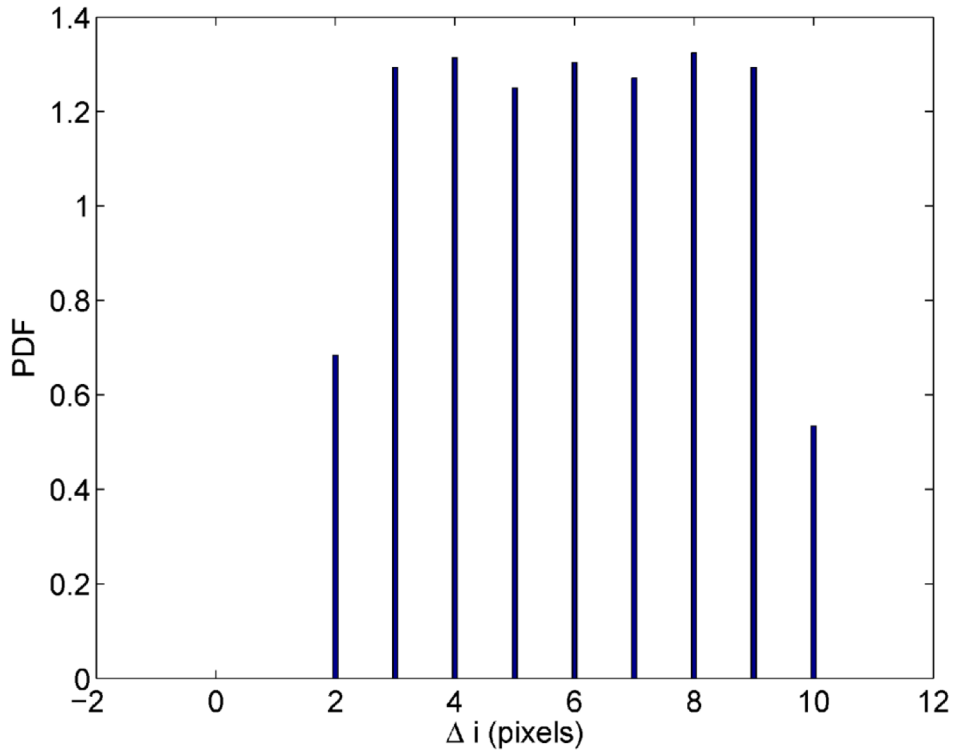
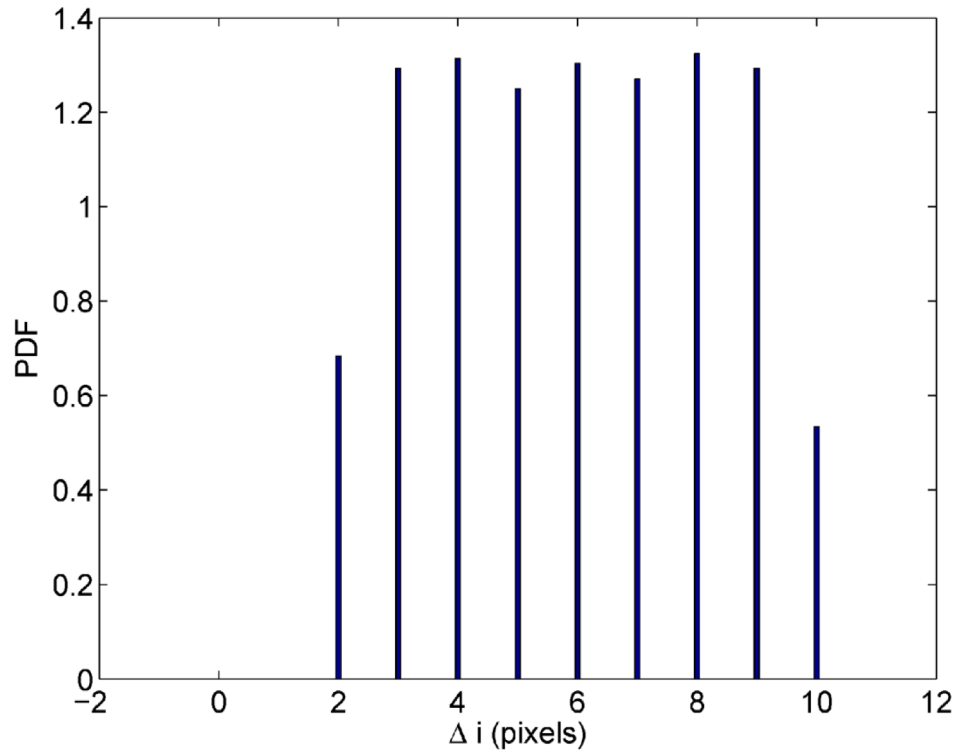
Peak locking

Consider a histogram of the dpx (Δi) or dpy (Δj) values attained from your cross correlation.



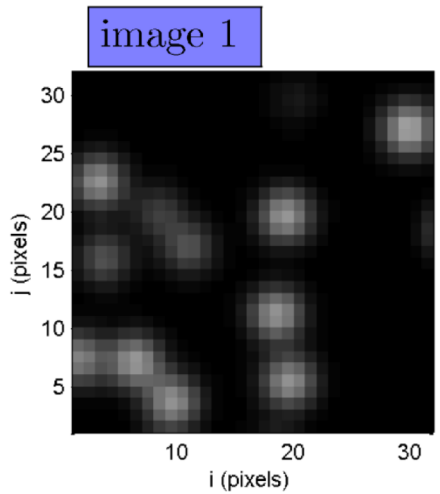
Peak locking

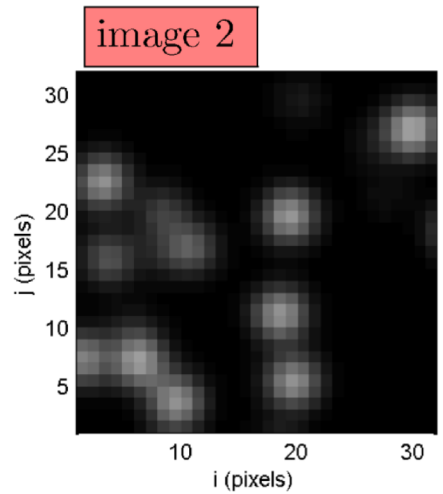
Everything looks good, but if we look at the histogram of i displacements we are 'locked' to integer displacements.



