Set 1:

1. If X has the probability distribution

x	-1	0	1	2
$p(x)$	0.3	0.1	0.4	0.2

Find $E(x)$, $E(x^2)$, $Var(X)$

x	$p(x)$	x^2	$E(x)$ $xp(x)$	$E(x^2)$ $x^2p(x)$
-1	0.3	1	-0.3	0.3
0	0.1	0	0	0
1	0.4	1	0.4	0.4
2	0.2	4	0.4	0.8
			0.5	1.5

$$E(x) = \sum_i x_i p(x_i) = 0.5$$

$$E(x^2) = \sum_i x_i^2 p(x_i) = 1.5$$

$$Var(X) = E(x^2) - [E(x)]^2$$

$$= 1.50 - 0.25$$

$$= 1.25$$

$P(X < 3)$

$P(0.5 < x < 2)$

2. A random variable X has the following probability distribution

0.1
0.067

x	-2	-1	0	1	2	3
$p(x)$	0.1	k	0.2	$2k$	0.3	$3k$
$F(x)$ 0	0.1	0.167	0.367			1

(i) Find k

(ii) Evaluate $P[X>2]$

(iii) $P[-2<X<2]$

(iv) Find the cummulative distribution function of X

(v) Mean of X & variance

(i) $\Sigma p(x) = 1$

$$\Rightarrow \left.\begin{array}{l} 0.1 + k + 0.2 + \\ 2k + 0.3 + 3k \end{array}\right\} = 1.$$

$$0.6 + 6k = 1.$$

$$k = \frac{0.4}{6} = \frac{4}{60} = \frac{1}{15} = 0.067$$

(ii) $P[X>2]$

$= P[X=3]$

$= 3k \quad = 3 \times \frac{1}{15} = \frac{1}{5}$

(iii) $P[-2<X<2]$

$= P[X=-1] + P[X=0] + P[X=1]$

$= 3k + 0.2$

$= 2/5$

(iv) CDF

When

$x < -2$ $\quad F(X) = 0$

$-2 \leq x < -1$ $\quad F(X) = P(X = -2) = 0.1$

$-1 \leq x < 0$ $\quad F(x) = \cancel{0.1 + 0.67 = 0.77}$

$= 0.1 + 0.067 = 0.167$

$0 \leq x < 1$ $\quad F(x) = 0.167 + 0.2 = 0.367$

$1 \leq x < 2$ $\quad F(x) = 0.367 + 0.133 = 0.500$

$2 \leq x < 3$ $\quad F(x) = 0.5 + 0.3 = 0.8$

$3 \leq x < \infty$ $\quad F(x) = 0.8 + 0.2 = 1$

(v)

x^2	x	$f(x)$	$xf(x)$	$x^2 f(x)$
4	-2	0.1	-0.2	+0.8
1	-1	0.067	-0.067	+0.067
0	0	0.2	0	0
1	1	0.133	0.133	0.133
4	2	0.3	0.6	~~1~~.4
9	3	0.2	0.6	~~5.4~~ 1.8
			2.263	~~7.066~~
			= 1.066	3.61

$$\boxed{\begin{aligned} &\text{Mean} \\ &= \sum_i x_i p(x_i) \\ &= 2.263 \end{aligned}}$$

$$\text{Variance} = E(x^2) - [E(x)]^2$$

$$= 7.066 - (2.263)^2$$

$$= 1.945 \qquad = 2.46.$$

3. The probability distribution of X is

x	0	2	4	6
$P(x)$	$1/6$	$1/3$	$1/8$	$3/8$

Find mean and variance

x	$p(x)$	x^2	$xp(x)$	$x^2p(x)$
0	$1/6$	0	0	0
2	$1/3$	4	$2/3$	$4/3$
4	$1/8$	16	$1/2$	2
6	$3/8$	36	$18/8$	$\frac{27}{2}$
			3.417	16.83

