Random Walks with the ARMA Model

Graeme Keleher

June 17, 2018

Load Packages

```
library(tidyverse)
library(MASS)
library(quantmod)
```

Characteristics of a True Random Walk

A true random walk is a martingale, meaning the expected value of the walk at time t+1 is the value at t.

Define a Basic Random Walk

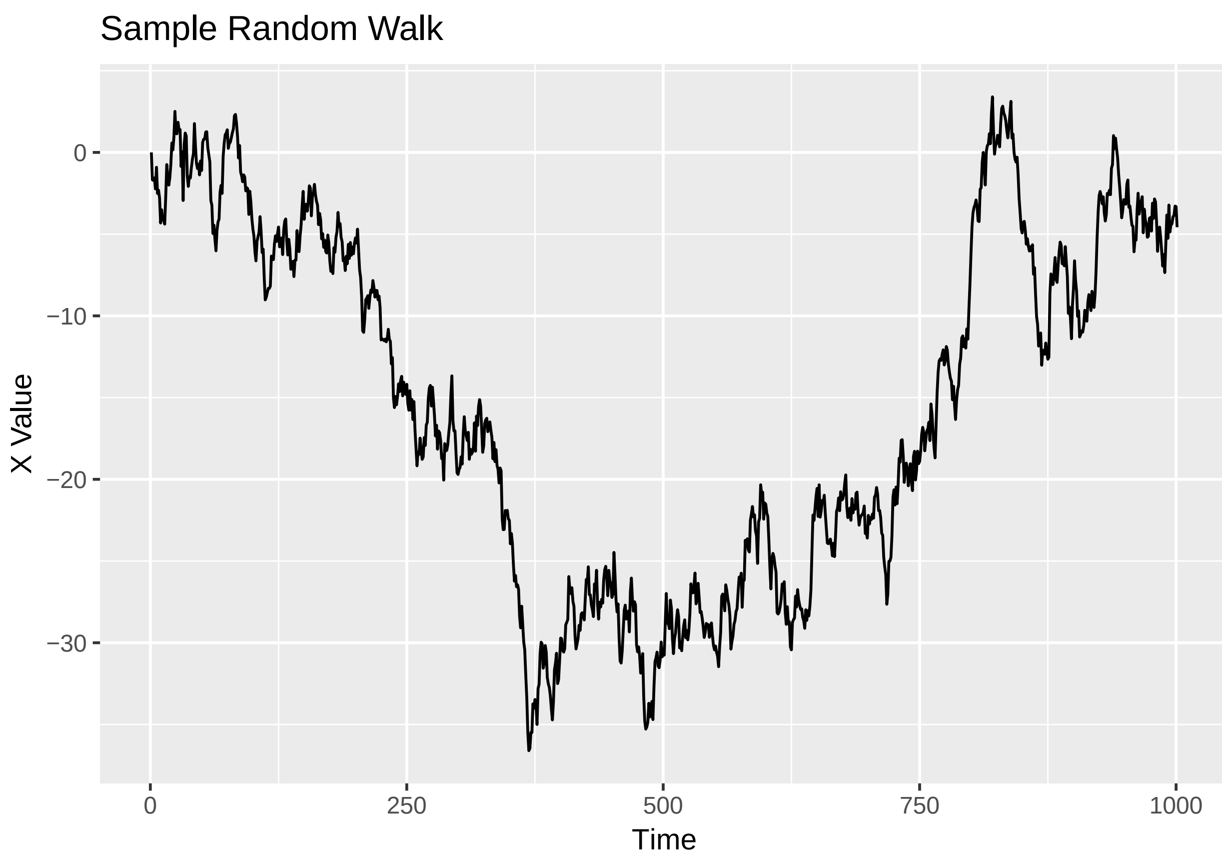
I am using the armia.sim function which is part of base R.

