SRM INSTITUTE OF SCIENCE AND TECHNOLOGY
RAMAPURAM CAMPUS
DEPARTMENT OF MATHEMATICS

Year/Sem : II/III
Branch: Common to All branches

Unit III – Applications of Partial Differential Equations

1.Write the possible solutions of one dimensional wave equation

(a) $y(x,t) = (\boldsymbol{c_1 e^{px} + c_2 e^{-px}})(\boldsymbol{c_3 e^{pat} + c_4 e^{-pat}})$

$$\boldsymbol{y(x,t) = (c_5 cospx + c_6 sinpx)(c_7 cospat + c_8 sinpat)}$$

$$\boldsymbol{y(x,t) = (c_9 x + c_{10})(c_{11} t + c_{12})}$$

(b) $y(x,t) = (c_1 e^{px} + c_2 e^{-px})(c_3 e^{pat} + c_4 e^{-pat})$

$$y(x,t) = (c_5 cospx - c_6 sinpx)(c_7 cospat + c_8 sinpat)$$

$$y(x,t) = (c_9 x + c_{10})(c_{11} t + c_{12} t^2)$$

(c) $y(x,t) = (c_1 e^{px} + c_2 e^{-px})(c_3 e^{at} + c_4 e^{-at})$

$$y(x,t) = (c_5 cospx + c_6 sinpx)(c_7 cospat + c_8 sinpat)$$

(d) $y(x,t) = (c_1 e^{px} + c_2 e^{-px})(c_3 e^{pat} + c_4 e^{-pat})$

$$y(x,t) = (c_9 x + c_{10} x^2)(c_{11} t + c_{12})$$

Solution:

The possible solution of one dimensional wave equation $\boldsymbol{\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}}$ ***are***

- $y(x,t) = (c_1 e^{px} + c_2 e^{-px})(c_3 e^{pat} + c_4 e^{-pat})$
- $y(x,t) = (c_5 cospx + c_6 sinpx)(c_7 cospat + c_8 sinpat)$
- $y(x,t) = (c_9 x + c_{10})(c_{11} t + c_{12})$

2. Write the all the possible solutions of one dimensional heat flow equation

(a) $u(x,t) = c_1(c_2x^2 + c_3)$,
$u(x,t) = c_3e^{\alpha^2p^2t}(c_4e^{px} + c_5e^{-px})$,
$u(x,t) = c_6e^{-\alpha^2p^2t}(c_7cospx + c_8sinpx)$

(b) $\boldsymbol{u(x,t) = c_1(c_2x + c_3)}$ **,**
$\boldsymbol{u(x,t) = c_3e^{\alpha^2p^2t}(c_4e^{px} + c_5e^{-px})}$ **,**
$\boldsymbol{u(x,t) = c_6e^{-\alpha^2p^2t}(c_7cospx + c_8sinpx)}$

(c) $u(x,t) = c_1(c_2x + c_3)$,
$u(x,t) = c_3e^{-\alpha^2p^2t}(c_4e^{px} + c_5e^{-px})$,
$u(x,t) = c_6e^{\alpha^2p^2t}(c_7cospx + c_8sinpx)$

(d) $u(x,t) = c_1(c_2x + c_3)$,
$u(x,t) = c_3e^{p^2t}(c_4e^{px} + c_5e^{-px})$,
$u(x,t) = c_6e^{-p^2t}(c_7cospx + c_8sinpx)$

Solution:

The possible solutions of one dimensional heat flow equation $\frac{\partial u}{\partial t} = \boldsymbol{\alpha^2 \frac{\partial^2 u}{\partial x^2}}$ are

- $u(x,t) = c_1(c_2x + c_3)$
- $u(x,t) = c_3e^{\alpha^2p^2t}(c_4e^{px} + c_5e^{-px})$
- $u(x,t) = c_6e^{-\alpha^2p^2t}(c_7cospx + c_8sinpx)$

