Measurement of the influence of dispersion on white-light interferometry

Pavel Pavliček and Jan Soubusta

White-light interferometry is a well-established method for measuring the height profiles of samples with rough as well as with smooth surfaces. Because white-light interferometry uses broadband light sources, the problem of dispersion arises. Because the optical paths in the two interferometer arms cannot be balanced for all wavelengths, the white-light correlogram is distorted, which interferes with its evaluation. We investigate the influence of setup parameters on the shape of the correlogram. Calculated values are compared with experimental results. © 2004 Optical Society of America

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1. Introduction

White-light interferometry is an established method for measuring the height profiles of objects with smooth as well as with rough surfaces.[1,2] By a rough surface is meant a surface whose microstructure is not resolved by the imaging system of a white-light interferometer.[3]

White-light interferometry measures the height profile of a rough surface by evaluating a white-light correlogram. In the evaluation, the maximum of the correlogram's envelope function is sought but the phase is not evaluated, unlike in conventional interferometry and in the white-light interferometry of a smooth surface.[4,5] Because white-light interferometry utilizes a broadband light source with a spectral width of tens of nanometers (LED, SLD) to hundreds of nanometers (incandescent lamp), the influence of dispersion must be taken into consideration.[6,7]

Dispersion has an effect on a correlogram if the geometrical path lengths of the light in dispersive elements are different in the two interferometer arms. Some reasons for these differences arise from the arrangement of the white-light interferometer, as shown in Fig. 1. The glass plate used for compensation of the neutral-density filter does not have exactly the same thickness as the filter, nor is it made from the same material. The beam-splitter cube may not be perfectly symmetrical. In some cases, a measurement is performed on a surface that is covered by a transparent coating.[8] In particular, the influence of dispersion must be kept in mind when one is designing a fiber-optical implementation of a white-light interferometer.[9]

Our aim in this paper is to investigate the influence of dispersion on the shape of a white-light correlogram and thereby on the precision of the measurement.

2. Theory

To calculate the influence of dispersion on the shape of a white-light correlogram we make two simplifying assumptions, as follows.

1. The spectrum of the light source has a Gaussian shape:

$$S(k) = \frac{1}{2\sqrt{\pi}\Delta k}\exp\left[-\left(\frac{k-k_0}{2\Delta k}\right)^2\right], \qquad (1)$$

where $k_0 = 2\pi/\lambda_0$ is the central wave number and $\Delta k = (2\pi/c)\Delta\nu$, where $\Delta\nu$ is the effective bandwidth.[10] Spectra of semiconductor light sources such as LEDs and SLDs are similar in shape to the Gaussian curve. Figure 2(a) shows a typical spectrum of a LED as measured by a spectrometer. In our case the central wavelength is $\lambda_0 = 866$ nm and the FWHM is 40 nm. Figure 2(b) shows the same spectrum displayed in wave-number units (solid curve) compared with the Gaussian curve (dotted dashed curve) given by Eq. (1).

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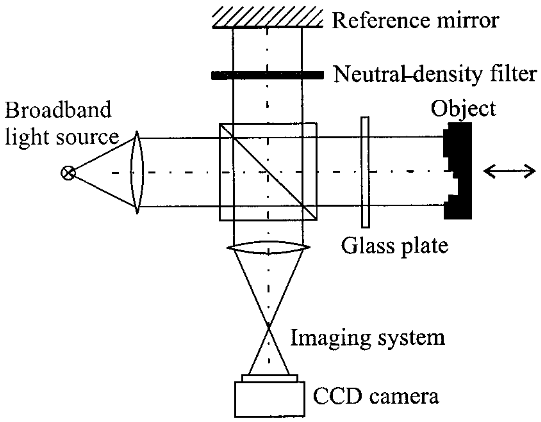

Fig. 1. Schematic of the white-light interferometer.

