COMMON FACTOR

A common factor of two or more numbers is a number which divides each of them exactly.

For example, 4 is a common factor of 8 and 12.

HIGHEST COMMON FACTOR

Highest common factor of two or more numbers is the greatest number that divides each one of them exactly. For example, 6 is the highest common factor of 12, 18 and 24. Highest Common Factor is also called *Greatest Common Divisor* or *Greatest Common Measure*.

Symbolically, these can be written as H.C.F. or G.C.D. or G.C.M., respectively.

METHODS OF FINDING H.C.F.

I. Method of Prime Factors

- Step 1 Express each one of the given numbers as the product of prime factors.

 [A number is said to be a *prime number* if it is exactly divisible by 1 and itself, but not by any other number, e.g., 2, 3, 5, 7, etc. are prime numbers]
- Step 2 Choose common factors.
- Step 3 Find the product of these common factors. This is the required H.C.F. of given numbers.

Illustration 1: Find the H.C.F. of 70 and 90.

Solution:
$$70 = 2 \times 5 \times 7$$

$$90 = 2 \times 5 \times 9$$

Common factors are 2 and 5.

:. H.C.F. =
$$2 \times 5 = 10$$
.

Illustration 2: Find the H.C.F. of 3332, 3724 and 4508.

Solution:
$$3332 = 2 \times 2 \times 7 \times 7 \times 17$$

$$3724 = 2 \times 2 \times 7 \times 7 \times 19$$

$$4508 = 2 \times 2 \times 7 \times 7 \times 23$$

$$\therefore$$
 H.C.F. = 2 × 2 × 7 × 7 = 196.

Illustration 3: Find the H.C.F. of 360 and 132.

Solution:
$$360 = 2^3 \times 3^2 \times 5$$

$$132 = 2^2 \times 3^1 \times 11$$

$$\therefore \text{ H.C.F.} = 2^2 \times 3^1 = 12.$$

Illustration 4: If $x = 2^3 \times 3^5 \times 5^9$ and $y = 2^5 \times 3^7 \times 5^{11}$, find H.C.F. of x and y.

Solution: The factors common to both x and y are 2^3 , 3^5 and 5^9 .

$$\therefore \text{ H.C.F.} = 2^3 \times 3^5 \times 5^9.$$

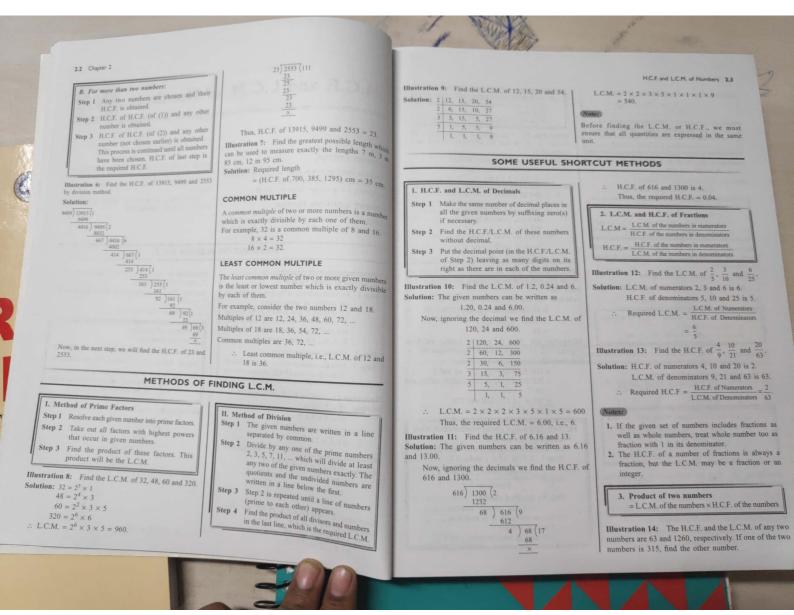
II. Method of Division

:. H.C.F. = 28.

A. For two numbers:

- Step 1 Greater number is divided by the smaller one.
- Step 2 Divisor of (1) is divided by its remainder.
- Step 3 Divisor of (2) is divided by its remainder.
 This is continued until no remainder is left.
 H.C.F. is the divisor of last step.

Illustration 5: Find the H.C.F. of 3556 and 3444.



ution: The required number
$$= \frac{L.C.M.\times H.C.F.}{First Number} = \frac{1260 \times 63}{315} = 252.$$

- 4. To find the greatest number that will exactly divide x, y and z.

