SRM INSTITUTE OF SCIENCE AND TECHNOLOGY
RAMAPURAM CAMPUS
DEPARTMENT OF MATHEMATICS

Year/Sem : II/III

Branch: Common to All branches

Unit 2 – Fourier Series

1. Write the formula for finding Euler's constants of a Fourier series in $0 \le x \le 2\pi$.

Solution:

Euler's constants of a Fourier series in $0 \le x \le 2\pi$ is given by

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\,\sin nx\,dx$$

2. Write the formula for finding Euler's constants of a Fourier series in $0 \le x \le 2l$.

Solution:

Euler's constants of a Fourier series in $0 \le x \le l$ is given by

$$a_0 = \frac{1}{l}\int_0^{2l} f(x)\,dx$$

$$a_n = \frac{1}{l}\int_0^{2l} f(x)\cos nx\,dx$$

$$b_n = \frac{1}{l}\int_0^{2l} f(x)\,\sin nx\,dx$$

3. Write the formula for Fourier constants for f(x) in the interval $-\pi \le x \le \pi$.

Solution:

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,\sin nx\,dx$$

4. Write the formula for Fourier constants for f(x) in the interval $-l \le x \le l$.

Solution:

$$a_0 = \frac{1}{l}\int_{-l}^{l} f(x)\,dx$$

$$a_n = \frac{1}{l}\int_{-l}^{l} f(x)\cos nx\,dx$$

$$b_n = \frac{1}{l}\int_{-l}^{l} f(x)\,\sin nx\,dx$$

5. Find the constant value a_0 of the Fourier series for the function f(x) = k, $0 \le x \le 2\pi$.

Solution:

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx \qquad = \frac{1}{\pi}\int_0^{2\pi} k\,dx \qquad = \frac{k}{\pi}(2\pi) \qquad = 2k$$

a) K   b) **2k**   c) 0   d) k/2

6. Find the constant value a_0 of the Fourier series for the function f(x) = x, $0 \le x \le \pi$.

Solution:

$$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx \qquad = \frac{2}{\pi}\int_0^{\pi} x\;dx \qquad = \frac{2}{\pi}\frac{(\pi)^2}{2} \qquad = \pi$$

a) π b)2 π c)0   d) π /2

7. $If\;\; f(x) = e^x \;\; in\; -\pi \le x \le \pi$, find a_n

Solution:

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx \qquad = \frac{1}{\pi}\int_{-\pi}^{\pi} e^x\cos nx\,dx \quad = \frac{1}{\pi}\left\{\frac{e^x}{\left(1+n^2\right)}\left(\cos nx + n\sin nx\right)\right\}_{-\pi}^{\pi}$$

$$= \frac{(-1)^n}{\pi\left(1+n^2\right)}\left(e^{\pi} - e^{-\pi}\right)$$

a) $\frac{(-1)^n}{\pi\left(1+n^2\right)}\left(e^{\pi} - e^{-\pi}\right)$ b) $\frac{(-1)^n}{\pi\left(1-n^2\right)}\left(e^{\pi} - e^{-\pi}\right)$

c) $\frac{(-1)^n}{\pi\left(1-n^2\right)}\left(e^{\pi} + e^{-\pi}\right)$ d) $\frac{(-1)^n}{\pi}\left(e^{\pi} - e^{-\pi}\right)$

8. a) $If\;\; f(x) = e^x \;\; in\; -\pi \le x \le \pi$, find b_n

Solution:

