SRM INSTITUTE OF SCIENCE AND TECHNOLOGY
RAMAPURAM CAMPUS
DEPARTMENT OF MATHEMATICS

Year/Sem : II/III

Branch: Common to All branches

Unit 4 – Fourier Transforms

1. The Fourier Transforms of $f(x) = 1 \; in \; a < x < b \; is$

   (a) $\boldsymbol{F[f(x)] = \frac{1}{is\sqrt{2\pi}}[e^{ibs} - e^{ias}]}$

   (b) $F[f(x)] = \frac{1}{\sqrt{2\pi}}\left[e^{ibs} - e^{ias}\right]$

   (c) $F[f(x)] = \frac{1}{i\sqrt{\pi}}\left[e^{ibs} - e^{ias}\right]$

   (d) $F[f(x)] = \frac{1}{is\sqrt{2\pi}}[e^{ias} - e^{ibs}]$

   Solution: $F[f(x)] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{isx}dx$

   $$= \frac{1}{\sqrt{2\pi}}\int_a^b 1.\, e^{isx}dx \;\; = \frac{1}{\sqrt{2\pi}}\left[\frac{e^{isx}}{is}\right]_a^b = \frac{1}{is\sqrt{2\pi}}[e^{ibs} - e^{ias}]$$

   Ans: a

2. The Fourier Transforms of $f(x) = e^{-a|x|}, a > 0 \; is$

   (a) $\sqrt{\frac{1}{2\pi}}\left[\frac{a}{a^2+s^2}\right]$ **(b)** $\boldsymbol{\sqrt{\frac{2}{\pi}}\left[\frac{a}{a^2+s^2}\right]}$ (c) $\sqrt{\frac{1}{\pi}}\left[\frac{1}{a^2+s^2}\right]$ (d) $\sqrt{\frac{2a}{\pi}}\left[\frac{s}{a^2+s^2}\right]$

   Solution: $F\left[e^{-a|x|}\right] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-a|x|}e^{isx}dx$

   $$= \frac{1}{\sqrt{2\pi}}\int_0^a (cos\, sx + isinsx)e^{-a|x|}dx$$

   $$= \frac{2}{\sqrt{2\pi}}\int_0^a cos\, sx\; e^{-ax}dx \;\; = \;\; \sqrt{\frac{\boldsymbol{2}}{\boldsymbol{\pi}}}\left[\frac{\boldsymbol{a}}{\boldsymbol{a^2 + s^2}}\right]$$

   Ans: b

3. $If \;\; F[f(x)] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{isx}dx = F(s), \; then \; F[\int_a^x f(x)dx] =$

   (a) $\frac{1}{is}F(s)$ (b) $-\frac{1}{2is}F(s)$ **(c)** $\boldsymbol{-\frac{1}{is}F(s)}$ (d) $-\frac{2}{is}F(s)$

   Solution:

   Let $\int_a^x f(x)dx = g(x) \;\; i.e., \;\; f(x)dx = g'(x)$

   If $F[g(x)] = G(s), \;\; F[g'(x)] = \; -isG(s)$

$F[f(x)] = -isF[g(x)] = -\frac{\mathbf{1}}{\boldsymbol{is}}\boldsymbol{F}(\boldsymbol{s})$

Ans: c

4. The Fourier Transforms of $f(x) = e^{ikx}, a < x < b\ is$

(a) $\frac{1}{i(ks)\sqrt{2\pi}}[e^{i(ks)b} - e^{i(ks)a}]$

(b) $\frac{1}{i(k-s)\sqrt{2\pi}}[e^{i(k-s)b} - e^{i(k-s)a}]$

(c) $\frac{2}{i(k+s)\sqrt{\pi}}[e^{i(k+s)b} - e^{i(k+s)a}]$

(d) $\boldsymbol{\frac{1}{i(k+s)\sqrt{2\pi}}[e^{i(k+s)b} - e^{i(k+s)a}]}$

Solution: $F[f(x)] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{isx}dx$

$$F\left[e^{ikx}\right] = \frac{1}{\sqrt{2\pi}}\int_a^b e^{ikx}e^{isx}dx$$

$$= \frac{1}{\sqrt{2\pi}}\int_a^b e^{i(k+s)x}dx \quad = \frac{1}{\sqrt{2\pi}}\left[\frac{e^{i(k+s)x}}{i(k+s)}\right]_a^b = \frac{1}{i(k+s)\sqrt{2\pi}}[e^{i(k+s)b} - e^{i(k+s)a}]$$

