

## SRM Institute of Science and Technology Department of Mathematics 18MAB204T-Probability and Queueing Theory Unit – I: Random Variables Tutorial Sheet - 3

S.No	Questions	Answers
Part – A		
1	State Tchebycheff's inequality.	
2	If the pdf of a RV X is $f_X(x) = 2x$ , $0 < x < 1$ .find the pdf of Y= $e^{-X}$	Ans: $f_y(Y) = \frac{-2}{y} \log y$ , $\frac{1}{e} < y < 1$
3	If the density function of a continuous RV X is given by $f_X(x)=e^{-x}, x>0$ find the density function of Y=2X+1.Hence or otherwise find $P(Y\geq 5)$ .	Ans: $f_y(Y) = \frac{1}{2}e^{-(y-1)/2}$ , y> 1, $P(Y \ge 5) = \frac{1}{e^2}$
4	If the pdf of a RV X is $f_X(x) = 2x$ , $0 < x < 1$ .find the density and distribution functions of Y=3X+1.	<b>Ans:</b> $f_y(Y) = \frac{2}{9}(y-1), 1 < y < 4$ $F_y(Y) = \frac{(y-1)^2}{9}$
5	If the density function of a continuous RV X is given by $f_X(x) = 2e^{-2x}, x > 0$ find the density function of Y=X <sup>2</sup>	<b>Ans:</b> $f_y(Y) = \frac{1}{\sqrt{y}} e^{-2\sqrt{y}}$ , y> 0
6	If Tchebycheff's inequality for a RV X is $P(-2 < X < 8) \ge \frac{21}{25}, \text{ find the mean and variance of X}.$	<b>Ans:</b> $\mu = 3,c=5$ $\sigma^2 = 4$
Part – B		
7	If X is a RV with $E(X) = 3$ and $E(X^2) = 13$ . Find the lower bound for $P(-2 < X < 8)$ , using Tchebycheff's inequality.	<b>Ans:</b> $P(-2 < X < 8) \ge \frac{21}{25}$
8	Two dice are thrown once. If X is the sum of the numbers showing up, prove that $P\{ X - 7  \ge 3\} \le \frac{35}{54}$ and compare this value with the exact probability.	Ans: $\frac{1}{3}$
9	Using Tchebycheff's inequality, find how many times a fair coin must be tossed in order that the probability that the ratio of the number of heads to the number of tosses will lie between 0.4 and 0.6 will be atleast 0.9.	Ans: n=250
10	A RV X has pdf $f(x) = e^{-x}$ , $x \ge 0$ use Tchebycheff's inequality to show that $P\{ X-1  > 1\} < \frac{1}{4}$ and show also that the actual probability is $e^{-3}$ .	