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx \qquad = \frac{1}{\pi}\int_{-\pi}^{\pi} e^x\sin nx\,dx \quad = \frac{1}{\pi}\left\{\frac{e^x}{\left(1+n^2\right)}\left(\sin nx - n\cos nx\right)\right\}_{-\pi}^{\pi}$$

$$= \frac{n(-1)^{n+1}}{\pi\left(1+n^2\right)}\left(e^{\pi} - e^{-\pi}\right)$$

a) $\frac{n(-1)^{n+1}}{\pi\left(1+n^2\right)}\left(e^{\pi} - e^{-\pi}\right)$ b) $\frac{1}{\pi\left(1-n^2\right)}\left(e^{\pi} - e^{-\pi}\right)$

c) $\frac{2}{\pi\left(1-n^2\right)}\left(e^{\pi} + e^{-\pi}\right)$ d) $\frac{(-1)^n}{\pi}\left(e^{\pi} - e^{-\pi}\right)$

b)

9. Check whether the function is even or odd, where $f(x)=\begin{cases}1+\dfrac{2x}{l}, & -l\le x\le 0\\ 1-\dfrac{2x}{l}, & 0\le x\le l\end{cases}$

Solution:

$$for\ -l\le\ x\le 0,\ \ f(-x)=1+\frac{2(-x)}{l}=1-\frac{2x}{l}=\ f(x), where\ 0\le x\le l$$

$\Rightarrow$ the given function is even function

a) Even function
b) odd function
c) constant function
d) neither even nor odd

10. Find the constant value a_n of the Fourier series for the function f(x) = x, $0\le x\le\pi$.

Solution:

$$a_n=\frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx\ =\ \frac{2}{\pi}\int_0^{\pi} x\cos nx\,dx$$

$$=\frac{2}{\pi}\left[x\frac{\sin nx}{n}+\frac{\cos nx}{n^2}\right]_0^{\pi}\ =\ \frac{2}{\pi}\left(\frac{(-1)^n-1}{n^2}\right)$$

a) $\frac{2}{\pi}\left(\frac{(-1)^n-1}{n^2}\right)$ b) 2π c) $\frac{2(-1)^n}{\pi}$ d) $\pi/2$

11. Find the constant value b_n of the Fourier series for the function f(x) = x, $-\pi\le x\le\pi$.

Solution:

$$b_n=\frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx\ =\ \frac{2}{\pi}\int_0^{\pi} x\sin nx\,dx$$

$$=\frac{2}{\pi}\left[x\frac{-\cos nx}{n}+\frac{\sin nx}{n^2}\right]_0^{\pi}\ =\ \frac{-2(-1)^n}{n}$$

a) $\frac{2}{\pi}\left(\frac{(-1)^n-1}{n^2}\right)$ b) $\frac{(-1)^n}{\pi}$ c) $\frac{-2(-1)^n}{n}$ d) $\pi/2$

12. Find the constant term of the Fourier series for the function f(x) = |x|, $-\pi\le x\le\pi$.

Solution:

$$a_0=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx\ =\ \frac{2}{\pi}\int_0^{\pi} x\,dx\ =\ \pi$$

a) π b) 2π c) 0 d) $\pi/2$

13. Find the Fourier coefficient b_n of the Fourier series for the function f(x) = x, $0\le x\le 2\pi$.

Solution:

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\,\sin nx\,dx \;=\; \frac{1}{\pi}\int_0^{2\pi} x\,\sin nx\,dx \;=\; \frac{1}{\pi}\left[x\frac{-\cos nx}{n} + \frac{\sin nx}{n^2}\right]_0^{2\pi}$$

$$= \left(-2(-1)^n\right)/n$$

a) π b) $\left(-2(-1)^n\right)/n$ c)0 d)3 π

14. Half-range cosine series for f(x) in $(0,\pi)$ is

a) $\dfrac{(a_0)}{2} + \sum_{n=1}^{\infty} a_n \cos nx$ b) $\sum_{n=1}^{\infty} b_n \cos nx$ c) $\sum_{n=1}^{\infty} a_n \cos nx$

d) $\dfrac{(a_0)^2}{4} + \dfrac{1}{2}\sum_{n=1}^{\infty} (a_n)^2 + (b_n)^2$

15. If f(x) = x^2 in $(-\pi,\pi)$ then the value of b_n is?

Solution:

