18MAB 204T -Probability and Queuing Theory

Assignment

Part-A

1. Range of the probability distribution.
2. The Random Variable X has a Moment generating function $\frac{3}{3-t}$. Find Mean.
3. Find the value of $Var(X)$ when $E(X)=1$ & $E(X(X-1))=4$.
4. A random variable X has a pdf $f(x)=ke^{-x}$ $0<x<\infty$. Find k
5. Find the value of $Var(X)$ When $E(X)=2$; $E(X^2)=8$
6. A random variable X has a pdf of $f(X)=\frac{1}{10}; x=10$ Find Mean.
7. In a probability distribution the sum of all the probabilities is always equal to___.
8. The Random variable X has Poisson distribution with mean2. Find $P(X=2)$
9. If a random variable X has a Poisson distribution such that P(X = 1) = P(X = 2), its mean is ____.
10. 4 Coins are tossed simultaneously. What is the probability of getting 2 heads?
11. The MGF of geometric, exponential, uniform distribution.
12. If X is uniform random variable in $(-2,2)$. The $Var(X)$ is ___.
13. The mean and variance of binomial distribution are 4 and 4/3, Find n
14. What is type-1 error
15. What is type-2 error.
16. The use of Chi-Square test is ____.
17. A sample is said to be large, if its sample size is ____.
18. The table value of z-test, t-test at 5% and 1% level of significance.
19. The LOS for t-test at $\alpha\%$ level of significance for one-tailed test is____.
20. In the Kendall's notation (a/b/c):(d/e) What is a,b,c,d,e?
21. If a customer has to wait in a $(M/M/1):(\infty/FIFO)$ queuing system, what is the waiting time in the queue if $\lambda=8$ per hour $\mu=8$ per hour?
22. In a $(M/M/1):(K/FIFO)$ queue, the effective arrival rate λ' is given by ___.
23. Consider a watch repair shop where customers arrive according to Poisson process at a rate of one for every 10 min with $\mu=60/8$ per hour the value of L_q is ____
24. Average number L_w of customer in a non empty queue is ____.
25. The Chapman-Kolmogorov theorem states the relation is _____.
26. Write the relation satisfied by the steady state distribution and the tpm of a regular Markov Chain.
27. If the GCD of reaching a state in n steps is 1 then it is said to be ____.
28. If the initial state distribution of a markov chain is $p^{(0)}=\left\{\frac{5}{6},\frac{1}{6}\right\}$ and the tpm is given by $\begin{pmatrix} 0 & 1 \\ 0.5 & 0.5 \end{pmatrix}$. Find the probability distribution after 2 steps

29. If all the states of a Markov chain communicate among them. The Markov chain is ___

Part-B&C

1. A random variable X has the probability function $f(x) = \frac{1}{2^x}, x = 1,2,3,....$ Find its MGF.
2. If X us continuous random variable whose pdf is given by $f(x) = kx(2-x); 0 < x < 2$ Find the value of $k \& P(X < 1)$
3. Find the mean and variance of the following density function of a random variable X

$$f(x) = \begin{cases} x \ , \ for\ 0 < x < 1 \\ 2 - x \ , \ for 1 < x < 2 \\ 0 \ , otherwise \end{cases}$$

4. A random variable X has the following probability distribution

X	0	1	2	3	4	5	6	7	8
P(X)	a	3a	5a	7a	9a	11a	13a	15a	17a

Determine and a , $P(X < 3), P(X \geq 3), P(0 < X < 5)$ CDF.

5. In a company the monthly break down of a machine is a random variable with Poisson distribution, with an average 1.2. Find the probability that the machine will function for a month with at most one breakdown.
6. The daily consumption of milk in a city in excess of 20,000 litres is exponentially distributed with an average of 3000 litres. The city has a daily stock of 35,000 litres. What is the probability that the stock is insufficient for two days selected at random?
7. If the probability of success is 0.2. What is the probability that the target would be first hit in more than 3 attempts?
8. Derive MGF, Mean and Variance of a binomial distribution.
9. Derive MGF, Mean and Variance of Poisson distribution.
10. State and Prove Memory less property of Geometric distribution.
11. State and prove memory less property of Exponential distribution.
12. If X is uniform random variable over $(0,5)$. Find the pdf of $Y = 2X - 3$
13. In a Normal distribution 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution.
14. If the probability that a certain test yields a positive reaction equals 0.4, What is the probability that fewer than 5 negative reaction occur before the first positive one.
15. If the probability of success is 0.58. What is the probability that the target would be first hit in the 4^{th} attempt?
16. A random variable has mean 12 & variance 9 for an unknown distribution Find $P(6 < x < 18)$
17. A simple sample of heights of 6400 English men has a mean of 170 cm & S.D of 6.4 cm, another simple sample 1600 Americans has a mean of 172cm & S.D as 6.3 cm. Do the data indicate that Americans on an average taller than English Men?
18. It is claimed that a random sample of 100 tyres with a mean life of 15629 kms drawn a population of tyres which has a mean life of 15200 kms and standard deviation of 1248 kms. Test the validity of the claim.

