RECURRENT NEURAL NETWORKS

{Grady Kurpasi}

{SSIE 616}

{Prof H. Lewis}

Multilayer Perceptrons

x, y

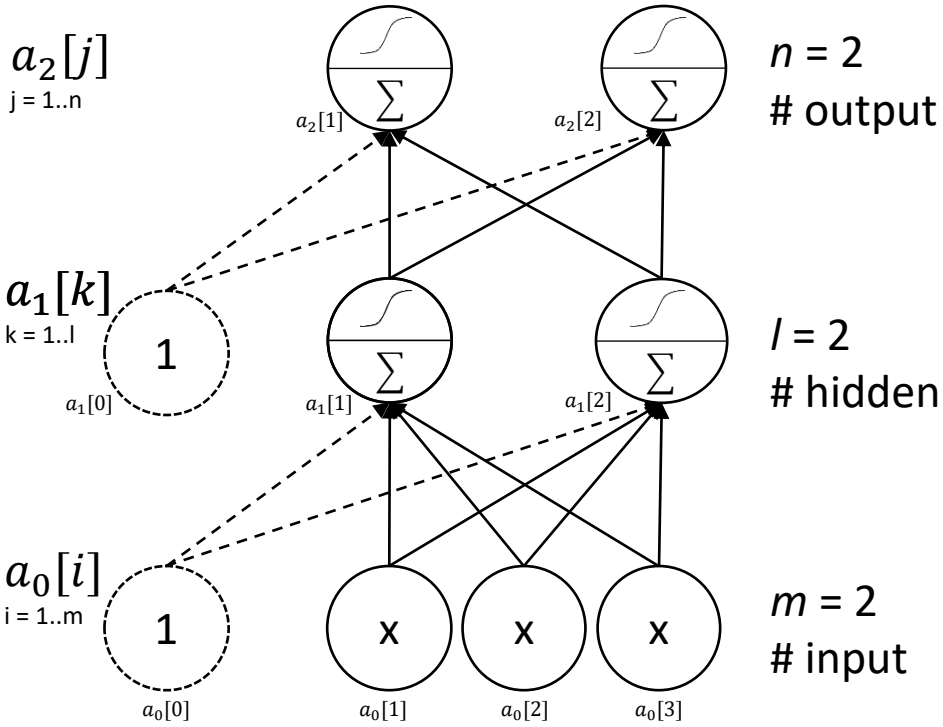
x_1	x_2	x_3	y_1	y_2
1.0000	4.0000	5.0000	0.1000	0.0500
0.1000	-5.0000	3.0000	0.1221	0.0964
6.0000	-5.5420	4.8970	0.1061	0.0702
4.0000	8.0000	9.0000	0.0996	0.0641
12.0000	-2.0000	0.0063	0.1110	0.0732
6.0000	-5.5000	4.8970	0.1060	0.0701

$w_1[i,k]$

i	w[i,1]	w[i,2]
0	0.5	0.5
1	0.1	0.2
2	0.3	0.4
3	0.5	0.6

$w_2[k,j]$

k	w[k,1]	w[k,2]
0	0.5	0.5
1	0.7	0.8
2	0.9	0.1



n = 2
output

l = 2
hidden

m = 2
input

3-2-2 Multilayer Perceptron

Multilayer Perceptrons

x, y

x_1	x_2	x_3	y_1	y_2
1.0000	4.0000	5.0000	0.1000	0.0500
0.1000	-5.0000	3.0000	0.1221	0.0964
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$w_2[k,j]$

k	w[k,1]	w[k,2]
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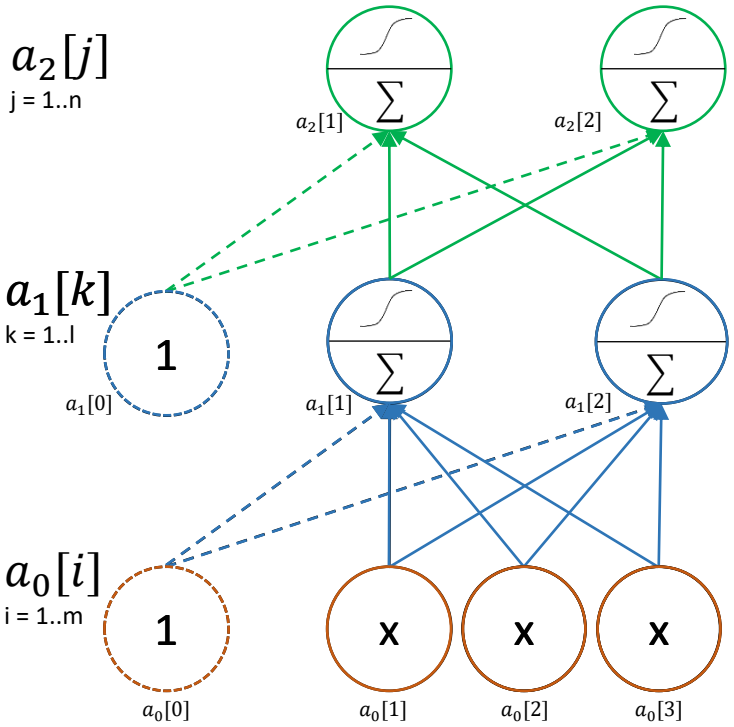
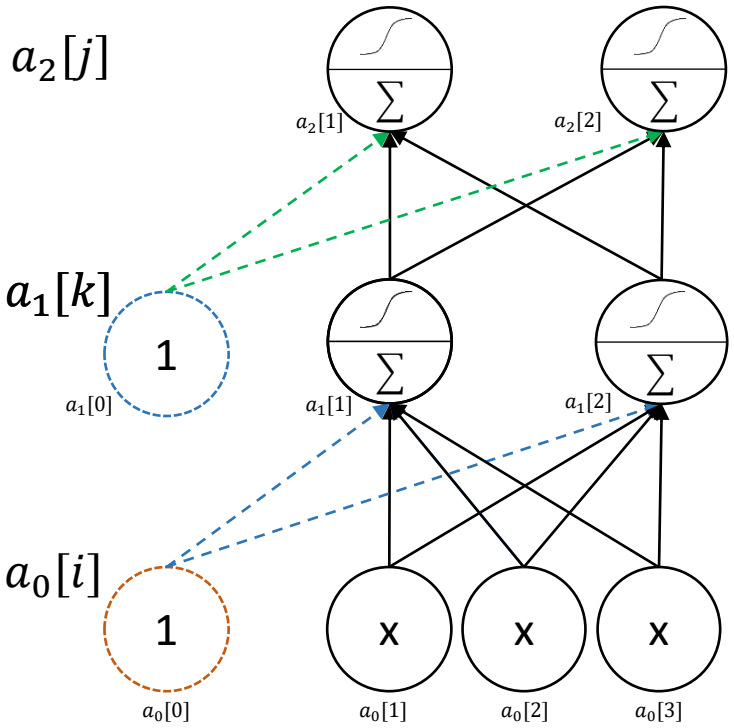


