H.C.F. and L.C.M. of Numbers 2

COMMON FACTOR

A *common factor* of two or more numbers is a number which divides each of them exactly.
For example, 4 is a common factor of 8 and 12.

HIGHEST COMMON FACTOR

Highest common factor of two or more numbers is the greatest number that divides each one of them exactly. For example, 6 is the highest common factor of 12, 18 and 24. Highest Common Factor is also called *Greatest Common Divisor* or *Greatest Common Measure*.

Symbolically, these can be written as H.C.F. or G.C.D. or G.C.M., respectively.

METHODS OF FINDING H.C.F.

I. Method of Prime Factors

Step 1 Express each one of the given numbers as the product of prime factors.
[A number is said to be a *prime number* if it is exactly divisible by 1 and itself, but not by any other number, e.g., 2, 3, 5, 7, etc. are prime numbers]

Step 2 Choose common factors.

Step 3 Find the product of these common factors. This is the required H.C.F. of given numbers.

Illustration 1: Find the H.C.F. of 70 and 90.

Solution: $70 = 2 \times 5 \times 7$

$90 = 2 \times 5 \times 9$

Common factors are 2 and 5.

$\therefore$ H.C.F. $= 2 \times 5 = 10$.

Illustration 2: Find the H.C.F. of 3332, 3724 and 4508.

Solution: $3332 = 2 \times 2 \times 7 \times 7 \times 17$

$3724 = 2 \times 2 \times 7 \times 7 \times 19$

$4508 = 2 \times 2 \times 7 \times 7 \times 23$

$\therefore$ H.C.F. $= 2 \times 2 \times 7 \times 7 = 196$.

Illustration 3: Find the H.C.F. of 360 and 132.

Solution: $360 = 2^3 \times 3^2 \times 5$

$132 = 2^2 \times 3^1 \times 11$

$\therefore$ H.C.F. $= 2^2 \times 3^1 = 12$.

Illustration 4: If $x = 2^3 \times 3^5 \times 5^9$ and $y = 2^5 \times 3^7 \times 5^{11}$, find H.C.F. of x and y.

Solution: The factors common to both x and y are 2^3, 3^5 and 5^9.

$\therefore$ H.C.F. $= 2^3 \times 3^5 \times 5^9$.

II. Method of Division

***A. For two numbers*:**

Step 1 Greater number is divided by the smaller one.

Step 2 Divisor of (1) is divided by its remainder.

Step 3 Divisor of (2) is divided by its remainder. This is continued until no remainder is left. H.C.F. is the divisor of last step.

Illustration 5: Find the H.C.F. of 3556 and 3444.

$$
\begin{array}{r}
3444\overline{)3556}(\ 1 \\
3444 \\
\hline
112\overline{)3444}(\ 30 \\
3360 \\
\hline
84\overline{)112}(\ 1 \\
84 \\
\hline
28\overline{)\ 84}(\ 3 \\
84 \\
\hline
\times
\end{array}
$$

$\therefore$ H.C.F. $= 28$.

B. For more than two numbers:

Step 1 Any two numbers are chosen and their H.C.F. is obtained.

Step 2 H.C.F. of H.C.F. (of (1)) and any other number is obtained.

Step 3 H.C.F. of H.C.F. (of (2)) and any other number (not chosen earlier) is obtained. This process is continued until all numbers have been chosen. H.C.F. of last step is the required H.C.F.

Illustration 6: Find the H.C.F. of 13915, 9499 and 2553 by division method.

Solution:

9499 | 13915 | 1
9499
4416 | 9499 | 2
8832
667 | 4416 | 6
4002
414 | 667 | 1
414
253 | 414 | 1
253
161 | 253 | 1
161
92 | 161 | 1
92
69 | 92 | 1
69
23 | 69 | 3
69
×

Now, in the next step, we will find the H.C.F. of 23 and 2553.

23 | 2553 | 111
23
25
23
23
23
×

Thus, H.C.F. of 13915, 9499 and 2553 = 23.

Illustration 7: Find the greatest possible length which can be used to measure exactly the lengths 7 m, 3 m 85 cm, 12 m 95 cm.

Solution: Required length
= (H.C.F. of 700, 385, 1295) cm = 35 cm.

COMMON MULTIPLE

A *common multiple* of two or more numbers is a number which is exactly divisible by each one of them.

For example, 32 is a common multiple of 8 and 16.

$8 \times 4 = 32$

$16 \times 2 = 32$.

LEAST COMMON MULTIPLE

The *least common multiple* of two or more given numbers is the least or lowest number which is exactly divisible by each of them.

For example, consider the two numbers 12 and 18.

Multiples of 12 are 12, 24, 36, 48, 60, 72, ...