Ans:d

5. The Fourier Transforms of $f(x) = 1, |x| \le a\ is$

(a) $\boldsymbol{\sqrt{\frac{2}{\pi}}\left(\frac{\sin as}{s}\right)}$ (b) $\sqrt{\frac{1}{\pi}}\left(\frac{\sin as}{s}\right)$ (c) $\sqrt{\frac{1}{2\pi}}\left(\frac{\cos as}{s}\right)$ (d) $\sqrt{\frac{2}{\pi}}\left(\frac{\cos as}{s}\right)$

Solution: $F[f(x)] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{isx}dx$

$$= \frac{1}{\sqrt{2\pi}}\int_{-a}^{a} 1.e^{isx}dx \quad = \frac{1}{\sqrt{2\pi}}\left[\frac{e^{isx}}{is}\right]_{-a}^{a} = \frac{1}{is\sqrt{2\pi}}[e^{isa} - e^{-ias}]$$

$$= \frac{1}{is\sqrt{2\pi}}[2i\sin as] = \sqrt{\frac{2}{\pi}}\left(\frac{\sin as}{s}\right)$$

Ans:a

6. If $f(x) = 1\ and \quad F[f(x)] = \sqrt{\frac{2}{\pi}}\left(\frac{\sin as}{s}\right)\ then\ by\ Parseval's\ Identity, \int_{-\infty}^{\infty}\left(\frac{\sin as}{s}\right)^2 ds\ is$

(a) $2\pi a$ **(b)** $\boldsymbol{\pi a}$ (c) π/a (d) 2π

Solution: $\int_{-a}^{a} 1dx = \int_{-\infty}^{\infty}\sqrt{\frac{2}{\pi}}\left(\frac{\sin as}{s}\right)^2 ds$

$2a = \frac{2}{\pi}\int_{-\infty}^{\infty}\left(\frac{\sin as}{s}\right)^2 ds \;=\; \int_{-\infty}^{\infty}\left(\frac{\sin as}{s}\right)^2 ds \;=\; \pi a$

Ans: b

7. $If\ F_s[x\,f(x)] = -\frac{d}{ds}F_c[\,f(x)],\ then\ F_s[x\,e^{-ax}]\ is$

(a) $\sqrt{\frac{2}{\pi}}\left[\frac{as}{(a^2+s^2)^2}\right]$ (b) $\sqrt{\frac{1}{\pi}}\left[\frac{2as}{(a^2+s^2)^2}\right]$ **(c)** $\sqrt{\frac{2}{\pi}}\left[\frac{2as}{(a^2+s^2)^2}\right]$ (d) $\sqrt{\frac{2}{\pi}}\left[\frac{2\pi}{(a^2+s^2)^2}\right]$

Solution: $F_s[x\,e^{-ax}] = -\frac{d}{ds}F_c[e^{-ax}]$

$$= -\frac{d}{ds}\sqrt{\frac{2}{\pi}}\left[\frac{a}{(a^2+s^2)}\right] = \sqrt{\frac{2}{\pi}}\left[\frac{2as}{(a^2+s^2)^2}\right]$$

Ans:c

8. $If\ F_c[x\,f(x)] = \frac{d}{ds}F_s[\,f(x)],\ then\ F_c[x\,e^{-ax}]\ is$

(a) $\sqrt{\frac{2}{\pi}}\left[\frac{a^2}{(a^2+s^2)^2}\right]$ (b) $\sqrt{\frac{2}{\pi}}\left[\frac{s^2}{(a^2+s^2)^2}\right]$

(c) $\sqrt{\frac{2}{\pi}}\left[\frac{a^2-s^2}{(a^2-s^2)^2}\right]$ **(d)** $\sqrt{\frac{2}{\pi}}\left[\frac{a^2-s^2}{(a^2+s^2)^2}\right]$

Solution: $F_c[x\,e^{-ax}] = \frac{d}{ds}F_s[e^{-ax}]$

$$= \frac{d}{ds}\sqrt{\frac{2}{\pi}}\left[\frac{s}{(a^2+s^2)}\right] = \sqrt{\frac{2}{\pi}}\left[\frac{a^2-s^2}{(a^2+s^2)^2}\right].$$

Ans: d

9. $If\ f(x) = e^{-ax}\ \ and\ F_c[x\,e^{-ax}] = \sqrt{\frac{2}{\pi}}\left[\frac{a}{(a^2+s^2)}\right]$
$then\ by\ Parseval's\ Identity\ \int_0^{\infty}\frac{1}{(a^2+s^2)^2}ds\ \ is$