2. The dependence of the refractive index of the dispersive elements on wave number k is linear:

$$n(k) = n(k_0) + \alpha(k - k_0), \tag{2}$$

where $\alpha = dn/dk$ is the slope of the linear dependence and acts as the dispersion parameter. In fact, α is dependent on the wavelength and therefore on k. Calculated values of α for two common optical materials (fused silica and BK7 glass) for four wavelengths are listed in Table 1. It is apparent from Table 1 that α varies only slightly with wavelength and can be considered constant in the spectral interval given by the FWHM of the LED.[7]

From assumptions (1) and (2) we calculate the shape of the white-light correlogram when a glass plate

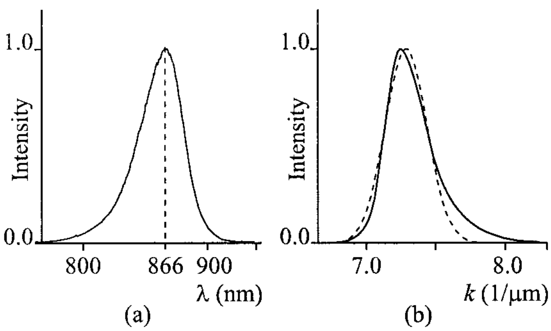

Fig. 2. Spectrum of a LED measured by a spectrometer (a) in wavelength units and (b) in wave-number units (solid curve). The dashed curve is a Gaussian curve.

Table 1. Dispersion Parameters α for Fused Silica and BK7[a]

	α (nm)	
λ (nm)	Fused Silica	BK7
600	1.909	2.262
700	1.777	2.074
800	1.761	2.021
900	1.840	2.076

[a]Calculated from the Sellmeier equation.

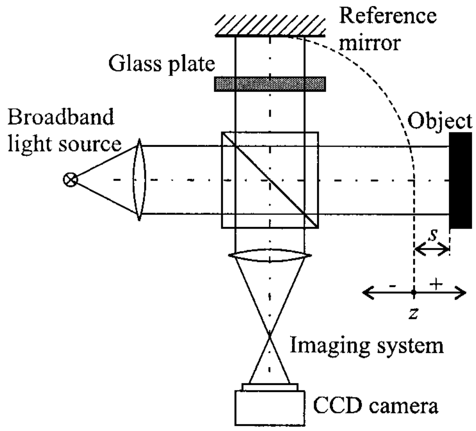

Fig. 3. Experimental schematic of the balanced white-light interferometer with a glass plate in the reference arm.

with thickness d is placed into the reference arm of the Michelson interferometer, as illustrated in Fig. 3:

$$I(z) = \int_0^\infty S(k)[1 + \cos(2k\{z - d[n(k) - 1]\} + \Delta\varphi)]dk, \tag{3}$$

where z is the distance of the object from the reference plane as depicted in Fig. 3 and $\Delta\varphi = \varphi_O - \varphi_R$, where φ_O and φ_R represent any phase change on reflection from the measured surface and the reference mirror, respectively.[4,11,12] These phase changes are due to the complex reflectance of a nondielectric material and amount to fractions of a radian (e.g., $\varphi = 0.21$ rad for Al and $\varphi = 0.37$ rad for Cu at $\lambda = 820$ nm).[13] In our model ($\Delta k \ll k_0$), we assume that φ_O and φ_R are wavelength independent.

Substituting $S(k)$ and $n(k)$ from Eqs. (1) and (2) into Eq. (3) and performing the integration, we obtain[14]

$$I(z) = I_0\left\{1 + (1 + \eta^2)^{-1/4} \exp\left[-\frac{1}{1 + \eta^2}\left(\frac{z - s}{l_c}\right)^2\right] \times \cos\left[\frac{\eta}{1 + \eta^2}\left(\frac{z - s}{l_c}\right)^2 + 2k_0(z - s) + \Phi_0 + \Delta\varphi\right]\right\}, \tag{4}$$

where we introduce the new parameters η, s, l_c, and Φ_0. The meaning of these parameters is described in what follows.