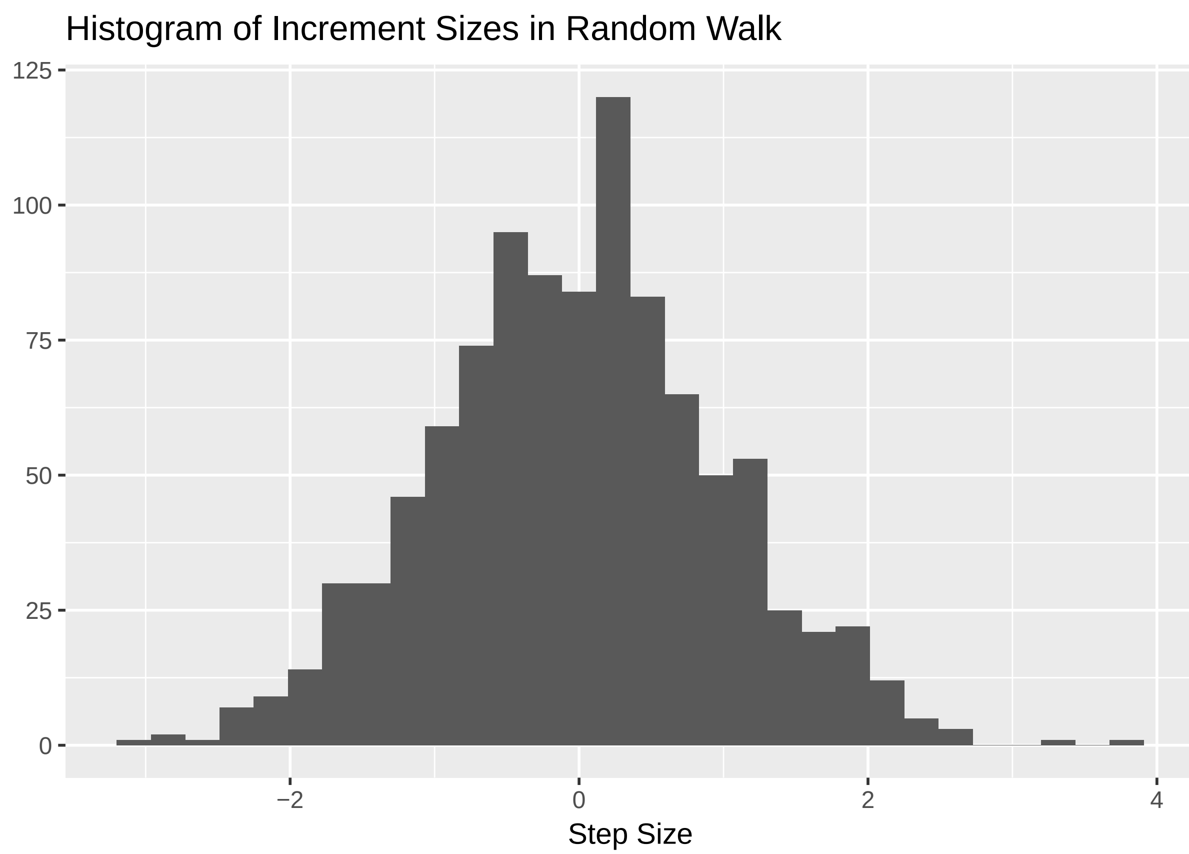
```
# Generate a random walk model witout drift and a standard deviation of 1.
random_walk <- arima.sim(model = list(order = c(0, 1, 0)), n = 1000, mean = 0, sd = 1)

# Calculate the first difference of the random walk, this will be white noise for a true random walk.
random_walk_diff <- diff(random_walk)
```

Plots of Random Walk and the Distribution of Increments

The random walk looks as it should and we get what looks like a rough normal distribution in the histogram of increments.


Sample Random Walk
0
−10
−20
−30
X Value
0
250
500
750
1000
Time



Fit Normal Distribtion to Increment Data

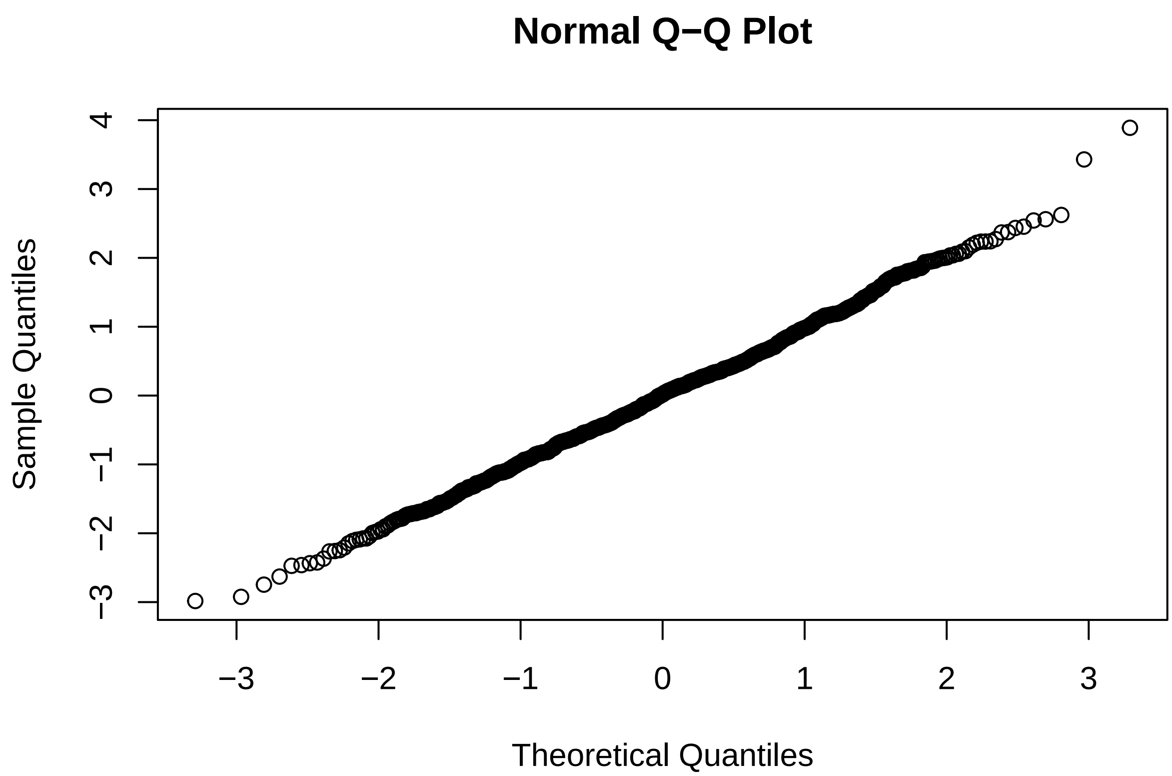
As expected, the fitdistr function fits the data to a normal distribution with a mean of ~0 and standard deviation of ~1.

```
Noise_Vec = as.vector(white_noise_df$x)
fitdistr(Noise_Vec, "normal")
```