Multiples of 18 are 18, 36, 54, 72, ...

Common multiples are 36, 72, ...

∴ Least common multiple, i.e., L.C.M. of 12 and 18 is 36.

METHODS OF FINDING L.C.M.

I. Method of Prime Factors

Step 1 Resolve each given number into prime factors.

Step 2 Take out all factors with highest powers that occur in given numbers.

Step 3 Find the product of these factors. This product will be the L.C.M.

Illustration 8: Find the L.C.M. of 32, 48, 60 and 320.

Solution: $32 = 2^5 \times 1$

$48 = 2^4 \times 3$

$60 = 2^2 \times 3 \times 5$

$320 = 2^6 \times 6$

∴ L.C.M. $= 2^6 \times 3 \times 5 = 960$.

II. Method of Division

Step 1 The given numbers are written in a line separated by common.

Step 2 Divide by any one of the prime numbers 2, 3, 5, 7, 11, ... which will divide at least any two of the given numbers exactly. The quotients and the undivided numbers are written in a line below the first.

Step 3 Step 2 is repeated until a line of numbers (prime to each other) appears.

Step 4 Find the product of all divisors and numbers in the last line, which is the required L.C.M.

Illustration 9: Find the L.C.M. of 12, 15, 20 and 54.

Solution:

2	12,	15,	20,	54
2	6,	15,	10,	27
3	3,	15,	5,	27
5	1,	5,	5,	9
	1,	1,	1,	9

L.C.M. $= 2 \times 2 \times 3 \times 5 \times 1 \times 1 \times 1 \times 9$
$= 540$.

Note:

Before finding the L.C.M. or H.C.F., we must ensure that all quantities are expressed in the same unit.

SOME USEFUL SHORTCUT METHODS

1. H.C.F. and L.C.M. of Decimals

Step 1 Make the same number of decimal places in all the given numbers by suffixing zero(s) if necessary.

Step 2 Find the H.C.F./L.C.M. of these numbers without decimal.

Step 3 Put the decimal point (in the H.C.F./L.C.M. of Step 2) leaving as many digits on its right as there are in each of the numbers.

Illustration 10: Find the L.C.M. of 1.2, 0.24 and 6.

Solution: The given numbers can be written as 1.20, 0.24 and 6.00.

Now, ignoring the decimal we find the L.C.M. of 120, 24 and 600.

2	120,	24,	600
2	60,	12,	300
2	30,	6,	150
3	15,	3,	75
5	5,	1,	25
	1,	1,	5

∴ L.C.M. $= 2 \times 2 \times 2 \times 3 \times 5 \times 1 \times 5 = 600$

Thus, the required L.C.M. = 6.00, i.e., 6.

Illustration 11: Find the H.C.F. of 6.16 and 13.

Solution: The given numbers can be written as 6.16 and 13.00.

Now, ignoring the decimals we find the H.C.F. of 616 and 1300.

616 | 1300 | 2
1232
68 | 616 | 9
612
4 | 68 | 17
68
×

∴ H.C.F. of 616 and 1300 is 4.

Thus, the required H.C.F. = 0.04.

2. L.C.M. and H.C.F. of Fractions

$$\text{L.C.M} = \frac{\text{L.C.M. of the numbers in numerators}}{\text{H.C.F. of the numbers in denominators}}$$

$$\text{H.C.F.} = \frac{\text{H.C.F. of the numbers in numerators}}{\text{L.C.M. of the numbers in denominators}}$$

Illustration 12: Find the L.C.M. of $\frac{2}{5}$, $\frac{3}{10}$ and $\frac{6}{25}$.

Solution: L.C.M. of numerators 2, 3 and 6 is 6.

H.C.F. of denominators 5, 10 and 25 is 5.

∴ Required L.C.M. $= \frac{\text{L.C.M. of Numerators}}{\text{H.C.F. of Denominators}}$

$= \frac{6}{5}$.

Illustration 13: Find the H.C.F. of $\frac{4}{9}$, $\frac{10}{21}$ and $\frac{20}{63}$.

Solution: H.C.F. of numerators 4, 10 and 20 is 2.

L.C.M. of denominators 9, 21 and 63 is 63.

∴ Required H.C.F $= \frac{\text{H.C.F. of Numerators}}{\text{L.C.M. of Denominators}} = \frac{2}{63}$.

Notes:

1. If the given set of numbers includes fractions as well as whole numbers, treat whole number too as fraction with 1 in its denominator.
2. The H.C.F. of a number of fractions is always a fraction, but the L.C.M. may be a fraction or an integer.

3. Product of two numbers

= L.C.M. of the numbers × H.C.F. of the numbers

Illustration 14: The H.C.F. and the L.C.M. of any two numbers are 63 and 1260, respectively. If one of the two numbers is 315, find the other number.