 Required number = H.C.F. of x, y and z.
- Illustration 15: Find the greatest number that will exactly divide 200 and 320.
- Solution: The required greatest number = H.C.F. of 200 and 320 = 40.
 - 5. To find the greatest number that will divide x, y and z leaving remainders a, b and c, respectively. Required number = H.C.F. of (x-a), (y-b) and

Illustration 16: Find the greatest number that will divide 148, 246 and 623 leaving remainders 4, 6 and

Solution: The required greatest number

- = H.C.F. of (148 4), (246 6) and (623 11), i.e., H.C.F. of 144, 240 and 612 = 12.
- divisible by x, y and z Required number = L.C.M. of x, y and z.

Illustration 17: What is the smallest number which is exactly divisible by 36, 45, 63 and 80?

Solution: The required smallest number

- = L.C.M. of 36, 45, 63 and 80
- To find the least number which when divided by x, y and z leaves the remainders a, b and crespectively. It is always observed that (x - a)
- = (y b) = (z c) = k(say) \therefore Required number = (L.C.M. of x, y and z) k.

Illustration 18: Find the least number which when divided by 36, 48 and 64 leaves the remainders 25, 37

Solution: Since, (36 - 25) = (48 - 37) = (64 - 53) = 11, therefore, the required smallest number

- = (L.C.M. of 36, 48 and 64) 11 = 576 - 11 = 565.

To find the least number which when divided by x, y and z leaves the same remainder r in each case. each case. Required number = (L.C.M. of x, y and z) +

Mustration 19: Find the least number which when divided by 12, 16 and 18, will leave in each case

Solution: The required smallest number

- = (L.C.M. of 12, 16 and 18) + 5 = 144 + 5 = 149.
- 9. To find the greatest number that will divide x, y
 - To find the greatest and z and z leaving the same remainder in each case. (a) When the value of remainder r is given: Required number = H.C.F. of (x r), (y r)
 - (b) When the value of remainder is not given: Required number = H.C.F. of |(x - y)|, |(y - y)||z| and |(z-x)|

Illustration 20: Find the greatest number which will divide 772 and 2778 so as to leave the remainder 5 in

Solution: The required greatest number

- = H.C.F. of (772 5) and (2778 5) = H.C.F. of 767 and 2773
- = 59

Illustration 21: Find the greatest number which on dividing 152, 277 and 427 leaves equal remainder.

- Solution: The required greatest number
 - = H.C.F. of |(x-y)|, |(y-z)| and |(z-x)|= H.C.F. of |(152-277)|, |(277-427)| and |(427-152)|
 - = H.C.F. of 125, 150 and 275

 - 10. To find the *n*-digit greatest number which, when divided by x, y and z,

(a) leaves no remainder (i.e., exactly divisible) Step 1 L.C.M. of x, y and z = L

- Step 2 L)n-digit greatest number (Remainder = R
- Step 3 Required number
- = n-digit greatest number R(b) leaves remainder K in each case Required number
 - Required number = (n-digit greatest number R) + K

Illustration 22: Find the greatest number of 4-digit number which, when divided by 12, 18, 21 and 28 leaves 3 as a remainder in each case.

Solution: L.C.M. of 12, 18, 21 and 28 = 252.

:. The required number = (9999 - 171) + 3 = 9931.

Illustration 23: Find the greatest number of four digits which, when divided by 12, 15, 20 and 35 leaves no remainder.

Solution: L.C.M. of 12, 15, 20 and 35 = 420.

$$420)9999(23)$$

$$9660$$

$$\overline{339}$$

- The required number = 9999 339
- 11. To find the n-digit smallest number which when divided by x, y and z(a) leaves no remainder (i.e., exactly divisible)
 - Step 1 L.C.M. of x, y and z = L

Step 2 L n-digit smallest number (

Remainder = R

H.C.F. and L.C.M. of Numbers 2.5

Step 3 Required number

- = n-digit smallest number + (L R).
- (b) leaves remainder K in each case.
 Required number
 = n-digit smallest number + (L R) + K.

Illustration 24: Find the least number of four digits which is divisible by 4, 6, 8 and 10. Solution: L.C.M. of 4, 6, 8 and 10 = 120.

> 120 1000 (8 40

 \therefore The required number = 1000 + (120 - 40)

= 1080. Illustration 25: Find the smallest 4-digit number, such that when divided by 12, 18, 21 and 28, it leaves

remainder 3 in each case. Solution: L.C.M. of 12, 18, 21 and 28 = 252.

252)1000 (3

- :. The required number
 - = 1000 + (252 244) + 3
 - = 1011.