Peak locking

Start-off with an unshifted image 1 and image 2

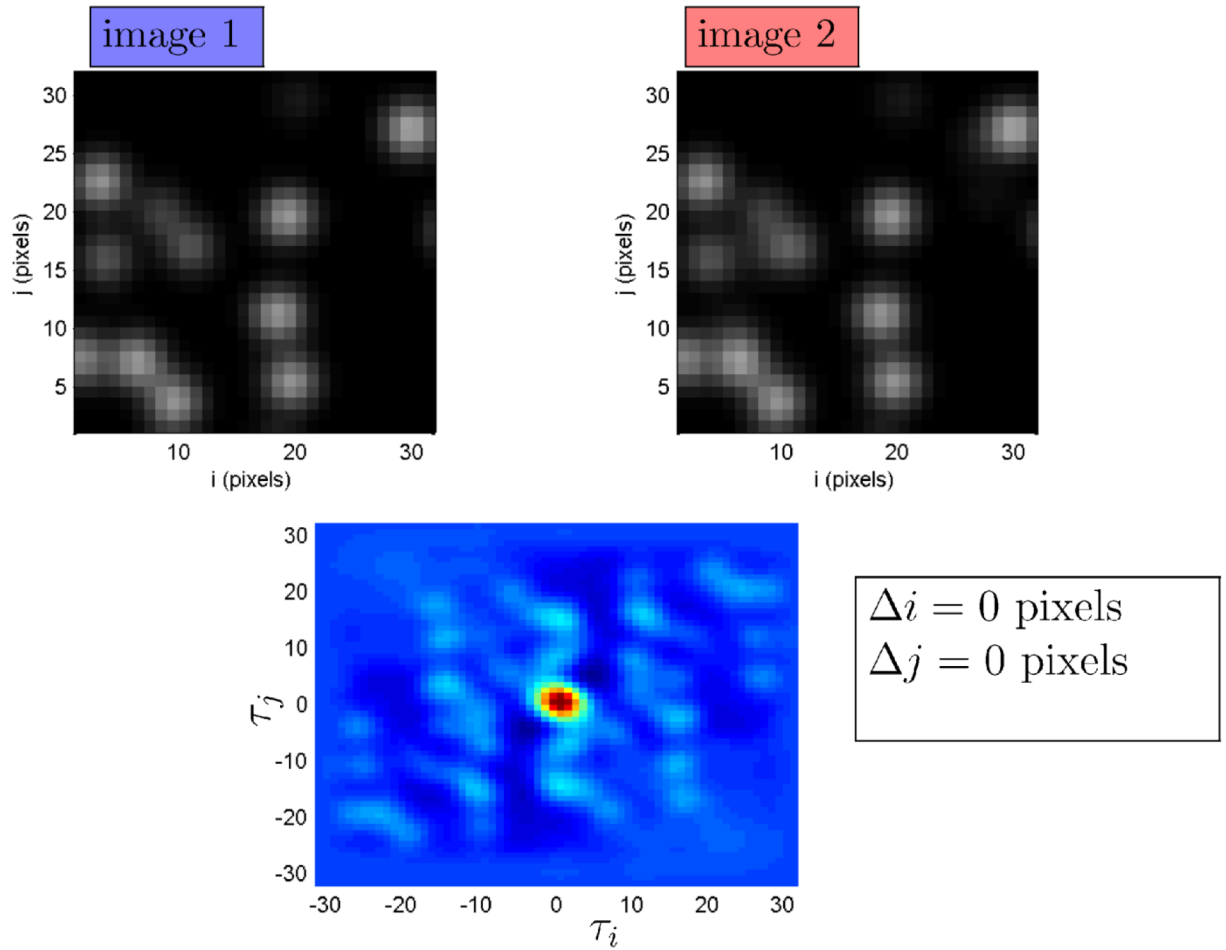




$\Delta i = 0$ pixels
$\Delta j = 0$ pixels

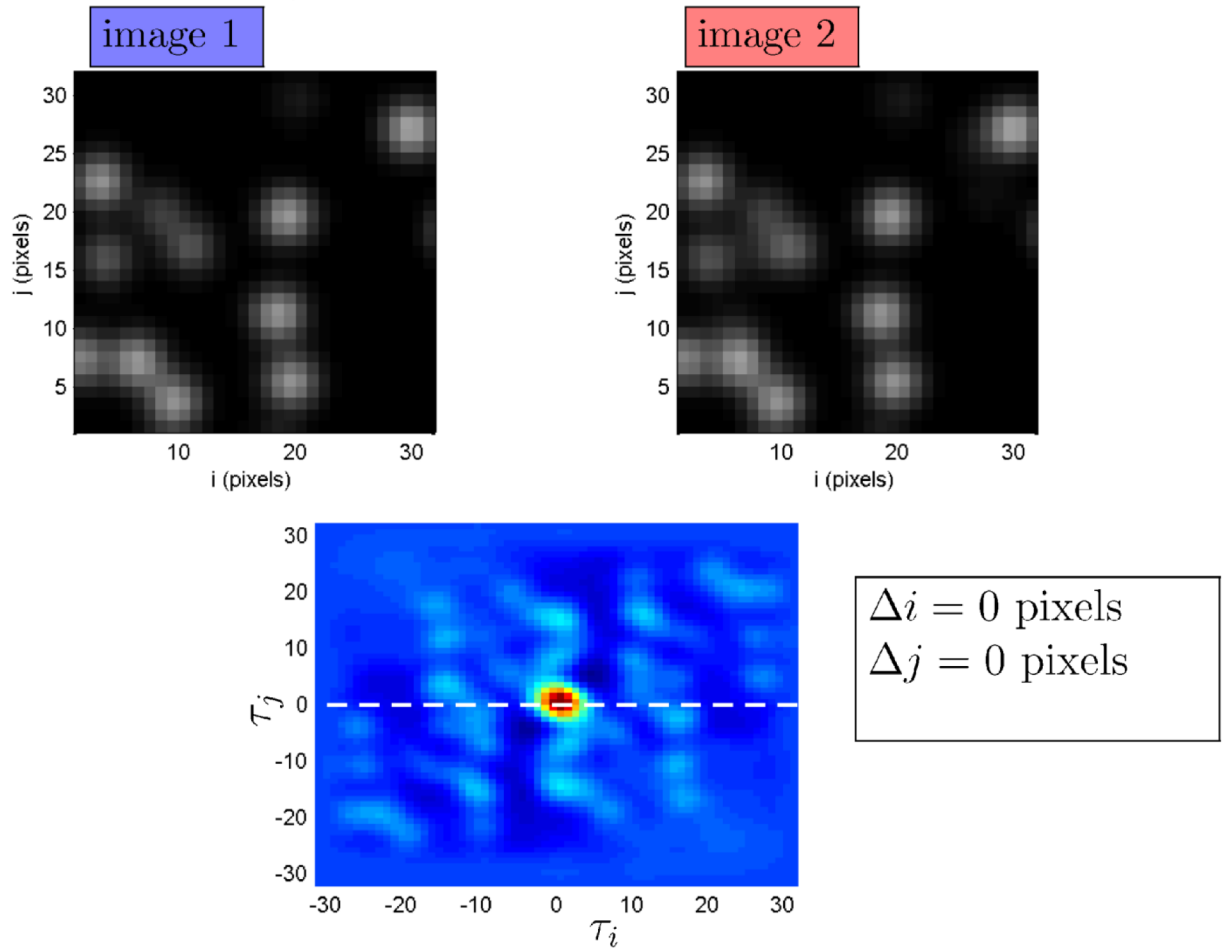
Peak locking

Calculate cross-correlation coefficient (R) and look at a line through R at $\tau_j = 0$



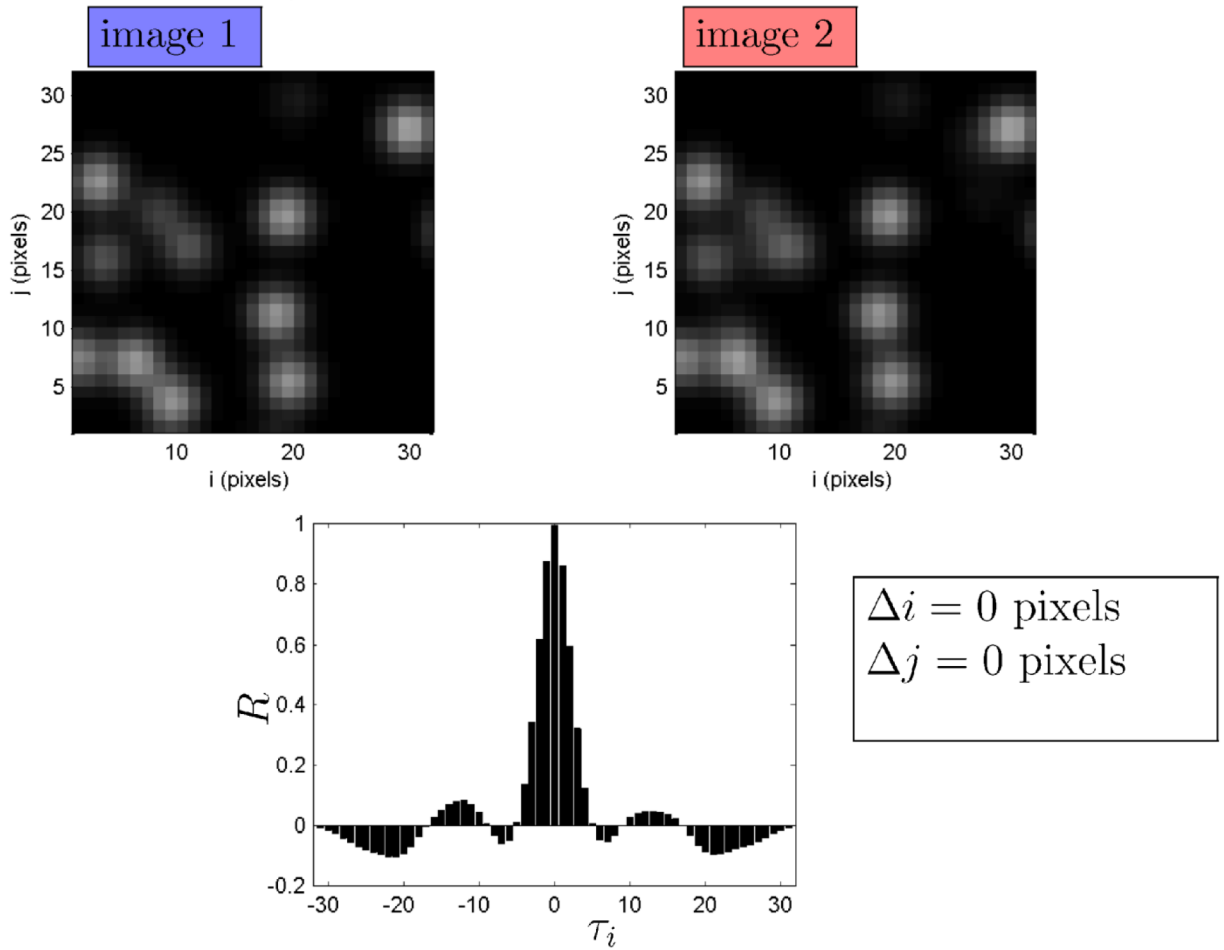
Peak locking

Calculate cross-correlation coefficient (R) and look at a line through R at $\tau_j = 0$



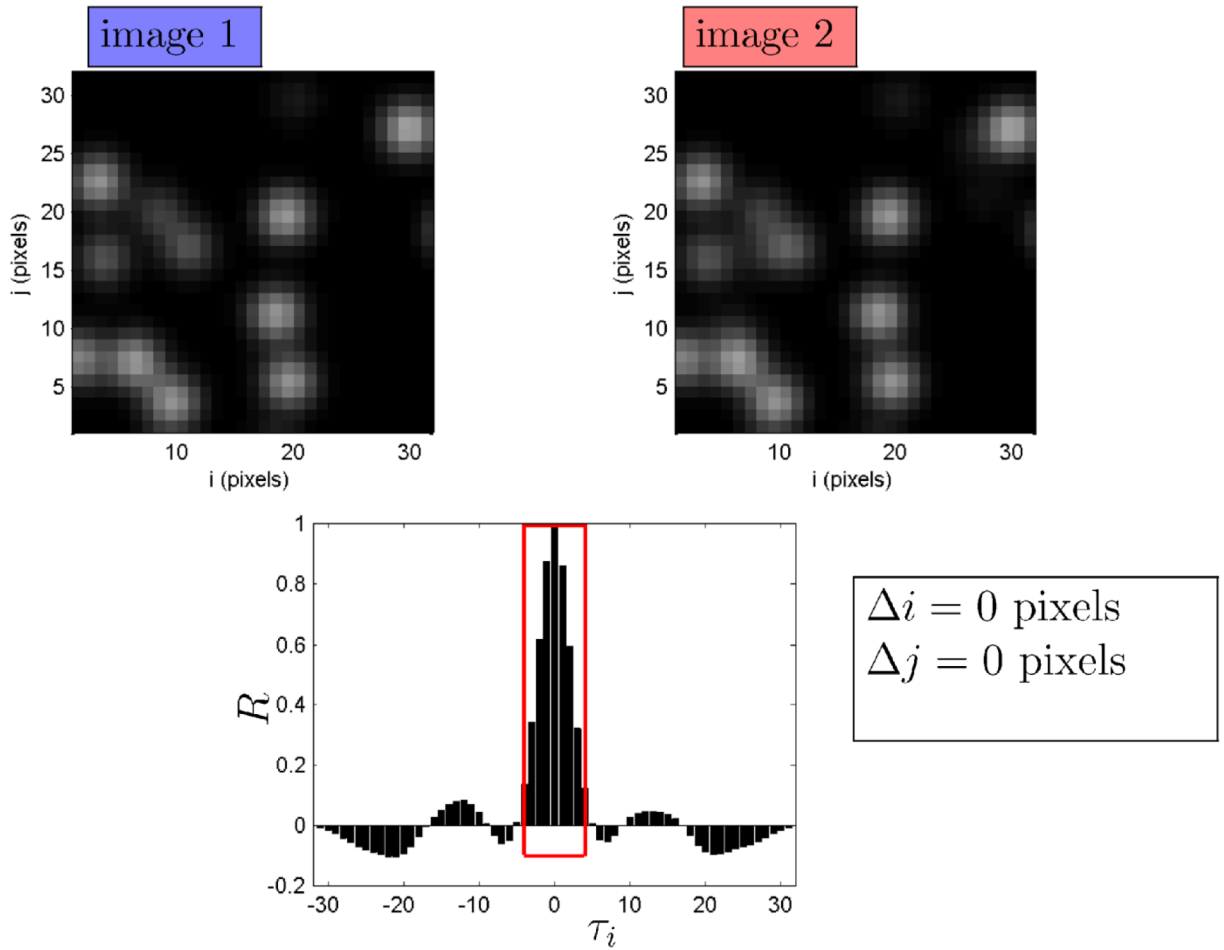
Peak locking

Calculate cross-correlation coefficient (R) and look at a line through R at $\tau_j = 0$