Solution: The required number

$$= \frac{\text{L.C.M.} \times \text{H.C.F.}}{\text{First Number}} = \frac{1260 \times 63}{315} = 252.$$

4. To find the greatest number that will exactly divide x, y and z.
Required number = H.C.F. of x, y and z.

Illustration 15: Find the greatest number that will exactly divide 200 and 320.

Solution: The required greatest number
= H.C.F. of 200 and 320 = 40.

5. To find the greatest number that will divide x, y and z leaving remainders a, b and c, respectively.
Required number = H.C.F. of $(x - a)$, $(y - b)$ and $(z - c)$.

Illustration 16: Find the greatest number that will divide 148, 246 and 623 leaving remainders 4, 6 and 11, respectively.

Solution: The required greatest number
= H.C.F. of $(148 - 4)$, $(246 - 6)$ and $(623 - 11)$,
i.e., H.C.F. of 144, 240 and 612 = 12.

6. To find the least number which is exactly divisible by x, y and z.
Required number = L.C.M. of x, y and z.

Illustration 17: What is the smallest number which is exactly divisible by 36, 45, 63 and 80?

Solution: The required smallest number
= L.C.M. of 36, 45, 63 and 80
= 5040.

7. To find the least number which when divided by x, y and z leaves the remainders a, b and c, respectively. It is always observed that $(x - a) = (y - b) = (z - c) = k$(say)
$\therefore$ Required number = (L.C.M. of x, y and z) $- k$.

Illustration 18: Find the least number which when divided by 36, 48 and 64 leaves the remainders 25, 37 and 53, respectively.

Solution: Since, $(36 - 25) = (48 - 37) = (64 - 53) = 11$, therefore, the required smallest number
= (L.C.M. of 36, 48 and 64) $- 11$
= $576 - 11 = 565$.

8. To find the least number which when divided by x, y and z leaves the same remainder r in each case.
Required number = (L.C.M. of x, y and z) $+ r$.

Illustration 19: Find the least number which when divided by 12, 16 and 18, will leave in each case a remainder 5.

Solution: The required smallest number
= (L.C.M. of 12, 16 and 18) + 5
= 144 + 5 = 149.

9. To find the greatest number that will divide x, y and z leaving the same remainder in each case.
(a) When the value of remainder r is given:
Required number = H.C.F. of $(x - r)$, $(y - r)$ and $(z - r)$.
(b) When the value of remainder is not given:
Required number = H.C.F. of $|(x - y)|$, $|(y - z)|$ and $|(z - x)|$

Illustration 20: Find the greatest number which will divide 772 and 2778 so as to leave the remainder 5 in each case.

Solution: The required greatest number
= H.C.F. of $(772 - 5)$ and $(2778 - 5)$
= H.C.F. of 767 and 2773
= 59.

Illustration 21: Find the greatest number which on dividing 152, 277 and 427 leaves equal remainder.

Solution: The required greatest number
= H.C.F. of $|(x - y)|$, $|(y - z)|$ and $|(z - x)|$
= H.C.F. of $|(152 - 277)|$, $|(277 - 427)|$ and $|(427 - 152)|$
= H.C.F. of 125, 150 and 275
= 25.

10. To find the n-digit greatest number which, when divided by x, y and z,
(a) leaves no remainder (i.e., exactly divisible)
Step 1 L.C.M. of x, y and $z = L$
Step 2 $L\overline{)n\text{-digit greatest number}}$ (
Remainder $= R$
Step 3 Required number
$= n$-digit greatest number $- R$
(b) leaves remainder K in each case
Required number
$= (n$-digit greatest number $- R) + K$.

Illustration 22: Find the greatest number of 4-digit number which, when divided by 12, 18, 21 and 28 leaves 3 as a remainder in each case.

Solution: L.C.M. of 12, 18, 21 and 28 = 252.

$$252\overline{)9999}\,(39$$
$$\underline{9828}$$
$$171$$

$\therefore$ The required number = $(9999 - 171) + 3 = 9931$.

Illustration 23: Find the greatest number of four digits which, when divided by 12, 15, 20 and 35 leaves no remainder.

Solution: L.C.M. of 12, 15, 20 and 35 = 420.

$$420\overline{)9999}\,(23$$
$$\underline{9660}$$
$$339$$

$\therefore$ The required number = $9999 - 339$
= 9663.

11. To find the n-digit smallest number which when divided by x, y and z
(a) leaves no remainder (i.e., exactly divisible)
Step 1 L.C.M. of x, y and $z = L$
Step 2 $L\overline{)n\text{-digit smallest number}}$ (
Remainder $= R$
Step 3 Required number
$= n$-digit smallest number $+ (L - R)$.
(b) leaves remainder K in each case.
Required number
$= n$-digit smallest number $+ (L - R) + K$.