3. Write all boundary conditions for one dimensional wave equation with zero initial velocity

(a) $y(0,t) = 0\,, for\ all\ t > 0$

$y(l,t) = l\,, for\ all\ t > 0$

$\frac{\partial y(x,0)}{\partial t} = 0$

$y(x,0) = f(x)$

(b) $y(0,t) = l\,, for\ all\ t > 0$

$y(l,t) = 0\,, for\ all\ t > 0$

$y(x,0) = 0$

$\frac{\partial y(x,0)}{\partial t} = f(x)$

(c) $y(0,t) = 0\,, for\ all\ t > 0$

$y(l,t) = 0\,, for\ all\ t > 0$

$y(x,0) = 0$

$\frac{\partial y(x,0)}{\partial t} = f(x)$

(d) $\boldsymbol{y(0,t) = 0\,, for\ all\ t > 0}$

$\boldsymbol{y(l,t) = 0\,, for\ all\ t > 0}$

$\boldsymbol{\frac{\partial y(x,0)}{\partial t} = 0}$

$\boldsymbol{y(x,0) = f(x)}$

Solution

One dimensional wave equation is $\boldsymbol{\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}}$**, boundary conditions are**

- $y(0,t) = 0\,, for\ all\ t > 0$
- $y(l,t) = 0\,, for\ all\ t > 0$
- $\frac{\partial y(x,0)}{\partial t} = 0\ (Initial\ Velocity = 0)$
- $y(x,0) = f(x)$

4. Write all boundary conditions for one dimensional heat flow equation

(a) $u(0,t) = l \; for \; all \; t > 0$
$u(l,t) = 0 \; for \; all \; t > 0$
$u(x,0) = f(x) \; for \; all \; x \; in \; (0,l)$

(b) $u(0,t) = 0 \; for \; all \; t > 0$
$u(l,t) = 0 \; for \; all \; t > 0$
$u(x,0) = 0 \; for \; all \; x \; in \; (0,l)$

(c) $\boldsymbol{u(0,t) = 0 \; for \; all \; t > 0}$
$\boldsymbol{u(l,t) = 0 \; for \; all \; t > 0}$
$\boldsymbol{u(x,0) = f(x) \; for \; all \; x \; in \; (0,l)}$

(d) $u(0,t) = 0 \; for \; all \; t > 0$
$u(l,t) = l \; for \; all \; t > 0$
$u(x,0) = f(x) \; for \; all \; x \; in \; (0,l)$

Solution:

One dimensional heat flow equation $\frac{\partial u}{\partial t} = \boldsymbol{\alpha^2 \frac{\partial^2 u}{\partial x^2} \; are}$

- $u(0,t) = 0 \; for \; all \; t > 0$
- $u(l,t) = 0 \; for \; all \; t > 0$
- $u(x,0) = f(x) \; for \; all \; x \; in \; (0,l)$

5. A string is stretched between two fixed points at a distance $2l$ apart and the points of the string are given velocity $f(x), x$ being the distance from end point. Formulate the problem to find the displacement of the string at any time

(a) $\boldsymbol{y(0,t) = 0\,, for\ all\ t > 0}$

$\boldsymbol{y(2l,t) = 0\,, for\ all\ t > 0}$

$\boldsymbol{y(x,0) = 0\ for\ all\ x\ in\ (0,2l)}$

$\boldsymbol{\frac{\partial y(x,0)}{\partial t} = f(x) forall\ x\ in\ (0,2l)}$

(a) $y(0,t) = 0\,, for\ all\ t > 0$

$y(2l,t) = 0\,, for\ all\ t > 0$

$\frac{\partial y(x,0)}{\partial t} = 0 forall\ x\ in\ (0,2l)$

$y(x,0) = f(x)\ for\ all\ x\ in\ (0,2l)$

(a) $y(0,t) = 2l\,, for\ all\ t > 0$

$y(2l,t) = 0\,, for\ all\ t > 0$

$y(x,0) = 0\ for\ all\ x\ in\ (0,2l)$

$\frac{\partial y(x,0)}{\partial t} = f(x) forall\ x\ in\ (0,2l)$

(a) $y(0,t) = 0\,, for\ all\ t > 0$

$y(2l,t) = 2l\,, for\ all\ t > 0$

$y(x,0) = 0\ for\ all\ x\ in\ (0,2l)$

$\frac{\partial y(x,0)}{\partial t} = f(x) forall\ x\ in\ (0,2l)$

Solution:

