SRM Institute of Science and Technology
Department of Mathematics
18MAB204T-Probability and Queueing Theory
Unit – I: Random Variables
Tutorial Sheet - I

S.No	Questions	Answers
	Part – A	
1	State the Properties of the cdf of a Random variable.	
2	If X represents the total number of heads obtained, when a fair coin is tossed 5 times. Find the probability distribution of X.	<table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>p_x</td><td>$\frac{1}{32}$</td><td>$\frac{5}{32}$</td><td>$\frac{10}{32}$</td><td>$\frac{10}{32}$</td><td>$\frac{5}{32}$</td><td>$\frac{1}{32}$</td></tr></table>
3	Verify whether $f(x) = \begin{cases} |x|, -1 \leq x \leq 1 \\ 0, \text{elsewhere} \end{cases}$ can be the pdf of a continuous RV X.	**Ans:** Yes
4	If the pdf of a RV X is given by $f(x) = \begin{cases} \frac{1}{4}, -2 < x < 2 \\ 0, \text{elsewhere} \end{cases}$ find $P\{|X| > 1\}$.	**Ans:** $\frac{1}{2}$
5	If the pdf of a RV X is $f(x) = \begin{cases} 2x, 0 < x < 1 \\ 0, \text{elsewhere} \end{cases}$ .Find the cdf of X.	**Ans:** $F(x) = \begin{cases} 0, & x < 0 \\ x^2, & 0 < x < 1 \\ 1, & x \geq 1 \end{cases}$
	Part – B	
6	A discrete RV X has the following probability distribution <table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td></tr><tr><td>P(x)</td><td>a</td><td>3a</td><td>5a</td><td>7a</td><td>9a</td><td>11a</td><td>13a</td><td>15a</td><td>17a</td></tr></table> Find the value of 'a', P(X < 3), variance and distribution function of X	**Ans:** a=$\frac{1}{18}$, P(X < 3) = $\frac{1}{9}$ V(X) = 4.4719 <table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>F(x)</td><td>$\frac{1}{81}$</td><td>$\frac{4}{81}$</td><td>$\frac{9}{81}$</td><td>$\frac{16}{81}$</td><td>$\frac{25}{81}$</td></tr></table> <table><tr><td>x</td><td>5</td><td>6</td><td>7</td><td>8</td></tr><tr><td>F(x)</td><td>$\frac{36}{81}$</td><td>$\frac{49}{81}$</td><td>$\frac{64}{81}$</td><td>1</td></tr></table>
7	If the probability mass function of a RV X is given by $P(X = r) = kr^3, r = 1,2,3,4$. Find (i) the value of k (ii)P $\left(\frac{1/2 < X < 5/2}{X > 1} \right)$ (iii) the mean and variance of X (iv)the distribution function of X.	**Ans:** (i) k =$\frac{1}{100}$, (ii)P = $\frac{8}{99}$, (iii)E[X] =3.54,V[X]=0.4684 (iv)F(x)=$\begin{cases} 0, & x < 1 \\ \frac{1}{100}, 1 \leq x < 2 \\ \frac{9}{100}, 2 \leq x < 3 \\ \frac{36}{100}, 3 \leq x < 4 \\ 1, & 4 \leq x \end{cases}$

8	The diameter of an electric cable X is a continuous RV with pdf $$f(x) = \begin{cases} kx(1-x), & 0 \le x \le 1 \\ 0, & \text{elsewhere} \end{cases}$$ Find (i) the value of k (ii) cdf of X (iii) the value of a such that $P(X < a) = 2P(X > a)$ and (iv) $P\left((X \le {}^1/_2 \,/\, \frac{1}{3} < X < \frac{2}{3}\right)$.	**Ans:** (i) k = 6 $$\text{(ii)}F(x)=\begin{cases} 0, & x < 0 \\ 3x^2-2x^3, & 0 \le x < 1 \\ 1, & 1 \le x \end{cases}$$ (iii) The root of the equation $6a^3 - 9a^2 + 2 = 0$ that lies between 0 and 1. (iv) $\frac{1}{2}$
9	A continuous RV X has the pdf $$f(x) = \begin{cases} kx^4, & -1 < x < 0 \\ 0, & \text{elsewhere} \end{cases}$$ Find the value of 'k' and $P\left(X > -1/2 \,/\, X < -1/4\right)$.	**Ans:** k = 5 , P = $\frac{1}{33}$
10	If the distribution function of a RV X is given by $$F(x) = \begin{cases} 0, & x < 0 \\ x, & 0 \le x < 1 \\ 1, & 1 \le x \end{cases}$$ Find the pdf of X. Also find $P(1/3 < X < 1/2)$ and $P(1/2 < X < 2)$ using the cdf of X.	**Ans:** $$f(x) = \begin{cases} 1, & 0 \le x \le 1 \\ 0, & \text{elsewhere} \end{cases}$$ P $=\frac{1}{6}$, P $=\frac{1}{2}$