Department of Mathematics

Sub Title: PROBABILITY AND QUEUING THEORY

Sub Code: 18MAB204T

Unit -III - Test of Hypothesis

BASIC DEFINITIONS

Population:

The study of any finite or infinite collection of individuals with which we are concerned,possessing a variable character, is called a population.

Sample:

A sample is a finite subset of the population.The number of elements in the sample is called the size of the sample.

Sample size: The number of elements in a sample is called the sample size.

Sampling : The process of selection of such samples is called sampling. For example, a housewife normally tests the cooked products to find if they are properly cooked & contain the proper quantity of salt.

Parameters and statistics

The statistical constant of the population namely mean μ variance σ^2 which are usually referred to as parameters.

The statistical measured computed form sample observation alone ex: mean $\bar{x}$, variance s^2 etc., are usually referred to as statistic.

Null hypothesis:

For applying the test of significant, we first set up a hypothesis a definite statement about the population parameters. Such as hypothesis is usually a hypothesis of no difference and it is called Null hypothesis. It is denoted by H_0.

Alternative hypothesis:

Any hypothesis which is complementary to the null hypothesis is called alternative hypothesis usually denoted by H_1

Critical Region : A region, corresponding to a statics ,, in the sample space S which amounts to rejection of the null hypothesis Ho is called as critical region or region of rejection. The region of the sample space S which amounts to the acceptance Ho is called acceptance region.

level of significance

The probability 'α' that a random values ,of the statistic belongs to the critical region is known as the level of significance. In other words, the level if significance is the size of the Type I error .

Standard Error (S.E.) : The standard deviation of sampling distribution of a statistic is known as its standard error.

S.No	Statistic t	Standard Error of t	Remark
1	Sample Mean $\bar{x}$	$\frac{\sigma}{\sqrt{n}}$	σ Population S.D n, size of the sample
2	Sample standard deviation s	$\frac{\sigma}{\sqrt{2n}}$	σ Population S.D n, size of the sample
3	Sample proportion p	$\sqrt{\frac{PQ}{n}}$	n, size of the sample P-Population proportion Q = 1 - P
4	Difference of sample means $\bar{x}_1 - \bar{x}_2$	$\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$	n_1, n_2 size of the samples σ_1 , σ_2 Population S.D
5	Difference of sample standard deviations $s_1 - s_2$	$\sqrt{\frac{\sigma_1^2}{2n_1} + \frac{\sigma_2^2}{2n_2}}$	n_1, n_2 size of the samples σ_1 , σ_2 Population S.D
6	Difference of proportions $p_1 - p_2$	$\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$	$P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$ Q = 1 - P

<u>Test of Significance</u> : A very important aspect of the sampling theory is the study of tests of significance which enable us to decide on the basis of the sample results if (i) The deviation between the observed sample statistic & the hypothetical

parameter value is significant.

(ii) The deviation between two sample statistics is significant.

<u>Error in Sampling</u>: The main aim of the sampling theory is to draw a valid conclusion or a valid inference about the population parameters on the basis of the sample results. For example, the mother at home tests the cooked products by taking and testing a small amount of cooked product. If this small amount of cooked product is good,

we accept the lot to be good. **Type I Error** : Reject Ho when it is true. **Type II Error** : Accept Ho when it is wrong.

<u>Large Samples</u>

When the size of the sample(n) is greater than 30, then that sample is called a large sample. There are 4 important test to test the significance of large samples.

Test used for large sample

(i) Test of significance of single proportion

(ii) Test of significance for difference of proportion

(iii) Test of significance of single means

(iv) Test of significance for difference of means

<u>PROCEDURE FOR TESTING OF HYPOTHESIS</u>

1. Set up the null hypothesis Ho.
2. Set up the alternative hypothesis H1 ". This will enable us to decide whether we have to use a single tailed (right or left) test or two tailed test.
3. Choose the appropriate level of significance(either 5% or 1% level). This is to be decided before sample is drawn.
4. Compute the test statistics $Z = \frac{t - E(t)}{S.E(t)}$
5. We compare the computed value of ÿ in step (4) with the tabulated value ÿˆ at given level of significance Ù. If the calculated value of ¤ is less than tabulated value ÿˆ then Ho accepted. If the calculated value of ¤ is greater than tabulated value ÿˆ then Ho rejected.

Critical vales (Z_α) of Z

critical value Z_α	Level of significance(α)			
	1%	2%	5%	10%
Two tailed test	$\lvert z_\alpha \rvert = 2.58$	$\lvert z_\alpha \rvert = 2.33$	$\lvert z_\alpha \rvert = 1.96$	$\lvert z_\alpha \rvert = 1.645$
Right tailed test	$\lvert z_\alpha \rvert = 2.33$	$\lvert z_\alpha \rvert = 2.055$	$\lvert z_\alpha \rvert = 1.645$	$\lvert z_\alpha \rvert = 1.28$
Left tailed test	$\lvert z_\alpha \rvert = -2.33$	$\lvert z_\alpha \rvert = -2.055$	$\lvert z_\alpha \rvert = -1.645$	$\lvert z_\alpha \rvert = -1.28$

Test of significance of single proportion

Suppose a large sample of size is taken from a normal population. To test the significant difference between the sample proportion p and the population proportion P , we use the statistic $Z = \dfrac{p-P}{\sqrt{\dfrac{PQ}{n}}}$

1. The probable limits for the observed proportion of successes p are given by $P \pm Z_\alpha \sqrt{\dfrac{PQ}{n}}$
2. If P is not known, the limits for the population proportion P are given by $p \pm Z_\alpha \sqrt{\dfrac{pq}{n}}$ wher q =1-p
3. If α is not given, we can take safely 3α limits. Hence, confidence limits for observed proportion p are

   $P \pm 3\sqrt{\dfrac{PQ}{n}}$ and confidence limits for the population proportion P are $p \pm 3\sqrt{\dfrac{PQ}{n}}$ where q = 1-p
4. 95% confidence limits for population proportion P are given by $p \pm 1.96\sqrt{\dfrac{pq}{n}}$ where q = 1-p
5. 99% confidence limits for population proportion P are given by $p \pm 2.58\sqrt{\dfrac{pq}{n}}$ where q = 1-p

PROBLEMS IN SINGLE PROPORTION

Two Tailed Test

1. A coin is tossed 256 times and 132 heads are obtained. Would you conclude that the coin is a biased one?

Here n = 256, = No. of success = 132, p = proportion of successes in the sample = $\frac{X}{n}$ = $\frac{132}{256}$ = 0.5156

P = populatin proportion = $\frac{1}{2}$, Q = 1-P = $\frac{1}{2}$

Null Hypothesis Ho : The coin is unbiased.

Alternative Hypothesis H1 : The coin is biased : P ≠ 0.5 (two tailed test)

Test statistic $Z = \frac{p-P}{\sqrt{\frac{PQ}{n}}}$

$$Z = \frac{0.5156-0.5}{\sqrt{\frac{0.5\times0.5}{256}}} = 0.4992 < 1.96$$

Ho is accepted. Hence the coin is unbiased.