Illustration 24: Find the least number of four digits which is divisible by 4, 6, 8 and 10.

Solution: L.C.M. of 4, 6, 8 and 10 = 120.

$$120\overline{)1000}(8$$
$$\underline{960}$$
$$40$$

$\therefore$ The required number = $1000 + (120 - 40)$
= 1080.

Illustration 25: Find the smallest 4-digit number, such that when divided by 12, 18, 21 and 28, it leaves remainder 3 in each case.

Solution: L.C.M. of 12, 18, 21 and 28 = 252.

$$252\overline{)1000}\,(3$$
$$\underline{756}$$
$$244$$

$\therefore$ The required number
= $1000 + (252 - 244) + 3$
= 1011.

EXERCISES

1. What is the H.C.F. of 27, 18 and 36?
(a) 7 (b) 11
(c) 9 (d) None of these

2. Determine the L.C.M of $\frac{2}{5}, \frac{3}{10}$ and $\frac{6}{25}$.
(a) $\frac{6}{5}$ (b) $\frac{11}{5}$
(c) $\frac{9}{5}$ (d) None of these

3. What is the L.C.M. of 25, 30, 35 and 40?
(a) 3800
(b) 4200
(c) 4400
(d) None of these

4. What is the greatest number which divides 852, 1065 and 1491 exactly?
(a) 193 (b) 183
(c) 223 (d) 213

5. What is the H.C.F. of $\frac{4}{9}, \frac{10}{21}$ and $\frac{20}{30}$?
(a) $\frac{4}{189}$ (b) $\frac{6}{23}$
(c) $\frac{2}{63}$ (d) None of these

6. Find the least number which when divided by 16, 18, 20 and 25 leaves 4 as remainder in each case but when divided by 7 leaves no remainder.
(a) 8004 (b) 13004
(c) 18004 (d) 18014

7. Area of three fields is 165 m^2, 195 m^2 and 85 m^2, respectively. In each of the fields a flower bed of equal length has to be made. If flower bed in each of the fields is 3 m wide then what is the maximum length of the flower bed in each of the fields?
(a) 7 m (b) 9 m
(c) 5 m (d) None of these

8. Find the greatest number which will divide 2112 and 2792 leaving the remainder 4 in each case.
(a) 78 (b) 68
(c) 65 (d) 63

9. The H.C.F. of two numbers is 12 and their difference is 12. The numbers are:
(a) 66, 78 (b) 70, 82
(c) 94, 106 (d) 84, 96

10. A merchant has 435 litres, 493 litres and 551 litres of three different kinds of milk. Find the least number of casks of equal size required to store all the milk without mixing.
(a) 51 (b) 61
(c) 47 (d) 45

11. Find the greatest number which will divide 25, 73 and 97 so as to leave the same remainder in each case.
(a) 12 (b) 18
(c) 24 (d) 32

12. The sum of two numbers is 216 and their H.C.F. is 27. The numbers are:
(a) 54, 162 (b) 108, 118
(c) 27, 189 (d) None of these

13. How often will five bells toll together in one hour if they start together and toll at intervals of 5, 6, 8, 12, 20 seconds, respectively?
(a) 29 (b) 30
(c) 31 (d) 120

14. Find the greatest number that will divide 964, 1238 and 1400 leaving remainders 41, 31 and 51, respectively.
(a) 71 (b) 81
(c) 61 (d) 73

15. Find the side of the largest square slabs which can be paved on the floor of a room 5 m 44 cm long and 3 m 74 cm broad.
(a) 56 (b) 42
(c) 38 (d) 34

16. The traffic lights at three different road crossings change after every 48 seconds, 72 seconds and 108 seconds, respectively. If they all change simultaneously at 8:20:00 hours; then they will again change simultaneously at:
(a) 8:27:12 hours
(b) 8:27:24 hours
(c) 8:27:36 hours
(d) 8:27:48 hours

17. The product of two numbers is 6760 and their H.C.F. is 13. How many such pairs can be formed?
(a) 2 (b) 3
(c) 4 (d) only one

18. Find the greatest number of four digits which when divided by 10, 15, 21 and 28 leaves 4, 9, 15 and 22 as remainders, respectively.
(a) 9654 (b) 9666
(c) 9664 (d) 9864

19. The number of prime factors in the expression $(6)^{10} \times (7)^{17} \times (11)^{27}$ is:
(a) 54 (b) 64
(c) 71 (d) 81

20. Find the greatest number which will divide 3962, 4085 and 4167 leaving the same remainder in each case.
(a) 37 (b) 39
(c) 41 (d) 43