(a) $\frac{\pi}{4a^3}$ (b) $\frac{\pi}{a^3}$ (c) $\frac{\pi}{2a^2}$ (d) $\frac{3\pi}{4a^3}$

Solution:

By Parseval's Identity, $\int_0^{\infty}|F_c[\,s]|^2 ds \;=\; \int_0^{\infty}|f(x)|^2 dx$

$$\frac{2a^2}{\pi}\int_0^{\infty}\frac{1}{(a^2+s^2)^2}ds \;=\; \int_0^{\infty}e^{-2ax}dx$$

$$\int_0^{\infty}\frac{1}{(a^2+s^2)^2}ds \;=\; \frac{\pi}{4a^3}$$

Ans: a

10. $If\ f(x) = e^{-ax}\ and\ F_s[\,e^{-ax}] = \sqrt{\frac{2}{\pi}}\left[\frac{s}{(a^2+s^2)}\right]$

$then\ by\ Parseval's\ Identity\ \int_0^\infty \frac{s^2}{(a^2+s^2)^2} ds\ \ is$

(a) $\frac{\pi}{4a^3}$ **(b)** $\frac{\pi}{4a}$ (c) $\frac{\pi}{2a^2}$ (d) $\frac{3\pi}{4a^3}$

Solution: By Parseval's Identity,

$$\int_0^\infty |F_c[\,s]|^2 ds = \int_0^\infty |f(x)|^2 dx$$

$$\frac{2}{\pi}\int_0^\infty \frac{s^2}{(a^2+s^2)^2} ds = \int_0^\infty e^{-2ax} dx$$

$$\int_0^\infty \frac{s^2}{(a^2+s^2)^2} ds = \frac{\pi}{4a}$$

Ans: b

11. If $\frac{2}{\pi}\int_0^\infty \frac{10}{(s^2+4)(s^2+25)} ds = \int_0^\infty e^{-7x} dx$,

then $\int_0^\infty \frac{dx}{(x^2+4)(x^2+25)} =$

(a) $\frac{\pi}{120}$ **(b)** $\frac{\pi}{160}$ **(c)** $\frac{\pi}{140}$ (d) $\frac{3\pi}{180}$

Solution:

$$\int_0^\infty \frac{dx}{(s^2+4)(s^2+25)} = \frac{\pi}{20}\int_0^\infty e^{-7x} dx = \frac{\pi}{140}$$

Ans: c

12. If $\int_0^\infty e^{-ax}\cos bx\, dx = \frac{a}{(a^2+b^2)}$, then FCT of $f(x) = 3e^{-5x} + 5e^{-2x}$ is

(a) $\sqrt{\frac{2}{\pi}}\left[\frac{25}{(25+s^2)} + \frac{10}{(4+s^2)}\right]$ (b) $\sqrt{\frac{2}{\pi}}\left[\frac{5}{(25+s^2)} + \frac{20}{(4+s^2)}\right]$

(c) $\sqrt{\frac{2}{\pi}}\left[\frac{10}{(25+s^2)} + \frac{15}{(4+s^2)}\right]$ **(d)** $\sqrt{\frac{2}{\pi}}\left[\frac{15}{(25+s^2)} + \frac{10}{(4+s^2)}\right]$

Solution:

$$F_c[f(x)] = 3F_c[e^{-5x}] + 5F_c[e^{-2x}]$$

$$= \sqrt{\frac{2}{\pi}}\left\{3\left[\frac{5}{(5^2+s^2)}\right] + 5\left[\frac{2}{(2^2+s^2)}\right]\right\}$$

$$= \sqrt{\frac{2}{\pi}}\left[\frac{15}{(25+s^2)} + \frac{10}{(4+s^2)}\right]$$

Ans: d

13. By Fourier Transforms identity, if

$$\frac{2ab}{\pi}\int_0^\infty \frac{1}{(s^2+a^2)(s^2+b^2)} ds = \int_0^\infty e^{-(a+b)x} dx\ ,$$

Then $\int_0^\infty \frac{\mathbf{1}}{(\boldsymbol{x^2+a^2})(\boldsymbol{x^2+b^2})}dx$ is

(a) $\frac{\boldsymbol{\pi}}{\boldsymbol{2ab(a+b)}}$ (b) $\frac{\boldsymbol{\pi}}{\boldsymbol{2b(a+b)}}$ (c) $\frac{\boldsymbol{\pi}}{\boldsymbol{2a(a+b)}}$ (d) $\frac{\boldsymbol{\pi}}{\boldsymbol{2ab(a-b)}}$