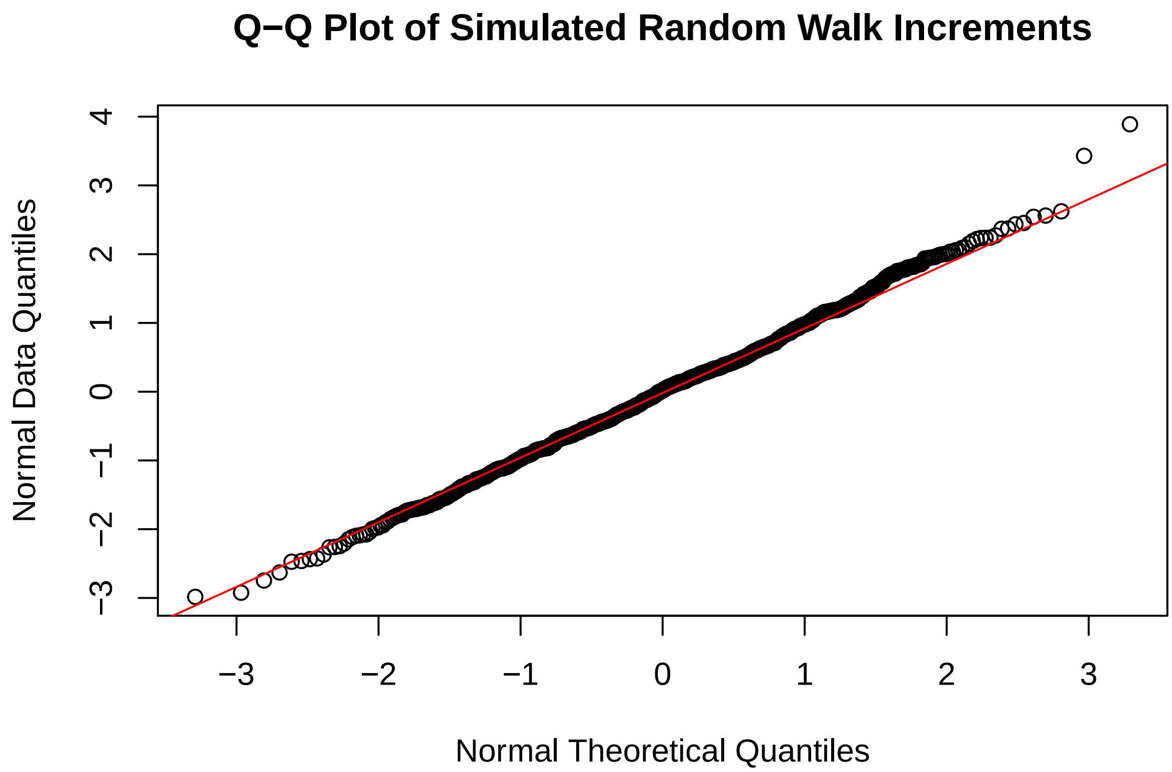
```
##        mean            sd
##   -0.004574482    0.988675779
##  ( 0.031264673) ( 0.022107463)
```

Q-Q plot

The QQ plot clearly shows that the increments of the simulated random walk are all but normally distributed.



