SRM INSTITUTE OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF MATHEMATICS

18MAB201T/Transforms and Boundary value problems

UNIT III - APPLICATIONS OF PARTIAL DIFFERENTIAL EQUATIONS

ANSWERS-TUTORIAL SHEET -1,2 AND 3

PART-B QUESTIONS

1. **The equation is parabolic at all points.**
2. **The equation is hyperbolic for all x, y.**
3. **The equation is elliptic.**
4. - **The motion takes place entirely in one plane. This plane is chosen as the xy plane.**
   - **In this plane, each particle of the string moves in a direction perpendicular to the equilibrium position of the string.**
   - **The tension T caused by stretching the string before fixing it at the end points is constant at all times at all points of the deflected string.**
   - **The tension T is very large compared with the wight of the string and hence the gravitational force may be neglected.**
   - **The effect of friction is negligible.**
   - **The string is perfectly flexible. It can transmit only tension but not bending or shearing forces.**
   - **The slope of the deflection curve is small at all points and at all times.**

PART-C QUESTIONS

4. $y(x,t) = \frac{3y_0}{4}\sin\frac{\pi x}{l}\cos\frac{\pi at}{l} - \frac{y_0}{4}\sin\frac{3\pi x}{l}\cos\frac{3\pi at}{l}$
5. $y(x,t) = \frac{k}{4}\left[\sin x\cos t + \sin 3x\cos 3t\right]$
6. $y(x,t) = \sum_{n=1}^{\infty}\frac{8k}{n^2\pi^2}\sin\frac{n\pi}{2}\sin\frac{n\pi x}{l}\cos\frac{n\pi ct}{l}$

TUTORIAL SHEET -2

PART-B QUESTIONS

1. **The possible solutions are**
   (i) $y(x,t) = (A_1e^{\lambda x} + B_1e^{-\lambda x})(C_1e^{\lambda at} + D_1e^{-\lambda at})$
   (ii) $y(x,t) = (A_2\cos\lambda x + B_2\sin\lambda x)(C_2\cos\lambda at + D_2\sin\lambda at)$
   (iii) $y(x,t) = (A_3x + B_3)(C_3t + D_3)$
   The correct solution is
   $y(x,t) = (A_2\cos\lambda x + B_2\sin\lambda x)(C_2\cos\lambda at + D_2\sin\lambda at)$.
2. **The initial and boundary conditions are**
   (i) $y(0,t) = 0$ **for** $t \geq 0$
   (ii) $y(l,t) = 0$ **for** $t \geq 0$
   (iii) $\left(\frac{\partial y}{\partial t}\right)$ **at (x,0)= 0 for** $0 \leq x \leq l$
   $$(iv)\quad y(x,0) = \begin{cases} \frac{2b}{l}x & \text{if } 0 \leq x \leq \frac{l}{2} \\ \frac{2b}{l}(l-x) & \text{if } \frac{1}{2} \leq x \leq l \end{cases}$$

3. **The initial and boundary conditions are**
   $(i)\quad y(0,t)=0\quad$ **for** $\quad t\geq 0$
   $(ii)\quad y(l,t)=0\quad$ **for** $\quad t\geq 0$
   $(iii)\quad y(x,0)=0\quad$ **for** $\quad 0\leq x\leq l$
   $(iv)\quad \left(\dfrac{\partial y}{\partial t}\right)$ **at t=0** $=\ 3x(l-x)\quad$ **for** $\quad 0\leq x\leq l$

PART-C QUESTIONS

4. $y(x,t)=\dfrac{3lv_0}{4\pi a}\sin\dfrac{\pi x}{l}\sin\dfrac{\pi at}{l}-\dfrac{v_0 l}{12\pi a}\sin\dfrac{3\pi x}{l}\sin\dfrac{3\pi at}{l}$

5. $y(x,t)=-\dfrac{4l^3}{\pi^3}\sum_{n=1}^{\infty}\left[\dfrac{1+2(-1)^n}{n^3}\right]\sin\dfrac{n\pi x}{l}.\cos\dfrac{n\pi ct}{l}$

6. $y(x,t)=\dfrac{16cl}{a\pi^3}\sum_{n=1}^{\infty}\dfrac{1}{n^3}\sin\dfrac{n\pi}{2}\sin\dfrac{n\pi x}{2l}\sin\dfrac{n\pi at}{2l}$

TUTORIAL SHEET -3

PART-B QUESTIONS

1.
   - **Heat flows from a higher to lower temperature**
   - **The amount of heat required to produce a given temperature change in a body is proportional to the mass of the body and to the temperature change. This constant of proportionality is known as the specific heat (c) of the conducting material.**
   - **The rate at which heat flows through an area is proportional to the area and to the temperature gradient normal to the area. This constant of proportionality is known as the thermal conductivity k of the material.**

2. **The possible solutions are**
   (i) $\quad u(x,t)=(A_1e^{\lambda x}+B_1e^{-\lambda x})(C_1e^{\alpha^2\lambda^2 t})$
   (ii) $\quad u(x,t)=(A_2\cos\lambda x+B_2\sin\lambda x)(C_2e^{-\alpha^2\lambda^2 t})$
   (iii) $\quad u(x,t)=(A_3x+B_3)C_3$
   The correct solution is
   $u(x,t)=(A_2\cos\lambda x+B_2\sin\lambda x)(C_2e^{-\alpha^2\lambda^2 t})$.

3. **The initial and boundary conditions are**
   $(i)\quad u(0,t)=0\quad$ **for all** $\quad t$
   $(ii)\quad u(l,t)=0\quad$ **for all** $\quad t$
   $$(iii)\quad u(x,0)=\begin{cases}\dfrac{2Tx}{l} & \text{if}\quad 0<x<\dfrac{l}{2}\\[2ex] \dfrac{2T}{l}(l-x) & \text{if}\quad \dfrac{1}{2}<x<l\end{cases}$$

4. $u(x)=2x+20$

PART-C QUESTIONS

5. $u(x,t)=\dfrac{4l}{\pi^2}\sum_{n=1}^{\infty}\dfrac{1}{n^2}\sin\dfrac{n\pi}{2}\sin\dfrac{n\pi x}{l}e^{\dfrac{-\alpha^2n^2\pi^2t}{l^2}}$

6. $u(x,t)=\sum_{n=1}^{\infty}\dfrac{200}{n\pi}(-1)^{n+1}\sin\dfrac{n\pi x}{l}e^{\dfrac{-\alpha^2n^2\pi^2t}{l^2}}$

7. $u(x,t)=u_s(x)+u_t(x,t)=50-4x-\dfrac{60}{\pi}\sum_{n=1}^{\infty}\dfrac{1}{n}\sin\dfrac{n\pi x}{5}e^{\dfrac{-\alpha^2n^2\pi^2t}{25}}$