Since the given function is even in the given interval, the Fourier coefficient b_n value is zero in this case.

a) 1 b)0 c)-1 d)2

16. In the Fourier series expansion of f(x) = sin x in $(-\pi,\pi)$. What is the value of a_n?

Solution:

The function $f(-x) = \sin(-x) = -sinx = -f(-x)\ so\ f(x) is\ odd\ function$

So $a_n = 0$

a) 1 b)0 c) π d) $-\pi$

17. Find the constant value a_0 from the following table:

X	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
F(x) = y	1	1.4	1.9	1.7	1.5	1.2	1

Solution:

$$a_0 = \frac{2\sum y}{n} = \frac{2(1+1.4+1.9+1.7+1.5+1.2)}{6} = 2.9$$

a) 1.9 b)2.9 c)4.9 d)6.9

18. What is the sum of the Fourier series at a point x = 1 where the function has a finite discontinuity.

Solution:

$$f(x) = \frac{f(x+x_0)+f(x-x_0)}{2}$$

Here $x_0 = 1\,, f(x) = \frac{f(x+1)+f(x-1)}{2}$

a) $\mathrm{f(x)} = \frac{\mathrm{f(x+1)+f(x-1)}}{2}$ b) $f(x) = \frac{f(x+1)}{2}$ c) $f(x) = \frac{f(x-1)}{2}$

d) $f(x) = f(x+1) + f(x-1)$

19. In the expansion of $f(x) = sinhx\ in\ (-\pi,\pi)$ as a Fourier Series, find the coefficient of a_n.

Solution:

$$f(x) = \sin h\,x = \frac{e^x - e^{-x}}{2}$$

$$f(-x) = \frac{e^{-x} - e^{x}}{2} = \frac{-(e^{x} - e^{-x})}{2} = -\sin h x = -f(x)$$

So, $f(x) is\ an\ odd\ function, the\ fourier\ coefficient\ \ a_n\ is\ 0$

a) 0 b)1 c)2 d)3

20. To what value, the Fourier series corresponding to f(x) = x^2 in (0, 2 π) converges at x = 0?

Solution:

The Fourier series converges to $\frac{f(0) + f(2\pi)}{2} = 2\pi^2$

a)π b)2π c)π^2 d)$\mathbf{2\pi^2}$

21. Examine whether the function $f(x) = \frac{1}{1-x}$, can be expanded in Fourier series in any interval including $x = 1$

Solution:

At $x = 1, the\ function\ f(x) = \frac{1}{1-x}\ \ is\ not\ continuous$

By Dirichlet`s condition, we cannot expand $f(x)$ as a Fourier series.

a) Can be expanded since f(x) is not continuous
b) Cannot be expanded since f(x) does not satisfies Dirichlet`s condition
c) Can be expanded since f(x) satisfies Dirichlet`s Condition
d) Cannot be expanded since f(x) is continuous

22. Find the constant term in the Fourier series corresponding to $f(x) = x - x^3\ in\ (-\pi, \pi)$

Solution:

$f(x) = x - x^3$

$f(-x) = -x + x^3 \quad = -(x - x^3) = -f(x)$

$f(x) is\ an\ odd\ function\ (-\pi, \pi)$

Hence Constant term $a_0 = 0$

a) 0 b)1 c)2 d)3

23. Find the R.M.S Value of the function $f(x) = x\ in\ (0, l)$.

Solution:

$$\mathrm{R.M.S} = \frac{\sqrt{\int_0^l \mathrm{x}^2 \mathrm{dx}}}{l} = \frac{\sqrt{\left(\frac{\mathrm{x}^3}{3}\right)_0^l}}{l} = \sqrt{\frac{l^3}{3l}} = \frac{l}{\sqrt{3}}$$

a) $\frac{l}{\sqrt{3}}$ b)$\frac{\mathrm{l}}{\sqrt{2}}$ c)$\frac{\mathrm{l}^2}{\sqrt{3}}$ d) $\frac{\mathrm{l}^2}{\sqrt{2}}$

