



SRM INSTITUTE OF SCIENCE AND TECHNOLOGY
RAMAPURAM CAMPUS
DEPARTMENT OF MATHEMATICS

Year/Sem : II/III

Branch: Common to All branches

Unit V – Z Transforms

1. Find $Z(n)$

(a) $\frac{z}{(z-1)^2}$ (b) $\frac{z}{(z+1)^2}$ (c) $\frac{z}{z-1}$ (d) $\frac{z}{z+1}$

Solution:

$$\begin{aligned} Z(n) &= \sum_{n=0}^{\infty} n z^{-n} \\ &= \frac{1}{z} + \frac{2}{z^2} + \frac{3}{z^3} + \dots \\ &= \frac{1}{z} \left(1 - \frac{1}{z} \right)^{-2} = \frac{z}{(z-1)^2} \end{aligned}$$

2. Find Z-Transform of na^n

(a) $\frac{z}{(z+a)^2}$ (b) $\frac{z}{(z-a)^2}$ (c) $\frac{az}{(z-a)^2}$ (d) $\frac{az}{(z+a)^2}$

Solution:

$$\begin{aligned} Z(nf(n)) &= -z \frac{d}{dz} Z(a^n) \\ &= -z \frac{d}{dz} \left(\frac{z}{z-a} \right) \\ &= -z \left(-\frac{a}{(z-a)^2} \right) \\ &= \frac{az}{(z-a)^2} \end{aligned}$$

3. Find Z-Transform of $a^n \frac{1}{n!}$

(a) $e^{-\frac{a}{z}}$ (b) $e^{\frac{a}{z}}$ (c) $e^{\frac{1}{z}}$ (d) $e^{-\frac{1}{z}}$

Solution

$$\begin{aligned} Z\left(a^n \frac{1}{n!}\right) &= Z\left(\frac{1}{n!}\right)_{z \rightarrow \frac{z}{a}} \\ &= \left(e^{\frac{1}{z}}\right)_{z \rightarrow \frac{z}{a}} \\ &= e^{\frac{a}{z}} \end{aligned}$$

4. Find Z-Transform of $r^n e^{in\theta}$ by eliminating arbitrary function

(a) $\frac{z}{z-re^{i\theta}}$ (b) $\frac{z}{z+re^{i\theta}}$ (c) $\frac{1}{z-re^{i\theta}}$ (d) $\frac{-1}{z-re^{i\theta}}$

Solution:

We know that $Z(a^n) = \frac{z}{z-a}$

Put $a = re^{i\theta}$

$$Z(r^n e^{in\theta}) = \frac{z}{z-re^{i\theta}}$$

5. Find $Z(n^2)$

(a) $-z \left[\frac{1+z}{(z-1)^3} \right]$

(b) $z \left[\frac{1+z}{(z-1)^3} \right]$

(c) $-z \left[\frac{1+z}{(z+1)^3} \right]$

(d) $z \left[\frac{1+z}{(z+1)^3} \right]$

Solution:

$$\begin{aligned} Z(n^2) &= -z \frac{d}{dz} Z(n) \\ &= -z \frac{d}{dz} \left(\frac{z}{(z-1)^2} \right) \\ &= -z \left[\frac{1+z}{(z-1)^3} \right] \end{aligned}$$

6. Find $Z(n(n-1))$

- (a) $\frac{z}{(z+1)^3}$ (b) $\frac{z}{(z-1)^3}$ (c) $\frac{2z}{(z+1)^3}$ (d) $\frac{2z}{(z-1)^3}$

Solution:

$$\begin{aligned} Z(n(n-1)) &= Z(n^2 - n) \\ &= Z(n^2) - Z(n) \\ &= \frac{z(z+1)}{(z-1)^3} - \frac{z}{(z-1)^2} \\ &= \frac{2z}{(z-1)^3} \end{aligned}$$

7. Find $Z(n(1+5^n))$

- (a) $\frac{z}{(z-1)^2} + \frac{5z}{(z-5)^2}$ (b) $\frac{z}{(z+1)^2} + \frac{5z}{(z+5)^2}$
 (c) $\frac{z}{(z-1)^2} + \frac{5z}{(z+5)^2}$ (d) $\frac{z}{(z+1)^2} + \frac{5z}{(z-5)^2}$

Solution:

$$\begin{aligned} Z(n(1+5^n)) &= Z(n) + Z(n \cdot 5^n) \\ &= \frac{z}{(z-1)^2} + \frac{5z}{(z-5)^2} \end{aligned}$$

8. Find the inverse Z-Transform of $\frac{z}{z+1} + \frac{7z}{z-3}$, for $n > 0$

- (a) $(-1)^n + 7(-3)^n$ (b) $(-1)^n - 7(3)^n$
 (c) $(-1)^n - 7(-3)^n$ (d) $(-1)^n + 7(3)^n$

