

DSP Lab 5 – Graeme Judge

Question 1)

1a)

$$6 \frac{d^3}{dt^3} y(t) - 2 \frac{d^2}{dt^2} y(t) + 5 \frac{d}{dt} y(t) = 2 \frac{d}{dt} x(t) + 3 \frac{d^2}{dt^2} x(t)$$

$$= (6s^3 - 2s^2 + 5s) y(e^{st}) = (3s^2 + 2s) x(e^{st})$$

$$= \frac{y(e^{st})}{x(e^{st})} = \frac{3s^2 + 2s}{6s^3 - 2s^2 + 5s}$$

$$\therefore H(s) = \frac{y(e^{st})}{x(e^{st})} = \frac{3s^2 + 2s}{6s^3 - 2s^2 + 5s}$$

1b)

$$\frac{d^2}{dt^2} y(t) + 5 \frac{d}{dt} y(t) + 6y(t) = \frac{d}{dt} x(t) + 6x(t)$$

$$\therefore H(s) = \frac{y(e^{st})}{x(e^{st})} = \frac{s+6}{s^2+5s+6}$$

Question 2) a)

2) a) i)

$$y(n) = x(n) - y(n-2)$$

$$= X(z) - Y(z)z^{-2}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^2}{1+z^2} \times \frac{z^2}{z^2} = \frac{z^2}{z^2+1}$$

ii)

DF I

DF II

iii)

$$z^2 = 0 \therefore \text{Zero @ } 0$$

$$z^2 + 1 = 0 \therefore \text{Pole @ } i$$

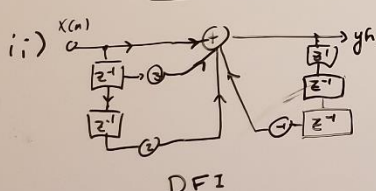
iv)

$$\text{mag} = \sqrt{i^2}$$

$$= i \therefore \text{it's marginally stable.}$$

Question 2) b)

2) b) i) $y(n) = x(n) - 3x(n-1) + 2x(n-2) - y(n-3)$
 $= x(z) - 3x(z)z^{-1} + 2x(z)z^{-2} - y(z)z^{-3}$
 $H(z) = \frac{y(z)}{x(z)} = \frac{1 + 3z^{-1} + 2z^{-2}}{1 + z^{-3}} \times \frac{z^3}{z^3}$
 $H(z) = \frac{z^3 + 3z^2 + 2z}{z^3 + 1}$

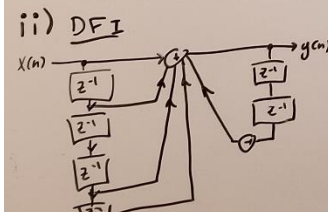
ii)  DFI

iii) $(z)(z+2)(z+1)$ Zeros @ $0, -1, -2$
 $z^3 + 1$ Poles @ -1

iv) mag of pole = $\sqrt{1^2}$
 $= 1$ ∴ marginally stable since mag = 1

Question 2) c)

2) c) i) $y(n) = x(n) + x(n-1) + x(n-3) + x(n-4) - y(n-2)$
 $= \frac{1 + z^{-1} + z^{-3} + z^{-4}}{1 + z^{-2}} \cdot \frac{z^4}{z^4}$
 $= \frac{z^4 + z^3 + z + 1}{z^4 + z^2}$ Synthetic division
 $\begin{array}{r|rrrrrr} & 1 & 0 & 1 & 0 & 1 & 1 \\ & & 1 & 0 & 0 & 0 & -1 \\ \hline & 1 & 1 & 0 & 0 & 1 & 0 \\ & & 1 & 0 & -1 & -1 & \\ \hline & 1 & 1 & 1 & 1 & 0 & \end{array}$
 $(z+1)(z+1)(z^2 - z + 1)$ ← quad formula
 $= \frac{-1 + \sqrt{3}i}{2} \cdot \frac{-1 - \sqrt{3}i}{2}$
 $z = -0.5 + 0.87i$ or $-0.5 - 0.87i$
 $\therefore H(z) = \frac{(z+1)(z+1)(z+0.5-0.87i)(z+0.5+0.87i)}{z^2(z^2+1)}$

ii) DFI  DFI

2c) iii) Zeros @ $-1, -1, -0.5 + 0.87i, -0.5 - 0.87i$

Poles @ $0, i$

iv) $\text{mag} = 1$ \circ marginally stable

Question 3)

$$3) H(z) = \frac{z^2 + 0.3z + 1}{z^2 - 0.3z + 0.8}$$

a) $\rightarrow z$ exponential then $\times z^{-n}$ (largest exp)

$$H(z) = \frac{z^2 + 0.3z + 1}{z^2 - 0.3z + 0.8} \times \frac{z^{-n}}{z^{-n}} = \frac{1 - 0.3z^{-1} - z^{-2}}{1 - 0.3z^{-1} - 0.8z^{-2}}$$

b) Find poles

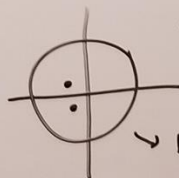
$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$z = \frac{-0.3 \pm \sqrt{0.09 - 4(1)(0.8)}}{2(1)}$$

$$z = -0.15 + 0.88i \quad z = -0.15 - 0.88i$$

$$\text{mag} = \sqrt{0.15^2 + 0.88^2}$$

$$= 0.89$$



\rightarrow Both inside

\therefore System is stable