

Divisibility test:-

2 → unit digits (0, 2, 4, 6, 8)

3 → sum of digits / 3

4 → last two digits → 00 or ^{last two digits.} divisible by 4.

5 → unit digit (0, 5)

6 → no. should verify for (2, 3).

7 →

| | |
|--|---|
| 34 (3) $3 \times 2 = 6$ $34 - 6$ $= 28$ divisible by 7 | 240 (1) $1 \times 2 = 2$ $240 - 2 = 238$ $8 \times 2 = 16$ $23 - 16 = 7$ divisible by 7. |
|--|---|

8 → last two digits 00 or divisible by 8.

9 → sum of digits / 9.

10 → unit 0.

11 → 3124 or ~~no~~

$$3 + 2 = 5$$

$$1 + 4 = 5$$

$5 - 5 = 0$ difference should be 0.
or multiple of 11.

12 → (4, 3) follow.

$$(12) \rightarrow 6 \times 2$$

$$\downarrow$$

$$(4 \times 3)$$

$$\downarrow$$

$$(4, 3)$$

$$99 \rightarrow 9 \times 11$$

$$\downarrow$$

$$(9, 11)$$

13 → A + 4B

14 → divisible by 2 & 7.

15 → divisible by 3 & 5.

16 → last 4 digits div. by 16.

17 → A - 5R

$$\begin{array}{r} 334 \\ 27 \overline{) 918} \\ \underline{54} \\ 378 \\ 36 \overline{) 378} \\ \underline{36} \\ 18 \end{array}$$

$$\begin{array}{r} 334 \\ 20 \overline{) 668} \\ \underline{40} \\ 268 \\ 20 \overline{) 268} \\ \underline{20} \\ 68 \end{array}$$

08/01/2020

Unit Digits

1. $1234 + 45678 + 7896 = 4 + 8 + 6 = 18 \rightarrow 8$ ✓

2. $1234 \times 45678 \times 7896 = 4 \times 8 \times 6 = 192 \rightarrow 2$

3. $(1230)^{123} \rightarrow 0 \rightarrow 0$ $(1231)^{123} \rightarrow 1 \rightarrow 1$ $(1235)^{123} \rightarrow 5 \rightarrow 5$ $(1236)^{123} \rightarrow 6 \rightarrow 6$

| | | |
|---|---|---|
| 0 | → | 0 |
| 1 | → | 1 |
| 5 | → | 5 |
| 6 | → | 6 |

$(1234)^{123}$

$4^1 = 4$
 $4^2 = 16$
 $4^3 = 64$
 $4^4 = 256$
 So, the pattern is like 4, 6, 4, 6, ...

| | |
|------|-----|
| odd | = 4 |
| even | = 6 |

Similarly, for $(1239)^{123}$

| | |
|------|-----|
| odd | = 9 |
| even | = 1 |

$9^1 = 9$
 $9^2 = 81$
 $9^3 = 729$
 $9^4 = 6561$

| | | | |
|------------|------------|--------------|--------------|
| $2^1 = 2$ | $3^1 = 3$ | $7^1 = 7$ | $8^1 = 8$ |
| $2^2 = 4$ | $3^2 = 9$ | $7^2 = 9$ | $8^2 = 4$ |
| $2^3 = 8$ | $3^3 = 27$ | $7^3 = 343$ | $8^3 = 512$ |
| $2^4 = 16$ | $3^4 = 81$ | $7^4 = 2401$ | $8^4 = 4096$ |

2

$$(2)^{75}$$

$$= (2^{72} \times 2^3)$$

$$4 \overline{) 13}$$

$$\underline{8}$$

$$5$$

$$\underline{4}$$

$$1$$

$$2^4$$

$$72/4 = 18$$

$$(2^4)^{18} \times 2$$

$$6 \times 2 = 12 \text{ Ans.}$$

$$(3)^{74}$$

$$4 \overline{) 18}$$

$$\underline{12}$$

$$6$$

$$(3^{72} \times 3^2)$$

$$3^2 \times 9 = 9 \text{ Ans.}$$

$$(7)^{75}$$

$$(7^{72} \times 7^3)$$

$$1 \times 3 = 3 \text{ Ans.}$$

3. >

$$225 \times 478 \times 574$$

$$2 \times 3$$

$$8 \times 3 = 24$$

$$4 \times 4 = 16$$

$$= 16$$

$$= 6 \text{ Ans.}$$

4. >

$$(101)^{101} + (105)^{105} + (106)^{106}$$

$$= 1 + 5 + 6 = 12 \text{ Ans.}$$

$$4 \overline{) 25}$$

$$\underline{20}$$

$$5$$

5. >

$$(101)^{1001} \times (102)^{102} \times (108)^{108}$$

$$1 \times 2^2 \times (8^4)$$

$$1 \times 2^2 \times 6$$

$$4 \times 6 = 24$$

$$= 4 \text{ Ans.}$$

$$4 \overline{) 1021}$$

$$\underline{8}$$

$$22$$

$$\underline{20}$$

$$2$$

$$4 \overline{) 108}$$

$$\underline{8}$$

$$28$$

$$\underline{24}$$

$$4$$

* Divisibility rule :-

Q. Find the least value of * if the no.

12473* is divisible by 9.