24. Find the value of $\mathrm{a_n}$in the cosine series expansion of $\mathrm{f(x) = k\ in\ (0,10)}$.

Solution:

$$a_n = \frac{1}{5}\int_0^{10} k\ cos\frac{n\pi x}{10}\ dx = \frac{k}{5}\left[\frac{sin\frac{n\pi x}{10}}{\frac{n\pi}{10}}\right]_0^{10} = 0$$

a) 0 b)10 c)20 d)30

25. Find the R.M.S Value of the function $f(x) = k\ in\ (-l, l)$.

Solution:

$$\mathrm{R.M.S} = \frac{\sqrt{\int_{-l}^{l} \mathrm{k}^2 \mathrm{dx}}}{2l} == \frac{\sqrt{\mathrm{k}^2(\mathrm{x})_{-l}{}^{l}}}{2l} = k$$

a) $\frac{\mathbf{l}}{\sqrt{3}}$ b) $\mathbf{k}$ c) $\frac{l^2}{\sqrt{3}}$ d) $\frac{l^2}{\sqrt{2}}$

26. In the Fourier series expansion of f(x) = |sin x| in $(-\pi, \pi)$. What is the value of b_n?

Solution:

$f(-x) = |\sin(-x)| = |-sinx| = |sinx| = f(x)$

$since\ f(x)\ is\ an\ odd\ function\ b_n = 0$

a) 1 b) 0 c) π d) $-\pi$

27. Find b_1, if f(x) = k in 0 < x < π .

Solution:

$$b_1 = \frac{2}{\pi}\int_0^{\pi} f(x)sinx\,dx = \frac{2}{\pi}\int_0^{\pi} ksinx\,dx$$

$$= \frac{2}{\pi}k(-cosx)_0{}^{\pi} = \frac{2}{\pi}(-(-1)+1) = \frac{4}{\pi}$$

a) $\frac{2}{\pi}$ b) $\frac{\mathbf{4}}{\boldsymbol{\pi}}$ c) $\frac{1}{2\pi}$ d) $\frac{1}{4\pi}$

28. Find the R.M.S Value of the function $f(x) = 2x\ in\ (0,3)$.

Solution:

$$R.M.S = \frac{\sqrt{\int_0^3 4x^2 dx}}{3} = \frac{\sqrt{4\left(\frac{x^3}{3}\right)_0^3}}{3} = \frac{6}{3} = 2$$

a) 2 b) 6 c) 9 d) 0

29. F(x) = x + x^2 in $(-\pi, \pi)$ is ____________

Solution:

$F(-x) = -x + (-x)^2 = -x + x^2 \neq -f(x)$ so it is neither ever nor odd function

a) Odd function b) Even function

c) Constant function d) Neither odd nor even

30. Find the series value $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} + \ldots$, if f(x) = x^2 in the interval ($-\pi$, π), $a_0 = 2\pi^2/3$, $a_n = 4(-1)^{n+1}/n^2$

Solution:

The Fourier cosine series is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4(-1)^{n+1}}{n^2}\cos nx$$

When x = 0,

$$f(0) = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4(-1)^{n+1}}{n^2}$$

$$\frac{f(-\pi) + f(\pi)}{2} = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4(-1)^{n+1}}{n^2}$$

$$\pi^2 = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4(-1)^{n+1}}{n^2} \quad \Rightarrow \quad \frac{\pi^2}{12} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$$

a) $\frac{\pi^2}{12}$ b) $\frac{\pi^2}{8}$ c) $\frac{\pi^2}{2}$ d)0

Answers

5	b	6	a	7	a	8	a	9	a	10	a	11	c	12	a	13	b
14	a	15	b	16	b	17	b	18	a	19	a	20	d	21	b	22	a
23	a	24	a	25	b	26	b	27	b	28	a	29	d	30	a		