3-2-2 Multilayer Perceptron

Multilayer Perceptrons



x, y

x_1	x_2	x_3	y_1	y_2
1.0000	4.0000	5.0000	0.1000	0.0500
0.1000	-5.0000	3.0000	0.1221	0.0964
6.0000	-5.5420	4.8970	0.1061	0.0702
4.0000	8.0000	9.0000	0.0996	0.0641
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2	0.3	0.4
3	0.5	0.6

$w_2[k,j]$

k	w[k,1]	w[k,2]
0	0.5	0.5
1	0.7	0.8
2	0.9	0.1

Forward Propagation

x, y

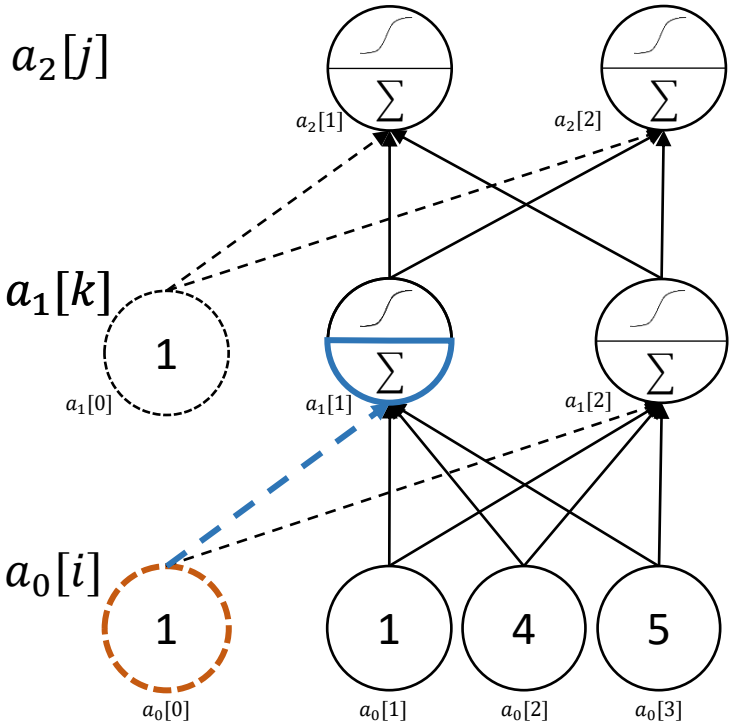
x_1	x_2	x_3	y_1	y_2
1.0000	4.0000	5.0000	0.1000	0.0500

$w_1[i,k]$

i	w[i,1]	w[i,2]
0	**0.5**	0.5
1	0.1	0.2
2	0.3	0.4
3	0.5	0.6

$w_2[k,j]$

k	w[k,1]	w[k,2]
0	0.5	0.5
1	0.7	0.8
2	0.9	0.1



$$s_1[k] = \sum_{i=0}^{m} w_1[i,k] * a_0 i \quad \text{for k} = 1..l$$

$$s_1[1] = w_1[0,1] * a_0[0] = .05 * 1$$

Forward Propagation

x, y

x_1	x_2	x_3	y_1	y_2
1.0000	4.0000	5.0000	0.1000	0.0500

$w_1[i,k]$

i	w[i,1]	w[i,2]
0	0.5	0.5
1	**0.1**	0.2
2	0.3	0.4
3	0.5	0.6

$w_2[k,j]$

k	w[k,1]	w[k,2]
0	0.5	0.5
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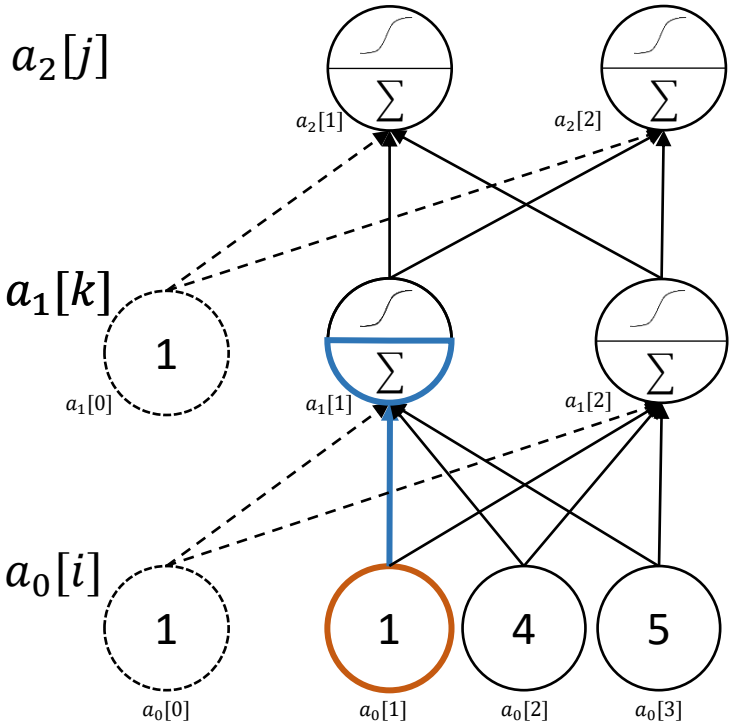


$$s_1[k] = \sum_{i=0}^{3} w_1[i,k] * a_0 i \quad \text{for k} = 1, 2$$

$$s_1[1] = w_1[0,1] * a_0[0] = .05 * 1$$

$$w_1[1,1] * a_0[1] = .1 * 1$$

Forward Propagation

x, y

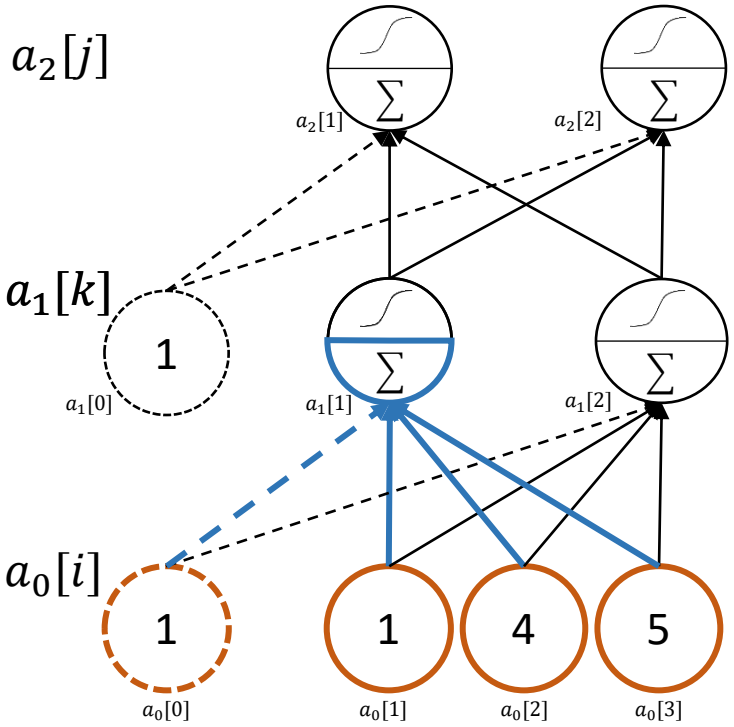
x_1	x_2	x_3	y_1	y_2
1.0000	4.0000	5.0000	0.1000	0.0500

