos x|$  in  $-\pi < x < \pi$  of periodicity  $2\pi$ .
10. Find the Fourier series for  $f(x) = |\sin x|$  in  $-\pi < x < \pi$  of periodicity  $2\pi$ .

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**TUTORIAL SHEET -2**

**PART B Questions**

1. Expand  $f(x) = (x-1)^2$  in  $0 < x < 1$  in a Fourier series of sine series only.

2. The Fourier series of the function  $f(x) = x + x^2, -\pi < x < \pi$  is

$$\frac{\pi^2}{3} + \sum_{n=1}^{\infty} (-1)^n \left( \frac{4}{n^2} \cos nx - \frac{2}{n} \sin nx \right). \text{ Then deduce that } \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

3. Find  $b_1$  for the function  $f(x) = \begin{cases} \sin x, 0 < x < \pi \\ 0, \pi < x < 2\pi \end{cases}$ .

4. The Fourier series of the function  $f(x) = (\pi - x)^2, 0 < x < 2\pi$  is

$$\frac{\pi^2}{3} + 4 \sum_{n=1}^{\infty} \left( \frac{1}{n^2} \cos nx \right). \text{ Then deduce the sum } \sum_{n=1}^{\infty} \frac{1}{n^2}$$

5. Express  $f(x) = x(\pi - x), 0 < x < \pi$ , as a Fourier series of periodicity  $2\pi$  containing sine terms only.

**PART C Questions**

6. Find the Fourier series of  $f(x) = x + x^2, -2 < x < 2$ . Hence find the sum of the series

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots$$

7. Find the half-range cosine series for the function  $f(x) = (x-1)^2, 0 < x < 1$ . Hence show that

$$\pi^2 = 6 \left\{ \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots \right\}.$$

8. If  $f(x) = \begin{cases} \frac{x}{l}, 0 < x < l \\ \frac{2l-x}{l}, l < x < 2l \end{cases}$ . Express  $f(x)$  as a Fourier series of periodicity  $2l$ .

9. Find the Fourier series of periodicity 2 for  $f(x) = \begin{cases} x, -1 < x < 0 \\ x+2, 0 < x < 1 \end{cases}$  and deduce the sum of

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$$

10. Express  $f(x) = x$  in half-range cosine series and sine series of periodicity  $2l$  in the range

$$0 < x < l \text{ and deduce the value of } \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \dots$$

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**TUTORIAL SHEET -3**

**PART B Questions**

1. Find R.M.S value of  $f(x) = x - x^2, -1 < x < 1$
2. Find R.M.S value of  $f(x) = x^2, -\pi < x < \pi$ .
3. Define Root Mean Square and find the RMS value of  $f(x) = 1 - x, 0 < x < 1$
4. Find the half-range Fourier sine series for  $f(x) = x, 0 < x < \pi$
5. Obtain the half –range cosine series for  $f(x) = x(\pi - x), 0 < x < \pi$

**PART C Questions**

6. Compute the first two harmonic of the Fourier series of  $f(x)$  given by the following table:

|      |   |                 |                  |       |                  |                  |        |
|------|---|-----------------|------------------|-------|------------------|------------------|--------|
| x    | 0 | $\frac{\pi}{3}$ | $\frac{2\pi}{3}$ | $\pi$ | $\frac{4\pi}{3}$ | $\frac{5\pi}{3}$ | $2\pi$ |
| f(x) | 1 | 1.4             | 1.9              | 1.7   | 1.5              | 1.2              | 1      |

7. Compute first three harmonics of the half-range cosine series of  $y = f(x)$  from

|      |   |   |    |   |   |   |
|------|---|---|----|---|---|---|
| x    | 0 | 1 | 2  | 3 | 4 | 5 |
| f(x) | 4 | 8 | 15 | 7 | 6 | 2 |

8. Compute the first two harmonic of the Fourier series of  $f(x)$  given by the following table:

|      |      |      |      |      |      |      |      |      |      |      |      |      |
|------|------|------|------|------|------|------|------|------|------|------|------|------|
| x    | 0°   | 30°  | 60°  | 90°  | 120° | 150° | 180° | 210° | 240° | 270° | 300° | 330° |
| f(x) | 6.82 | 7.97 | 8.02 | 7.20 | 5.67 | 3.67 | 1.76 | 0.55 | 0.26 | 0.90 | 2.49 | 4.73 |
| )    | 4    | 6    | 6    | 4    | 6    | 4    | 4    | 2    | 2    | 4    | 2    | 6    |

9. The values of  $x$  and the corresponding values of  $f(x)$  over period  $T$  are given below .Show that

$$f(x) = 0.75 + 0.37\cos\theta + 1.004\sin\theta \text{ where } \theta = \frac{2\pi x}{T}.$$

|      |      |               |               |               |                |                |      |
|------|------|---------------|---------------|---------------|----------------|----------------|------|
| x    | 0    | $\frac{T}{6}$ | $\frac{T}{3}$ | $\frac{T}{2}$ | $\frac{2T}{3}$ | $\frac{5T}{6}$ | T    |
| f(x) | 1.98 | 1.30          | 1.05          | 1.30          | -0.88          | -0.25          | 1.98 |

10. Expand  $f(x) = x - x^2$  as a Fourier series in  $-1 < x < 1$  and using this series find the RMS value of  $f(x)$  in the interval.