21. A wholesale tea dealer has 408 kilograms, 468 kilograms and 516 kilograms of three different qualities of tea. He wants it all to be packed into boxes of equal size without mixing. Find the capacity of the largest possible box.
(a) 50 (b) 36
(c) 24 (d) 12

22. A room is 4 m 37 cm long and 3 m 23 cm broad. It is required to pave the floor with minimum square slabs. Find the number of slabs required for this purpose.
(a) 485 (b) 431
(c) 391 (d) 381

23. The least perfect square number which is divisible by 3, 4, 5, 6 and 8:
(a) 900 (b) 1200
(c) 2500 (d) 3600

24. Find the least number of five digits which when divided by 12, 16, 21, 36 and 40 leaves remainder 8 in each case.
(a) 10088 (b) 10072
(c) 10080 (d) None of these

25. Three pieces of timber 42 m, 49 m and 63 m long have to be divided into planks of the same length. What is the greatest possible length of each plank?
(a) 7 m (b) 14 m
(c) 42 m (d) 63 m

26. Three men start together to travel the same way around a circular track of 11 kilometres in circumference. Their speeds are 4, $5\frac{1}{2}$ and 8 Km/h, respectively. When will they meet at the starting point?
(a) 11 hours (b) 12 hours
(c) 23 hours (d) 22 hours

27. Five bells begin to toll together and toll at intervals of 36, 45, 72, 81 and 108 seconds. After what interval of time will they keep on tolling together?
(a) 3240 seconds (b) 3080 seconds
(c) 3140 seconds (d) 3200 seconds

28. Three different containers contain different quantities of mixture of milk and water, whose measurements are 403 Kg, 434 Kg and 465 Kg. What biggest measure must be there to measure all the different quantities exactly?
(a) 1 Kg (b) 7 Kg
(c) 31 Kg (d) 41 Kg

29. The L.C.M. and G.C.D. of two numbers are 1530 and 51, respectively. Find how many such pairs are possible?
(a) 2 (b) 3
(c) 4 (d) Only one

30. Find the least number of five digits which when divided by 63, 56 and 42 leaves remainder 1 in each case.
(a) 10082 (b) 10081
(c) 10001 (d) 10071

31. The H.C.F. and L.C.M. of two numbers are 44 and 264, respectively. If the first number is divided by 2, the quotient is 44. The other number is:
(a) 33 (b) 66
(c) 132 (d) 264

32. The largest natural number which exactly divides the product of any four consecutive natural numbers, is:
(a) 6 (b) 12
(c) 24 (d) 120

33. Find the least number of six digits which is exactly divisible by 15, 21 and 28:
(a) 100480 (b) 100270
(c) 100380 (d) 100340

34. Find the greatest number of five digits which when divided by 12, 15, 21, 25 and 28 leaves 5, 8, 14, 18 and 21 as remainders, respectively.
(a) 98696 (b) 98700
(c) 97693 (d) 98693

35. What is the smallest number which when increased by 3 is divisible by 16, 24, 30 and 32?
(a) 480 (b) 475
(c) 472 (d) 477

36. The least number of square tiles required to cover the ceiling of a room 15 m 17 cm long and 9 m 2 cm broad, is:
(a) 656 (b) 738
(c) 814 (d) 902

37. Find the least number which when divided by 2, 3, 4, 5 and 6 leaves 1, 2, 3, 4 and 5 as remainders, respectively, but when divided by 7 leaves no remainder.
(a) 210 (b) 119
(c) 126 (d) 154

38. Find the greatest number of five digits which when divided by 4, 6, 10 and 15 leaves the same remainder 3 in each case.
(a) 99993 (b) 99063
(c) 90093 (d) 99963

39. Find the least number which is a multiple of 31 and when divided by 15, 24 and 32 leaves the remainders 2, 11 and 19, respectively.
(a) 2418 (b) 2387
(c) 2356 (d) 2325

40. Find the two largest numbers of four digits having 531 as their H.C.F.
(a) 9231, 9762
(b) 9027, 9558
(c) 9037, 9568
(d) 9127, 9658

41. Find the greatest number of five digits which becomes exactly divisible by 10, 12, 15 and 18 when 3769 is added to it.
(a) 99811 (b) 99911
(c) 98911 (d) 99011

42. Find the least number which when decreased by 11 is divisible by 14, 15, 21, 32 and 60.
(a) 4371 (b) 3271
(c) 3371 (d) 3360

43. Find the least number of five digits which when divided by 8, 12, 16 and 20 leaves remainders 1, 5, 9 and 13, respectively.
(a) 10003 (b) 10093
(c) 10073 (d) 10013

44. The H.C.F. of two numbers is 11 and their L.C.M. is 693. If one of the numbers is 77, find the other.
(a) 909 (b) 119
(c) 66 (d) 99

