Quantum Algorithms Homework

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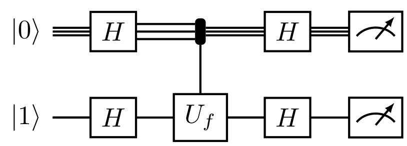
1. Draw the n-variable Deustch Oracle circuit.

   It is clear to see that this is given by:

   where the input $|0\rangle$ is understood to mean $|0\rangle^{\otimes n}$, and U_f is the function to be tested.

2. Realize a full adder using:

   (a) Decoders and OR gates

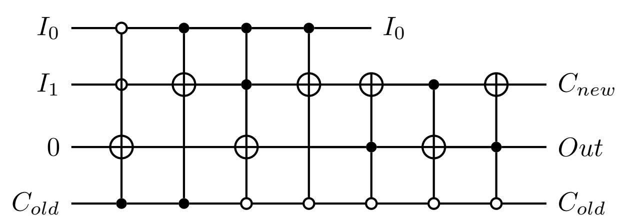
   (b) Reversible logic

   (c) Multiplexers, with 1 address variable

   (d) Arbitrary multiplexers

   (a) This is attached separately, because making classical circuits in latex is way worse than quantum.

   (b) This is given by:

   This runs on a single ancilla, at the cost of having to do a CSWAP at the end to get the outputs and carries in the right spots. I don't expect there to be a no ancillae solution since we have a product or sum no matter what.

3. Realize the functions:

   (a) $f = ab + a'c$ using reversible logic

   (b) $f = abc + de'f' + a'b'c' + d'f$ with multiplexers

   (a) We get:

4. Implement the 3x3 majority gate in an efficient way.

   Proof. We do this by taking a spectral decomposition of the majority function. The attached FPRMSpectrum.py python file takes the Karnaugh map of an arbitrary n variable function and returns the spectrum of minimal cost. Of course, we could also just take this since it's not very hard to compute, but this approach generalizes well. We get a spectrum of

   $$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

   which is equivalent to $ab \oplus bc \oplus ca$. $\square$