EECE72425 DSP Assignment #6
Combining Systems, Bilinear Transform

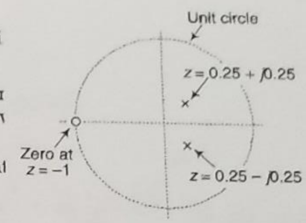
Due Monday April 20 at noon. Upload files to eConestoga dropbox
The solution will be posted at noon so you have it available for study.

(30 marks, equal weighting for 3 questions on this page)

1 Given the z-plane pole/zero plot, associated with a 2nd-order IIR digital fi ter, in Figure P6–28:

(a) What is the $H(z)$ transfer function, in terms of z^{-1} and z^{-2}, of the Figur P6–28 filter having two poles and a single zero on the z-plane? Show ho you arrived at your answer.

(b) Draw the Direct Form I block diagram of the $H(z)$ filter that implemen the transfer function arrived at in Part (a) of this problem.



2 A 1st-order analog highpass filter's s-domain transfer function is

$$H(s) = \frac{s}{s + \omega_o}.$$

Determine a digital filter's $H(z)$ z-domain transfer function that simulates $H(s)$ using the bilinear transform process. Given that frequency $\omega_o = 62.832$ radians/second, assume that the digital filter's sample rate is $f_s = 100$ Hz. Manipulate your final $H(z)$ expression so that it is in the following form:

$$H(z) = \frac{A + Bz^{-1}}{1 + Cz^{-1}}$$

3 Due to its simplicity, the 1st-order analog lowpass filter shown in Figure P6–47(a) is often used to attenuate high-frequency noise in a $v_{in}(t)$ input signal voltage. This lowpass filter's s-domain transfer function is

$$H(s) = \frac{1}{1 + RCs}.$$

Determine a digital filter's $H_{bt}(z)$ z-domain transfer function that simulates $H(s)$, using the bilinear transform process. Draw the digital filter's Direct Form II block diagram where the coefficients are in terms of R and C. Again, assume that $t_s = 1$.

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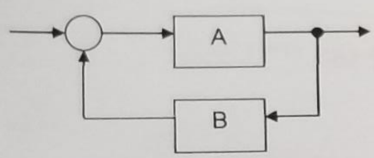
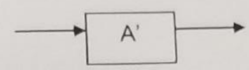
(15 marks, 3 marks for each part a and b, 4 marks part c, 5 marks part d)

Reduce each of the following block diagrams to the two forms shown below – **Simplified** Form & **Reduced** form. For each form, express A and B in terms of the G's, H's & K's. Be sure to reduce each fraction to a fraction with a single numerator and denominator.

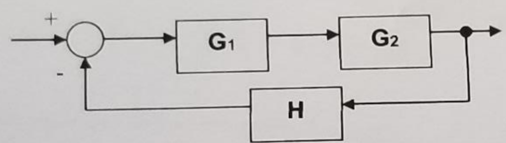
Simplified form



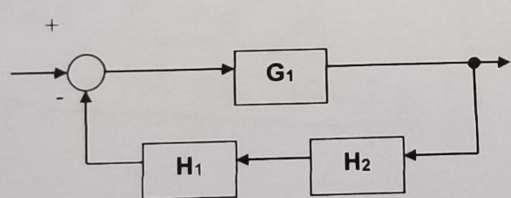
Reduced form



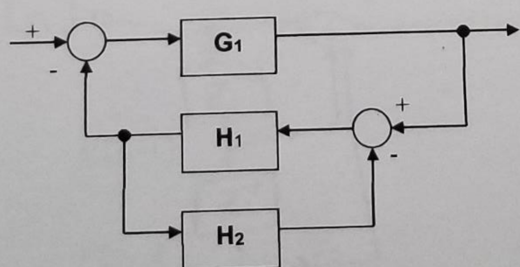
a)



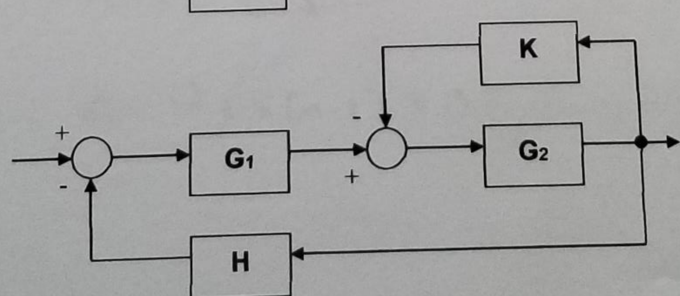
b)



c)



d).