$$\frac{1+2+4+7+3+\alpha}{9}$$

$$\frac{17+\alpha}{9}$$

$$\alpha \geq 1$$

should be

Q. What least value in the place of *

56412* ÷ by 33.

$$\begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$\frac{56412}{33}$$

$$\frac{36412}{33}$$

$$\frac{36412}{33}$$

$$\begin{aligned} 3+4+2 &= 9 \\ 6+1+2 &= 9 \end{aligned}$$

$$9-9=0$$

$$15+\alpha$$

$$11 \times 2$$

$$\begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$\frac{8}{56}$$

Perfectly satisfies

Q.

12875* ÷ 8

$$\begin{pmatrix} 6 \\ 0 \end{pmatrix}$$

$$\begin{aligned} 8+4+2 &= 14 \\ 6+1+2 &= 9 \end{aligned}$$

$$\frac{38412}{8}$$

20/01/2020

Factors

$$6 \Rightarrow 1, 2, 3, 6$$

↓
2' x 3'

$$24 \Rightarrow 1, 2, 3, 4, 6, 8, 12, 24$$

$$N = a^x \times b^y \times c^z$$

Total no. of factors.

$$= (x+1)(y+1)(z+1)$$

$$4^2 \times 2^3 \times 3^1 \times 5^1$$

$$120$$

$$2^3 \times 3^1 \times 5^1$$

Total no. of factors = $4 \times 2 \times 2$
 $= 16$

$$2^3 \times 3^2 \times 5^2$$

$\frac{2^3-1}{2-1} \times \frac{3^2-1}{3-1} \times \frac{5^2-1}{5-1}$
 $= \frac{8-1}{1} \times \frac{9-1}{2} \times \frac{25-1}{4}$
 $= 7 \times 4 \times 6$
 $= 168$

Philly
Sara
Anthony

odd x odd = odd
 odd x even = even
 even x even = even
 even x odd = even

$$120$$

12×10
 \downarrow
 $2^2 \times 3 \times 2 \times 5$
 $2^3 \times 3^1 \times 5^1$
 $\frac{2^3-1}{2-1} \times \frac{3^1-1}{3-1} \times \frac{5^1-1}{5-1}$
 $= 7 \times 2 \times 4$
 $= 56$

120
 No. of odd factors.

$$= 2^0 \times 3^0 \times 5^0$$

$$= 1 \times (1+1) \times (1+1)$$

$$= 2 \times 2$$

$$= 4$$

120
 No. of even factors = $2^3 \times 3^1 \times 5^1$

(excl. 20)
 (take 2, 2, 2)

$$= 3 \times (1+1) \times (1+1)$$

$$= 3 \times 2 \times 2$$

$$= 12$$

$$8 \times 320 = 2 \times 5$$

$$7 \times 2 = 14 \quad \text{f.n.f.}$$

$$\text{odd. } 1 \times 2 = 2 \frac{2}{2}$$

$$2 \frac{10}{5}$$

$$\text{even } 6 \times 2 = 12$$

Sum of factors.

$$\text{Sum of factors} = \left(\frac{a^{x+1} - 1}{a - 1} \right) \left(\frac{b^{y+1} - 1}{b - 1} \right) \left(\frac{c^{z+1} - 1}{c - 1} \right)$$

$$\text{Product of factors} = (N)^{\frac{\text{No. of factors}}{2}}$$

(multiply the factors and many 24 we get)

$$24 = (24)^{\frac{1}{2}} = (24)^{\frac{4}{2}} = (24)^2$$

Ans.

Q. In how many different ways can you write the no. as a product of 2 of its factors?

* To find highest power of prime p in $n!$

$$\text{Formula} = \left(\frac{n}{p^1}\right) + \left(\frac{n}{p^2}\right) + \left(\frac{n}{p^3}\right) + \dots$$

Q.1 Highest power of 2 in 5!

$$5! = 5 \times 4 \times 3 \times 2 \times 1 \\ = 2 \times 2 \times 3 \times 2 \times 1 \\ \therefore \text{Ans} = 3$$

Formula method

Q.2 Highest power of 2 in 100!

$$n = 100 \\ p = 2$$

$$\left(\frac{100}{2}\right) + \left(\frac{100}{4}\right) + \left(\frac{100}{8}\right) + \left(\frac{100}{16}\right) + \left(\frac{100}{32}\right) + \left(\frac{100}{64}\right) + \left(\frac{100}{128}\right)$$

$$= 50 + 25 + 12 + 6 + 3 + 1$$

$$= 97 \text{ Ans.} \therefore 2^{97} \text{ in } 100!$$

Q.3 3 in 100!

$$= \frac{100}{3} + \frac{100}{9} + \frac{100}{27} + \frac{100}{81}$$

$$= 33 + 11 + 3 + 1 = 48 \text{ Ans.}$$

Q.4 8 in 100!

