TEST -3 Material Z-Test

Critical vales (Z_α) of Z

critical value Z_α	Level of significance(α)			
	1%	2%	5%	10%
Two tailed test	$\lvert z_\alpha \rvert = 2.58$	$\lvert z_\alpha \rvert = 2.33$	$\lvert z_\alpha \rvert = 1.96$	$\lvert z_\alpha \rvert = 1.645$
Right tailed test	$\lvert z_\alpha \rvert = 2.33$	$\lvert z_\alpha \rvert = 2.055$	$\lvert z_\alpha \rvert = 1.645$	$\lvert z_\alpha \rvert = 1.28$
Left tailed test	$\lvert z_\alpha \rvert = -2.33$	$\lvert z_\alpha \rvert = -2.055$	$\lvert z_\alpha \rvert = -1.645$	$\lvert z_\alpha \rvert = -1.28$

PROBLEMS IN SINGLE PROPORTION

Two Tailed Test

1. A coin is tossed 256 times and 132 heads are obtained. Would you conclude that the coin is a biased one?

Here n = 256, = No. of success = 132, p = proportion of successes in the sample = $\frac{X}{n}$ = $\frac{132}{256}$ = 0.5156

P = populatin proportion = $\frac{1}{2}$, Q = 1-P = $\frac{1}{2}$

Null Hypothesis Ho : The coin is unbiased.

Alternative Hypothesis H1 : The coin is biased : P ≠ 0.5 (two tailed test)

Test statistic $Z = \frac{p-P}{\sqrt{\frac{PQ}{n}}}$

$$Z = \frac{0.5156-0.5}{\sqrt{\frac{0.5\times 0.5}{256}}} = 0.4992 < 1.96$$

Ho is accepted. Hence the coin is unbiased.

2. A coin is tossed 400 times and its turns up head 216 times. Discuss whether the coin may unbiased one at 5% level if significance.

Solution:

Given n= 400, P = $\frac{1}{2}$

$\Rightarrow$Q = 1-P

= 1-1/2 $\Rightarrow$1/2

Q = 1/2

n = number of success

X = 216, p = proportion of successes in the sample = $\frac{X}{n}$ = $\frac{216}{400}$ = 0.54

Null hypothesis H_0: The coin is unbiased

Alternative hypothesis H_1 : The coin is biased

$\alpha = 5\,\% = 0.05$

Test Statistic

$$Z = \frac{p-P}{\sqrt{\frac{PQ}{n}}}$$

$$Z = \frac{0.54-0.5}{\sqrt{\frac{0.5\times 0.5}{400}}}$$

Z_{cal} = 1.6

At $\alpha = 5\,\%$ Z_{tab} = 1.96

$\therefore$ $Z_{cal} < Z_{tab}$ (ie) 1.6 <1.96

Hence we accept null hypothesis H_o (ie) The coin is unbiased

3. A random sample of 400 mangoes was taken from a large consignment and 40 were found to be bad. In this a sample from a consignment with proportion of bad mangoes 7.5%?

Here n = 400, p = sample proportion of bad mangoes = $\frac{X}{n}$ = $\frac{40}{400} = 0.1$

P = populatin proportion of bad mangoes = 7.5% = 0.075 , Q = 1 –P = 1- 0.075 = 0.925

Ho : P= 0.075, H_1 : P ≠ 0.075 (two tailed test); Z α at 1% level l = 2.58

Test Statistic

$$Z = \frac{p-P}{\sqrt{\frac{PQ}{n}}}$$

$$Z = \frac{0.1-0.075}{\sqrt{\frac{0.075\times 0.925}{400}}}$$ = 1.89 < 2.58

Null hypothesis Ho is accepted at 1% level.

4. In a city, a sample of 1000 people were taken & out of them 540 are vegetarians & the rest are non vegetarians. Can we say that both habits of eating are equally popular in the city at 1% & 5% level of significance?

Here n = 1000, p = sample proportion of vegetarians = $\frac{540}{1000} = 0.54$

P = populatin proportion of vegetarians = $\frac{1}{2}$

Q = 1 – P = 0.5

Ho : P = 0.5 Both habits are equally popular in the city). H1" : P ≠ 0.5 (two tailed test)

Test Statistic

$$Z = \frac{p-P}{\sqrt{\frac{PQ}{n}}}$$

$$Z = \frac{0.54-0.5}{\sqrt{\frac{0.5\times 0.5}{1000}}}$$ = 2.5298 > 1.96

Ho rejected at 5% level of significance. Both types of eaters are popular at 1% level and not so at 5% level of significance.

DIFFERENCE OF MEAN

Let $\overline{x_1}$ be the mean of a sample of size n_1 from a population with mean μ_1 and variance σ_1^2. Let $\overline{x_2}$ be the mean of a sample of size n_2 from a population with mean μ_2 and variance σ_2^2.