$$\text{Mean} = \sum_i x p(x) = E(x)$$

$$= 3.417$$

$$\text{Variance} = \Sigma(x^2) - E(x)$$

$$= 16.833 - 11.676$$

$$= 5.157$$

4. If the random variable X takes the values 1, 2, 3, 4 such that $2P(X=1) = 3P(X=2) = P(X=3) = 5P(X=4)$.

Find the probability distribution function of X and the cummulative distribution function of X

$$2P(X=1) = 3P(X=2) = P(X=3) = 5P(X=4) = k$$

$$P(X=1) = k/2 = \frac{15k}{30}$$

$$P(X=2) = k/3 = \frac{10k}{30}$$

$$P(X=3) = k = \frac{30k}{30}$$

$$P(X=4) = k/5 = \frac{6k}{30}$$

$$\frac{61k}{30}$$

WKT

$$\sum_{1} p(x) = 1$$

$$\frac{61k}{30} = 1$$

$$k = \frac{30}{61}$$

x	$p(x)$
1	$15/61$
2	$10/61$
3	$30/61$
4	$6/61$

$F(x)$ - CDF

$x < 1$	$F(x) = 0$.
$1 \leq x < 2$	$F(x) = 15/61$
$2 \leq x < 3$	$F(x) = \frac{15}{61} + \frac{10}{61} = \frac{25}{61}$
$3 \leq x < 4$	$F(x) = \frac{25}{61} + \frac{30}{61} = \frac{55}{61}$
$4 \geq x$	$F(x) = 1$.

5. A discrete random variable X has the following probability distribution

x	0	1	2	3	4	5	6	7	8
$p(x)$	a	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$

Find

(i) the value of a

(ii) $P(0 < X < 3)$

(iii) $P(X \geq 3)$

(iv) the distribution function of X

(v) $P\left(\frac{3 < X < 6}{X > 4}\right)$

(vi) $P(0 < X \leq 6)$

(vii) $P(X > 3)$

(viii) $P\left(\frac{1.5 < X < 4.5}{X > 2}\right)$

(ix) $E(x^2)$

(i) W.K.T $\sum p(x) = 1$.

$$\left.\begin{array}{l} a + 3a + 5a + 7a + 9a \\ + 11a + 13a + 15a + 17a \end{array}\right\} = 1.$$

$$81a = 1$$

$$a = 1/81$$

iv)

x	0	1	2	3	4	5	6	7	8
$p(x)$	$\frac{1}{81}$	$\frac{3}{81}$	$\frac{5}{81}$	$\frac{7}{81}$	$\frac{9}{81}$	$\frac{11}{81}$	$\frac{13}{81}$	$\frac{15}{81}$	$\frac{17}{81}$

(ix)

x^2	0	1	4	9	16	25	36	49	64
$x^2 p(x)$	0	$\frac{3}{81}$	$\frac{20}{81}$	$\frac{63}{81}$	$\frac{144}{81}$	$\frac{275}{81}$	$\frac{468}{81}$	$\frac{735}{81}$	$\frac{1088}{81}$