DSP Assingment #6

1) a) Zeros: $z = -1$

Poles: $z = 0.25 + j0.25$

$z = 0.25 - j0.25$

$$H(z) = \frac{(z+1)}{(0.25 + j0.25 - z)(0.25 - j0.25 - z)}$$

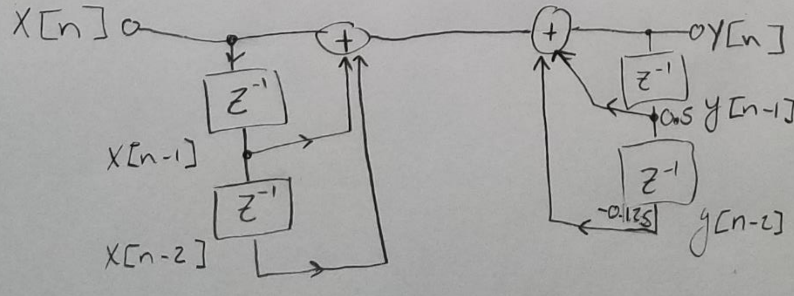
$$= \frac{(z+1)}{0.0625 \cancel{- 0.0625j} - 0.25z + \cancel{0.0625j} - 0.0625j^2 - \cancel{zj0.25} - \cancel{z0.25} + \cancel{zj0.25} + z^2}$$

+0.0625. (-1)

$$H(z) = \frac{(z+1)}{z^2 - 0.5z + 0.125} \times \frac{z^{-2}}{z^{-2}}$$

$$\boxed{H(z) = \frac{z^{-1} + z^{-2}}{1 - 0.5z^{-1} + 0.125z^{-2}}}$$

b)



$$y[n] = x[n-1] + x[n-2] + 0.5y[n-1] - 0.125y[n-2]$$

2) $H(s) = \frac{s}{s+\omega_0}$

$\omega_0 = 62.832$

$T = \frac{1}{fs} = \frac{1}{100} = 0.01$

$$H(z) = \frac{\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}{\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + \omega_0} = \frac{2(1-z^{-1})}{2(1-z^{-1}) + \omega_0 T(1+z^{-1})} = \frac{1-z^{-1}}{1-z^{-1} + \frac{\omega_0 T}{2} + \frac{\omega_0 T z^{-1}}{2}}$$

$$= \frac{1-z^{-1}}{1-z^{-1} + \left(\frac{62.832 \times 0.01}{2}\right) + \left(\frac{62.832 \times 0.01}{2}\right)z^{-1}}$$

$$H(z) = \frac{1-z^{-1}}{(1-z^{-1}) + (0.314) + (0.314)z^{-1}}$$

$$= \frac{1-z^{-1}}{1.314 - 0.686z^{-1}} \times \frac{\left(\frac{1}{1.314}\right)}{\left(\frac{1}{1.314}\right)}$$

$$\boxed{H(z) = \frac{0.761 - 0.761\,z^{-1}}{1.314 - 0.522\,z^{-1}}}$$

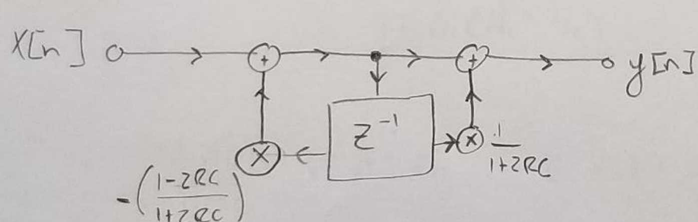
3) $H(s) = \frac{1}{1+RCs}$

$t_s = 1$

$s = 2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$

$$H(z) = \frac{(1+z^{-1})}{(1+z^{-1}) + 2Rc(1-z^{-1})} = \frac{1+z^{-1}}{1+z^{-1} + 2RC - 2RCz^{-1}} = \frac{1+z^{-1}}{(1+2RC)+(1-2RC)z^{-1}}$$

$$\boxed{H(z) = \frac{\frac{1}{(1+2RC)} + \frac{1}{(1+2RC)}z^{-1}}{1 + \left(\frac{1-2RC}{1+2RC}\right)z^{-1}}}$$



4a) Simplified form: $A = G_1 \cdot G_2$ $\quad B = H$

Reduced form: $A' = \dfrac{G_1 \cdot G_2}{1 + H(G_1 \cdot G_2)}$

b) Simplified form: $A = G_1$ $\quad B = H_1 H_2$

Reduced form: $A' = \dfrac{G_1}{1 + G_1(H_1 \cdot H_2)}$

c) Simplified form: $A = G_1$ $\quad B = 1 + \dfrac{H_1}{H_1 \cdot H_2}$

Reduced form: $A' = \dfrac{A}{1+AB} = \dfrac{G_1}{1 + G_1\left(\dfrac{H_1}{1+H_1 \cdot H_2}\right)}$

$$= \dfrac{G_1}{1 + \dfrac{G_1 H_1}{1 + H_1 + H_2}}$$

D) Simplified form: $A = G_1 \cdot \left(\dfrac{G_2}{1+G_2 k}\right)$ $\quad B = H$

Reduced form: $A' = \dfrac{A}{1+AB} = \dfrac{G_1 \cdot \left(\dfrac{G_2}{1+G_2 k}\right)}{1 + H\left(G_1\left(\dfrac{G_2}{1+G_2 k}\right)\right)} \times \dfrac{1+G_2 K}{1+G_2 K}$

$$= \dfrac{G_1 + G_1 G_2 k + G_2}{1 + G_2 k + H(G_1 + G_1 G_2 k + G_2)}$$