1. Consider the function

$$f(x, y) = (1.5 - x + xy)^2 + (2.25 - x + xy^2)^2 + (2.625 - x + xy^3)^2.$$

Write python code to determine its minimum with $-4.5 \leq x, y \leq 4.5$.

2. The data in data_q2.csv are believed to be drawn from a Gaussian distribution. Using python, plot a histogram of these data and superimpose a Gaussian with the appropriate mean and variance.

3. We make measurements of a quantity y which is believed to be related to another variable x. We take N observations which are modeled as

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i$$

where ϵ_i is observation noise for $i = 1, \ldots, N$. β_0 and β_1 are real constants which we need to estimate. We estimate them as

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{N}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

and

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

where $\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$ and $\bar{y} = \frac{1}{N}\sum_{i=1}^{N} y_i$.

The data in data_q3.csv correspond to x_i and y_i for $i = 1, \ldots, N$. Write python code to determine $\hat{\beta}_0$ and $\hat{\beta}_1$ and plot the data as well as the line $y = \hat{\beta}_0 + \hat{\beta}_1 x$.