$$E(x^2) = 2796/81 = 34.52$$

(ii) $P(0<X<3)$

$= P(X=1) + P(X=2)$

$= \frac{3}{81} + \frac{5}{81}$

$= \frac{8}{81}$

(iii) $P(X \geqslant 3)$

$= 1 - P(X<3)$

$= 1 - \left[P[X=0] + P[X=1] + P[X=2]\right]$

$= 1 - \frac{9}{81} \quad = 1 - \frac{1}{9}$

$= \frac{8}{9}$

(v) $3<X<6 \;= \{4, 5\}$ $\qquad P(B/A) = \frac{P(A \cap B)}{P(A)}.$

$X>4 \;= \{5, 6, 7, 8\}$

$$P\left(\frac{3<X<6}{X>4}\right) = \frac{P((3<X<6) \cap (X>4))}{P(X>4)}.$$

$$= \frac{11/81}{P(X=5) + P(X=6) + P(X=7) + P(X=8)}$$

$$= \frac{11/81}{56/81}$$

$$= \frac{11}{56}.$$

(vi) $P(0 < x \leq 6)$

$= 1 - [P(x=0) + P(x=7) + P(x=8)]$

$= 1 - \frac{33}{81}$

$\begin{array}{r} \cancel{8}^{7}1 \\ 33 \\ \hline 48 \end{array}$

$= \frac{48}{81}$

(vi) $P(x>3)$

$= P(x \geq 3) - P(x=3)$

$= \frac{8}{9} - \frac{7}{81}$

$\begin{array}{r} {}^{6}\cancel{7}2 \\ 7 \\ \hline 65 \end{array}$

$= \frac{72-7}{81}$

$= \frac{65}{81}$

(viii) $1.5 < x < 4.5 = \{2, 3, 4\}$

$x > 2 = \{3, 4, 5, 6, 7, 8\}$

$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$

$= \frac{P(x=3) + P(x=4)}{8/9} = \frac{16/81}{72/81} = \frac{16}{72} = \frac{2}{9}$

6. If X denote the number in a throw of a fair die, find

$E(x)$, $E(9x+2)$, $Var(X)$

x_i	$p(x_i)$	x_i^2	$x_i p(x_i)$	$x_i^2 p(x_i)$
1	1/6	1	1/6	1/6
2	1/6	4	2/6	4/6
3	1/6	9	3/6	9/6
4	1/6	16	4/6	16/6
5	1/6	25	5/6	25/6
6	1/6	36	6/6	36/6
			21/6 $= 7/2$	91/6

$E(ax+b) = aE(x) + b$

$Var(ax+b) = a^2 E(x^2)$

$$E(x) = \sum_i x_i p(x_i) = \frac{7}{2} = 3.5$$

$$E(9x+2) = 9E(x) + 2 = \frac{67}{2} = 33.5$$

$$Var(X) = 9\,E(x^2) - [E(x)]^2$$

$$= \frac{91}{6} - \left(\frac{7}{2}\right)^2 = \frac{497}{36} = 13.81$$

7. The probability function of an infinite discrete distribution is given by

$P(X=j) = \frac{1}{2^j}$ $(j = 1, 2, \ldots \infty)$. Verify that the total probability is 1 and find the mean and variance of the distribution.

Total probability

$$\sum_i p_i = 1$$

$$\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots = 1$$

$$S_\infty = \frac{a}{1-r}$$

$$= \frac{1/2}{1-1/2} = \frac{1/2}{1/2} = 1$$

Mean $= \mu$

$$E(x) = \sum_i x_i \, p(x_i)$$

$$= \frac{1}{2} + 2 \cdot \frac{1}{2^2} + 3 \cdot \frac{1}{2^3} + \cdots$$

$$= \frac{1}{2}\left[1 + 2 \cdot \frac{1}{2} + 3 \cdot \frac{1}{2^2} + \cdots\right]$$

$$= \frac{1}{2}\left[1 - 1/2\right]^{-2} = \frac{1}{2}\left[1 + 2 \cdot \frac{1}{2}\right]$$

$$= \frac{1}{2} \times 4 = 2 = \mu$$

Variance $= \sigma^2$

$E(x^2) = \sum_i x_i^2 p(x_i)$

$= \frac{1}{2} + 4 \cdot \frac{1}{2^2} + 9 \frac{1}{2^3} + \cdots$

$=$

8. A die is cast until 6 appears. What is the probability that it must be cast more than 3 times.

Let A be an event of appearing 6 on a die.

$P(A) = 1/6$.

Each cast of a die are independent events. So, a die to be cast more than 3 times $\rightarrow$ event B

$P(B) = P(\bar{A}) \cdot P(\bar{A}) \cdot P(\bar{A})$

$= \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6}$.

$= \frac{125}{216}$

$= 0.58$