Overview of Foundations

IS 365: Artificial Intelligence

Contents

- Probability Theory
- Matrix Calculus
- Python
- Time Complexity
- Recurrence Relations

Probability Theory – Random Variables

- **Discrete Variable:** random variable taking on discrete values with a probability (mass) distribution $P(A = a)$ OR $P_A(a)$
- **Continuous Variable:** random variable taking on a range of values with a probability density distribution
- **Probability Density Function(PDF):** used to calculate the probability of a random variable falling within a particular range of values.
- Probability density function $f_A(A) \rightarrow \Pr[x \leq A \leq y] = \int_x^y f_A da$
- Cumulative Distribution Function: the probability that the random variable will take a value less than or equal to some value $F_A(a) \rightarrow$ $F_A(a) = \Pr[\mathrm{A} \leq a] = \int_{-\infty}^{a} f_A(a)(u)du$

Probability Theory – Independence and Expectation

Independence:

$$\forall_{a,b} \quad P(A = a, B = b) = P(A = a)P(B = b)$$

$$\forall_{a,b} \quad f_{A,B}(a, b) = f_A(a)f_B(b)$$

Expectation:

$$E[f(A)] = \sum_a f(a)P(A = a)$$

$$E[f(A)] = \int_a f(a)P(A = a)$$

- **Problem 1:** Independence and Expectation

	A = 0	A = 1	A = 2	A = 3
B = 0	0.1	0.25	0.1	0.05
B = 1	0.15	0	0.15	0.2

- Are A and B independent?
- What are E[A], E[B], E[A+B]?
- Linearity of expectation: E[A+B] = E[A] + E[B]
- True even when A and B are dependent.

- **Problem 2:** Expectation

Suppose n hatted IS 365 students toss their hats into the air and pick up one hat at random. In expectation, how many students get their own hats back?

Hint: linearity of expectation

- **Solution:**
- $X = X_1 + X_2 + \ldots + X_n$
- $X_i = \begin{cases} 1 \; if \; i \; selects \; own \; hat \\ 0 \qquad\qquad\qquad otherwise \end{cases}$
- $P[X_i = 1] = \frac{1}{n}$
- $E[X_i] = \frac{1}{n}$
- $E[X] = E[X_1 + X_2 + \cdots + EX_n] = E[X_1] + E[X_2] + \cdots + E[X_n] = 1$
- Note that the X_i are not independent of each other