EXERCISES

- 1. What is the H.C.F. of 27, 18 and 36?
 - (a) 7 (c) 9
- (b) 11
- (d) None of these
- 2. Determine the L.C.M of $\frac{2}{5}$, $\frac{3}{10}$ and $\frac{6}{25}$
 - (a) $\frac{6}{5}$

- (d) None of these
- 3. What is the L.C.M. of 25, 30, 35 and 40?
 - (a) 3800
 - (b) 4200
 - (c) 4400
 - (d) None of these

- 4. What is the greatest number which divides 852, 1065 and 1491 exactly? (b) 183
 - (a) 193
- 5. What is the H.C.F. of $\frac{4}{9}$, $\frac{10}{21}$ and $\frac{20}{30}$?
 - (a) 189
- (b) $\frac{6}{23}$
- (d) None of these
- 6. Find the least number which when divided by 16, 18, 20 and 25 leaves 4 as remainder in each case but when divided by 7 leaves no remainder.
 - (a) 8004
- (b) 13004
- (d) 18014 (c) 18004

21. A wholesale tea dealer has 408 kilograms, 468 kilograms and 516 kilograms of three different 20. Find the greatest number which will divide 3962, 4085 and 4167 leaving the same remainder in each 19. The number of prime factors in the expression 18. Find the greatest number of four digits which when divided by 10, 15, 21 and 28 leaves 4, 9, 15 and 22 The traffic lights at three different road crossings change after every 48 seconds, 72 seconds and The product of two numbers is 6760 and their H.C.F. is 13. How many such pairs can be formed? simultaneously at 8:20:00 hours; then they will again 108 seconds, respectively. If they all change (b) 39 (d) 43 (b) 64 (d) 9864 (b) 9666 (d) only one (d) 81

qualities of tea. He wants it all to be packed into boxes of equal size without mixing. Find the capacity

(c) 10001

(a) 33 (c) 132 (b) 66

32. The largest natural number which exactly divides the product of any four consecutive natural numbers, is:

(c) 24 (a) 6 (b) 12 (d) 120

33. Find the least number of six digits which is exactly divisible by 15, 21 and 28:

(d) 34

(c) 10080

(d) None of these

(c) 100380

(d) 100340

(c) 3371

What is the greatest possible length of each plank? Three pieces of timber 42 m, 49 m and 63 m long have to be divided into planks of the same length. (d) 63 m (b) 14 m

25.

26. Three men start together to travel the same way around a circular track of 11 kilometres in circumference

(a) 11 hours Their speeds are 4, $5\frac{1}{2}$ and 8 Km/h, respectively. When will they meet at the starting point? (b) 12 hours

27. Five bells begin to toll together and toll at intervals of time will they keep on tolling together? of 36, 45, 72, 81 and 108 seconds. After what interval (c) 23 hours (d) 22 hours

(c) 814

(d) 902

measure must be there to measure all the different are 403 Kg, 434 Kg and 465 Kg. What biggest of mixture of milk and water, whose measurements (c) 3140 seconds (a) 3240 seconds (b) 3080 seconds (d) 3200 seconds

28. Three different containers contain different quantities quantities exactly?

(a) 1 Kg (c) 31 Kg (b) 7 Kg (d) 41 Kg

29. The L.C.M. and G.C.D. of two numbers are 1530 possible? and 51, respectively. Find how many such pairs are

(a) 2 (c) 4 (d) Only one (b) 3

30. Find the least number of five digits which when case. divided by 63, 56 and 42 leaves remainder 1 in each

(c) 2356

(a) 10082

31. The H.C.F. and L.C.M. of two numbers are 44 and 264, respectively. If the first number is divided by 2, the quotient is 44. The other number is: (b) 10081 (d) 10071

(c) 9037, 9568

(a) 100480 (b) 100270

> 34. Find the greatest number of five digits which when divided by 12, 15, 21, 25 and 28 leaves 5, 8, 14, 18 and 21 as remainders, respectively. (a) 98696 (b) 98700

(a) 480 (b) 475

(c) 97693

(d) 98693

35. What is the smallest number which when increased by 3 is divisible by 16, 24, 30 and 32? (c) 472

36. The least number of square tiles required to cover (a) 656 cm broad, is: the ceiling of a room 15 m 17 cm long and 9 m 2

37. Find the least number which when divided by 2, 3, 4, 5 and 6 leaves 1, 2, 3, 4 and 5 as remainders, respectively, but when divided by 7 leaves no remainder.

(a) 210 (c) 126

38. Find the greatest number of five digits which when 3 in each case. divided by 4, 6, 10 and 15 leaves the same remainder

