Geometric Topology Homework

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June 6, 2023

1) Let $f^{(n)}$ be the nth magic knitting. Show that $f^{(n)} - 1$ is an element of the subalgebra of $TL_n(A)$ generated by the e_i. That is, show that the idempotent really does behave like a projection.

Proof. First, remember that as an algebra, $TL_n(A)$ is generated by 1 and the e_i. Further, $f^{(n)}1 = f^{(n)} \neq 0$ for $n \geq 1$, but $f^{(n)}(f^{(n)} - 1) = f^{(n)} - f^{(n)} = 0$. Since $1e_i = e_i1 = e_i$, and any nontrivial addend of some element of $TL_n(A)$ can be written without 1s by condensing them. □