Solution:

$$\int_0^\infty \frac{\mathbf{1}}{(\boldsymbol{s^2+a^2})(\boldsymbol{s^2+b^2})}ds = \frac{\boldsymbol{\pi}}{\boldsymbol{2ab}}\int_0^\infty e^{-(a+b)x}dx\ ,$$

$$= \frac{\boldsymbol{\pi}}{\boldsymbol{2ab(a+b)}}$$

Ans: a

14. If $F_c\left[\frac{e^{-ax}}{x}\right] = -\frac{1}{\sqrt{2\pi}}\log(a^2+s^2)$ then $F_c\left[\frac{e^{-ax}-e^{-bx}}{x}\right]$ is

(a) $\frac{1}{\sqrt{2\pi}}\log\left(\frac{b^2}{a^2+s^2}\right)$ **(b)** $\boldsymbol{\frac{1}{\sqrt{2\pi}}\log\left(\frac{b^2+s^2}{a^2+s^2}\right)}$

(c) $\frac{1}{\sqrt{2\pi}}\log\left(\frac{a^2}{a^2+s^2}\right)$ (d) $\frac{1}{\sqrt{2\pi}}\log\left(\frac{b^2-s^2}{a^2+s^2}\right)$

Solution:

$F_c\left[\frac{e^{-ax}-e^{-bx}}{x}\right] = -\frac{1}{\sqrt{2\pi}}\log(a^2+s^2) + \frac{1}{\sqrt{2\pi}}\log(b^2+s^2)$

$$= \frac{1}{\sqrt{2\pi}}\log\left(\frac{b^2+s^2}{a^2+s^2}\right)$$

Ans:b

15. If $F_s\left[\frac{e^{-ax}}{x}\right] = -\sqrt{\frac{2}{\pi}}\,tan^{-1}\left(\frac{a}{s}\right)$ then $\int_0^\infty\left(\frac{e^{-ax}-e^{-bx}}{x}\right)\sin sx\,dx$ is

(a) $\left(tan^{-1}\left(\frac{a}{s}\right) - tan^{-1}\left(\frac{b}{s}\right)\right)$ (b) $\left(tan^{-1}\left(\frac{b}{s}\right) + tan^{-1}\left(\frac{a}{s}\right)\right)$

(c) $\boldsymbol{\left(tan^{-1}\left(\frac{b}{s}\right) - tan^{-1}\left(\frac{a}{s}\right)\right)}$ (d) $\left(tan^{-1}\left(\frac{a}{s}\right) + tan^{-1}\left(\frac{b}{s}\right)\right)$

Solution:

$F_c\left[\frac{e^{-ax}-e^{-bx}}{x}\right] = \sqrt{\frac{2}{\pi}}\left(tan^{-1}\left(\frac{b}{s}\right) - tan^{-1}\left(\frac{a}{s}\right)\right)$

$\sqrt{\frac{2}{\pi}}\int_0^\infty\left(\frac{e^{-ax}-e^{-bx}}{x}\right)\sin sx\,dx = \sqrt{\frac{2}{\pi}}\left(tan^{-1}\left(\frac{b}{s}\right) - tan^{-1}\left(\frac{a}{s}\right)\right)$

$\int_0^\infty\left(\frac{e^{-ax}-e^{-bx}}{x}\right)\sin sx\,dx = \left(tan^{-1}\left(\frac{b}{s}\right) - tan^{-1}\left(\frac{a}{s}\right)\right)$

Ans: c

16. If $f(x) = \frac{\boldsymbol{2a}}{\boldsymbol{\pi}}\int_{\mathbf{0}}^\infty \frac{\boldsymbol{\cos sx}}{\boldsymbol{s^2+a^2}}\boldsymbol{ds}$ (where $f(x) = e^{-ax}, a > 0$)

then by Inverse Fourier Transform $\int_0^\infty \frac{\cos mx}{x^2+a^2}dx$ is

(a) $\frac{\pi}{2a}e^{-ms}$ (b) $\frac{\pi}{2m}e^{-ma}$ (c) $\frac{\pi}{4a}e^{-ma}$ **(d)** $\frac{\pi}{2a}e^{-ma}$