45. Find the greatest number of four digits which is exactly divisible by 24, 28, 30 and 35.
(a) 9225 (b) 9240
(c) 9250 (d) 9260

46. Find the greatest number of four digits which must be added to 5231 so that the final number becomes exactly divisible by 12, 15, 27, 32 and 40.
(a) 7929 (b) 7829
(c) 9729 (d) 7729

47. A heap of stones can be made up into groups of 16, 20, 25 and 21. When made up into groups of 45, there are 3 stones left in each case. How many stones at least can there be in the heap?
(a) 7203 (b) 2403
(c) 3603 (d) 4803

48. Find the greatest number of five digits which when divided by 8, 9 and 10 leaves 3 as remainder in each case.
(a) 99996
(b) 99723
(c) 99983
(d) None of these

49. What is the least number of cut pieces of equal length that can be cut out of two lengths 10 m 857 mm and 15 m 87 mm?
(a) 174 (b) 172
(c) 164 (d) 184

ANSWER KEYS

1. (c)	2. (a)	3. (b)	4. (d)	5. (c)	6. (c)	7. (c)	8. (b)	9. (d)	10. (a)	11. (c)	12. (c)	13. (c)
14. (a)	15. (d)	16. (a)	17. (a)	18. (a)	19. (b)	20. (c)	21. (d)	22. (c)	23. (d)	24. (a)	25. (a)	26. (d)
27. (a)	28. (c)	29. (c)	30. (b)	31. (c)	32. (c)	33. (c)	34. (d)	35. (d)	36. (c)	37. (b)	38. (d)	39. (b)
40. (b)	41. (b)	42. (c)	43. (c)	44. (d)	45. (b)	46. (d)	47. (a)	48. (b)	49. (d)			

EXPLANATORY ANSWERS

1. (c) H.C.F. of 27, 18 and 36

$18\overline{)27}\,(1$, 18, remainder 9

$9\overline{)18}\,(2$, 18, remainder 0

∴ H.C.F. of 27 and 18 is 9

Now, H.C.F. of 9 and 36

$9\overline{)36}\,(4$, 36, remainder 0

∴ H.C.F. of 9 and 36 is 9

Therefore, the required H.C.F of 27, 18 and 36 is 9.

2. (a) L.C.M. of $\frac{2}{5}, \frac{3}{10}$ and $\frac{6}{25}$

$$= \frac{\text{L.C.M. of 2, 3 and 6}}{\text{H.C.F. of 5, 10 and 25}}$$

∵ L.C.M. of 2, 3 and 6 = 6

and, H.C.F. of 5, 10 and 25 = 5

∴ Required L.C.M. $= \frac{6}{5}$.

3. (b)

2	25,	30,	35,	40
5	25,	15,	35,	20
	5,	3,	7,	4

∴ Required L.C.M. $= 2 \times 5 \times 5 \times 3 \times 7 \times 4$
$= 4200.$

4. (d) H.C.F. of 852 and 1065 is 213.
H.C.F. of 213 and 1491 is 213.

5. (c) H.C.F. of $\frac{4}{9}, \frac{10}{21}$ and $\frac{20}{63}$

$$= \frac{\text{H.C.F. of 4, 10 and 20}}{\text{L.C.M. of 9, 21 and 63}}$$

∵ H.C.F. of 4, 10 and 20 = 2

and L.C.M. of 9, 21 and 63 = 63

∴ Required H.C.F. $= \frac{2}{63}$.

6. (c) L.C.M. of 16, 18, 20 and 25 is 3600.
Required number $= 3600 \times K + 4$
$= (7 \times 514 + 2)K + 4$
$= (7 \times 514)K + 2K + 4$
Now $(2K + 4)$ is divisible by 7 for $K = 5$.
∴ Required number $= 5 \times 3600 + 4$
$= 18004.$

7. (c) H.C.F. of 165, 195 and 85 will be maximum area of each of the flower beds.
H.C.F. of 165 and 195:

$165\overline{)195}\,(1$, 165, remainder 30

$30\overline{)165}\,(2$, 150, remainder 15

$15\overline{)35}\,(2$, 30, remainder 5

∴ H.C.F. of 165 and 195 is 15.
Also, now, H.C.F. of 15 and 85 is 5.

8. (b) Subtract 4 from each of the numbers 2112 and 2792 and then take the H.C.F, i.e., H.C.F of 2108 and 2788.

9. (d) The difference of requisite numbers must be 12 and each one must be divisible by 12. So, the numbers are 84, 96.