One dimensional wave equation is $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$**, boundary conditions are**

- $y(0,t) = 0\ , for\ all\ t > 0$

- $y(2l,t) = 0\ , for\ all\ t > 0$
- $y(x,0) = 0\ for\ all\ x\ in\ (0,2l)$
- $\frac{\partial y(x,0)}{\partial t} = f(x) forall\ x\ in\ (0,2l)$

6.A string of length $2l$ is stretched to a constant tension T, is fastened at both the ends and hence fixed. The mid points of the string is taken to a height 'b' and then released from rest in that position. Find the equation of the string in its initial position

(a) $y(x,0) = \begin{cases} \frac{bx}{l} & when\ 0 < x < l \\ \frac{b}{l}(2l + x) & when\ l < x < 2l \end{cases}$

(b) $\boldsymbol{y(x,0) = \begin{cases} \frac{bx}{l} & when\ 0 < x < l \\ \frac{b}{l}(2l - x) & when\ l < x < 2l \end{cases}}$

(c) $y(x,0) = \begin{cases} -\frac{bx}{l} & when\ 0 < x < l \\ \frac{b}{l}(2l - x) & when\ l < x < 2l \end{cases}$

(d) $y(x,0) = \begin{cases} -\frac{bx}{l} & when\ 0 < x < l \\ \frac{b}{l}(2l + x) & when\ l < x < 2l \end{cases}$

Solution:

The initial displacement of the string is in the form

$$y(x,0) = \begin{cases} \frac{bx}{l} & when\ 0 < x < l \\ \frac{b}{l}(2l - x) & when\ l < x < 2l \end{cases}$$

7.Classify the equation $U_{xx} - y^4 U_{yy} = 2y^3 U_y$

(a) Elliptic **(b) Hyperbolic** (c)Parabolic (d) Concentric

Solution:

Here $A = 1,\ B = 0, C = -y^4$

$$B^2 - 4AC = 0 - 4(1)(-y^4) = 4y^4 > 0$$

The equation is hyperbolic

8. Classify the equation $x^2 f_{xx} + (1 - y^2) f_{yy} = 0\ when\ x = 0$

(a) Elliptic (b) Hyperbolic **(c)Parabolic** (d) Concentric

Solution:

Here $A = x^2,\ B = 0, C = 1 - y^2$

$$B^2 - 4AC = 0 - 4(x^2)(1 - y^2) = 0 \quad (since\ x = 0)$$

The equation is parabolic

9. Classify the equation $4U_{xx} + 4U_{xy} + U_{yy} + 2U_x - U_y = 0$

(a) Elliptic (b) Hyperbolic **(c)Parabolic** (d) Concentric

Solution:

Here $A = 4,\ B = 4, C = 1$

$$B^2 - 4AC = 16 - \ 4(4)(1) = 0$$

The equation is parabolic

10. Classify $U_{xx} + U_{yy} = 0$

(a) Elliptic (b) Hyperbolic (c)Parabolic (d) Concentric

Solution:

Here $A = 1,\ B = 0, C = 1$

$$B^2 - 4AC = -1 < 0$$

The equation is elliptic

11.Classify $U_{xx} + 5U_{xy} + 4U_{yy} + U_x + U_y = 0$

(a) Elliptic **(b) Hyperbolic** (c)Parabolic (d) Concentric

Solution:

Here $A = 1,\ B = 5, C = 4$

$$B^2 - 4AC = 25 - 16 > 0$$

The equation is hyperbolic

12.Classify $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$

(a) Elliptic **(b) Hyperbolic** (c)Parabolic (d) Concentric

Solution:

Here $A = 1,\ B = 0, C = -c^2$

$$B^2 - 4AC = 0 - 4(1)(-c^2) = 4c^2 > 0$$

The equation is hyperbolic

13. Classify $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$

(a) Elliptic (b) Hyperbolic **(c)Parabolic** (d) Concentric

Solution:

Here $A = c^2,\ B = 0, C = 0$

$$B^2 - 4AC = 0 - 4(0)(c^2) = 0$$

The equation is parabolic

14. Write the conditions for classification of PDE to be hyperbolic, parabolic and hyperbolic

(a) $\boldsymbol{B^2 - 4AC < 0\ [Elliptic\ Equation]}$

$\boldsymbol{B^2 - 4AC = 0\ [Parabolic\ Equation]}$

$\boldsymbol{B^2 - 4AC > 0\ [Hyperbolic\ Equation]}$

(a) $B^2 - 4AC > 0\ [Elliptic\ Equation]$

$B^2 - 4AC < 0\ [Parabolic\ Equation]$

$B^2 - 4AC = 0\ [Hyperbolic\ Equation]$

(a) $B^2 - 4AC = 0\ [Elliptic\ Equation]$

$B^2 - 4AC > 0\ [Parabolic\ Equation]$

$B^2 - 4AC < 0$ $[Hyperbolic\ Equation]$

(a) $B^2 - 4AC = 0$ $[Elliptic\ Equation]$
$B^2 - 4AC > 0$ $[Parabolic\ Equation]$
$B^2 - 4AC < 0$ $[Hyperbolic\ Equation]$

Solution:
Let a second order PDE in the function u of the two independent variables x, y be of the form

$$A(x,y)\frac{\partial^2 u}{\partial x^2} + B(x,y)\frac{\partial^2 u}{\partial x \partial y} + C(x,y)\frac{\partial^2 u}{\partial y^2} + f\left(x, y, u, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}\right) = 0 \quad --\rightarrow (1)$$

Equation (1) is classified as elliptic, parabolic or hyperbolic at the points of a given region R depending on whether

$$B^2 - 4AC < 0\ [Elliptic\ Equation]$$
$$B^2 - 4AC = 0\ [Parabolic\ Equation]$$
$$B^2 - 4AC > 0\ [Hyperbolic\ Equation]$$

15.Classify $x^2 U_{xx} + 2xyU_{xy} + (1 + y^2)U_{yy} = 0$
(a) Elliptic (b) Hyperbolic (c)Parabolic (d) Concentric

Solution:
Here $A = x^2$, $B = 2xy, C = (1 + y^2)$

$$B^2 - 4AC = 2xy - 4(x^2)(1 + y^2) = -4x^2 < 0$$

The equation is elliptic

16. Classify $\frac{\partial^2 u}{\partial x \partial y} = \left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial u}{\partial y}\right) + xy$
(a) Elliptic **(b) Hyperbolic** (c)Parabolic (d) Concentric

Solution:
Here $A = 0$, $B = 1, C = 0$

$$B^2 - 4AC = 1 - 4(0)(0) = 1 > 0$$

The equation is hyperbolic

17. *A rod of length l cm whose one side is kept at $20°C$ and the other end is kept at $50°C$ is maintained until steady state prevails. Find the steady state temperature.*

(a) $u(x) = \frac{20}{l}x - 20$

(b) $u(x) = \frac{20}{l}x + 30$

(c) $\boldsymbol{u(x) = \frac{30}{l}x + 20}$

(d) $u(x) = \frac{30}{l}x - 20$

Solution:

$$u(x) = ax + b$$

When $x = 0, u(0) = b \Rightarrow b = 20C$

When $x = l, u(l) = al + 20 \Rightarrow 50C = al + 20$

$$al = 30 \Rightarrow a = \frac{30}{l}$$

$$so, u(x) = \frac{30}{l}x + 20$$

18. *A rod of length l cm whose one side is kept at $0°C$ and the other end is kept at $100°C$ is maintained until steady state prevails. Find the steady state temperature.*