$w_1[i,k]$

i	w[i,1]	w[i,2]
0	**0.5**	0.5
1	**0.1**	0.2
2	**0.3**	0.4
3	**0.5**	0.6

$w_2[k,j]$

k	w[k,1]	w[k,2]
0	0.5	0.5
1	0.7	0.8
2	0.9	0.1



$$s_1[k] = \sum_{i=0}^{3} w_1[i,k] * a_0 i \quad \text{for k} = 1, 2$$

$$s_1[1] = w_1[0,1] * a_0[0] = .05 * 1 +$$
$$w_1[1,1] * a_0[1] = .1 * 1 +$$
$$w_1[2,1] * a_0[2] = .3 * 4 +$$
$$w_1[3,1] * a_0[3] = .5 * 5$$
$$= 4.3$$

Forward Propagation

x, y

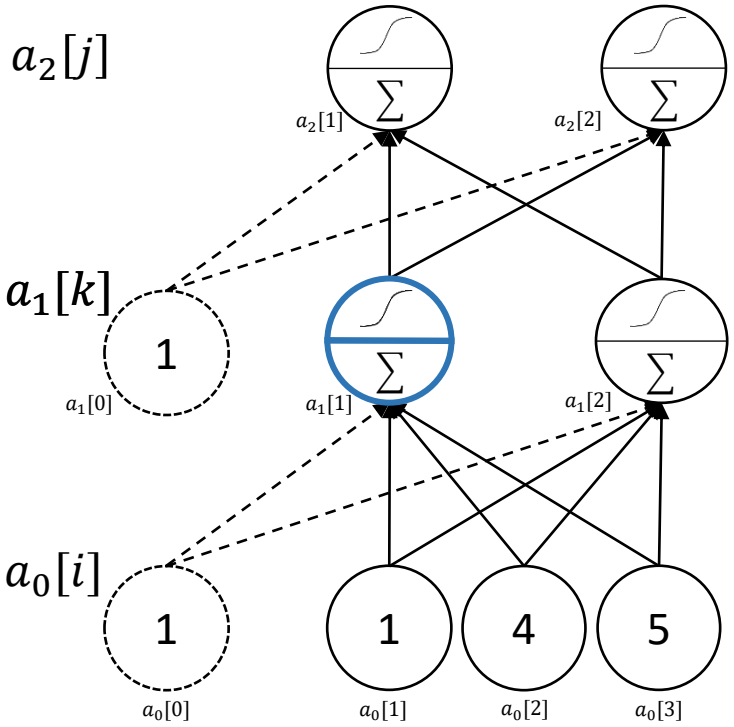
x_1	x_2	x_3	y_1	y_2
1.0000	4.0000	5.0000	0.1000	0.0500

$w_1[i,k]$

i	w[i,1]	w[i,2]
0	0.5	0.5
1	0.1	0.2
2	0.3	0.4
3	0.5	0.6

$w_2[k,j]$

k	w[k,1]	w[k,2]
0	0.5	0.5
1	0.7	0.8
2	0.9	0.1



$$a_1[k] = \frac{1}{1 + e^{-s_1[k]}} \quad \text{for k} = 1, 2$$

$$a_1[1] = \frac{1}{1+e^{-4.3}} = .9866$$

Forward Propagation

x, y

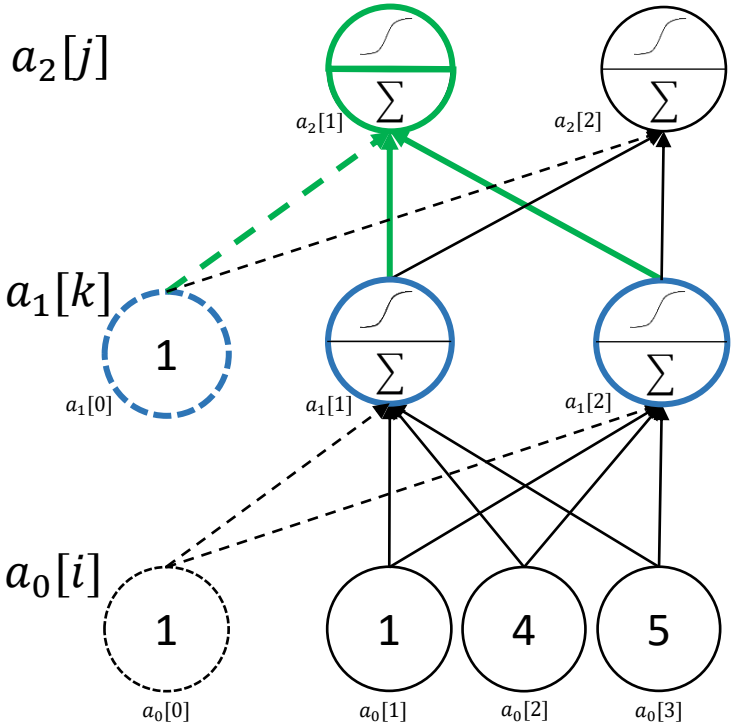
x_1	x_2	x_3	y_1	y_2
1.0000	4.0000	5.0000	0.1000	0.0500