Solution:

$$Z^{-1}\left(\frac{z}{z+1} + \frac{7z}{z-3}\right) = (-1)^n + 7(3)^n$$

9. Find $Z^{-1}\left(\frac{2z}{2z-1}\right)$

- (a) $\left(\frac{1}{2}\right)^n$ (b) $\left(\frac{1}{2}\right)^n$
 (c) $\left(\frac{1}{2}\right)^n$ (d) $\left(\frac{1}{2}\right)^n$

Solution:

$$Z^{-1}\left(\frac{2z}{2z-1}\right) = \frac{2}{2} Z^{-1}\left(\frac{z}{z-\frac{1}{2}}\right) = \left(\frac{1}{2}\right)^n$$

10. Find $Z^{-1}\left(\frac{z}{4z+1}\right)$

(a) $(-1)^n \left(\frac{1}{3}\right)^{n+1}$

(b) $(-2)^n \left(\frac{1}{3}\right)^{n+1}$

(c) $(-1)^n \left(\frac{1}{4}\right)^{n+1}$

(d) $(-2)^n \left(\frac{1}{4}\right)^{n+1}$

Solution:

$$\begin{aligned} Z^{-1}\left(\frac{z}{4z+1}\right) &= \frac{1}{4} Z^{-1}\left(\frac{z}{z+\frac{1}{4}}\right) \\ &= \frac{1}{4} \left(-\frac{1}{4}\right)^n = (-1)^n \left(\frac{1}{4}\right)^{n+1} \end{aligned}$$

11. Find $Z(3^n(1+n))$

(a) $\frac{z}{z+3} + \frac{3z}{(z+3)^2}$

(b) $\frac{z}{z+3} + \frac{3z}{(z-3)^2}$

(c) $\frac{z}{z-3} + \frac{3z}{(z-3)^2}$

(d) $\frac{z}{z-3} + \frac{3z}{(z+3)^2}$

Solution:

$$\begin{aligned} Z(3^n(1+n)) &= Z(3^n) + Z(n \cdot 3^n) \\ &= \frac{z}{z-3} + \frac{3z}{(z-3)^2} \end{aligned}$$

12. If $F(z)z^{n-1} = \frac{z^n}{(z-1)(z-2)}$ then find residue at largest pole

(a) 2^{-n}

(b) 3^{-n}

(c) 3^n

(d) 2^n

Solution:

$$\begin{aligned} \text{Res}_{z=2} F(z)z^{n-1} &= \lim_{z \rightarrow 2} (z-2) \frac{z^n}{(z-1)(z-2)} \\ &= \frac{2^n}{2-1} = 2^n \end{aligned}$$

13. If $F(z)z^{n-1} = \frac{z^n}{(z-1)(z-2)}$ then find residue at smallest pole

(a) -1

(b) $(-1)^n$

(c) 1

(d) $(-2)^n$

Solution:

$$\begin{aligned} \operatorname{Res}_{z=1} F(z) z^{n-1} &= \lim_{z \rightarrow 2} (z-1) \frac{z^n}{(z-1)(z-2)} \\ &= \frac{1}{-1} = -1 \end{aligned}$$

14. Find $y(z)$ for the difference Equation $y_{n+1} - y_n = 0, y_0 = 1$

$$\begin{array}{ll} \text{(a)} y(z) = \frac{z}{z+1} & \text{(b)} y(z) = \frac{z}{1-z} \\ \text{(c)} y(z) = \frac{z}{z-1} & \text{(d)} y(z) = \frac{-z}{z+1} \end{array}$$

Solution:

$$\begin{aligned} Z(y_n) - zy_0 &= 0 \\ zy(z) - zy_0 - y(z) &= 0 \\ (z-1)y(z) &= z \\ y(z) &= \frac{z}{z-1} \end{aligned}$$

15. Find $Z[(e^n)^{100} + (e^n)^{200}]$

$$\begin{array}{ll} \text{(a)} \frac{z}{z-e^{100}} + \frac{z}{z-e^{200}} & \text{(b)} \frac{z}{z+e^{100}} + \frac{z}{z-e^{200}} \\ \text{(c)} \frac{z}{z-e^{100}} + \frac{z}{z+e^{200}} & \text{(d)} \frac{z}{z+e^{100}} + \frac{z}{z+e^{200}} \end{array}$$

Solution:

$$\begin{aligned} Z[(e^n)^{100} + (e^n)^{200}] &= Z((e^{100})^n) + Z((e^{200})^n) \\ &= \frac{z}{z-e^{100}} + \frac{z}{z-e^{200}} \end{aligned}$$

16. Using Final value theorem evaluate $f(z) = \frac{1+z^{-1}}{1-0.25z^{-1}}$

$$\begin{array}{ll} \text{(a)} 0 & \text{(b)} 1 \\ \text{(c)} 2 & \text{(d)} 3 \end{array}$$

Solution:

$$\text{Let } F(z) = \frac{z+1}{z-0.25}$$

$$\lim_{z \rightarrow 1} (z-1)F(z) = \lim_{z \rightarrow 1} (z-1) \frac{z+1}{z-0.25} = 0$$