(a) 99993 (c) 90093 (d) 99963 (b) 99063

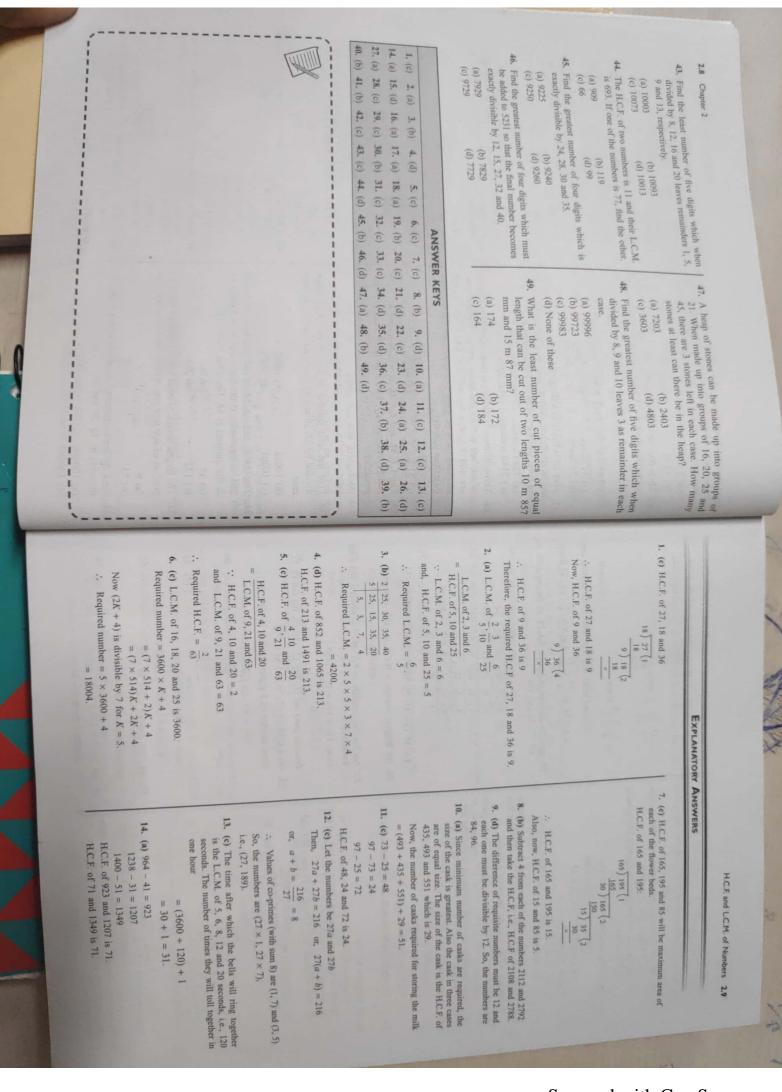
39. Find the least number which is a multiple of 31 and (a) 2418 2, 11 and 19, respectively. when divided by 15, 24 and 32 leaves the remainders (b) 2387

40. Find the two largest numbers of four digits having (a) 9231, 9762 (b) 9027, 9558 531 as their H.C.F.

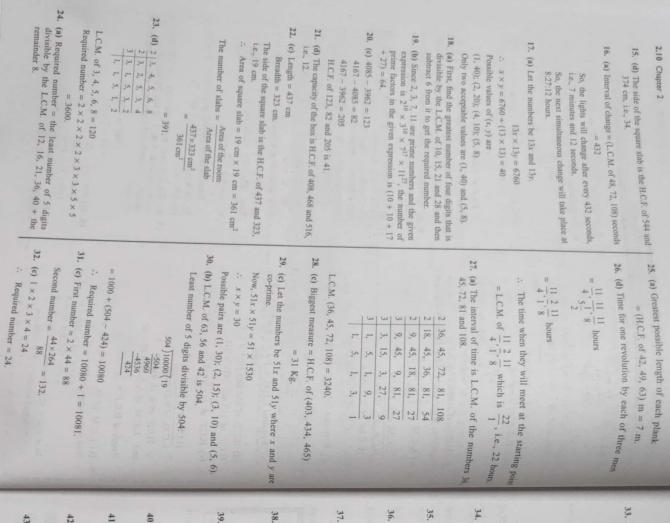
41. Find the greatest number of five digits which becomes exactly divisible by 10, 12, 15 and 18 when 3769 is (d) 9127, 9658 added to it.

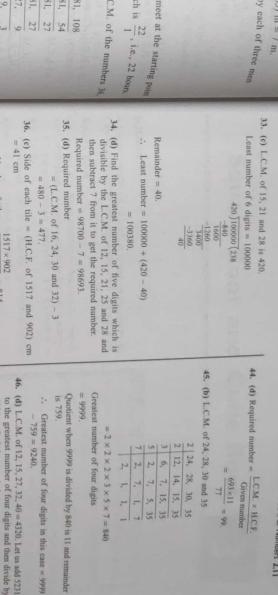
(d) 99011

42. Find the least number which when decreased by 11 is divisible by 14, 15, 21, 32 and 60.



Required number = 10080 - 7 = 10073.





4320 15230 (3

= 3603 which is not divisible by 21 = 7203 which is divisible by 21.

360) 99999 (277

 $=\frac{693\times11}{77}=99.$

Given number

H.C.F. and LC.M. of Numbers 2.11