$w_1[i,k]$

i	w[i,1]	w[i,2]
0	0.5	0.5
1	0.1	0.2
2	0.3	0.4
3	0.5	0.6

$w_2[k,j]$

k	w[k,1]	w[k,2]
0	0.5	0.5
1	0.7	0.8
2	0.9	0.1



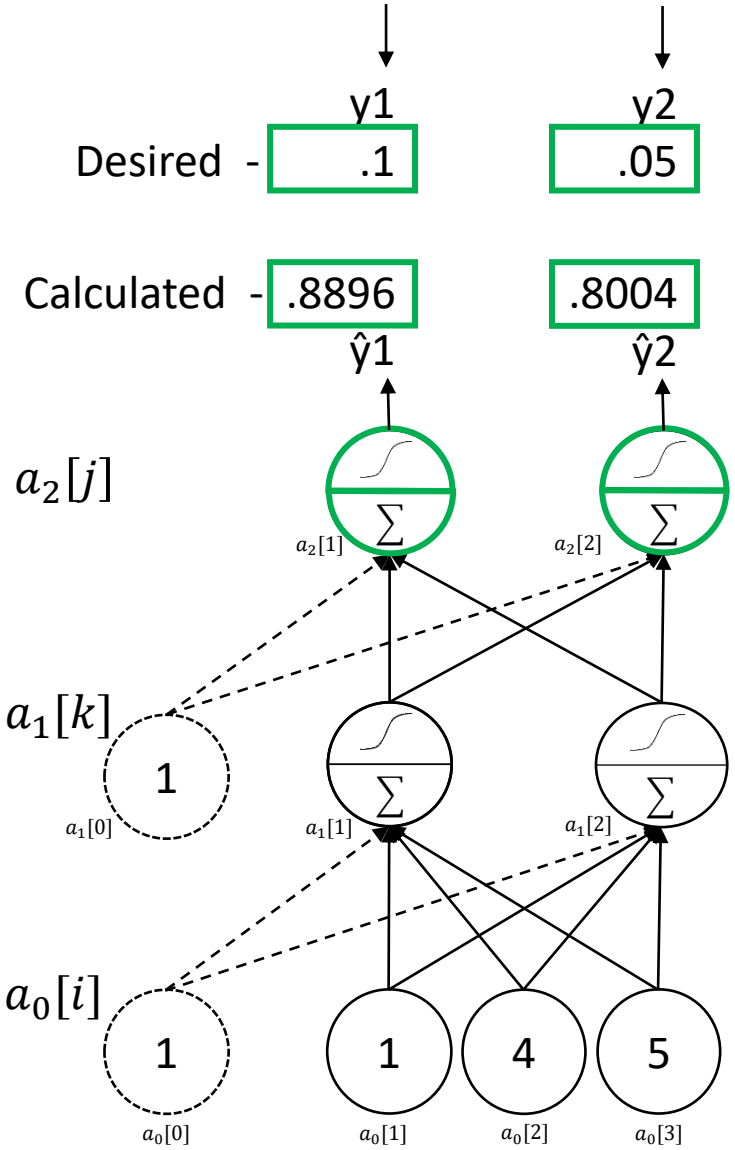
$$s_2[j] = \sum_{k=0}^{l} w_2[k,j] * a_1 i \quad \text{for j} = 1..\text{m}$$

$$a_2[j] = \frac{1}{1 + e^{-s_2[j]}} \quad \text{for j} = 1, 2$$

$$s_2[1] = w_2[0,2] * a_1[0] = .5 * 1$$
$$w_2[1,2] * a_1[1] = .7 * .9866$$
$$w_2[2,2] * a_1[2] = .9 * .9950$$
$$= 2.0862$$

$$a_2[1] = \frac{1}{1+e^{-2.0862}} = .8896 = \hat{y}$$

Error



x, y

x_1	x_2	x_3	y_1	y_2
1.0000	4.0000	5.0000	0.1000	0.0500

$w_1[i,k]$

i	w[i,1]	w[i,2]
0	0.5	0.5
1	0.1	0.2
2	0.3	0.4
3	0.5	0.6

$w_2[k,j]$

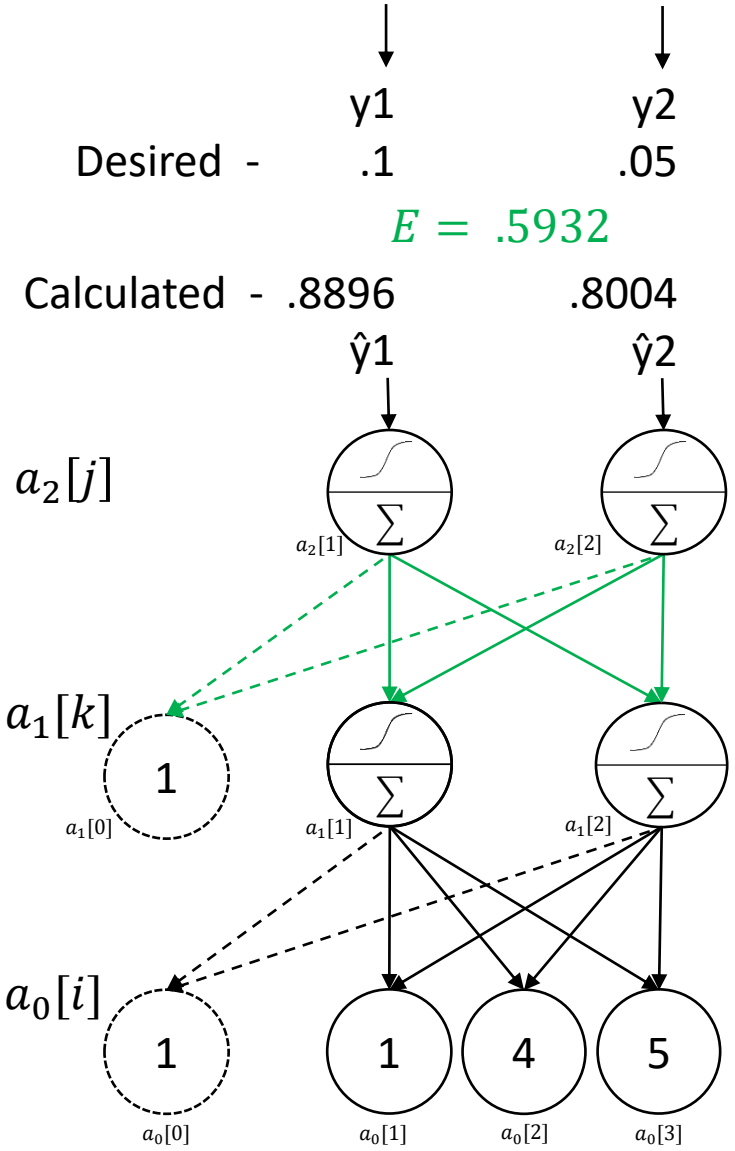
k	w[k,1]	w[k,2]
0	0.5	0.5
1	0.7	0.8
2	0.9	0.1

