Department of Mathematics

Sub Title: PROBABILITY AND QUEUING THEORY

Sub Code: 18MAB204T

Unit -II - Theoretical Distributions

1. A discrete R.V X has moment generating function $M_x(t) = (\frac{1}{4} + \frac{3}{4} e^t)^5$. Then E(X) and Var(X) is

a) $\frac{15}{4}, \frac{15}{4}$ b) $\frac{15}{4}, \frac{15}{16}$ c) $\frac{1}{4}, \frac{5}{4}$ d) $\frac{1}{4}, \frac{3}{4}$ **Ans: (b)**

2. Mean and Variance of Binomial Distribution is

a) np, npq b) nq, n/q c) pq , p+q =1, d) p+q,p-q **Ans: (a)**

3. If on an average, 9 ships out of 10 arrive safely to a port then the variance of the number of ships returning safely out of 150 ships is

a) 135 b) 13.5 c) 1.35 d) 12 **Ans: (b)**

4. If X and Y are independent Poisson variates with parameters λ_1 and λ_2, then X+Y is also a Poisson variate with parameter

a) $\lambda_1 + \lambda_2$ b) $\lambda_1 - \lambda_2$ c) λ_1 / λ_2 d) $\lambda_1 . \lambda_2$ **Ans: (a)**

5. Let X be a random variable following Poisson distribution such that P(X=2) = 9P(X=4) + 90P(X=6), then the mean of X is

a)1 b) 2 c)0 d)5 **Ans: (a)**

6. If X is a random variable with geometric distribution, then P[X > s+t / X > s] =

a) P[X > s] b) P[X > t] c) P[X < t] d) P[X < s] **Ans: (b)**

7. If the probability of success on each trial is 1/3 , then the expected number of trials required for the first success is

a) 2/3 b) 3 c) 2 d)1/3 **Ans: (b)**

8. A typist types 2 letters errorneously for every 100 letters. Then the probability that the tenth letter typed is the first letter with error is

a) 0.0167 b) 2.335 c) .0001 d) 0.1 **Ans: (a)**

9. Four coins are tossed simultaneously the probability of getting 2 heads is

a) 3/4 b)11/16 c)3/8 d)3 **Ans: (c)**

10. *Poisson distribution is a limiting case of*

a)Binomial distribution b) uniform distribution

c) Geometric distribution d) Normal distribution. **Ans: (a)**

11. The mean and variance of poisson distribution is

a)λ b) λ^2 c) λ^3 d) pq **Ans: (a)**

12. If the moment generating function of the random variable is $e^{4(e^t-1)}$. Find $P(X = \mu + \sigma)$

where μ and σ^2 are the mean and variance of poisson

a)$\frac{e^4 4^6}{6!}$ b) $\frac{e^{-4} 4^6}{6!}$ c) $\frac{e^{-6} 6^4}{4!}$ d) $\frac{e^6 6^4}{4!}$ **Ans: (b)**

13. Variance of Exponential distribution is

a)$\frac{1}{\lambda}$ b) $\frac{1}{\lambda^2}$ c)$\frac{1}{\sqrt{\lambda}}$ d)λ **Ans: (b)**

14. Memory less property is satisfied by

a) Exponential distribution b) Uniform distribution C) Normal distribution

d) Binomial distribution **Ans: (a)**

15. Moment generating function of exponential disturibution is ____________

16. All odd order moments of a Normal distribution about its mean are

a) Zero b) one c) infinity d) uniform **Ans: (a)**

17. Total area under the standard normal curve is equal to

a)0 b) 1 c)2 d)∞ **Ans: (b)**

18. If for a poisson variate, $E(X^2) = 6$, what is E(X)

a)1 b) 2 c) 6 d)3 **Ans: (b)**

19. If X has uniform distribution in (-3,3) Then $P(|x-2| < 2)$ IS

a) 0 b)1 c)1/2 d)2 **Ans: (c)**

20. Which of the following distribution satisfies Memoryless Property?

a) Binomial distribution b) Poisson distribution c) Geometric distribution

d) Normal distribution. **Ans: (c)**

PART-B

1.The mean and variance of the Binomial distribution are 4 and 3 respectively. Find P(X=0).