2. A coin is tossed 400 times and its turns up head 216 times. Discuss whether the coin may unbiased one at 5% level if significance.

Solution:

Given n= 400, P = $\frac{1}{2}$

$\Rightarrow$Q = 1-P

= 1-1/2 $\Rightarrow$1/2

Q = 1/2

n = number of success

X = 216, p = proportion of successes in the sample = $\frac{X}{n}$ = $\frac{216}{400}$ = 0.54

Null hypothesis H_0: The coin is unbiased

Alternative hypothesis H_1 : The coin is biased

$\alpha = 5\,\% = 0.05$

Test Statistic

$$Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}}$$

$$Z = \frac{0.54 - 0.5}{\sqrt{\frac{0.5 \times 0.5}{400}}}$$

$Z_{cal} = 1.6$

At $\alpha = 5\%$ $Z_{tab} = 1.96$

$\therefore$ $Z_{cal} < Z_{tab}$ (ie) 1.6 <1.96

Hence we accept null hypothesis H_o (ie) The coin is unbiased

3. A random sample of 400 mangoes was taken from a large consignment and 40 were found to be bad. In this a sample from a consignment with proportion of bad mangoes 7.5%?

Here n = 400, p = sample proportion of bad mangoes = $\frac{X}{n}$ = $\frac{40}{400} = 0.1$

P = populatin proportion of bad mangoes = 7.5% = 0.075 , Q = 1 –P = 1- 0.075 = 0.925

Ho : P= 0.075, H_1 : P ≠ 0.075 (two tailed test); Z α at 1% level l = 2.58

Test Statistic

$$Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}}$$

$$Z = \frac{0.1 - 0.075}{\sqrt{\frac{0.075 \times 0.925}{400}}} = 1.89 < 2.58$$

Null hypothesis Ho is accepted at 1% level.

4. In a city, a sample of 1000 people were taken & out of them 540 are vegetarians & the rest are non vegetarians. Can we say that both habits of eating are equally popular in the city at 1% & 5% level of significance?

Here n = 1000, p = sample proportion of vegetarians = $\frac{540}{1000} = 0.54$

P = populatin proportion of vegetarians = $\frac{1}{2}$

Q = 1 – P = 0.5

Ho : P = 0.5 Both habits are equally popular in the city). H1" : P ≠ 0.5 (two tailed test)

Test Statistic

$$Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}}$$

$$Z = \frac{0.54 - 0.5}{\sqrt{\frac{0.5 \times 0.5}{1000}}} = 2.5298 > 1.96$$

Ho rejected at 5% level of significance. Both types of eaters are popular at 1% level and not so at 5% level of significance.

DIFFERENCE OF MEAN

Let $\overline{x_1}$ be the mean of a sample of size n_1 from a population with mean μ_1 and variance σ_1^2. Let $\overline{x_2}$ be the mean of a sample of size n_2 from a population with mean μ_2 and variance σ_2^2.

To test whether there is any significant difference between $\overline{x_1}$ and $\overline{x_2}$ we have to use the statistic $z = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$

Note: If the samples have been drawn from the same population then $\sigma_1^2 = \sigma_2^2 = \sigma^2$, $z = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{\frac{\sigma^2}{n_1} + \frac{\sigma^2}{n_2}}}$

If σ is not known we can use a estimate of σ^2 given by $\sigma^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2}$

Two Tailed Test

1. ***In a random sample of size 500, the mean is found to be 20. In another independent sample of size 400, the mean is 15. Could the samples have been drawn from the same population with S.D. 4?***

   Solution: $n_1 = 500,\ n_2 = 400,\ \overline{x_1} = 20,\ \overline{x_2} = 15,\ \sigma = 4$

   Null Hypothesis $H_0 : \bar{x}_1 = \bar{x}_2$, *Alternative Hypothesis* $H_1 : \bar{x}_1 \neq \bar{x}_2$ (two tailed test)

   Test statistics $z = \frac{\bar{x}_1 - \bar{x}_2}{\sigma \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{20-15}{4\sqrt{\frac{1}{500} + \frac{1}{400}}} = 18.6$; $z_\alpha = 2.58$ at 1%, $|z| > z_\alpha$, H_0 is rejected.

   That is, the samples could not have been drawn from the same population.

2. ***The mean of 2 large samples 1000 and 2000 members are 67.5 inches and 68.0 inches respectively. Can the samples the regarded as drawn from the same population of S.D. 2.5 inches.***

   Solution: $n_1 = 1000,\ n_2 = 2000,\ \overline{x_1} = 67.5,\ \overline{x_2} = 68,\ \sigma = 2.5$

   Null Hypothesis $H_0 : \bar{x}_1 = \bar{x}_2$, *Alternative Hypothesis* $H_1 : \bar{x}_1 \neq \bar{x}_2$ (two tailed test)

   Test statistics $z = \frac{\bar{x}_1 - \bar{x}_2}{\sigma \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{67.5-68}{2.5\sqrt{\frac{1}{1000} + \frac{1}{2000}}} = -5.1,\ |z| = 5.1$

   $z_\alpha = 1.96$ at 1% level of significance, $|z| > z_\alpha$, H_0 is rejected and H_1 is accepted.

3. ***In a survey of buying habits, 400 women shoppers are chosen at random in super market A located in a certain section of the city. Their average weekly food expenditure is Rs. 250 with a S.D. of Rs. 40. For 400 women shoppers chosen at random in super market B in another section of the city, the average weekly food expenditure is Rs. 220 with a S.D. of Rs. 55. Test at 1% level of significance whether the average weekly food expenditure of the two populations of shopper are equal.***

Solution: $n_1 = 400,\ n_2 = 400, \overline{x_1} = 250,\ \overline{x_2} = 220,\ s_1 = 40, s_2 = 55$

Null Hypothesis $H_0 : \bar{x}_1 = \bar{x}_2$, *Alternative Hypothesis* $H_1 : \bar{x}_1 \neq \bar{x}_2$ (two tailed test)

Test statistics$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$, Since σ_1 & σ_2 the population S.D. are not known, we can take $\sigma_1^2 = s_1^2\ and\ \sigma_2^2 = s_2^2$,

Test statistics $z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{250-220}{\sqrt{\frac{40^2}{400} + \frac{55^2}{400}}} = 8.82$; $z_\alpha = 2.58$ at 1% level of significance, $|z| > z_\alpha$, H_0 is rejected.

4. ***Test the significance of difference between the means of the samples, drawn from two normal populations with the same S.D. from the following data:***

	Size	***Mean***	***S.D.***
Sample 1	***100***	***61***	***4***
Sample 2	***200***	***63***	***6***

Solution: $n_1 = 100,\ n_2 = 200, \overline{x_1} = 61,\ \overline{x_2} = 63,\ s_1 = 4, s_2 = 6$

Null Hypothesis $H_0 : \bar{x}_1 = \bar{x}_2$, *Alternative Hypothesis* $H_1 : \bar{x}_1 \neq \bar{x}_2$ (two tailed test)

Test statistics$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$, Since σ_1 & σ_2 the population S.D. are not known, we can take $\sigma_1^2 = s_1^2\ and\ \sigma_2^2 = s_2^2$,

Test statistics $z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{61-63}{\sqrt{\frac{4^2}{100} + \frac{6^2}{200}}} = -3.02, |z| = 3.02; z_\alpha = 1.96$ at 5%, $|z| > z_\alpha$, H_0 is rejected.