$$E = \frac{1}{2}\sum_{j=1}^{n}(a_2[j] - y_r[j])^2$$

$$= \frac{1}{2}\left((.8896 - .1)^2 + (.8004 - .5)^2\right)$$

$$= .5932$$

Backpropagation



x, y

x_1	x_2	x_3	y_1	y_2
1.0000	4.0000	5.0000	0.1000	0.0500

$w_1[i,k]$

i	w[i,1]	w[i,2]
0	0.5	0.5
1	0.1	0.2
2	0.3	0.4
3	0.5	0.6

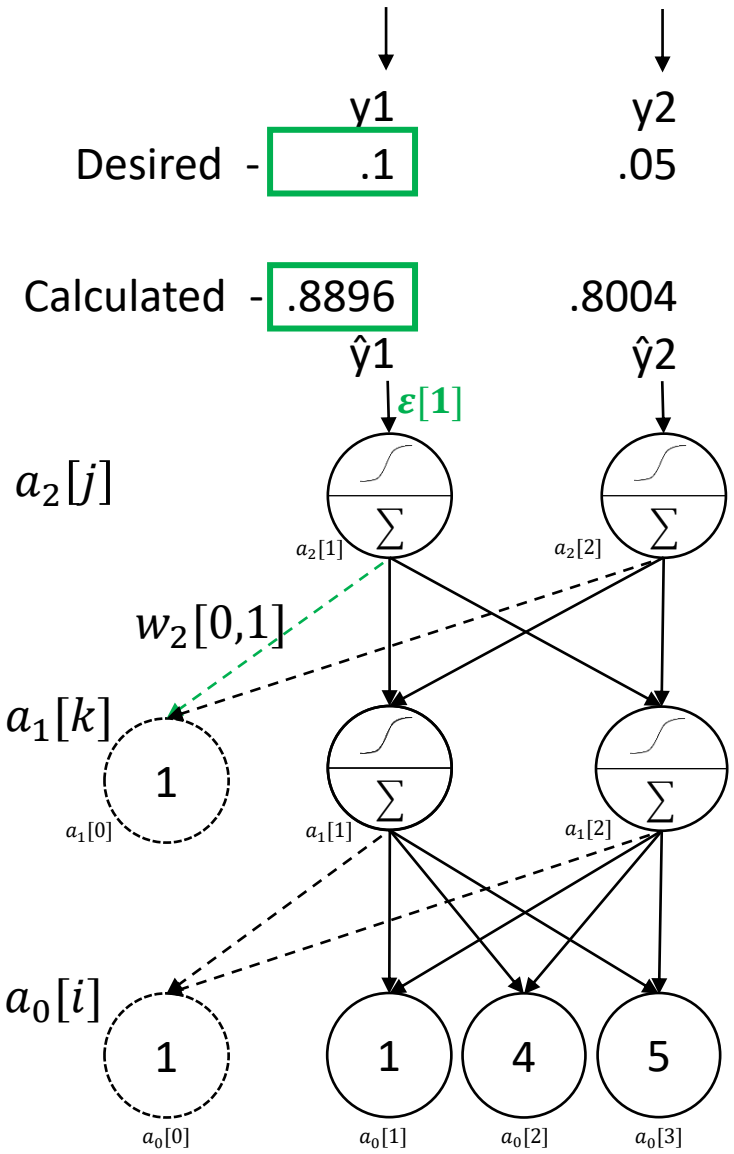
$w_2[k,j]$

k	w[k,1]	w[k,2]
0	0.5	0.5
1	0.7	0.8
2	0.9	0.1

$$\frac{\partial E}{\partial w_2[k,j]} = \frac{\partial E}{\partial a_2[j]} * \frac{\partial a_2[j]}{\partial s_2[j]} * \frac{\partial s_2[j]}{\partial w_2[k,j]}$$

$$= \{\varepsilon[j]\} * \{a_2[j](1 - a_2[j])\} * \{a_1[k]\}$$

Backpropagation



x, y

x_1	x_2	x_3	y_1	y_2
1.0000	4.0000	5.0000	0.1000	0.0500

$w_1[i,k]$

i	w[i,1]	w[i,2]
0	0.5	0.5
1	0.1	0.2
2	0.3	0.4
3	0.5	0.6

$w_2[k,j]$

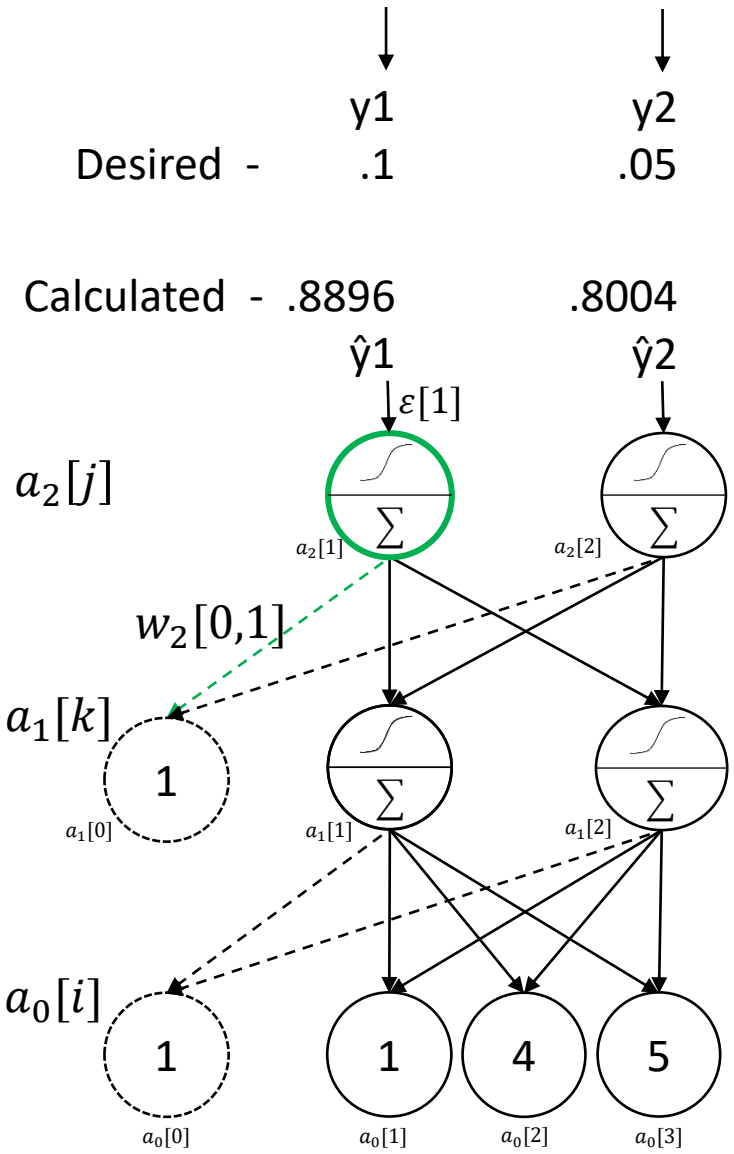
k	w[k,1]	w[k,2]
0	0.5	0.5
1	0.7	0.8
2	0.9	0.1

$$\frac{\partial E}{\partial w_2[0,1]} = \frac{\partial E}{\partial a_2[1]} * \frac{\partial a_2[1]}{\partial s_2[1]} * \frac{\partial s_2[1]}{\partial w_2[0,1]}$$

$$= \{\varepsilon[1]\} * \{a_2[1](1 - a_2[1])\} * \{a_1[0]\}$$

$$= .8896 - .1$$

Backpropagation



x, y

x_1	x_2	x_3	y_1	y_2
1.0000	4.0000	5.0000	0.1000	0.0500

$w_1[i,k]$

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0	0.5	0.5
1	0.1	0.2
2	0.3	0.4
3	0.5	0.6