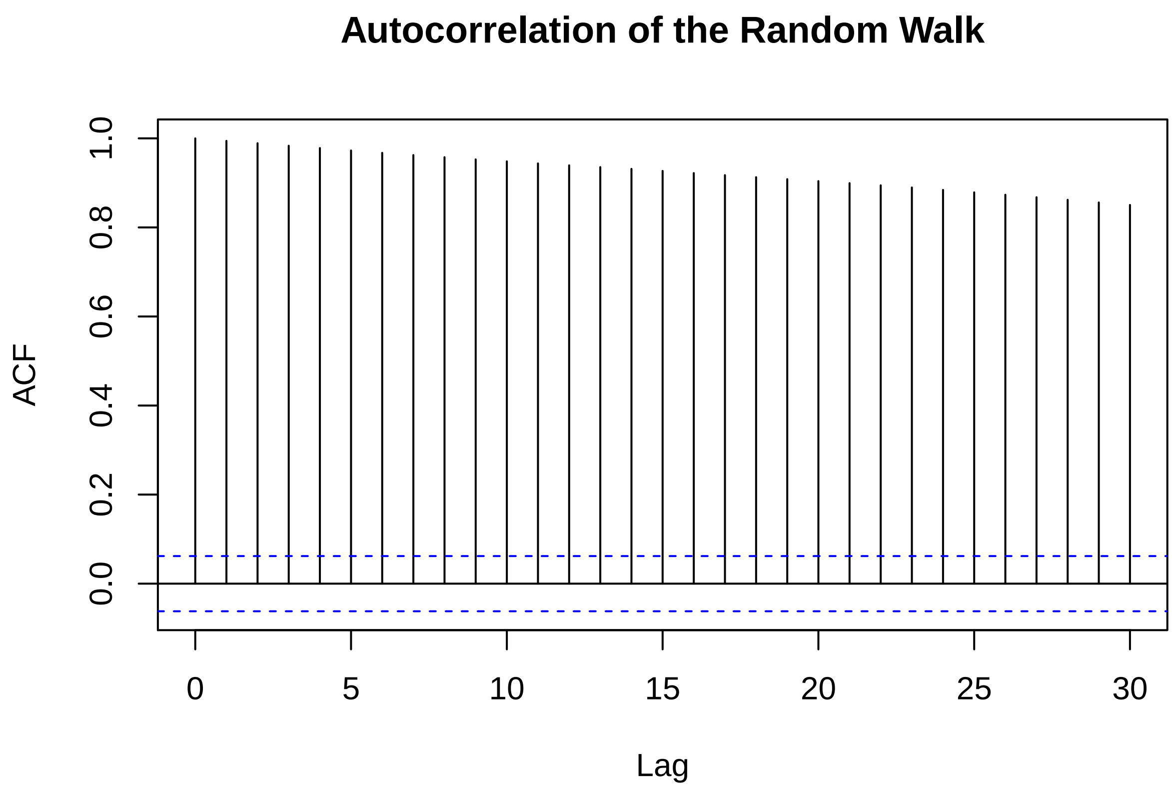

```
qqline(Noise_Vec,col = "red")
```



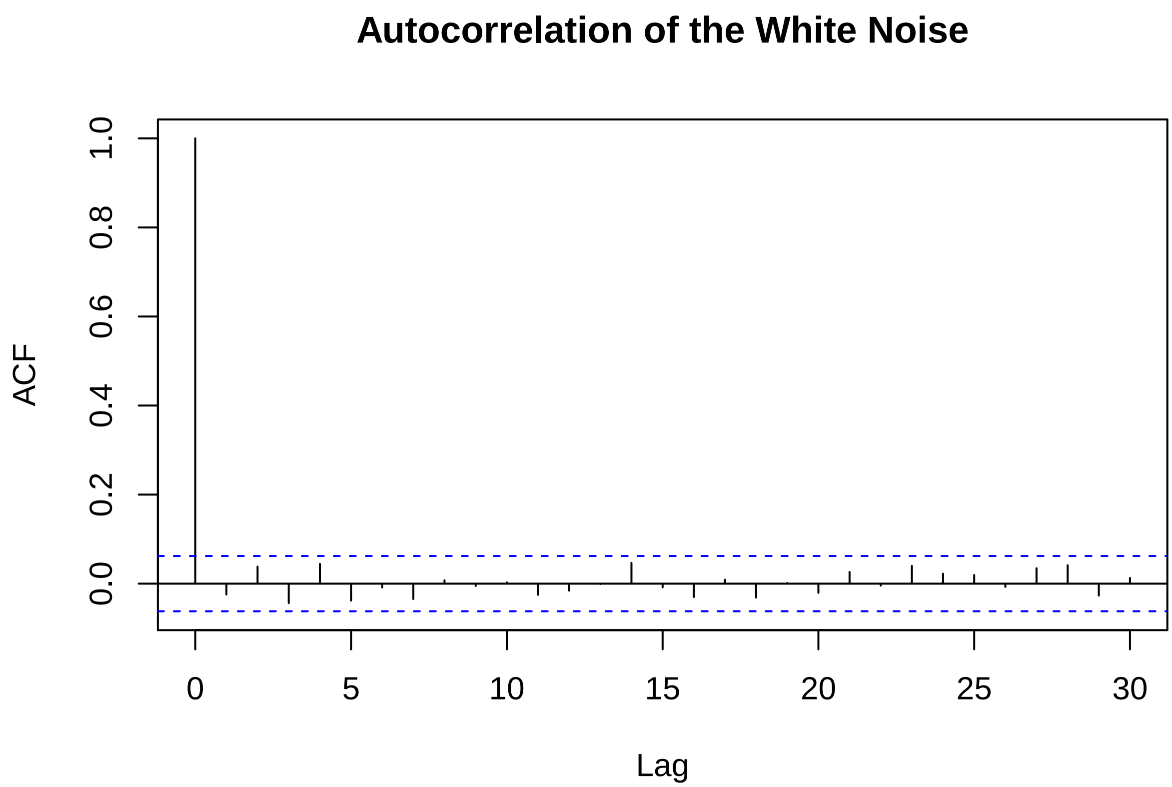
Plots of Autocorrelation

Below are autocorrelation plots for the origional random walk and the derived white noise. As expected that is a very high level of persistence for the random walk. However, there is no meaningful autocorrelation for the white noise.

```
#for random walk
acf(random_walk, main = "Autocorrelation of the Random Walk")
```




```r
#for random walk increment size
acf(random_walk_diff, main =  "Autocorrelation of the White Noise")
```



Is the Stock Market a Random Walk?

Famously, it has been hypothesized that the stock market can be modeled as a random walk. More specifically, the log of the price can be modeled as a random walk with drift. Below, I compare the characteristics of s&P 500 data with those of the 'pure' random walk above.