Given , mean = np = 4, Variance = npq = 3

$$q = \frac{3}{4}, \qquad p = 1 - \frac{3}{4} = \frac{1}{4}, \qquad np = 4 \Rightarrow n = 16$$

$$P(X=0) \quad = {}_nC_0 p^0 q^{n-0} = 16C_0 p^0 q^{16-0} = \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^{16} = \left(\frac{3}{4}\right)^{16}$$

2. Find p for a Binimial variate X if n=6,and 9P(X=4)=P(X=2).

Sol: $9P(X=4) = P(X=2) \Rightarrow 9\left({}_6C_4 p^4 q^2\right) = {}_6C_2 p^2 q^4$

$$\Rightarrow 9p^2 = q^2 = (1-p)^2 \therefore 8p^2 + 2p - 1 = 0$$

$$\therefore p = \frac{1}{4}\left(\because p \neq -\frac{1}{2}\right)$$

3. If X is a Poisson variate such that P(X=2)=9P (X = 4) + 90 P(X=6), find the variance

Ans : $P[X=x] = \dfrac{e^{-\lambda}\lambda^x}{x!}$

Given P(X=2)=9P (X = 4) + 90 P(X=6)

$$\therefore \frac{e^{-\lambda}\lambda^2}{2!} = 9\frac{e^{-\lambda}\lambda^4}{4!} + 90\frac{e^{-\lambda}\lambda^6}{6!}$$

$$\Rightarrow \frac{1}{2} = \frac{9}{24}\lambda^2 + \frac{90}{720}\lambda^4 \Rightarrow \lambda^4 + 3\lambda^2 - 4 = 0$$

$$\Rightarrow (\lambda^2+4)(\lambda^2-1) = 0$$

$$\Rightarrow \lambda^2 = -4 \quad \text{or} \quad \lambda^2 = 1$$

hence $\lambda = 1 [\because \lambda^2 \neq -4]$ Variance=1.

4. If X is a Poisson variate such that P(X = 1) = 3/10 and P(X = 2) = 1/5 Find P(X = 0) and P(X

Sol: $P(X=1) = \dfrac{3}{10} \Rightarrow \dfrac{e^{-\lambda}\lambda}{1} = \dfrac{3}{10}$ ....(1)

$P(X=2) = \dfrac{1}{5} \Rightarrow \dfrac{e^{-\lambda}\lambda^2}{2} = \dfrac{1}{5}$ ....(2)

$$\frac{(2)}{(1)} \Rightarrow \frac{\lambda}{2} = \frac{10}{15} \Rightarrow \lambda = \frac{4}{3} \qquad \therefore P(X=0) = \frac{e^{-\frac{4}{3}}\left(\frac{4}{3}\right)^0}{0!} = 0.2636$$

$$\therefore P(X=3) = \frac{e^{-\frac{4}{3}}\left(\frac{4}{3}\right)^3}{3!}$$

5. A Certain Blood Group type can be find only in 0.05% of the people. If the population of a randomly selected group is 3000.What is the Probability that atleast a people in the group have this rare blood group.

p=0.05% =0.0005 n=3000 $\therefore \lambda = np = 1.5$

$P(X \geq 2) = 1 - P(X < 2) = 1 - P(X = 0) - P(X = 1)$

$$= 1 - e^{-1.5}\left[1 + \frac{1.5}{1}\right] = 0.4422.$$

6. If the probability that a target is destroyed on any one shot is 0.5, what is the probability that it would

be destroyed on 6th attempt?

Given p = 0.5 q = 0.5 By Geometric distribution

$P[X=x] = q^x p$, x = 0,1,2………

since the target is destroyed on 6th attempt x = 5 ∴Required probability = qx p = (0.5)6 = 0.0157

7. If a boy is throwing stones at a target, what is the probability that his 10th throw is his 5th hit, if the

probability of hitting the target at any trial is ½?