Right Tailed Test

5. ***The average marks scored by 32 boys is 72 with a S.D. of 8, while that for 36 girls is 70 with a S.D. of 6. Test at 1% level of significance whether the boys perform better than girls.***

Solution: $n_1 = 32,\ n_2 = 36, \overline{x_1} = 72,\ \overline{x_2} = 70,\ s_1 = 8,\ s_2 = 6$

Null Hypothesis $H_0 : \bar{x}_1 = \bar{x}_2\ (or\ \mu_1 = \mu_2)$,

Alternative Hypothesis $H_1 : \bar{x}_1 > \bar{x}_2$ (right tailed test)

Test statistics $z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{72-70}{\sqrt{\frac{64}{32} + \frac{36}{36}}} = 1.15$, $z_\alpha = 2.33$ at 1% level of significance

$|z| < z_\alpha$, H_0 is accepted. That is, we cannot conclude that boys perform better than girls.

6. ***A random sample of 100 bulb from a company A showed a mean life 1300 hours and standard deviation 82 hours. Another random sample of 100 bulbs from company B showed a mean life 1248 hours and standard deviation of 93 hours. Are the bulbs of company A superior to bulbs of company B at 5% level of significance.***

Solution: $n_1 = 100,\ n_2 = 100, \overline{x_1} = 1300,\ \overline{x_2} = 1248,\ s_1 = 82,\ s_2 = 93$

Null Hypothesis $H_0 : \bar{x}_1 = \bar{x}_2\ (or\ \mu_1 = \mu_2)$,

Alternative Hypothesis $H_1 : \bar{x}_1 > \bar{x}_2$ (right tailed test)

Test statistics $z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{1300-1248}{\sqrt{\frac{82^2}{100} + \frac{93^2}{100}}} = 4.19$, $z_\alpha = 1.645$ at 1%, $|z| > z_\alpha$, H_0 is rejected.

Left Tailed Test

7. ***The average hourly wage of a sample of 150 workers in plant A was Rs. 2.56 with a S.D. of Rs. 1.08. The average wage of a sample of 200 workers in plant B was Rs. 2.87 with a S.D. of Rs. 1.28. Can an applicant safely assume that the hourly wage paid by plant B are higher than those paid by plant A?***

   Solution: $n_1 = 150,\ n_2 = 200, \overline{x_1} = 2.56,\ \overline{x_2} = 2.87,\ s_1 = 1.08, s_2 = 1.28$

   Null Hypothesis $H_0 : \bar{x}_1 = \bar{x}_2$, *Alternative Hypothesis* $H_1 : \bar{x}_1 < \bar{x}_2$ (left tailed test)

   Test statistics$z = \frac{\bar{x}_1-\bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}}$, Since σ_1 & σ_2 the population S.D. are not known, we can take $\sigma_1^2 = s_1^2\ and\ \sigma_2^2 = s_2^2$, Test statistics $z = \frac{\bar{x}_1-\bar{x}_2}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}} = \frac{2.56-2.87}{\sqrt{\frac{(1.08)^2}{150}+\frac{(1.28)^2}{200}}} = -2.46$; $|z| = 2.46$; $z_\alpha = 1.645$ at 5% LoS, $|z| > z_\alpha$, H_0 is rejected.

   Conclusion: The hourly wage paid by plant B are higher than those paid by plant A.

8. ***A simple sample of heights of 6400 English men has a mean of 170 cm and a S.D. of 6.4 cm, while a simple sample of heights of 1600 Americans has a mean of 172 cm and a S.D. of 6.3 cm. Do the data indicate that Americans are on the average, taller than the Englishmen?***

   Solution: $n_1 = 6400,\ n_2 = 1600, \overline{x_1} = 170,\ \overline{x_2} = 172,\ s_1 = 6.4, s_2 = 6.3$

   Null Hypothesis $H_0 : \bar{x}_1 = \bar{x}_2$, *Alternative Hypothesis* $H_1 : \bar{x}_1 < \bar{x}_2$ (left tailed test)

   Test statistics$z = \frac{\bar{x}_1-\bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}}$, Since σ_1 & σ_2 the population S.D. are not known, we can take $\sigma_1^2 = s_1^2\ and\ \sigma_2^2 = s_2^2$, Test statistics $z = \frac{\bar{x}_1-\bar{x}_2}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}} = \frac{170-172}{\sqrt{\frac{(6.4)^2}{6400}+\frac{(6.3)^2}{1600}}} = -11.32$; $|z| = 11.32$; $z_\alpha = 2.33$ at 1% LoS, $|z| > z_\alpha$, H_0 is rejected.

Students 't' – Test - *SINGLE MEAN*

Standard deviation given directly: $t = \frac{\bar{x}-\mu}{\left(\frac{s}{\sqrt{n-1}}\right)}$, where $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$,

$\bar{x}$ - sample mean, μ – population mean, n – sample size, s^2 – sample variance

Standard deviation not given directly: $t = \frac{\bar{x}-\mu}{\left(\frac{S}{\sqrt{n}}\right)}$, where $S^2 = \frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1}$

$\bar{x}$ - sample mean, μ – population mean, n – sample size, S^2 – population variance σ^2, ***Degree of freedom :*** $n-1$

DIFFERENCE MEAN

$t = \frac{\bar{x}-\bar{y}}{\sqrt{S^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$, where $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$, $\bar{y} = \frac{\sum_{j=1}^{n} y_j}{n}$, ***Degree of freedom :*** $n_1 + n_2 - 2$

Standard deviation given directly: $s^2 = \frac{[n_1 s_1^2 + n_2 s_2^2]}{n_1+n_2-2}$

Standard deviation not given directly: $s^2 = \frac{\sum(x-\bar{x})^2 + \sum(y-\bar{y})^2}{n_1+n_2-2}$,

Confidence or Fiducial Limits for μ:

95% Confidence limits for μ : $\bar{x} - t_{0.05}\left(\frac{S}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.05}\left(\frac{S}{\sqrt{n}}\right)$

99% Confidence limits for μ : $\bar{x} - t_{0.01}\left(\frac{S}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.01}\left(\frac{S}{\sqrt{n}}\right)$

Application of t – distribution:

The t – distribution has a wide number of applications in statistics, some of which are enumerated below

(i) To test if the sample mean $\bar{x}$ differs significantly from the hypothetical value μ of the population mean.
(ii) To test the significance of the difference between two sample means.
(iii) To test the significance of an observed sample correlation coefficient and sample regression coefficient
(iv) To test the significance of observed partial correlation coefficient.