10. (a) Since minimum number of casks are required, the size of the cask is greatest. Also the cask in three cases are of equal size. The size of the cask is the H.C.F. of 435, 493 and 551 which is 29.
Now, the number of casks required for storing the milk
$= (493 + 435 + 551) \div 29 = 51.$

11. (c) $73 - 25 = 48$
$97 - 73 = 24$
$97 - 25 = 72$
H.C.F. of 48, 24 and 72 is 24.

12. (c) Let the numbers be $27a$ and $27b$
Then, $27a + 27b = 216$ or, $27(a + b) = 216$
or, $a + b = \frac{216}{27} = 8$
∴ Values of co-primes (with sum 8) are (1, 7) and (3, 5)
So, the numbers are $(27 \times 1, 27 \times 7)$,
i.e., (27, 189).

13. (c) The time after which the bells will ring together is the L.C.M. of 5, 6, 8, 12 and 20 seconds, i.e., 120 seconds. The number of times they will toll together in one hour
$= (3600 \div 120) + 1$
$= 30 + 1 = 31.$

14. (a) $964 - 41 = 923$
$1238 - 31 = 1207$
$1400 - 51 = 1349$
H.C.F. of 923 and 1207 is 71.
H.C.F. of 71 and 1349 is 71.

15. **(d)** The side of the square slab is the H.C.F. of 544 and 374 cm, i.e., 34.

16. **(a)** Interval of change = (L.C.M. of 48, 72, 108) seconds
= 432

So, the lights will change after every 432 seconds, i.e., 7 minutes and 12 seconds.

So, the next simultaneous change will take place at 8:27:12 hours.

17. **(a)** Let the numbers be $13x$ and $13y$.

$13x \times 13y = 6760$

$\therefore \quad x \times y = 6760 \div (13 \times 13) = 40$

Possible values of (x, y) are

(1, 40); (2, 20); (4, 10); (5, 8)

Only two acceptable values are (1, 40) and (5, 8).

18. **(a)** First, find the greatest number of four digits that is divisible by the L.C.M. of 10, 15, 21 and 28 and then subtract 6 from it to get the required number.

19. **(b)** Since 2, 3, 7, 11 are prime numbers and the given expression is $2^{10} \times 3^{10} \times 7^{17} \times 11^{27}$, the number of prime factors in the given expression is (10 + 10 + 17 + 27) = 64.

20. **(c)** 4085 − 3962 = 123

4167 − 4085 = 82

4167 − 3962 = 205

H.C.F. of 123, 82 and 205 is 41.

21. **(d)** The capacity of the box is H.C.F. of 408, 468 and 516, i.e., 12.

22. **(c)** Length = 437 cm

Breadth = 323 cm.

The side of the square slab is the H.C.F. of 437 and 323, i.e., 19 cm.

$\therefore$ Area of square slab = 19 cm × 19 cm = 361 cm^2

The number of slabs $= \dfrac{\text{Area of the room}}{\text{Area of the slab}}$

$= \dfrac{437 \times 323\ cm^2}{361\ cm^2}$

= 391.

23. **(d)**

2	3,	4,	5,	6,	8
2	3,	2,	5,	3,	4
3	3,	1,	5,	3,	2
	1,	1,	5,	1,	2

L.C.M. of 3, 4, 5, 6, 8 = 120

Required number = 2 × 2 × 2 × 2 × 3 × 3 × 5 × 5
= 3600.

24. **(a)** Required number = the least number of 5 digits divisible by the L.C.M. of 12, 16, 21, 36, 40 + the remainder 8.

25. **(a)** Greatest possible length of each plank
= (H.C.F. of 42, 49, 63) m = 7 m.

26. **(d)** Time for one revolution by each of three men

$= \dfrac{11}{4}, \dfrac{11}{5\frac{1}{2}}, \dfrac{11}{8}$ hours

$= \dfrac{11}{4}, \dfrac{2}{1}, \dfrac{11}{8}$ hours

$\therefore$ The time when they will meet at the starting point
= L.C.M. of $\dfrac{11}{4}, \dfrac{2}{1}, \dfrac{11}{8}$ which is $\dfrac{22}{1}$, i.e., 22 hours.

27. **(a)** The interval of time is L.C.M. of the numbers 36, 45, 72, 81 and 108.

2	36,	45,	72,	81,	108
2	18,	45,	36,	81,	54
2	9,	45,	18,	81,	27
3	9,	45,	9,	81,	27
3	3,	15,	3,	27,	9
3	1,	5,	1,	9,	3
	1,	5,	1,	3,	1

L.C.M. (36, 45, 72, 108) = 3240.

28. **(c)** Biggest measure = H.C.F. of (403, 434, 465)
= 31 Kg.

29. **(c)** Let the numbers be $51x$ and $51y$ where x and y are co-prime.

Now, $51x \times 51y = 51 \times 1530$

$\therefore \quad x \times y = 30$

Possible pairs are (1, 30); (2, 15); (3, 10) and (5, 6).