Solution:

Since 10th throw should result in the 5th successes ,the first 9 throws ought to have resulted in 4 successes and 5 faliures.

n = 5,r = 5, p = 1/2 = q

∴Required probability = P(X=5)= (5+5-1)C5 (1/2)5 (1/2)5

=9C4 (1/210) = 0.123

8. Find the M.G.F of Poisson Distribution.

$$M_X(t) = \sum_{x=0}^{\infty} e^{tx} \frac{\lambda^x e^{-\lambda}}{x!} = \sum_{x=0}^{\infty} \frac{(e^t \lambda)^x e^{-\lambda}}{x!} = \quad e^{-\lambda} \sum_{x=0}^{\infty} \frac{(e^t \lambda)^x}{x!} = e^{-\lambda} e^{\lambda e^t} = e^{\lambda(e^t - 1)}$$

9. Find the M.G.F of Geometric distribution

$P(X=x) = p(x) = q^{x-1}$; $x = 1, 2, \ldots\ldots, 0 < p < 1,$ Where $q = 1-p$

To find MGF

$$\begin{aligned}
M_X(t) &= E[e^{tx}] \\
&= \sum e^{tx} p(x) \\
&= \sum_{x=1}^{\infty} e^{tx} q^{x-1} p \\
&= \sum_{x=1}^{\infty} e^{tx} q^{x} q^{-1} p \\
&= \sum_{x=1}^{\infty} e^{tx} q^{x} p / q \\
&= p / q \sum_{x=1}^{\infty} e^{tx} q^{x} \\
&= p / q \sum_{x=1}^{\infty} (e^{t} q)^{x} \\
&= p / q \left[(e^t q)^1 + (e^t q)^2 + (e^t q)^3 + \ldots \right]
\end{aligned}$$

Let $\quad x = e^t q \; = p / q \left[x + x^2 + x^3 + \ldots \right]$

$$= \frac{p}{q}(1-x)^{-1}$$

$$= \frac{p}{q}qe^t\left[1-qe^t\right] = pe^t\,[1-qe^t]^{-1}$$

$$\therefore M_X(t) = \frac{pe^t}{1-qe^t}$$

10. Find the mean and variance of the distribution $P[X=x]=2^{-x}$, x=1,2,3….

$$P[X=x]= \frac{1}{2^x} = \left(\frac{1}{2}\right)^{x-1}\frac{1}{2}, x = 1,2,3.....$$

$$\therefore p = \frac{1}{2} \text{ and } q = \frac{1}{2}$$

$$Mean = \frac{q}{p} = 1; \; Variance = \frac{q}{p^2} = 2$$

11. Find the MGF of a uniform distribution in (a, b)?

Ans :

$$M_X(t) = \frac{1}{b-a}\int_a^b e^{tx}dx \qquad = \frac{e^{bt}-e^{at}}{(b-a)t}$$

12.

The time (in hours) required to repair a machine is exponentially distributed with parameter $\lambda = \frac{1}{3}$ what is the probability that the repair time exceeds 3 hours?

Ans : X – represents the time to repair the machine

$$\therefore f(x) = \frac{1}{3}e^{-x/3} > 0$$

$$P(X > 3) = \int_3^\infty \frac{1}{3}e^{-x/3}dx = e^{-1} = 0.3679$$

13. **The time (in hours) required to repair a machine is exponentially distributed with parameter $\lambda = \frac{1}{2}$ what is the probability that a repair takes at least 10 hours given that its duration exceeds 9 hours?**

Ans :

Let X be the RV which represents the time to repair machine.

$P[X \geq 10/ X \geq 9] = P[X \geq 1]$ (by memory less property)

$$= \int_1^{\infty} \frac{1}{2} e^{-x/2} dx = 0.6065$$

14. What are the properties of Normal distribution

(1) The curve is symmetrical about mean.