The dimensionless parameter

$$\eta = 8\alpha d(\Delta k)^2 \tag{5}$$

represents the change in the shape of the correlogram. The compensating shift

$$s = d[n(k_0) + \alpha k_0 - 1] = d[N(k_0) - 1] \tag{6}$$

is the distance by which the reference mirror must be shifted to balance the interferometer for white-light interferometry (Fig. 3). The presence of group refractive index N in Eq. (6), where $N(k) = n(k) + [\mathrm{d}n(k)/\mathrm{d}k]k$, reveals the fact that the interferometric methods measure group refractive index.[15] Coherence length

$$l_c = (2\Delta k)^{-1} \tag{7}$$

is defined in Ref. 10, and

$$\begin{aligned}\Phi_0 &= 2\alpha d k_0{}^2 - \tfrac{1}{2} \arctan \eta \\ &= 2k_0 d[N(k_0) - n(k_0)] - \tfrac{1}{2} \arctan \eta\end{aligned} \tag{8}$$

is the phase shift of the correlogram maximum caused by the dispersion. The term $2\alpha d k_0{}^2$ in Eq. (8) is dominant and reflects the fact that one balances the interferometer by shifting the object by a distance $s = d(N - 1)$, whereas the additional optical path length in the glass amounts to $d(n - 1)$. The term $-\tfrac{1}{2}$ arctan η is a consequence of the nonlinearity introduced by the dispersion.

Substituting $d = 0$, we get the special case without dispersion; Eqs. (5)–(8) yield $\eta = 0$, $s = 0$, and $\Phi_0 = 0$; and Eq. (4) acquires the standard form[10]

$$I(z) = I_0\{1 + \exp[-(z/l_c)^2]\cos(2k_0 z + \Delta\varphi)\}. \tag{9}$$

In this case the correlogram exhibits a contrast of 1, a width $w = l_c$, and a period $p = \lambda_0/2$. The width of the correlogram, w, is defined by the decrease of the contrast of the correlogram to 1/e of its maximal value.

Theoretical inspection of Eq. (4) and its comparison with Eq. (9) yield the following explanation of the structure of the terms in Eq. (4). The primed quantities indicate a relation to dispersion.

1. The term $(1 + \eta^2)^{-1/4}$ gives the loss of contrast of the correlogram:

$$\frac{C}{C'} = (1 + \eta^2)^{1/4}. \tag{10}$$

2. The term $1/(1 + \eta^2)$ in the exponent gives a broadening of the correlogram:

$$w'/w = \sqrt{1 + \eta^2}. \tag{11}$$

3. The quadratic term $[\eta/(1 + \eta^2)][(z - s)/l_c]^2$ in the argument of the cosine in Eq. (4) is responsible for a change in period of the correlogram with translation. Period p' as a function of the shift $z - s$ is given by[16]

$$p' \approx p\left[1 - \frac{\eta}{1 + \eta^2}\frac{p}{\pi w^2}(z - s)\right]. \tag{12}$$

4. When there is no dispersion, the correlogram's maximum is shifted by $\Delta\varphi$ with respect to the maximum of the correlogram's envelope function. If dispersion takes place, the phase shift between the correlogram's maximum and the maximum of the correlogram's envelope function changes because of the presence of the constant term Φ_0 in the argument of the cosine in Eq. (4). For common light sources such that $k_0 > \sqrt{2}\,\Delta k$, $\Phi_0 > 0$. Equation (8) gives $\Phi_0 = 2\pi$ if thickness d of the glass plate takes the value of several tens of a micrometer. Because thickness d and refractive index n of the glass plate are not known precisely, for thickness d larger than ~30 μm the phase shift gets some random value from the whole interval $[0, 2\pi)$.

The change in the shape of the correlogram caused by dispersion is characterized by parameter η. Equation (5) yields the dependence of η on the thickness d of the glass plate, on dispersion parameter α, and spectral width Δk of the light source. Because spectral width Δk need not necessarily be known, it is useful to express parameter η in terms of width w of the correlogram. According to Eqs. (7) and (9) we get

$$\eta = 2\alpha d(1/w^2). \tag{13}$$

In this way it is possible to calculate the broadening of the correlogram, w'/w, the loss of contrast, C/C', and the magnitude of the period, p', from the known or measured parameters α, d, and w.