Peak locking

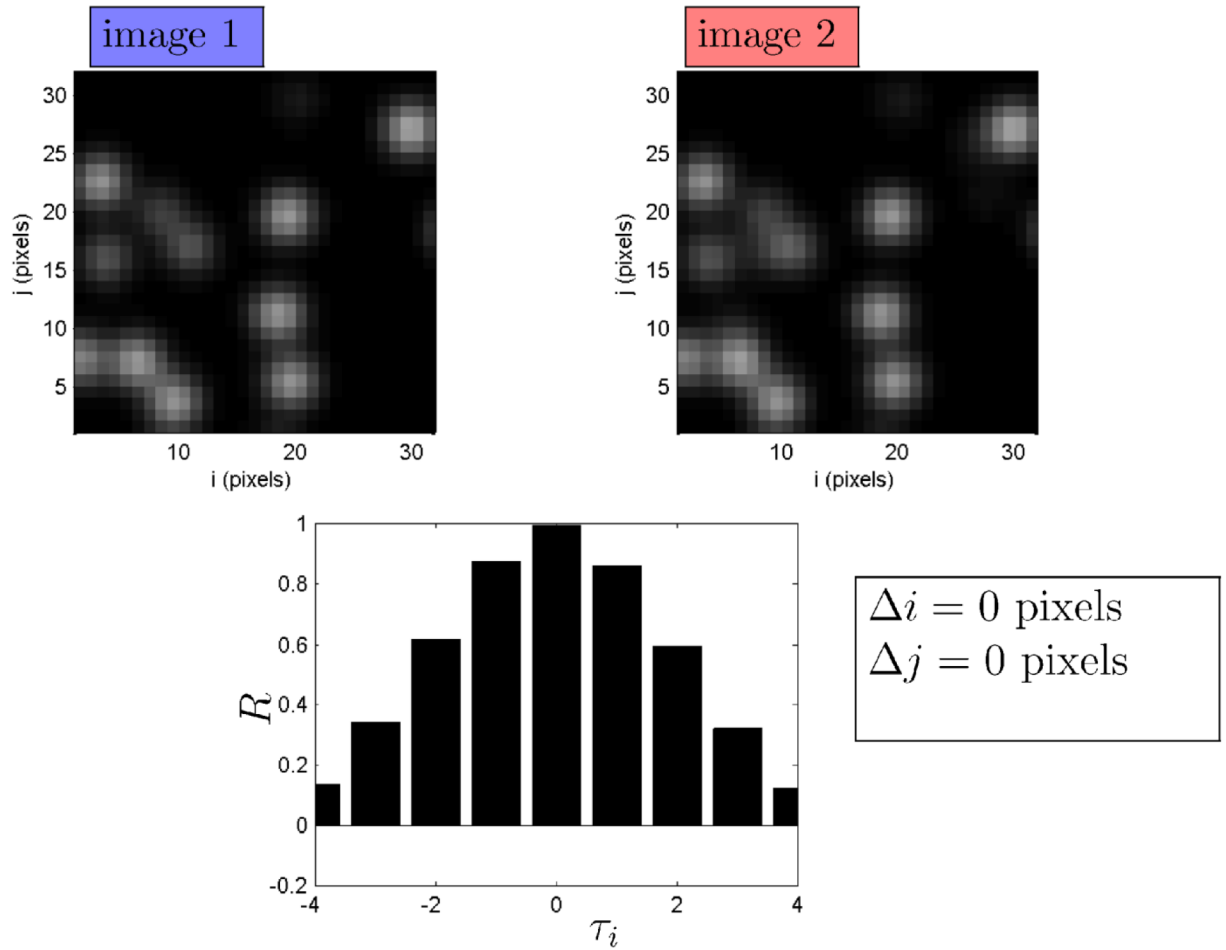
Now zoom on area close to peak in R



$\Delta i = 0$ pixels
$\Delta j = 0$ pixels

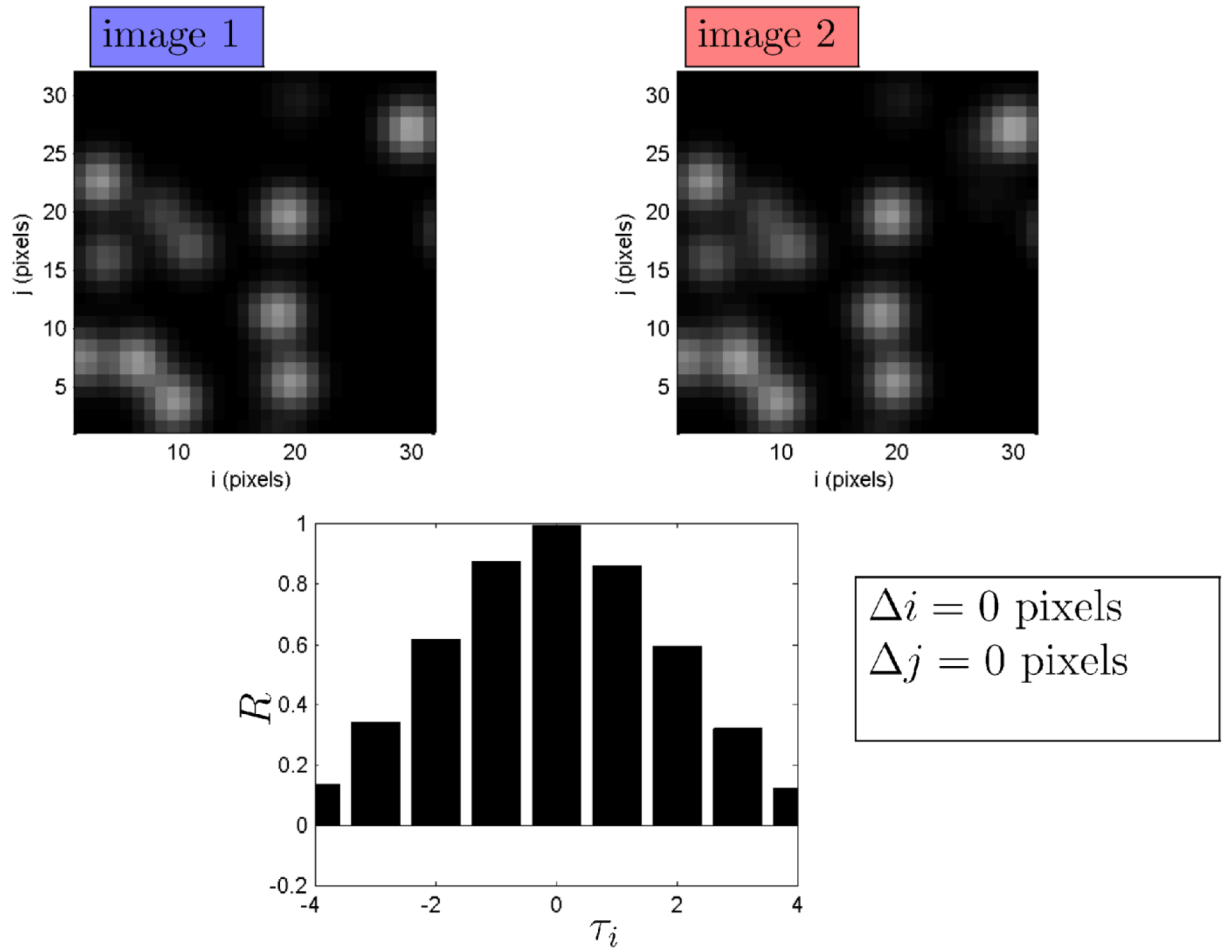
Peak locking

Now zoom on area close to peak in R



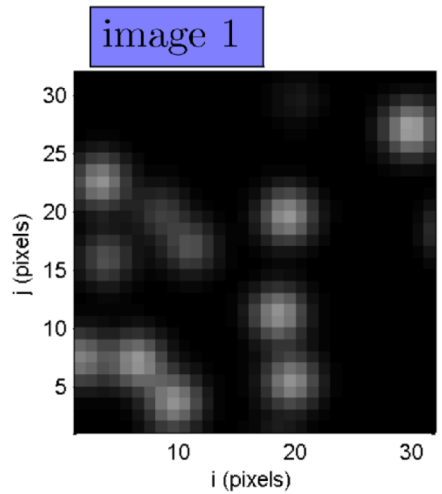
Peak locking

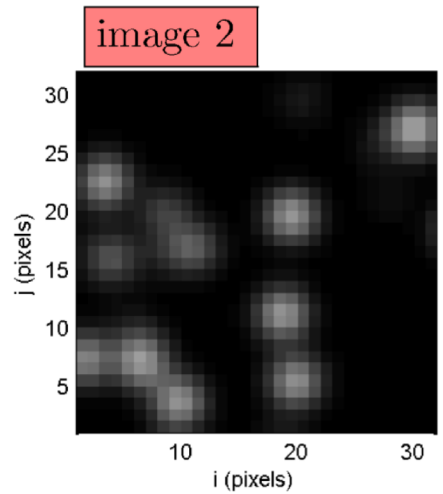
Now closely watch the correlation peak as we increase the shift between image 1 and 2 (as we increase Δi).

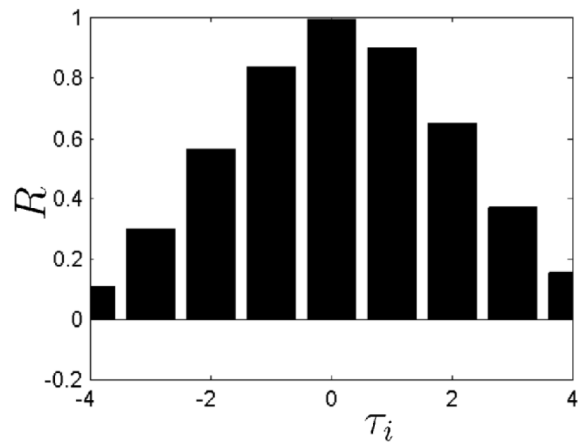


Peak locking

Now closely watch the correlation peak as we increase the shift between image 1 and 2 (as we increase Δi).