Assumption for students's t – test:

(i) The parent population from which the sample is drawn is normal.
(ii) The sample observations are independent, that is, the sample is random.
(iii) The population standard deviation σ is unknown.

PROBLEMS

Students ‘t’ – Test : Single Mean and Standard Deviation Given Directly

1. ***The mean weekly sales of soap bars in departmental stores was 146.3 bars per store. After an advertising campaign the mean weekly sales in 22 stores for a typical week increased to 153.7 and showed a S.D. of 17.2. Was the advertising campaign successful?***

   Solution: $n = 22, \ \bar{x} = 153.7, \ \mu = 146.3 \ , s = 17.2$

   Null Hypothesis $\boldsymbol{H_0}$: $\mu = 146.3$, *i.e.* The advertising campaign is not successful.

   Alternate Hypothesis $\boldsymbol{H_1}$ **:** $\mu > 146.3$ *(Right tailed)*

   $t = \frac{\bar{x}-\mu}{\left(\frac{s}{\sqrt{n-1}}\right)} = \frac{153.3-146.3}{\left(\frac{17.2}{\sqrt{22-1}}\right)} = 1.96$**,** degree of freedom: $n - 1 = 21$ at 5% level of significance = 1.72.

   Calculate value t > Tabulated t. $\mathbf{H_0}$ **is rejected.**

   Conclusion: The advertising campaign was definitely successful in promoting sales.

2. ***A sample of 20 items has mean 42 units and S.D. 5 units. Test the hypothesis that it is a random sample from a normal population with mean 45 units.***

   Solution: $n = 20, \ \bar{x} = 42, \ \mu = 45, \ s = 5$

   Null Hypothesis $\boldsymbol{H_0}$: $\mu = 45$, ***Alternate Hypothesis*** $\boldsymbol{H_1}$ **:** $\mu \neq 45$ *(Two tailed)*

   $t = \frac{\bar{x}-\mu}{\left(\frac{s}{\sqrt{n-1}}\right)} = \frac{42-45}{\left(\frac{5}{\sqrt{20-1}}\right)} = 2.615$**,** degree of freedom: $n - 1 = 19$ at 5% level of significance = 2.09.

   Calculate value t > Tabulated t. $\mathbf{H_0}$ **is rejected.** The sample could not have come from this population.

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3. ***A spare part manufacturer is making spare parts with axle diameter of 0.700 inch. A random sample of 10 parts shows a mean diameter of 0.742 inch with a S.D. of 0.040 inch. Verify whether the work satisfies the specifications.***

   Solution: $n = 10, \ \bar{x} = 0.742, \ \mu = 0.700, \ s = 0.040$

   Null Hypothesis $\boldsymbol{H_0}$: $\mu = 0.700$, ***Alternate Hypothesis*** $\boldsymbol{H_1}$ **:** $\mu \neq 0.700$ *(Two tailed)*

   $t = \frac{\bar{x}-\mu}{\left(\frac{s}{\sqrt{n-1}}\right)} = \frac{0.742-0.700}{\left(\frac{0.040}{\sqrt{10-1}}\right)} = 3.15$, degree of freedom: $n - 1 = 9$ at 5% level of significance = 2.26.

   Calculate value t > Tabulated t. $\mathbf{H_0}$ **is rejected.** ***Conclusion:*** The product is not meeting the specifications.

4. ***The mean life time of a sample of 25 bulbs is found as 1550 hours with a S.D. of 120 hours. The company manufacturing the bulbs claims that the average life of their bulbs is 1600 hours. Is the claim acceptable at 5% level of significance?***

   Solution: $n = 25, \ \bar{x} = 1550, \ \mu = 1600, \ s = 120$

   Null Hypothesis $\boldsymbol{H_0}$: $\bar{x} = \mu$, ***Alternate Hypothesis*** $\boldsymbol{H_1}$ **:** $\bar{x} < \mu$ *(Left Tailed)*

   $t = \frac{\bar{x}-\mu}{\left(\frac{s}{\sqrt{n-1}}\right)} = \frac{1550-1600}{\left(\frac{120}{\sqrt{25-1}}\right)} = -2.04$, degree of freedom: $n - 1 = 24$ at 5% level of significance = 1.71.

   Calculate value t > Tabulated t. $\mathbf{H_0}$ **is rejected.** The claim of the company cannot be accepted at 5% LOS.

Students 't' – Test : Single Mean and Standard Deviation Not Given Directly

5. ***A random sample of 10 boys had the following I.Q.'s 70, 120, 110, 101, 88, 83, 95, 98, 107, 100. Do these date support the assumption of a population mean I.Q.'s of 100? Find a reasonable range in which most of the mean I.Q. values of samples of 10 boys lie.***

Solution: $n = 10,\ \mu = 100,\ \boldsymbol{H_0} : \mu = 100,\ \boldsymbol{H_1} :\ \mu \neq 100$ *(Two tail)*

$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{972}{10} = 97.2, \quad S^2 = \frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1} = \frac{1833.6}{10-1} = 203.73$

x	$x - \bar{x}$	$(x - \bar{x})^2$
70	-27.2	739.84
120	22.8	519.84
110	12.8	163.84
101	3.8	14.44
88	-9.2	84.64
83	-14.2	201.64
95	-2.2	4.84
98	0.8	0.64
107	9.8	96.04
100	2.8	7.84
$\sum x = 972$		$\sum(x-\bar{x})^2 = 1833.60$

$t = \frac{\bar{x}-\mu}{\left(\frac{S}{\sqrt{n}}\right)} = \frac{97.2-100}{\left(\sqrt{\frac{203.73}{10}}\right)} = \frac{-2.8}{(\sqrt{20.373})} = -0.62 \Rightarrow |\boldsymbol{t}| = \mathbf{0.62}$; d.f.= n − 1 = 10 − 1 = 9 at 5% LOS = 2.262.

Calculate value t < Tabulated t. $\mathbf{H_0}$ **is accepted.**

Conclusion: The data are consistent with the assumption for a mean I.Q. of 100 in population.

95% Confidence Limits for μ: $\bar{x} - t_{0.05}\left(\frac{S}{\sqrt{n}}\right) \leq \mu \leq \bar{x} + t_{0.05}\left(\frac{S}{\sqrt{n}}\right)$

$97.2 - 2.262\left(\sqrt{\frac{203.73}{10}}\right) \leq \mu \leq 97.2 + 2.262\left(\sqrt{\frac{203.73}{10}}\right)$

$86.99 \leq \mu \leq 107.41$, Hence the required 95% confidence interval is $[86.99,\ 107.41]$.