To test whether there is any significant difference between $\overline{x_1}$ and $\overline{x_2}$ we have to use the statistic $z = \dfrac{\overline{x_1}-\overline{x_2}}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}}$

Note: If the samples have been drawn from the same population then $\sigma_1^2 = \sigma_2^2 = \sigma^2$, $z = \dfrac{\overline{x_1}-\overline{x_2}}{\sqrt{\frac{\sigma^2}{n_1}+\frac{\sigma^2}{n_2}}}$

If σ is not known we can use a estimate of σ^2 given by $\sigma^2 = \dfrac{n_1 s_1^2 + n_2 s_2^2}{n_1+n_2}$

Two Tailed Test

1. ***In a random sample of size 500, the mean is found to be 20. In another independent sample of size 400, the mean is 15. Could the samples have been drawn from the same population with S.D. 4?***

   Solution: $n_1 = 500,\ n_2 = 400,\ \overline{x_1} = 20,\ \overline{x_2} = 15,\ \sigma = 4$

   Null Hypothesis $H_0 : \bar{x}_1 = \bar{x}_2$, *Alternative Hypothesis* $H_1 : \bar{x}_1 \neq \bar{x}_2$ (two tailed test)

   Test statistics $z = \dfrac{\bar{x}_1-\bar{x}_2}{\sigma\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \dfrac{20-15}{4\sqrt{\frac{1}{500}+\frac{1}{400}}} = 18.6$; $z_\alpha = 2.58$ at 1%, $|z| > z_\alpha$, H_0 is rejected.

   That is, the samples could not have been drawn from the same population.

2. ***The mean of 2 large samples 1000 and 2000 members are 67.5 inches and 68.0 inches respectively. Can the samples the regarded as drawn from the same population of S.D. 2.5 inches.***

   Solution: $n_1 = 1000,\ n_2 = 2000,\ \overline{x_1} = 67.5,\ \overline{x_2} = 68,\ \sigma = 2.5$

   Null Hypothesis $H_0 : \bar{x}_1 = \bar{x}_2$, *Alternative Hypothesis* $H_1 : \bar{x}_1 \neq \bar{x}_2$ (two tailed test)

   Test statistics $z = \dfrac{\bar{x}_1-\bar{x}_2}{\sigma\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \dfrac{67.5-68}{2.5\sqrt{\frac{1}{1000}+\frac{1}{2000}}} = -5.1,\ |z| = 5.1$

   $z_\alpha = 1.96$ at 1% level of significance, $|z| > z_\alpha$, H_0 is rejected and H_1 is accepted.

3. ***In a survey of buying habits, 400 women shoppers are chosen at random in super market A located in a certain section of the city. Their average weekly food expenditure is Rs. 250 with a S.D. of Rs. 40. For 400 women shoppers chosen at random in super market B in another section of the city, the average weekly food expenditure is Rs. 220 with a S.D. of Rs. 55. Test at 1% level of significance whether the average weekly food expenditure of the two populations of shopper are equal.***

   Solution: $n_1 = 400,\ n_2 = 400,\ \overline{x_1} = 250,\ \overline{x_2} = 220,\ s_1 = 40, s_2 = 55$

   Null Hypothesis $H_0 : \bar{x}_1 = \bar{x}_2$, *Alternative Hypothesis* $H_1 : \bar{x}_1 \neq \bar{x}_2$ (two tailed test)

   Test statistics$z = \dfrac{\bar{x}_1-\bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}}$, Since σ_1 & σ_2 the population S.D. are not known, we can take $\sigma_1^2 = s_1^2$ *and* $\sigma_2^2 = s_2^2$,

   Test statistics $z = \dfrac{\bar{x}_1-\bar{x}_2}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}} = \dfrac{250-220}{\sqrt{\frac{40^2}{400}+\frac{55^2}{400}}} = 8.82$; $z_\alpha = 2.58$ at 1% level of significance, $|z| > z_\alpha$, H_0 is rejected.

4. ***Test the significance of difference between the means of the samples, drawn from two normal populations with the same S.D. from the following data:***

	Size	*Mean*	*S.D.*
Sample 1	***100***	***61***	***4***
Sample 2	***200***	***63***	***6***

   Solution: $n_1 = 100,\ n_2 = 200,\ \overline{x_1} = 61,\ \overline{x_2} = 63,\ s_1 = 4, s_2 = 6$

   Null Hypothesis $H_0 : \bar{x}_1 = \bar{x}_2$, *Alternative Hypothesis* $H_1 : \bar{x}_1 \neq \bar{x}_2$ (two tailed test)

   Test statistics$z = \dfrac{\bar{x}_1-\bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}}$, Since σ_1 & σ_2 the population S.D. are not known, we can take $\sigma_1^2 = s_1^2$ *and* $\sigma_2^2 = s_2^2$,

   Test statistics $z = \dfrac{\bar{x}_1-\bar{x}_2}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}} = \dfrac{61-63}{\sqrt{\frac{4^2}{100}+\frac{6^2}{200}}} = -3.02, |z| = 3.02$; $z_\alpha = 1.96$ at 5%, $|z| > z_\alpha$, H_0 is rejected.

