18MAB204T – Probability and Queuing Theory

Assignment-II

PART – B

1. Find the MGF of binomial distribution.
2. If X is Poisson Random Variable such that
   $P[X = 2] = 9P[X = 4] + 90P[X = 6]$, Find the variance
3. If X is a Poisson variate such that $P(X = 1) = \frac{3}{10}$ and $(X = 2) = \frac{1}{5}$, Find $P(x = 0)$ and $P(X = 3)$.
4. State and prove memoryless property of exponential distribution
5. Find the MGF, mean and variance of Geometric distribution.
6. Find the MGF of exponential distribution and hence find its mean and variance.
7. A random variable X has a uniform distribution over (-3,3), Compute $P(|X| < 2),\ P(|X - 2| < 2)$
8. If X is a normal variate with mean 30 and S.D 5. Find (i) $P(26 \leq X \leq 40)$ (ii) $P(X \geq 45)$
9. Buses arrive at a specified stop at 15min intervals starting at 7am that is, they arrive at 7,7.15,7.30,7.45 and so on. If a passenger arrives at the stop at a random time that is uniformly distributed between 7 and 7.30 am. Probability that he waits less than 5min for a bus.
10. Trains arrive at a station at 15 minutes intervals starting at 4am. If a passenger arrive to the station at a time that is uniformly distributed between 9.00 and 9.30 find the probability that he has to (i) less than 6 minutes (ii) more than 10 minutes
11. The mileage which car owner get with a certain kind of radial tyre is a random variable having an exponential distribution with mean 40,000km. Find the probabilities that one of these tyres will last
    (i) atleast 20,000kms (ii) atmost 30,000 kms
12. Suppose that the probability of a dry day following a rainy day is ${}^{1}/_{3}$ and that the probability of a rainy day following a dry day is ${}^{1}/_{2}$ Given that May 1 is a dry day. Find the probability that May 3 is a dry day and also May 5 is a dry day.
13. If the transition probability matrix of a Markov Chain is $\begin{pmatrix} 0 & 1 \\ {}^{1}/_{2} & {}^{1}/_{2} \end{pmatrix}$, find the steady state distribution of the chain after 2 steps, if the initial state probability distribution of a Markov chain $P^{(0)} = \left(\frac{3}{4}, \frac{1}{4}\right)$
14. P.T the matrix $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ {}^{1}/_{2} & {}^{1}/_{2} & 0 \end{pmatrix}$ is the tpm of an irreducible Markov chain.
15. A man tosses a fair coin until 3 heads occur in a row. Let X_n denotes the longest string of heads ending at the nth trial. Show that the process is Markovian. Find the transition matrix and classify the state.

PART – C

16. Fit Poisson distribution for the following data

x	0	1	2	3	4	5	Total
f(x)	142	156	69	27	5	1	400

Find the probability mass function and then find theoretical frequencies

17. Fit a binomial distribution for the following data

x	0	1	2	3	4	5	6	Total
f(x)	5	18	28	12	7	6	4	80

18. If X has geometric distribution, then for any two positive integers m and n, prove that $P[X > m + n/X > m] = P[X > n]$

19. A and B shoot independently until each has hit his own target. The probabilities of their hitting the target at each shot are $\frac{3}{5}$ and $\frac{5}{7}$ respectively. Find the probability that B will require more shots than A

20. In a test 2000 electric bulbs, it was found that the life of a particular make was normally distributed with an average of 2040 hours and SD of 60 hours. Estimate the number of bulbs like to burn for (i) more than 2150 hours (ii) less than 1950 hours (iii) more than 1920 hours but less than2160 hours

21. If X is normal variate with mean 30 and standard deviation 5. Find the probabilities that (i) $26 \le X \le 40$ (ii) $X \ge 45$ (iii) $|X - 30| \le 5$

22. The transition probability matrix of a Markov chain $\{X_n\}; n = 1,2,3 \ldots..$ having 3 states 0, 1,2 is $P = \begin{pmatrix} 0.2 & 0.3 & 0.5 \\ 0.1 & 0.6 & 0.3 \\ 0.4 & 0.3 & 0.3 \end{pmatrix}$. Initial distribution is $P^{(0)} = (0.5 \quad 0.3 \quad 0.2)$.

    Find (i) $P\{X_2 = 2\}$ (ii) $P\{X_3 = 3, X_2 = 2, X_1 = 1, X_0 = 2\}$

23. The transition probability matrix of a Markov chain $\{X_n\}; n = 1,2,3 \ldots..$ having 3 states 1,2 and 3 is $P = \begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$. Initial distribution is $P^{(0)} = (0.7 \quad 0.2 \quad 0.1)$.
    Find (i) $P\{X_2 = 3\}$ (ii) $P\{X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2\}$.

24. Three boys A, B and C are throwing a ball to each other. A always throws the ball to B and B always throws the ball to C, but C is just as likely to throw the ball to B as to A. Show that the process is Markovian. Find the transition matrix and classify the states.

25. A man either drives a car or catches a bus to go to office each day. He never goes to 2 days in a row by bus but if he drives one day, then the next day he is just as likely to drive again as he is to travel by bus. Now suppose that on the first day of the week, the man tossed a fair dice and drove to work if and only if a 6 appeared. Find i) The probability that he takes a bus on the fourth day ii) The probability that he travels by bus in the long run.

26. A salesman's territory consists of 3 cities A,B and C. He never sells in the same city on successive days. If he sells in city A, then the next day he sells in B. However, if he sells either in B or C, then the next day he is twice as likely to sell in city A as in the other city. How often does he sell in each of the cities in the steady state?

27. A gambler has Rs.2/-. He bets Re.1 at a time and wins Re1 with probability$\frac{1}{2}$. He stops playing if he loses Rs.2 or wins Rs.4
    (i) What is the tpm of the related Markov Chain?
    (ii) What is the probability that he has lost his money at the end of 5 plays?
    (iii) What is the probability that the game lasts more than 7 plays?