19. The data given below relate to two random samples of employees from two different state. Test the hypothesis that variances of the population are equal.

	Mean	Variance	Size
State 1	28	40	16
State 2	19	42	25

20. The nicotine contents in two random samples of tobacco are given below.

Sample 1	21	24	25	26	27	
Sample 2	22	27	28	30	31	36

Can you say that the two samples come from the same population?

21. Two independent sample of 8&7 items respectively had the following values of the variable. Do the two estimate of population variance differ significantly.

Sample I	9	11	13	11	15	9	12	14
Sample 2	10	12	10	14	9	8	10	

22. Fit a Poisson distribution for the following distribution and also test for the goodness of fit.

x	0	1	2	3	4	5	**Total**
f	142	156	69	27	5	1	400

23. A salesman in a departmental store claims that at most 60 percent of the shoppers entering the store leaves without making a purchase. A random sample of 50 shoppers showed that 35 of them left without making a purchase. Are these sample results consistent with the claim of the salesman?
24. A sample of 900 items has a mean 3.4 and S.D. 2.61. Can it be regarded as drawn from a population with mean 3.25 at 5% level?
25. Three boys A, B & C throw ball to each other. A will throw the ball to B. B will throw the ball to C. C will throw the ball to A or B. Write the tpm and prove that the chain is irreducible Markov chain.
26. Arrivals at the telephone booth are considered to be Poisson with an average time of 12 min. Between one arrival and the next. The length of a phone call is assumed to be distributed exponentially with mean 4 min.
    i) Find the average number of persons waiting in the system .
    ii) What is the probability that a person arriving at the booth will have to wait in the queue?
    iii) What is the probability that it will take him more than 10 min altogether to wait for the phone and complete the call?
    iv) Estimate the fraction of the day when the phone will be in use.
27. The local one-person barber shop can accommodate a maximum of 5 people at a time (4waiting and 1 getting hair-cut). Customers arrive according to a Poisson distribution with mean 5 per hour. The barber cuts hair at an average rate of 4 per hour (Exponential service time)
    i) What percentage of time is the barber idle?
    ii) What fraction of the potential customers are turned away?

iii) What is the expected number of customers waiting for a hair-cut?

iv) How much time can a customer expect to spend in the barber shop?.

28. If the customer has to wait in a (M/M/1): (∞/FIFO) queue system, what is his average waiting time in the queue, if $\lambda = 8$ per hour and $\mu = 12$ per hour.
29. A student's study habits are as follows. If the studies one nigh, he is 70% sure not to study the next night, where as if he doesn't study one day then next day he is 0.6 sure not to study. Find the tpm and the limiting probability.
30. The tpm of a Markov chain having 3 state 1,2,3 is $P = \begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$ and

    $P^{(0)} = (0.7, 0.2, 0.1)$

    Find $P(X_2 = 3)$ $P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2)$
31. A man either drives a car or catches a train to go to office each day. He never goes 2 days in a row by train but if he drives one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair dice and drove to work if and only if a 6 appeared. Find the probability that he drives car on the third day. Also find the probability of the man travelling by car in long run.
32. The tpm of a Markov chain having 3 states 0,1,2 is

    $P = \begin{pmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{pmatrix}$ and $P^{(0)} = (1/3, 1/3, 1/3)$ Find

    $P(X_2 = 2), P(X_3 = 1, X_2 = 2, X_1 = 1, X_0 = 2)$
33. A gambler has Rs.2/- . He bets one Re.1/- at a time and wins Re.1/- with probability 1/2/ He stops playing if he loses Rs.2 or wins Rs.4.

    i) Wrtie the tpm of the related Markov chain?

    ii) What is the proability that he has lost his money at the end of 5 plays?