Right Tailed Test

5. ***The average marks scored by 32 boys is 72 with a S.D. of 8, while that for 36 girls is 70 with a S.D. of 6. Test at 1% level of significance whether the boys perform better than girls.***

Solution: $n_1 = 32,\ n_2 = 36, \overline{x_1} = 72,\ \overline{x_2} = 70,\ s_1 = 8,\ s_2 = 6$

Null Hypothesis $H_0 : \bar{x}_1 = \bar{x}_2\ (or\ \mu_1 = \mu_2)$,

Alternative Hypothesis $H_1 : \bar{x}_1 > \bar{x}_2$ (right tailed test)

Test statistics $z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{72-70}{\sqrt{\frac{64}{32} + \frac{36}{36}}} = 1.15$, $z_\alpha = 2.33$ at 1% level of significance

$|z| < z_\alpha,\ H_0$ is accepted. That is, we cannot conclude that boys perform better than girls.

6. ***A random sample of 100 bulb from a company A showed a mean life 1300 hours and standard deviation 82 hours. Another random sample of 100 bulbs from company B showed a mean life 1248 hours and standard deviation of 93 hours. Are the bulbs of company A superior to bulbs of company B at 5% level of significance.***

Solution: $n_1 = 100,\ n_2 = 100, \overline{x_1} = 1300,\ \overline{x_2} = 1248,\ s_1 = 82,\ s_2 = 93$

Null Hypothesis $H_0 : \bar{x}_1 = \bar{x}_2\ (or\ \mu_1 = \mu_2)$,

Alternative Hypothesis $H_1 : \bar{x}_1 > \bar{x}_2$ (right tailed test)

Test statistics $z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{1300-1248}{\sqrt{\frac{82^2}{100} + \frac{93^2}{100}}} = 4.19$, $z_\alpha = 1.645$ at 1%, $|z| > z_\alpha,\ H_0$ is rejected.

Left Tailed Test

7. ***The average hourly wage of a sample of 150 workers in plant A was Rs. 2.56 with a S.D. of Rs. 1.08. The average wage of a sample of 200 workers in plant B was Rs. 2.87 with a S.D. of Rs. 1.28. Can an applicant safely assume that the hourly wage paid by plant B are higher than those paid by plant A?***

Solution: $n_1 = 150,\ n_2 = 200, \overline{x_1} = 2.56,\ \overline{x_2} = 2.87,\ s_1 = 1.08, s_2 = 1.28$

Null Hypothesis $H_0 : \bar{x}_1 = \bar{x}_2$, *Alternative Hypothesis* $H_1 : \bar{x}_1 < \bar{x}_2$ (left tailed test)

Test statistics$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$, Since σ_1 & σ_2 the population S.D. are not known, we can take $\sigma_1^2 = s_1^2\ and\ \sigma_2^2 = s_2^2$, Test statistics $z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{2.56-2.87}{\sqrt{\frac{(1.08)^2}{150} + \frac{(1.28)^2}{200}}} = -2.46$; $|z| = 2.46$; $z_\alpha = 1.645$ at 5% LoS, $|z| > z_\alpha,\ H_0$ is rejected.

Conclusion: The hourly wage paid by plant B are higher than those paid by plant A.

8. ***A simple sample of heights of 6400 English men has a mean of 170 cm and a S.D. of 6.4 cm, while a simple sample of heights of 1600 Americans has a mean of 172 cm and a S.D. of 6.3 cm. Do the data indicate that Americans are on the average, taller than the Englishmen?***

Solution: $n_1 = 6400,\ n_2 = 1600, \overline{x_1} = 170,\ \overline{x_2} = 172,\ s_1 = 6.4, s_2 = 6.3$

Null Hypothesis $H_0 : \bar{x}_1 = \bar{x}_2$, *Alternative Hypothesis* $H_1 : \bar{x}_1 < \bar{x}_2$ (left tailed test)

Test statistics$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$, Since σ_1 & σ_2 the population S.D. are not known, we can take $\sigma_1^2 = s_1^2\ and\ \sigma_2^2 = s_2^2$, Test statistics $z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{170-172}{\sqrt{\frac{(6.4)^2}{6400} + \frac{(6.3)^2}{1600}}} = -11.32$; $|z| = 11.32$; $z_\alpha = 2.33$ at 1% LoS, $|z| > z_\alpha,\ H_0$ is rejected.