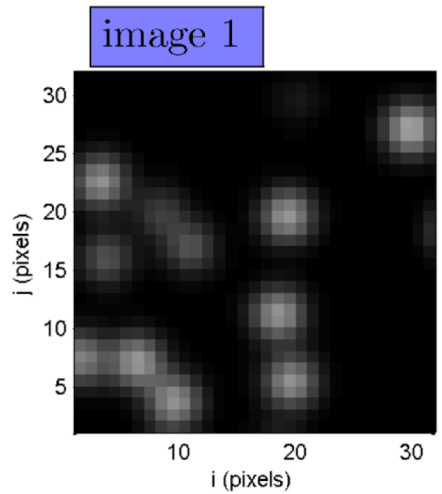


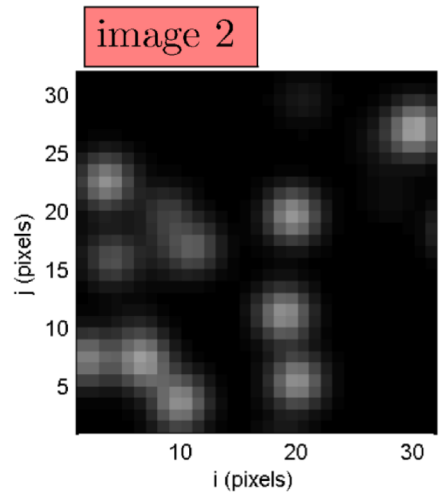


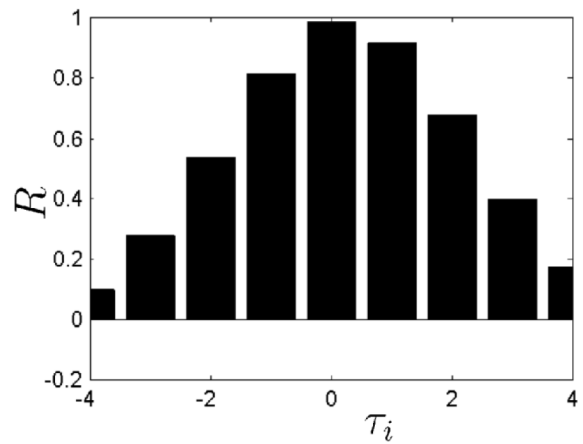
$\Delta i = 0.1$ pixels
$\Delta j = 0$ pixels

Peak locking

Now closely watch the correlation peak as we increase the shift between image 1 and 2 (as we increase Δi).



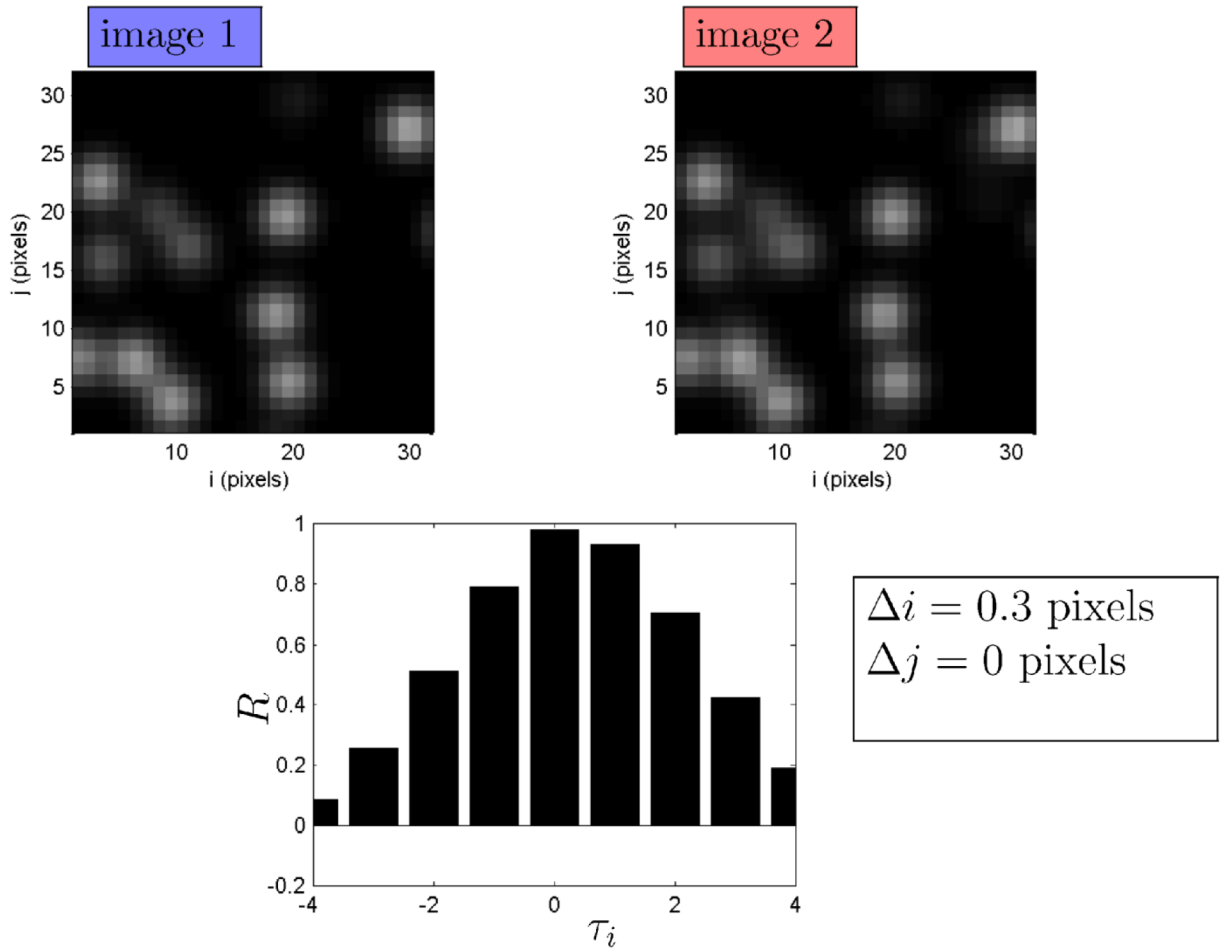




$\Delta i = 0.2$ pixels
$\Delta j = 0$ pixels

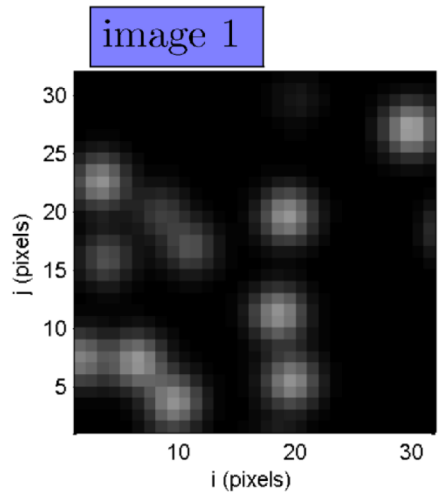
Peak locking

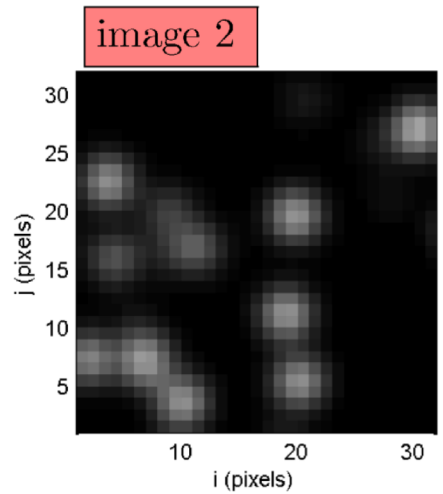
Now closely watch the correlation peak as we increase the shift between image 1 and 2 (as we increase Δi).

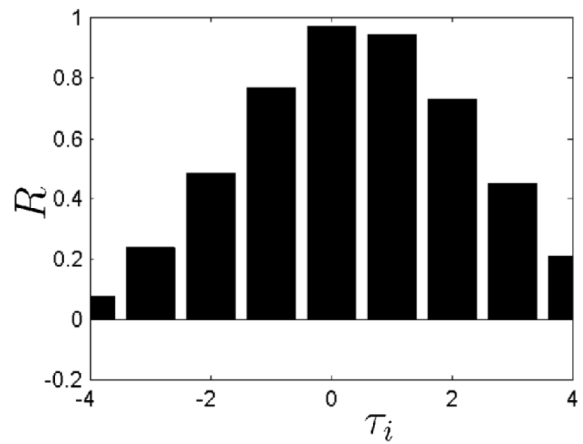


Peak locking

Now closely watch the correlation peak as we increase the shift between image 1 and 2 (as we increase Δi).



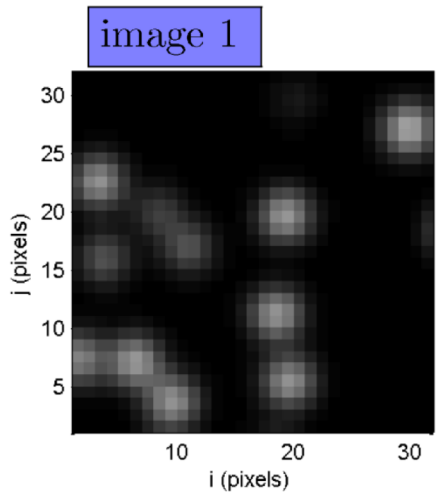


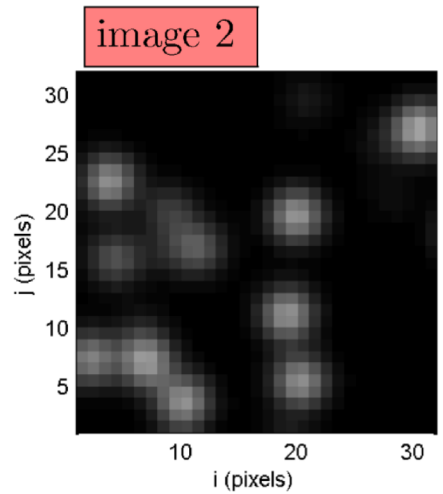


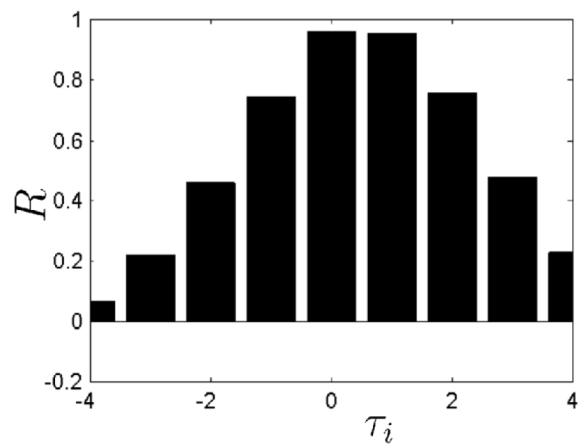
$\Delta i = 0.4$ pixels
$\Delta j = 0$ pixels

Peak locking

Now closely watch the correlation peak as we increase the shift between image 1 and 2 (as we increase Δi).



