1) Mutually exclusive - $P(A \cap B) = 0$

2) Independent event - $P(A \cap B) = P(A) \cdot P(B)$

3) Addition law - $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

when its ↳

- mutually exclusive - $P(A \cup B) = P(A) + P(B)$
- independent event - $P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B)$

4) Conditional Probability :-

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$P(B) \neq 0$

$\Rightarrow \boxed{P(A \cap B) = P(A|B) \cdot P(B)}$ → Multiplication law

5) Random variable

$X : S \longrightarrow R$

6) Probability mass function (pmf) → discrete

→ P_i (or) $P\{x = x\}$

i) $P_i \geq 0$

ii) $\Sigma P_i = 1$

7) Probability density function (pdf) → continuous

→ $f(x)$

i) $f(x) \geq 0$

ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

8) Cumulative distribution function (cdf)

$F(x) = P\{X \leq x\}$

Properties
1) $F(-\infty) = 0$
2) $F(\infty) = 1$

9) Relationship between pdf and cdf

$$f(x) = \frac{d}{dx} F(x)$$

10) Expectation

$$E(x) = \begin{cases} \Sigma x p(x) \text{ or } \Sigma x_i p_i & \rightarrow x \text{ is discrete} \\ \int x f(x) dx & \rightarrow x \text{ is continuous} \end{cases}$$

$\downarrow$
average

$$E(x^2) = \begin{cases} \Sigma x^2 p_i & \rightarrow x \text{ is discrete} \\ \int x^2 f(x) dx & \rightarrow x \text{ is continuous} \end{cases}$$

11) Variance (x) or Var (x) :-

$$Var(x) = E(x^2) - [E(x)]^2$$

$\boxed{\because Var(x) \geq 0}$

12) Standard Deviation (SD):-

$$SD(x) = \sqrt{Var(x)}$$

NOTE :-

i) $E(a) = 0$

ii) $Var(a) = 0$

iii) $E(ax + by) = aE(x) + bE(y)$

iv) $Var(ax + by) = a^2 Var(x) + b^2 Var(y)$

v) $E(xy) = E(x) \cdot E(y)$, if x and y are independent

MOMENTS

1) About origin

$$E(x^r) = \begin{cases} \sum x^r p_i, & \text{if } x \text{ is discrete} \\ \int x^r f(x)\,dx, & \text{if } x \text{ is continuous} \end{cases}$$

r^{th} moment about origin

$r=1$, first moment is $E(x)$

$r=2$, second moment is $E(x^2)$

2) About mean :-

$$E(x-\bar{x})^r = \begin{cases} \sum (x-\bar{x})^r p_i \\ \int (x-\bar{x})^r f(x)\,dx \end{cases}$$

r^{th} moment about mean

3) About any point A :-

$$E(x-A)^r = \begin{cases} \sum (x-A)^r p_i \\ \int (x-A)^r f(x)\,dx \end{cases}$$

MOMENT GENERATING FUNCTION :-

$$M_x(t) = E\left[e^{tx}\right] = \begin{cases} \sum e^{tx} p_i \\ \int e^{tx} f(x)\,dx \end{cases}$$

$$E(x) = \frac{d}{dt} M_x(t) \Big|_{t=0}$$

$$E(x^2) = \frac{d^2}{dt^2} M_x(t) \Big|_{t=0}$$

$$\boxed{M_x(t) = E\left(e^{tx}\right)}$$

$$\boxed{e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\cdots}$$

$$\Rightarrow E\left(1 + \frac{tx}{1!} + \frac{t^2x^2}{2!} + \frac{t^3x^3}{3!} + \cdots\cdots\right)$$

→ **FUNCTIONS OF RANDOM VARIABLE:-**

$X_{r.v} \longrightarrow$ pmf (or) pdf is given

$y = g(x)$

pmf (or) pdf

$$f_y(y) = g_x(x) \left|\frac{dx}{dy}\right|$$

$$\boxed{f_y(y) = f_x(x) \left|\frac{dx}{dy}\right|}$$

CHEBYCHEV'S INEQUALITY:-

$P\{|x-\mu| \geq k\sigma\} \leq \frac{1}{k^2}$ (upper bound)

$P\{|x-\mu) < k\sigma\} \geq 1 - \frac{1}{k^2}$ (lower bound)

$\mu \to$ Mean

$\sigma^2 =$ variance

$$P\{|x-\mu| \geq c\} \leq \frac{\sigma^2}{c^2}$$

$$P\{|x-\mu| < c\} \geq 1 - \frac{\sigma^2}{c^2}$$

for $c > 0$

★ If two inbetween values are given like $P\{6 < x < 18\}$
→ lower bound

★ If one value are given like $P\{x \geq 6\}$ or $P\{x < 6\}$
→ upper bound