(2) Mean = Median = Mode

(3) All odd moments vanish

(4) x - axis is an asymptote of the normal curve

15. If X is normally distributed RV with mean 12 and SD 4.Find $P[X \leq 20]$.

Sol: $P[X \leq 20] = P[Z \leq 2]$ where $Z = \frac{X-12}{4}$ $\left\{\because Z = \frac{X-\mu}{\sigma}\right\}$

$= P[-\infty \leq Z \leq 0] + P[0 \leq Z \leq 2]$

$=0.5+0.4772$

$=0.9772.$

16. If X is a Normal variate with mean30 and SD 5.Find $P[26<X<40]$.

Sol: $P[26<X<40] = P[-0.8 \leq Z \leq 2]$ where $Z = \frac{X-30}{5}$ $\left\{\because Z = \frac{X-\mu}{\sigma}\right\}$

$= P[0 \leq Z \leq 0.8] + P[0 \leq Z \leq 2]$

$= 0.2881+0.4772$

$=0.7653.$

17.Find the Moment Generating Function (MGF) of a binomial distribution about origin.

WKT $M_X(t) = \sum_{x=0}^{n} e^{tx} p(x)$

Let 'X' be a random variable which follows binomial distribution then MGF about origin is given by

$$E[e^{tX}] = M_X(t) = \sum_{x=0}^{n} e^{tx} p(x)$$

$$= \sum_{x=0}^{n} e^{tx}\, nC_x p^x q^{n-x} \qquad \left[\because p(x) = nC_x p^x q^{n-x}\right]$$

$$= \sum_{x=0}^{n} (e^{tx})\, p^x nC_x q^{n-x}$$

$$= \sum_{x=0}^{n} (pe^t)^x\, nC_x q^{n-x}$$

$$\therefore M_X(t) = (q+pe^t)^n$$

18.

Find the mean and variance of binomial distribution.

Solution

$$M_X(t) = (q+pe^t)^n$$

$$\therefore M_X'(t) = n(q+pe^t)^{n-1}.pe^t$$

Put $t = 0$, we get

$$M_X'(0) = n(q+p)^{n-1}.p$$

$$\text{Mean} = E(X) = np \qquad [\because (q+p)=1] \qquad \left[\text{Mean } M_X'(0)\right]$$

$$M_X''(t) = np\left[(q+pe^t)^{n-1}.e^t + e^t(n-1)(q+pe^t)^{n-2}.pe^t\right]$$

Put $t = 0$, we get

$$M_X''(t) = np\left[(q+p)^{n-1} + (n-1)(q+p)^{n-2}.p\right]$$

$$= np[1+(n-1)p]$$

$$= np + n^2p^2 - np^2$$

$$= n^2p^2 + np(1-p)$$

$$M_X''(0) = n^2p^2 + npq \qquad [\because 1-p = q]$$

$$M_X''(0) = E(X^2) = n^2p^2 + npq$$

$$\text{Var}(X) = E(X^2) - [E(X)]^2 = n^2/p^2 + npq - n^2/p^2 = npq$$

$$\text{Var}(X) = npq$$

$$\text{S.D} = \sqrt{npq}$$

19. Find the MGF of geometric distribution.

$P(X=x) = p(x) = q^{x-1}$; $x = 1, 2, \ldots\ldots, 0 < p < 1,$ Where $q = 1-p$

To find MGF

$$M_X(t) = E[e^{tx}]$$
$$= \sum e^{tx} p(x)$$
$$= \sum_{x=1}^{\infty} e^{tx} q^{x-1} p$$
$$= \sum^{\infty} e^{tx} q^x q^{-1} p$$
$$= \frac{p}{q} x \left[1 + x + x^2 + \ldots\right] \quad = \frac{p}{q}(1-x)^{-1}$$
$$= \frac{p}{q} qe^t \left[1 - qe^t\right] \quad = pe^t \, [1 - qe^t]^{-1}$$
$$\therefore M_X(t) = \frac{pe^t}{1 - qe^t}$$

20. Find the MGF of exponential distribution.

$$F(x) = \begin{cases} \lambda e^{-\lambda x} & x > a \\ 0 & \text{otherwise} \end{cases}$$

To find MGF

$$M_X(t) = \int_{-\infty}^{\infty} e^{tx} f(x) dx$$
$$= \int_0^{\infty} e^{tx} \lambda e^{-\lambda x} dx \qquad = \lambda \int_0^{\infty} e^{-(\lambda - t)x} dx$$
$$= \lambda \left[\frac{e^{-(\lambda - t)x}}{\lambda - t}\right]_0^{\infty}$$
$$= \frac{\lambda}{-(\lambda - t)} \left[e^{-\infty} - e^{-0}\right] \qquad = \frac{\lambda}{\lambda - t}$$
$$\therefore \text{MGF of } x = \frac{\lambda}{\lambda - t}, \lambda > t$$