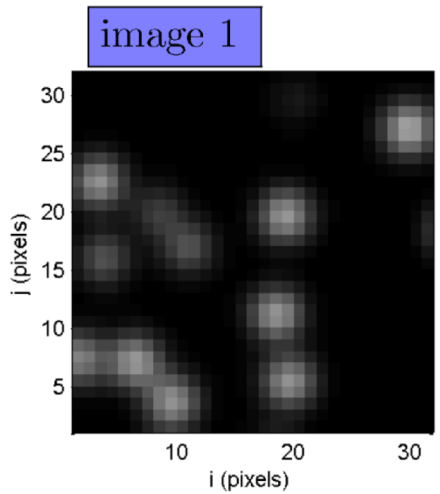


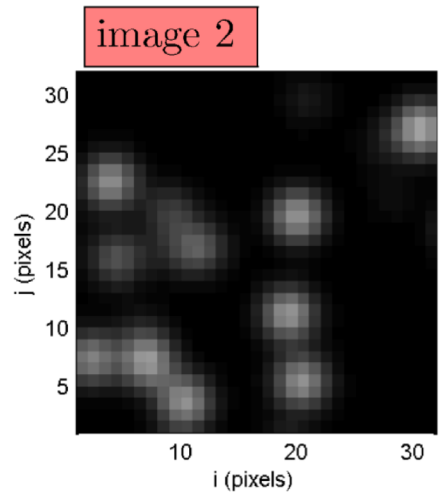


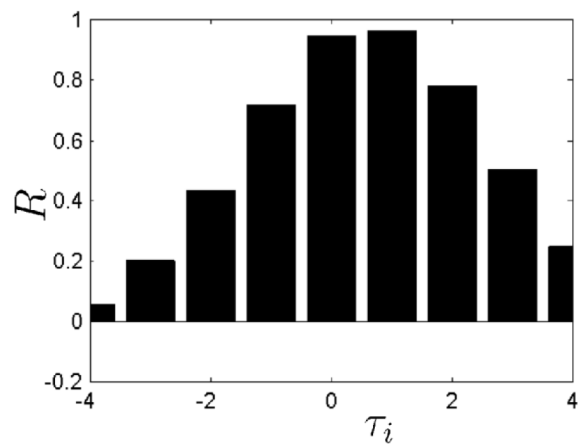
$\Delta i = 0.5$ pixels
$\Delta j = 0$ pixels

Peak locking

Now closely watch the correlation peak as we increase the shift between image 1 and 2 (as we increase Δi).



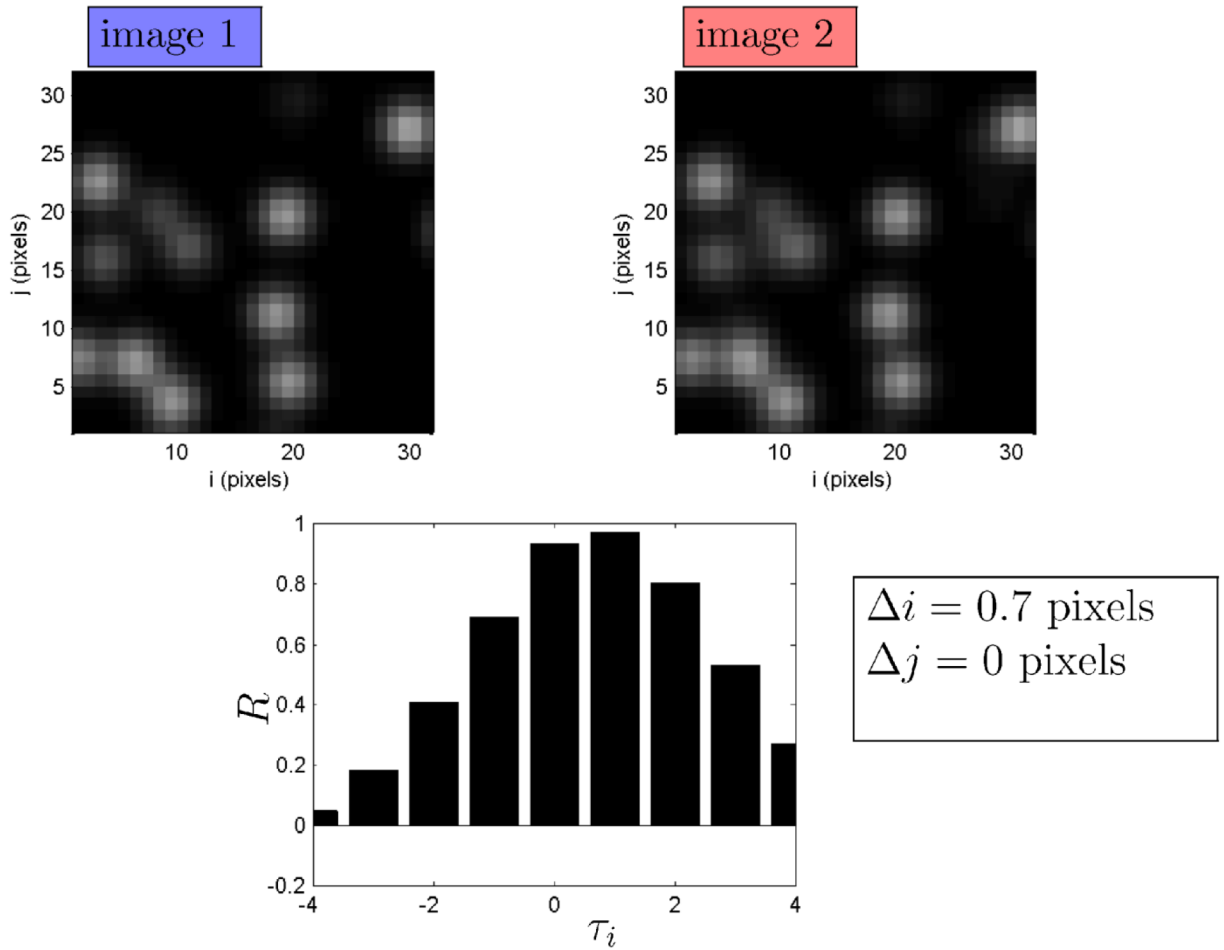




$\Delta i = 0.6$ pixels
$\Delta j = 0$ pixels

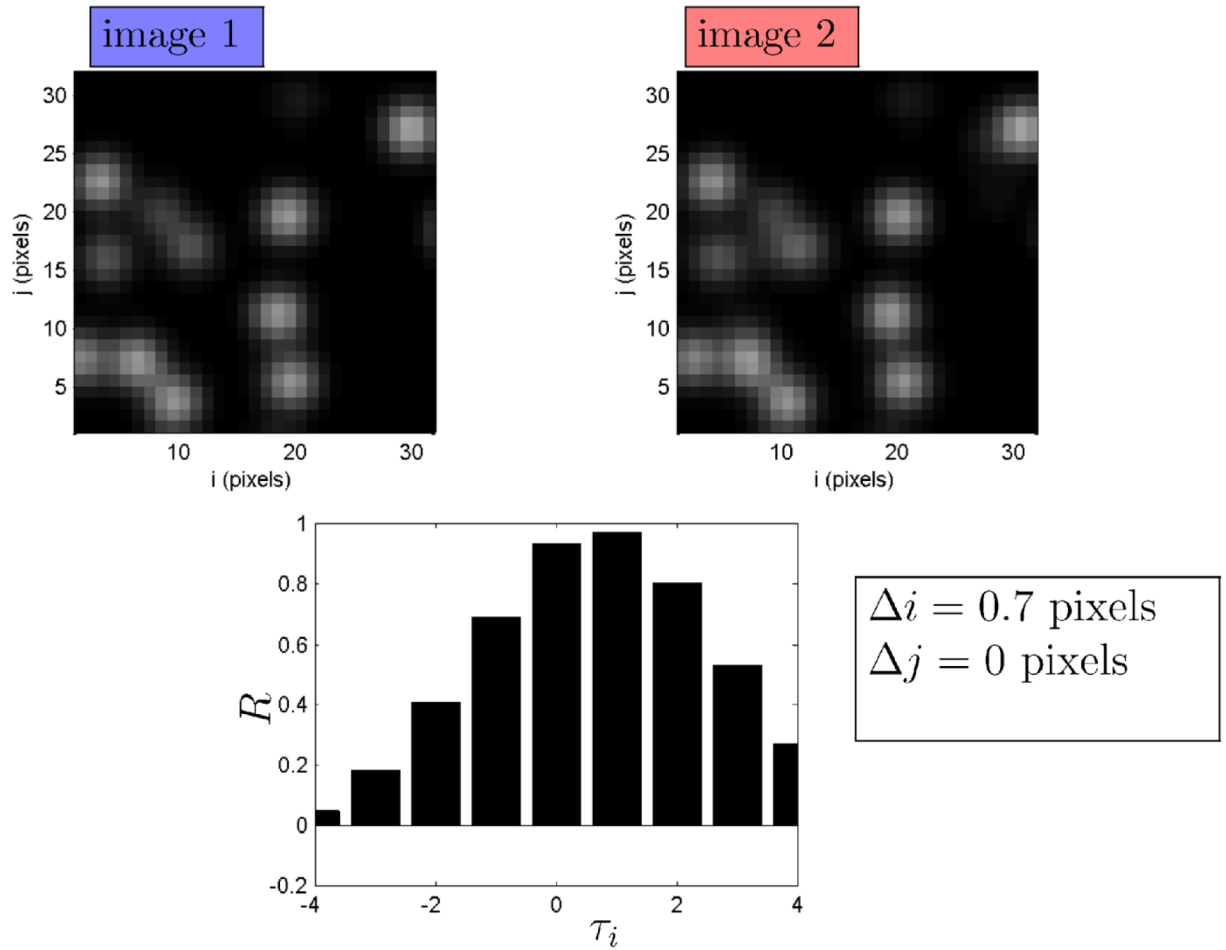
Peak locking

Now closely watch the correlation peak as we increase the shift between image 1 and 2 (as we increase Δi).