Create a Time Series Object of S&P 500 Closing Prices

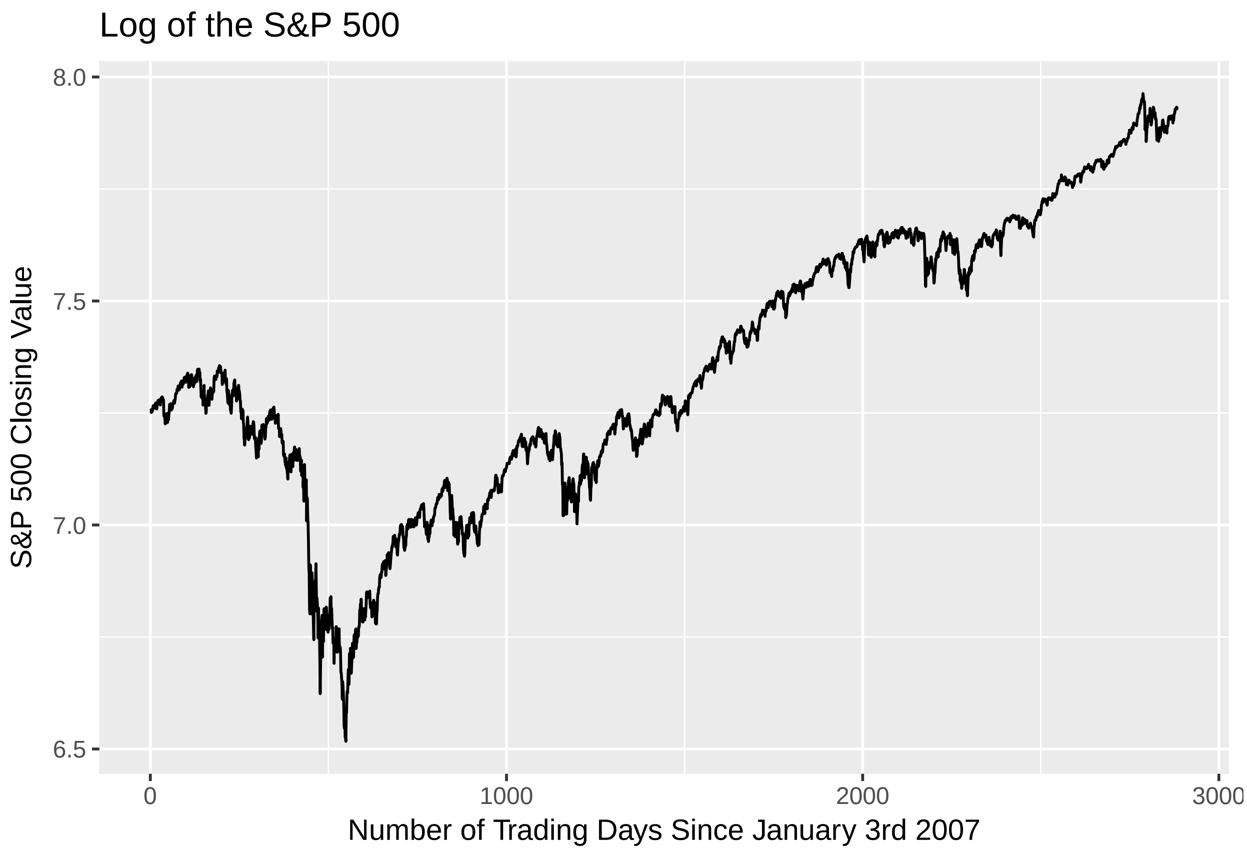
For simplicity I am disregrarding actual dates and working with the data as trading days since January 3rd 2007.The price that I am choosing to work with is the log of the closing value for a given day

```
getSymbols("^GSPC", src = "yahoo")
```

```
## [1] "GSPC"
```

```
SP_Close = log(as.ts(GSPC$GSPC.Close))
```