$w_2[k,j]$

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1	0.7	0.8
2	0.9	0.1

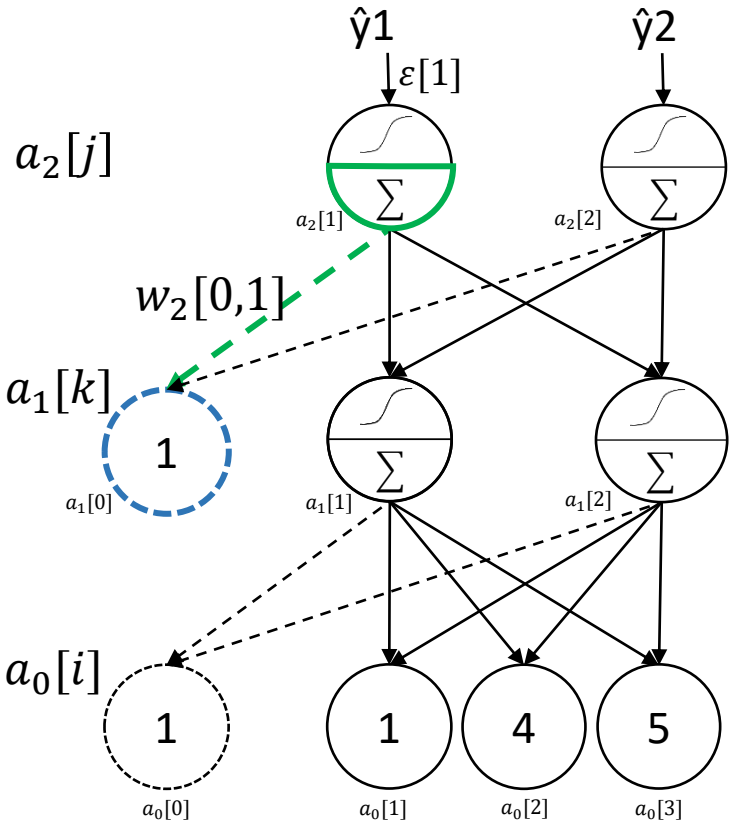
$$\frac{\partial E}{\partial w_2[0,1]} = \frac{\partial E}{\partial a_2[1]} * \frac{\partial a_2[1]}{\partial s_2[1]} * \frac{\partial s_2[1]}{\partial w_2[0,1]}$$

$$= \{\varepsilon[1]\} * \{a_2[1](1 - a_2[1])\} * \{a_1[0]\}$$

$$= .8896 - .1 * .8896(1 - .8896)$$

Backpropagation

	y1	y2
Desired -	.1	.05
Calculated -	.8896	.8004



x, y

x_1	x_2	x_3	y_1	y_2
1.0000	4.0000	5.0000	0.1000	0.0500

$w_1[i,k]$

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0	0.5	0.5
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2	0.3	0.4
3	0.5	0.6

$w_2[k,j]$

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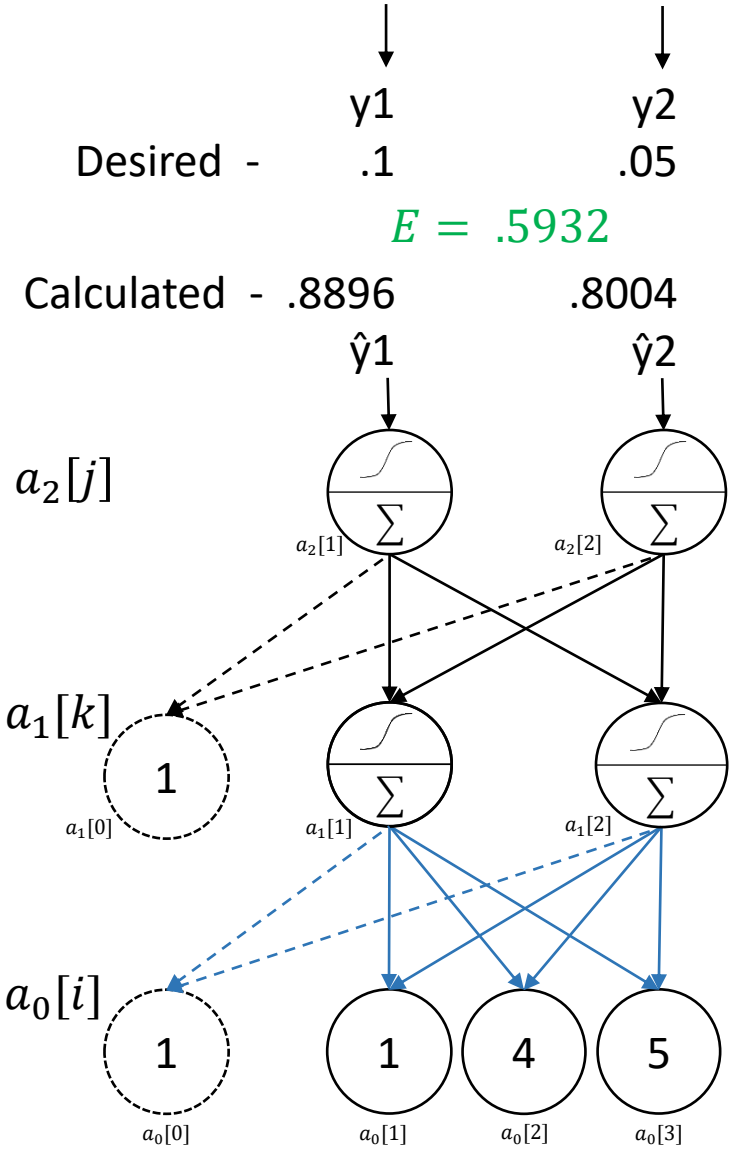
$$\frac{\partial E}{\partial w_2[0,1]} = \frac{\partial E}{\partial a_2[1]} * \frac{\partial a_2[1]}{\partial s_2[1]} * \frac{\partial s_2[1]}{\partial w_2[0,1]}$$
$$= \{\varepsilon[1]\} * \{a_2[1](1 - a_2[1])\} * \{a_1[0]\}$$
$$= .8896 - .1 * .8896(1 - .8896) * 1$$
$$= .0775$$

$c_2[k,j]$

k	c[k,1]	c[k,2]
0	.0775	0
1	0	0
2	0	0

Backpropagation



x, y

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1.0000	4.0000	5.0000	0.1000	0.0500

$w_1[i,k]$

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0	0.5	0.5
1	0.1	0.2
2	0.3	0.4
3	0.5	0.6