(a) $\boldsymbol{u(x) = \frac{100}{l}x}$

(b) $u(x) = \frac{10}{l}x$

(c) $u(x) = \frac{-100}{l}x$

(d) $u(x) = \frac{100}{l}x^2$

Solution:

$$u(x) = ax + b$$

When $x = 0, u(0) = b \Rightarrow b = 0$

When $x = l, u(l) = al + 0 \Rightarrow 100 = al$

$$al = 100 \Rightarrow a = \frac{100}{l}$$

$$so, u(x) = \frac{100}{l}x$$

19. *A rod of length l cm whose one side is kept at $0°C$ and the other end is kept at $120°C$ is maintained until steady state prevails. Find the steady state temperature.*

(a) $u(x) = -\frac{120}{l}x$

(b) $\boldsymbol{u(x) = \frac{120}{l}x}$

(c) $u(x) = \frac{120}{l}x^2$

(d) $u(x) = \frac{120}{l}$

Solution:

$$u(x) = ax + b$$

When $x = 0, u(0) = b \Rightarrow b = 0$

When $x = l, u(l) = al + 0 \Rightarrow 120 = al$

$$al = 120 \Rightarrow a = \frac{120}{l}$$

$$so, u(x) = \frac{120}{l}x$$

20. *A rod of length 20 cm whose one side is kept at $30°C$ and the other end is kept at $70°C$ is maintained until steady state prevails. Find the steady state temperature.*

(a) $u(x) = 3x - 20$

(b) $u(x) = 2x - 30$

(c) $\boldsymbol{u(x) = 2x + 30}$

(d) $u(x) = 3x + 30$

Solution:

$$u(x) = ax + b$$

When $x = 0, u(0) = b \Rightarrow b = 30$

When $x = 20, u(20) = 70 = 20a + 30$

$$\Rightarrow 40 = 20a$$

$$\Rightarrow a = 2$$

$$so, u(x) = 2x + 30$$

21. A Uniform string of length `l` is struck in such a way that an initial velocity of V_0 is imparted to the portion of the string between $\frac{l}{4}$ $and \frac{3l}{4}$ while the string is in its equilibrium position. Write the wave equation and its boundary conditions

(a) $\boldsymbol{\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}}$,

$$\boldsymbol{y(0,t) = 0, t > 0}$$
$$\boldsymbol{y(l,t) = 0, t > 0}$$
$$\boldsymbol{y(x,0) = 0}$$
$$\boldsymbol{\frac{\partial y(x,0)}{\partial t} = \begin{cases} 0 & 0 < x < \frac{l}{4} \\ v_0 & \frac{l}{4} \le x \le \frac{3l}{4} \\ 0 & \frac{3l}{4} < x \le l \end{cases}}$$

(b) $\frac{\partial^2 y}{\partial x} = a^2 \frac{\partial^2 y}{\partial t^2}$,

$$y(0,t) = 0, t > 0$$
$$y(l,t) = 0, t > 0$$
$$y(x,0) = x$$
$$\frac{\partial y(x,0)}{\partial t} = \begin{cases} v_0 & 0 < x < \frac{l}{4} \\ 1 & \frac{l}{4} \le x \le \frac{3l}{4} \\ 0 & \frac{3l}{4} < x \le l \end{cases}$$

(c) $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$,

$y(0,t) = 0, t > 0$

$y(l,t) = 0, t > 0$

$\frac{\partial y(x,0)}{\partial t} = 0$

$$y(x,0) = \begin{cases} 0 & 0 < x < \frac{l}{4} \\ v_0 & \frac{l}{4} \leq x \leq \frac{3l}{4} \\ 0 & \frac{3l}{4} < x \leq l \end{cases}$$