6. ***The wages of 10 workers taken at random from a factory are given as Wages: 578, 572, 570, 568, 572, 578, 570, 572, 596, 584. Is it possible that the mean wage of all workers of this factory could be Rs. 580***

Solution: $n = 10,\ \mu = 580,\ \boldsymbol{H_0} : \mu = 580, \boldsymbol{H_1} :\ \mu \neq 580$ *(Two tailed)*

$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{5760}{10} = 576, \quad S^2 = \frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1} = \frac{656}{10-1} = 72.89$

d.f.= n − 1 = 10 − 1 = 9 at 5% LoS = 2.262.

x	$x-\bar{x}$	$(x-\bar{x})^2$
578	2	4
572	-4	16
570	-6	36
568	-8	64
572	-4	16
578	2	4
570	-6	36
572	-4	16
596	20	400
584	8	64
$\sum x = 5760$		$\sum(x-\bar{x})^2 = 656$

$t = \frac{\bar{x}-\mu}{\left(\frac{S}{\sqrt{n}}\right)} = \frac{576-580}{\left(\sqrt{\frac{72.89}{10}}\right)} = \frac{-4}{(\sqrt{7.289})} = -1.48 \Rightarrow |t| = \mathbf{1.48}$; Calculate value t < Tabulated t. **H_0 is accepted.**

Conclusion: Is it possible that the mean wage of all workers of this factory could be Rs. 580.

Students 't' – Test : Difference Mean and Standard Deviation Given Directly

7. ***Sample of two types of electric light bulbs were tested for length of life and following data were obtained***

	Type I	***Type II***
Sample Number	$n_1 = 8$	$n_2 = 7$
Sample Means	$\overline{x_1} = 1234\ hrs$	$\overline{x_2} = 1036\ hrs$
Sample S.D.	$s_1 = 36\ hrs$	$s_2 = 40\ hrs$

Is the difference in the means sufficient to warrant that type I is superior to type II regarding length of life

Solution: Null Hypothesis : $H_0: \overline{x_1} = \overline{x_2}$, **i.e.** The two types I and II of electric bulbs are identical.

Alternate Hypothesis : $H_1: \overline{x_1} > \overline{x_2}$ (Right tail)

Standard deviation given directly: $s^2 = \frac{[n_1 s_1^2 + n_2 s_2^2]}{n_1+n_2-2} = \frac{[8(36)^2+7(40)^2]}{8+7-2} = 1659.08$

$t = \frac{\overline{x_1}-\overline{x_2}}{\sqrt{S^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{1234-1036}{\sqrt{1659.08\left(\frac{1}{8}+\frac{1}{7}\right)}} = 9.39$, degree of freedom $= n_1 + n_2 - 2 = 3$ at 5% = 1.77.

Calculate value t > Tabulated t. **H_0 is rejected.** The type I is definitely superior to type II regarding length of life.

Students 't' – Test : Difference Mean and Standard Deviation Not Given Directly

8. ***Below are given the gain in weights (in kgs) of pigs fed on two diets A and B.***

Diet A	25	32	30	34	24	14	32	24	30	31	35	25	-	-	-
Diet B	44	34	22	10	47	31	40	30	32	35	18	21	35	29	22

Test if the two diets differ significantly as regards their effect on increase in weight.

Solution: Null Hypothesis $H_0: \overline{x_1} = \overline{x_2}$, **i.e.** There is no significant difference between the mean increase in weight due to diets A & B, ***Alternate Hypothesis :*** $H_1: \overline{x_1} \neq \overline{x_2}$ (Two tailed)

x	$x-\bar{x}$	$(x-\bar{x})^2$	y	$y-\bar{y}$	$(y-\bar{y})^2$
25	-3	9	44	14	196
32	4	16	34	4	16
30	2	4	22	-8	64
34	6	36	10	-20	400
24	-4	16	47	17	289
14	-14	196	31	1	1
32	4	16	40	10	100
24	-4	16	30	0	0
30	2	4	32	2	4
31	3	9	35	5	25
35	7	49	18	-12	144
25	-3	9	21	-9	81
$\sum x = 336$		$\sum(x-\bar{x})^2 = 380$	35	5	25
			29	-1	1
			22	-8	64
			$\sum y = 450$		$\sum(y-\bar{y})^2 = 1410$

$n_1 = 12,\ n_2 = 15,\ \ \bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{336}{12} = 28,\ \bar{y} = \frac{\sum_{j=1}^{n} y_j}{n} = \frac{450}{15} = 30$

Standard deviation not given directly: $s^2 = \frac{\sum(x-\bar{x})^2+\sum(y-\bar{y})^2}{n_1+n_2-2} = 71.6$

$t = \frac{\bar{x}-\bar{y}}{\sqrt{s^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{28-30}{\sqrt{71.6\left(\frac{1}{12}+\frac{1}{15}\right)}} = -0.609 \Rightarrow |t| = \mathbf{0.609}$, d.f.$= n_1 + n_2 - 2 = 25$ at 5% LOS = 2.06.

Calculate value t < Tabulated t. **H_0 is accepted.**

Conclusion: The two diets do not differ significantly as regards their effect on increase in weight.

9. ***A group of five patients treated with medicine A weigh 42, 39, 48, 60 and 41 kg.; a second group of 7 patients from the same hospital treated with medicine B weigh 38, 42, 56, 64, 68, 69 and 62 kg. Do you agree with the claim that medicine B increases the weight significantly?***

Solution: Null Hypothesis : $H_0: \mu_1 = \mu_2$, ***Alternate Hypothesis*** : $H_1: \mu_1 < \mu_2$

x	$x-\bar{x}$	$(x-\bar{x})^2$	y	$y-\bar{y}$	$(y-\bar{y})^2$
42	-4	16	38	-19	361
39	-7	49	42	-15	225
48	2	4	56	-1	1
60	14	196	64	7	49
41	-5	25	68	11	121
$\sum x = 230$		$\sum(x-\bar{x})^2 = 290$	69	12	144
			62	5	25
			$\sum y = 399$		$\sum(y-\bar{y})^2 = 926$

$n_1 = 5,\ n_2 = 7,\ \ \bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{230}{5} = 46,\ \bar{y} = \frac{\sum_{j=1}^{n} y_j}{n} = \frac{399}{7} = 57$

Standard deviation not given directly: $s^2 = \frac{\sum(x-\bar{x})^2+\sum(y-\bar{y})^2}{n_1+n_2-2} = \frac{290+926}{5+7-2} = \frac{1216}{10} = 121.6$

$t = \frac{\bar{x}-\bar{y}}{\sqrt{s^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{46-57}{\sqrt{121.6\left(\frac{1}{5}+\frac{1}{7}\right)}} = \frac{46-57}{\sqrt{41.69}} = \frac{-11}{6.46} = -1.703 \Rightarrow |t| = \mathbf{1.703}$, d.f.$= n_1 + n_2 - 2 = 10$ at 5% = 1.81.

Calculate value t < Tabulated t. **H_0 is accepted.** Medicines A and B do not differ significantly.

10. The table below represent the values of protein content from cow's milk and buffalo's milk at a certain level. Examine if these differences are significant.

