

# LOGARITHMS

If  $x$ ,  $a$  and  $m$  are any three numbers connected by the relation:  
 $m = a^x$  ( $a > 0$ ,  $a \neq 1$ ), then,

“ $x$ ” is defined as the logarithm of “ $m$ ” to the base “ $a$ ” and is written as:

$$\log_a m = x$$

Logarithm means power of base  $m = a^x$

**Important properties:**

$$\log_a a = 1$$

$$\log_a (m^n) = n \cdot \log_a m$$

$$\log_a 1 = 0$$

$$\log_a (m \times n) = \log_a m + \log_a n$$

$$\log_a (m/n) = \log_a m - \log_a n$$

$$x = \log_a (a^x)$$

$$\log_{a^b} m^x = \frac{x}{b} \log_n m.$$

$$\log_b a \times \log_c b = (\log_c a) \dots \text{Chain rule}$$

$$\log_a m = (\log_b m) / (\log_b a) \dots \text{Change of base theorem}$$

$$\log_a m = 1 / (\log_m a)$$

$$\log_a b * \log_b a = 1$$

1.The value of  $\log_{343} 7$

Solution:

$$\log_{7^3} 7^1 = 1/3 \quad \log_7 7 = 1/3.$$

2. Find  $\log_5 \frac{1}{125}$

Solution:

$$= \log_5 5^{-3}$$

$$= -3 \log_5 5$$

$$= -3$$

3. Find the value of  $\text{Log}\sqrt{8}/\log 8$

Solution:

$$\log\sqrt{8} / \log 8$$

$$\log 8^{1/2} / \log 8$$

$$= 1/2 \log 8 / \log 8$$

$$= 1/2 .$$

We used the formula,  $\log a^b = b \log a$

4. FIND THE VALUE OF X

$$\text{Log}_{10} 20X = 4$$

SOLUTION:

$$10^4 = 20X$$

$$X = \frac{10^4}{20} = 500$$

## 5. FIND THE VALUE OF X

$$\log(x+3)+\log(x-3)=\log 72$$

$$\log[(x+3)(x-3)]=\log 72.$$

apply the exponential function on both sides of the equation :

$$(x+3)(x-3)=72$$

$$x^2-9=72$$

$$x^2=81,$$

$$X=+9,-9$$

-9 NOT APPLICABLE SO +9



Find the value of

$$= \frac{1}{\log_{xy}(xyz)} + \frac{1}{\log_{yz}(xyz)} + \frac{1}{\log_{zx}(xyz)}$$

$$= \log_{xyz}(xy) + \log_{xyz}(yz) + \log_{xyz}(zx)$$

$$= \log_{xyz}(x^2 y^2 z^2)$$

$$= 2$$

7. FIND THE VAULE OF X :

$$\log_{27} 8 \cdot \log_x 3 = 1$$

SOLUTION:

$$\log_3^3 2^3 \cdot \log_x 3 = 1$$

$$= \frac{3}{3} \log_3 2 \cdot \log_x 3 = 1$$

$$= \log_3 2 \cdot \log_x 3 = 1$$

$$\text{hint}(\log_a b * \log_b a = 1)$$

$$X = 2$$

8. FIND THE VALUE OF

$$\frac{1}{2}\log(11+4\sqrt{7})=\log(2+x)$$

$$\log(11+4\sqrt{7})=\log(2+x)^2$$

$$11+4\sqrt{7}=(2+x)^2$$

$$11+4\sqrt{7}=4+4x+x^2$$

$$7+4\sqrt{7}=x^2+4x$$

Comparing both the side,

$$x=\sqrt{7}.$$

Find the value of

$$= \log_a a + \log_{a^{\frac{1}{2}}} a + \log_{a^{\frac{1}{3}}} a + \dots + \log_{a^{\frac{1}{20}}} a, a > 1$$

$$= 1 + \frac{1}{\frac{1}{2}} + \frac{1}{\frac{1}{3}} + \dots + \frac{1}{\frac{1}{20}}$$

$$= 1 + 2 + 3 + \dots + 20 = \frac{20 \times 21}{2} = 210.$$

## 10. FIND THE VALUE OF

$$\log_2 \log_2 \log_3 \log_3 27^3$$

**Solution**

$$= \log_2 \log_2 \log_3 (3 \log_3 3^3)$$

$$= \log_2 \log_2 \log_3 9$$

$$= \log_2 \log_2 2$$

$$= \log_2 1 = 0$$

11. The value of  $\log_2 3 \times \log_3 2 \times \log_3 4 \times \log_4 3$  is ?

1.1

2.2

3.3

4.4

SOLUTION:

$$\text{hint}(\log_a b * \log_b a = 1)$$

$$= \log_2 3 \times \log_3 2 \times \log_3 4 \times \log_4 3$$

$$= (\log 3 / \log 2) \times (\log 2 / \log 3) \times (\log 4 / \log 3) \times (\log 3 / \log 4) = 1$$

12. If  $\log 2 = 0.3010$ , then the number of digits in  $2^{64}$  is ?

SOLUTION

$$\text{Required answer} = [64 \log_{10} 2]$$

$$= [64 \times 0.3010]$$

$$= 19.264$$

$$= 19 + 1$$

$$= 20$$

**13. Given that  $\log_{10} 2 = 0.3010$ , then  $\log_2 10$  is equal to ?**

**1. 0.3010**

**2. 0.6990**

**3.  $1000 / 301$**

**4.  $699 / 301$**

**SOLUTION**

$$\log_2 10 = \log 10 / \log 2$$

$$= 1 / \log 2$$

$$= 1.0000 / 0.3010$$

$$= 1000 / 301$$



14. The value of  $\log 9/8 - \log 27/32 + \log 3/4$  is ?

SOLUTION:

$$\text{Given Exp.} = \log \left[ \left\{ \frac{9}{8} \right\} / \left( \frac{27}{32} \right) \right] \times \frac{3}{4}$$

$$= \log \left[ \left( \frac{9}{8} \right) \times \left( \frac{3}{4} \right) \times \left( \frac{32}{27} \right) \right]$$

$$= \log 1$$

16. If  $\log_{10} 2 = 0.3010$  and  $\log_{10} 7 = 0.8451$ , then find the value of  $\log_{10} 2.8$  ?

1. 0.4471

2. 1.4471

3. 2.4471

4. 14.471

SOLUTION:

$$\log_{10} 2.8 = \log_{10} (28/10)$$

$$= \log 28 - \log 10$$

$$= \log (7 \times 4) - \log 10$$

$$= \log 7 + 2 \log 2 - \log 10$$

$$= 0.8451 + 2 \times 0.3010 - 1$$

$$= 0.8451 + 0.6020 - 1$$

$$= 0.4471$$

17.If  $a^x = b$ ,  $b^y = c$ ,  $c^z = a$ , then the value of  $xyz$  is ?

SOLUTION

$$\because a^x = b$$

$$\Rightarrow \log_a b = x$$

$$\because b^y = c$$

$$\Rightarrow \log_b c = y$$

$$\because c^z = a$$

$$\Rightarrow \log_c a = z$$

$$\therefore xyz$$

$$= \log_a b \times \log_b c \times \log_c a$$

$$= 1$$

18. If  $\log_x 4 = 0.4$  then the value of  $x$  is ?

SOLUTION:

$$\log_x 4 = \log 4 / \log x = 2/5$$

$$\Rightarrow 2\log 2 / \log x = 2/5$$

$$\Rightarrow \log x = 5\log 2 = \log 2^5$$

$$\Rightarrow \log x = \log 32$$

$$= 32$$

Thank You