Brownian Motion with Ito and Stratonovich

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Install Packages

The focus of this demonstration is the "Sim.DiffPrc" package which is designed for simulating diffusion processes. Click the link for full documentation:

https://cran.r-project.org/web/packages/Sim.DiffProc/Sim.DiffProc.pdf

ggplot2 is a package for graphing, tidyr and dplyr are for data manipulation.

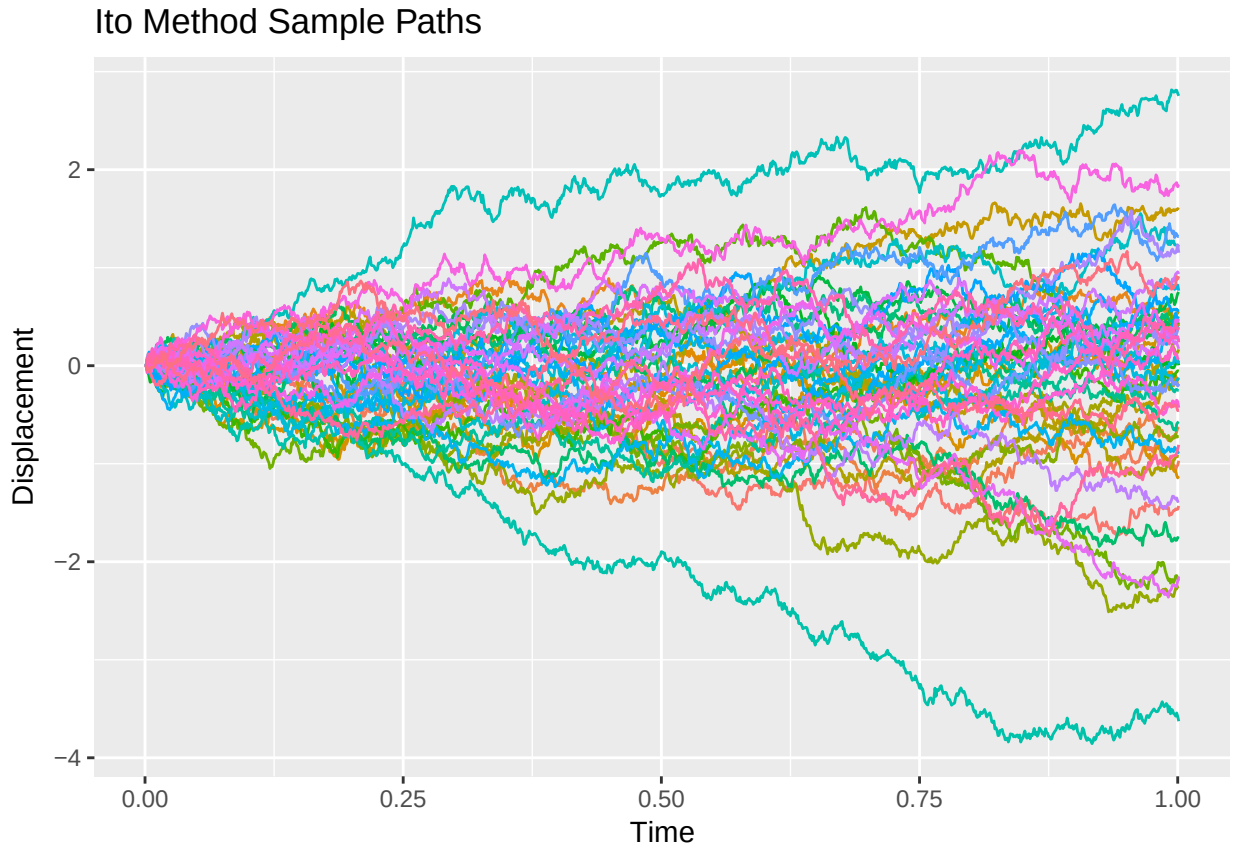
```
library(Sim.DiffProc)
library(ggplot2)
library(dplyr)
library(tidyr)
```