Cow's milk	***1.82***	***2.02***	***1.88***	***1.61***	***1.81***	***1.54***
Buffalo's milk	***2***	***1.83***	***1.86***	***2.03***	***2.19***	***1.88***

Solution: Null Hypothesis $H_0: \overline{x_1} = \overline{x_2}$ ***, Alternate Hypothesis :*** $H_1: \overline{x_1} \neq \overline{x_2}$ (Two tailed)

x_1	x_1^2	x_2	x_2^2
1.82	3.3124	2	4.0040
2.02	4.0804	1.83	3.3489
1.88	3.5344	1.86	3.4596
1.61	2.5921	2.03	4.1209
1.81	3.2761	2.19	4.761
1.54	2.3716	1.88	3.5344
$\sum x_1 = 10.68$	$\sum x_1^2 = 19.1670$	$\sum x_2 = 11.79$	$\sum x_2^2 = 23.2599$

$n_1 = 6,\ n_2 = 6,\ \overline{x_1} = \frac{\sum x_1}{n_1} = \frac{10.68}{6} = 1.78;\ \ \overline{x_2} = \frac{\sum x_2}{n_2} = \frac{11.79}{6} = 1.965$

Sample S.D.: $s_1^2 = \frac{\sum x_1^2}{n_1} - (\overline{x_1})^2 = \frac{19.167}{6} - (1.78)^2 = 0.0261;\ \ s_2^2 = \frac{\sum x_2^2}{n_2} - (\overline{x_2})^2 = \frac{25.2599}{6} - (11.79)^2 = 0.0154$

$t = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{\left(\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}\right)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{1.78 - 1.965}{\sqrt{\left(\frac{6\times 0.0261 + 6\times 0.0154}{6+6-2}\right)\left(\frac{1}{6} + \frac{1}{6}\right)}} = -2.03 \Rightarrow |\boldsymbol{t}| = \mathbf{2.03}$, d.f.$= n_1 + n_2 - 2 = 10$ at 5% $= 2.26$.

Calculate value t < Tabulated t. $\mathbf{H_0}$ **is accepted.**

Conclusion: The difference between the mean protein values of the two varieties of milk is not significant at 5%.

Paired Student 't' – Test

$t = \frac{\bar{d}}{\left(\frac{s}{\sqrt{n-1}}\right)}$, where $s^2 = \frac{\sum d^2}{n} - (\bar{d})^2$; $d = x_1 - x_2$; $\bar{d} = \frac{\sum d}{n}$; $d.f = n-1$; ***Degree of freedom* : $\mathbf{n-1}$**

11. The following data relate to the marks obtained by 11 students in two tests, one held at the beginning of a year and the other at the end of the year after intensive coaching. Do the data indicate that the students have benefited by coaching?

Test I	19	23	16	24	17	18	20	18	21	19	20
Test II	17	24	20	24	20	22	20	20	18	22	19

Solution: Null Hypothesis $H_0: \overline{x_1} = \overline{x_2}$; ***Alternate Hypothesis :*** $H_1: \overline{x_1} < \overline{x_2}$ (left tailed)

x_1	x_2	$d = x_1 - x_2$	d^2
19	17	2	4
23	24	-1	1
16	20	-4	16
24	24	0	0
17	20	-3	9
18	22	-4	16
20	20	0	0
18	20	-2	4
21	18	3	9
19	22	-3	9
20	19	1	1
		$\sum d = -11$	$\sum d^2 = 69$

$n_1 = 11,\ n_2 = 11,\ \bar{d} = \frac{\sum d}{n} = \frac{-11}{11} = -1;\ s^2 = \frac{\sum d^2}{n} - (\bar{d})^2 = \frac{69}{11} - (-1)^2 = 5.27;\ s = 2.296$

$t = \frac{\bar{d}}{\left(\frac{s}{\sqrt{n-1}}\right)} = -\frac{1}{\left(\frac{2.296}{\sqrt{11-1}}\right)} = -1.38$; $\Rightarrow |t| = 1.38$, d.f.$= n-1 = 10$ at 5% $= 1.81$.

Calculate value t < Tabulated t. **H_0 is accepted.** ***Conclusion:*** The students have not benefitted by coaching.

A certain pesticide is packed into bags by a machine. A random sample of 10 bags is chosen and the contents of the bags is found to have the following weight (in kg) 50,49,52,44,45,48,46,45,49,and 45. Test if the average quantity packed be taken as 50 kg.

Solution:

Calculation for sample mean and S.D:

x	$x - \bar{x}$ $=(x-47.3)$	$(x-\bar{x})^2$
50	2.7	7.29
49	1.7	2.89
52	4.7	22.09
44	-3.3	10.89

45	-2.3	5.29
48	0.7	0.49
46	-1.3	1.69
45	-2.3	5.29
49	1.7	2.89
45	-2.3	5.59
Σ=473		64.10

Mean = $\bar{x} = \frac{\sum x}{n} = \frac{473}{10} = 47.3$

We know that $S^2 = \frac{1}{n-1} \sum (x-\bar{x})^2$

$= \frac{64.10}{10-1}$

$S^2 = 7.12$

S = 2.67

Null hypothesis Ho : The average pack is 50kg (ie) μ= 50

Alternative hypothesis H_1 : $\mu \neq 50$

Test statistic is $t = \frac{\bar{x}-\mu}{S/Vn} \rightarrow t = \frac{47.3-50}{\left(2.67/\sqrt{10}\right)}$

|t| =3.19

Calculated value of t for (n-1)= 9 d.f is 3.19

Tabulated value of 't' for 9 d.f is 2.262

$\rightarrow t_{cal} > t_{tab}$

Hence,we reject the null hypothesis H_0 (i.e)The average packing is not 50 Kgs.

7. Given $\overline{X_1}$ = 72, $\overline{X_2}$ =74, S_1 = 8, S_2 = 6, n_1 = 32, n_2 = 36. Test if the means are significant

Solution:

With the given data, it is determine that this test is large sample test to perform difference between sample means.

Null hypothesis H_o : There is no significant difference between sample means (ie) $\overline{X}_1 = \overline{X}_2$

Alternative hypothesis $H_1 = \overline{X}_1 \neq \overline{X}_2$

Test statistic $Z = \dfrac{\overline{x}_1 - \overline{x}_2}{\sqrt{\dfrac{S_1^2}{n_1} + \dfrac{S_2^2}{n_2}}}$

$$Z = \frac{72 - 74}{\sqrt{\dfrac{8}{32} + \dfrac{6}{36}}}$$

Z = -3.0984

|Z|=3.0984

At 5% level of significance |Z|= 1.96

$\rightarrow |Z|_{cal} > |Z|_{tab}$

(ie) 3.0984 > 96

We reject null hypothesis H_o

(ie) there is no significant difference between $\overline{x_1}$ and $\overline{x_2}$

8. A mathematics test was given to 50 girls and 75 boys. The girls made an average grade of 76 with S.D of 6 and the boys made an average grade of 82 with S.D of 2 . Test whether there is any difference between the performance of boys and girls.