Plot S&P 500 Closing Prices



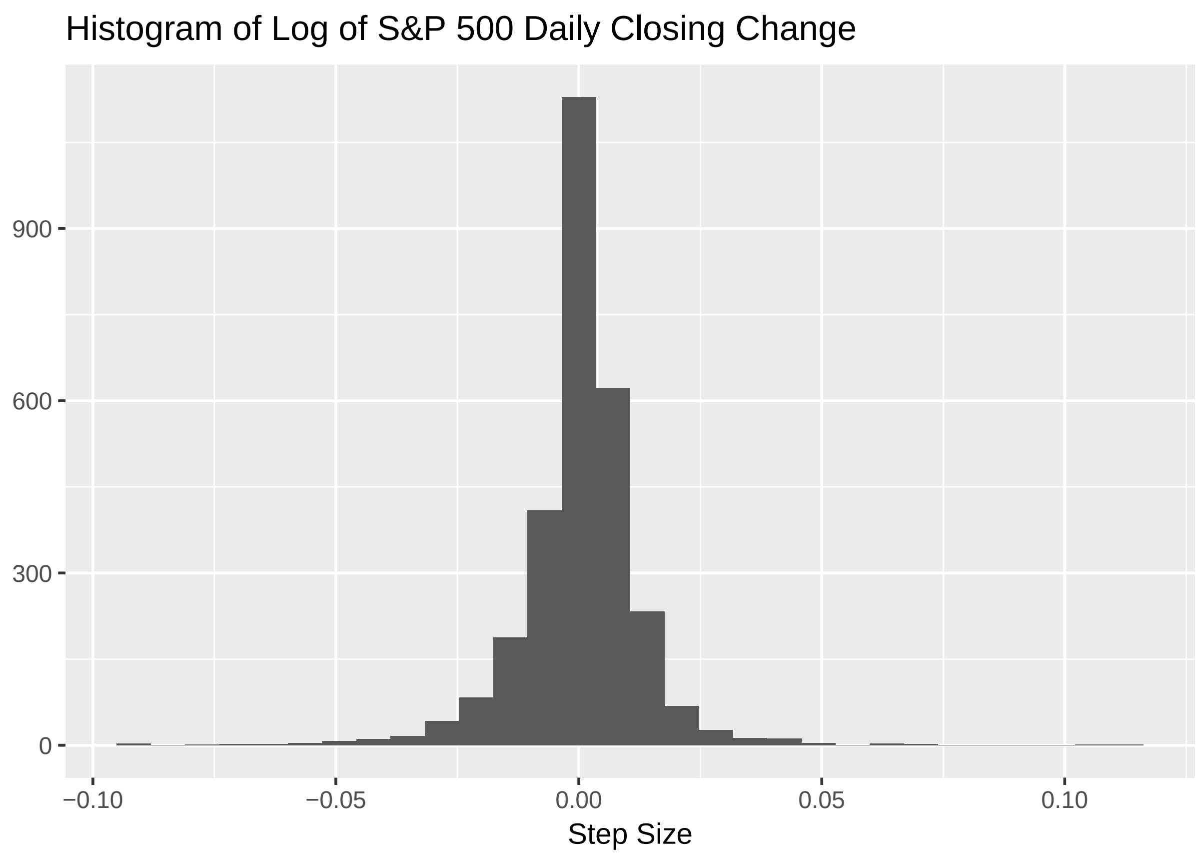
Create Differences

```
SP_Diff = diff(SP_Close)
```

Create Increment Histogram

Again, the shape looks decently normal.

```
## Don't know how to automatically pick scale for object of type ts. Defaulting to continuous.
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```