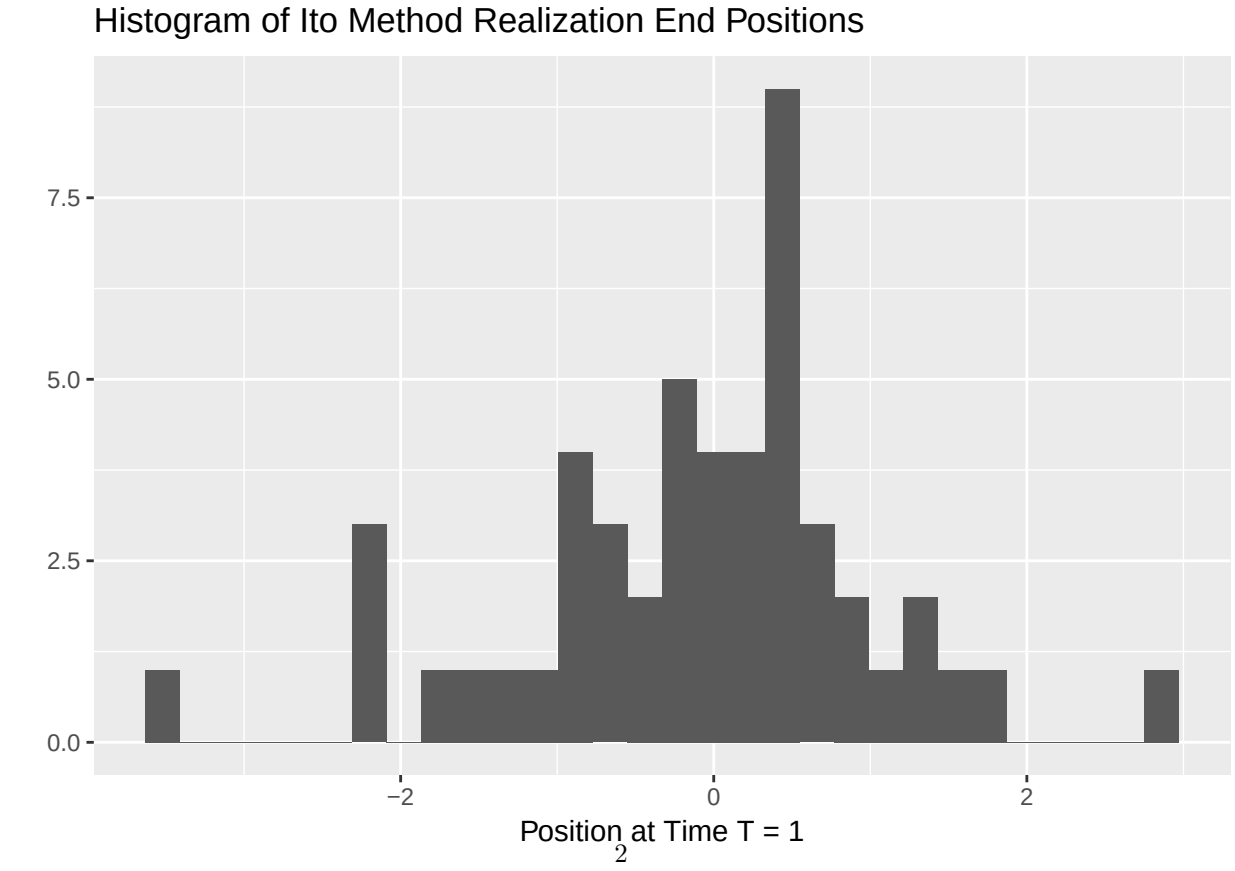
Ito Method Realizations of Basic Browian Motion

For basic Brownian motion with the martingale property there is no drift. My choice of 1 as the diffusion coefficent is arbitrary.

```
#Specify the drift and diffusion coefficents
f = expression(0)
g = expression(1)

#Specify desired number of realizations
number = 50

#Returns an 'snssde1d' object containing realizations of the specified diffusion process
Realizations = snssde1d(drift=f,diffusion=g,M=number, type = "ito")

#Returns the final positions of the realizations (i.e. at position t = 1)
End_Position = as.data.frame(rsde1d(Realizations, at = 1))
colnames(End_Position) = c("Position")

#Extract the time series' from the 'snssde1d' object and transform to a data frame
#to make plotting with ggplot2 possible
TS =  as.data.frame(Realizations$X)
```