Solution:

$$\int_0^\infty \frac{\cos sx}{x^2+a^2}ds = \frac{\pi}{2a}e^{-ax}$$

By changing the dummy variables of integration s=x and x=m then

$$\int_0^\infty \frac{\cos mx}{x^2+a^2}dx = \frac{\pi}{2a}e^{-ma}$$

Ans:d

17. If $f(x) = \frac{2}{\pi}\int_0^\infty \frac{s \sin sx}{s^2+a^2}ds$ (where $f(x) = e^{-ax}, a > 0$)

then by Inverse Fourier Transform $\int_0^\infty \frac{x \sin mx}{x^2+a^2}dx$ is

(a) $\frac{\pi}{2}e^{-am}$ (b) $\frac{\pi}{2m}e^{-ma}$ (c) $\frac{\pi}{4a}e^{-ma}$ (d) $\frac{\pi}{2a}e^{-ma}$

Solution:

$$\int_0^\infty \frac{s \sin sx}{x^2+a^2}ds = \frac{\pi}{2}e^{-ax}$$

By changing the dummy variables of integration s=x and x=m then

$$\int_0^\infty \frac{x \sin mx}{x^2+a^2}dx = \frac{\pi}{2}e^{-am}$$

Ans: a

18. If $\int_0^\infty \frac{\sin t}{t}dt = \frac{\pi}{2}$, then $F_s\left[\frac{1}{x}\right]$ is

(a) $\sqrt{\frac{2}{\pi}}$ **(b)** $\sqrt{\frac{\pi}{2}}$ (c) $\sqrt{\frac{1}{\pi}}$ (d) $\sqrt{\frac{1}{2\pi}}$

Solution: $F_s\left[\frac{1}{x}\right] = \sqrt{\frac{2}{\pi}} \int_0^\infty \frac{1}{x}\sin sx\, dx$

$$Put\ sx = \theta\ then\ \frac{1}{x} = \frac{s}{\theta}$$

$dx = \frac{d\theta}{s}$, then $F_s\left[\frac{1}{x}\right] = \sqrt{\frac{\pi}{2}}$

Ans: b

19. If $\int_0^\infty e^{-ax}\cos bx\, dx = \frac{a}{(a^2+b^2)}$, then FCT of $f(x) = e^{-2x} + 3e^{-x}$ is

(a) $\sqrt{\frac{2}{\pi}}\left[\frac{2}{(24+s^2)} + \frac{3}{(9+s^2)}\right]$ (b) $\sqrt{\frac{2}{\pi}}\left[\frac{1}{(25+s^2)} + \frac{2}{(4+s^2)}\right]$

(c) $\sqrt{\frac{2}{\pi}}\left[\frac{2}{(4+s^2)}+\frac{3}{(1+s^2)}\right]$ (d) $\sqrt{\frac{2}{\pi}}\left[\frac{5}{(25+s^2)}+\frac{1}{(4+s^2)}\right]$

Solution: $F_c[f(x)] = F_c[e^{-2x}] + 3F_c[e^{-x}]$

$$\sqrt{\frac{2}{\pi}}\left\{\left[\frac{2}{(4+s^2)}\right]+3\left[\frac{1}{(1+s^2)}\right]\right\}$$

$$=\sqrt{\frac{2}{\pi}}\left[\frac{2}{(4+s^2)}+\frac{3}{(1+s^2)}\right]$$

Ans:c

20. The Fourier sine Transforms of $f(x) = 1, 0 < x < 1\ is$

(a) $\sqrt{\frac{2}{\pi}}\left[\frac{\cos s}{s}\right]$ (b) $\sqrt{\frac{2}{\pi}}\left[\frac{1+\cos s}{s}\right]$ (c) $\sqrt{\frac{1}{\pi}}\left[\frac{2-\cos s}{s}\right]$ **(d)** $\sqrt{\frac{2}{\pi}}\left[\frac{1-\cos s}{s}\right]$

Solution: $F_s[f(x)] = \sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\sin sx\ dx = \sqrt{\frac{2}{\pi}}\int_0^1 \sin sx\ dx =$

$$\sqrt{\frac{2}{\pi}}\left[-\frac{\cos sx}{s}\right]_0^1 = \sqrt{\frac{2}{\pi}}\left[\frac{1-\cos s}{s}\right]$$

$$=\frac{1}{is\sqrt{2\pi}}[2i\sin as] = \sqrt{\frac{2}{\pi}}\left(\frac{\sin as}{s}\right)$$