$w_2[k,j]$

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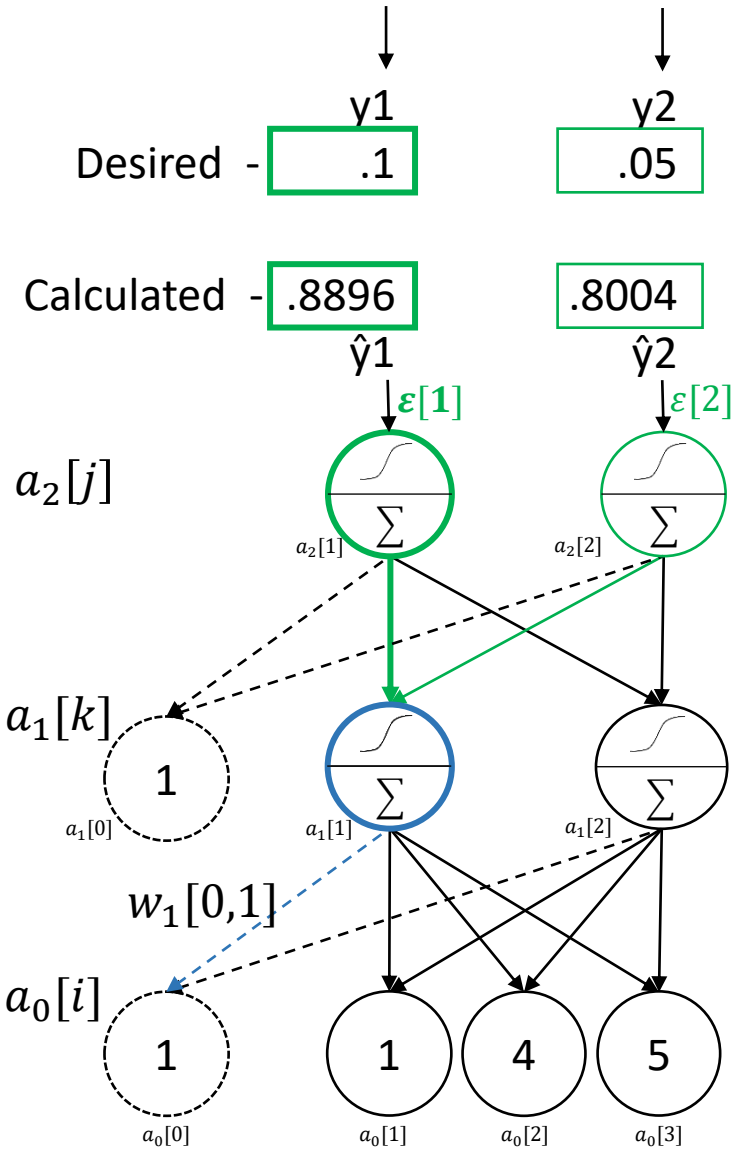
0	0.5	0.5
1	0.7	0.8
2	0.9	0.1

$$\frac{\partial E}{\partial w_1[i,k]} = \frac{\partial E}{\partial a_1[k]} * \frac{\partial a_1[k]}{\partial s_1[k]} * \frac{\partial s_1[k]}{\partial w_1[i,k]}$$

$$= \frac{\partial E}{\partial a_1[k]} * \{a_1[j](1 - a_1[j])\} * \{a_0[i]\}$$

Backpropagation



x, y

x_1	x_2	x_3	y_1	y_2
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$w_2[k,j]$

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1	0.7	0.8
2	0.9	0.1

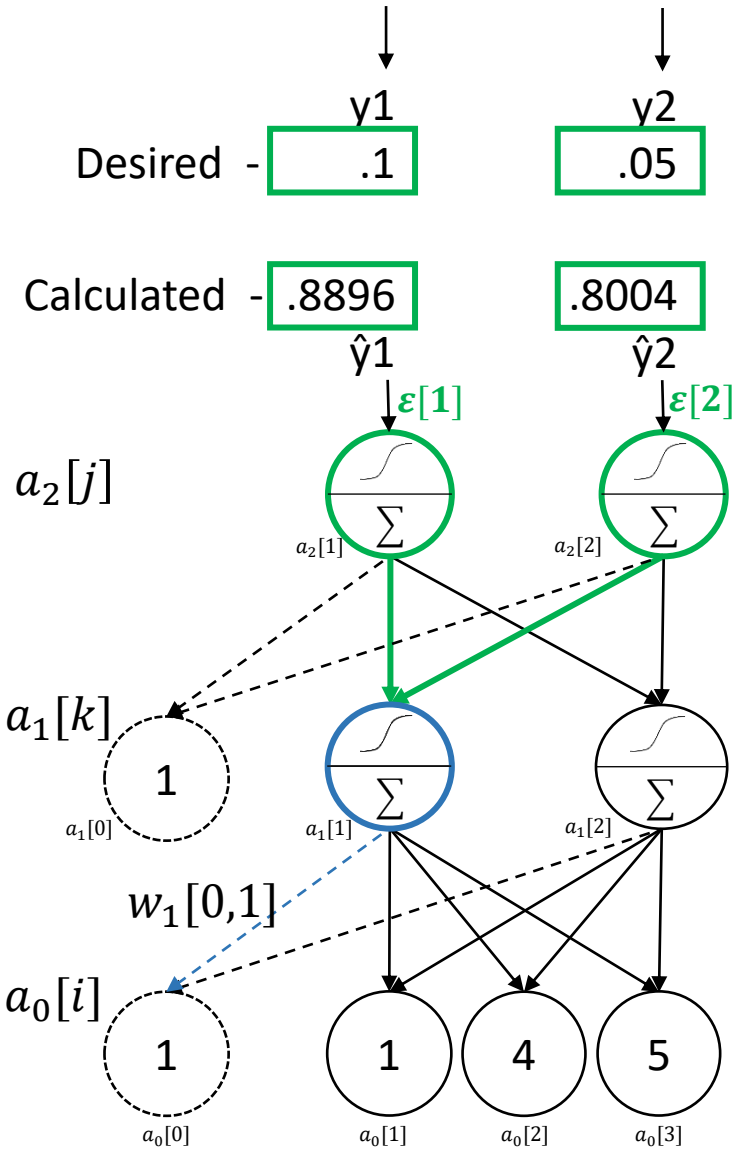
$$\frac{\partial E}{\partial w_1[i,k]} = \frac{\partial E}{\partial a_1[k]} * \frac{\partial a_1[k]}{\partial s_1[k]} * \frac{\partial s_1[k]}{\partial w_1[i,k]}$$

$$= \frac{\partial E}{\partial a_1[k]} * \{a_1[k](1 - a_1[k])\} * \{a_0[i]\}$$

$$\frac{\partial E}{\partial a_1[k]} = \sum_{j=1}^{n} \frac{\partial E}{\partial a_2[j]} * \frac{\partial a_2[j]}{\partial s_2[j]} * \frac{\partial s_2[j]}{\partial a_1[k,j]}$$

$$= \sum_{j=1}^{n} \{\varepsilon[j]\} * \{a_2[j](1 - a_2[j])\} * \{w_2[k,j]\}$$

Backpropagation



x, y

x_1	x_2	x_3	y_1	y_2
1.0000	4.0000	5.0000	0.1000	0.0500

$w_1[i,k]$

i	w[i,1]	w[i,2]
0	0.5	0.5
1	0.1	0.2
2	0.3	0.4
3	0.5	0.6

$w_2[k,j]$

k	w[k,1]	w[k,2]
0	0.5	0.5
1	**0.7**	**0.8**
2	0.9	0.1

$$\frac{\partial E}{\partial w_1[0,1]} = \frac{\partial E}{\partial a_1[1]} * \frac{\partial a_1[1]}{\partial s_1[1]} * \frac{\partial s_1[1]}{\partial w_1[0,1]}$$

$$= \frac{\partial E}{\partial a_1[1]} * \{a_1[1](1 - a_1[1])\} * \{a_0[0]\}$$

$$\frac{\partial E}{\partial a_1[k]} = \sum_{j=1}^{n} \frac{\partial E}{\partial a_2[j]} * \frac{\partial a_2[j]}{\partial s_2[j]} * \frac{\partial s_2[j]}{\partial a_1[k,j]}$$