Plots For Ito Realizations



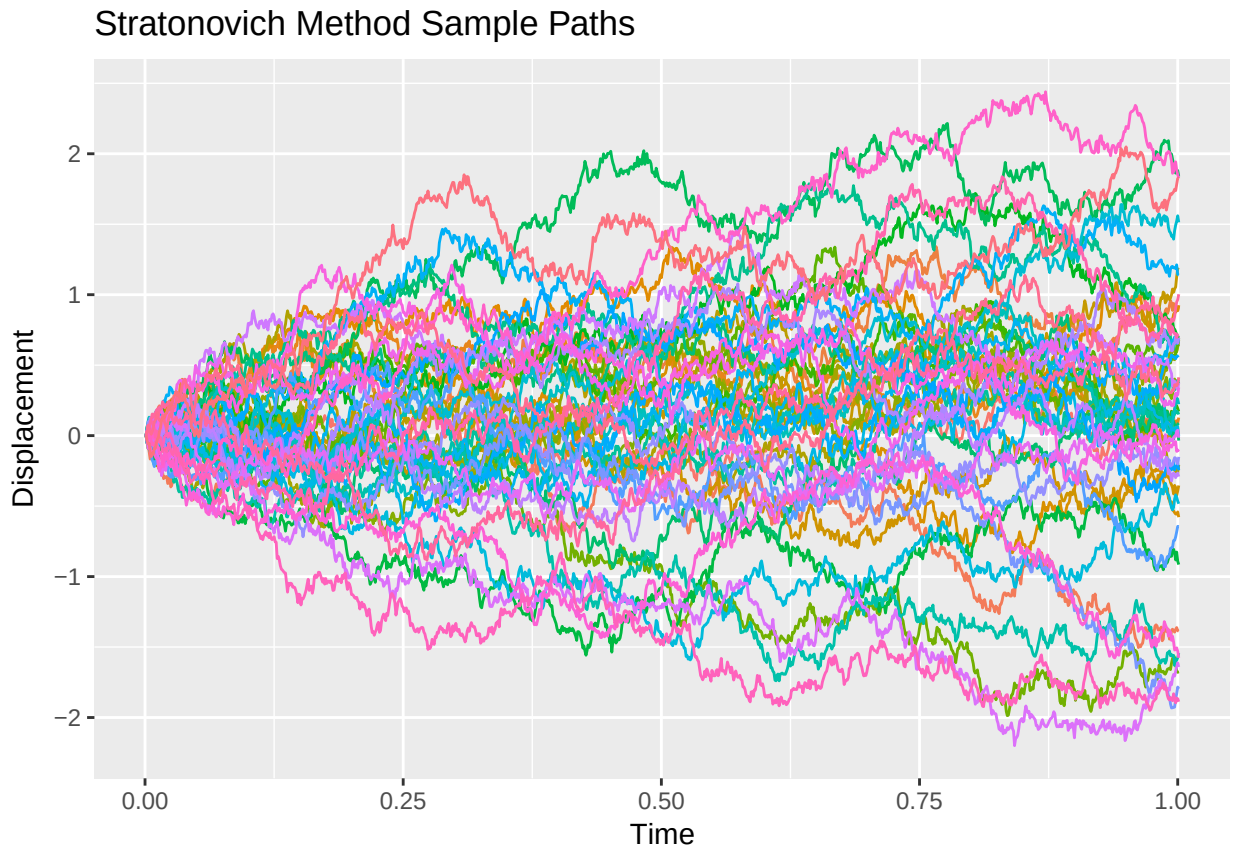


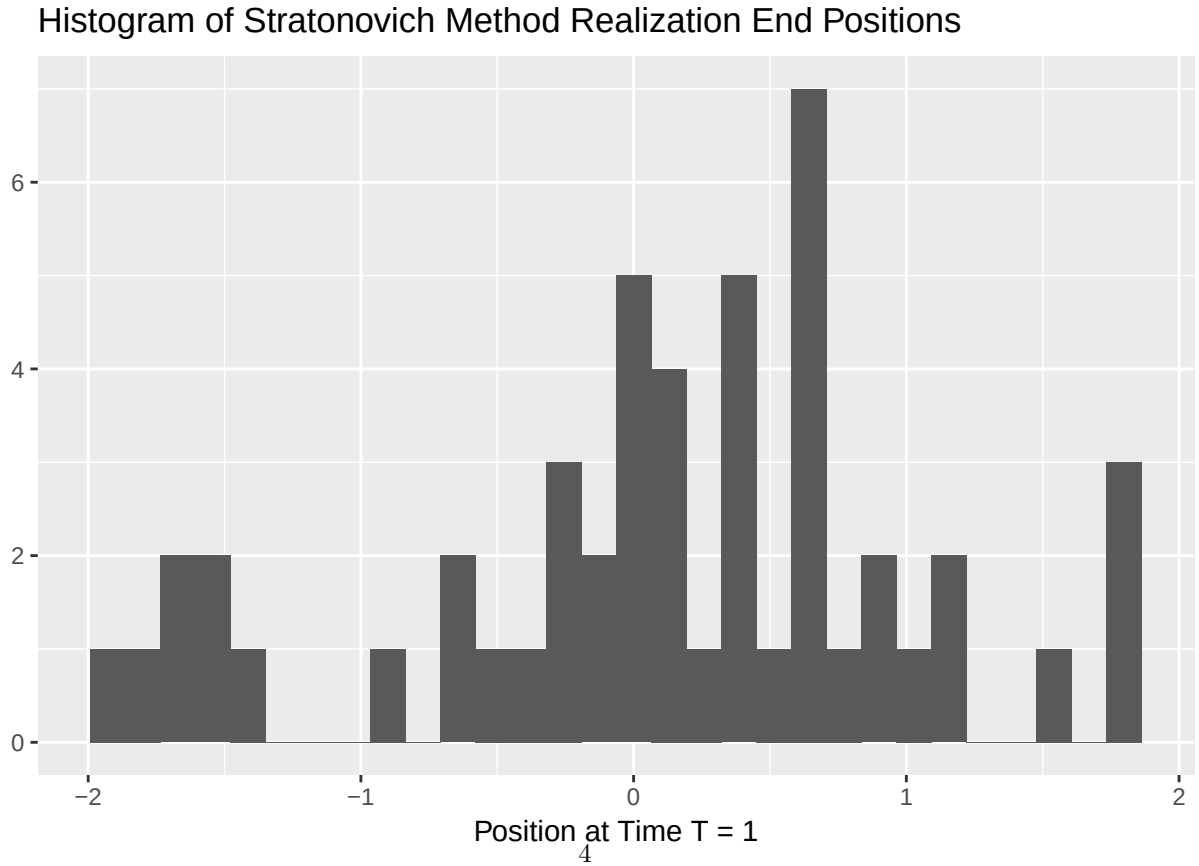
Stratonovich Method Realizations of Basic Browian Motion

I am replicating the above example with the only change being the method.

```
#Specify the drift and diffusion coefficents
f = expression(0)
g = expression(1)

#Specify desired number of realizations
number = 50

#Returns an 'snssde1d' object containing realizations of the specified diffusion process
#The type has changed from ito to Stratonovich
Realizations <- snssde1d(drift=f,diffusion=g,M=number, type = "str")

#Returns the final positions of the realizations (i.e. at position t = 1)
End_Position <- as.data.frame(rsde1d(Realizations, at = 1))
colnames(End_Position) = c("Position")

#Extract the time series' from the 'snssde1d' object and transform to a data frame
#to make plotting with ggplot2 possible
TS =  as.data.frame(Realizations$X)
```

Plots For Stratonovich Method Realizations





Ito Method Realizations of Geometric Browian Motion

Geometric Brownian motion has drift and diffusion coefficients that are functions of position. Because of this, the process cannot start at zero if we want non-trivial results.

```
#Specify the drift and diffusion coefficents
f = expression(3*x)
g = expression(.5*x)

#Specify desired number of realizations
number = 50

#Returns an 'snssde1d' object containing realizations of the specified diffusion process.
#Note that the x0 value is no longer 0.
Realizations = snssde1d(drift=f,diffusion=g,M=number, type = "ito", x0 = 1)

#Returns the final positions of the realizations (i.e. at position t = 1)
End_Position = as.data.frame(rsde1d(Realizations, at = 1))
colnames(End_Position) = c("Position")

#Extract the time series' from the 'snssde1d' object and transform to a data frame
#to make plotting with ggplot2 possible
TS =  as.data.frame(Realizations$X)
```

Plots For Ito Realizations Of Geometric Brownian Motion