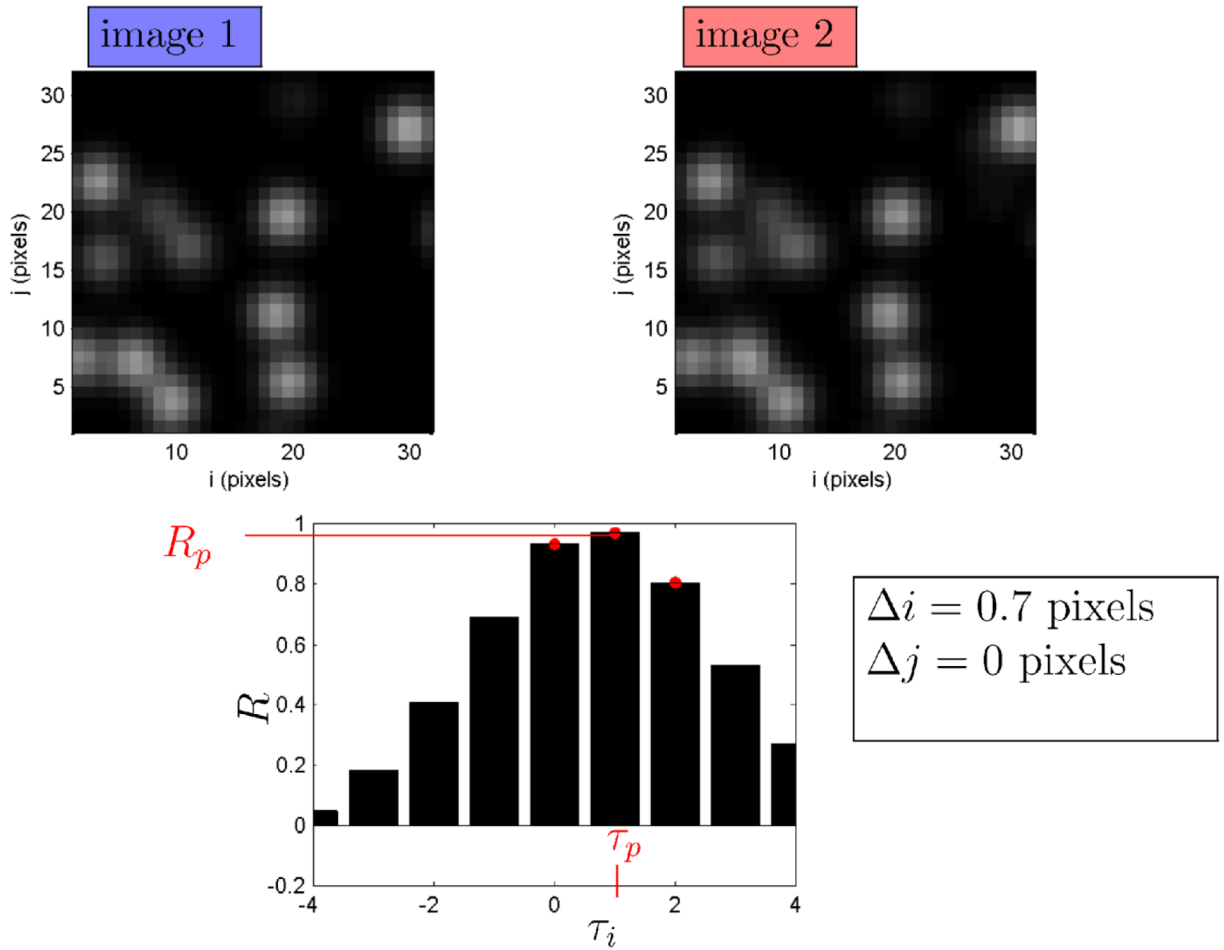
Peak locking

What we should do here is use the information in the indices adjacent to the peak in R



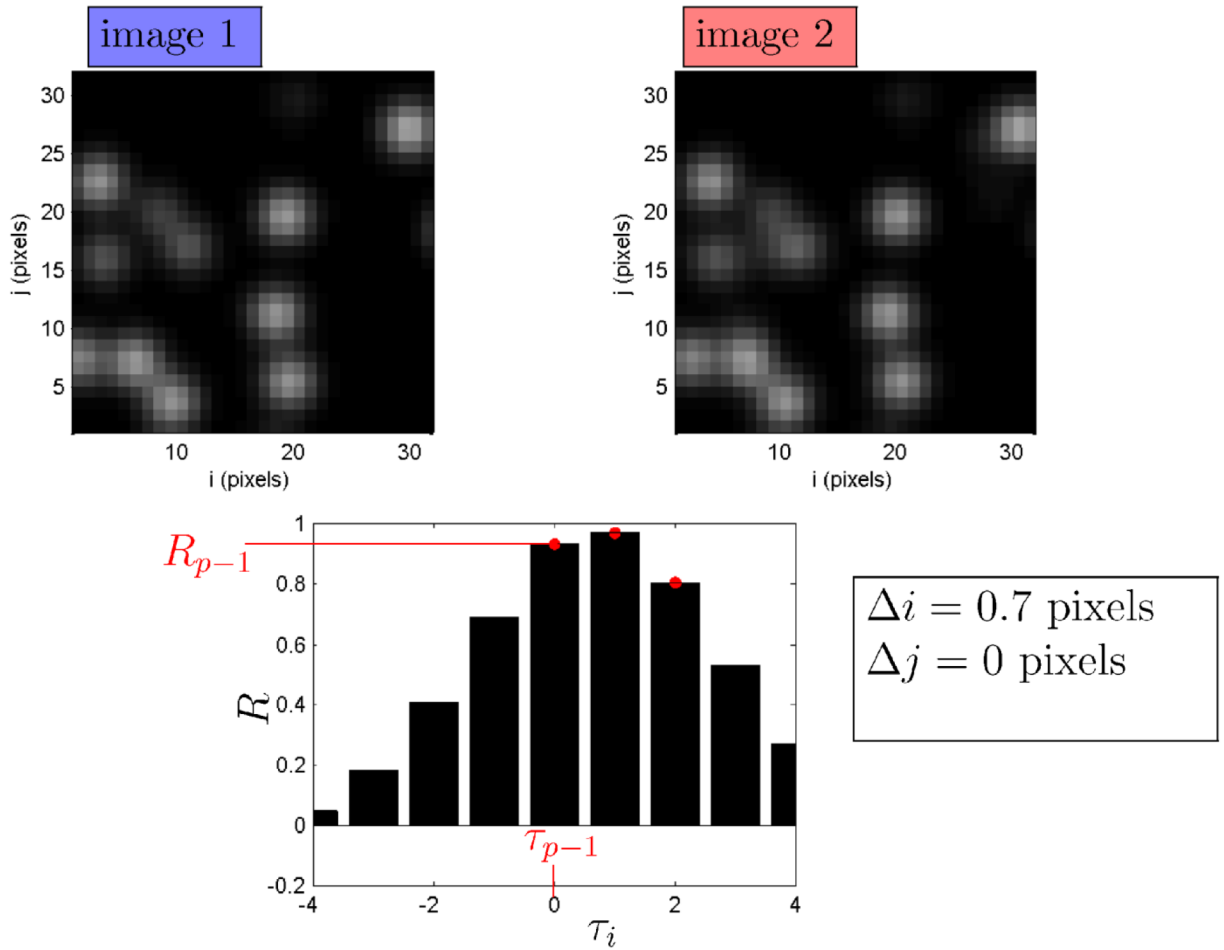
Peak locking

Take the value of R before and after the peak



Peak locking

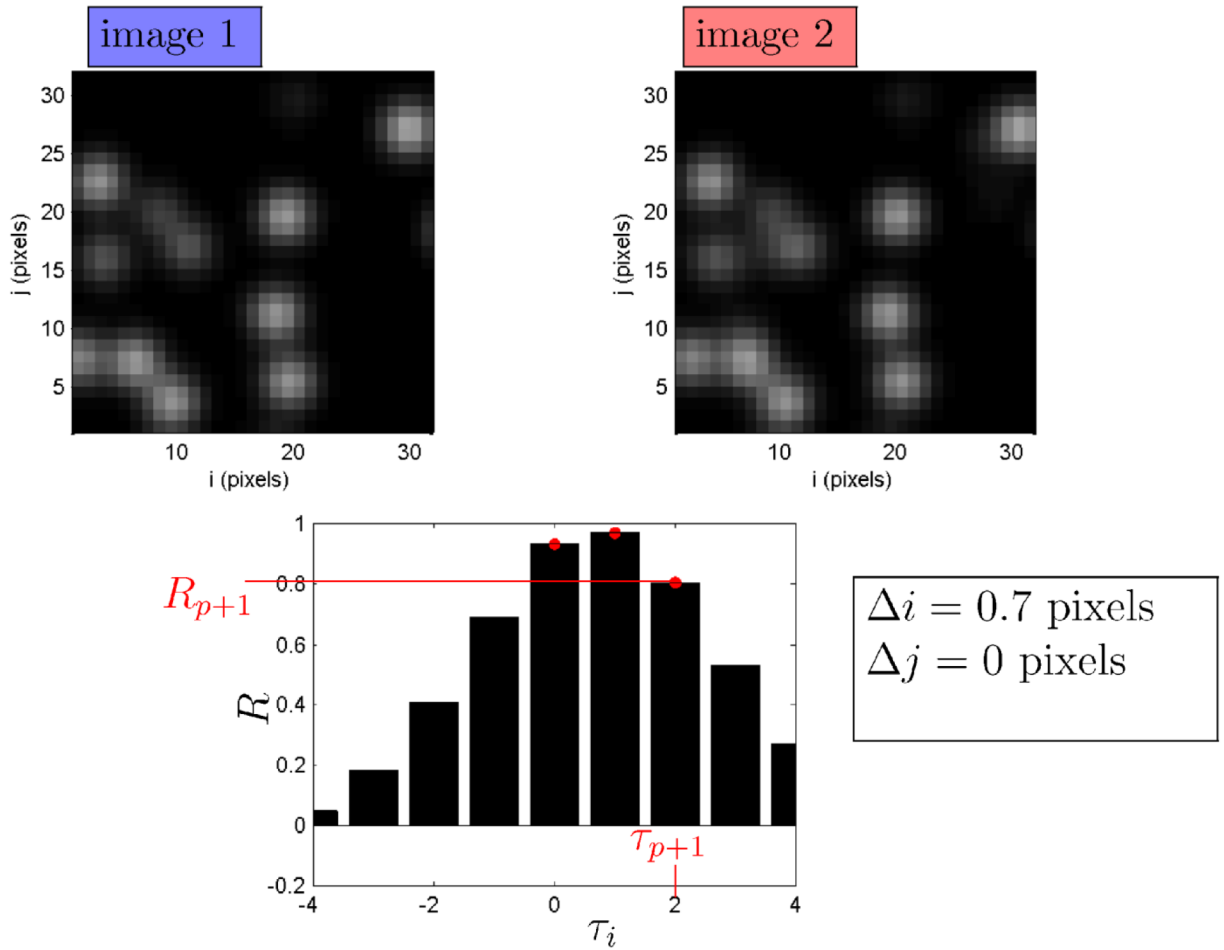
Take the value of R before and after the peak