Solution:

Given $n_1 = 50$ $\quad$ $n_2 = 75$

$\overline{x_1} = 76$ $\quad$ $\overline{x_2} = 82$

$\sigma_1 = 6$ $\quad$ $\sigma_2 = 2$

Null Hypothesis H_o : there is no significant difference between sample means (ie) $\overline{x_1} = \overline{x_2}$

Alternative Hypothesis H_1 : $\overline{x_1} \neq \overline{x_2}$

Test Statistic $Z = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$

$|Z| = \dfrac{76-82}{\sqrt{\dfrac{36}{50} + \dfrac{4}{75}}}$

$|Z| = 6.8229$

At 5% level of significance, $|Z| = 1.96$

$\rightarrow |Z|_{cal} > |Z|_{tab}$

$\rightarrow H_o$ is rejected

(ie) there is significant difference between performance of girls and boys

9. Theory predicts the proportion of beans in the group A,B,C,D as 9:3:3:1. In an experiment among beans the numbers in the groups were 882,313,287 and 118. Does the experiment support theory?

Solution:

Null hypothesis Ho : The experimental result support the theory.

If we divide 1600 in the ratios 9:3:3:1, we get the expected frequencies as 900,300,300,100.

Observed frequency (0)	Expected frequency (E)	(0-E)	$\frac{(0-E)^2}{E}$
882	900	-18	0.360
313	300	13	0.563
287	300	-13	0.563
118	100	18	0.324
1600			4.726

$\therefore\ y^2 = \sum \dfrac{(0-E)^2}{E}$

$y^2 = 4.726$

(ie) Calculated value of χ^2 =4.726 for 3 d.f. At 5% level, χ^2_{tab} =7.81 for 3 d.f

$\rightarrow \chi^2_{cal} = \chi^2_{tab}$

$\rightarrow$ we accept null hypothesis

(i.e) The experimental results support the theory.

10. 400 men and 600 women were asked whether they would like to have a flyover near their residence 200 men and 325 women were in favour of the proposal. Test whether these two proportions are same.

Solution:

Given sample sizes $n_1 = 400$, $n_2 = 600$

proportion of men $P_1 = \frac{200}{400} = 0.5$

proportion of women $P_2 = \frac{325}{600} =0.541$

Null hypothesis H_o : Assume that there is no significant difference between the option of men and woman as for as proposal of flyover is concerned (ie) $P_1 = P_2$

Alternative hypothesis H1 : $P_1 \neq P_2$

The test statistic $Z = \frac{P_1 - P_2}{\sqrt{Pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$

Where $P = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2}$

$$= \frac{400\left(\frac{200}{400}\right) + 600\left(\frac{325}{600}\right)}{400 + 600} = \frac{525}{1000}$$

$$P = 0.525 ,Q = 1\text{-}P= 1\text{-}0.525$$

$$\therefore Z = \frac{0.5 - 0.541}{\sqrt{(0.525)(0.475)\left(\frac{1}{400} + \frac{1}{600}\right)}}$$

$$= \frac{-0.041}{0.032} = -1.34$$

$|Z| = 1.34$

since $|Z| = 1.96$, at 5% level of significant, (ie) $|Z|_{cal} < |Z|_{tab}$,we accept the null hypothesis H_o

(ie) There is no difference of opinion between men and women as far as proposal of flyover is concerned.

11. The IQ's of 10 girls are respectively 120, 110,70,88,101,100,83,98,95,107. Test whether the population mean IQ is 100.

Solution:

Here S.D and mean of the sample is not given directly, we have to determine S.D and mean as follows.

x	$\left(x-\bar{x}\right)$ = (x-97.2)	$\left(x-\bar{x}\right)^2$
120	-27.2	739.84
110	22.8	519.84
70	12.8	163.84
88	3.8	14.44
101	-9.2	84.64
100	-14.2	201.64
83	-2.2	4.84
98	0.8	0.64
95	9.8	96.04
107	2.8	7.84
Σ= 972		1833.60

mean $\bar{x} = \frac{\sum x}{n} = \frac{972}{10} = 97.2$

We know that, $S^2 = \frac{1}{n-1}\Sigma\left(x-\bar{x}\right)^2$

$= \frac{1833.60}{9} = 203.73$

S=14.27

Null Hypothesis Ho : The data support the assumption of a population mean I.Q of 100 in the population.

Alternative hypothesis H_1: $\mu \neq 100$

Test statistic $t = \frac{\bar{x}-\mu}{S/vn}$

$= \frac{97.2-100}{14.27/\sqrt{10}}$

t = -0.62

Calculated value of |t| = 2.26 for (10-1) = 9.d.f at 5% level of significance

Since $t_{cal} < t_{tab}$,we accept the null hypothesis H_o (ie) the data support the assumption of mean IQ of 100 in the population.

PART-A

1. If θ_0 is a population parameter and θ is the corresponding sample statistic and if we set up the null hypotheses $H_0: \theta = \theta_0$ then the right-tailed alternative hypotheses is

(a) $H_1: \theta = \theta_0$ (b) $H_1: \theta > \theta_0$ (c) $H_1: \theta < \theta_0$ (d) $H_1: \theta \neq \theta_0$ **Ans: (b)**

2. The size of large sample is :

(a)Exact (b) Less than 30 (c) Greater than 30 (d) Equal to 30 **Ans: (c)**

3. The statistic to test the significance difference between sample proportion and population proportion is

(a) $\frac{p-P}{\sqrt{\frac{p}{n}}}$ (b) $\frac{p+P}{\sqrt{\frac{pQ}{n}}}$ (c) $\frac{p-P}{\sqrt{\frac{pQ}{n}}}$ (d) $\frac{p-P}{\sqrt{\frac{Q}{n}}}$

Ans: (c)

4. The statistic to test the significance difference between the sample mean and population mean is

(a) $Z = \frac{\bar{X}-\mu}{\frac{\sigma}{\sqrt{n}}}$ (b) $Z = \frac{\bar{X}+\mu}{\frac{\sigma}{\sqrt{n}}}$ (c) $Z = \frac{\bar{X}}{\frac{\sigma}{\sqrt{n}}}$ (d) $Z = \frac{\bar{X}-\mu}{\frac{\sigma}{n}}$

Ans: (a)

5. If σ_1 and σ_2 are equal and not known then the test statistic is

(a) $Z=\dfrac{\bar{X}_1-\bar{X}_2}{\sqrt{\dfrac{s_1^2}{n_2}+\dfrac{s_2^2}{n_1}}}$ (b) $Z=\dfrac{\bar{X}_1+\bar{X}_2}{\sqrt{\dfrac{s_1^2}{n_2}+\dfrac{s_2^2}{n_1}}}$ (c) $Z=\dfrac{\bar{X}_1-\bar{X}_2}{\sqrt{\dfrac{s_1^2}{n_2}-\dfrac{s_2^2}{n_1}}}$ (d) $Z=\dfrac{\bar{X}_1-\bar{X}_2}{\sqrt{\dfrac{s_1^2}{n_1}+\dfrac{s_2^2}{n_2}}}$

Ans: (a)