(d) $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$,

$y(0,t) = 0, t > 0$

$y(l,t) = 0, t > 0$

$\frac{\partial y(x,0)}{\partial t} = 0$

$$y(x,0) = \begin{cases} 0 & 0 < x < \frac{l}{4} \\ -v_0 & \frac{l}{4} \leq x \leq \frac{3l}{4} \\ 1 & \frac{3l}{4} < x \leq l \end{cases}$$

Solution:

The wave equation is $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$

The boundary conditions are

$y(0,t) = 0, t > 0$

$y(l,t) = 0, t > 0$

$y(x,0) = 0$

$$\frac{\partial y(x,0)}{\partial t} = \begin{cases} 0 & 0 < x < \frac{l}{4} \\ v_0 & \frac{l}{4} \leq x \leq \frac{3l}{4} \\ 0 & \frac{3l}{4} < x \leq l \end{cases}$$

22. Write the one dimensional wave equation and also general solution for the displacement $y(x,t)$ of the string l vibrating between fixed end points with initial zero and initial displacement $f(x)$.

(a) $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$, $\sum B_n \cos\left(\frac{n\pi x}{l}\right) \sin\left(\frac{n\pi at}{l}\right)$

(a) $\boldsymbol{\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}}$ **,** $\boldsymbol{\sum B_n \sin\left(\frac{n\pi x}{l}\right) \cos\left(\frac{n\pi at}{l}\right)}$

(a) $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$, $\sum \sin\left(\frac{n\pi x}{l}\right) \cos\left(\frac{n\pi at}{l}\right)$

(a) $\frac{\partial^2 y}{\partial x^2} = a^2 \frac{\partial^2 y}{\partial t^2}$, $\sum B_n \sin\left(\frac{n\pi x}{l}\right) \cos\left(\frac{n\pi at}{l}\right)$

Solution:

The one dimensional wave equation is given by $\boldsymbol{\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}}$

The general solution for the displacement $y(x,t) is\ given\ by$

$$\sum B_n \sin\left(\frac{n\pi x}{l}\right) \cos\left(\frac{n\pi at}{l}\right)$$

23. Classify $x^2 f_{xx} + (1-y^2) f_{yy}) = 0\ for -1 < y < 1, -\infty < x < \infty$

$$and\ x \neq 0$$

(a) Elliptic (b) Hyperbolic (c)Parabolic (d) Concentric

Solution:

Here $A = x^2,\ B = 0, C = 1 - y^2$

$$B^2 - 4AC = 0 - 4(x^2)(1-y^2) = 4x^2(y^2-1) < 0$$

(Since $x^2 is\ always\ positive\ in\ -\infty < x < \infty\ \&$

$$in\ -1 < y < 1, y^2 - 1\ is\ negative)$$

The equation is elliptic

24. Classify the one dimensional wave equation

(a) Elliptic **(b) Hyperbolic** (c)Parabolic (d) Concentric

Solution:

The one dimensional wave equation is given by $\boldsymbol{\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}}$

Here $A = a^2,\ B = -1, C = 0$

$$B^2 - 4AC = (-1)^2 - 4(a^2)(0) = 1 > 0$$

The equation is hyperbolic.

25. Write the one dimensional heat flow equation and also general solution for $u(x,t)$ where $u(x,0) = f(x)$.

(a) $\boldsymbol{\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2},\ \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{l}}$

(b) $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2},\ \sum_{n=1}^{\infty} B_n \cos\frac{n\pi x}{l}$

(c) $\frac{\partial u}{\partial x} = \alpha^2 \frac{\partial^2 u}{\partial t^2},\ \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{l}$

(d) $\frac{\partial u}{\partial x} = \alpha^2 \frac{\partial^2 u}{\partial t^2},\ \sum_{n=1}^{\infty} B_n \cos\frac{n\pi x}{l}$

Solution:

One dimensional heat flow equation $\frac{\partial u}{\partial t} = \boldsymbol{\alpha^2 \frac{\partial^2 u}{\partial x^2}}$

Solution is $u(x,t) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{l}$

where $B_n = b_n = \frac{2}{l}\int f(x)\ \sin\frac{n\pi x}{l}\ dx$