3. Measurement

All effects described theoretically in Section 2 were tested by measurement of correlograms both without dispersion and with dispersion. The measurement setup is shown in Fig. 3. Several types of LED with the wavelengths ranging from 600 to 864 nm and several spectral width (FWHM) from 22 to 45 nm were used as light sources. Dispersion is introduced by a BK7 glass plate in the reference arm of the interferometer. Glass plates with thicknesses of 0.4 to 10 mm were used. The light from the broadband light source illuminates a Michelson interferometer formed by the reference mirror and a copper plate with a matte finish serving as a measured object. The copper plate is moved by a translation stage in the z direction. A CCD camera is used as a detector that records the correlogram.

The measured correlograms are plotted in Fig. 4. The light source in this case is a LED with central wavelength $\lambda_0 = 866$ nm and spectral width (FWHM) 40 nm (its spectrum is displayed in Fig. 2). Figure 4(a) shows the correlogram without dispersion. The correlogram is symmetric, except for phase shift $\Delta\varphi$, because of the symmetry of the cosine function in respect to the origin that represents the correlogram for a single wave number.

Figure 4(b) shows the correlogram when a plate of BK7 glass with a thickness of 10 mm is placed in the reference arm. It appears that, according to our theoretical assumptions, a broadening of the correlogram, a loss of the contrast, and an additional phase shift between the correlogram's maximum and the maximum of the correlogram's envelope function

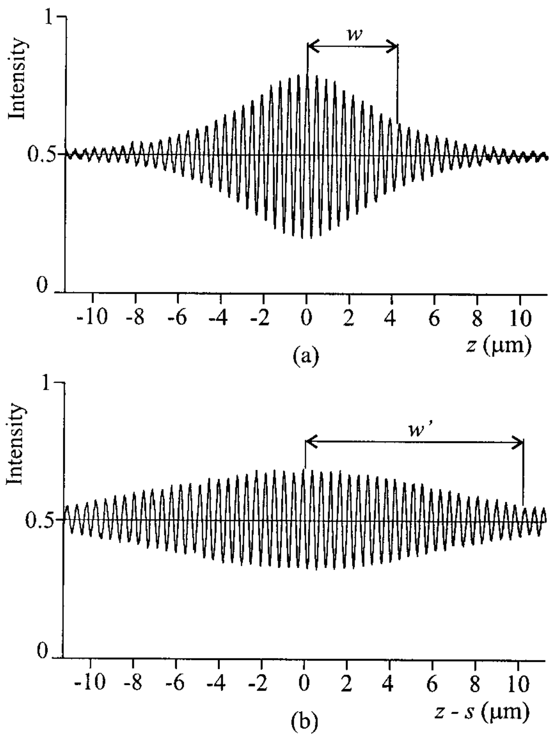

Fig. 4. Correlogram $I(z)$ measured with the LED (λ_0 = 866 nm; FWHM, 40 nm): (a) without dispersion, (b) with dispersion (10-mm-thick plate of BK7).

