AVERAGES

INTRODUCTION

Important Formulae

- To find the average (also called Arithmetic Mean)of n given quantities, we use the following formula

$$\text{Average of n quantities} = A = \frac{(\text{Sum of n quantities})}{N}$$

$$\Rightarrow \text{Sum of these n quantities} = N \times A$$

- If the each quantity is increased or decreased by the same value **B,** then

$$\text{New Average} = A \pm B$$

- If each quantity is multiplied or divided by the same value **M** $(M \neq 0)$, then

$$\text{New Average} = A \times M \text{ or } A \div M$$

SHORTCUT

Example 1

Q. Find the average of 12, 14, 22, 28, 42, 44

(a) 25 (b) 26 (c) 27 (d) 28

Solution:

- Method 1

$$\text{Average} = \left(\frac{12 + 14 + 22 + 28 + 42 + 44}{6}\right) = \frac{162}{6} = 27$$

- Method 2

Let us assume that the average of above numbers to be 28, then

$$\text{Average} = 28 + \left(\frac{-16 - 14 - 6 + 0 + 14 + 16}{6}\right) = 28 - 1 = 27$$

Example 2

A student was asked to find the arithmetic mean of numbers 3, 11, 7, 9, 15, 13, 8, 19, 17, 21, 14 and X. He found the mean to be 12. What should be the number in place of X?

(a) 6 (b) 7 (c) 8 (d) 9

Solution:

$$3 + 11 + 7 + 9 + 15 + 13 + 8 + 19 + 17 + 21 + 14 + X = 12 \times 12$$

$$137 + X = 144$$

$$X = 144 - 137 = 7$$

Example 3

The average of four consecutive even numbers is 27. Find the largest of these numbers.

(a) 24 (b) 26 (c) 28 (d) 30

Solution:

Let the numbers be $2x$, $(2x + 2)$, $(2x + 4)$, $(2x + 6)$, where x is a natural number

$$2x + (2x + 2) + (2x + 4) + (2x + 6) = 4 \times 27$$

$$8x + 12 = 108$$

$$8x = 96$$

$$x = 12$$

The largest number is $2x + 6 = 30$

Example 3

The average of four consecutive even numbers is 27. Find the largest of these numbers.

(a) 24 (b) 26 (c) 28 (d) 30

Solution:

Let the numbers be $2x - 2, 2x, 2x + 2, 2x + 4$, where x is a natural number

$$(2x - 2) + 2x + (2x + 2) + (2x + 4) = 4 \times 27$$

$$8x + 4 = 108$$

$$8x = 104$$

$$x = 13$$

The largest number is $2x + 4 = 30$

- If the average of a group of **N** quantities is **A,** and one of these quantities, say P, is replaced by a new quantity **Q** then the new average will be $\frac{Q-P}{N}$ more/less than A.

Example 4

The average weight of 18 soldiers is increased by 1 kg when one of them, who weighs 65 kg, is replaced by a new soldier. Find the weight of the new soldier.

(a) 77 (b) 91 (c) 69 (d) 83

Solution: Let the initial average of 18 men be A and the weight of new man be Q then

- Method 1: Initial Sum of Weights = $18 \times A$ and New Average is $(A + 1)$ and New Sum = $18 \times (A + 1)$, therefore

$$18 \times A - 65 + Q = 18 \times (A + 1)$$

$$18A - 65 + Q = 18A + 18$$

$$Q = 65 + 18 = 83 \text{ kg.}$$

Example 4

- **Method 2:** Using Shortcut Formula (Note that the new average is 1 kg more than the initial average A)

$$\frac{Q - 65}{18} = 1$$

$$\Rightarrow Q - 65 = 18 \times 1$$

$$\Rightarrow Q = 65 + 18 = 83$$

Example 5

The average marks of a group of 20 students on a test is reduced by 4 when the topper who scored 90 marks is replaced by a new student. How many marks did the new student have?

(a) 10 (b) 15 (c) 20 (d) None of these

Solution: Let the initial average of 20 students be A and the marks of new student be Q then

- **Method 1:** Initial Sum of Marks = 20 X A and New Average is (A – 4) and New Sum of Marks = 20 X (A - 4), therefore

$$20 \times A - 90 + Q = 20 \times (A - 4)$$

$$20A - 90 + Q = 20A - 80$$

$$Q = 10 \text{ Marks}$$

Example 5

- **Method 2:** Using Shortcut Formula (Note that the new average is 4 less than the initial average)

$$\frac{Q - 90}{20} = -4$$

$$\Rightarrow Q - 90 = -80$$

$$\Rightarrow Q = 90 - 80 = 10$$

- If the average of a group of **N** quantities is **A** and one of the quantities **Q** is dropped from the group, then the new average is more/less than A by $\frac{A-Q}{N-1}$.