6. The sample is said to be small if

(a) n > 30 (b) n > 100 (c) n < 60 (d) n < 30 **Ans: (d)**

7. The t – distribution is used to test the significance of the difference between

(a)Mean of two small samples (b) Variance of two small samples

(c) Mean of two large samples (d) Variance of two large samples **Ans: (a)**

8. If $n_1 = n_2 = n$, then the degrees of freedom to test mean of the two small samples is

(a) $n_1 + n_2 - 2$ (b) $n_1 + n_2 + 2$ (c) 2n - 2 (d) 2n + 2 **Ans: (c)**

9. The statistics to test the significance difference between means of two samples is

(a) $\dfrac{\overline{x_1}-\overline{x_2}}{\sqrt{\left(\dfrac{n_1 s_1^2+n_2 s_2^2}{n_1+n_2-2}\right)\left(\dfrac{1}{n_1}+\dfrac{1}{n_2}\right)}}$ (b) $\dfrac{\overline{x_1}+\overline{x_2}}{\sqrt{\left(\dfrac{n_1 s_1^2+n_2 s_2^2}{n_1+n_2-2}\right)\left(\dfrac{1}{n_1}+\dfrac{1}{n_2}\right)}}$

(c) $\dfrac{\overline{x_1}-\overline{x_2}}{\sqrt{\left(\dfrac{n_1 s_1^2+n_2 s_2^2}{n_1+n_2-2}\right)\left(\dfrac{1}{n_1}-\dfrac{1}{n_2}\right)}}$ (d) $\dfrac{\overline{x_1}-\overline{x_2}}{\sqrt{\left(\dfrac{n_1 s_1^2+n_2 s_2^2}{n_1+n_2-2}\right)}}$ **Ans: (a)**

10. 18. A statement about a population developed for the purpose of testing is called:
(a) <u>Hypothesis</u> (b) Hypothesis testing (c) Level of significance (d) Test-statistic

11. Any hypothesis which is tested for the purpose of rejection under the assumption that it is true is called: **(a) <u>Null hypothesis</u>** (b) Alternative hypothesis
(c) Statistical hypothesis (d) Composite hypothesis

12. A statement about the value of a population parameter is called:
(a) <u>Null hypothesis</u> (b) Alternative hypothesis
(c) Simple hypothesis (d) Composite hypothesis

13.Any statement whose validity is tested on the basis of a sample is called:
(a) Null hypothesis (b) Alternative hypothesis
(c) <u>Statistical hypothesis</u> (b) Simple hypothesis

14. A quantitative statement about a population is called:
(a) Research hypothesis (b) Composite hypothesis (c) Simple hypothesis **(d) <u>Statistical hypothesis</u>**

15.A statement that is accepted if the sample data provide sufficient evidence that the null hypothesis is false is called: (a) Simple hypothesis (b) Composite hypothesis
(c) Statistical hypothesis **(d) <u>Alternative hypothesis</u>**

16. A hypothesis may be classified as:
(a) Simple (b) Composite (c) Null **(d) <u>All of the above</u>**

17.The dividing point between the region where the null hypothesis is rejected and the region where it is not rejected is said to be:
(a) Critical region **(b) <u>Critical value</u>** (c) Acceptance region (d) Significant region

18. If the critical region is located equally in both sides of the sampling distribution of test-statistic, the test is called: (a) One tailed **(b) <u>Two tailed</u>** (c) Right tailed (d) Left tailed

19. The choice of one-tailed test and two-tailed test depends upon:
(a) Null hypothesis **(b) <u>Alternative hypothesis</u>** (c) None of these (d) Composite hypotheses

270. Test of hypothesis Ho: $\mu = 50$ against H_1: $\mu > 50$ leads to:
(a) Left-tailed test **(b) <u>Right-tailed test</u>** (c) Two-tailed test (d) one tailed test

21. Testing Ho: $\mu = 25$ against H_1: $\mu \neq 20$ leads to:
(a) <u>Two-tailed test</u> (b) Left-tailed test (c) Right-tailed test (d) Neither (a), (b) and (c)

22. The range of test statistic-Z is:
(a) 0 to 1 (b) -1 to +1 (c) 0 to ∞ **(d) <u>$-\infty$ to $+\infty$</u>**

23. The range of test statistic-t is:
(a) 0 to ∞ (b) 0 to 1 **(c) <u>$-\infty$ to $+\infty$</u>** (d) -1 to +1

24. If H_o is true and we reject it is called:
(a) <u>Type-I error</u> (b) Type-II error (c) Standard error (d) Sampling error

25.The probability associated with committing type-I error is:
(a) β **(b) <u>α</u>** (c) $1 - \beta$ (d) $1 - \alpha$

26. A failing student is passed by an examiner, it is an example of:
(a) Type-I error **(b) <u>Type-II error</u>** (c) Unbiased decision (d) Sampling error

27. A passing student is failed by an examiner, it is an example of:
(a) <u>Type-I error</u> (b) Type-II error (c) Best decision (d) All of the above

28. A null hypothesis is rejected if the value of a test statistic lies in the:
(a) <u>Rejection region</u> (b) Acceptance region (c) Both (a) and (b) (d) Neither (a) nor (b)

29. The test statistics is equal to:

$(a)\dfrac{Sample - Population}{s\tan dard\ error}$ $(b)\dfrac{Sample\ statistic - Parameter}{Standard\ error\ of\ the\ statistic}$

$(c)\dfrac{Sample\ mean - Population\ mean}{Population\ s\tan dard\ deviation}$ $(e)\dfrac{Statistics\ - E(Statistic)}{Variance\ of\ the\ statistic}$

30. Level of significance is also called:
(a) Power of the test **(b) <u>Size of the test</u>** (c) Level of confidence (d) Confidence coefficient

31. Level of significance α lies between:
(a) -1 and +1 **(b) <u>0 and 1</u>** (c) 0 and n (d) -∞ to +∞

32. Critical region is also called:
(a) Acceptance region **(b) <u>Rejection region</u>** (c) Confidence region (d) Statistical region

33. An example in a two-sided alternative hypothesis is:
(a) H_1: $\mu < 0$ (b) H_1: $\mu > 0$ (c) H_1: $\mu \geq 0$ **(d) <u>H_1: $\mu \neq 0$</u>**

34. Student's t-test is applicable only when:
(a) <u>n≤30 and σ is known</u> (b) n>30 and σ is unknown (c) n=30 and σ is known (d) All of the above

35. The degree of freedom for paired t-test based on n pairs of observations is:
(a) 2n - 1 (b) n - 2 (c) 2(n - 1) **(d) <u>n - 1</u>**

36. The mean difference between 16 paired observations is 25 and the standard deviation of differences is 10. The value of statistic-**t** is:
(a) 4 **(b) <u>10</u>** (c) 16 (d) 25

37. The number of independent values in a set of values is called:
(a) Test-statistic **(b) <u>Degree of freedom</u>** (c) Level of significance (d) Level of confidence

45. When σ is known, the hypothesis about population mean is tested by:
(a) t-test **(b) <u>Z-test</u>** (c) χ^2-test (d) F-test

38. Given H_o: $\mu = \mu_o$, H_1: $\mu \neq \mu_o$, $\alpha = 0.05$ and we reject H_o; the absolute value of the Z-statistic must have equalled or been beyond what value?
(a) 1.96 (b) 1.65 **(c) <u>2.58</u>** (d) 2.33