Ans:d

21. If $\int_0^\infty e^{-ax}\sin bx\,dx = \frac{b}{(a^2+b^2)}$, then FST of $f(x) = e^{-2x}+3e^{-x}$ is

(a) $\sqrt{\frac{2}{\pi}}\left[\frac{s}{(4+s^2)}+\frac{3s}{(1+s^2)}\right]$ (b) $\sqrt{\frac{2}{\pi}}\left[\frac{s}{(25+s^2)}+\frac{2s}{(4+s^2)}\right]$

(c) $\sqrt{\frac{2}{\pi}}\left[\frac{2}{(4+s^2)}-\frac{3}{(1+s^2)}\right]$ (d) $\sqrt{\frac{2}{\pi}}\left[\frac{5s}{(25+s^2)}+\frac{s}{(4+s^2)}\right]$

Solution: $F_s[f(x)] = F_c[e^{-2x}] + 3F_c[e^{-x}]$

$$\sqrt{\frac{2}{\pi}}\left\{\left[\frac{s}{(4+s^2)}\right]+3\left[\frac{s}{(1+s^2)}\right]\right\}$$

$$=\sqrt{\frac{2}{\pi}}\left[\frac{s}{(4+s^2)}+\frac{3s}{(1+s^2)}\right]$$

Ans:a

22. The Fourier cosine Transforms of $f(x) = 1, 0 < x < 1\ is$

(a) $\sqrt{\frac{2}{\pi}}\left[\frac{\cos s}{s-a}\right]$ **(b)** $\sqrt{\frac{2}{\pi}}\left[\frac{\sin s}{s}\right]$ (c) $\sqrt{\frac{1}{\pi}}\left[\frac{\cos s}{s}\right]$ **(d)** $\sqrt{\frac{2}{\pi}}\left[\frac{1-\sin s}{s}\right]$

Solution: $F_c[f(x)] = \sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\cos sx\,dx = \sqrt{\frac{2}{\pi}}\int_0^1 \cos sx\ dx$

$$= \sqrt{\frac{2}{\pi}}\left[\frac{\sin sx}{s}\right]_0^1 = \sqrt{\frac{2}{\pi}}\left[\frac{\sin s}{s}\right]$$

Ans:b

23. If $\frac{2}{\pi}\int_0^\infty \frac{s}{(s^2+4)(s^2+25)}ds = \int_0^\infty e^{-4x}dx$,

then $\int_0^\infty \frac{x\,dx}{(x^2+4)(x^2+25)} =$

(a) $\frac{\pi}{12}$ (b) $\frac{\pi}{16}$ **(c)** $\frac{\pi}{14}$ **(d)** $\frac{\pi}{8}$

Solution:

$$\int_0^\infty \frac{sds}{(s^2+4)(s^2+25)} = \frac{\pi}{2}\int_0^\infty e^{-4x}dx = \frac{\pi}{8}$$

Put s=x.

Ans: d

24. By Fourier Transforms identity, if $\frac{2ab}{\pi}\int_0^\infty \frac{1}{(s^2+a^2)(s^2+b^2)}ds = \int_0^\infty e^{-(a+b)x}dx$,

Then $\int_0^\infty \frac{1}{(x^2+9)(x^2+16}dx$ is

(a) $\frac{\pi}{162}$ (b) $\frac{\pi}{186}$ **(c)** $\frac{\pi}{168}$ (d) $\frac{\pi}{816}$

Solution:

$$\int_0^\infty \frac{1}{(s^2+3^2)(s^2+4^2)}ds = \frac{\pi}{2(3)(4)}\int_0^\infty e^{-7x}dx = \frac{\pi}{168}$$

Ans: c$e^{-7x}dx$

25. The Fourier Transforms of $f(x) = 1\ in\ 1 < x < 2\ is$

(a) $F[f(x)] = \frac{1}{is\sqrt{2\pi}}[e^{i2s} - e^{is}]$

(b) $F[f(x)] = \frac{1}{\sqrt{2\pi}}[e^{is} + e^{i2s}]$

(c) $F[f(x)] = \frac{1}{i\sqrt{\pi}}[e^{ib} - e^{ia}]$

(d) $F[f(x)] = \frac{1}{is\sqrt{2\pi}}[e^{is} - e^{i2s}]$

Solution: $F[f(x)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx} dx$

$$= \frac{1}{\sqrt{2\pi}} \int_{1}^{2} 1.\, e^{isx} dx \quad = \frac{1}{\sqrt{2\pi}} \left[\frac{e^{isx}}{is}\right]_1^2 = \frac{1}{is\sqrt{2\pi}} [e^{i2s} - e^{is}]$$

Ans: a