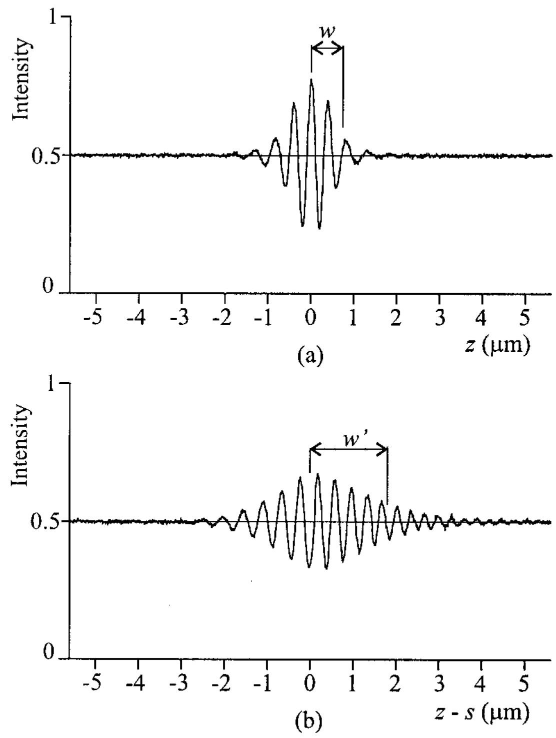

Fig. 5. Correlogram $I(z)$ measured with an incandescent lamp (a) without dispersion, (b) with dispersion (0.4-mm-thick plate of BK7 glass).

occur. The calculated change in period with translation is too small to be confirmed by measurement. Though the correlogram is not symmetric, the correlogram's envelope function is symmetric because the spectrum of the light source is symmetric, a condition that is approximately fulfilled in the case of the LED.

A comparison of the calculated values with the measured results is made in Table 2. Our measured results for the several types of LED are in good agreement with the calculated values, a result that supports the validity of our model, which uses the Gaussian spectrum of the light source.

As a complement to the measurement with the LED, we performed a measurement in which an incandescent lamp was used as the light source. The spectrum of the incandescent lamp is wider than that of the LED, and its shape differs markedly from a Gaussian curve. Figure 5(a) shows the correlogram without dispersion. The correlogram is symmetric, except for phase shift $\Delta\varphi$ [like that in Fig. 4(a)], and it is evident that the coherence length of the incandescent lamp is substantially less than that of the LED.

Figure 5(b) shows the correlogram that results when a BK7 glass plate with a thickness of 0.4 mm is placed in the reference arm. All effects described in Section 2 occur also in this case, i.e., broadening of the correlogram, loss of contrast, change in period of the correlogram with translation, and an additional phase shift between the correlogram's maximum and the maximum of the correlogram's envelope function. The correlogram's envelope function is asymmetric because the spectrum of the incandescent lamp is asymmetric.

A comparison of the calculated values with the measured results is given in Table 3. The measured broadening of the correlogram, w'/w, and the loss of contrast, C/C', are less than the calculated values. The reason is that the shape of the incandescent lamp's spectrum differs markedly from the Gaussian curve, and therefore the real correlogram is narrower than it would be with a Gaussian spectrum with the same spectral width. The spectral width calculated from the correlogram's width w is then too high, and this causes the calculated values w'/w and C/C' to be higher than the measured values.

Table 2. Calculated Values and Measured Results for the Correlogram of the LED[a]

Measured						Given		Calculated			
w (μm)	p (μm)	w' (μm)	w'/w	C/C'	$(p' - p)/(z - s)$	α (nm)	d (mm)	η	w'/w	C/C'	$(p' - p)/(z - s)$
4.2	0.432	10.2	2.4	1.6	0.0	2.04	10.0	2.3	2.5	1.6	−0.001

[a]As shown in Fig. 4.

Table 3. Calculated Values and Measured Results for the Correlogram of the Incandescent Lamp[a]

Measured						Given		Calculated			
w (μm)	p (μm)	w' (μm)	w'/w	C/C'	$(p' - p)/(z - s)$	α (nm)	d (mm)	η	w'/w	C/C'	$(p' - p)/(z - s)$
0.70	0.420	1.8	2.6	1.6	−0.036	2.03	0.4	3.3	3.4	1.8	−0.032

[a]As shown in Fig. 5.

4. Conclusions

A theoretical analysis of the influence of dispersion on the shape of a white-light correlogram was performed based on the following assumptions:

1. The spectrum of the light source has a Gaussian shape and
2. The refractive index of the dispersive elements in the interferometer is a linear function of the wave number.

Dispersion has following effects on the shape of the correlogram:

1. Broadening of the correlogram,
2. Loss of contrast,
3. Nearly linear change in period with translation, and
4. Additional phase shift between the correlogram's maximum and the maximum of the correlogram's envelope function.

