30/12/19

18PDH101T

1. Number system
   (a) Divisibility
   (b) Unit digit
   (c) LCM and HCF
   (d) Simplification
   (e) Surds & indices.
   (f) factors
   (g) remainders
2. Percentage
3. Profit, loss and discount
4. Simple, compound interest
5. log
6. linear equation

Q1 If $N^2 = 12345676 54321$

$N = ?$

$N = 7 \times 1$

Q2 If $N^2 = 12321$

$N = 3 \times 1$

Q3 If $y = 7 - 4\sqrt{3}$, then $\left(\sqrt{y} + \frac{1}{\sqrt{y}}\right)^2$

$= y + \frac{1}{y} + 2 = 7 - 4\sqrt{3} + 7 + 4\sqrt{3} + 2$

$= 16$

Q4

① $1+2+3+4+ \cdots n = \frac{n(n+1)}{2}$

② $1^2+2^2+3^2+ \cdots n^2 = \frac{n(n+1)(2n+1)}{6}$

③ $1^3+2^3+3^3+ \cdots n^3 = \left[\frac{n(n+1)}{2}\right]^2$

④ sum of 'n' odd nos $= n^2$

Q4 sum of square of first 20 even natural numbers?

$2^2+4^2+ \cdots 40^2 ?$

$= 2^2(1^2+2^2+ \cdots 20^2)$

$= 2^2 \frac{(20\times21\times41)}{6}$

Q5 sum of odd natural nos. upto 100.

$1+3+5+ \cdots 99$

$\frac{50}{2}(1+99) = \frac{50}{2}\times100$

$= 2500$

sum $= \frac{n}{2}(a+l)$

By 4

Sum of last two digits divisible.

By 5

0 or 5 in units place

By 6

Divisible by both 2 and 3.

By 7

$A - 2B$ → should be 0 or div. by 7. where B is unit place digit, A is remaining digit.

eg: 945

B = 5

$A = 94$

$A - 2B = 94 - 5 \times 2 = 94 - 10 = \underline{84}$

By 8

Last 3 digits div. by 8.

By 9

Sum of all digits divisible by 9.

By 11

Even place digits − odd place digits = Div. by 11

eg: $\boxed{4}8\boxed{3}2\boxed{7}1\boxed{8}$

$22 - 11 = 11$

By 12

Div. by 3 and 4.

Scanned by CamScanner

By 13

$A + 4B \longrightarrow$ Div. by 13 or 0

~~31149~~

(1) 314(6)
$\underline{314}$ A, 6 B

$314 + 4 \times 6$

$= 314 + 20 = 334$

(2) 140(4)
4 B

$140 + 4 \times 4 = 140 + 16$

$= 156$

By 14

Div. by 2 and 7

By 15

Div. by 3 and 5.

By 16

Last 4 digits div. by 16.

By 17

$A - 5B$ by 17

eg: 2278

Q. Which div. by 3, 7, 9, 11?

(a) 639 (b) 2079 (c) 3791 (d) 37911

Q. If 481y is div. by 9, find smallest nat. value of y?

$13 + y =$ div. by 9 $\Rightarrow$ $y = 5$

$$\begin{array}{r} 16 \\ 15 \\ 12 \\ \hline 43 \end{array}$$

Q. What is the value of $x+y$ if $789432x64y$ is div. by both 8 and 9?

$64y \rightarrow$ div. by 8

$y = 0$ (y can be both 0 and 8)

$x = 2$

If $y = 8$

$x = 3$

$x + y = 2 + 0 = 2$

$x + y = 11$

CT1 Topics
(1) Divisibility
(2) Unit digit
(3) factors

Unit Digit

Find the last digit of 2^{35}

$2^1 = 2$

$2^2 = 4$

$2^3 = 8$

$2^4 = 16$

$2^5 = 32$

group	I	II			III
	0, 1, 5, 6	Pow	even	odd	2, 3, 7, 8 power is 4 multi-ple
		4	6	4	
		9	1	9	