30. **(b)** L.C.M. of 63, 56 and 42 is 504.

Least number of 5 digits divisible by 504:

$$504\overline{)10000}(19$$
$$-504$$
$$4960$$
$$-4536$$
$$424$$

= 1000 + (504 − 424) = 10080

$\therefore$ Required number = 10080 + 1 = 10081.

31. **(c)** First number = 2 × 44 = 88

Second number $= \dfrac{44 \times 264}{88} = 132.$

32. **(c)** 1 × 2 × 3 × 4 = 24

$\therefore$ Required number = 24.

33. **(c)** L.C.M. of 15, 21 and 28 is 420.

Least number of 6 digits = 100000

$$420\overline{)100000}(238$$
$$-840$$
$$1600$$
$$-1260$$
$$3400$$
$$-3360$$
$$40$$

Remainder = 40.

$\therefore$ Least number = 100000 + (420 − 40)
= 100380.

34. **(d)** Find the greatest number of five digits which is divisible by the L.C.M. of 12, 15, 21, 25 and 28 and then subtract 7 from it to get the required number.

Required number = 98700 − 7 = 98693.

35. **(d)** Required number
= (L.C.M. of 16, 24, 30 and 32) − 3
= 480 − 3 = 477.

36. **(c)** Side of each tile = (H.C.F. of 1517 and 902) cm = 41 cm

$\therefore$ Number of tiles $= \dfrac{1517 \times 902}{41 \times 41} = 814.$

37. **(b)** L.C.M. of 2, 3, 4, 5, 6 is 60.

One of the numbers satisfying the first condition is 60 − 1 = 59

60 + 59 = 119, etc.

But 119 is also divisible by 7.

38. **(d)** L.C.M. of 4, 6, 10, 15 = 60

Greatest number of five digits which is divisible by 60 = 99960.

$\therefore$ Required number = 99960 + 3 = 99963.

39. **(b)** L.C.M. of 15, 24, 32 is 480

Required number = $480K - 13$
$= 15 \times 31K + (15K - 13)$

$(15K - 13)$ is divisible by 31 for $K = 5$

$\therefore$ Least number = 480 × 5 − 13 = 2387.

40. **(b)** The greatest number of four digits divisible by 531 is 9558, so the other number is 9558 − 531 = 9027. Thus, the numbers are 9558 and 9027.

41. **(b)** L.C.M. of 10, 12, 15 and 18 = 540. Dividing (99999 + 3769) by 540, the remainder is 88.

$\therefore$ Required number = 99999 − 88 = 99911.

42. **(c)** Required number
= (L.C.M. of 14, 15, 21, 32, 60) + 11
= 3360 + 11 = 3371.

43. **(c)** Least number of five digits divisible by L.C.M. of 8, 12, 16, 20 is 10080.

$\therefore$ Required number = 10080 − 7 = 10073.

44. **(d)** Required number $= \dfrac{\text{L.C.M.} \times \text{H.C.F.}}{\text{Given number}}$

$= \dfrac{693 \times 11}{77} = 99.$

45. **(b)** L.C.M. of 24, 28, 30 and 35

2	24,	28,	30,	35
2	12,	14,	15,	35
3	6,	7,	15,	35
5	2,	7,	5,	35
7	2,	7,	1,	7
	2,	1,	1,	1

= 2 × 2 × 2 × 3 × 5 × 7 = 840

Greatest number of four digits
= 9999.

Quotient when 9999 is divided by 840 is 11 and remainder is 759.

$\therefore$ Greatest number of four digits in this case = 9999 − 759 = 9240.

46. **(d)** L.C.M. of 12, 15, 27, 32, 40 = 4320. Let us add 5231 to the greatest number of four digits and then divide by 4320 to find the remainder.

$$4320\overline{)15230}(3$$
$$12960$$
$$2270$$

Required greatest number of four digits
= 9999 − 2270
= 7729.

47. **(a)** L.C.M. of 16, 20, 25, 45 = 3600

1st number = 3600 × 1 + 3
= 3603 which is not divisible by 21.

2nd number = 3600 × 2 + 3
= 7203 which is divisible by 21.

48. **(b)** L.C.M. of 8, 9, 10 = 360

$$360\overline{)99999}(277$$
$$720$$
$$2799$$
$$2520$$
$$2799$$
$$2520$$
$$279$$

Greatest number of five digits which is divisible by 360
= 99999 − 279 = 99720

$\therefore$ Required number = 99729 + 3 = 99723.

49. **(d)** H.C.F. of 10857 and 15087 is 141.

The least number of cut pieces
= (10857 + 15087) ÷ 141
= 184.

Hari Mohan Prasad
Meenakshi Upadhyay