$\Delta i = 0.7$ pixels
$\Delta j = 0$ pixels

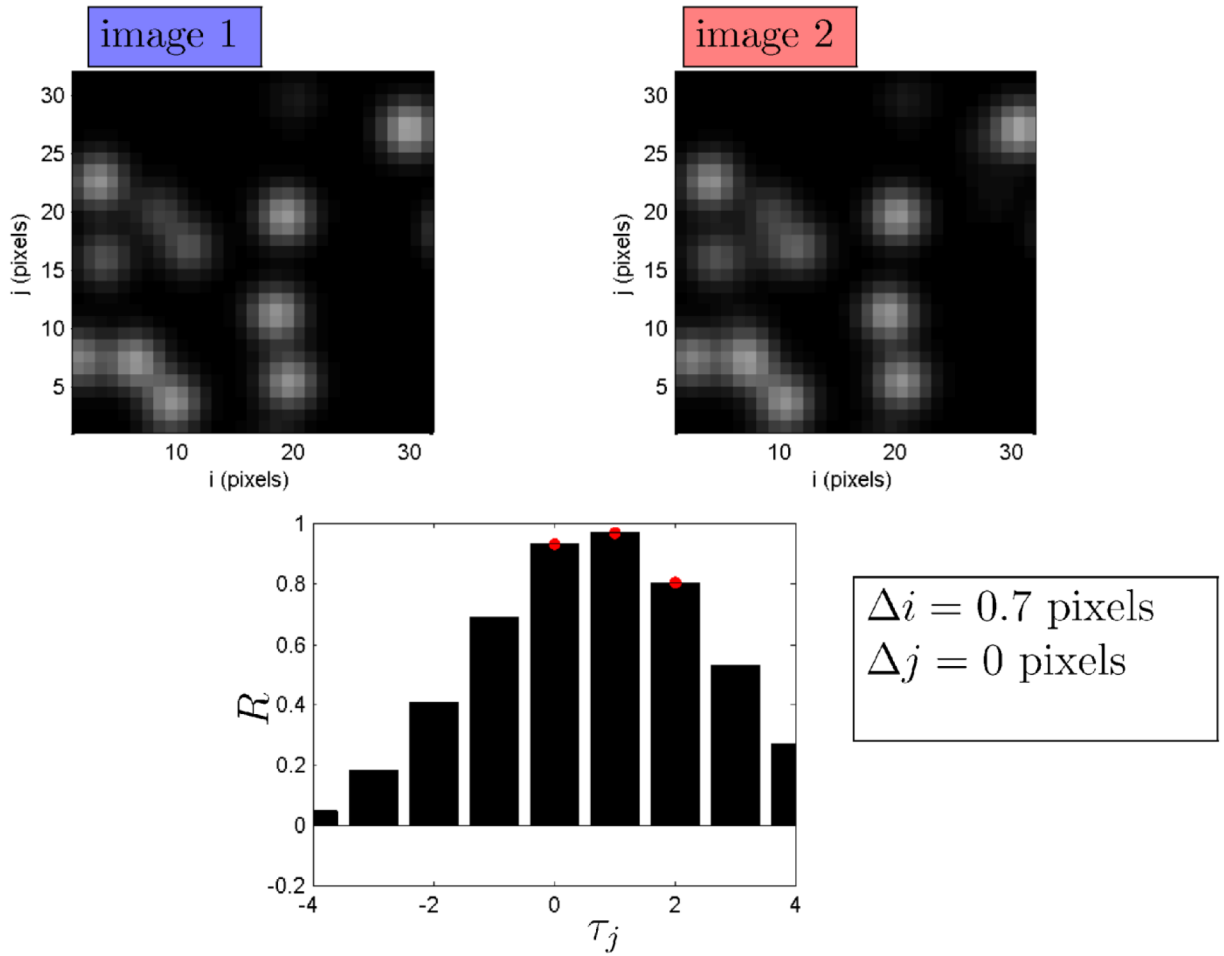
Peak locking

Take the value of R before and after the peak



Peak locking

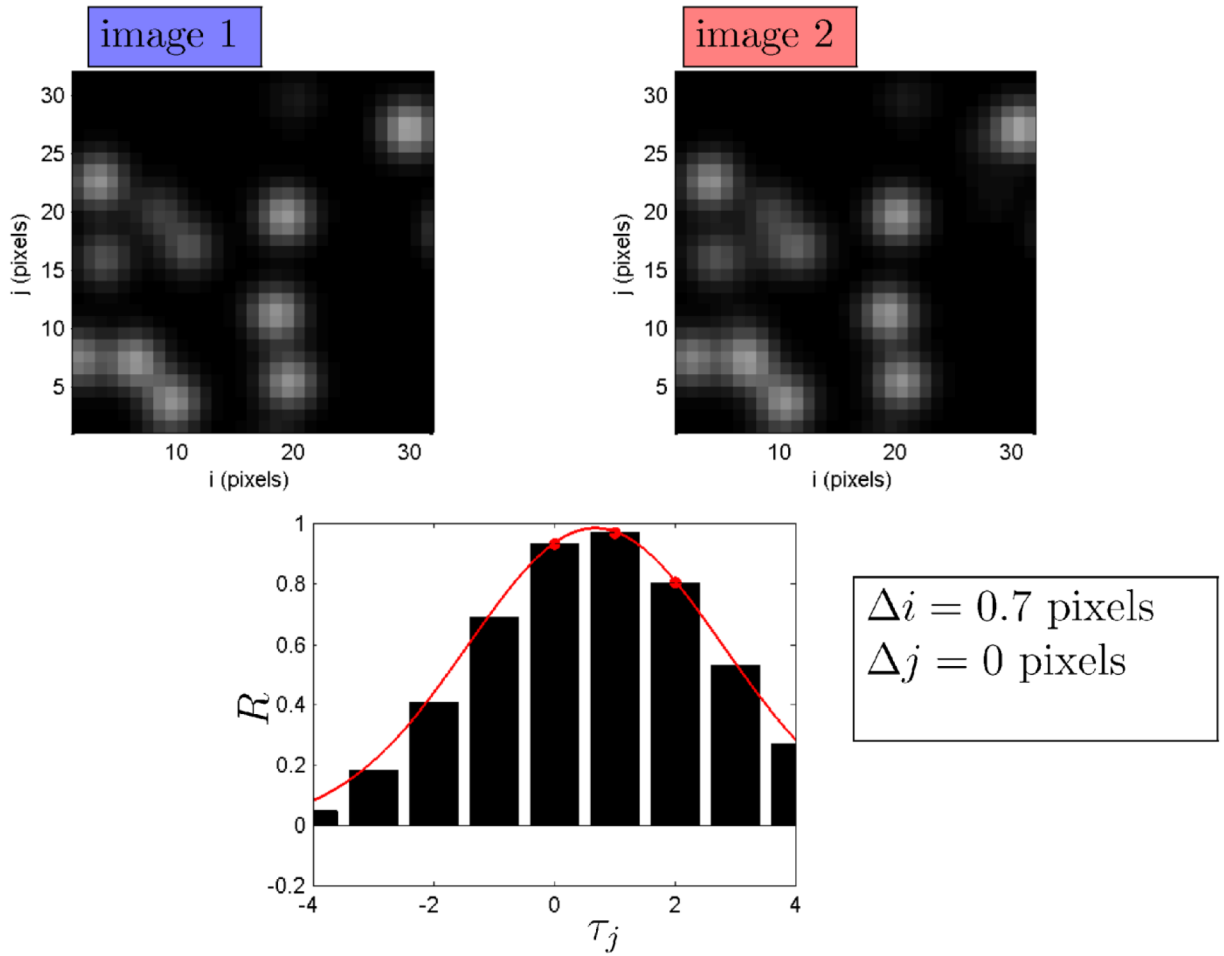
Take the value of R before and after the peak, and fit a Gaussian to these points