$$= \sum_{j=1}^{n} \{\varepsilon[j]\} * \{a_2[j](1 - a_2[j])\} * \{w_2[k,j]\}$$

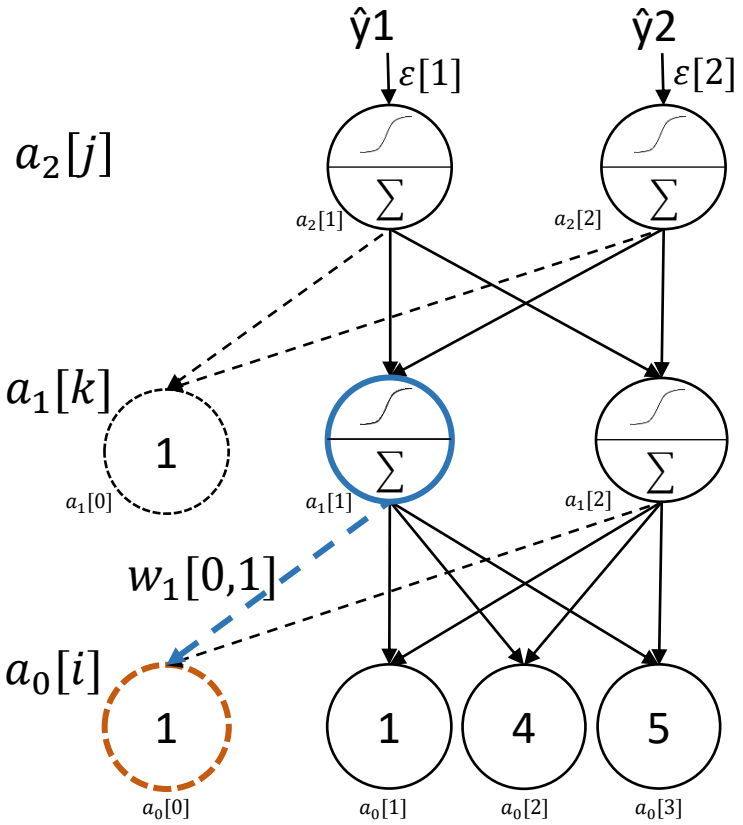
$$= .8896 - .1 * .8896(1 - .8896) * .7$$

$$+ .8004 - .05 * .8004(1 - .8004) * .8$$

$$= .1502$$

Backpropagation

	y1	y2
Desired -	.1	.05
Calculated -	.8896	.8004



x, y

x_1	x_2	x_3	y_1	y_2
1.0000	4.0000	5.0000	0.1000	0.0500

$w_1[i,k]$

i	w[i,1]	w[i,2]
0	0.5	0.5
1	0.1	0.2
2	0.3	0.4
3	0.5	0.6

$w_2[k,j]$

k	w[k,1]	w[k,2]
0	0.5	0.5
1	0.7	0.8
2	0.9	0.1

$$\frac{\partial E}{\partial w_1[0,1]} = \frac{\partial E}{\partial a_1[1]} * \frac{\partial a_1[1]}{\partial s_1[1]} * \frac{\partial s_1[1]}{\partial w_1[0,1]}$$

$$= \frac{\partial E}{\partial a_1[1]} * \{a_1[1](1 - a_1[1])\} * \{a_0[0]\}$$

$$= .1502 * .9866(1 - .9866) * 1$$

$$= .0020$$

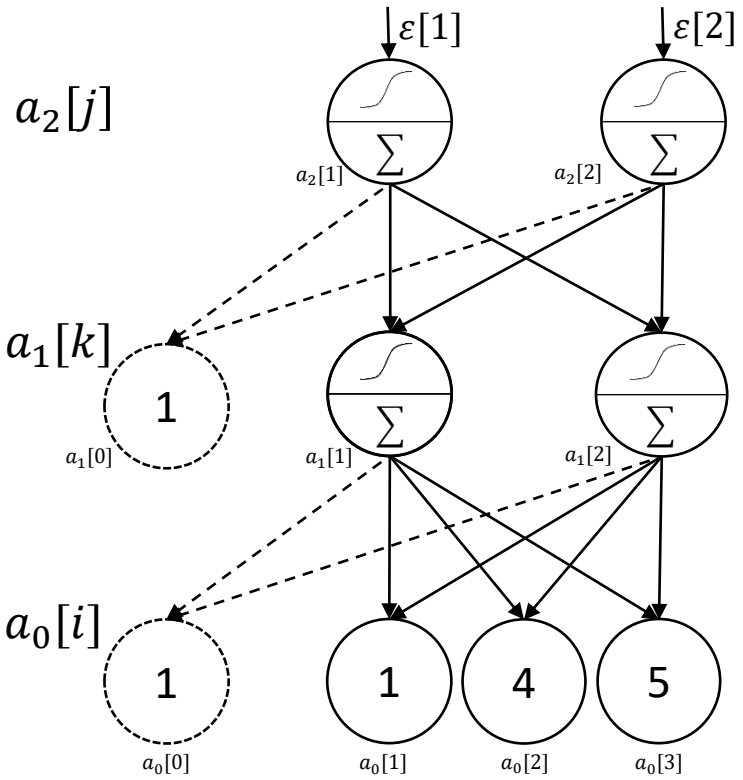
$c_1[i,k]$

i	w[i,1]	w[i,2]
0	.0020	0
1	0	0
2	0	0
3	0	0

Backpropagation

Desired - y1 y2

Calculated - ŷ1 ŷ2



x, y

	x_1	x_2	x_3	y_1	y_2
r =1	1.0000	4.0000	5.0000	0.1000	0.0500
⋮	0.1000	-5.0000	3.0000	0.1221	0.0964
	6.0000	-5.5420	4.8970	0.1061	0.0702
r =4	4.0000	8.0000	9.0000	0.0996	0.0641
	12.0000	-2.0000	0.0063	0.1110	0.0732
	6.0000	-5.5000	4.8970	0.1060	0.0701

Learning rate α = .01

$w^* = w - \alpha * c$

$w_1[i,k]$

i	w[i,1]	w[i,2]
0	0.5	0.5
1	0.1	0.2
2	0.3	0.4
3	0.5	0.6

$w_2[k,j]$

k	w[k,1]	w[k,2]
0	0.5	0.5
1	0.7	0.8
2	0.9	0.1

$c_1[i,k]$

i	w[i,1]	w[i,2]
0	.0604	.0332
1	.1134	.0427
2	-.2929	-.1658
3	.2193	.1129

$c_2[k,j]$

k	c[k,1]	c[k,2]
0	.3447	.4816
1	.2929	.4176
2	.2930	.4191

$w_1{}^*[i,k]$

i	w[i,1]	w[i,2]
0	.4994	.4997
1	.0989	.1996
2	.3029	.4017
3	.4978	.5989

$w_2{}^*[k,j]$

k	w[k,1]	w[k,2]
0	.4966	.0039
1	.4971	.0038
2	.0021	.0004