Example 6

The average age of a class of 30 students and a teacher reduces by 0.5 years if we exclude the teacher. If the initial average is 14 years, find the age of the class teacher.

(a) 42 (b) 37 (c) 33 (d) 29

Solution: Let the age of teacher be Q years.

- Method 1: The initial Sum = $14 \times 31 = 434$

New Sum of Ages = $30 \times (14 - 0.5) = 405$

$\therefore 434 - X = 405$

$X = 434 - 405 = 29$ Years

Example 6

- **Method 2:** Using Shortcut Formula (Note that the new average is 0.5 less than the initial average 14)

$$\text{New average is } \frac{14 - Q}{31 - 1} \text{ less than } 14$$

$$\Rightarrow \frac{14 - Q}{30} = -0.5$$

$$\Rightarrow 14 - Q = -15$$

$$\Rightarrow Q = 15 + 14 = 29$$

- If the average of a group of **N** quantities is **A** and a new quantity **Q** is added to the group, , then the new average is more/less than A by $\frac{Q-A}{N+1}$.

Example 7

Rishi has a certain average for 9 innings. In the tenth inning, he scores 100 runs thereby increasing his average by 8 runs. His new average is?

(a) 20 (b) 28 (c) 38 (d) 40

Solution: Let the initial average of 9 innings is A and the new Average be Y then

- Method 1: Initial Sum of Runs = 9 X A

  New Average Y = (A + 8)

  New Sum of Runs = 10 Y = 10 X (A + 8),

$$\therefore 10 \times (A + 8) = 9 \times A + 100$$

$$10A + 80 = 9A + 100$$

$$10A - 9A = 100 - 80 = 20$$

$$Y = A + 8 = 28 \text{ Runs}$$

Example 7

- **Method 2:** Using Shortcut Formula (Note that the new average is 8 more than the initial average A)

$$\text{New average is } \frac{100 - A}{10} \text{ more than A}$$

$$\Rightarrow \frac{100 - A}{10} = 8$$

$$\Rightarrow 100 - A = 80$$

$$\Rightarrow A = 100 - 80 = 20$$

$$\Rightarrow Y = 20 + 8 = 28$$

Some Important Averages to Remember

The average of first *n* natural numbers i.e. average of 1, 2, 3, ..., n is

$$\frac{n+1}{2}$$

The average of square of first n natural numbers average of $1^2, 2^2, 3^2, \ldots\ldots, n^2$ is

$$\frac{(n+1)(2n+1)}{6}$$

The average of cubes of first n natural numbers i.e. average of $1^3, 2^3, 3^3, \ldots\ldots$, n^3 is

$$\frac{n\,(n+1)^2}{4}$$

Some Important Averages to Remember

- The average of first n consecutive even numbers i.e. average of 2, 4, 6, 8, ..., 2n is **(n + 1).**
- The average of first *n* consecutive odd numbers i.e. average of 1, 3, 5, 7, ..., (2n – 1) is **n.**
- The average of n consecutive numbers is
  - Always the middle number, if n is odd. e.g. average of 1, 2, 3, 4, 5, 6, 7 is 4.
  - Always the average of the middle two numbers, if n is even. e.g. average of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 is (6 + 7)/2 = 6.5.


Mean, Median,
Mode, & Range

- **Median :** (middle)

- The "Median" of a data set is dependent on whether the number of elements in the data set is odd or even.
- First reorder the data set from the smallest to the largest
- Mark off high and low values until you reach the **middle**.
- If there 2 middles, add them and **divide** by 2.

- **Examples : Odd Number of Elements**
- Data Set = 2, 5, 9, 3, 5, 4, 7
- Reordered = 2, 3, 4, 5, 5, 7, 9
  ^
- Median = 5

- **Examples : Even Number of Elements**
- Data Set = 2, 5, 9, 3, 5, 4
- Reordered = 2, 3, 4, 5, 5, 9
  ^ ^

  Median = (4 + 5) / 2 = 4.5

- **Mode : (most often)**
- The "Mode" for a data set is the element that occurs the most often.
- It is not uncommon for a data set to have more than one mode.
- This happens when two or more elements occur with equal frequency in the data set.

- **Example :**
- Data Set = 2, 5, 9, 3, 5, 4, 7
- Mode = 5

- **Example:**
- Data Set = 2, 5, 2, 3, 5, 4, 7
- Modes = 2 and 5

- **Range :**

- The "Range" for a data set is the difference between the largest value and smallest value contained in the data set.

- First **reorder** the data set from smallest to largest then **subtract** the first element from the last element.

Examples :

- Data Set = 2, 5, 9, 3, 5, 4, 7
- Reordered = 2, 3, 4, 5, 5, 7, 9
- Range = (9 - 2) = 7

THANK YOU

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