Fit Normal Distribtion to S&P 500 Increment Data

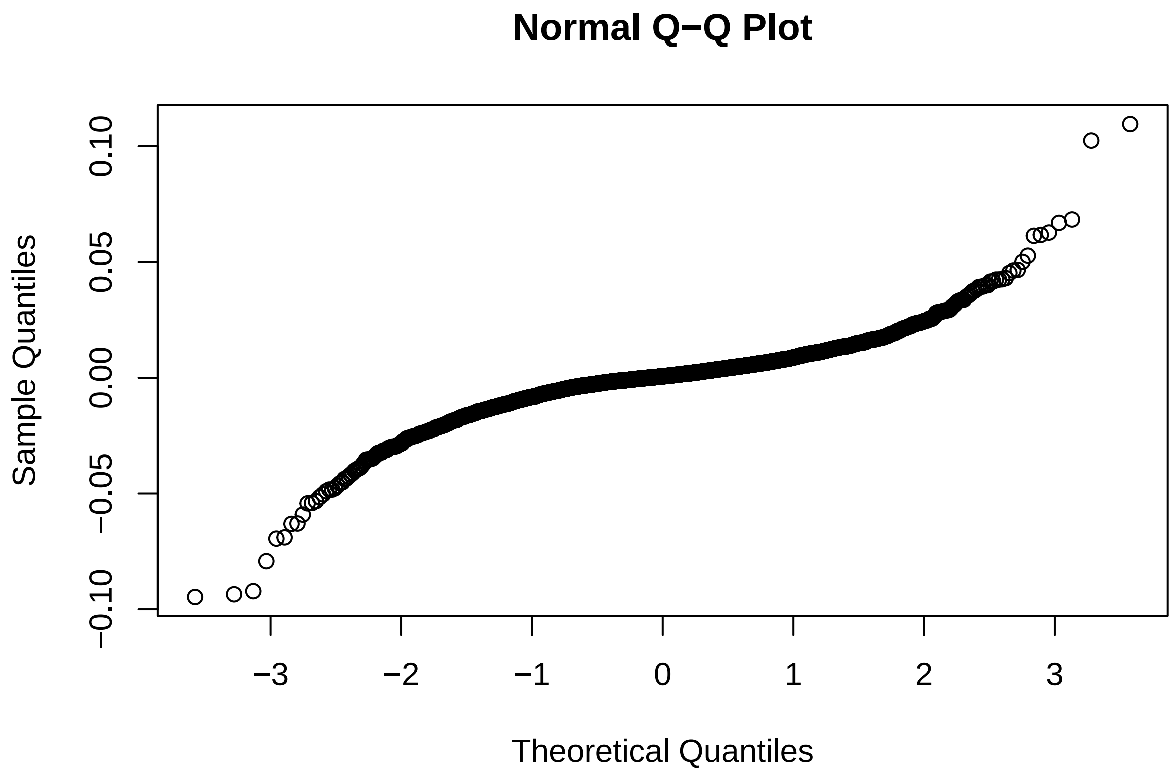
This result shows that the S&P 500 is not a pure random walk. A better model would be a random walk with drift.

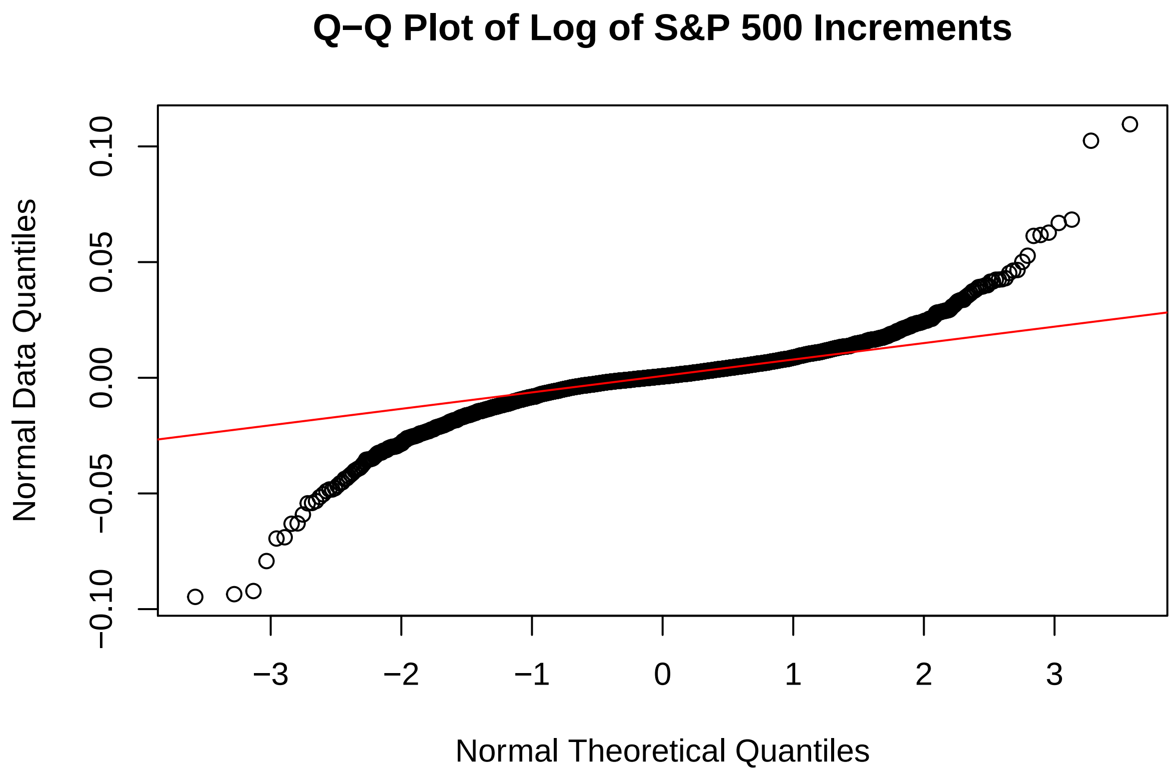
```
SP_Vec = as.vector(SP_df$x)
fitdistr(SP_Vec, "normal")
```