$\Delta i = 0.7$ pixels
$\Delta j = 0$ pixels

Peak locking

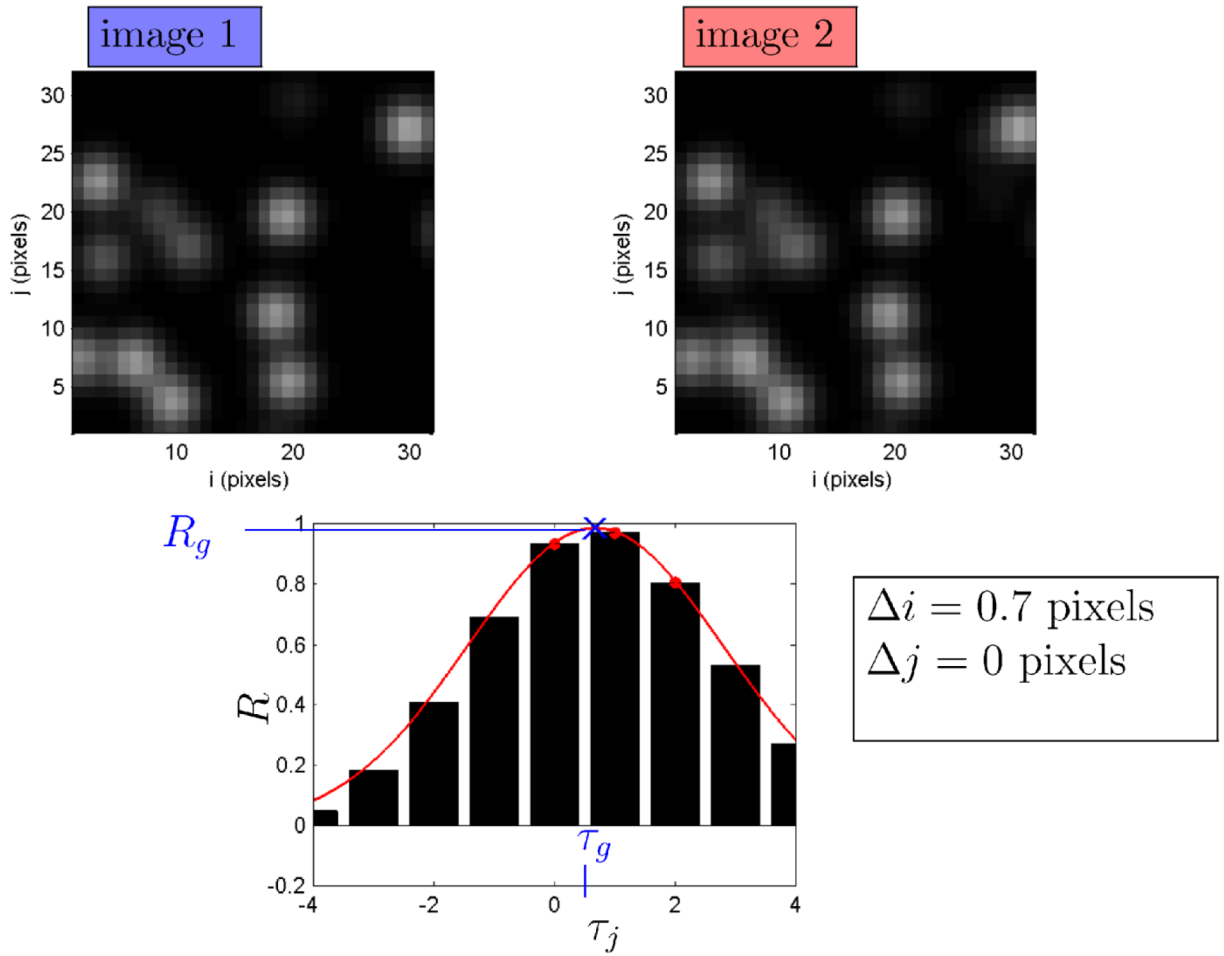
Take the value of R before and after the peak, and fit a Gaussian to these points



$\Delta i = 0.7$ pixels
$\Delta j = 0$ pixels

Sub-pixel displacement

From which we can find the true estimated peak.



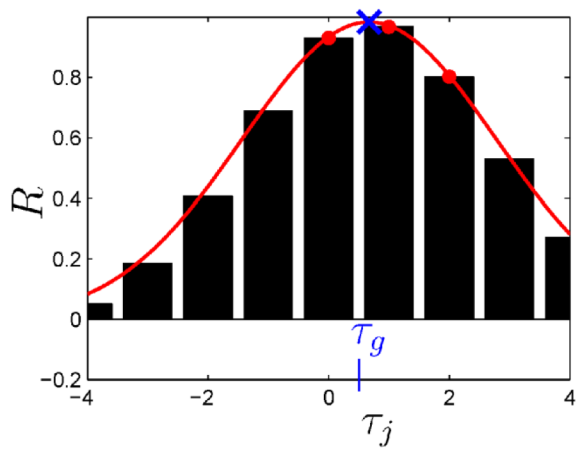
$\Delta i = 0.7$ pixels
$\Delta j = 0$ pixels

Sub-pixel displacement

In reality there is a simple analytical solution for a three point Gaussian fit. If we assume that,

$$R = A \exp\left[-B(\tau - \tau_g)^2\right]$$

where τ_g is the true peak shift, and A and B are fitting parameters.

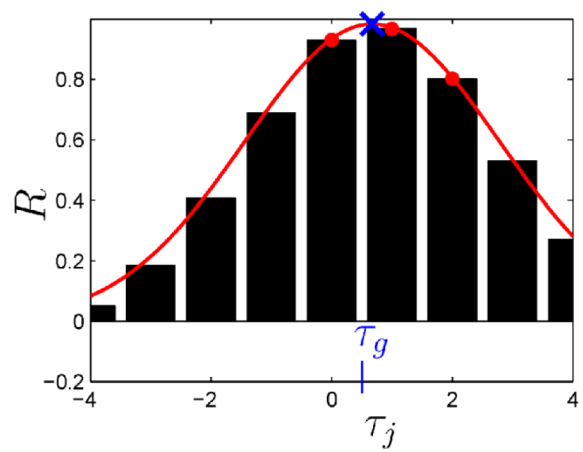


Sub-pixel displacement

For the three points we know that:

$$\begin{aligned} R_p &= A\exp\left[-B(\tau_p - \tau_g)^2\right] \\ R_{p-1} &= A\exp\left[-B(\tau_p - 1 - \tau_g)^2\right] \\ R_{p+1} &= A\exp\left[-B(\tau_p + 1 - \tau_g)^2\right] \end{aligned}$$

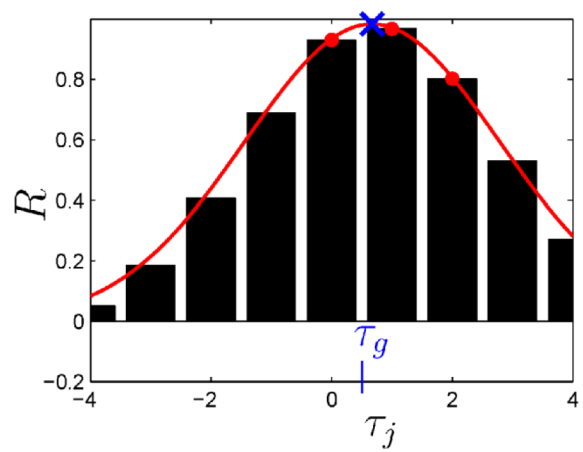
Three equations, three unknowns $\rightarrow$ solve for A, B and τ_g.



Sub-pixel displacement

From which we can get an expression for the sub-pixel displacement

$$\tau_g - \tau_p = \varepsilon = \frac{1}{2}\frac{(\log R_{p-1} - \log R_{p+1})}{(\log R_{p-1} + \log R_{p+1} - 2\log R_p)}$$

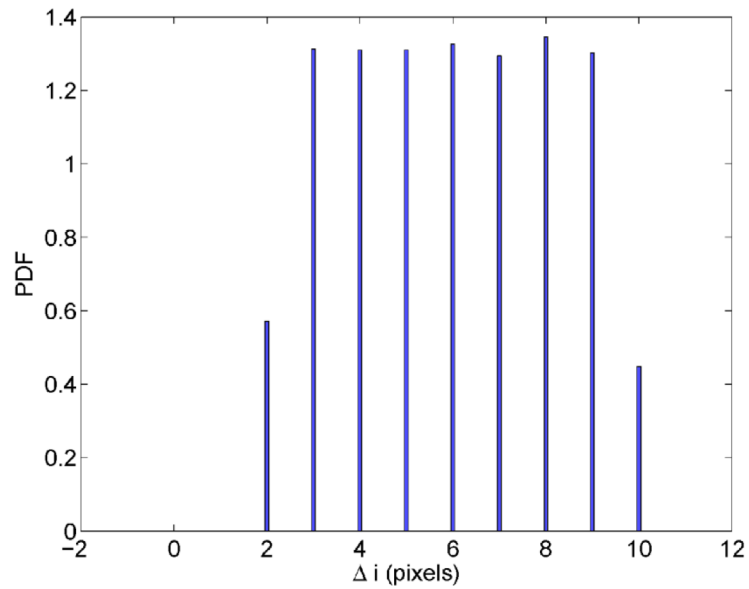


Sub-pixel displacement

From which we can get an expression for the sub-pixel displacement

$$\tau_g - \tau_p = \varepsilon = \frac{1}{2}\frac{(\log R_{p-1} - \log R_{p+1})}{(\log R_{p-1} + \log R_{p+1} - 2\log R_p)}$$

If we look back at the original histogram distribution,



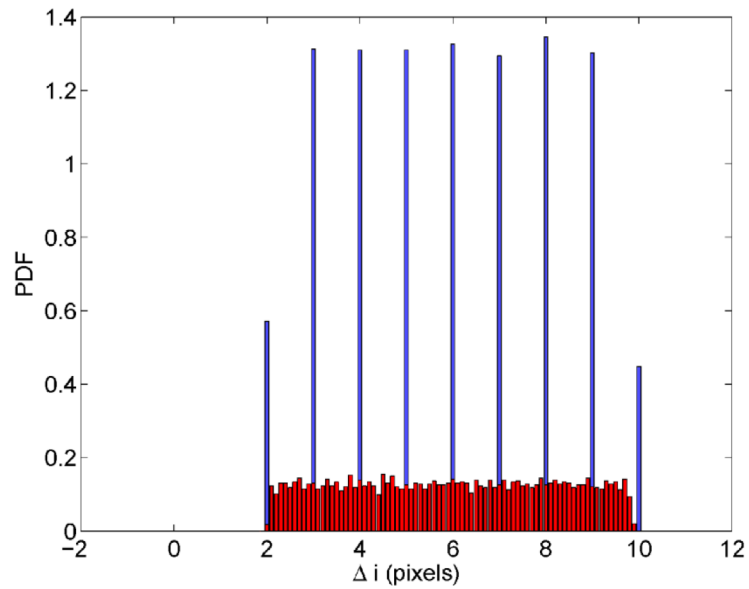
peak-locked

Sub-pixel displacement

From which we can get an expression for the sub-pixel displacement

$$\tau_g - \tau_p = \varepsilon = \frac{1}{2}\frac{(\log R_{p-1} - \log R_{p+1})}{(\log R_{p-1} + \log R_{p+1} - 2\log R_p)}$$

If we look back at the original histogram distribution,



peak-locked
sub pixel