Measurements with various light sources and with glass plates of various thicknesses confirmed the validity of the model presented here.

In addition, we used a light source whose spectrum does not fulfill the aforementioned assumption of a Gaussian spectrum. Nevertheless, there is a certain agreement between the calculated values and the measured results, even for such light sources. Moreover, the correlogram envelope function is asymmetric if the spectrum of the light source is asymmetric.

The influence of dispersion on the shape of the correlogram increases strongly (quadratically) with the spectral width of the light source. Whereas a glass plate with a thickness of several millimeters may be irrelevant in a white-light interferometer with a LED, a plate with a thickness of several tenths of a millimeter may significantly distort the correlogram in an interferometer with an incandescent lamp.

All effects mentioned above impair the evaluation of the correlogram and hence have a negative effect on measurement accuracy. Therefore there is an effort to eliminate, or at least to minimize, dispersion in white-light interferometers. Nevertheless, the relations derived in our paper are certainly useful for estimating the influence of the dispersion on the white-light correlogram in a concrete interferometer arrangement in various experiments.

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References

1. G. S. Kino and S. S. C. Chim, "Mirau correlation microscope," Appl. Opt. **29,** 3775–3783 (1990).
2. T. Dresel, G. Häusler, and H. Venzke, "Three-dimensional sensing of rough surfaces by coherence radar," Appl. Opt. **31,** 919–925 (1992).
3. G. Häusler, P. Ettl, M. Schenk, G. Bohn, and I. Laszlo, "Limits of optical range sensors and how to exploit them," in *International Trends in Optics and Photonics ICO IV*, T. Asakura, ed., Vol. 74 of Springer Series in Optical Sciences (Springer Verlag, Berlin, 1999), pp. 328–342.
4. K. G. Larkin, "Efficient nonlinear algorithm for envelope detection in white light interferometry," J. Opt. Soc. Am. A **13,** 832–843 (1996).
5. P. de Groot, X. Colonna de Lega, J. Kramer, and M. Turzhitsky, "Determination of fringe order in white-light interference microscopy," Appl. Opt. **41,** 4571–4578 (2002).
6. A. Pförtner and J. Schwider, "Dispersion error in white-light Linnik interferometers and its implication for evaluation procedures," Appl. Opt. **40,** 6223–6228 (2001).
7. P. de Groot, "Chromatic dispersion effects in coherent absolute ranging," Opt. Lett. **17,** 898–900 (1992).
8. B. L. Danielson and C. Y. Boisrobert, "Absolute optical ranging using low coherence interferometry," Appl. Opt. **30,** 2975–2979 (1991).
9. R. Ulrich and A. Koch, "Faseroptischer Antastsensor für rauhe Oberflächen," F & M **100,** 521–524 (1992).
10. M. Born and E. Wolf, *Principles of Optics* (Pergamon, Oxford, 1980).
11. P. Hariharan, K. G. Larkin, and M. Roy, "The geometric phase: interferometric observation with white light," J. Mod. Opt. **41,** 663–667 (1994).
12. J. F. Biegen, "Determination of the phase change on reflection from two-beam interference," Opt. Lett. **19,** 1690–1692 (1994).
13. M. Bass, ed., *Handbook of Optics* (McGraw-Hill, New York, 1995).
14. I. S. Gradsteyn and I. M. Ryzhik, *Table of Integrals, Series and Products* (Academic, New York, 1980).
15. M. Haruna, M. Ohmi, T. Mitsuyama, H. Tajiri, H. Maruyama, and M. Hashimoto, "Simultaneous measurement of the phase and group indices and the thickness of transparent plates by low-coherence interferometry," Opt. Lett. **23,** 966–968 (1998).
16. Z. Ge, F. Kobayashi, S. Matsuda, and M. Takeda, "Coordinate-transform technique for closed-fringe analysis by the Fourier-transform method," Appl. Opt. **40,** 1649–1657 (2001).