```
##         mean              sd
##   0.0002337782   0.0125752277
##  (0.0002342036) (0.0001656069)
```

Q-Q Plot of S&P 500 Increments

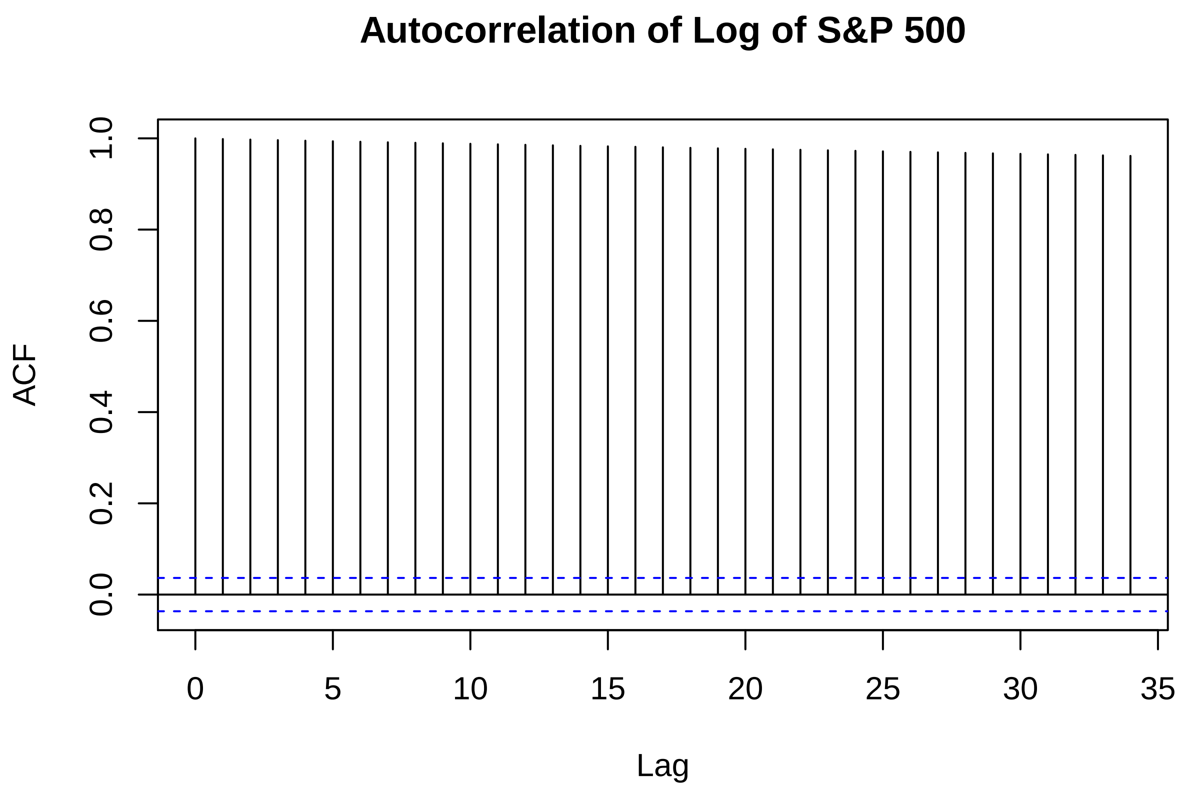
The Q-Q plot clearly shows that if the S&P 500 can be modeled as a random walk the noise term should definately not be modeled as normal distribution. The tails are extreamly heavy.


Normal Q−Q Plot
Sample Quantiles
0.10
0.05
0.00
−0.05
−0.10
−3
−2
−1
0
1
2
3
Theoretical Quantiles

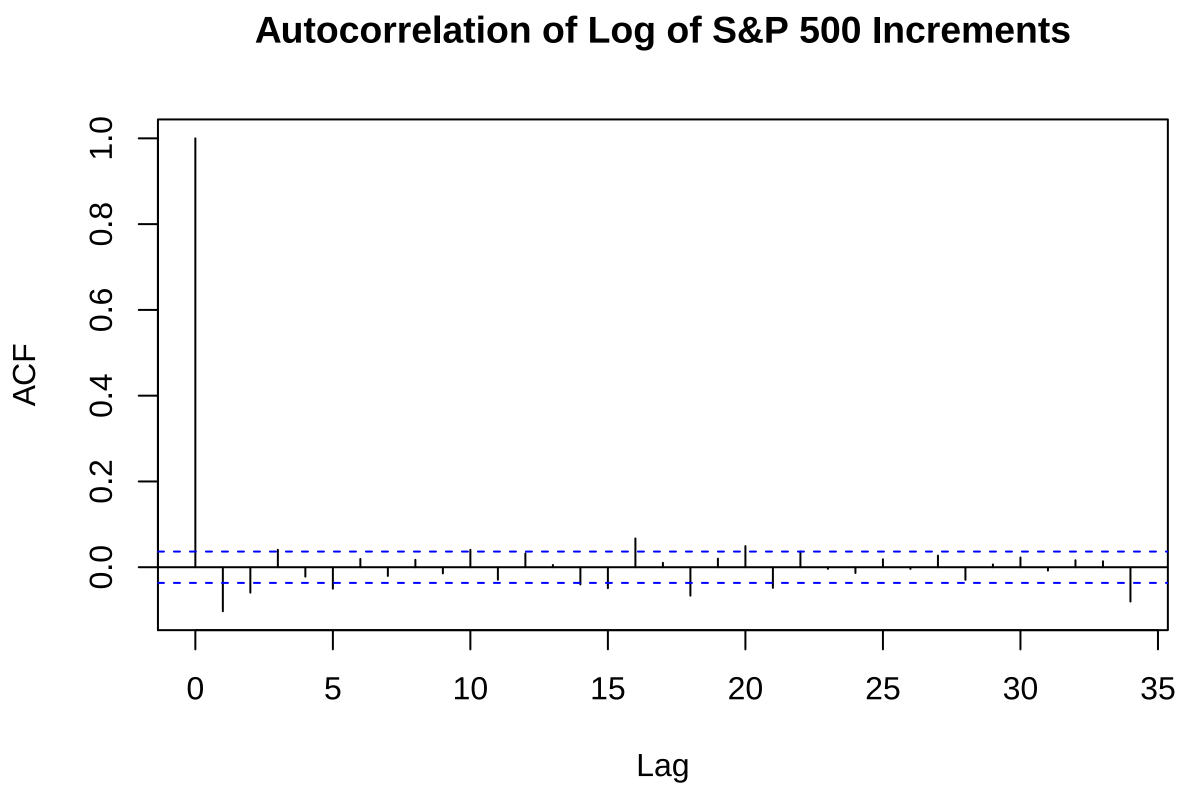


Plots of Autocorrelation for S&P 500

There appears to be great persistence for the S&P 500 itself while possible none for it's incremenets. From the chart, there does appear to be a slight negative autocorrelation with lag of one trading day. I would suspect this is not actually true as someone else would have noticed this by now and traded accordingly to make the correlation dissapear. Additionally, if there truely is some correlation, then the S&P 500 cannot be modeled as a random walk.



Autocorrelation of Log of S&P 500
ACF
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
20
25
30
35
Lag


Autocorrelation of Log of S&P 500 Increments
ACF
0.0
0.2
0.4
0.6
0.8
1.0
0
5
10
15
20
25
30
35
Lag