Scanned by CamScanner

Q. Find the unit digit of the following:—

$\rightarrow 2^{5\times4+1} \rightarrow$ multiple of 4

(1) $2^4 \times 6^8 \times 9^6 \times 5^6 \times 7^7 = 0$

(2) $2^{21} + 6^4 + 8^9 + 9^5 + 5^{12} + 3^{21}$

$2 + 6 + 8 + 9 + 5 + 3 = 33$

(3) $(22)^{42} \times (68)^{21} \times (49)^{21} \times (63)^{20}$

$42 \rightarrow 2^{2+40}$

$(2)^{42} \times (8)^{21} \times (9)^{21} \times (3)^{20}$

$= 4 \times 8 \times 9 \times 1 = 288$

(4) $35 \times 89 \times 51 \times 32 = 0$

(5) $1^1 + 2^2 + 3^3 + 4^4 + 5^5 + 6^6 + 7^7 + 8^8 + 9^9 + 10^{10}$

$1 \quad 4 \quad 7 \quad 6 \quad 5 \quad 6 \quad 3 \quad 6 \quad 9 \quad 0$

$= 47$

(6) $1^2 + 2^2 + 3^2 + --- 10^2$

(7) $(1!)^1 + (2!)^2 + (3!)^2 + (4!)^4 + (5!)^5 ——— (10!)^2$

$(1)^1 + (2)^2 + (6)^3 + (24)^4 + (120)^5 + ----$

$= 1 + 4 + 6 + 6 \Rightarrow 17 = 7$

(8) $222^{888} + 888^{222}$

$(2)^{888} + (8)^{222} = 2^4 + 8^2 = 6 + 4 = 10 = 0$

(9) $32^{32^{32}} \rightarrow (2)^{32^{32}} \rightarrow 2^{32\times32\times--32 \text{ times}}$

$= 2^4 = 16 = 16$

Scanned by CamScanner

Prime numbers

(2, 3, 5, 7, 11, 13), 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67.

211 is prime? check divisibility by these nos.

$211 < 225$

$(15)^2$ (perfect square)

Q. Which is prime?

(a) 161/7 → nearest square = 169 (13)

(b) 221/13 → " " = 225 (15)

(c) 373

(d) 437/19 → " " → 441 (21)

$$\begin{array}{r} 26 \\ \times 26 \\ \hline 156 \\ 52\times \\ \hline 676 \end{array}$$

3 | 1

Q. Which among 667, 559, 901 is prime.

$(667) < (26)^2$

$\frac{559}{13}$, $\frac{901}{17}$

$$\begin{array}{r} 19 \\ \times 19 \\ \hline 171 \\ 19\times \\ \hline 361 \end{array}$$

Q. The sum of three prime no. is 100. If one of them exceeds other by 36, then one of the nos. is →

$A + B + C = 100$ — ①

$B = C + 36$ — ②

$A + C + 36 + C = 100$

$A + 2C + 36 = 100$

$A + 2C = 64$

$A + C = 100 - B$

$- \quad - \quad -$

$C = 64 - 100 + B$

$\boxed{A = 31 \quad C = 2 \quad B = 67}$

prime factors of 1936

(1) $1936 = 2^4 \times 11^2$

2	1936
2	968
2	484
2	242
11	121
	11

(2) 1726

2	1726
	$\bar{8}6\bar{3}$

14/1/20

Factors

$N = 12$

no. of factors $= \{1, 2, 3, 4, 6, 12\}$

$N = 12 = 2^2 \times 3$

$2^0 \times 3^0 \quad 2^1 \times 3^0 \quad 2^2 \times 3^0$

$2^0 \times 3^1 \quad 2^1 \times 3^1 \quad 2^2 \times 3^1$

$N = a^p \times b^q \times c^r \times \ldots$

No. of factors $= (p+1)(q+1)(r+1) \ldots$

sum of all the factors $= \left(\frac{a^{p+1}-1}{a-1}\right)\left(\frac{b^{q+1}-1}{b-1}\right)\left(\frac{c^{r+1}-1}{c-1}\right)$