Part-C

1. Fitting a binomial distribution for the following data.

x	0	1	2	3	4	5	6	Total
f(x)	5	18	28	12	7	6	4	80

Solution:

x	0	1	2	3	4	5	6	Total
f(x)	5	18	28	12	7	6	4	80
fx	0	18	56	36	28	30	24	192

Mean = np = $\dfrac{\sum fx}{\sum f} = \dfrac{192}{80} = 2.4$

$6p = 2.4 \Rightarrow p = 0.4$

$q = 1 - p = 1 - 0.4 = 0.6$

The probability distribution is $P(X = x) = nC_x p^x q^{n-x}, x = 0,1,2,3,.....n$

$$= 6C_x (0.4)^x (0.6)^{6-x}, x = 0,1,2,3,....6$$

The theoretical frequencies are given by $NP(X = x) = N\left(nC_x p^x q^{n-x}\right), x = 0,1,2,3,.....n$

ie) $80\left(6C_x (0.4)^x (0.6)^{6-x}\right), x = 0,1,2,3,....6$

Calculation of expected frequencies:

E(0) = 80 P (X=0) = $80\left(6C_0 (0.4)^0 (0.6)^{6-0}\right) = 3.732 \approx 4$

E(1) = 80 P (X=1) = $80\left(6C_1 (0.4)^1 (0.6)^{6-1}\right) = 14.92 \approx 15$

E(2) = 80 P (X=2) = $80\left(6C_2 (0.4)^2 (0.6)^{6-2}\right) = 24.88 \approx 25$

E(3) = 80 P (X=3) = $80\left(6C_3 (0.4)^3 (0.6)^{6-3}\right) = 22.11 \approx 22$

E(4) = 80 P (X=4) = $80\left(6C_4 (0.4)^4 (0.6)^{6-4}\right) = 11.05 \approx 11$

E(5) = 80 P (X=5) = $80\left(6C_5 (0.4)^5 (0.6)^{6-5}\right) = 2.94 \approx 3$

E(6) = 80 P (X=6) = $80\left(6C_6 (0.4)^6 (0.6)^{6-6}\right) = 0.32 \approx 0$

$\therefore$ The fitted binomial distribution is P (X = x) $= 6C_x (0.4)^x (0.6)^{6-x}, x = 0,1,2,3,....6$

The expected frequencies are

x	0	1	2	3	4	5	6	Total
Observed frequencies	5	18	28	12	7	6	4	80
Expected frequencies	4	15	25	22	11	3	0	80

2. Fitting a poisson distribution for the following data.

x	0	1	2	3	4	5	Total
f(x)	142	156	69	27	5	1	400

Solution:

x	0	1	2	3	4	5	Total
f(x)	142	156	69	27	5	1	400
fx	0	156	138	81	20	5	400

Mean = λ = $\dfrac{\sum fx}{\sum f} = \dfrac{400}{400} = 1$

The probability distribution is $P(X = x) = \dfrac{e^{-\lambda}\lambda^x}{x!}, x = 0,1,2,3,.....\infty$

$$P(X = x) = \frac{e^{-1}1^x}{x!} = \frac{e^{-1}}{x!}, x = 0,1,2,3,.....\infty$$

The theoretical frequencies are given by $NP(X = x) = N\left(\dfrac{e^{-1}}{x!}\right), x = 0,1,2,3,.....\infty$

ie) $400\left(\dfrac{e^{-1}}{x!}\right), x = 0,1,2,3,....\infty$

Calculation of expected frequencies:

___ E(0) = 400 P (X=0) = $400\left(\frac{e^{-1}}{0!}\right) = 147.15 \approx 147$

E(1) = 400 P (X=1) = $400\left(\frac{e^{-1}}{1!}\right) = 147.15 \approx 147$

E(2) = 400 P (X=2) = $400\left(\frac{e^{-1}}{2!}\right) = 73.57 \approx 74$

E(3) = 400 P (X=3) = $400\left(\frac{e^{-1}}{3!}\right) = 24.52 \approx 25$

E(4) = 400 P (X=4) = $400\left(\frac{e^{-1}}{4!}\right) = 6.13 \approx 6$

E(5) = 400 P (X=5) = $400\left(\frac{e^{-1}}{5!}\right) = 1.22 \approx 1$

$\therefore$ The fitted binomial distribution is $P(X = x) = \frac{e^{-1}1^x}{x!} = \frac{e^{-1}}{x!}, x = 0,1,2,3,.....\infty$

The expected frequencies are

x	0	1	2	3	4	5	Total
Observed frequencies	142	156	69	27	5	1	400
Expected frequencies	147	147	74	25	6	1	400

3. A and B independently until each has hit his own target.The probabilities of their hitting a target at each shot are $\frac{3}{5}$ and $\frac{5}{7}$ respectively.Find the probability that B will require more shots than A.

Solution:

Let X denote the number of trials required by A to get his first success.Then X follows a geometric distribution given by

$$P(X = x) = p_1 q_1^{\;r-1} = \frac{3}{5}\left(\frac{2}{5}\right)^{r-1}, r = 1,2,3,......\infty$$

Let Y denote the number of trials required by B to get his first success.Then X follows a geometric distribution given by

$$P(Y=y)=p_2 q_2^{\,r-1}=\frac{5}{7}\left(\frac{2}{7}\right)^{r-1}, r=1,2,3,......\infty$$

P(B requires to get his first success than A requires to get his first success)

$$=\sum_{r=1}^{\infty} P(X=r \;\; and \; Y=r+1 \,\text{or}\, \mathrm{r}+2.........\infty)$$

$$=\sum_{r=1}^{\infty}\left[P(X=r) \; and \; \mathrm{P}(\mathrm{Y}=r+1 \,\text{or}\, \mathrm{r}+2.........\infty)\right] \quad \text{Since X and Y are independent.}$$

$$=\sum_{r=1}^{\infty}\frac{3}{5}\left(\frac{2}{5}\right)^{r-1} \quad \sum_{\mathrm{k}=1}^{\infty}\frac{5}{7}\left(\frac{2}{7}\right)^{r+k-1}$$

$$=\sum_{r=1}^{\infty}\frac{3}{7}\left(\frac{4}{35}\right)^{r-1} \quad \sum_{\mathrm{k}=1}^{\infty}\left(\frac{2}{7}\right)^{k}$$

$$=\sum_{r=1}^{\infty}\frac{3}{7}\left(\frac{4}{35}\right)^{r-1}\left(\frac{\frac{2}{7}}{1-\frac{2}{7}}\right)=\frac{6}{35}\sum_{r=1}^{\infty}\left(\frac{4}{35}\right)^{r-1}=\frac{6}{35}\left(\frac{1}{1-\frac{4}{35}}\right)=\frac{6}{31}$$

4. Establish the memory less property of geometric distribution.

A geometric random variable X has the memoryless property if for all nonnegative

integers m and n ∈ N, P(X > m + n | X > m)= P (X > n)

The probability mass function for a geometric random variable X is

$$P(X=x)=pq^{x-1}, x=1,2,3,....$$

Now, P(X> k) = $\sum_{x=k+1}^{\infty} pq^{x-1} = pq^{k}+pq^{k+1}+pq^{k+2}+..........$

$$=pq^{k}\left(1+q^{1}+q^{2}+..........\right)=pq^{k}(1-q)^{-1}=pq^{k}\left(\frac{1}{p}\right)=q^{k}$$