17. Find the initial value of $F(z) = \frac{z}{2z^2 - 3z + 1}$

- (a) 0 (b) 1 (c) -1 (d) 2

Solution:

$$f(0) = \lim_{z \rightarrow \infty} F(z)$$

$$0 = \frac{1}{z \left(2 - \frac{3}{z} + \frac{1}{z^2} \right)} = \frac{z}{z^2 \left(2 - \frac{3}{z} + \frac{1}{z^2} \right)}$$

18. Evaluate $Z[(k-1)a^{k-1}]$

(a) $\frac{a}{(z-a)^2}$

(b) $\frac{a}{(z+a)^2}$

(c) $\frac{1}{(z-a)^2}$

(d) $\frac{1}{(z+a)^2}$

Solution

$$\begin{aligned} Z[(k-1)a^{k-1}] &= z^{-1}Z[ka^k] \\ &= z^{-1} \left(\frac{az}{(z-a)^2} \right) = \frac{a}{(z-a)^2} \end{aligned}$$

19. Find $Y(z)$ for $y_{n+2} + 4y_n = 0$, $y_0 = 0$, $y_1 = -2$

(a) $Y(z) = \frac{z}{z^2+4}$ (b) $Y(z) = \frac{2z}{z^2-4}$

(c) $Y(z) = \frac{2z}{z^2+4}$ (d) $Y(z) = \frac{-2z}{z^2+4}$

Solution:

$$z^2Y(z) - z^2y(0) - zy(1) - 4Y(z) = 0$$

$$z^2Y(z) - z(2) + 4Y(z) = 0$$

$$Y(z) = \frac{2z}{z^2 + 4}$$

20. Find $Z\{e^{3n}\}$

- (a) $-\cos x$ (b) $\cos x$ (c) $-\sin x$ (d) $\sin x$

Solution:

$$Z\{e^{3n}\} = Z\{(e^3)^n\} = \frac{z}{z - e^3}$$

21. Find $Z\{(-3)^n\}$

- (a) $\frac{z}{z+3}$ (b) $\frac{-z}{z+3}$ (c) $\frac{z}{z-3}$ (d) $\frac{2z}{z+3}$

Solution:

$$Z\{a^n\} = \frac{z}{z - a}$$

$$Z\{(-3)^n\} = \frac{z}{z - (-3)} = \frac{z}{z + 3}$$

22. Find $Z\{2^n + (-5)^n\}$

- (a) $\frac{z}{z+2} + \frac{z}{z-5}$ (b) $\frac{z}{z-2} + \frac{z}{z-5}$ (c) $\frac{z}{z-2} + \frac{z}{z+5}$ (d) $\frac{z}{z+2} + \frac{z}{z-5}$

Solution:

$$\begin{aligned} Z\{2^n + (-5)^n\} &= Z\{2^n\} + Z\{(-5)^n\} \\ &= \frac{z}{z - 2} + \frac{z}{z + 5} \end{aligned}$$

23. Find $Z\{4.8^n + (-4)(-6)^n\}$

(a) $\frac{4z}{z-8} + \frac{4z}{z+6}$

(b) $\frac{4z}{z-8} - \frac{4z}{z+6}$

(c) $\frac{4z}{z+8} - \frac{4z}{z-6}$

(d) $\frac{4z}{z+8} + \frac{4z}{z-6}$

Solution:1

$$\begin{aligned}
 1Z\{4.8^n + (-4)(-6)^n\} &= 4Z\{8^n\} + (-4)Z\{(-6)^n\} \\
 &= \frac{4z}{z-8} - \frac{4z}{z+6}
 \end{aligned}$$

24. Find $Z^{-1}\left\{\frac{z}{z+7} + 2.\frac{z}{z-3}\right\}$

(a) $(7)^n + 2(-3)^n$

(b) $(-7)^n - 2(3)^n$

(c) $(7)^n + 2(3)^n$

(d) $(-7)^n + 2(3)^n$

Solution:

$$\begin{aligned}
 Z^{-1}\left\{\frac{z}{z+7} + 2.\frac{z}{z-3}\right\} &= Z^{-1}\left\{\frac{z}{z+7}\right\} + 2 Z^{-1}\left\{\frac{2}{z-3}\right\} \\
 &= (-7)^n + 2(3)^n
 \end{aligned}$$

25. Find $Z^{-1}\left\{\frac{z}{z-8} + 3.\frac{z}{z+3}\right\}$

(a) $(8)^n - 3(-3)^n$

(b) $(-8)^n + 3(3)^n$

(c) $(-8)^n + 3(-3)^n$

(d) $(8)^n + 3(-3)^n$

Solution:

$$\begin{aligned}
 Z^{-1}\left\{\frac{z}{z-8} + 3.\frac{z}{z+3}\right\} &= Z^{-1}\left\{\frac{z}{z-8}\right\} + 3 Z^{-1}\left\{\frac{2}{z+3}\right\} \\
 &= (8)^n + 3(-3)^n
 \end{aligned}$$