Q1. 288

$$\begin{array}{r|l} 2 & 288 \\ \hline 2 & 144 \\ \hline 2 & 72 \\ \hline 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

16×2

$N = 288 = 2^5 \times 3^1$

$a = 2 \quad b = 3$

$p = 5 \quad q = 1$

No. of factors $= (5+1)(1+2) = 6 \times 3 = 18$

sum $= \frac{(2^6-1)}{2-1} \times \frac{(3^2-1)}{3-1} = \frac{63}{1} \times \frac{8}{2}$

$= \frac{504}{2} = 252$

(b) 1094

$= 2^1 \times 547^1$

$p=1 \quad q=1$

No. of ~~p~~ factors $= (1+1)(1+1)$

$= 4$

Sum $= 1644$

<u>Even, Odd factors</u>

$N = 12 = 2^2 \times 3^1$

Odd $= 1+1 = 2$

Even $= 2+1 = 3$.

Q Find no. of even & odd factors of

~~2 | 196~~ 196

~~2×7×7×2~~

$2^2 \times 7^2$

Odd factors $= 2+1 = 3$.

Even $2(2+1) = 6$

Q. 2088

$\longrightarrow 2^3 \times 3^2 \times 29^1$ $(4 \times 3 \times 2)$

Total no. of factors $= 24$

$(4 \times 3 \times 2)$

Even $3 \times 3 \times 2 = 18$

Odd $3 \times 2 = 6$ $(24-18)$

Find the highest power of prime p in $n!$

→ Highest power of 2 in 5!?

$\Rightarrow 5! = 5\times4\times3\times2$

$= 5\times3\times2^3$

Formula → $\left[\frac{n}{p}\right]+\left[\frac{n}{p^2}\right]+\left[\frac{n}{p^3}\right]+\cdots$

Q. Highest power of 2 in 100!

$n=100$

$p=2$

$= \left[\frac{100}{2}\right]+\left[\frac{100}{4}\right]+\left[\frac{100}{8}\right]+\cdots\left[\frac{100}{128}\right]$

$= 50+25+12+6+3+1 = 97$

$\Rightarrow 2^{97}$ contained in 100!

Q. 3 in 100!

$= \frac{100}{3}+\frac{100}{9}+\frac{100}{27}+\frac{100}{81}$

$= 33+11+3+1 = 48$ //

3^{48} contained in 100!

Q. 6 in 100!

6^{48} contained in 100!

$5! = 5\times4\times(3\times2)\times1 \quad (6^1)$

$5! = 5\times4\times(3)\times2\times1 \quad (2^1)$

(Not prime)

Q. 24 in 100! → Minimum power.

$2^3 \times 3^1$ in 100!

$2^{97} \times 3^{48}$

$(2^3)^{32} \cdot 2 \times 3^{48}$

$\cancel{81}$

Q. Find the highest power of 7 in 126!

$= \frac{126}{7} + \frac{126}{49} = 18 + 2 = 20.$

27×3

Q. 12 in 200!

$12 = 2^2 \times 3$

$= \frac{200}{3} + \frac{200}{9} + \frac{200}{27} + \frac{200}{81}$

$= 66 + 22 + \cancel{6+2} + 7 + 2$

$= 88 + 9 = 97$

$9 \overline{)200}$ 22, -18, 20

Q 42 in 122!

2	42
3	21
7	7
	1

$\Rightarrow 2 \times 3 \times 7$ (Greatest no. with the lowest power)

$= \frac{122}{7} + \frac{122}{49}$

$= 17 + 2 = 19$

$49 \times 7 = 343$

$7 \overline{)122}$ = 17, -7, 52

Q. 175 in 344!

5	175
5	35
7	7
	1

$= 5^2 \times 7$

$= \frac{344}{7} + \frac{344}{49} + \frac{344}{343}$

$= 49 + 7 + 1$

$= 49 + 8 = 57$

$\frac{344}{5}$ ....