P(X > m + n | X > m)= $\dfrac{P((X>m+n)\cap(X>m))}{P(X>m)}$

$$=\frac{P(X>m+n)}{P(X>m)}=\frac{q^{m+n}}{q^{m}}=q^{n}$$

$= P(X > n)$

5. Establish the memory less property of exponential distribution

An exponential random variable X has the memoryless property if for all nonnegative integers s and t ∈ N, P(X > s + t | X > s)= P (X > t)

$$P(X = x) = \lambda e^{-\lambda x}, x > 0$$

Now, $P(X > s) = \int_s^\infty \lambda e^{-\lambda x} dx = \lambda \left(\frac{e^{-\lambda x}}{-\lambda}\right)_s^\infty = e^{-\lambda s}$

$$P(X > s + t \mid X > s) = \frac{P((X > s+t) \cap (X > s))}{P(X > s)}$$

$$= \frac{P(X > s+t)}{P(X > s)} = \frac{\lambda^{-(s+t)}}{\lambda^s} = \lambda^{-t}$$

$$= P(X > t)$$

6. Starting at 5.00 am every half an hour there is a flight from San Fransisco airport to Losangles .. A person who wants to fly to Losangles arrive at a random time between 8.45 am and 9.45 am. Find the probability that he waits

(a) Atmost 10 min (b) atleast 15 min

Soln: Let X be the uniform r.v. over the interval (0,60)

Then the pdf is given by

$$f(x) = \frac{1}{b-a}, \quad a < x < b$$

$$= \frac{1}{60}, \quad 0 < x < 60$$

(a) The passengers will have to wait less than 10 min. if she arrives at the airport

$$= p(5 < x < 15) + p(35 < x < 45)$$

$$= \int_{5}^{15} \frac{1}{60}\, dx + \int_{35}^{45} \frac{1}{60}\, dx$$

$$= \frac{1}{60}\left[x\right]_{5}^{15} + \frac{1}{60}\left[x\right]_{35}^{45}$$

$$= \frac{5}{12}$$

(b)The probability that she has to wait atleast 15 min.

$$= p(15 < x < 30) + p(45 < x < 60)$$

$$= \int_{15}^{30} \frac{1}{60}\, dx + \int_{45}^{60} \frac{1}{60}\, dx$$

$$= \frac{1}{60}\left[x\right]_{15}^{30} + \frac{1}{60}\left[x\right]_{45}^{60}$$

$$= \frac{1}{2}$$

7. Buses arrive at specified stop at 15 minute intervals starting at 7.00 A.M that is they arrive at 7.00,7.15,7.30 and 7.45 and so on .If a passenger arrives at the specified stop at a random time that is uniformly distributed between 7 and 7.30 A.M. Find the probability that

(a) he waits less than 5 minutes for a bus. (b) atleast 12 minutes for a bus

Let X be the uniform r.v. over the interval (0,30)

Then the pdf is given by

$$f(x) = \frac{1}{b-a} \quad , a < x < b$$

$$= \frac{1}{30} \quad , \quad 0 < x < 30$$

(a) The passenger will have to wait less than 5 minutes if he arrive at the stop between 7.10 and 7.15 or 7.25 and 7.30

$$\text{Re}\,\textit{quired}\ \text{probability} = p(10 < x < 15) + p(25 < x < 30)$$

$$= \int_{10}^{15} \frac{1}{30}\, dx + \int_{25}^{30} \frac{1}{30}\, dx$$

$$= \frac{1}{30}\left[x\right]_{10}^{15} + \frac{1}{30}\left[x\right]_{25}^{30}$$

$$= \frac{1}{3}$$

(b) The passenger will have to wait atleast 12 minutes if he arrive at the stop between 7.00 and 7.03 or 7.15 and 7.18

$$\text{Required probability} = p(0 < x < 3) + p(15 < x < 18)$$

$$= \int_0^3 \frac{1}{30}\,dx + \int_{15}^{18} \frac{1}{30}\,dx$$

$$= \frac{1}{30}\left[x\right]_0^3 + \frac{1}{30}\left[x\right]_{15}^{18}$$

$$= \frac{1}{5}$$

8. The time (in hours) required to repair a machine is exponentially distributed with parameter $\lambda = \frac{1}{2}$.

(a)What is the probability that the repair time exceeds 2 hrs ?

(b)What is the conditional probability that a repair takes atleast 11 hrs given that Its direction exceeds 8 hrs ?