$$= \frac{100}{8} + \frac{100}{64} + \frac{100}{512}$$

$$= 12 + 1 + 0 \\ = 13$$

08/02/2020

$$11^2 = 121$$

$$12^2 = 144$$

$$13^2 = 169$$

$$14^2 = 196$$

$$15^2 = 225$$

$$16^2 = 256$$

$$17^2 = 289$$

$$18^2 = 324$$

$$19^2 = 361$$

$$20^2 = 400$$

$$21^2 = 441$$

$$22^2 = 484$$

$$23^2 = 529$$

$$24^2 = 576$$

$$25^2 = 625$$

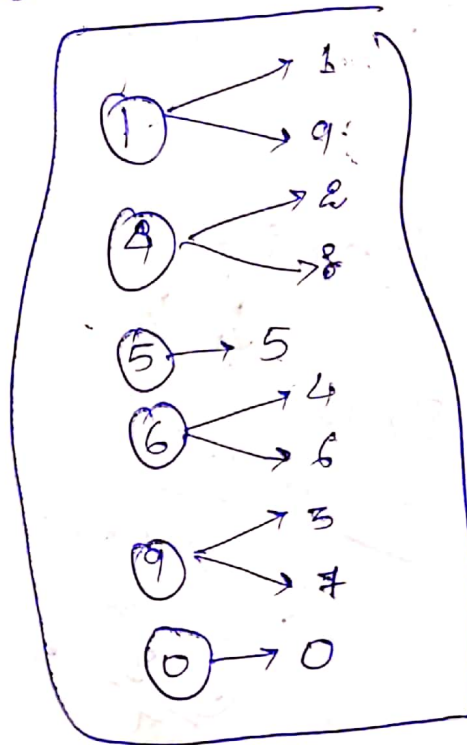
$$26^2 = 676$$

$$27^2 = 729$$

$$28^2 = 784$$

$$29^2 = 841$$

$$30^2 = 900$$



$$(44)^2$$

$$4^2 = 16$$

$$2 \times 4 \times 4 = 32$$

$$7^2 = 49$$

Q. No. $\sqrt{841}$ ignore it.

$\swarrow \searrow$
1 9

$\underline{21}$ or $\underline{29}$

$\therefore 29$ Ans.

$$2 \times 3 = 6$$

10/02/2017

Remainder Theorem

Q.1) Find the remainder when 1801×1803 is divided by 18? $\rightarrow 3$

Q.2) Find the remainder when $1691 \times 1692 \times 1694$ is divided by 13? $\rightarrow 8$

Q.3) Find the remainder when $181 \times 182 \times 183 \times 185$ is divided by 18? $\rightarrow 12$

Q.4) Find the remainder when $1603 \times 1604 \times 1605$ is divided by 16? $\rightarrow 12$

Q.5) Find the remainder when 1398×1399 is divided by 7? $\rightarrow 2$

Q.6) Find the remainder when 3598×3597 is divided by 18? $\rightarrow 12$

Q.7) Find the remainder when 2^{32} is divided by 15? $\rightarrow 1$

Q.8) Find the remainder when 2^{256} is divided by 31? $\rightarrow 2$

Q.9) Find the remainder when 2^{72} is divided by 63? $\rightarrow 63$

Q.10) Find the remainder when $2^{75} \div 63$? $\rightarrow 2$

Q.11) Find the remainder when $2^{68} \div 17$? $\rightarrow 16$

Q.12) Find the remainder when $2^{69} \div 17$? $\rightarrow 1$

Q.13) Find the remainder when $1! + 2! + 3! + 4! + \dots + 1000!$ is divided by 24? $\rightarrow 20$

Q15) Find the remainder when $1! + 2! + 3! + 4! + 5! + \dots + 100!$
 $\div 8$?

$$\frac{100}{9} = 11 \rightarrow \frac{10 \times 10}{9} = 1 \times 1 = 1$$

$$\frac{11 \times 12}{9} = 2 \times 3 = 6$$

$$\frac{12 \times 15}{9} = 3 \times 4 = \frac{12}{9} = 3$$

$$\frac{44 \times 45}{23} = \frac{21 \times 22}{23} = -2 \times -1 = 2$$

$$\frac{43 \times 44 \times 45}{23} = -3 \times -2 \times -1 = -6$$

*** when we get -ve remainder
 add the no. by which we are
 dividing.

$$\therefore -6 + 23 = 17$$

Percentage : ~

① Let the population of town be P now & suppose it increases at the rate of $r\%$ per annum, then

(i) population after n years $= P \left(1 + \frac{r}{100}\right)^n$

(ii) population after n years ago $= \frac{P}{\left(1 + \frac{r}{100}\right)^n}$

② Let the present value of the machine be P and if it depreciates at the rate of $r\%$ per annum, then

(i) value of the machines after n years $= P \left(1 - \frac{r}{100}\right)^n$

(ii) value of the machines n years ago $= \frac{P}{\left(1 - \frac{r}{100}\right)^n}$