83

$\frac{83}{2}$

$= \boxed{41}$

$5 \overline{)175}$ = 35, -15, 25

$49 \times 7 = 343$

$7 \overline{)344}$ = 49, -28, 64

$49 \times 7 = 343$

Q. Find maxm value of n such that 157! is perfectly divisible by 12^n.

$2^2 \times 3$

$\frac{157}{3} + \frac{157}{9} + \frac{157}{27} + \frac{157}{81} = 52 + 17 + 5 + 1$

$= 75$

$\frac{157}{2} + \frac{157}{4} + \frac{157}{8} + \frac{157}{16} + \frac{157}{32} + \frac{157}{64} + \frac{157}{128}$

$= 151$

$2^2 \times 3$

Highest power of 2 in 200!

$= \left[\frac{200}{2}\right] + \left[\frac{200}{2^2}\right] + \dots = 197$

" " " of 2^2 in in 200!

$= \frac{197}{2}$

" " " of 3 in 200!

$= 97$

2) 146! is divisible by 5^n, then maximum value of n

↳ equivalent of highest no. of 5 in 146!

3) Max no. of zeros in end of

$222^{555} \times 555^{222}$

Ans = 222 (smallest exponent)

ex:

$2^1 \times 5^1 = 10$,

$2^2 \times 5^1 = 20$

$2^2 \times 5^3 = 500$

How many zeroes end with $10! \Rightarrow 2$

↳ (Same as highest order of 5)

end 79!

↳ $\frac{79}{5} + \frac{79}{25} + \cancel{\frac{79}{125}}$

$\Rightarrow$ 18

end with $27! \times 129!$

$\left[\frac{27}{5}\right] + \left[\frac{27}{25}\right]$ $\qquad$ $\frac{129}{5} + \frac{129}{25} + \frac{129}{125}$

$5 + 1$ $\qquad$ $25 + 5 + 1$

$6 \quad + \quad 31$

$= 37$

(1) ~~Find~~ For 7200, find

(a) The sum and no. of all factors

72×100

$2^3 \times 3^2 5^2 \times 2^2$

$\boxed{2^5 \times 3^2 \times 5^2}$

$\cancel{2^6}$

$6 \times 3 \times 3 = 54$

2	72
2	36
2	18
3	9
3	3
	1

(b) $5 \times 3 \times 3 = 45$ (Even factors)

(c) 9 (odd factors)

sum of factors $= \left(\frac{2^6 - 1}{2-1}\right) * \left(\frac{3^3-1}{3-1}\right) * \left(\frac{5^3-1}{5-1}\right)$

$= \cancel{16926} = 25389$

sum of odd factors = 403

sum of even factors = Total − Odd = 24986

2) Find number of zeros.

1) $13 \times 15 \times 22 \times 125 \times 44 \times 35 \times 11$

$\Rightarrow 5^5 \times 2^3$ $\boxed{Ans = 3}$

2) $144! \times (5 \times 15 \times 22 \times 11 \times 44 \times 135)$

$\frac{144}{5} + \frac{144}{25} + \frac{144}{125}$ Ans = 3

$28 + 3 + 1$

$32 + 3$

35

n! has 23 zeroes what is max possible value of n?

$\frac{n}{5} + \frac{n}{25} + \frac{n}{125} = 23$

$\frac{25n + 5n + n}{125}$ $31n = 125 \times 23$

42.7/93

21 → 2

22

29

92.74

no number of 5s till 94, so

94

$\frac{26n}{25} = 31$

$\frac{n}{5} + \frac{n}{25} = \frac{25n + n}{}$

6n

4) Find

1^2

5^5

4) Find the number of zeroes in Products of

$1^2 \times 2^2 \times 3^3 \times 4^4 \text{ ------ } 49^{49}$.

Find all 5_5

$5^5 \times 10^{10} \times 15^{15} \times 20^{20} \times 25^{25} \times 30^{30} \times 35^{35} \times 40^{40} \times 45^{45}$.

$5 + 10 + 15 \times 20 + 25 + 30 + 35 + 40 + \text{------}$

$= 5$