Soln: If X represents the time to repair the machine, the density function Of X is given by

$$f(x) = \lambda e^{-\lambda x} \quad , x \ge 0$$

$$= \frac{1}{2} e^{-x/2} \quad x \ge 0$$

(a)

$$p(x > 2) = \int_2^\infty f(x)\,dx = \int_2^\infty \lambda e^{-\lambda x}\,dx$$

$$= \int_2^\infty \frac{1}{2} e^{\frac{-x}{2}}\,dx = \frac{1}{2}\left[\frac{e^{-\frac{x}{2}}}{-\frac{1}{2}}\right]_2^\infty$$

$$= -\left[0 - e^{-1}\right] = 0.3679$$

b)

$$p[x \geq 11/x > 8] = p[x > 3]$$

$$= \int_3^\infty f(x)\,dx = \int_3^\infty \lambda\, e^{-\lambda x}\,dx$$

$$= \int_3^\infty \frac{1}{2} e^{\frac{-x}{2}}\,dx = \frac{1}{2}\left[\frac{e^{-\frac{x}{2}}}{-\frac{1}{2}}\right]_3^\infty$$

$$= -\left[0 - e^{-\frac{3}{2}}\right] = e^{-\frac{3}{2}} = 0.2231$$

9. The number of personel computer (pc) sold daily at a computer world is uniformly

distributed with a minimum of 2000 pc and a maximum of 5000 pc. Find

(1) The probability that daily sales will fall between 2500 and 3000 pc

(2)What is the probability that the computer world will sell atleast 4000 pc's?

(3) What is the probability that the computer world will sell exactly 2500 pc's?

Soln: Let X~U(a , b) , then the pdf is given by

$$f(x) = \frac{1}{b-a} \quad , a < x < b$$

$$= \frac{1}{5000-2000} \ , \quad 2000 < x < 5000$$

$$= \frac{1}{3000} \quad , \quad 2000 < x < 5000$$

$$p[2500 < x < 3000] = \int_{2500}^{3000} f(x)\,dx$$

(1)

$$= \int_{2500}^{3000} \frac{1}{3000}\,dx = \frac{1}{3000}[x]_{2500}^{3000}$$

$$= \frac{1}{3000}[3000 - 2500] = 0.166$$

(2)

$$p[x \geq 4000] = \int_{4000}^{5000} f(x)\,dx$$

$$= \int_{4000}^{5000} \frac{1}{3000}\,dx = \frac{1}{3000}[x]_{4000}^{5000}$$

$$= \frac{1}{3000}[5000 - 4000] = 0.333$$

(3) $p[x = 2500] = 0$, (i.e) it is particular point ,the value is zero.

10. Suppose that a trainee soldier shoots a target in an independent fashion. If the probability

That the target is shot on any one shot is 0.7.

(i) What is the probability that the target would be hit in 10 th attempt?
(ii) What is the probability that it takes him less than 4 shots?
(iii)What is the probability that it takes him an even no. of shots?
(iv) What is the average no. of shots needed to hit the target?

Soln: Let X denote the no. of shots needed to hit the target and X follows geometric

distribution with pmf $p[X = x] = p\,q^{x-1}$, $x = 1,2,....$

Given p=0.7 , and q=1-p=0.3

(i) $p[x = 10] = (0.7)(0.3)^{10-1} = 0.0000138$

(ii)

$$\begin{aligned} p[x < 4] &= p(x = 1) + p(x = 2) + p(x = 3) \\ &= (0.7)(0.3)^{1-1} + (0.7)(0.3)^{2-1} + (0.7)(0.3)^{3-1} \\ &= 0.973 \end{aligned}$$

(iii)

$$\begin{aligned} p[x\, is\, an\, even\, number] &= p(x = 2) + p(x = 4) + ... \\ &= (0.7)(0.3)^{2-1} + (0.7)(0.3)^{4-1} + ... \\ &= (0.7)(0.3)[1 + (0.3)^2 + (0.3)^4 ....] \\ &= 0.21\left[1 + \left((0.3)^2\right) + \left((0.3)^2\right)^2 + ...\right] \\ &= 0.21\left[1 - (0.3)^2\right]^{-1} = (0.21)\,(0.91)^{-1} \\ &= \frac{0.21}{0.91} = 0.231 \end{aligned}$$

(iv) Average no. of shots $= E(X) = \dfrac{1}{p} = \dfrac{1}{0.7} = 1.4286$